Credit, Capital and Crises: A Theory of Counter-Cyclical Macroprudential Policy*

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Abstract

This paper examines the role of macroprudential capital requirements in preventing inefficient credit booms. If banks care about their reputations, unprofitable banks have strong incentives to invest in risky assets and generate inefficient credit booms when macroeconomic fundamentals are good. We show that across-the-system counter-cyclical capital requirements that deter credit booms are constrained optimal when fundamentals are within an intermediate range. We also show that when fundamentals are deteriorating, a public announcement of that fact can itself play a powerful role in preventing inefficient credit booms, providing an additional channel through which macroprudential policies can improve outcomes.

JEL Classifications:

Keywords: Macroprudential policy, credit booms, Basel III

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1 Introduction

One of the key elements of the Basel III framework is the countercyclical capital buffer. According to the Basel Committee on Banking Supervision (BCBS 2010), the primary aim of the countercyclical capital buffer regime is to use a buffer of capital to achieve the broader macroprudential goal of protecting the banking sector from periods of excess aggregate credit growth that have often been associated with the build up of system-wide risk. In enhancing the resilience of the banking sector over the credit cycle, the countercyclical capital buffer regime may also help to lean against credit in the build-up phase of the cycle in the first place.

In the United Kingdom, the recent financial crisis has led to the creation of a new macroprudential framework. Under the new framework, the Financial Policy Committee (FPC) within the Bank of England is given the responsibility to operate macroprudential (i.e. 'across the system') policy instruments, in order to moderate credit cycles and enhance banking sector resilience (HMT 2010). Although the range of instruments at the FPC's disposal is yet to be determined, one of the main tools under consideration is counter-cyclical capital adequacy requirements.

This paper considers the role of macroprudential counter-cyclical capital adequacy regulation in moderating credit cycles and enhancing banking sector resilience using a global games model.[1] In our model, banks not only care about returns on their investment, but also their reputations. In particular, banks are assumed to suffer a bigger reputational loss if they fail to make money when macroeconomic fundamentals are good than when they are bad. This is because when fundamentals are good, high ability banks are more likely to earn high profits, such that markets attribute low profits to the low ability of bank managers. The fear of getting a bad market reputation gives low-ability bank managers the incentive to hide low profits and extend excessive credit in a bid to 'gamble for resurrection' when macroeconomic fundamentals are good, thus generating socially inefficient credit booms.

Our analysis suggests that there is a case for counter-cyclical capital adequacy requirements because the presence of the reputational effect means that banks' incentives to gamble are strongest when macroeconomic fundamentals are good. By helping to reduce the incidence of inefficient credit booms, which ultimately lead to bank losses, counter-cyclical capital adequacy requirements help to meet the dual objectives of moderating credit cycles and enhancing banking sector resilience. Higher aggregate capital adequacy requirements come at a cost however, as they also impose higher funding costs for those profitable, high-ability banks that do not have incentives to gamble. When the policymaker cannot observe banks' types *ex ante*, this generates a policy trade-off.

[1] See Morris & Shin (2003) for a discussion of the theory of global games, and Morris & Shin (2000) for applications to macroeconomics.

In the absence of more targeted instruments, we demonstrate that, given this trade-off, counter-cyclical macroprudential capital adequacy regulation is constrained socially optimal when macroeconomic fundamentals are within an intermediate range, such that a higher capital requirement can deter gambling by some banks without imposing excessive costs on the rest of the banking sector. Intuitively, when fundamentals are very weak, most banks will be unprofitable and they have little need to gamble for resurrection in order to preserve their reputation. In this case, the regulator does not need to raise the capital adequacy requirement in response to a modest improvement in the fundamentals. By contrast, when fundamentals are very good, most banks are profitable, and the remaining few unprofitable banks are not easily deterred from gambling by a moderate increase in capital requirements. Since the regulator cannot deter gambling by the minority without imposing a high cost on the rest of the banking sector, an increase in aggregate capital adequacy regulation is not socially optimal when the fundamentals are very strong. In this case, our analysis suggests that instruments targeted with greater precision are needed.

We are also able to separate out the two effects of counter-cyclical capital requirements on banks' risk-taking incentives, namely (i) the direct effect of raising the cost of risk taking, and (ii) the indirect effect of making information about the state of macroeconomic fundamentals public. We demonstrate that the latter can have a powerful effect in reducing banks' risk-taking incentives when the fundamentals are rapidly deteriorating. The publicly announced results of stress tests that look 'through the cycle' can therefore also help to achieve macroprudential policy objectives.

Our paper is related to a number of existing papers which analyse the impact of strategic interdependence on banks' risk-taking incentives, including Acharya (2009), Acharya & Yorulmazer (2008), and Aikman, Haldane & Nelson (2010). Our main contribution to this theoretical literature is to model credit and the role of capital adequacy regulation explicitly, so that we can characterize the optimal counter-cyclical capital adequacy regulation. In our model, the rationale for counter-cyclical capital regulation arises because banks' risk-taking incentives which generate inefficient credit booms rise during macroeconomic upswings in the presence of reputational considerations (Rajan (1994), Gorton & He (2008), Scharfstein & Stein (1990), Froot, Scharfstein & Stein (1992), Thakor (2006)). This rationale is related but distinct from those articulated by Bianchi (2010) and Lorenzoni (2008), who suggest that counter-cyclical capital requirements – or higher capital requirements on assets with higher correlation with macroeconomic shocks – could be desirable if private agents' failure to internalise the pecuniary cost of increasing leverage on *ex post* asset prices and others' collateral constraints leads to *ex ante* overborrowing. Our theory also offers rationales for some existing empirical findings. For instance, our analysis suggests that the convergence of bank profits during credit booms – found by Aikman e*t al.* (2010) for US and UK banks – could be an outcome of low ability banks' attempt to hide their low returns in order to mimic the high ability banks. Similarly, the finding by Drehmann, Borio, Gambacorta, Jiménez & Trucharte (2010) that credit-to-GDP ratio is a good leading indicator of banking crises can be explained by our theory that suggests that inefficient credit booms preceding banking crises are associated with gambling by those banks trying to mimic profitable banks.

This paper is organized as follows. Section 2 provides the most basic set up of the model, in which banks receive noisy signals about the macroeconomic fundamentals in deciding whether to gamble for resurrection or not. The analytical solution in this section helps us to illustrate how capital adequacy requirement affects banks' incentives to gamble and hence the credit cycles. We also discuss the empirical implications of our analysis. Section 3 explicitly analyzes the optimal counter-cyclical capital adequacy regulation, using a model in which banks receive both public and private signals about the macroeconomic fundamentals. Section 4 discusses the policy implications of our analysis. Section 5 considers the effect of public announcement of the macroeconomic fundamentals – or 'moral suasion' – on banks' risk-taking incentives. Section 6 concludes.

2 The Model

We first set up a simple global games model in which those banks receiving low returns in the interim decide whether to gamble for resurrection in order to preserve their reputations, based on a private signal they receive about the macroeconomic fundamentals. This simple set up helps to illustrate the impact of the reputational considerations on banks' incentives to gamble, and how capital adequacy requirement affects these. We will characterize the optimal counter-cyclical capital adequacy requirement more explicitly in Section 3.

2.1 Set up

The model consists of three dates, $t = 0, 1, 2$, and there is a continuum of *ex ante* identical banks. Each bank invests 1 at $t = 0$ in a risky project. A fraction k of the investment is funded by equity, while fraction $1 - k$ is funded by debt. We normalize the cost of debt to zero. The equity premium over debt is c, such that the unit investment costs ck to fund. The cost of equity is taken as given by the bank, and for the moment, we assume that k is exogenous. As we will illustrate in the next section, k can be used as a policy tool to prevent inefficient credit booms.

At $t = 1$, banks privately observe the return from an initial investment made in $t = 0$. A fraction α of banks are high ability and observe high returns R_H with probability $f(\theta)$ (such that in the population as a whole, a fraction $\alpha f(\theta)$ of banks observe R_H). The remaining fraction of high ability banks observe low returns $R_L < R_H$. The parameter θ

indexes macroeconomic fundamentals, which determine the fraction of high ability banks that observe high returns in the first period. We assume that $f'(\theta) > 0$, such that the fraction of high ability banks receiving high return increases as the macroeconomic fundamental, θ, increases. High ability banks observing R_H publicly announce these returns, raise new finance, and invest 1 unit for another period, at cost ck. Banks that have observed R_H from their $t = 0$ investments can be sure that their $t = 1$ investments will return R_H at $t = 2$.

