Real-Time Measurement of Business Conditions

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Our Approach

- *Dynamic factor model*, treating business conditions as an unobserved variable, related to observed indicators
- Explicitly incorporate business conditions indicators measured at *different frequencies*
- Explicitly incorporate indicators measured at *high frequencies*
- Extract and forecast latent business conditions using linear yet statistically optimal procedures, which involve *no approximations*

Daily Dynamic Factor Structure

Factor:

$$x_t = \rho_1 x_{t-1} + \rho_2 x_{t-2} + \ldots + \rho_p x_{t-p} + e_t$$

Observables:

$$y_t^i = c_i + \beta_i x_t + \delta_{i1} w_t^1 + \ldots + \delta_{ik} w_t^k + \gamma_{i1} y_{t-D_i}^i + \ldots + \gamma_{in} y_{t-nD_i}^i + u_t^i$$

Observed Stock Data

Stock variable measurement equation:

$$\tilde{y}_t^i = \begin{cases} y_t^i = c_i + \beta_i x_t + \delta_{i1} w_t^1 + \ldots + \delta_{ik} w_t^k + \gamma_{i1} y_{t-D_i}^i + \ldots + \gamma_{in} y_{t-nD_i}^i + u_t^i \\ NA \end{cases}$$

Observed Flow Data

Flow variable measurement equation:

$$\tilde{y}_t^i = \begin{cases} \sum_{j=0}^{D_i-1} y_{t-j}^i \\ NA \end{cases}$$

$$\tilde{y}_t^i = \begin{cases} \sum_{j=0}^{D_i-1} c_i + \beta_i \sum_{j=0}^{D_i-1} x_{t-j}^i + \delta_{i1} \sum_{j=0}^{D_i-1} w_{t-j}^1 + \ldots + \delta_{ik} \sum_{j=0}^{D_i-1} w_{t-j}^k \\ + \gamma_{i1} \sum_{j=0}^{D_i-1} y_{t-D_i-j}^i + \ldots + \gamma_{in} \sum_{j=0}^{D_i-1} y_{t-nD_i-j}^i + u_t^{*i} \\ NA \end{cases}$$

Trend

$$\sum_{j=0}^{D_i-1}\left[c_i+\delta_{i1}\left(t-j\right)+\ldots+\delta_{ik}\left(t-j\right)^k\right]\equiv c_i^*+\delta_{i1}^*t+\ldots+\delta_{ik}^*t^k$$

Assembling it All

Stocks:

$$\tilde{y}_t^i = \begin{cases} c_i^* + \beta_i x_t^i + \delta_{i1}^* t + \ldots + \delta_{ik}^* t^k + \gamma_{i1} \tilde{y}_{t-D_i}^i + \ldots + \gamma_{in} \tilde{y}_{t-nD_i}^i + u_t^{*i} \\ NA \end{cases}$$

Flows:

$$\tilde{y}_t^i = \begin{cases} c_i^* + \beta_i \sum_{j=0}^{D_i - 1} x_{t-j}^i + \delta_{i1}^* t + \ldots + \delta_{ik}^* t^k + \gamma_{i1} \tilde{y}_{t-D_i}^i + \ldots + \gamma_{in} \tilde{y}_{t-nD_i}^i + u_t^{*i} \\ NA \end{cases}$$

State Space Representation

$$y_t = Z_t \alpha_t + \Gamma_t w_t + \varepsilon_t$$

$$\alpha_{t+1} = T\alpha_t + R\eta_t$$

$$\varepsilon_t \sim (0, H_t), \eta_t \sim (0, Q)$$

$$t = 1, ..., \mathscr{T}$$

Nuance I: Time-varying system matrices
Nuance II: High-dimensional state

Signal Prediction and Extraction

Kalman filter:

$$
\begin{aligned}
a_{t|t} &= a_t + P_t Z_t' F_t^{-1} v_t \\
P_{t|t} &= P_t - P_t Z_t' F_t^{-1} Z_t P_t' \\
a_{t+1} &= T a_{t|t} \\
P_{t+1} &= T P_{t|t} T' + RQR',
\end{aligned}
$$

