

Conventional and Unconventional Monetary Policy*

Vasco Cúrdia
Federal Reserve Bank of New York

Michael Woodford
Columbia University

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Abstract

We extend a standard New Keynesian model to incorporate heterogeneity in spending opportunities and two sources of (potentially time-varying) credit spreads, and to allow a role for the central bank's balance sheet in equilibrium determination. We use the model to investigate the implications of imperfect financial intermediation for familiar monetary policy prescriptions, and to consider additional dimensions of central-bank policy — variations in the size and composition of the central bank's balance sheet, and payment of interest on reserves — alongside the traditional question of the proper choice of an operating target for an overnight policy rate. We also give particular attention to the special problems that arise when the zero lower bound for the policy rate is reached. We show that it is possible to provide criteria for the choice of policy along each of these possible dimensions, within a single unified framework, and to provide policy prescriptions that apply equally when financial markets work efficiently and when they are subject to substantial disruptions, and equally when the zero bound is reached and when it is not a concern.

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The recent global financial crisis has confronted central banks with a number of questions beyond the scope of many conventional accounts of the theory of monetary policy. For example, do projections of the paths of inflation and of aggregate real activity under some contemplated path for policy provide a sufficient basis for monetary policy decisions, or must financial conditions be given independent weight in such deliberations? (The fact that the Fed began aggressively cutting its target for the federal funds rate, in late 2007 and early 2008, while inflation was arguably increasing and real GDP was not yet known to be contracting — and has nonetheless often been criticized as responding too slowly in this period — suggests that familiar prescriptions that focus on inflation and real GDP alone, such as the Taylor (1993) rule or common accounts of “flexible inflation targeting” (Svensson, 1997), may be inadequate to circumstances of the kind recently faced.¹ As a further, more specific question, how should a central bank’s interest rate policy be affected by the observation that other key interest rates no longer co-move with the policy rate (the federal funds rate, in the case of the US) in the way that they typically have in the past? (The dramatically different behavior of the LIBOR-OIS spread, shown in Figure 1, since August 2007, has drawn particular comment. Indeed, John Taylor himself (Taylor, 2008) has suggested that movements in this spread should be taken into account in an extension of his famous rule.)

In addition to such new questions about traditional interest-rate policy, the very focus on interest-rate policy as the central question about monetary policy has been called into question. The explosive growth of base money in the US since September 2008 (shown in Figure 2) has led many commentators to suggest that the main instrument of US monetary policy has changed, from an interest-rate policy to one often described as “quantitative easing.” Does it make sense to regard the supply of bank reserves (or perhaps the monetary base) as an alternative or superior operating target for monetary policy?

¹See Mishkin (2008) for discussion of some of the considerations behind the Fed’s relatively aggressive rate cuts in the early part of the crisis.

Does this (as some would argue) become the only important monetary policy decision once the overnight rate (the federal funds rate) has reached the zero lower bound, as it effectively has in the US since December 2008 (Figure 3)? And now that the Federal Reserve has legal authorization to pay interest on reserves (under the Emergency Economic Stabilization Act of 2008), how should this additional potential dimension of policy be used?

The past two years have also seen dramatic developments with regard to the composition of the asset side of the Fed's balance sheet (Figure 4). Whereas the Fed had largely held Treasury securities on its balance sheet prior to the fall of 2007, other kinds of assets — including both a variety of new “liquidity facilities” and new programs under which the Fed has essentially become a direct lender to certain sectors of the economy — have rapidly grown in importance, and decisions about the management of these programs have occupied much of the attention of policymakers during the recent period. How should one think about the aims of these programs, and the relation of this new component of Fed policy to traditional interest-rate policy? Is Federal Reserve credit policy a substitute for interest-rate policy, or should it be directed to different goals than those toward which interest-rate policy is directed?

These are clearly questions that a theory of monetary policy adequate to our present circumstances must address. Yet not only have they been the focus of relatively little attention until recently, but the very models commonly used to evaluate the effects of alternative prescriptions for monetary policy have little to say about them. Many New Keynesian models abstract entirely from the role of financial intermediation in the economy (by assuming a representative household), or assume perfect risk-sharing (to facilitate aggregation), so that the consequences of financial disruptions cannot be addressed. Many models include only a single interest rate (or at any rate, only a single interest rate of a given maturity, with long rates tied to short rates through a no-arbitrage condition), and hence cannot say anything about the proper response to changes in spreads. And many models abstract entirely from the balance sheet of the central bank, so that questions about the additional dimensions of policy resulting from the possibility of varying the size

and composition of the balance sheet cannot be addressed.²

The aim of the research summarized here is to show how issues of these kinds can be addressed in a DSGE framework. We extend a basic New Keynesian model in directions that are crucial for analysis of the questions just posed: we introduce non-trivial heterogeneity in spending opportunities, so that financial intermediation matters for the allocation of resources; we introduce imperfections in private financial intermediation, and the possibility of disruptions to the efficiency of intermediation for reasons taken here as exogenous; and we introduce additional dimensions of central bank policy by explicitly considering the role of the central bank's balance sheet in equilibrium determination, and by allowing central-bank liabilities to supply transactions services. Unlike some other recent approaches to the introduction of financial intermediation into New Keynesian DSGE models³ — which arguably include some features that allow for greater quantitative realism — our aim has been to develop a model that departs from a standard (representative-household) model in only the most minimal ways necessary to address the issues raised above. In this way, we can nest the standard (and extensively studied) model as a special case of our model, facilitating understanding of the sources of our results and the precise significance of the various new model elements that are introduced.

²In a representative-household model, abstraction from the role of the central bank's balance sheet in equilibrium determination is relatively innocuous; in particular, Eggertsson and Woodford (2003) show that introducing both a large range of possible choices about the composition of the balance sheet and transactions frictions that accord a special role to central-bank liabilities need not imply any additional channels through which monetary policy can affect the economy when the zero lower bound is reached. However, we wish to reconsider this question in a framework where financial intermediation is both essential and costly.

³This has been a very active literature of late. See *e.g.*, Christiano *et al.* (2007), Faia and Monacelli (2007), Gerali *et al.* (2008), and Gertler and Karadi (2009).

1 A Model with Multiple Dimensions of Monetary Policy

Here we sketch the key elements of our model, which extends the model introduced in Cúrdia and Woodford (2009a) to introduce the additional dimensions of policy associated with the central bank's balance sheet. (The reader is referred to our earlier paper, and especially its technical appendix, for more details.) We stress the similarity between the model developed there and the basic New Keynesian model, and show how the standard model is recovered as a special case of the extended model.

1.1 Heterogeneity and the Allocative Consequences of Credit Spreads

Our model is a relatively simple generalization of the basic New Keynesian model used for the analysis of optimal monetary policy in sources such as Goodfriend and King (1997), Clarida *et al.* (1999), and Woodford (2003). The model is still highly stylized in many respects; for example, we abstract from the distinction between the household and firm sectors of the economy, and instead treat all private expenditure as the expenditure of infinite-lived household-firms, and we similarly abstract from the consequences of investment spending for the evolution of the economy's productive capacity, instead treating all private expenditure as if it were non-durable consumer expenditure (yielding immediate utility, at a diminishing marginal rate).

We depart from the assumption of a representative household in the standard model, by supposing that households differ in their preferences. Each household i seeks to maximize a discounted intertemporal objective of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u^{\tau_t(i)}(c_t(i); \xi_t) - \int_0^1 v^{\tau_t(i)}(h_t(j; i); \xi_t) dj \right], \quad (1.1)$$

where $\tau_t(i) \in \{b, s\}$ indicates the household's "type" in period t . Here $u^b(c; \xi)$ and $u^s(c; \xi)$ are two different period utility functions, each of which may also be shifted by the vector of aggregate taste shocks ξ_t , and $v^b(h; \xi)$ and $v^s(h; \xi)$ are correspondingly two different functions indicating the period disutility

from working. As in the basic NK model, there is assumed to be a continuum of differentiated goods, each produced by a monopolistically competitive supplier; $c_t(i)$ is a Dixit-Stiglitz aggregator of the household's purchases of these differentiated goods. The household similarly supplies a continuum of different types of specialized labor, indexed by j , that are hired by firms in different sectors of the economy; the additively separable disutility of work $v^\tau(h; \xi)$ is the same for each type of labor, though it depends on the household's type and the common taste shock.

Each agent's type $\tau_t(i)$ evolves as an independent two-state Markov chain. Specifically, we assume that each period, with probability $1 - \delta$ (for some $0 \leq \delta < 1$) an event occurs which results in a new type for the household being drawn; otherwise it remains the same as in the previous period. When a new type is drawn, it is b with probability π_b and s with probability π_s , where $0 < \pi_b, \pi_s < 1, \pi_b + \pi_s = 1$. (Hence the population fractions of the two types are constant at all times, and equal to π_τ for each type τ .) We assume moreover that

$$u_c^b(c; \xi) > u_c^s(c; \xi)$$

for all levels of expenditure c in the range that occur in equilibrium. (See Figure 5, where these functions are graphed in the case of the calibration used in the numerical work reported here.) Hence a change in a household's type changes its relative impatience to consume, given the aggregate state ξ_t ; in addition, the current impatience to consume of all households is changed by the aggregate disturbance ξ_t . We also assume that the marginal utility of additional expenditure diminishes at different rates for the two types, as is also illustrated in the figure; type b households (who are borrowers in equilibrium) have a marginal utility that varies less with the current level of expenditure, resulting in a greater degree of intertemporal substitution of their expenditures in response to interest-rate changes. Finally, the two types are also assumed to differ in the marginal disutility of working a given number of hours; this difference is calibrated so that the two types choose to work the same number of hours in steady state, despite their differing marginal utilities of income. For simplicity, the elasticities of labor supply of the two types are not assumed to differ.

The coexistence of the two types with differing impatience to consume

creates a social function for financial intermediation. In the present model, as in the basic New Keynesian model, all output is consumed either by households or by the government; hence intermediation serves an allocative function only to the extent that there are reasons for the intertemporal marginal rates of substitution of households to differ in the absence of financial flows. The present model reduces to the standard representative-household model in the case that one assumes that $u^b(c; \xi) = u^s(c; \xi)$ and $v^b(h; \xi) = v^s(h; \xi)$.

We assume that most of the time, households are able to spend an amount different from their current income *only* by depositing funds with or borrowing from financial intermediaries, that the same nominal interest rate i_t^d is available to all savers, and that a (possibly) different nominal interest i_t^b is available to all borrowers,⁴ independent of the quantities that a given household chooses to save or to borrow. For simplicity, we also assume that only one-period riskless nominal contracts with the intermediary are possible for either savers or borrowers. The assumption that households cannot engage in financial contracting other than through the intermediary sector represents one of the key financial frictions. We also allow households to hold one-period riskless nominal government debt, but since government debt and deposits with intermediaries are perfect substitutes as investments, they must pay the same interest rate i_t^d in equilibrium, and the decision problem of the households is the same as if they have only a decision about how much to deposit with or borrow from the intermediaries.

Aggregation is simplified by assuming that households are able to sign state-contingent contracts with one another, through which they may insure one another against both aggregate risk and the idiosyncratic risk associated with a household's random draw of its type, but that households are *only intermittently* able to receive transfers from the insurance agency; between the infrequent occasions when a household has access to the insurance agency, it can only save or borrow through the financial intermediary sector mentioned in the previous paragraph. The assumption that households are *eventually* able to make transfers to one another in accordance with an insurance contract signed earlier means that they continue to have identical

⁴Here “savers” and “borrowers” identify households according to whether they choose to save or borrow, and not by their “type”.

expectations regarding their marginal utilities of income far enough in the future, regardless of their differing type histories.