All low ability banks, together with the 'unlucky' fraction $1-f(\theta)$ of high ability banks observe $R_L < R_H$ at $t = 1$ (such that in the populations as a whole, a fraction $1 - \alpha f(\theta)$ of all banks observe R_L). If a bank chooses to announce the true profit of R_L, it is unable to raise new finance to invest at $t = 1$. But given that interim returns are observed privately, banks observing R_L can mimic lucky high ability types by announcing R_H, too. They can then raise new finance at cost ck, and invest 1 unit: this investment constitutes 'gambling for resurrection'. In particular, having observed low returns, investing in a subsequent project yields a $t = 2$ return of $2R_H - R_L$ with probability $b \in [0, 1]$, such that at $t = 2$, total announced profits are $2(R_H - ck)$, which are exactly the same as those of the lucky high ability banks. But the gamble could fail. With probability $1 - b$, banks lose all of their $t = 1$ profits, such that they have to amount zero profits in $t = 2$ and become insolvent. We assume that the probability of the $t = 1$ gamble being successful is independent of banks' ability, whereas the probability of the $t = 0$ investment being successful depends on banks' ability.

Banks that fail to announce a final profit of $2(R_H - ck)$ at $t = 2$ suffer reputational damage $p(\theta, l)$, where $l \in [0, 1]$ is the proportion of banks that take the risky gamble after having observed initial returns of R_L. A banker's reputation is assessed by the market who cannot observe fundamentals. Hence, reputational damage has the following properties: (a) $\frac{\partial p(\theta,l)}{\partial \theta} > 0$, so that as fundamentals improve, the reputational cost for announcing low returns increases; and (b) $\frac{\partial p(\theta,l)}{\partial l} > 0$, so that as the proportion of banks taking the risky gamble increases, the reputational damage of announcing high returns increases. Property (a) follows from the observation that as θ rises, high ability types are more likely to receive high initial returns. In the extreme case where $f(\theta) = 1$, all high types always announce high returns; so announcing low returns is a sure signal that ability is low. Property (b) follows from the fact that as the proportion of banks announcing interim low returns for sure decreases, the reputational penalty to any remaining bank doing so increases, as this signals low ability for sure.

In making a decision about whether to gamble for resurrection, banks have to make an assessment of whether other banks will also gamble, as their reputational cost of announcing low returns will depend on what others will do. In making this decision, each bank $i \in [0, 1]$ receives a noisy private signal x_i about the fundamental at $t = 1$:

$$x_i = \theta + \sigma\varepsilon_i, \quad \sigma > 0,$$

where the noise terms are distributed with density $g(.)$ with support on the real line. Given this set up, a bank's expected payoff from gambling at $t = 1$ is:

$$b[2(R_H - ck)] + (1 - b)[-2ck - p(\theta, l)],$$

whereas the payoff to playing safe is

$$R_L - ck - p(\theta, l).$$

From a social perspective, gambling for resurrection is inefficient if

$$b < \frac{R_L + ck}{2R_H}, \tag{1}$$

i.e. if the gamble is sufficiently risky. We assume condition (1) holds throughout our analysis. Taken together, the game gives a banker's marginal payoff to gambling $\pi(\theta, l)$ as

$$\pi(\theta, l) = b[2R_H + p(\theta, l)] - R_L - ck. \tag{2}$$

Figure 1 summarizes the timing and the payoffs of the game. Note that in our set up, reputational considerations generate a source of strategic interdependence between banks' actions: each banker has a stronger incentive to gamble when (s)he believes that others are doing the same. So in this set up, the reputational consideration is the friction which induces banks to take the socially inefficient action of gambling for resurrection and generates inefficient credit booms: in its absence, banks will never choose to gamble, as $\pi(\theta, l)$ will always be negative.

'Reputation' should be interpreted as a metaphor, which is designed to capture bankers' aversion to admitting to bad results when everyone else is doing well. There are several reasons why bankers may behave in this way. First, their compensation, promotion and dismissal – as well as their ability to secure another job – may be implicitly or explicitly linked to their performance relative to others in the industry: indeed, a banker's performance relative to others in the industry is a good signal of their ability when the banking industry is subject to a common shock.[2] Murphy (1999), updating Gibbons & Murphy (1990), finds that CEO pay in financial services is likely to be evaluated relative to market and industry returns among S&P500 financial services companies. Explicit relative performance evaluation is used by 57% of the financial services firms in

[2] Holmstrom (1982) argues that relative performance evaluation is useful if agents face some common uncertainty, such that other agents' performance reveals information about an agent's unobservable choices that cannot be inferred from his or her own measured performance.

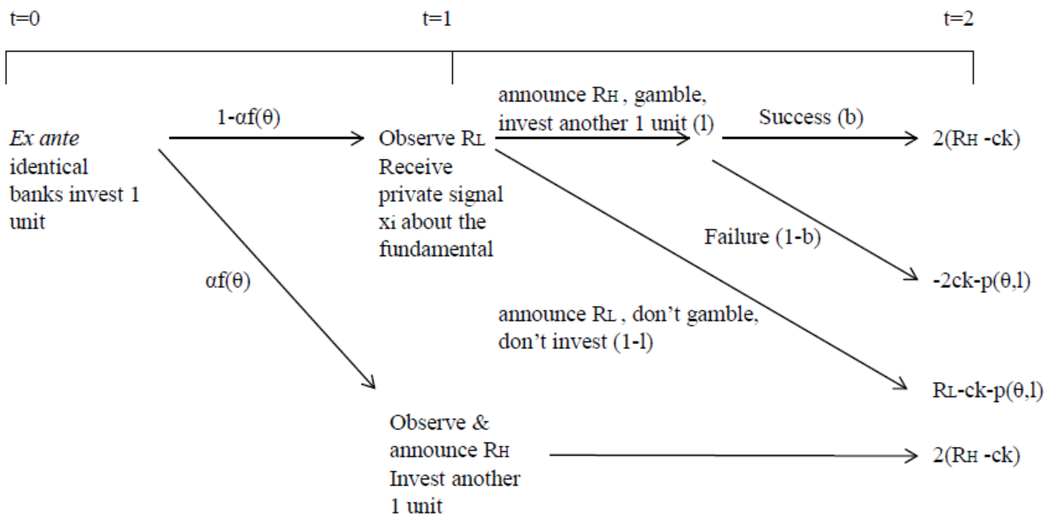

Figure 1: The timing and payoffs of the game

Murphy's (1999) survey.[3] Second, policymakers' inclination to bail out banks when they fail together than when they fail in isolation – due to their concerns about systemic risk associated with multiple bank failures – may also give bankers the incentive to avoid failure by gambling when other banks are doing well.[4]

Our story also relies on imperfect information. Empirically, Slovin, Sushka & Polonchek (1992) find that market participants take individual bank stock issuance as signals of value for other banking firms. In particular, commercial bank equity issues are associated with a significant negative valuation effect of -0.6% on rival commercial banking firms. Slovin et al. (1992) interpret this as evidence that an individual bank's issuance conveys not just institution-specific information to the market, but industry-wide information too. That is, information released by one bank conveys information to the market about industry value, which triggers a re-appraisal of other banks' market values. Rajan (1994) also finds evidence in favour of cross-bank informational effects.[5] When benchmarking in compensation ties individual incentives to relative performance, these informational externalities generate strong incentives to herd.

[3]See Table 9, p. 2538.

[4]See, for example, Acharya & Yorulmazer (2008).

[5]Rajan examines the cross-bank effects resulting from Bank of New England Corp.'s announcement that, prompted by the regulator, it would boost loan loss reserves in response to growing losses in 1989. Banks with headquarters in one state in New England suffered disproportionate cumulative abnormal returns of -8%. Using data on real estate firms, Rajan argues that the announcement conveyed information to the market about the state of the New England real estate sector in general, rather than conveying only institution-specific information in particular.

2.2 The symmetric switching equilibrium

We analyze the problem faced by a bank who has observed low initial returns. At this juncture, it has to choose an action {gamble, safe} to maximize its expected payoff. Suppose that a bank that has received R_L and signal x_i at $t = 1$ uses the following switching strategy:

$$s(\bar{\theta}) = \quad \{\text{gamble if } x_i \geqslant \theta^*, \text{ don't if } x_i < \theta^*\}.$$

Using equation (2), we can prove the following:

Proposition 1 *The unique symmetric switching equilibrium θ^* above which banks coordinate on gambling following low initial returns is given implicitly by:*

$$\int_0^1 p(\theta^*, l)dl = \frac{R_L + ck - 2bR_H}{b}.$$

Proof. See Annex. ■

Consider a simple example, in which $p(\theta, l) = \theta + l - 1$. Then θ^* is given by

$$\theta^* = \frac{1}{2} + \frac{R_L + ck}{b} - 2R_H \tag{3}$$

Note that the gambling threshold θ^* is increasing in k, the capital held by banks. This is very intuitive: a bank has a weaker incentive to gamble if it has to finance a higher proportion of the new lending by costly capital, as it diminishes the expected return from gambling relative to playing safe. Thus, a bank with a higher level of capital tends to play safe even if their private signal points to relatively strong fundamentals. Were the gamble to pay off with a higher probability (i.e. b is high), this effect would be mitigated: banks would then choose to gamble even if their private signal suggests fundamentals are low, as they are more likely to be able to avoid a reputational penalty; thus, θ^* would fall. Note that our model assumes unlimited liability, so the mechanism via which higher capital reduces risk taking in our model is different from that in Furlong & Keeley (1989) and Tanaka & Hoggarth (2006), in which banks' risk-taking incentives arises from the implicit subsidy from (mispriced) deposit insurance or limited liability.