where

$$
\begin{aligned}
v_t &= y_t - Z_t a_t - \Gamma_t w_t \\
F_t &= Z_t P_t Z_t' + H_t
\end{aligned}
$$

$$
a_{t|t} \equiv E(\alpha_t|\mathscr{Y}_t),\ P_{t|t} = var(\alpha_t|\mathscr{Y}_t),\ a_t \equiv E(\alpha_t|\mathscr{Y}_{t-1}),\\
P_t = var(\alpha_t|\mathscr{Y}_{t-1}),\ \mathscr{Y}_t \equiv \{y_1, \ldots, y_t\},\ t = 1, \ldots, \mathscr{T}
$$

Nuance III: Missing Data

All of y_t missing (skip updating):

$$\begin{aligned} a_{t+1} &= Ta_t \\ P_{t+1} &= TP_tT' + RQR \end{aligned}$$

Some of y_t missing (update w/ modified measurement eqn.):

$$\begin{aligned} y_t^* &= Z_t^*\alpha_t + \Gamma_t^* w_t + \varepsilon_t^* \\ \varepsilon_t^* &\sim N(0, H_t^*) \end{aligned}$$

$y_t^* = W_t y_t,\ Z_t^* = W_t Z_t,\ \Gamma_t^* = W_t \Gamma_t,\ \varepsilon_t^* = W_t \varepsilon_t,\ H_t^* = W_t H_t W_t'$

Nuance IV: Likelihood

$$\ln L = \sum_{t=1}^{\mathscr{T}} lnl_t$$

where

$\ln l_t = 0$, if no elements of y_t are observed

$\log l_t = -\frac{1}{2}\left[N^* \log 2\pi + \left(\log|F_t^*| + v_t^{*\prime} F_t^{*-1} v_t^*\right)\right]$, otherwise

Prototype Implementation

- Daily, April 1, 1962 - April 7, 2008
- Four indicators:
  Term premium, initial claims, payroll employment, GDP
- Initial detrending
- AR(1) factor dynamics

System Structure

$$\underbrace{\begin{bmatrix} \tilde{y}_t^1 \\ \tilde{y}_t^2 \\ \tilde{y}_t^3 \\ \tilde{y}_t^4 \end{bmatrix}}_{y_t} = \underbrace{\begin{bmatrix} \beta_1 & \beta_2 & \beta_3 & \beta_4 \\ 0 & \beta_2 & 0 & \beta_4 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \beta_2 & 0 & \beta_4 \\ 0 & 0 & 0 & \beta_4 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \beta_4 \text{ or } 0 \\ 0 & 0 & 0 & \beta_4 \text{ or } 0 \\ 0 & 0 & 0 & \beta_4 \text{ or } 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}'}_{Z_t} \underbrace{\begin{bmatrix} x_t \\ x_{t-1} \\ \vdots \\ x_{t-\bar{q}-1} \\ x_{t-\bar{q}} \\ u_t^1 \end{bmatrix}}_{\alpha_t} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ \gamma_2 & 0 & 0 \\ 0 & \gamma_3 & 0 \\ 0 & 0 & \gamma_4 \end{bmatrix}}_{\Gamma_t} \underbrace{\begin{bmatrix} \tilde{y}_{t-W}^2 \\ \tilde{y}_{t-M}^3 \\ \tilde{y}_{t-q}^4 \end{bmatrix}}_{w_t} + \underbrace{\begin{bmatrix} 0 \\ u_t^{*2} \\ u_t^3 \\ u_t^{*4} \end{bmatrix}}_{\varepsilon_t}$$

$$\underbrace{\begin{bmatrix} x_{t+1} \\ x_t \\ \vdots \\ x_{t-\bar{q}} \\ x_{t-\bar{q}+1} \\ u_{t+1}^1 \end{bmatrix}}_{\alpha_{t+1}} = \underbrace{\begin{bmatrix} \rho & 0 & \cdots & 0 & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & \gamma_1 \end{bmatrix}}_{T} \underbrace{\begin{bmatrix} x_t \\ x_{t-1} \\ \vdots \\ x_{t-\bar{q}-1} \\ x_{t-\bar{q}} \\ u_t^1 \end{bmatrix}}_{\alpha_t} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}}_{R} \underbrace{\begin{bmatrix} e_t \\ \zeta_t \end{bmatrix}}_{\eta_t} \tag{1}$$