It then turns out that in equilibrium, the marginal utility of a given household at any point in time depends *only* on its type $\tau_t(i)$ at that time; hence the entire distribution of marginal utilities of income at any time can be summarized by two state variables, λ_t^b and λ_t^s , indicating the marginal utilities of each of the two types. The expenditure level of type τ is similarly the same for all households of that type, and can be obtained by inverting the marginal-utility functions (graphed in Figure 5) to yield an expenditure demand function $c^\tau(\lambda; \xi_t)$ for each type. Aggregate demand Y_t for the Dixit-Stiglitz composite good can then be written as

$$Y_t = \pi_b c^b(\lambda_t^b; \xi_t) + \pi_s c^s(\lambda_t^s; \xi_t) + G_t + \Xi_t, \quad (1.2)$$

where G_t indicates the (exogenous) level of government purchases and Ξ_t indicates resources consumed by intermediaries (the sum of two components, $X i_t^p$ representing costs of the private intermediaries and Ξ_t^{cb} representing costs of central-bank activities, each discussed further below). Thus the effects of financial conditions on aggregate demand can be summarized by tracking the evolution of the two state variables λ_t^τ . The marginal-utility ratio $\Omega_t \equiv \lambda_t^b / \lambda_t^s \geq 1$ provides an important measure of the inefficiency of the allocation of expenditure owing to imperfect financial intermediation, since in the case of frictionless financial markets we would have $\Omega_t = 1$ at all times.

In the presence of heterogeneity, instead of a single Euler equation each period, relating the path of the marginal utility of income of the representative household to “the” interest rate, we instead have *two* Euler equations each period, one for each of the two types, and each involving a different interest rate — i_t^b in the case of the Euler equation for type b (who choose to borrow in equilibrium) and i_t^d in the case of the Euler equation for type s (who choose to save). If we log-linearize these Euler equations,⁵ and combine them with a log-linearized version of (1.2), we obtain a structural relation of

⁵Here and in the case of all other log-linearizations discussed below, we log-linearize around a deterministic steady state in which the inflation rate is zero and aggregate output is constant.

the form

$$\begin{aligned}\hat{Y}_t = & -\bar{\sigma}(\hat{i}_t^{avg} - E_t\pi_{t+1}) + E_t\hat{Y}_{t+1} - E_t\Delta g_{t+1} - E_t\Delta\hat{\Xi}_{t+1} \\ & -\bar{\sigma}s_\Omega\hat{\Omega}_t + \bar{\sigma}(s_\Omega + \psi_\Omega)E_t\hat{\Omega}_{t+1},\end{aligned}\tag{1.3}$$

generalizing the “intertemporal IS relation” of the basic New Keynesian model. Here $\hat{Y}_t \equiv \log(Y_t/\bar{Y})$ measures the percentage deviation of aggregate output from its steady-state level;

$$\hat{i}_t^{avg} \equiv \pi_b\hat{i}_t^b + \pi_s\hat{i}_t^d$$

is the average of the interest rates that are relevant (at the margin) for all of the savers and borrowers in the economy, where we define $\hat{i}_t^\tau \equiv \log(1 + i_t^\tau/1 + \bar{i}^\tau)$ for $\tau \in \{b, d\}$;⁶ g_t is a composite “autonomous expenditure” disturbance as in Woodford (2003, pp. 80, 249), taking account of exogenous fluctuations in G_t as well as exogenous variation in the spending opportunities facing the two types of households (reflected in the dependence of the functions $u^\tau(c; \xi_t)$ on the state vector ξ_t); $\hat{\Xi}_t \equiv (\Xi_t - \bar{\Xi})/\bar{Y}$ measures departures of the quantity of resources consumed by the intermediary sector from its steady-state level;⁷ and $\hat{\Omega}_t \equiv \log(\Omega_t/\bar{\Omega})$ measures the gap between the marginal utilities of the two types.

Note that the first four terms on the right-hand side of (1.3) are exactly as in the basic New Keynesian model, with the only differences that instead of “the” interest rate we have an average interest rate; $\bar{\sigma}$ is no longer the intertemporal elasticity of substitution for the representative household, but instead a weighted average of the corresponding parameters for the two types; and the composite disturbance g_t similarly averages the changes in spending opportunities for the two types. The crucial differences are the presence of the new terms involving $\hat{\Xi}_t$ and $\hat{\Omega}_t$, that only exist in the case of financial frictions. The sign of the coefficient s_Ω depends on the asymmetry of the degrees of interest-sensitivity of expenditure by the two types; in the case shown in

⁶One can show that, to a log-linear approximation, the average marginal utility of income in the population depends only on the expected path of this particular average of the interest rates in the economy.

⁷We adopt this notation so that $\hat{\Xi}_t$ is defined even when the model is parameterized so that $\bar{\Xi} = 0$.

Figure 5 (which we regard as the empirically relevant case), $s_\Omega > 0$ because the intertemporal elasticity of expenditure is higher for type b .⁸ In this case, a larger value of $\hat{\Omega}_t$ reduces aggregate demand, for given expectations about the forward path of average real interest rates; this can be thought of as representing “financial headwinds” of a kind sometimes discussed within the Federal Reserve system.

Log-linearization of the two Euler equations also implies that

$$\hat{\Omega}_t = \hat{\omega}_t + \hat{\delta} E_t \hat{\Omega}_{t+1}. \quad (1.4)$$

where $\omega_t \equiv \hat{i}_t^b - \hat{i}_t^d$ is the short-term *credit spread* and $\hat{\delta}$ is a coefficient satisfying $0 < \hat{\delta} < 1$. Thus the marginal-utility gap $\hat{\Omega}_t$ is a forward-looking moving average of the expected path of the short-term credit spread. Alternatively, we can view $\hat{\Omega}_t$ itself as a credit spread, a positive multiple of the spread between two long-term yields,

$$r_t^\tau \equiv (1 - \hat{\delta})^{-1} \sum_{j=0}^{\infty} \hat{\delta}^j E_t \hat{i}_{t+j}^\tau$$

for $\tau \in \{b, d\}$. Hence the terms in (1.3) involving $\hat{\Omega}_t$ indicate that variations in credit spreads are relevant to aggregate demand. Credit spreads are also relevant to the relation between the path of the policy rate⁹ and aggregate expenditure because of the identity

$$\hat{i}_t^{avg} = \hat{i}_t^d + \pi_b \hat{\omega}_t \quad (1.5)$$

connecting the policy rate to the interest rate that appears in (1.3).

Under an assumption of Calvo-style staggered price adjustment, we similarly obtain an aggregate supply relation that is only slightly different from the “New Keynesian Phillips curve” of the representative-household model. Specifically, we obtain

$$\pi_t = \kappa(\hat{Y}_t - \hat{Y}_t^n) + \beta E_t \pi_{t+1} + u_t + \kappa_\Omega \hat{\Omega}_t - \kappa_\Xi \hat{\Xi}_t, \quad (1.6)$$

⁸In our calibration, ψ_Ω is a small negative quantity, but because it is small its sign is not of great importance.

⁹The identification of \hat{i}_t^d with the policy rate is discussed below in section 1.3.

where \hat{Y}_t^n (the “natural rate of output”) is a composite exogenous disturbance that depends on technology, preferences, and government purchases; u_t (the “cost-push shock”) is another composite exogenous disturbance that depends on variations in distorting taxes and in the degree of market power in labor or product markets; and the coefficients satisfy $\kappa, \kappa_{\Xi} > 0$, and, in the case that we regard as realistic, $\kappa_{\Omega} > 0$ as well. Here the first three terms on the right-hand side are identical to those of the standard “New Keynesian Phillips curve,” subject to similar comments as above about the dependence of κ on a weighted average of the intertemporal elasticities of substitution of the two types and the dependence of \hat{Y}_t^n on a weighted average of the preference shocks of the two types, while the final two terms appear only as a result of credit frictions. We note in particular that increases in credit spreads shift *both* the aggregate-supply and aggregate-demand relations in our model.

In the presence of heterogeneity, household behavior results in one further structural equation, that has no analog in the representative-household model. This is a law of motion for b_t , the per capita level of private borrowing. This depends on the fluctuations in the levels of expenditure of the two types, and hence on the fluctuations in both marginal utilities λ_t^r . Details of this additional relationship are provided in Cúrdia and Woodford (2009a). We also suppose that the government issues one-period riskless nominal debt, the real value of which at the end of period t is denoted b_t^g . We treat $\{b_t^g\}$ as an exogenous process; this is one of three independent fiscal disturbances that we allow for.¹⁰ We suppose that government debt can be held either by saving households or by the central bank,¹¹ and in equilibrium we suppose that at least part of the public debt is always held by households. Since government debt is a perfect substitute for deposits with the intermediaries in our model, from the standpoint of saving households, in equilibrium the yield on government debt must always equal i_t^d , the competitive interest rate on deposits.

¹⁰The other two are exogenous variations in government purchases G_t of the composite good and exogenous variations in the proportional sales tax rate τ_t .

¹¹We could also allow intermediaries to hold government debt, but they will choose not to, as long as $i_t^b > i_t^d$, as is always true in the equilibria that we consider.

1.2 Financial Intermediaries

We assume an intermediary sector made up of identical, perfectly competitive firms. Intermediaries take deposits, on which they promise to pay a riskless nominal return i_t^d one period later, and make one-period loans on which they demand a nominal interest rate of i_t^b . An intermediary also chooses a quantity of reserves M_t to hold at the central bank, on which it will receive a nominal interest yield of i_t^m . Each intermediary takes as given all three of these interest rates. We assume that arbitrage by intermediaries need not eliminate the spread between i_t^b and i_t^d , for either of two reasons. On the one hand, resources are used in the process of loan origination; and on the other hand, intermediaries may be unable to tell the difference between good borrowers (who will repay their loans the next period) and bad borrowers (who will be able to disappear without having to repay), and as a consequence have to charge a higher interest rate to good and bad borrowers alike.

We suppose that origination of good loans in real quantity L_t requires an intermediary to also originate bad loans in quantity $\chi_t(L_t)$, where $\chi_t', \chi_t'' \geq 0$, and the function $\chi_t(L)$ may shift from period to period for exogenous reasons. (While the intermediary is assumed to be unable to discriminate between good and bad loans, it is able to predict the fraction of loans that will be bad in the case of any given scale of lending activity on its part.) This scale of operations also requires the intermediary to consume real resources $\Xi_t^p(L_t; m_t)$ in the period in which the loans are originated, where $m_t \equiv M_t/P_t$, and $\Xi_t^p(L; m)$ is a convex function of its two arguments, with $\Xi_{L_t}^p \geq 0$, $\Xi_{m_t}^p \leq 0$, $\Xi_{L_t m_t}^p \leq 0$. We further suppose that for any scale of operations L , there exists a finite *satiation level* of reserve balances $\bar{m}_t(L)$, defined as the lowest value of m for which $\Xi_{m_t}^p(L; m) = 0$. (Our convexity and sign assumptions then imply that $\Xi_{m_t}^p(L; m) = 0$ for all $m \geq \bar{m}_t(L)$.) We assume the existence of a finite satiation level of reserves in order for an equilibrium to be possible in which the policy rate is driven to zero, a situation of considerable practical relevance at present.

Given an intermediary's choice of its scale of lending operations L_t and reserve balances m_t to hold, we assume that it acquires real deposits d_t in the maximum quantity that it can repay (with interest at the competitive rate)

from the anticipated returns on its assets (taking into account the anticipated losses on bad loans). Thus it chooses d_t such that

$$(1 + i_t^d)d_t = (1 + i_t^b)L_t + (1 + i_t^m)m_t.$$

The deposits that it does not use to finance either loans or the acquisition of reserve balances,

$$d_t - m_t - L_t - \chi_t(L_t) - \Xi_t^p(L_t; m_t),$$

are distributed as earnings to its shareholders. The intermediary chooses L_t and m_t each period so as to maximize these earnings, given i_t^d, i_t^b, i_t^m . This implies that L_t and m_t must satisfy the first-order conditions

$$\Xi_{L_t}^p(L_t; m_t) + \chi_{L_t}(L_t) = \omega_t \equiv \frac{i_t^b - i_t^d}{1 + i_t^d}, \quad (1.7)$$

$$-\Xi_{m_t}^p(L_t; m_t) = \delta_t^m \equiv \frac{i_t^d - i_t^m}{1 + i_t^d}. \quad (1.8)$$

Equation (1.7) can be viewed as determining the equilibrium credit spread ω_t ¹² as a function $\omega_t(L_t; m_t)$ of the aggregate volume of private credit and the real supply of reserves. As indicated above, a positive credit spread exists in equilibrium to the extent that $\Xi_t^p(L; m)$, $\chi_t(L)$, or both are increasing in L . Equation (1.8) similarly indicates how the equilibrium differential δ_t^m between the interest paid on deposits and that paid on reserves at the central bank is determined by the same two aggregate quantities.