These results are quite general for $p(.)$ with the properties we described above. Therefore, we write $\theta^* = \theta^*(k)$, in which:

$$\frac{\partial \theta^*(k)}{\partial k} > 0,$$

such that higher bank capital raises the threshold level of the private signal above which banks take the gambling option.

2.3 Empirical implications

This simple private signals model has a number of empirical implications. We focus on two. First, reputational incentives drive low ability banks to gamble when macro fundamentals are sufficiently high. This generates an inefficient credit boom in the model, which is followed by the realization of large scale losses. In other words, credit booms should precede crises, and even small changes in fundamentals can have a large impact on the path for credit. Work by Drehmann et al (2010) supports this view, arguing that the ratio of credit to GDP can be a useful indicator of subsequent distress. In Figure 2, we plot the ratio of credit to GDP for the UK, since 1963. The series have been filtered using a band-pass filter, which isolates variation in the ratio over a particular frequency range. Consistent with Drehmann et al (2010) and Aikman et al (2010), we show variation in the ratio of credit to GDP over the 1-20 year frequency range.[6] Shaded regions indicate periods of banking distress, namely, the 1973-5 secondary banking crisis, the 1990-4 small banks crisis[7], and the recent episode. The figure illustrates that a medium-term build up in the ratio of credit to GDP has tended to lead crisis periods.

Second, on the microeconomic level, the efforts of low ability banks to mimic their high ability counterparts implies a compression in the distribution of announced profits during credit booms. It is during these periods that standing out from the crowd is most damaging to reputation. Figure 3 plots the cross-sectional dispersion of equity returns for major UK banks and the top 100 UK private non-financial corporations (PNFCs) for 1997-2009. It is striking that the cross sectional dispersion tended to be lower for banks versus PNFCs for much of the period, despite banks operating at much higher levels of leverage. Further, this compression reached its nadir in the boom years of 2004-7. This phase maps our model, which says that standing out from the crowd is worst for reputation in a boom, to the micro data. A similar story is told in Figure 4, which shows the cross-sectional dispersion in the return on equity (ROE) for major UK banks versus PNFCs.

We turn next to an examination of what policy actions might contribute to mitigating the inefficient credit booms that the model predicts. To do that, we extend our model to include a policymaker explicitly.

[6] This is equivalent to passing a relatively 'smooth' trend through the series. An HP filter with a high value of the smoothing parameter would achieve this. We use a band pass filter because it allows us to be more precise about the band of the frequency domain over which the filter returns cyclical variation.

[7] In the early 1990s, the Bank of England provided liquidity support to a few small banks in order to prevent a widespread loss of confidence in the banking system. 25 banks failed or closed during this period. The emergency liquidity assisstance provided by the Bank is regarded as having safeguarded the system as a whole, which was vulnerable to a tightening in wholesale markets. See Logan (2000) for discussion.

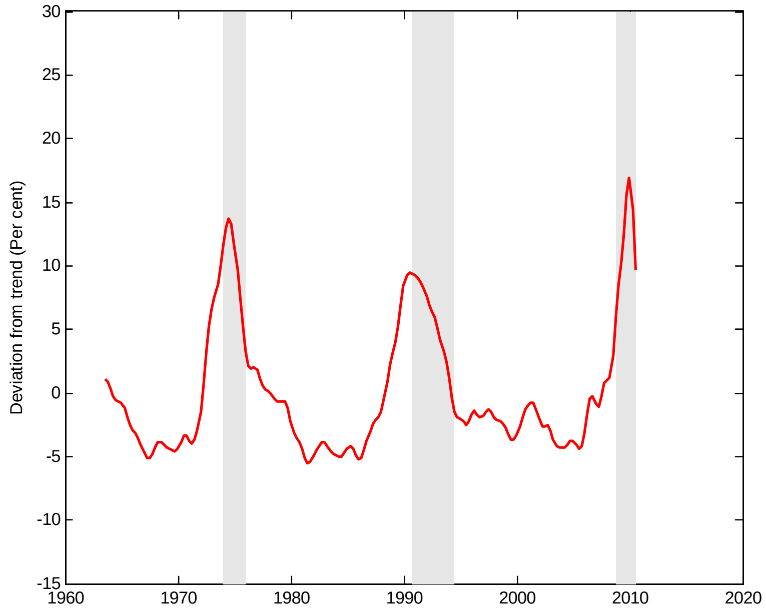

Figure 2: Band pass filtered ratio of UK credit:GDP, 1963Q2-2010Q2. The credit series is M4 Lending, which comprises monetary financial institutions' sterling net lending to private sector. The filter returns cyclical variation in the ratio over the 1-20 year frequency range. Shaded regions indicate periods of distress: 1973Q4-1975Q4 (secondary banking crisis); 1990Q3-1994Q4 (small banks crisis); 2008Q3–2010Q2.

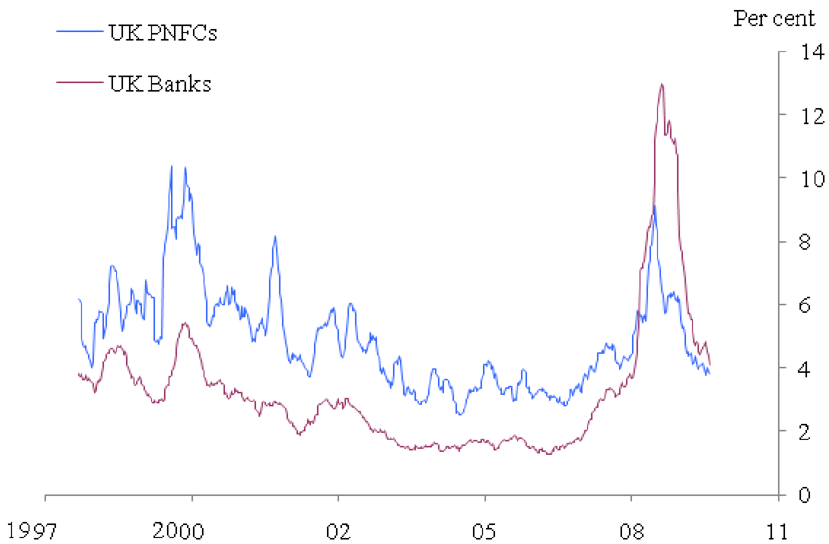

Figure 3: Dispersion of equity returns of major UK banks and top UK 100 PNFCs (by market cap)

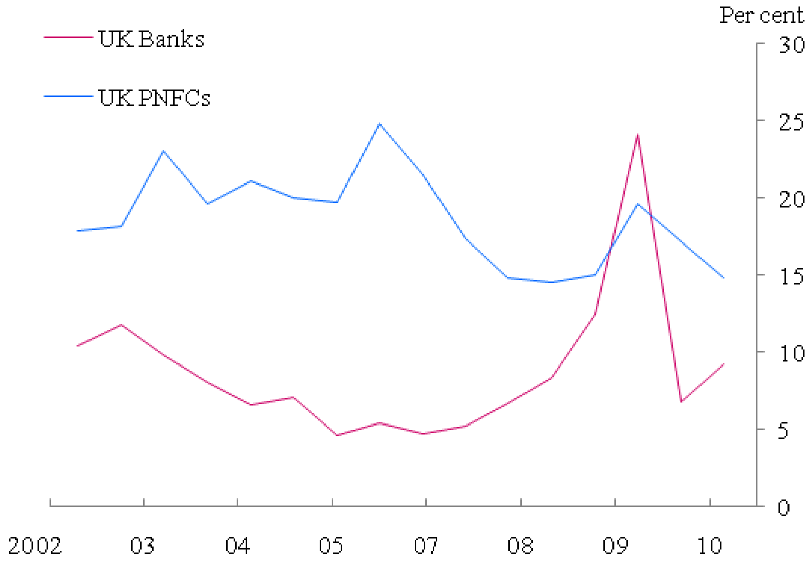

Figure 4: Dispersion of ROE of top 10 UK banks and top 10 UK PNFCs (by market cap)

3 Counter-cyclical capital adequacy regulation

3.1 Game with public and private signals

Let us now consider how the regulator may set k, which can be interpreted as the regulatory capital adequacy requirement. To do that, the regulator needs to know the distribution of θ, such that it can estimate what proportion of banks would receive low returns and hence would potentially have incentives to gamble at time $t = 1$. So suppose now that $\theta \sim N(y, \tau)$. The regulator observes this and sets the capital adequacy requirement, k^*, at $t = 0$, which applies to investments made at both $t = 0$ and $t = 1$, so as to maximize the social welfare. We also assume that the regulator reveals the distribution of θ truthfully to banks in order to explain the choice of k^*: thus, y acts as a public signal of the fundamentals.[8] The rest of the game's set up is as before, as illustrated in Figure 5.