$$\begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix} \sim N\left(\begin{bmatrix} 0_{4\times 1} \\ 0_{2\times 1} \end{bmatrix}, \begin{bmatrix} H_t & 0 \\ 0 & Q \end{bmatrix} \right), \quad H_t = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \sigma_{2t}^{*2} & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_{4t}^{*2} \end{bmatrix}, \quad Q = \begin{bmatrix} 1 & 0 \\ 0 & \sigma_1^2 \end{bmatrix}$$

Questions

- How does our real activity indicator look over time?
- Does it track NBER recessions well?
- Does high-frequency data add much?
- Where is real activity *right now*?

Figure 1
Smoothed Real Activity Factor, Full Model (GEIS)

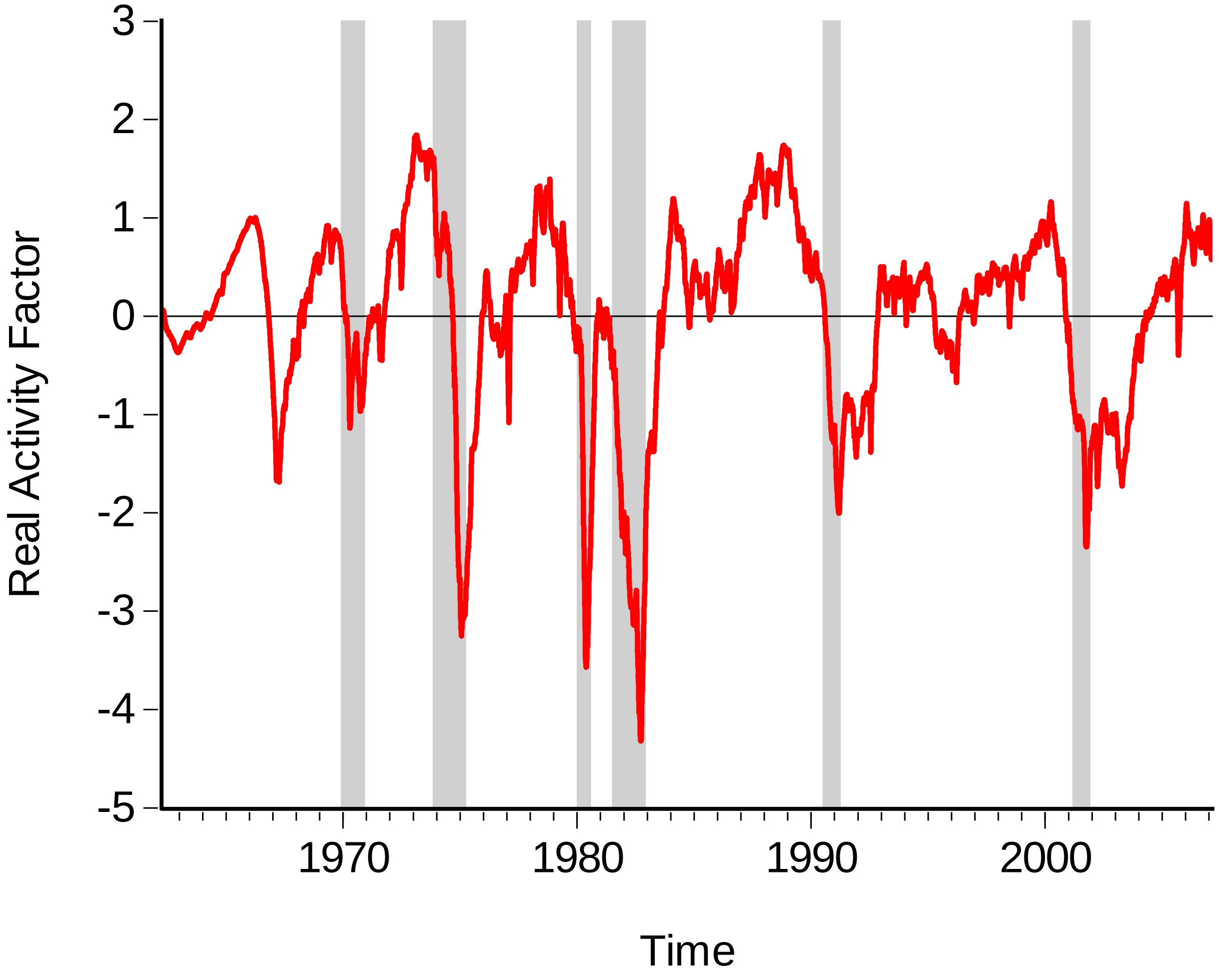


Figure 2
Smoothed Real Activity Factors: GE (Interval) and GEI (Point)

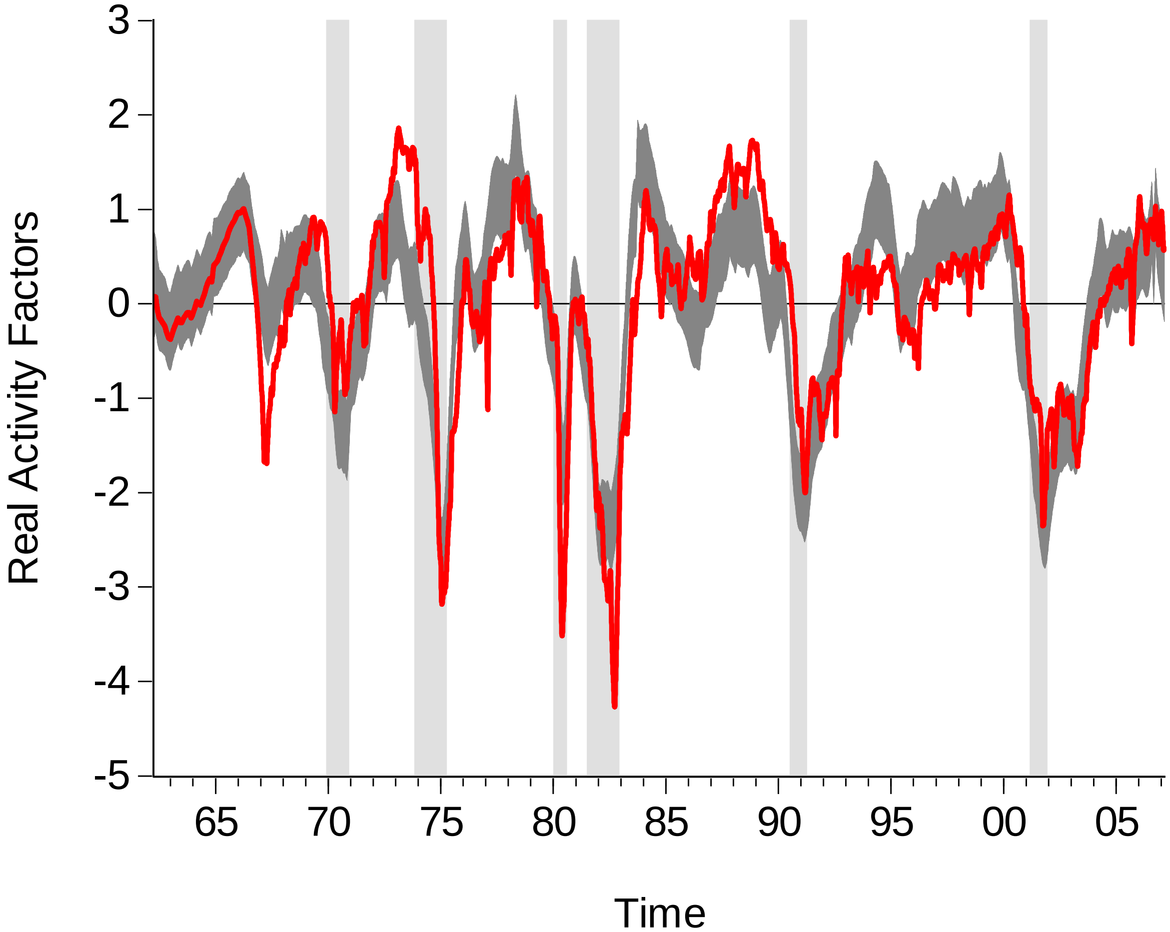


Figure 3
Smoothed Real Activity Factors Around NBER Recessions
GE (Interval) and GEI (Point)