In addition to these two equilibrium conditions that determine the two interest-rate spreads in the model, the absolute level of (real) interest rates must be such as to equate the supply and demand for credit. Market-clearing in the credit market requires that

$$b_t = L_t + L_t^{cb}, \quad (1.9)$$

where L_t^{cb} represents real lending to the private sector by the central bank, as discussed next.

¹²Note that in terms of this definition of the credit spread, the previous defined deviation corresponds to $\hat{\omega}_t \equiv \log(1 + \omega_t/1 + \bar{\omega})$.

1.3 The Central Bank and Interest-Rate Determination

In our model, the central bank's liabilities consist of the reserves M_t (which also constitute the monetary base), on which it pays interest at the rate i_t^m . These liabilities in turn fund the central bank's holdings of government debt, and any lending by the central bank to type b households. We let L_t^{cb} denote the real quantity of lending by the central bank to the private sector; the central bank's holdings of government debt are then given by the residual $m_t - L_t^{cb}$. We can treat m_t (or M_t) and L_t^{cb} as the bank's choice variables, subject to the constraints

$$0 \leq L_t^{cb} \leq m_t. \quad (1.10)$$

It is also necessary that the central bank's choices of these two variables satisfy the bound

$$m_t < L_t^{cb} + b_t^g,$$

where b_t^g is the total outstanding real public debt, so that a positive quantity of public debt remains in the portfolios of households. In the calculations below, however, we shall assume that this last constraint is never binding. (We confirm this in our numerical examples.)

We assume that central-bank extension of credit other than through open-market purchases of Treasury securities consumes real resources, just as in the case of private intermediaries, and represent this resource cost by a function $\Xi^{cb}(L_t^{cb})$, that is increasing and at least weakly convex, with $\Xi^{cb}(0) > 0$, as is discussed further in section 4. The central bank has one further independent choice to make each period, which is the rate of interest i_t^m to pay on reserves. We assume that if the central bank lends to the private sector, it simply chooses the amount that it is willing to lend and auctions these funds, so that in equilibrium it charges the same interest rate i_t^b on its lending that private intermediaries do; this is therefore not an additional choice variable for the central bank. Similarly, the central bank receives the market-determined yield i_t^d on its holdings of government debt.

The interest rate i_t^d at which intermediaries are able to fund themselves is determined each period by the joint inequalities

$$m_t \geq m_t^d(L_t, \delta_t^m), \quad (1.11)$$

$$\delta_t^m \geq 0, \tag{1.12}$$

together with the “complementary slackness” condition that at least one of (1.11) and (1.12) must hold with equality each period; here $m_t^d(L, \delta^m)$ is the demand for reserves defined by (1.8), defined to equal the satiation level $\bar{m}_t(L)$ in the case that $\delta^m = 0$. (Condition (1.11) may hold only as an inequality, as intermediaries will be willing to hold reserves beyond the satiation level as long as the opportunity cost δ_t^m is zero.) We identify the rate i_t^d at which intermediaries fund themselves with the central bank’s *policy rate* (e.g., the federal funds rate, in the case of the US).

The central bank can influence the policy rate through two channels, its control of the supply of reserves and its control of the interest rate paid on them. By varying m_t , the central bank can change the equilibrium differential δ_t^m , determined as the solution to (1.11)–(1.12). And by varying i_t^m , it can change the level of the policy rate i_t^d that corresponds to a given differential. Through appropriate adjustment on both margins, the central bank can control i_t^d and i_t^m separately (subject to the constraint that i_t^m cannot exceed i_t^d). We also assume that for institutional reasons, it is not possible for the central bank to pay a negative interest rate on reserves. (We may suppose that intermediaries have the option of holding currency, earning zero interest, as a substitute for reserves, and that the second argument of the resource cost function $\Xi_t^p(b; m)$ is actually the sum of reserve balances at the central bank plus vault cash.) Hence the central bank’s choice of these variables is subject to the constraints

$$0 \leq i_t^m \leq i_t^d. \tag{1.13}$$

There are thus three independent dimensions along which central-bank policy can be varied in our model: variation in the quantity of reserves M_t that are supplied; variation in the interest rate i_t^m paid on those reserves; and variation in the breakdown of central-bank assets between government debt and lending L_t^{cb} to the private sector. Alternatively, we can specify the three independent dimensions as *interest-rate policy*, the central bank’s choice of an operating target for the policy rate i_t^d ; *reserve-supply policy*, the choice of M_t , which in turn implies a unique rate of interest that must be paid on reserves in order for the reserve-supply policy to be consistent with the

bank’s target for the policy rate;¹³ and *credit policy*, the central bank’s choice of the quantity of funds L_t^{cb} to lend to the private sector. We prefer this latter identification of the three dimensions of policy because in this case our first dimension (interest-rate policy) corresponds to the sole dimension of policy emphasized in many conventional analyses of optimal monetary policy, while the other two dimensions are additional dimensions of policy introduced by our extension of the basic New Keynesian model.¹⁴ Changes in central-bank policy along each of these dimensions has consequences for the bank’s cash flow, but we abstract from any constraint on the joint choice of the three variables associated with cash-flow concerns. (We assume that seignorage revenues are simply turned over to the Treasury, where their only effect is to change the size of lump-sum transfers to the households.)

Given that central-bank policy can be independently varied along each of these three dimensions, we can independently discuss the criteria for policy to be optimal along each dimension. Below, we take up each of the three dimensions in turn.

1.4 The Welfare Objective

In considering optimal policy, we take the objective of policy to be the maximization of average expected utility. Thus we can express the objective as

¹³We might choose to call the second dimension variation in the interest rate paid on reserves, which would correspond to something that the Board of Governors makes an explicit decision about under current US institutional arrangements, as is also true at most other central banks. But description of the second dimension of policy as “reserve-supply policy” allows us to address the question of the value of “quantitative easing” under this heading as well.

¹⁴Goodfriend (2009) similarly describes central-bank policy as involving three independent dimensions, corresponding to our first three dimensions, and calls the first of those dimensions (the quantity of reserves, or base money) “monetary policy.” We believe that this does not correspond to standard usage of the term “monetary policy,” since the traditional focus of FOMC deliberations about monetary policy has been the choice of an operating target for the policy rate, as is generally the case at other central banks as well. Reis (2009) also distinguishes among the three dimensions of policy in terms similar to ours.

maximization of

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} U_t \quad (1.14)$$

where the welfare contribution U_t each period weights the period utility of each of the two types by their respective population fractions at each point in time. As shown in Cúrdia and Woodford (2009a),¹⁵ this can be written as

$$U_t = U(Y_t, \Omega_t, \Xi_t, \Delta_t; \xi_t). \quad (1.15)$$

Here Δ_t is an index of price dispersion in period t , taking its minimum possible value of 1 when the prices of all goods are identical; for any given total quantity Y_t of the composite good that must be produced, the total disutility of working indicated in (1.1) is greater the more dispersed are prices, as this implies a correspondingly less uniform (and hence less efficient) composition of output.

The total disutility of working is also a decreasing function of Ω_t , since a larger gap between the marginal utilities of the two types implies a less efficient division of labor effort between the two types. The average utility from consumption is smaller, for given aggregate output Y_t , the larger is Ξ_t , since only resources $Y_t - G_t - \Xi_t$ are consumed by households. And the average utility from consumption is also decreasing in Ω_t , since a larger marginal-utility gap implies a less efficient division of expenditure between the two types. Thus the derived utility $U(\cdot)$ is a concave function of Y_t that reaches an interior maximum for given values of the other arguments, and a monotonically decreasing function of Ω_t, Ξ_t , and Δ_t . The dependence of $U(\cdot)$ on Y_t and Δ_t is the same as in the representative-household model of Benigno and Woodford (2005), while the dependence on Ω_t and Ξ_t indicates new distortions resulting from the credit frictions in our model.

¹⁵Cúrdia and Woodford (2009a) analyze a special case of the present model, in which central-bank lending and the role of central-bank liabilities in reducing the transactions costs of intermediaries are abstracted from. However, the form of the welfare measure (1.15) depends only on the nature of the heterogeneity in our model, and the assumed existence of a credit spread and of resources consumed by the intermediary sector; the functions that determine how Ω_t and Ξ_t are endogenously determined are irrelevant for this calculation, and those are the only parts of the model that are generalized in this paper. Hence the form of the welfare objective in terms of these variables remains the same.

As in Benigno and Woodford, the assumption of Calvo-style price adjustment implies that the index of price dispersion evolves according to a law of motion of the form

$$\Delta_t = h(\Delta_{t-1}, \pi_t),$$

where for a given value of Δ_{t-1} , $h(\Delta_{t-1}, \cdot)$ has an interior minimum at an inflation rate that is near zero when Δ_{t-1} is near 1. Thus for given paths of the variables $\{\Omega_t, \Xi_t\}$, welfare is maximized by trying (to the extent possible) to simultaneously keep aggregate output near the (time-varying) level that maximizes U and inflation near the (always low) level that minimizes price dispersion. Hence our model continues to justify concerns with output and inflation stabilization of a kind that are familiar from the New Keynesian literature. However, it *also* implies that welfare can be increased by reducing credit spreads and the real resources consumed in financial intermediation. These latter concerns make the effects of policy on the evolution of aggregate credit and on the supply of bank reserves also relevant to monetary policy deliberations. We now turn to the question of how each of the three dimensions of central-bank policy can effect these several objectives.

2 Optimal Policy: The Supply of Reserves

We shall first consider optimal policy with regard to the supply of reserves, taking as given (for now) the way in which the central bank chooses its operating target for the policy rate i_t^d , and the state-contingent level of central-bank lending L_t^{cb} to the private sector. Under fairly weak assumptions, we obtain a very simple result: *optimal policy requires that intermediaries be satiated in reserves, i.e., that $M_t/P_t \geq m_t(\bar{L}_t)$ at all times.*

For levels of reserves below the satiation point, an increase in the supply of reserves has two effects that are relevant for welfare: on the one hand, the resource cost of financial intermediation Ξ_t^p is reduced (for a given level of lending by the intermediary sector); and on the other hand, the credit spread ω_t is reduced (again, for a given level of lending) as a consequence of (1.7). Each of these effects raises the value of the objective (1.14); note that reductions in credit spreads increase welfare because of their effect on

the path of the marginal-utility gap Ω_t , as a consequence of (1.4). Hence an increase in the supply of reserves is unambiguously desirable, in any period in which they remain below the satiation level.¹⁶ Once reserves are at or above the satiation level, however, further increases reduce neither the resource costs of intermediaries nor equilibrium credit spreads (as in this case $\Xi_{mt}^p = \Xi_{Lmt}^p = 0$), so that there would be no further improvement in welfare. Hence policy is optimal along this dimension if and only if $M_t/P_t \geq m_t(\bar{L}_t)$ at all times,¹⁷ so that

$$\Xi_{mt}^p(L_t; m_t) = 0. \quad (2.1)$$

This is just another example in which the familiar “Friedman Rule” for “the optimum quantity of money” (Friedman, 1969) applies. Note, however, that our result has no consequences for *interest-rate policy*. While the Friedman rule is sometimes taken to imply a strong result about the optimal control of short-term nominal interest rates — namely, that the nominal interest rate should equal zero at all times — the efficiency condition (2.1), together with the equilibrium relation (1.8), implies only that the interest-rate *differential* δ_t^m should equal zero at all times. With zero interest on reserves, this would also require that $i_t^d = 0$ at all times; but given that the central bank is free to set any level of interest on reserves consistent with (1.13), the efficiency condition (2.1) actually implies *no* restriction upon either the average level of the degree of state-contingency of the central bank’s target for the policy rate i_t^d .