We solve the model backwards, first working out banks' strategies at $t = 1$ given that they now observe a public signal about $\theta \sim N(y, \tau)$ (namely, its distribution) in addition to the private signal, which we now assume follows the process $x_i = \theta + \varepsilon_i$, where $\varepsilon_i \sim N(0, 1)$. Given these two signals, a bank's posterior belief of θ conditional on the two signals will be normal with a mean of:

[8] Note that it is important to assume that banks can observe y perfectly, rather than infer it from the regulator's choice of k^*, in order to ensure the uniqueness of the symmetric switching equilibrium. As we show later, the regulator chooses $k^* = 0$ both for very low y and very high y, such that banks observing $k^* = 0$ cannot infer y precisely. This can generate multiple equilibria. For further discussion of this issue, see Angeletos, Hellwig & Pavan (2006).

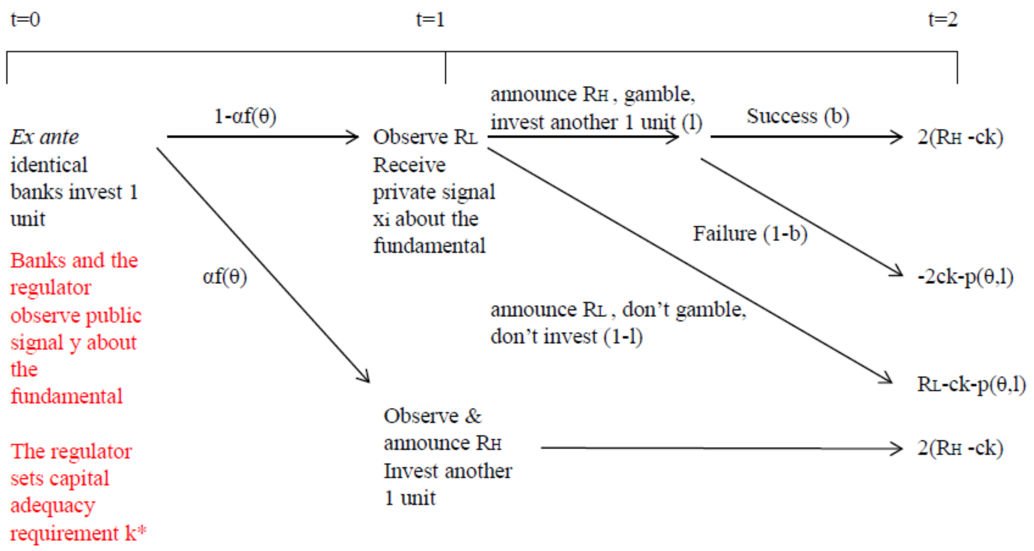

Figure 5: The timing and the payoffs of the game with optimal capital adequacy regulation

$$\bar{\theta} = \frac{\sigma^2 y + \tau^2 x}{\sigma^2 + \tau^2}, \tag{4}$$

and standard deviation

$$\sqrt{\frac{\sigma^2 \tau^2}{\sigma^2 + \tau^2}}.$$

Suppose banks that have received R_L at $t = 1$ uses the following switching strategy:

$$s(\bar{\theta}) = \left\{ \text{gamble if } \bar{\theta} \geqslant \theta^*, \text{ don't if } \bar{\theta} < \theta^* \right\} \tag{5}$$

To solve for the equilibrium, assume a simple functional form for bank reputation, $p(\theta, l) = \theta + l - 1$. Following the solution method used by Morris and Shin (2001), we can prove the following:

Proposition 2 *There exists a unique symmetric switching equilibrium with cut-off θ^*, where θ^* solves the equation:*

$$\theta^*(k, y) = \Phi(\sqrt{\gamma}(\theta^* - y)) + \frac{R_L + ck}{b} - 2R_H, \tag{6}$$

as long as $\gamma \equiv \frac{\sigma^2}{\tau^4} \left(\frac{\sigma^2 + \tau^2}{\sigma^2 + 2\tau^2} \right) \leq 2\pi$.

Proof. See Annex. ■

The condition $\gamma \leq 2\pi$ implies that the unique equilibrium exists only when the public signal is quite noisy relative to the private signal; Morris & Shin (2003) show that when this condition is violated, multiple equilibria can arise. The expression (6) defines banks' reaction function to the public signal about the fundamental, y, and the capital adequacy requirement, k. It can be shown that, by totally differentiating (6),

$$\frac{d\theta^*(k,y)}{dk} = \frac{c/b}{1-\phi[\sqrt{\gamma}(\theta^*(k,y)-y)]\sqrt{\gamma}} > 0, \tag{7}$$

and

$$\frac{d\theta^*(k,y)}{dy} = \frac{-\phi(.)\sqrt{\gamma}}{1-\phi[\sqrt{\gamma}(\theta^*(k,y)-y)]\sqrt{\gamma}} < 0. \tag{8}$$

The expression (7) says that, as before, a higher capital adequacy requirement increases the threshold of the private signal above which banks start gambling, and hence it helps to reduce the incidence of gambling. In addition, (8) says that a higher public signal y reduces the threshold of private signal at which banks start gambling. This is because the higher y, the more likely it is that other banks will also choose to gamble, and since all banks observe y, all banks know this. As such, each bank has an increased incentive to gamble even if his own private signal is low. Thus, a high public signal makes it more likely that banks will coordinate on the gambling equilibrium, all else equal.

3.2 The optimal capital requirement

We now consider how the policymaker might set the *aggregate* capital requirement which applies system-wide, to all banks. In setting the capital requirement, the policymaker faces the following trade-off. On the one hand, raising capital requirement deters gambling by those banks that have received low profits in the interim, and thus prevents inefficient investments. On the other hand, it also increases the funding cost for banks and thus reduces their payoffs, including for those which have received high profits in the interim and therefore have no incentive to gamble. Capital requirements set too high will also affect lending: beyond a certain point, raising k makes all payoffs negative, even those of lucky high ability banks.

To examine the optimal capital requirement, let us assume that the policymaker chooses k to maximize social welfare, S, consisting of a weighted sum of banks' expected returns given their reaction function (6).

$$\begin{aligned}
\max_k S(k,y) &= (1-\delta)\alpha f(y)[2(R_H-ck)] + \delta[1-\alpha f(y)]X(k,\theta^*), \qquad (9)\\
&\quad \text{s.t. } \theta^* = \theta^*(k,y),\\
\text{where } X(k,\theta^*) &\equiv [1-\Pr(\text{safe})][b(2R_H-2ck)+(1-b)(-2ck)] + \Pr(\text{safe})(R_L-ck).
\end{aligned}$$

The parameter $\delta \in [0,1]$ captures the relative weight (s)he places on the outcome of the gambling game played by low return banks: $\delta = 1/2$ characterizes a risk-neutral policymaker who cares equally about the returns of profitable and unprofitable banks, whereas $\delta = 1$ characterizes a highly risk-averse policymaker who cares only about deterring gambling by unprofitable banks. The function $X(k, \theta^*)$ is the expected payoff of unprofitable banks, where Pr[safe] defines the probability of unprofitable banks taking the safe strategy, given the public and private signals about the fundamental, and the capital requirement. We show in the Annex that:

$$\Pr(\text{safe}) = \theta^*(k, y) - \frac{R_L + ck}{b} + 2R_H.$$

Note that the social welfare function (9) is not a weighted sum of banks' utility functions. This is because the reputational effect, $p(\theta, l)$, is a private cost which induces banks to gamble for resurrection, so that the policymaker does not place any weight on it. Thus, the policymaker's objective as formulated in (9) can be interpreted as minimizing the banks' expected losses caused by gambling and inefficient credit booms, while avoiding the imposition of excessive funding costs on the entire banking system.

Solving for the policymaker's first order condition, the optimal capital requirement k^* – and hence the regulator's optimal choice of $\theta^*(k^*, y)$ – is given by the solution to the following (see Annex):

$$\{(1-\delta)\alpha f(y) 2c + \delta[1 - \alpha f(y)][-2c + c \Pr(\text{safe})]\} + \delta[1 - \alpha f(y)] \frac{\partial \Pr(\text{safe})}{\partial k}(u^s - u^g) = 0, \tag{10}$$

where $u^g \equiv b(2R_H - 2ck) + (1 - b)(-2ck)$ and $u^s \equiv R_L - ck$ are banks' returns from gambling and safe options, respectively, and:

$$\begin{aligned} \frac{\partial \Pr(\text{safe})}{\partial k} &= \frac{d\theta^*(k, y)}{dk} - \frac{c}{b} > 0, \\ u^s - u^g &= R_L - 2bR_H + ck > 0. \end{aligned}$$

Note that (10) equates the marginal cost of increasing capital requirement with the marginal benefit. In the Annex, we show that the second order condition is negative – and an interior solution exists – only when γ is sufficiently close to 2π, and k^* which solves (10) gives rise to $\theta^*(k^*, y) > y$. Otherwise, we will have a corner solution, as we will illustrate later using simulations.