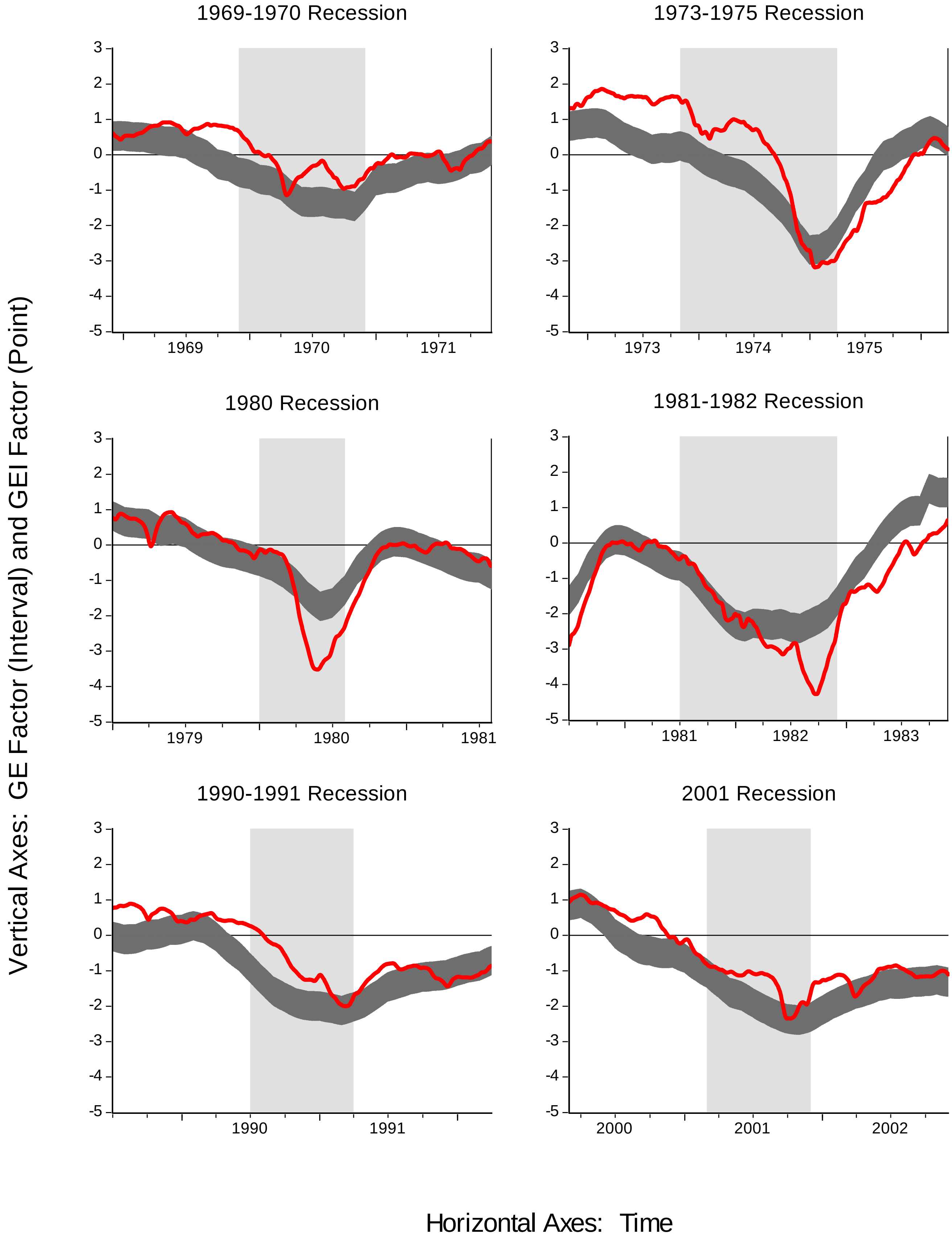