2.1 Is a Reserve Supply Target Needed?

Our result about the importance of ensuring an adequate supply of reserves might suggest that the question of the correct target level of reserves at each point in time should receive the same degree of attention at meetings of the FOMC as the question of the correct operating target for the federal

¹⁶The discussion here assumes that the upper bound in (1.10) is not a binding constraint. But if that constraint does bind, then an increase in the supply of reserves relaxes the constraint, and this too increases welfare, so that the conclusion in the text is unchanged.

¹⁷To be more precise, policy is optimal if and only if (2.1) is satisfied *and* the upper bound in (1.10) does not bind. Both conditions will be satisfied by any quantity of reserves above some finite level.

funds rate. But deliberations of that kind are not needed in order to ensure fulfillment of the optimality criterion (2.1). For the efficiency condition can alternatively be stated (using (1.8)) as requiring that $i_t^d = i_t^m$ at all times. Reserves should be supplied to the point at which the policy rate falls to the level of the interest rate paid on reserves, or, in a formulation that is more to the point, *interest should be paid on reserves at the central bank's target for the policy rate.*

Given a rule for setting an operating target for i_t^d (discussed in the next section), i_t^m should be chosen each period in accordance with the simple rule

$$i_t^m = i_t^d. \quad (2.2)$$

When the central bank acts to implement its target for the policy rate through open-market operations, it will automatically have to adjust the supply of reserves so as to satisfy (2.1). But this does not require a central bank's monetary policy committee (the FOMC in the case of the US) to deliberate about an appropriate target for reserves at each meeting; once the target for the policy rate is chosen (and the interest rate to be paid on reserves is determined by that, through condition (2.2), the quantity of reserves that must be supplied to implement the target can be determined by the bank staff in charge of carrying out the necessary interventions (the trading desk at the New York Fed, in the case of the US), on the basis of a more frequent monitoring of market conditions than is possible on the part of the monetary policy committee.

One obvious way to ensure that the efficiency condition (2.2) is satisfied is to adopt a routine practice of automatically paying interest on reserves at a rate that is tied to the current operating target for the policy rate. This is already the practice of many central banks outside the US. At some of those banks, the fixed spread between the target for the policy rate and the rate paid on overnight balances at the central bank is quite small (for example, 25 basis points in the case of the Bank of Canada); in the case of New Zealand, the interest rate paid on overnight balances is the policy rate itself. There are possible arguments (relating to considerations not reflected in our simple model) why the optimal spread might be larger than zero, but it is likely in any event to be desirable to maintain a constant small spread,

rather than treating the question of the interest rate to be paid on reserves as a separate, discretionary policy decision to be made at each meeting of the policy committee. Apart from the efficiency gains modeled here, such a system should also help to facilitate the central bank’s control of the policy rate (Goodfriend, 2002; Woodford, 2003, chap. 1, sec. 3).

2.2 Is there a Role for “Quantitative Easing”?

While our analysis implies that it is desirable to ensure that the supply of reserves never falls below a certain lower bound $\bar{m}_t(L_t)$, it also implies that there is no benefit from supplying reserves *beyond* that level. There is, however, one important exception to this assertion: it can be desirable to supply reserves beyond the satiation level if this is necessary in order to make the optimal quantity of central bank lending to the private sector L_t^{cb} consistent with (1.10). This qualification is important in thinking about the desirability of the massive expansion in the supply of reserves by the Fed since September 2008, as shown in Figure 2. As can be seen from Figure 4, the increase in reserves occurred only once the Fed decided to expand the various newly-created liquidity and credit facilities beyond the scale that could be financed simply by reducing its holdings of Treasury securities (as had been its policy over the previous year).¹⁸

Some have argued, instead, that further expansion of the supply of reserves beyond the level needed to bring the policy rate down to the level of interest paid on reserves is an important additional tool of policy in its own right — one of particular value precisely when a central bank is no longer able to further reduce its operating target for the policy rate, owing to the zero lower bound (as at present in the US and many other countries). It is sometimes proposed that when the zero lower bound is reached, it is desirable for a central bank’s policy committee to shift from deliberations about an interest-rate target to a target for the supply of bank reserves, as under the Bank of Japan’s policy of “quantitative easing” during the period between

¹⁸Bernanke (2009) distinguishes between the Federal Reserve policy of “credit easing” and the type of “quantitative easing” practiced by the Bank of Japan earlier in the decade, essentially on this ground.

March 2001 and March 2006.

Our model provides no support for the view that such a policy should be effective in stimulating aggregate demand. Indeed, it is possible to state an *irrelevance proposition* for quantitative easing in the context of our model. Let the three dimensions of central-bank policy be described by functions that specify the operating target for the policy rate, the supply of reserves, the interest rate to be paid on reserves, and the quantity of central-bank credit as functions of macroeconomic conditions.

For the sake of concreteness, we may suppose that each of these variables is to be determined by a Taylor-type rule,

$$\begin{aligned} i_t^d &= \phi^{id}(\pi_t, Y_t, L_t; \xi_t), \\ M_t/P_t &= \phi^m(\pi_t, Y_t, L_t; \xi_t), \\ i_t^m &= \phi^{im}(\pi_t, Y_t, L_t; \xi_t), \\ L_t^{cb} &= \phi^L(\pi_t, Y_t, L_t; \xi_t), \end{aligned}$$

where the functions are such that constraints (1.10)–(1.13) are satisfied for all values of the arguments. (Here the vector of exogenous disturbances ξ_t upon which the reaction functions may depend includes the exogenous factors that shift the function $\Xi_t^p(L; m)$.) Then our result is that given the three functions $\phi^{id}(\cdot)$, $\phi^{im}(\cdot)$, and $\phi^L(\cdot)$, the set of processes $\{\pi_t, Y_t, L_t, b_t, i_t^d, i_t^b, \Omega_t, \Delta_t\}$ that constitute possible rational expectations equilibria is *the same*, independently of the choice of the function $\phi^m(\cdot)$, as long as the specification of $\phi^m(\cdot)$ is consistent with the other three functions (in the sense that (1.10) and (1.11) are necessarily satisfied, and that (1.11) holds with equality in all cases where (1.12) is a strict inequality).¹⁹

Of course, the stipulation that $\phi^m(\cdot)$ be consistent with the other functions uniquely determines what the function must be for all values of the arguments for which the functions $i^d(\cdot)$ and $i^m(\cdot)$ imply that $\delta_t^m > 0$. However the class of policies considered allows for an arbitrary degree of expansion of reserves beyond the satiation level in the region where those functions imply that $\delta_t^m = 0$, and in particular, for an arbitrary degree of quantitative

¹⁹This result generalizes the irrelevance result for quantitative easing in Eggertsson and Woodford (2003) to a model with heterogeneity and credit frictions.

easing when the zero bound is reached (*i.e.*, when $i_t^d = i_t^m = 0$). The class of policies considered includes the popular proposal under which the quantity of excess reserves should depend on the degree to which a standard Taylor rule (unconstrained by the zero bound) would call for a negative policy rate. Our result implies that there should be no benefits from such policies.

Our result might seem to be contradicted by the analysis of Auerbach and Obstfeld (2004), in which an open market operation that expands the money supply is found to stimulate real activity even when the economy is at the zero bound at the time of the monetary expansion. But their thought experiment does not correspond to pure quantitative easing of the kind contemplated in the above proposition, because they specify monetary policy in terms of a path for the money supply, and the policy change that they consider is one that *permanently* increases the money supply, so that it remains higher after the economy has exited from the “liquidity trap” in which the zero bound is temporarily binding. The contemplated policy change is therefore *not* consistent with an unchanged reaction function $\phi^{id}(\cdot)$ for the policy rate, and the effects of the intervention can be understood to be the consequences of the commitment to a different future interest-rate policy.

Our result only implies that quantitative easing should be irrelevant under two conditions: that the increase in reserves finances an increase in central-bank holdings of Treasury securities, rather than an increase in central-bank lending to the private sector; and that the policy implies no change in the way that people should expect future interest-rate policy to be conducted. Our model *does* allow for real effects of an increase in central-bank lending L_t^{cb} financed by an increase in the supply of reserves, in the case that private-sector financial intermediation is inefficient;²⁰ but the real effects of the increased central-bank lending in that case are the same whether the lending is financed by an increase in the supply of reserves or by a reduction in central-bank holdings of Treasury securities. Our model also allows for real effects of an announcement that interest-rate policy in the future will

²⁰This result differs from that obtained in Eggertsson and Woodford (2003), where changes in the composition of the assets on the central bank’s balance sheet are *also* shown to be irrelevant. That stronger result depends on the assumption of a representative household as in the earlier paper, or alternatively, frictionless financial intermediation.

be different, as in the case where a central bank commits itself not to return immediately to its usual Taylor rule as soon as the zero bound ceases to bind, but promises instead to maintain policy accommodation for some time after it would become possible to comply with the Taylor rule (as discussed in the next section). But such a promise (if credible and correctly understood by the private sector) should increase output and prevent deflation to the same extent even if it implies no change in policy during the period when the zero lower bound binds.

While our definition of quantitative easing may seem a narrow one, the policy of the Bank of Japan during the period 2001-2006 fits our definition fairly closely. The BOJ's policy involved the adoption of a series of progressively higher quantitative targets for the supply of reserves, and the aim of the policy was understood to be to increase the monetary base, rather than to allow the BOJ to acquire any particular type of assets. The assets purchased were almost entirely Japanese government bonds, since credit allocation to malfunctioning markets was not a goal. There was no suggestion that the targets of policy after the end of the zero-interest-rate period would be any different than before; there was no commitment to maintain the increased quantity of base money in circulation permanently, and indeed, once it was judged time to end the zero-interest-rate policy, the supply of reserves was rapidly contracted again (Figure 6).

Our theory suggests that expansion of the supply of reserves under such circumstances should have little effect on aggregate demand, and this seems to have been the case. For example, as is also shown in Figure 6, despite an increase in the monetary base of 60 percent during the first two years of the quantitative easing policy, and an eventual increase of nearly 75 percent, nominal GDP never increased at all (relative to its March 2001 level) during the entire five years of the policy.²¹

²¹As indicated in Figure 6, over the first two years of the quantitative easing policy, nominal GDP fell by more than 4 percent, despite extremely rapid growth of base money. While nominal GDP recovered thereafter, it remained below its 2001:Q1 level over the entire period until 2006:Q4, three quarters after the official end of quantitative easing, by which time the monetary base had been reduced again by more than 20 percent. Moreover, even if the growth of nominal GDP after 2003:Q1 is regarded as a delayed effect of the growth in the monetary base two years earlier, this delayed nominal GDP growth was

3 Optimal Policy: Interest-Rate Policy

We turn now to a second dimension of policy, the approach taken by the central bank in determining its operating target for the policy rate (the federal funds rate, in the case of the Federal Reserve). In this section, we take for granted that reserve-supply policy is being conducted in the way recommended in the previous section, *i.e.*, that the rate of interest on reserves will satisfy (2.2) at all times. In this case, we can replace the function $\Xi_t^p(L_t; m_t)$ by

$$\bar{\Xi}_t^p(L_t) \equiv \Xi_t^p(L_t; \bar{m}_t(L_t))$$

and the function $\omega_t(L_t; m_t)$, defined by the left-hand side of (1.7), by

$$\bar{\omega}_t(L_t) \equiv \omega_t(L_t; \bar{m}_t(L_t)),$$

since there will be satiation in reserves at all times.²² Using these functions to specify the equilibrium evolution of Ξ_t^p and ω_t as functions of the evolution of aggregate private credit, we can then write the equilibrium conditions of the model without any reference to the quantity of reserves or to the interest rate paid on reserves. We shall also take as given the state-contingent evolution of central-bank lending $\{L_t^{cb}\}$, and ask how the central bank's target for the policy rate should be adjusted in response to shocks to the economy. In this case the problem considered is of the form considered in Cúrdia and Woodford (2009a).