Is the optimal capital adequacy requirement counter-cyclical? We show in the Annex that it is, as long as the macroeconomic fundamentals are within a certain range:

Proposition 3 *When the public signal about the macroeconomic fundamentals, y, is within a range, $y \in [\underline{y}, \bar{y}]$, and the public signal is neither too noisy nor too informative, $\gamma \in (\underline{\gamma}, 2\pi]$, the policymaker's optimal capital requirement k^* is procyclical, such that*

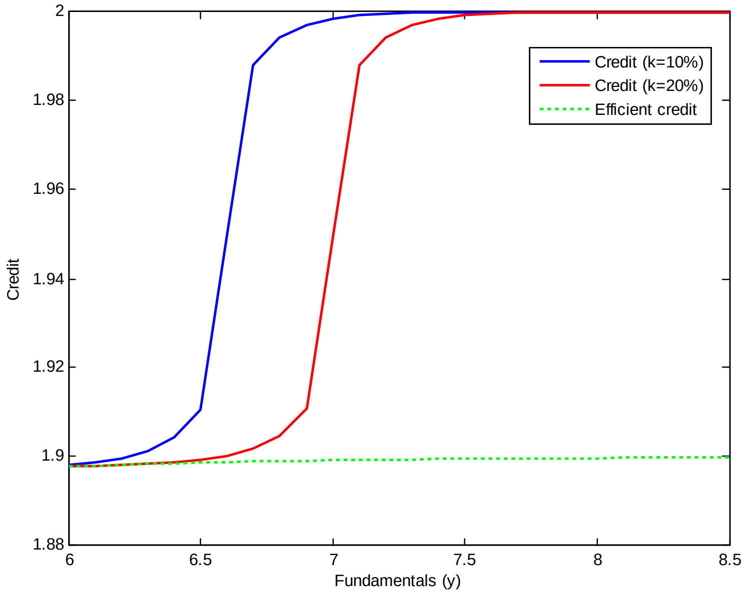

Figure 6: Aggregate credit supply

$\frac{dk^*}{dy} > 0$.

Proof. See Annex. ■

3.3 Simulations

We now show our results graphically in order to illustrate the intuition behind them. Figure 6 plots the aggregate credit supply (expected at $t = 0$ for different values of y) under our baseline calibration ($\delta = 0.5$, $\alpha = 0.8$, $b = 0.09$, $c = 0.15$, $R_L = 1$, $R_H = 2$, $\sigma = 0.5$, $\tau = 0.414$, $f(y) = \frac{1}{1+e^{-y}}$): this illustrates how a higher capital adequacy requirement can mitigate inefficient credit booms. The green dotted line in Figure 6 represents the efficient, 'no gambling' level of credit supply, given by $\alpha f(y) * 2 + (1 - \alpha f(y)) * 1$, which rises gently with y. The blue and the red lines show the aggregate credit supply with gambling, $\alpha f(y) * 2 + (1 - \alpha f(y)) * [\Pr(\text{safe}) * 1 + (1 - \Pr(\text{safe})) * 2]$, for different levels of capital requirements, $k = 10\%$ and $k = 20\%$, respectively. As the blue and the red lines show, banks' gambling incentives generate inefficient credit booms when fundamentals are high; and a higher capital requirement mitigates inefficient credit booms by increasing the range of fundamentals in which banks choose not to gamble, and reducing gambling for any given level of fundamentals.

Figure 7 plots the optimal capital adequacy requirement k^*, for a different range of the public signal about the fundamentals, y, under our baseline calibration. As this shows, the optimal capital requirement is zero when y is below a threshold, but pro-cyclical

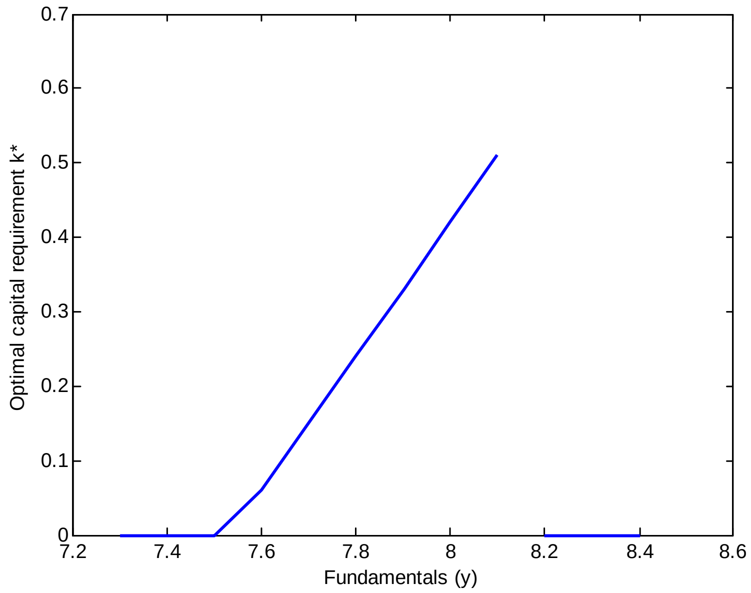

Figure 7: Optimal capital adequacy requirement, k^*

for an intermediate range of y, and then becomes zero again when y is above a certain threshold.

To understand why this is the case, note that capital requirements have a non-linear impact on banks' incentives to gamble, as Figure 8 illustrates. When the capital requirement is low, almost all banks gamble in expectation, whereas when it is high, almost all of them are expected to choose to play safe. In the intermediate range of k, a small increase in capital requirements will lead to a rapid reduction in gambling as banks switch from gambling to playing safe. As y becomes larger, banks' incentives to gamble becomes greater, and hence a higher capital requirement is needed to deter gambling.

As a result, the social benefit of increasing k is non-linear. By contrast, the cost of increasing k is linear given our assumption that the cost of raising capital is a constant, c. Consequently, the social welfare function (9), is not globally concave, as shown in Figure 9. This is why we have corner solutions for some range of y.

The comparative statics are intuitive, too. For instance, as the cost of raising equity, c, falls, it becomes optimal for the regulator to set a higher capital requirement for any given y (see Figure 10). Moreover, the optimal capital requirement becomes more strongly counter-cyclical as c falls. Similarly, we can show that as δ rises – i.e. the regulator becomes more concerned about the social cost associated with gambling – the optimal capital adequacy requirement becomes more stringent *and* more strongly counter-cyclical.

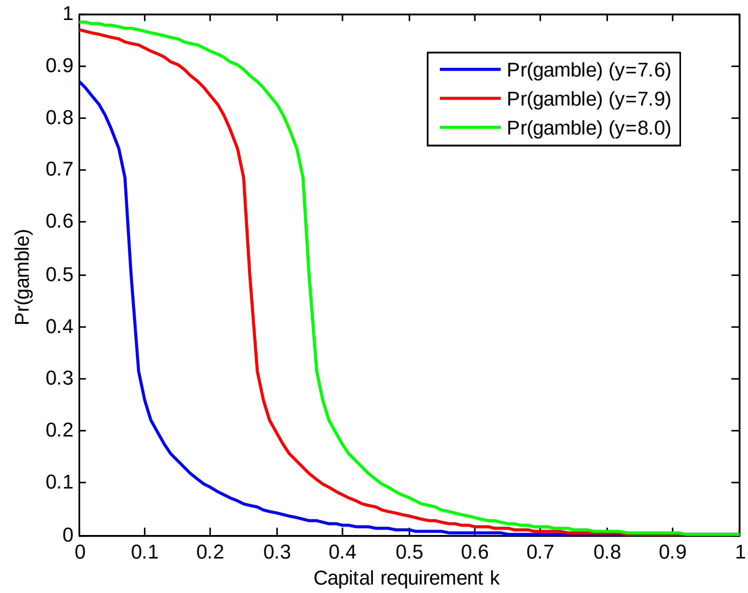

Figure 8: The impact of capital adequacy requirement on banks' incentives to gamble