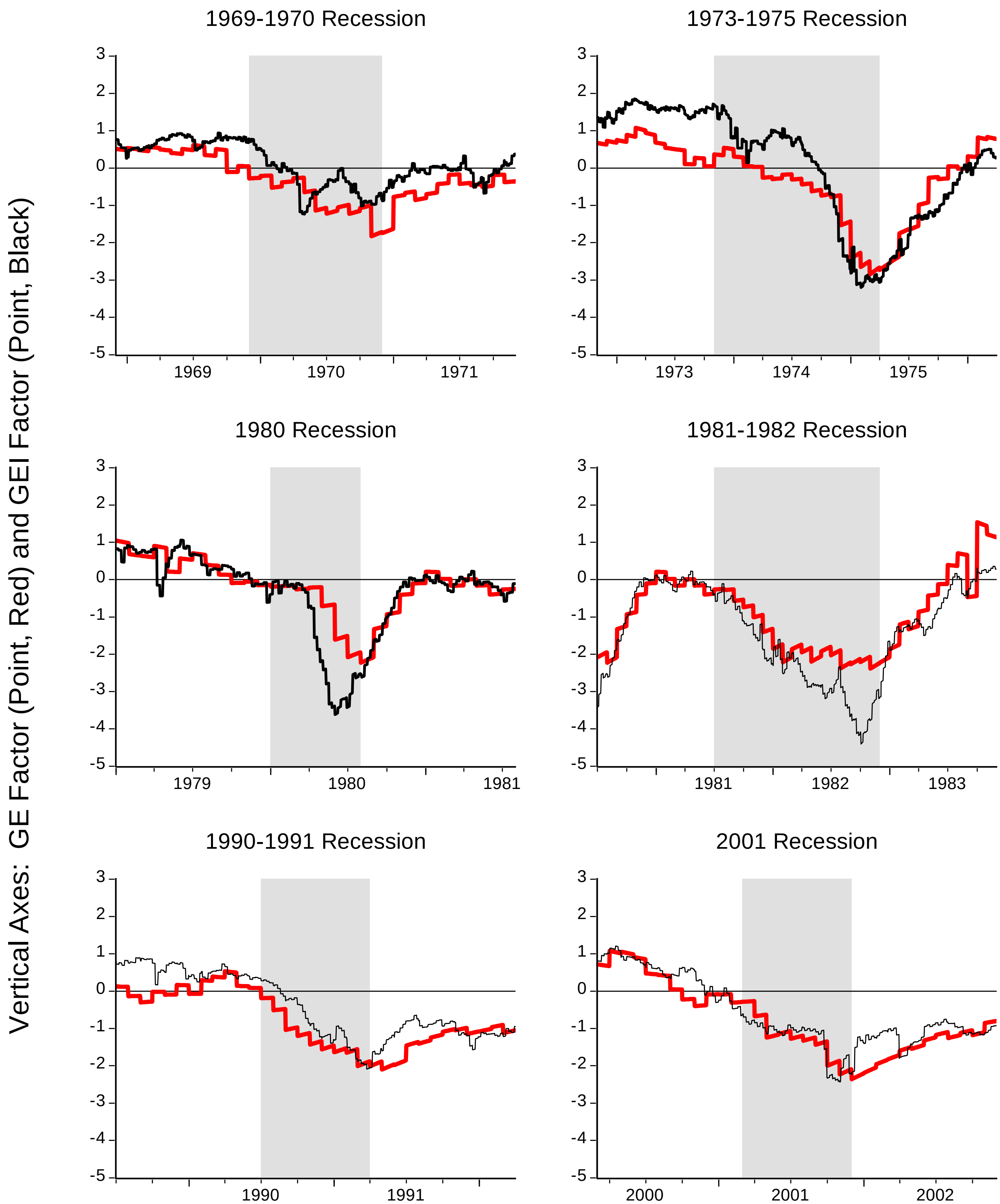