As in a representative-household model with no financial frictions, a consideration of optimal interest-rate policy requires taking into account the desired evolution of aggregate output and of inflation (which affects the objective (1.14) because of the consequences of inflation for the evolution of the price dispersion index Δ_t), given the tradeoff between variations in these two variables implied by the aggregate-supply relation. While our model implies that in presence of credit frictions, interest-rate policy also has consequences for the evolution of Ξ_t and Ω_t (which are also arguments of (1.15)), owing to its effects on the volume of lending by the intermediary sector, the most important effects are the effects on the paths of output and inflation. The

quite modest relative to the size of the expansion in base money.

²²Even if at some times m_t exceeds $\bar{m}_t(L_t)$, this will not affect the values of Ξ_t^p or ω_t .

way in which the paths of output and inflation matter for welfare are essentially the same as in a model without financial frictions, and the nature of the aggregate-supply tradeoff indicated by (1.6) remains the same as well, with credit frictions appearing mainly as a source of additional shift terms (like the “cost-push shocks” emphasized in treatments like that of Clarida *et al.*, 1999). Hence some of the important conclusions of the standard literature continue to apply, at least approximately, even in an environment where credit frictions are non-trivial and time-varying.

3.1 The Robustness of Flexible Inflation (or Price-Level) Targeting

Benigno and Woodford (2005) show that in the representative-household version of our model, optimal interest-rate policy can be characterized by the requirement that interest rates be adjusted so that a certain *target criterion* is satisfied each period.²³ To a log-linear approximation, the optimal target criterion can be expressed as

$$\pi_t + \phi(x_t - x_{t-1}) = 0, \tag{3.1}$$

regardless of the degree of steady-state distortions due to market power or distorting taxes, where $x_t \equiv \hat{Y}_t - \hat{Y}_t^*$, \hat{Y}_t^* is a function of the exogenous disturbances to preferences, technology, fiscal policy, and markups,²⁴ and ϕ is a positive coefficient. This can be viewed as a form of “flexible inflation targeting” in the sense of Svensson (1997): the acceptable near-term inflation projection should be adjusted by an amount proportional to the projected change in the output gap. (Farther in the future, there will never be continuing forecastable changes in the output gap; so the criterion will always

²³For further discussion of targeting regimes as an approach to the conduct of monetary policy, see Svensson (1997, 2005), Svensson and Woodford (2005), or Woodford (2007).

²⁴In the case that the steady-state level of output under flexible prices (or with zero inflation) is efficient, \hat{Y}_t^* corresponds to variations in the efficient level of output. When the steady-state level of output under flexible prices is not efficient, the two concepts differ somewhat; for the more general definition of \hat{Y}_t^* , and discussion of its relation to the efficient level of output and to the flexible-price equilibrium level of output, see Woodford (2009, sec. 2).

require that the projected path of inflation a few years in the future will equal an unchanging long-run target value, here equal to zero.)

The optimal target criterion in the representative-household model can alternatively be expressed in the form

$$\tilde{p}_t \equiv p_t + \phi x_t = p^*, \quad (3.2)$$

where p_t is the log of the general price index at time t . (Note that (3.1) simply states that the first difference of \tilde{p}_t should be zero each period, so that \tilde{p}_t must never be allowed to change.) This is an output-gap-adjusted price-level target, or the commitment to a rule of the form (3.2) is an example of what Hall (1984) calls an “elastic price standard.” In the case that the target criterion can be fulfilled precisely each period, the two target criteria are equivalent; but if it is sometimes not possible for the central bank to satisfy the target criterion (as in the case of the zero lower bound, discussed below), the two commitments are no longer equivalent. In the latter case, there are actually advantages to the price-level formulation, as we discuss below.

Cúrdia and Woodford (2009a) show that in a special limiting case, the target criterion (3.1) — or alternatively, (3.2) — continues to be necessary and sufficient for the optimality of interest-rate policy, even in the model with heterogeneity and credit frictions. This is the special case in which steady-state distortions (including the steady-state credit spread $\bar{\omega}$) are negligible, though we allow for shocks that temporarily increase credit spreads relative to the steady-state level; real resources used in financial intermediation are negligible (so that the shocks that increase credit spreads are purely due to an increase in the perceived fraction of bad loans); and the time-varying fraction loans that are bad is independent of intermediaries’ scale of operations. In this case, there are no variations in Ξ_t and the fluctuations in Ω_t are essentially exogenous, so that the welfare-relevant effects of interest-rate policy relate only to its effects on output and inflation, as in a model without credit frictions; and the additional terms in the aggregate-supply tradeoff (1.6) are purely exogenous disturbance terms, so that the derivation of the optimal target criterion proceeds as in the representative-household model.

More generally, the target criterion (3.1) will not correspond precisely to

optimal policy; but our numerical investigations of calibrated models suggest that it can easily continue to provide a reasonably good approximation to the optimal Ramsey policy, making the prescription of “flexible inflation targeting” still a useful practical rule of thumb. Figures 7-10 illustrate this, for one illustrative calibration of our model, now allowing both Ξ_t^p and ω_t to vary endogenously with the volume of private lending.²⁵ Each figure plots the impulse responses (under a log-linear approximation to the model dynamics) of several of the endogenous variables to a particular type of exogenous disturbance, under each of four different possible specifications of monetary policy. The four monetary policies considered are a simple “Taylor rule,” using the coefficients proposed by Taylor (1993); a “strict inflation targeting” regime (the dashed line labeled “PiStab” in each figure), under which interest-rate policy is used to ensure that inflation never deviates from its target level (zero) in response to any disturbance; a “flexible inflation targeting” regime (the line labeled “Flex Target”), under which interest-rate policy ensures that (3.1) holds each period; and fully optimal policy (the solution to the Ramsey policy problem).

In each of the cases shown (as well as for a large number of other types of disturbances that we have considered), the “flexible inflation targeting” regime remains a good approximation to the fully optimal policy, even if it is no longer precisely the optimal policy. Both types of inflation targeting

²⁵The calibration is discussed further in Cúrdia and Woodford (2009a). The model parameters that are shared with the representative-household version of the model are calibrated as in Woodford (2003, chap. 6), on the basis of the empirical estimates of Rotemberg and Woodford (1997). The degree of heterogeneity of the consumption preferences of the two types is as shown in Figure 5, while the disutility of labor is the same for the two types, except for a multiplicative factor chosen so that in steady state the two types work the same amount. The steady-state credit spread is calibrated to equal 2 percent per annum, as in Mehra *et al.* (2008), and is attributed entirely to the marginal resource cost of private financial intermediation, so as to make the endogeneity of Ξ_t as great as possible given the average size of the spread. A highly convex function $\bar{\Xi}^p(L)$ is also assumed in the numerical results presented here, to make the endogeneity of the credit spread as great as possible. If we assume a less convex function for $\bar{\Xi}^p(L)$ or that a smaller fraction of the steady-state credit spread is due to real resource costs, then the special case in which (3.1) is optimal is an even better approximation than in the case shown in the figures.

regimes are closer to the optimal policy than is the Taylor rule, which mechanically responds to observed variations in real activity without taking account of the types of disturbances that are responsible for those variations.²⁶ (The Taylor rule tightens policy too much in response to the increases in output resulting from productivity growth or increased government purchases, while it does not tighten enough in the case of the wage-markup shock, which causes output to fall even as inflation increases.) But especially in the case of the wage-markup shock and the shock to government purchases, the flexible inflation targeting regime provides a better approximation to optimal policy than would a strict inflation target.

The target criterion (3.1) continues to provide a good approximation to optimal policy in the case of a “purely financial” disturbance as well, even though such disturbances are not allowed for in the analysis of Benigno and Woodford (2005). Figure 10 shows the impulse responses to an exogenous increase in the function $\chi_t(L)$, corresponding to an increase in the fraction of loans that are expected to be bad loans, that then gradually shifts back to its steady-state value. Such a shock temporarily shifts up the value of $\omega_t(L)$ for any value of L , and so represents a contraction of loan supply for reasons internal to the financial sector. In equilibrium, such a disturbance results both in a contraction of private lending (and hence in equilibrium borrowing b_t , as shown in the bottom left panel) and in an increase in the equilibrium credit spread ω_t (as shown in the second panel on the right). If the central bank follows the Taylor rule, such a shock results in both an output contraction and deflation, but an optimal policy would allow little of either to occur.²⁷ And again the flexible inflation targeting regime pro-

²⁶Here we assume a rule in which the intercept term representing the equilibrium real funds rate is a constant, and the output gap is defined as output relative to a deterministic trend, as in Taylor (1993). A more sophisticated variant, in which the intercept varies with variations in the “natural rate of interest,” and the output gap is defined relative to variations in the “natural rate of output” (defined as in equation (1.6)), provides a better approximation to optimal policy, but still less close an approximation than that provided by the flexible inflation targeting rule. The responses to exogenous disturbances under the more sophisticated form of Taylor rule are discussed in Cúrdia and Woodford (2009b).

²⁷Interestingly, the optimal policy does not involve a much larger cut in the policy rate than occurs under the Taylor rule. The difference is that under the Taylor rule, the central

vides a reasonable approximation to what would happen under an optimal policy commitment. (We obtain a very similar figure in the case that the disturbance is instead an exogenous increase in the marginal resource cost of private financial intermediation.)

These results provide an answer to one of the questions posed in the introduction: does keeping track of the projected paths of inflation and output alone provide a sufficient basis for judgments about whether monetary policy (by which interest-rate policy is here intended) remains on track, even during times of financial turmoil. Our results suggest that, while the target criterion (3.1) involving only the projected paths of inflation and the output gap is not complex enough to constitute a fully optimal policy in our extended model, ensuring that (3.1) holds at all times would in fact ensure that policy is not too different from a fully optimal policy commitment — not only in an environment in which financial intermediation is imperfect, but even when the main disturbances to the economy originate in the financial sector and imply large increases in the size of credit spreads.

It is important to note, however, that our results do *not* imply that there is no need for a central bank to monitor or respond to financial conditions. Under the targeting regime recommended here, it is necessary to keep track of the various exogenous disturbances affecting the economy, in order to correctly forecast the evolution of inflation and output under alternative paths for the policy rate — and this includes keeping track of financial disturbances, when these are important. The simple Taylor rule, which does not require the central bank to use *information* about any variables other than inflation and real GDP, would *not* be an adequate guide to policy.

3.2 A Spread-Adjusted Taylor Rule?

Might a Taylor rule instead be a sufficient basis for setting interest-rate policy, if the standard Taylor rule is augmented, as proposed by Taylor (2008), by an adjustment for observed variations in a credit spread such as one of the

bank is unwilling to cut the policy *except* to the extent that this can be justified by a fall in inflation or output, and so in equilibrium those must occur; under the optimal policy, the central bank is willing to cut the policy rate without requiring inflation or output to decline, and in equilibrium they do not.

LIBOR-OIS spreads shown in Figure 1? In the case of the kind of disturbance considered in Figure 10, this type of adjustment would allow the policy rate to be cut by more than a full percentage point even in the absence of any decline in inflation or output — which is exactly what is necessary to allow the kind of equilibrium responses associated with the optimal policy commitment.

Cúrdia and Woodford (2009b) consider modified Taylor rules of this kind in the context of the same calibrated structural model used in Figures 7-10. While they find that the type of spread adjustment proposed by Taylor would be beneficial under some circumstances — such as the type of disturbance considered in Figure 10 — the desirable degree of adjustment (and even sometimes the sign of the adjustment) of the policy rate in response to a change in credit spreads is not independent of the nature of the disturbance that causes spreads to change. Even in the case of “purely financial” disturbances, like the kind considered in Figure 10, the optimal degree of response to changes in the credit spread depends on the degree of anticipated persistence of the disturbance.