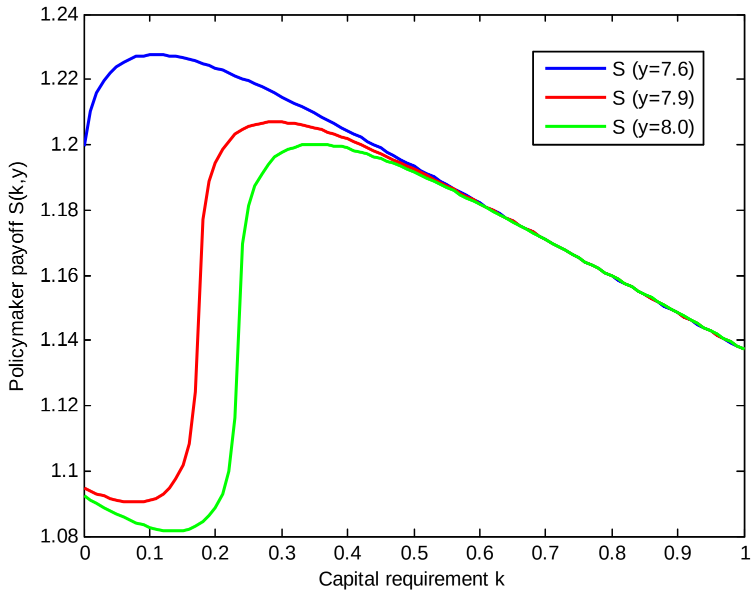

Figure 9: Social welfare function

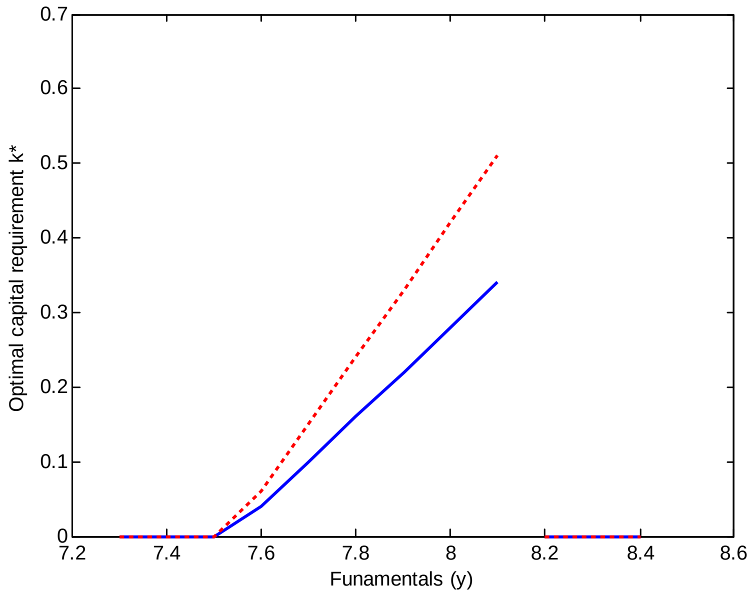

Figure 10: The effect of lower costs of raising equity on optimal capital adequacy requirements, blue ($c = 15\%$), red dashed ($c = 10\%$).

4 Discussion

Our analysis clearly illustrates the trade-off facing the policymaker in setting an aggregate countercyclical capital adequacy requirement. In our model, as fundamentals improve, more banks become genuinely profitable, and this gives those banks that turned out to be unprofitable the incentive to gamble for resurrection to preserve their reputation. This triggers an inefficient credit boom. To prevent this, the policymaker can raise the aggregate capital adequacy requirement which raises the cost of gambling for banks. However, this also raises the cost of investment for all banks, including those successful high ability banks, which do not have the incentive to gamble. Although it is optimal for policymakers to raise the capital adequacy requirement as macroeconomic fundamentals improve for some range of fundamentals, there will be a point at which the marginal benefit of deterring gambling by some banks through a higher capital adequacy requirement becomes less than the marginal cost of increasing funding cost for all banks. This is where an aggregate capital adequacy requirement loses traction.

Thus, 'across-the-system' counter-cyclical capital requirements can only achieve a 'constrained' optimum; and so, when macroeconomic fundamentals are very strong, instruments which target specific risk-taking activities may be needed in order to prevent an inefficient credit boom. In the context of our model, it would of course be more efficient to increase capital requirements only for those banks that have the incentives to gamble (i.e. those that have observed R_L in the interim), than imposing a higher requirement across the banking sector; but this requires the policymaker to be able to observe banks' balance sheets accurately and determine which subset of banks are likely to gamble. Although obtaining detailed information about banks' balance sheets and investment strategies is likely to be a costly exercise, our analysis highlights the limitation of aggregate counter-cyclical capital requirements and suggests that investing in acquiring more detailed information in order to design targeted instruments may be particularly desirable during boom times.

It is also worth noting that, in our model, we treat c as constant, and, in particular, as invariant to k. Were c to fall with k, capital regulation would have traction only to the extent that $c(k)k$ increases in k. Note also that $c'(k) < 0$ would mitigate the costs of increasing k on non-gambling banks. For this reason the range of fundamentals over which procyclical capital regulation is optimal would expand when $c'(k) < 0$. And since a given increase in k would have a smaller marginal effect on the incentive to gamble, the optimal level of k would rise for all values of fundamentals.

5 The role of public information: can 'moral suasion' work?

Finally, we separate out the two effects of counter-cyclical capital requirements on banks' risk-taking incentives, namely (i) the direct effect of raising the cost of risk taking, and (ii) the indirect effect of making information about the state of macroeconomic fundamentals public – for example, via the publication of the *Financial Stability Review*. In our set up, banks do not observe y directly but find out y only because the regulator announces it in order to explain their choice of counter-cyclical capital requirements (and that the regulator can be trusted to announce the true state of y). If so, capital adequacy requirements will affect banks' gambling incentives through two distinct channels: first, higher capital adequacy requirements will increase the cost of gambling directly; and second, information about y will play a role in coordinating banks' actions between gambling and non-gambling equilibria.

To distinguish these two effects, Figure 11 plots the switching point, θ^*, in the game where banks only have private information (given by (3)), and in the game where they are also given public information about y (given by (6)); all the other parameters, including k, are held constant. Thus, the gap between the two lines gives us the marginal effect of public information on banks' risk-taking incentives for different values of y. As the figure illustrates, public information has a powerful effect in deterring gambling when y is low. This suggests that 'moral suasion' – i.e. telling banks to stop taking risks – can potentially act as a powerful deterrence when the fundamentals are deteriorating and the policymakers' warning is thought to reveal the true information about the fundamentals.

By contrast, telling banks that fundamentals are currently good can have a counter-

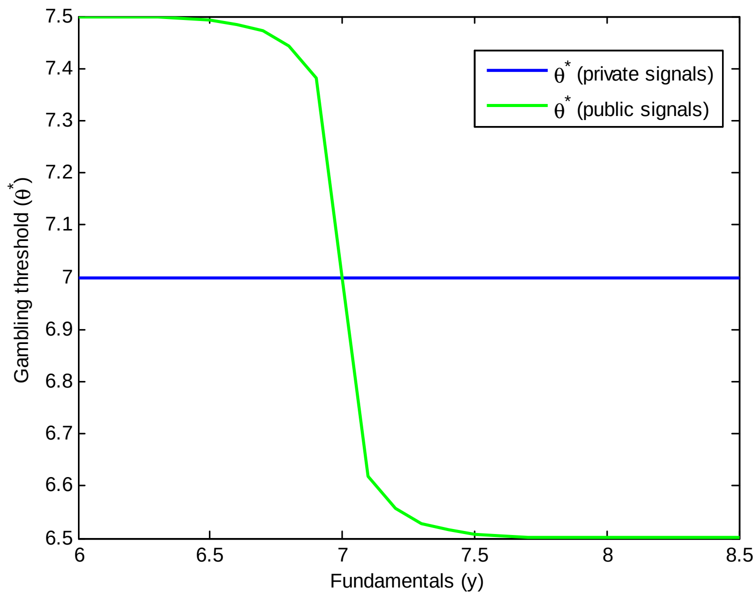

Figure 11: The role of public information

productive effect of encouraging them to coordinate to the gambling equilibrium, when the lack of detailed information about banks' risk-taking activities prevents policymakers from implementing a targeted policy. So how should policymakers communicate when fundamentals are good? If future fundamentals are affected by banks' current risk-taking decisions – as assumed by Aikman *et al.* (2010) – then an effective communication strategy for policymakers might be to highlight the future risks to the banking system created by banks' current risk-taking. For instance, the public release of stress test results could serve this purpose. Although our static framework does not allow us to model explicitly the impact of future fundamentals on banks' current risk-taking incentives, banks in the real world make long-term investments which are affected by current as well as future fundamentals, and it is plausible that future fundamentals are endogenous to banks' current risk-taking, as losses caused by unproductive investments could ultimately lead to a banking crisis and a large output loss. In this sense, publicly announcing the results of stress tests can serve as a macroprudential policy tool in itself to the extent that stress tests 'look through' contemporaneous exuberance to reveal underlying fragilities. The macroprudential toolkit can therefore operate both directly on costs (through k), and indirectly on beliefs, which affect outcomes in a world of imperfect information.

6 Conclusions

This paper contributes to the nascent literature on macroprudential regulation by articulating the trade-off faced by policymakers in setting counter-cyclical capital adequacy requirements when banks have the incentives to make high-risk, high-return investments in order to maintain their reputations. We show that counter-cyclical capital adequacy requirements are socially optimal for an intermediate range of fundamentals but not when fundamentals are either very weak or very strong. In the intermediate range, improved fundamentals imply high ability banks perform well. In order to safeguard their reputations, low ability banks then have an increased incentive to gamble – to 'keep up with the Goldmans'. Optimal macroprudential policy works against this incentive by raising the cost of gambling as fundamentals improve.