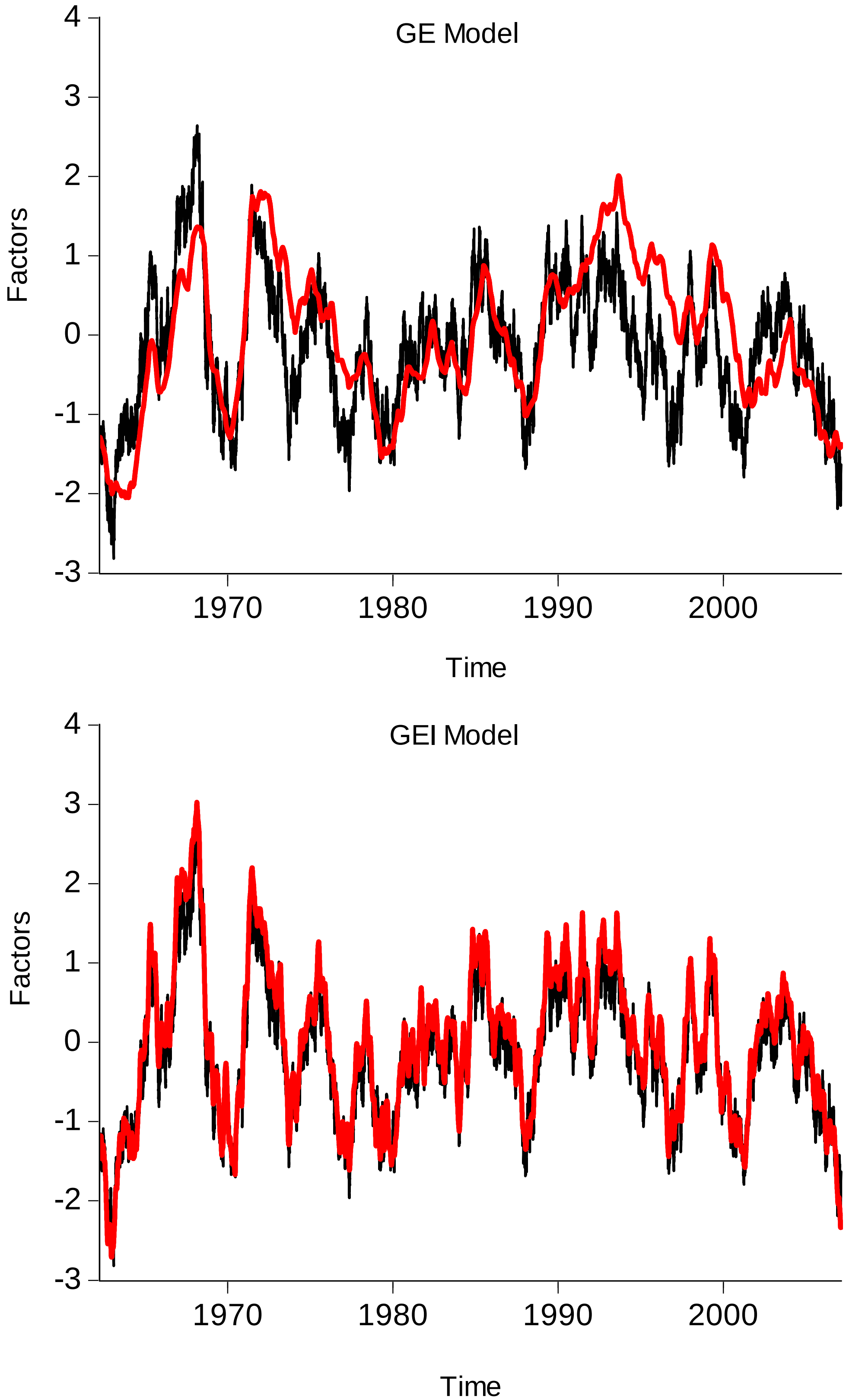