In fact, the targeting regime that we propose above automatically involves a spread adjustment of the general type proposed by Taylor. Given that a change in the credit spread (and in the anticipated future path of credit spreads, which determines the marginal-utility gap Ω_t , because of (1.4)) affects aggregate demand, for any given anticipated path of the policy rate — both because of the difference between i_t^{avg} and the policy rate, indicated in (1.5), and because of the $\hat{\Omega}_t$ terms in (1.3) — the consequences of a given path of the policy rate for the inflation and output projections will be different when the path of credit spreads changes, and so the path for the policy rate required in order to produce projections that conform to the target criterion will be different. Since larger credit spreads (now and in the future) reduce aggregate demand leading to lower inflation, the policy rate will generally need to be reduced in order to offset this effect.

Moreover, we believe that the targeting approach represents a conceptually superior way of introducing these considerations into decisions about interest-rate policy. The Taylor proposal requires that one specify which particular measure of credit spreads will be taken into account in the modified

reaction function. (Taylor has proposed one very specific spread, the LIBOR-OIS spread.) But in fact central banks monitor many different credit spreads; and while in our highly stylized model there is only a single credit spread, a more empirically realistic model would have to include several (as indeed the FRB-US model already does). Under our proposed targeting regime, *each* of these would be relevant to setting interest-rate policy: variations in each of the different spreads would be taken into account to the extent that they enter the equations of the model used to project the paths of inflation and output. Furthermore, under the targeting approach, the adjustment of the policy rate would not have to be a mechanical (and purely contemporaneous) function of the change in the credit spread; instead, one would automatically respond differently depending on the nature of the disturbance, and to changes in the expected future path of spreads as well as to the current spread.

3.3 Policy When the Zero Lower Bound is Reached

In the discussion above (and in the simulations presented in Figures 7-10), it is assumed that the zero lower bound on the policy rate is never reached, and our theoretical model implies that it should not be, in the case of *small enough* shocks. But it is theoretically possible for it to bind in the case of large enough shocks of certain types, and recent events in the US and elsewhere have shown that one cannot presume that the constraint will never bind in practice. (As a practical matter, it seems that it is most likely to bind following severe disruptions of the financial sector, as in the case of the Great Depression, Japan during the 1990s, and at present.)

When the zero bound is a binding constraint, it may not be possible for the central bank to use interest-rate policy to ensure fulfillment of the target criterion (3.1) in all periods. Does this affect the validity of our recommendation of this policy? The fact that it may not be possible to fulfill the target criterion at all times does not in itself imply that it is not desirable to adjust interest-rate policy so as to fulfill the criterion when it *can* be satisfied — or that the question whether there exists an interest-rate path that would satisfy the target criterion is not at all times the question that one should wish to ask when deliberating about interest-rate policy.

But the fact that the lower bound is sometimes a binding constraint also has consequences for the appropriate policy target even under certain circumstances when the zero lower bound would *not* prevent one from achieving the target criterion (3.1). The reason is that the severity of the distortions during the period when the lower bound is binding should depend on the way in which policy is expected to be conducted *after* the constraint ceases to bind. Hence the policy that a central bank should commit to follow in such a period should be chosen with a view to the consequences of the anticipation of that policy during the period when the zero bound binds.

Eggertsson and Woodford (2003) analyze this issue in a model that is equivalent to a special case of the model considered here. Let us again consider the special case, mentioned in section 3.1, in which there are no steady-state distortions, no resources are used in financial intermediation, and the fraction of loans that are bad is independent of the scale of lending. Because in this case both the credit spread and the marginal-utility gap evolve exogenously, a second-order Taylor series approximation to the objective function (1.14), expanding around the optimal (zero-inflation) steady state, is exactly the same quadratic function of inflation and the output gap as in the case of a representative-household model (the case considered by Eggertsson and Woodford). The “intertemporal IS relation” (1.3) and the aggregate-supply relation (1.6) are also identical to those of the representative-household model, except for the presence of additional additive disturbance terms involving $\hat{\omega}_t$ and $\hat{\Omega}_t$.

The optimal policy problem — which can be stated as the choice of processes for inflation, output, and the policy rate consistent with (1.3), (1.6) and (1.13) each period, so as to maximize the welfare measure written in terms of output and inflation — is then of the same form as the one analyzed by Eggertsson and Woodford (2003), except with additional possible interpretations of the exogenous disturbance terms. In particular, the extension of the model to incorporate credit frictions provides a more empirically realistic interpretation of the disturbance hypothesized by Eggertsson and Woodford, that makes the real policy rate needed to maintain a constant zero output gap temporarily negative; rather than having to postulate a sudden, temporary disappearance of real spending opportunities or a temporary reduction

in the rate of time preference, we can instead attribute the situation to a temporary increase in credit spreads as a result of disruption of the financial sector.

Eggertsson and Woodford show that it can be a serious mistake for a central bank to be expected to return immediately to the pursuit of its normal policy target as soon as the zero bound no longer prevents it from hitting that target. For example, Figure 11 (reproduced from their paper) compares the dynamic paths of the policy rate, the inflation rate, and aggregate output under two alternative monetary policies, in the case of a real disturbance (here interpreted as an exogenous increase in the probability that loans are bad, requiring intermediaries to increase the credit spread by several percentage points) that begins in period zero and lasts for 15 quarters, before real fundamentals permanently return to their original (“normal”) state.

In the case considered, if the financial disturbance were never to occur, optimal policy would involve maintaining a zero inflation rate, as this would also imply a zero output gap in every period. After the disturbance dissipates, one of the feasible policies is an immediate return to this zero-inflation steady state (under the parameterization assumed in the figure, this involves a nominal interest rate of 4 percent), and this is optimal from the point of view of welfare in all of the periods after the financial disturbance dissipates. It is not, however, possible to maintain the zero-inflation steady state at all times, because during the financial disturbance this would require the policy rate to equal minus 2 percent, which would violate the zero lower bound.

One of the policies considered in the figure (the dashed lines) is strict (forward-looking) inflation targeting: the central bank uses interest-rate policy to maintain a zero inflation rate whenever it is not prevented by the zero lower bound on the policy rate; and when undershooting the inflation target cannot be avoided, the policy rate is maintained at the lower bound. The other policy (the solid lines) is the optimal Ramsey policy, when the zero lower bound is included among the constraints on the set of possible equilibria. The forward-looking inflation targeting policy is clearly much worse, as it involves both a much more severe output contraction and much more severe deflation during the period when the zero bound constrains policy.

The problem with the forward-looking inflation targeting policy is that

because the central bank simply targets zero inflation from the time that it again becomes possible to do so, all of the deflation that occurs while the zero bound binds is fully accommodated by the subsequent policy: the central bank continues to maintain the price level at whatever level it has fallen to. This results in expected deflation during the entire period of the financial disturbance, for deflation will continue as long as the financial disruption continues, while no inflation will be allowed even if the disturbance dissipates; this expected deflation makes the zero bound on nominal interest rates a higher lower bound on the *real* policy rate, making the contraction and deflation worse, giving people reason to expect more deflation as long as the disruption continues, and so on in a vicious circle.

The outcome would be even worse in the case that the central bank were to seek to achieve the target criterion (3.1) each period, as soon as it becomes possible to do so. This is because, once credit spreads contract again, this policy would require the central bank to target *negative* inflation and/or a negative output gap (even though zero inflation and a zero output gap would now be achievable), simply because there had been a large negative output gap in the recent past (when the zero bound was a binding constraint); but the expectation of such policy would make the contraction while the zero bound constrained policy even more severe (justifying even tighter policy immediately following the “exit” from the “liquidity trap”, and so on).

Under the optimal policy, there is instead a commitment to maintain accommodative conditions for a brief interval, even though the reduction in credit spreads means that this level for the policy rate is now expansionary, leading to a mild boom and temporary inflation above the long-run target level (of zero). The expectation that this will occur during the “exit” from the trap results in much less contraction of economic activity and much less deflation, because it makes the perceived real rate of interest lower at all times while the policy rate is at zero (given that there is in each period some probability that credit spreads will shrink again in the next period, allowing mild inflation to occur). This expectation results in less deflation and higher real activity while the lower bound binds; and the expectation that continuation of the financial stress will have less drastic consequences is itself a substantial factor in making those consequences much less drastic —

in a “virtuous circle” that exactly reverses the logic of our analysis above.

While our analysis implies that it is desirable for people to be able to expect that the “exit” from the trap will involve mild inflation, it does *not* follow that the possibility of occasionally hitting the zero lower bound on the policy rate is a reason to aim for a substantial positive rate of inflation at all times (as proposed by Summers, 1991), simply in order to ensure that a zero nominal interest rate will always mean a sufficiently negative value of the real policy rate. To the extent that a history-dependent inflation target of the kind called for Eggertsson and Woodford can be made credible and understood by the public, it suffices that the central bank be committed to bring about a *temporarily* higher rate of inflation only on the particular occasions when the zero lower bound has bound in the recent past, and not all of the time.²⁸

This analysis implies that a commitment to maintain policy accommodation can play an important role in mitigating the effects of the zero lower bound on interest rates. One might reasonably ask for what length of time it is sensible to commit to keep rates low, and in particular whether it is really prudent to make any lengthy commitment when it is hard for a central bank to be certain that recovery may not come much sooner than anticipated. The answer is that the best way to formulate such a commitment is not in terms of a *period of time* that can be identified with certainty in advance, but rather in terms of targets that must be met in order for the removal of policy accommodation to be appropriate.

In fact, Eggertsson and Woodford (2003) show that in the representative-household model (and hence similarly in the special case described above), optimal policy can be precisely described, regardless of the nature of the exogenous disturbances,²⁹ by a target criterion involving only the path of

²⁸Eggertsson and Woodford (2003) compare the welfare levels associated with alternative constant inflation targets, and find that the existence of an occasionally binding zero lower bound does indeed make the optimal inflation target higher than it would otherwise be, if one must choose from among this very restrictive class of policies. But they show that even the best policy in that class involves much larger average distortions than a price-level targeting policy, even though the price-level targeting policy implies a long-run average inflation rate of zero.

²⁹Their analysis allows for both exogenous variations in the “natural rate of interest”

the output-gap-adjusted price level defined in (3.2). Under the optimal rule, the central bank has a target each period for \tilde{p}_t , that depends only on the economy’s history through period $t - 1$, and must use interest-rate policy to achieve the target, if this is possible without violation of the zero lower bound; if the target is undershot even with a zero policy rate, the policy rate is at any rate reduced to zero — and the target for \tilde{p}_{t+1} is increased in proportion to the degree of undershooting. In periods when the zero bound does not bind, the target for the gap-adjusted price level is not adjusted, and the target criterion is the same as the one discussed in section 3.1.

Actually, the adjustments of the target are not of great importance, even when the zero bound does bind: Eggertsson and Woodford show that almost all of the improvement in stabilization achievable under the optimal policy commitment can be obtained by simply committing to a target criterion of the form (3.2) with a constant target p^* . The crucial feature of the optimal policy is that the target for \tilde{p}_t must not be allowed to *fall* as a result of having undershot the target in past periods. Hence one of the approximate characterizations of optimal policy proposed in section 3.1 continues to provide a good approximation to optimal policy even when the zero lower bound sometimes binds: it is simply important that the commitment be to the *level* form of the target criterion (3.2) rather than to the *growth rate* form (3.1).