When fundamentals are very weak however, few banks make profits and hence unprofitable banks have no incentive to gamble in order to preserve their reputations; thus, there is no need to increase capital adequacy requirements in response to a small improvement in fundamentals. And when fundamentals are very strong, most high ability banks make profits and hence the unprofitable banks have very strong incentives to gamble in order to avoid being labelled as 'low ability'; in this case, policymakers cannot deter gambling by the unprofitable banks without also imposing excessively high funding costs on high ability banks, which have no incentive to gamble. This suggests that, when fundamentals are very strong, the need for policymakers to invest in obtaining detailed information about banks' balance sheets and their investment strategies in order to devise targeted instruments is particularly strong.

Our analysis also clarifies the role of central bank communication in deterring gambling via its impact on banks' beliefs. In particular, we show that a warning by policymakers that the fundamentals are deteriorating can be effective in preventing inefficient credit booms when that warning is seen to reveal the true state of the fundamentals and thus helps to coordinate banks' beliefs to the efficient equilibrium. When fundamentals are good, policymakers may wish to focus on communicating the potential damage to future fundamentals and banks' profitability caused by their current risk-taking activities – for example by releasing stress test results or regular conjunctural analysis of financial stability issues.

Our analysis focuses on a particular role for capital adequacy requirements, namely, that of preventing banks from investing in risky projects that have negative net present value. There are other rationales for counter-cyclical capital adequacy requirements which we have not considered here, including enhancing loss absorbance. Our analysis also focuses on the role of capital adequacy requirements in preventing inefficient credit booms, and does not examine its potential role in preventing inefficient credit crunches. Examining all these aspects of counter-cyclical capital requirements in a single framework is left for future research.

A Annex

Proof of Proposition 1:

Our model already satisfies two conditions set out in Morris & Shin (2003), whose technology we subsequently employ, namely:

Condition 1: *Action Monotonicity*: By $\frac{\partial p(\theta,l)}{\partial l} > 0$, $\pi(\theta, l)$ is non-decreasing in l;

Condition 2: *State Monotonicity*: By $\frac{\partial p(\theta,l)}{\partial \theta} > 0$, $\pi(\theta, l)$ is non-decreasing in θ;

and we impose that $p(\theta, l)$ is such that:

Condition 3: *Strict Laplacian State Monotonicity*: there exists a unique θ^* solving

$$\int_{l=0}^{1} \pi(\theta^*, l)dl = 0;$$

holds. Next, suppose $p(.)$ implies that

Condition 4: There exist $\underline{\theta} \in \mathbb{R}, \overline{\theta} \in \mathbb{R}$ and $\varepsilon \in \mathbb{R}_{++}$, such that (a) $\pi(\theta, l) \leq -\varepsilon$ for all l and for $\theta < \underline{\theta}$; and (b) $\pi(\theta, l) > \varepsilon$ for all l and $\theta > \overline{\theta}$.

This condition implies that, for sufficiently low (high) values of fundamentals, choosing the safe (risky) option having observed low returns is a dominant action regardless of the aggregate proportion of banks that do so too. In the intervening interval, the dominant action depends on the proportion of banks that follow that action too. Finally, we require that

Condition 5: *Continuity*: $\int_{l=0}^{1} g(l)\pi(x, l)dl$ is continuous with respect to signal x and density $g(.)$.

Condition 6: *Finite expectations of signals*: $\int_{z=-\infty}^{\infty} zf(z)dz$ is well defined.

These six conditions ensure the model complies with the generic formulation of Morris & Shin (2003). We therefore use the following result, taken from their paper:

Lemma 1 *(Morris & Shin (2003), Prop. 2.2): Let θ^* be defined by Condition 3. For any $\delta > 0$, there exists $\overline{\sigma} > 0$ such that for all $\sigma < \overline{\sigma}$, if strategy s survives iterated deletion of strictly dominated strategies, then $s(x) = \{safe\}$ for all $x \leq \theta^* - \delta$ and $s(x) = \{gamble\}$ for all $x \geq \theta^* + \delta$.*

(We refer readers to Morris and Shin (2001) for the proof.). In words, this says that the support of fundamentals can be divided into two regions: one, for which $\theta < \theta^*$, in which banks coordinate on choosing the safe option conditional on observing low initial returns. Intuitively, fundamentals are not sufficiently high to cause severe reputational damage to announcing low returns when all other banks do so too. In the second region, in which $\theta > \theta^*$, high fundamentals imply a large degree of reputational damage to announcing low returns. Hence, all banks coordinate on the gambling option to minimize the reputational downside to having made a bad initial investment.

Lemma 1 and the expression 2 together proves Proposition 1.

Proof of Proposition 2:
(Morris & Shin (2003), Prop. 3.1): A banker who has observed a private signal x_i believes that any other player's signal x' is distributed normally with mean $\bar{\theta}$ and standard deviation

$$\sqrt{\frac{2\sigma^2\tau^2+\sigma^4}{\sigma^2+\tau^2}}.$$

Suppose that the bank believed that all other banks played a switching strategy of gambling if and only if $\frac{\sigma^2 y+\tau^2 x'}{\sigma^2+\tau^2} > \theta^*$. Thus, the bank's belief about other banks' probability of gambling – and hence his best guess about l – is given by:

$$l = 1-\Phi\left(\frac{\theta^*-\bar{\theta}+\frac{\sigma^2}{\tau^2}(\theta^*-y)}{\sqrt{\frac{2\sigma^2\tau^2+\sigma^4}{\sigma^2+\tau^2}}}\right). \tag{11}$$

We know that each bank's payoff from gambling is given by (2). Assuming a simple functional form $p(\theta,l)=\theta+l-1$, its expected payoff from gambling is now given by:

$$\pi(\bar{\theta},\kappa) = b\left\{2R_H+\bar{\theta}-\Phi\left(\frac{\theta^*-\bar{\theta}+\frac{\sigma^2}{\tau^2}(\theta-y)}{\sqrt{\frac{2\sigma^2\tau^2+\sigma^4}{\sigma^2+\tau^2}}}\right)\right\}-R_L-ck.$$

A symmetric equilibrium with switching point θ occurs exactly when $\pi^*(\theta,\theta)\equiv\pi(\theta,\theta)=0$, where

$$\begin{aligned}\pi^*(\theta,\theta) &= b\left\{2R_H+\theta^*-\Phi\left(\frac{\sigma^2(\theta^*-y)}{\tau^2\sqrt{\frac{2\sigma^2\tau^2+\sigma^4}{\sigma^2+\tau^2}}}\right)\right\}-R_L-ck \\ &= b\left\{2R_H+\theta^*-\Phi(\sqrt{\gamma}(\theta^*-y))\right\}-R_L-ck,\end{aligned} \tag{12}$$

where

$$\gamma \equiv \frac{\sigma^2}{\tau^4}\left(\frac{\sigma^2+\tau^2}{\sigma^2+2\tau^2}\right).$$

As Morris and Shin (2001) illustrate, this game has a unique equilibrium if and only if (12) is strictly increasing in θ^*. The necessary condition for this is

$$\frac{d\pi^*}{d\theta^*} = b(1-\sqrt{\gamma}\phi(\theta^*-y)) \geq 0.$$

Since the normal density $\phi(x)$ reaches its maximum at $\frac{1}{\sqrt{2\pi}}$, the above condition holds as long as $\gamma \leq 2\pi$. Assuming that this condition holds, the unique switching equilibrium θ^* solves:

$$\theta^* = \Phi(\sqrt{\gamma}(\theta^*-y)+\frac{R_L+ck}{b}-2R_H.$$

Derivation of $\Pr(\textbf{safe})$**:**

Given banks' strategy, the probability of a bank gambling in the symmetric switching equilibrium is given by l in (11) when $\theta^* = \overline{\theta}$. Thus, the probability of a bank which has observed R_L at $t = 1$ choosing to gamble is:

$$\Pr(\text{gamble}) = 1 - \Phi\left(\sqrt{\gamma}(\theta^* - y)\right),$$

where $\gamma \equiv \frac{\sigma^2}{\tau^4}\frac{\sigma^2+\tau^2}{\sigma^2+2\tau^2}$, and θ^* is given by (6). Rearranging (6) and substituting into the above gives:

$$\begin{aligned}\Pr(\text{gamble}) &= 1 - \left[\theta^*(k,y) - \frac{R_L + ck}{b} + 2R_H\right],\\ \Pr(\text{safe}) &= \theta^*(k,y) - \frac{R_L + ck}{b} + 2R_H.\end{aligned}$$

The first and the second order conditions of the policymaker's maximization problem:

The policymaker's first order condition is given by:

$$\frac{\partial S(k,y)}{\partial k} = -(1-\delta)\alpha f(y)2c + \delta[1 - \alpha f(y)]\frac{\partial X(k,\theta^*)}{\partial k} = 0, \tag{13}$$

where:

$$\frac{\partial X(k,\theta^*)}{\partial k} = -2c + c\Pr[\text{safe}] + \frac{\partial \Pr[\text{safe}]}{\partial k}(u^s - u^g).$$

Re-arranging (13), we obtain (10). The second order condition for the maximization problem is satisfied if and only if:

$$\frac{\partial^2 S(k,y)}{\partial k^2} = \delta[1 - \alpha f(y)]\frac{\partial^2 X(k,\theta^*)}{\partial k^2} < 0,$$

where

$$\frac{\partial^2 X(k,\theta^*)}{\partial k^2} = 2c\frac{\partial \Pr[\text{safe}]}{\partial k} + \frac{\partial^2 \Pr[\text{safe}]}{\partial k^2}(u^s - u^g),$$

Substituting in $\frac{\partial \Pr[\text{safe}]}{\partial k} = \frac{d\theta^*(k,y)}{dk} - \frac{c}{b}$ and $\frac{\partial^2 \Pr[\text{safe}]}{\partial k^2} = \frac{d^2\theta^*(k,y)}{dk^2}$, the SOC is satisfied iff:

$$\frac{\partial^2 X(k,\theta^*)}{\partial k^2} = 2c\left(\frac{d\theta^*(k,y)}{dk} - \frac{c}{b}\right) + (u^s - u^g)\frac{d^2\theta^*(k,y)}{dk^2} < 0. \tag{14}$$

From (**??**),

$$\frac{d^2\theta^*(k,y)}{dk^2} = \frac{-c/b * \gamma^{3/2}(\theta^*(k,y) - y) * \phi[\sqrt{\gamma}(\theta^*(k,y) - y)]}{\left(1 - \phi[\sqrt{\gamma}(\theta^*(k,y) - y)]\sqrt{\gamma}\right)^2} * \frac{d\theta^*(k,y)}{dk}.$$

The last line uses $\phi'(x) = -x\phi(x)$. The LHS of (14) becomes

$$\frac{\sqrt{\gamma}\frac{c^2}{b}\phi[\sqrt{\gamma}(\theta^*(k,y) - y)]}{1 - \phi[\sqrt{\gamma}(\theta^*(k,y) - y)]\sqrt{\gamma}}\left\{2 - (u^s - u^g)\frac{(1/b) * \gamma(\theta^*(k,y) - y)}{\left(1 - \phi[\sqrt{\gamma}(\theta^*(k,y) - y)]\sqrt{\gamma}\right)^2}\right\},$$

where $\frac{\sqrt{\gamma}\frac{c^2}{b}\phi[\sqrt{\gamma}(\theta^*(k,y)-y)]}{1-\phi[\sqrt{\gamma}(\theta^*(k,y)-y)]\sqrt{\gamma}} \geq 0$. If $\gamma = 0$, this becomes zero; but as $\gamma \to 2\pi$, the SOC becomes negative as long as $\theta^*(k,y) > y$. In other words, as long as γ is sufficiently large (i.e. the public signal is quite precise relative to the private signal), the SOC is satisfied of the policymaker's optimal choice is to set k^* such that $\theta^*(k^*,y) > y$.

Proof of Proposition 3:

For there to be a case for countercyclical capital adequacy requirement, i.e. $\frac{dk^*}{dy} > 0$, it must be the case for the relevant range of y (i.e. $y < \bar{y}$) that $\frac{\partial^2 S(k,y)}{\partial k \partial y} > 0$. From (13),

$$\frac{\partial^2 S(k,y)}{\partial k \partial y} = -(1-\delta)\alpha f'(y)2c - \delta\alpha f'(y)\frac{\partial X(k,\theta^*)}{\partial k} + \delta[1-\alpha f(y)]\frac{\partial^2 X(k,\theta^*)}{\partial k \partial y},$$

where $f'(y) > 0$. Evaluated at k^* given by FOC (13),

$$\frac{\partial X(k,\theta^*)}{\partial k} = -\frac{(1-\delta)\alpha f'(y)2c}{\delta\alpha f'(y)}.$$

So

$$\frac{\partial^2 S(k,y)}{\partial k \partial y} = \delta[1-\alpha f(y)]\frac{\partial^2 X(k,\theta^*)}{\partial k \partial y}.$$

So the necessary and sufficient condition for countercyclical capital adequacy requirement is $\frac{\partial^2 X(k,\theta^*)}{\partial k \partial y} > 0$.

From (??), $\frac{\partial X(k,\theta^*)}{\partial k} = -2c + c\Pr[\text{safe}] + \frac{\partial \Pr[\text{safe}]}{\partial k}(u^s - u^g)$

$$\frac{\partial^2 X(k,\theta^*)}{\partial k \partial y} = c\frac{\partial \Pr[\text{safe}]}{\partial y} + \frac{\partial^2 \Pr[\text{safe}]}{\partial k \partial y}(u^s - u^g),$$

where

$$\begin{aligned}
\frac{\partial \Pr[\text{safe}]}{\partial y} &= \frac{d\theta^*(k,y)}{dy} = \frac{-\phi(.)\sqrt{\gamma}}{1-\phi[\sqrt{\gamma}(\theta^*(k,y)-y)]\sqrt{\gamma}} < 0, \\
\frac{\partial^2 \Pr[\text{safe}]}{\partial k \partial y} &= \frac{d^2\theta^*(k,y)}{dkdy}, \\
\frac{d\theta^*(k,y)}{dk} &= \frac{c/b}{1-\phi[\sqrt{\gamma}(\theta^*(k,y)-y)]\sqrt{\gamma}} > 0, \\
\frac{d^2\theta^*(k,y)}{dkdy} &= \frac{-(c/b)\sqrt{\gamma}\sqrt{\gamma}(\theta^*(k,y)-y)\phi[\sqrt{\gamma}(\theta^*(k,y)-y)]}{\left(1-\phi[\sqrt{\gamma}(\theta^*(k,y)-y)]\sqrt{\gamma}\right)^2}\sqrt{\gamma}\left(\frac{d\theta^*(k,y)}{dy} - 1\right),
\end{aligned}$$

in which

$$\frac{d\theta^*(k,y)}{dy} - 1 = \frac{-1}{1-\phi[\sqrt{\gamma}(\theta^*(k,y)-y)]\sqrt{\gamma}} < 0.$$

So

$$\frac{d^2\theta^*(k,y)}{dkdy} = \frac{(c/b)\sqrt{\gamma}\sqrt{\gamma}(\theta^*(k,y)-y)\phi[\sqrt{\gamma}(\theta^*(k,y)-y)]}{\left(1-\phi[\sqrt{\gamma}(\theta^*(k,y)-y)]\sqrt{\gamma}\right)^2}\sqrt{\gamma}\left(\frac{1}{1-\phi[\sqrt{\gamma}(\theta^*(k,y)-y)]\sqrt{\gamma}}\right),$$

which is positive iff $\theta^*(k,y) - y > 0$. Use this in $\frac{\partial^2 X(k,\theta^*)}{\partial k \partial y}$ to give

$$\frac{\partial^2 X(k,\theta^*)}{\partial k \partial y} = c\frac{\phi(.)\sqrt{\gamma}}{1-\phi(.)\sqrt{\gamma}}\left\{-1+\frac{(1/b)\sqrt{\gamma}\sqrt{\gamma}(\theta^*(k,y)-y)}{\left(1-\phi(.)\sqrt{\gamma}\right)^2}(u^s-u^g)\right\},$$

which is positive iff

$$\frac{(1/b)\gamma(\theta^*(k,y)-y)}{\left(1-\phi(.)\sqrt{\gamma}\right)^2}(u^s-u^g) > 1.$$

A necessary condition for this is $\theta^*(k,y)-y>0$. For this, since $\frac{d\theta^*(k,y)}{dy}-1<0$, there exists a value of y, $\bar{y}$, such that $\theta^*(k,y)-y>0$ for $y<\bar{y}$. Then as $\gamma \longrightarrow 2\pi$, $\phi(.)\sqrt{\gamma} \longrightarrow 1$, such that when $y \in [\underline{y},\bar{y}]$ there exists some γ, $\underline{\gamma} < 2\pi$, such that, for $\gamma \in (\underline{\gamma}, 2\pi]$, $\frac{\partial^2 X(k,\theta^*)}{\partial k \partial y} > 0$. The lower bound on the noise ratio, $\underline{\gamma}$, solves:

$$\frac{(1/b)\underline{\gamma}(\theta^*(k,y)-y)}{\left\{1-\phi\left[\sqrt{\underline{\gamma}}(\theta^*(k,y)-y)\right]\sqrt{\underline{\gamma}}\right\}}(u^s-u^g) = 1.$$

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