Figure 4
Filtered Real Activity Factors Around NBER Recessions
GE (Point, Red) and GEI (Point, Black)

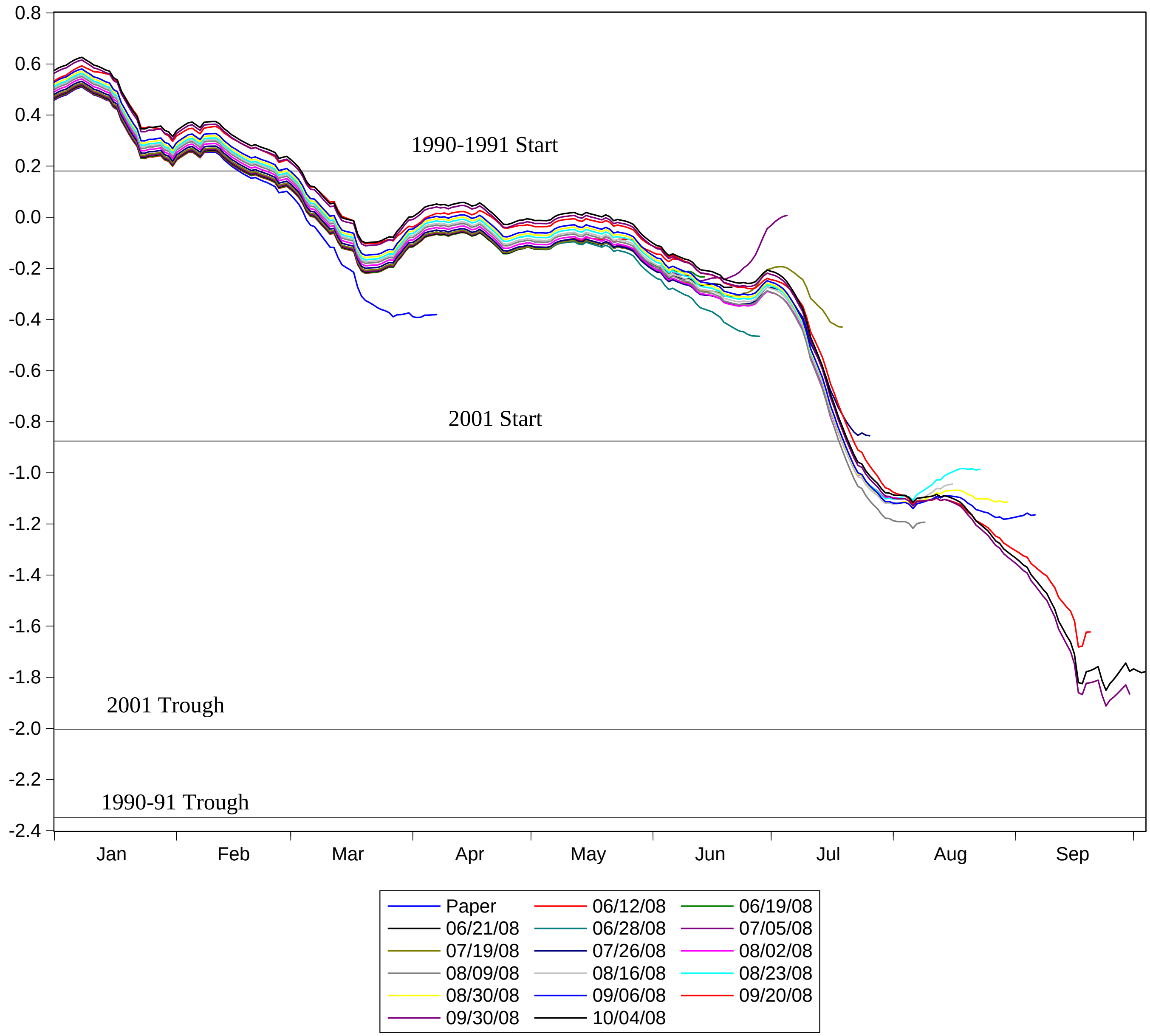


Horizontal Axes: Time

Figure 5
Simulated Real Activity Factors
Smoothed (Red) and True (Black)



Aruoba-Diebold-Scotti Real Activity Index (01/01/2008-10/04/2008)
Tentacle Plot: Extracted at Various Dates



The Future...

- Incorporation of regime switching
- Incorporation of direct indicators of daily activity
- Links between real activity and the bond market