4 Optimal Policy: Credit Policy

We turn now to the final of our three independent dimensions of central-bank policy, namely, adjustment of the composition of the asset side of the central bank’s balance sheet, taking as given the overall size of the balance sheet (determined by the reserve-supply decision discussed in section 2). According to the traditional doctrine of “Treasuries only,” the central bank should *not* (which means an additive exogenous term in the intertemporal IS relation) and in the “cost-push” term (which means an additive exogenous term in the Phillips-curve tradeoff), evolving according to arbitrary stochastic processes. Since the effects of the financial frictions in (1.3) and (1.6) are to add additional terms involving $\hat{\Omega}_t$ that can be viewed as a combination of these two types of shifts, the optimal target criterion derived by Eggertsson and Woodford continues to apply — under the special assumptions stated above — even in the presence of time-varying credit frictions.

vary the composition of its balance sheet as a policy tool; instead, it should avoid both balance-sheet risk and the danger of politicization by only holding (essentially riskless) Treasury securities at all times, while varying the size of its balance sheet to achieve its stabilization goals for the aggregate economy.³⁰

And even apart from these prudential concerns, if private financial markets can be relied upon to allocate capital efficiently, it is hard to argue that there would be any substantial value to allowing the central bank this additional dimension of policy. Eggertsson and Woodford (2003) present a formal irrelevance proposition in the context of a representative-household general-equilibrium model; in their model, the assets purchased by the central bank have no consequences for the equilibrium evolution of output, inflation or asset prices — and this is true regardless of whether the central bank purchases long-term or short-term assets, nominal or real assets, riskless or risky assets, and so on. And even in a model with heterogeneity of the kind considered here, the composition of the central bank’s balance sheet would be irrelevant if we were to assume frictionless private financial intermediation, since private intermediaries would be willing to adjust their portfolios to perfectly offset any changes in the portfolio of the central bank.

This irrelevance result does not hold, however, in the presence of credit frictions of the kind assumed in section 1, and so we can also consider the optimal use of this additional dimension of policy, if we are willing to suppose that the prudential arguments against the central bank’s involvement in the allocation of credit should not be determinative, at least in the case of sufficiently severe financial disruptions. In our model, an increase in L_t^{cb} can improve welfare on two grounds: for a given volume of private borrowing b_t , an increase in L_t^{cb} allows the volume of private lending L_t to fall, which should reduce both the resources Ξ_t^p consumed by the intermediary sector and the equilibrium credit spread ω_t (due to equilibrium relation (1.7)). Under plausible conditions, our model implies both a positive shadow value $\varphi_{\Xi,t}$ of reductions in Ξ_t (the Lagrange multiplier associated with the resource constraint (1.2)) and a positive shadow value $\varphi_{\omega,t}$ of reductions in ω_t (the Lagrange multiplier associated with the constraint (1.4)); hence an increase

³⁰See Goodfriend (2009) for discussion of this view and a warning about the dangers of departing from it.

in L_t^{cb} should be desirable on both grounds.

In the absence of any assumed cost of central-bank credit policy, one can easily obtain the result that it is always optimal for the central bank to lend in amount sufficient to allow an equilibrium with $L_t = 0$, *i.e.*, the central bank should substitute for private credit markets altogether. Of course, we do not regard this as a realistic conclusion. As a simple way of introducing into our calculations the fact that the central bank is unlikely to have a comparative advantage at the activity of credit allocation under normal circumstances, we assume that central-bank lending consumes real resources in a quantity $\Xi^{cb}(L_t^{cb})$, by analogy with our assumption that real resources Ξ_t^p are consumed by private intermediaries. The function $\Xi^{cb}(L)$ is assumed to be increasing and at least weakly convex; in particular, we assume that $\Xi^{cb'}(0) > 0$, so that there is a positive marginal resource cost of this activity, even when the central bank starts from a balance sheet made up entirely of Treasury securities.

4.1 When Is Active Credit Policy Justified?

The first-order conditions for optimal choice of L_t^{cb} then become:

$$\varphi_{\Xi,t}[\bar{\Xi}_t^{p'}(b_t - L_t^{cb}) - \Xi^{cb'}(L_t^{cb})] + \varphi_{\omega,t}[\bar{\Xi}_t^{p''}(b_t - L_t^{cb}) + \chi_t''(b_t - L_t^{cb})] \leq 0, \quad (4.1)$$

$$L_t^{cb} \geq 0, \quad (4.2)$$

together with the complementary slackness condition that at least one of conditions (4.1) or (4.2) must hold with equality in each period. (Here the first expression in square brackets in (4.1) is the partial derivative of Ξ_t with respect to L_t^{cb} , holding constant the value of total borrowing b_t , while the second expression in square brackets is the partial derivative of ω_t with respect to L_t^{cb} under the same assumption.)

A “Treasury only” policy is optimal in the event of a corner solution, in which (4.1) is an inequality, as will be the case if $\Xi^{cb'}(0)$ is large enough. In our view, it is probably most reasonable to calibrate the model so that this is true in steady state. Then not only will the optimal policy involve “Treasury only” in the steady state, but (assuming that the inequality is strict at the steady state) this will continue to be true in the case of

any stochastic disturbances that are small enough. However, it will remain possible for the optimal policy to require $L_t^{cb} > 0$ in the case of certain large enough disturbances. This is especially likely to be true in the case of large enough disruptions of the financial sector, of a type that increase the marginal resource cost of private intermediation (the value of $\bar{\Xi}^{p'}$) and/or the degree to which increases in private credit require a larger credit spread (the value of $\bar{\omega}'$).

However, not all “purely financial” disturbances — by which we mean exogenous shifts in the functions $\bar{\Xi}_t^p(L)$ or $\chi_t(L)$ of a type that increase the equilibrium credit spread $\bar{\omega}_t(L)$ for a given volume of private credit — are equally likely to justify an active central-bank credit policy on the grounds just mentioned.³¹ To illustrate this, let us consider four different possible purely financial disturbances, each of which will be assumed to increase the value of $\bar{\omega}_t(\bar{L})$ by the same number of percentage points. By an *additive* shock, we mean one that translates the schedule $\bar{\omega}_t(L)$ vertically by a constant amount; a *multiplicative* shock will instead multiply the entire schedule $\bar{\omega}_t(L)$ by some constant factor greater than 1. We shall also distinguish between disturbances that change the function $\bar{\Xi}_t(L)$ (“ Ξ shocks”) and disturbances that change the function $\chi_t(L)$ (“ χ shocks”). Thus a “multiplicative χ shock” is a change in the function $\chi_t(L)$ as a consequence of which the schedule $\bar{\omega}_t(L)$ is multiplied by a factor greater than 1 for all values of L , and so on.

Figure 12 plots the dynamic response of the sum of the three positive terms on the left-hand side of (4.1) to each of these four types of purely financial disturbance, when the model is calibrated as in the numerical exercises reported in Figures 7-10. In these simulations, both interest-rate policy and reserve-supply policy are assumed to be optimal, as discussed in sections 2 and 3. We assume in each case that there is no central-bank lending to the private sector in the equilibrium being computed, but we ask (in the equilibrium computed under this assumption) what the *smallest* value of $\Xi^{cb'}(0)$

³¹Our result here is quite different from that in section 3, where the consequence of a “purely financial” disturbance for optimal interest-rate policy, taking as given the path of central-bank lending to the private sector, depends (to a first approximation) only on the size of the shift in $\bar{\omega}_t(\bar{L})$, which is why we do not bother to show the optimal responses to more than one type of purely financial disturbance.

is at each point in time for which this would be consistent with the first-order condition (4.1). (Thus an increase in the quantity plotted means that the marginal benefit of central-bank credit policy is increased, even if in our calculations no central-bank lending actually occurs.) We divide the sum of the three terms by the value of $\varphi_{\Xi,t}$, so that the quantity plotted is precisely the threshold value of $\Xi^{cb'}(0)$, expressed in terms of an interest rate spread. (Since it is an interest rate spread, we multiply by 4, so that the quantity on the vertical axis of the figure is in units of percentage points per annum.) In the figure, each of the four disturbances is of a size that increases the value of $\omega_t(\bar{L})$ by 4 percentage points per annum (*i.e.*, from 2.0 percent to 6.0 percent).

In the absence of any disturbances, the steady-state value of this quantity is a little less than 3.5 percentage points per annum. This means that a marginal resource cost of central-bank loan origination of 3.5 percent or higher will suffice to justify our proposal above, that in the steady state the optimal quantity of central-bank credit is zero.³² Let us suppose that $\Xi^{cb'}(0)$ is equal to 4.0 percent. Then in the absence of shocks, a corner solution with “Treasury only” is optimal. However, either a “multiplicative Ξ shock” or an “additive Ξ shock” of the size assumed would cause condition (4.1) to be violated in the case of a corner solution; hence optimal policy would require a positive quantity of central-bank lending. (In the case of the “multiplicative Ξ shock,” this would be true even if $\Xi^{cb'}(0)$ were equal to 5.0 percentage points.)

On the other hand, even in the case of the “multiplicative Ξ shock,” the threshold required to justify a corner solution is only above 4 percent in the quarter of the shock and the quarter immediately following it — despite the fact that in our numerical experiment, the disturbance is assumed to have an autocorrelation coefficient of 0.9, so that the shift in the $\omega_t(L)$ schedule is still 65 percent of its initial magnitude a year later. This suggests that even in the case of those disturbances for which the welfare benefits of central-bank

³²Note that this quantity is well above the marginal resource cost of private lending in the steady state, which we have calibrated at 2.0 percent per annum. The reason is because our baseline calibration implies a relatively inelastic private supply of credit: $\bar{\omega}_t(L)$ is steeply increasing with L .

credit policy are greatest, departure from the corner solution is likely to be justified only for a relatively brief period of time.

4.2 An Example with Active Credit Policy

As an example of how optimal credit policy can, under some circumstances, substantially alter the economy's response to a financial disruption, Figure 13 considers the optimal response to a "multiplicative Ξ shock," under a calibration in which $\Xi^{cb'}(0)$ is assumed to be low enough so that even in the steady state, a corner solution is not optimal.³³ (While this is not the case that we regard as most realistic, it simplifies the calculations reported in Figure 13, since it implies that constraint (4.2) never binds. We leave for future work analysis of the more interesting case, in which (4.2) binds in some periods and not in others.) The figure plots the impulse responses under two alternative assumptions about policy: in the "Credit" case, central-bank policy is optimal along all three dimensions (and L_t^{cb} varies over time), while in the "No Credit" case, L_t^{cb} is constrained to equal the steady-state value \bar{L}^{cb} at all times,³⁴ while interest-rate policy and reserve-supply policy are optimal. In addition to the responses of the five variables plotted in Figures 7-10, Figure 13 also plots the response of an additional variable, \hat{L}_t^{cb} , indicating the deviation of central-bank credit from its steady-state value, expressed as a fraction of total steady-state credit \bar{b} (in percentage points).

Under an optimal use of credit policy, central-bank lending to the private sector increases substantially in response to the financial disturbance (central-bank lending increases from 5 percent of total credit to a little over 9 percent). As a result, the large increase in the credit spread that would otherwise occur as a result of the shock is essentially prevented from occurring (so that the credit spread remains close to its steady-state level of 2.0 percent per annum). As a further consequence of this, it is not necessary under this policy to cut the policy rate sharply as would otherwise be required by an optimal interest-rate policy. The substantial contraction of credit that would

³³This alternative calibration is chosen to imply that in the steady state, only 5 percent of total credit b_t is supplied by the central bank.

³⁴We impose the constraint that L_t^{cb} must equal \bar{L}^{cb} , rather than zero, in the "No Credit" case, so that the steady state is the same under both policies.

otherwise occur (an eventual contraction of aggregate credit by more than 2 percent, a year after the shock) is largely avoided, and the modest effects on output and inflation that would occur even under an optimal interest-rate policy in the absence of an active credit policy are also largely avoided.

This example indicates that under at least some circumstances, our model would support a fairly aggressive use of active credit policy for stabilization purposes. We must caution, however, that these results are quite dependent upon assumptions about the nature of the financial disturbance. It is equally possible to conclude that central-bank credit should be *contracted* (assuming that it would be positive to begin with) in response to a disturbance that increases credit spreads. If the only form of purely financial disturbance is an “additive χ disturbance,” and we assume that $\Xi^{cb}(L^{cb})$ is a linear function, then none of the functions $\Xi^{p'}(L)$, $\Xi^{p''}(L)$ or $\chi'(L)$ is time-varying, and Ξ^{cb} is a constant. In this case, the requirement that (4.1) hold with equality determines the volume of private credit L_t as a time-invariant function of $\varphi_{\omega,t}/\varphi_{\Xi,t}$. In the case of a disturbance that increases the credit spread, the resulting decline in credit demand b_t means that, in order for credit supply L_t to be stabilized, L_t^{cb} would have to contract; so unless $\varphi_{\omega,t}/\varphi_{\Xi,t}$ changes to such an extent that the value of L_t consistent with (4.1) falls as much as b_t does,³⁵ it is optimal for L_t^{cb} to contract (as Figure 12 would also suggest). In a case of this kind, active credit policy would actually cause credit to contract by more (and credit spreads to increase by more) than they would in the case that the supply of central-bank credit did not respond to the shock.

4.3 Segmented Credit Markets

In the simple model expounded above, there is a single credit market and single borrowing rate i_t^b charged for loans in this market; our discussion of central-bank credit policy has correspondingly simply referred to the optimal quantity of central-bank lending to the private sector overall, as if the allocation of this credit is not an issue. In reality, of course, there are many distinct credit markets, and many different parties to which the central bank might consider lending. Moreover, since there is only a potential case to

³⁵Our numerical experiments indicate that this can easily fail to be the case.

be made for central-bank credit policy when private financial markets are severely impaired, it does not make sense to assume efficient allocation of credit among different classes of borrowers by the private sector, so that only the total credit extended by the central bank would matter. Our simple discussion here has sought merely to clarify the connection that exists, in principle, between decisions about credit policy and the other dimensions of credit policy. An analysis of credit policy that could actually be used as a basis for credit policy decisions would instead have to allow for multiple credit markets, with imperfect arbitrage between them.

We do not here attempt an extension of our model in that direction. (A simple extension would be to allow for multiple types of “type b ” households, each only able to borrow in a particular market with its own borrowing rate, and market-specific frictions for the intermediaries lending in each of these markets.) We shall simply note that in such an extension, there would be a distinct first-order condition, analogous to conditions (4.1)–(4.2), for each of the segmented credit markets. There would be no reason to assume that the question whether active credit policy is justified should have a single answer at a given point in time: lending might be justified in one or two specific markets while the corner solution remained optimal in the other markets.

The conditions that should be appealed to in order to justify central-bank lending are more microeconomic than macroeconomic: they relate to the severity of the distortions that have arisen in particular markets and to the costs of intervention in those particular markets, rather than to aggregate conditions. Thus the main determinants of whether central-bank credit policy is justified — when it is justifiable to initiate active policy, and when it would be correct to phase out such programs — should not be questions such as whether the zero lower bound on interest-rate policy binds, or whether the central bank continues to undershoot the level of real GDP that it would like to attain. While aggregate conditions will be one factor that affects the shadow value of marginal reductions in the size of credit spreads (represented by the multiplier $\varphi_{\omega,t}$ in (4.1)), the value of this multiplier will likely be different for different markets, and the main determinants of variations in it are likely to be market-specific. This will be even more true of the other variables that enter into the first-order condition (4.1).

Hence it would be a mistake to think of credit policy as a substitute for interest-rate policy, an alternative tool that can be used to achieve the same goals, and that should be used to achieve the central bank’s target criterion for inflation and the output gap when interest-rate policy alone is unable to. Such a conception would be dangerous for two reasons. On the one hand, it would direct attention away from the most relevant costs and benefits when thinking about the appropriate scale, timing, and allocation of active credit policy. And on the other hand, it could also allow the central bank to avoid recognition of the extent to which the correct target criterion for interest-rate policy needs to be modified as a result of the zero lower bound — in particular, to avoid the challenge of shaping expectations about interest-rate policy after the lower bound ceases to bind, on the ground that credit policy (or “quantitative easing”) should allow the bank’s usual target criterion to be achieved continuously, without any need for signalling about unconventional future interest-rate policy as a compensation for past target misses.

5 Conclusions

We have shown that a canonical New Keynesian model of the monetary transmission mechanism can be extended in a fairly simple way to allow analysis of additional dimensions of central bank policy that have been at center stage during the recent global financial crisis: variations in the size and composition of the central-bank balance sheet, and in the interest rate paid on reserves, alongside the traditional monetary policy issue of the choice of an operating target for the federal funds rate (or some similar overnight inter-bank rate elsewhere). We have also considered the consequences for monetary policy analysis both of non-zero credit spreads all of the time and of financial disruptions that greatly increase the size of those spreads for a period of time, and we have also considered the consequences of the fact the zero lower bound for short-term nominal interest rates is sometimes a binding constraint on interest-rate policy.

One of our most important conclusions is that these issues can be addressed in a framework that represents a straightforward extension of the kind of model often used for monetary policy analysis in the past. This al-

lows both the considerations emphasized in the traditional literature and the more novel considerations brought to the fore by recent events to be taken into account, within a single coherent framework. This integration is particularly important, in our view, for clear thinking about the way in which the transition from the current emergency policy regime to a more normal policy framework should be handled, as financial conditions normalize. Because of the importance of expectations regarding future policy in determining market outcomes now, we believe that clarity about “exit strategy” is important for the success of policy even during periods of severe disruption of financial markets.

Another important implication of our model is that interest-rate policy should continue to be a central focus of monetary policy deliberations, despite the existence of the other dimensions of policy discussed here, and despite the existence of time-varying credit frictions that complicate the relationship between the central bank’s policy rate and financial conditions more broadly. While welfare can also be affected by reserve-supply policy, we have argued that this dimension of policy should be determined by a simple principle that does not require any discretionary adjustments in light of changing economic conditions: intermediaries should be satiated in reserves at all times, by maintaining an interest rate on reserves at or close to the current target for the policy rate.

And while welfare can similarly be affected by central-bank credit policy, to the extent that non-trivial credit frictions exist, we nonetheless believe that under normal circumstances, a corner solution (“Treasuries only”) is likely to represent the optimal composition of the central-bank balance sheet, so that decisions about active credit policy will be necessary only under relatively unusual circumstances, and it will be desirable to phase out special credit programs relatively rapidly after the disturbances that have justified their introduction. We thus do not anticipate that it should be necessary to make state-contingent adjustments of central-bank policy along multiple dimensions as an ongoing affair, even if recent events suggest that it is desirable for central banks to have the power to act along additional dimensions under sufficiently exigent circumstances.

Finally, our results suggest that the traditional emphasis in interest-rate

policy deliberations on the consequences of monetary policy for the projected evolution of inflation and aggregate real activity is not mistaken, even taking into account the consequences for the monetary transmission mechanism of time-varying credit frictions. At least in the context of the simple model of credit frictions proposed here, optimal interest-rate policy can be characterized to a reasonable degree of approximation by a target criterion that involves the paths of inflation and of an appropriately defined output gap, but no other endogenous target variables. This does not mean that central banks should remain indifferent toward changes in financial conditions; to the contrary, credit spreads (and perhaps other measures of financial market distortions as well) should be closely monitored, and taken into account in judging the forward path of interest-rate policy necessary for conformity with the target criterion; but financial variables need not be taken themselves as *targets* of monetary policy.

The main respect in which the appropriate target criterion for interest-rate policy should be modified to take account of the possibility of financial disruptions is by aiming at a target path for the price level (ideally, for an output-gap-adjusted price level), rather than for a target rate of inflation looking forward, as a forward-looking inflation target accommodates a permanent decline in the price level after a period of one-sided target misses due to a binding zero lower bound on interest rates. Our analysis implies that a credible commitment to the right kind of “exit strategy” should substantially improve the ability of monetary policy to deal with the unusual challenges posed by a binding zero lower bound during a deep financial crisis, and to the extent that this is true, the development of an integrated framework for policy deliberations, suitable both for crisis periods and for more normal times, is a matter of considerable urgency for the world’s central banks.

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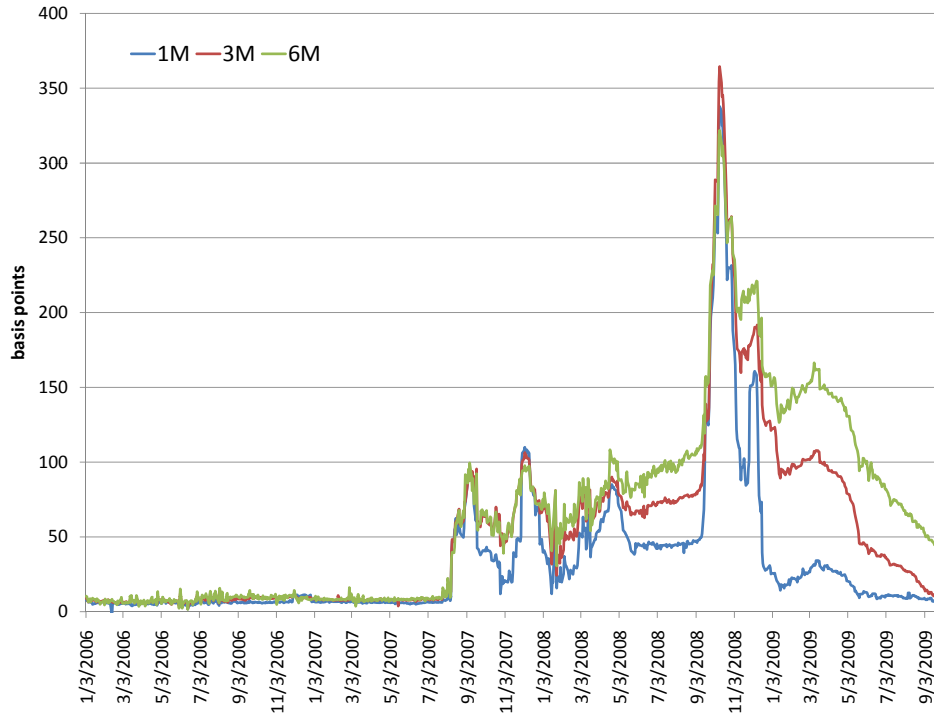


Figure 1: Spread between the U.S. dollar LIBOR rate and the corresponding OIS rate, for three different terms. (Source: Bloomberg.)

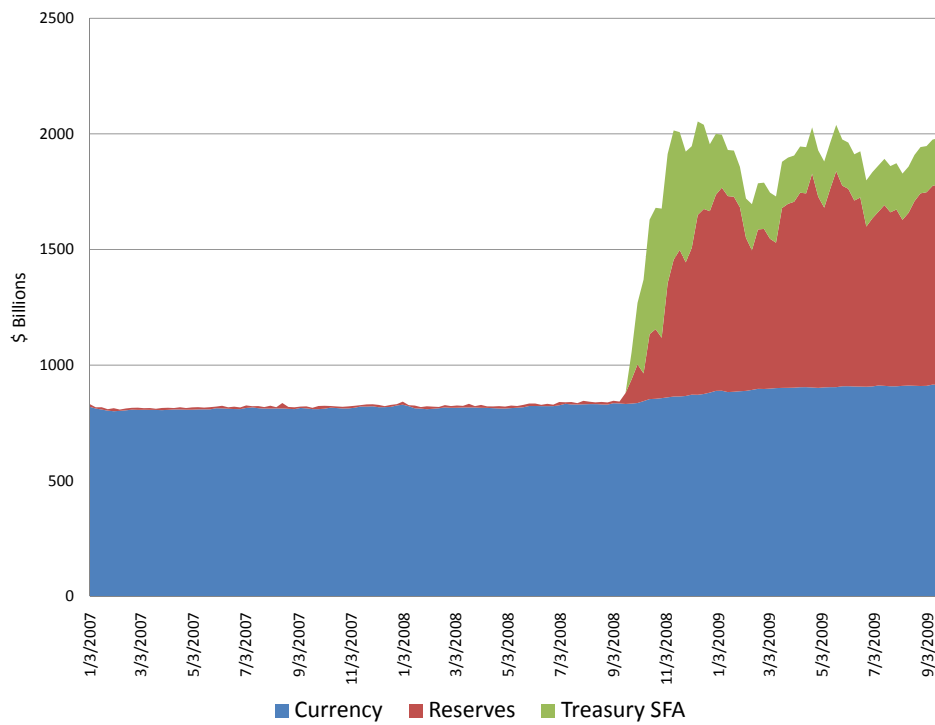


Figure 2: Liabilities of the Federal Reserve. (Source: Federal Reserve Board.)

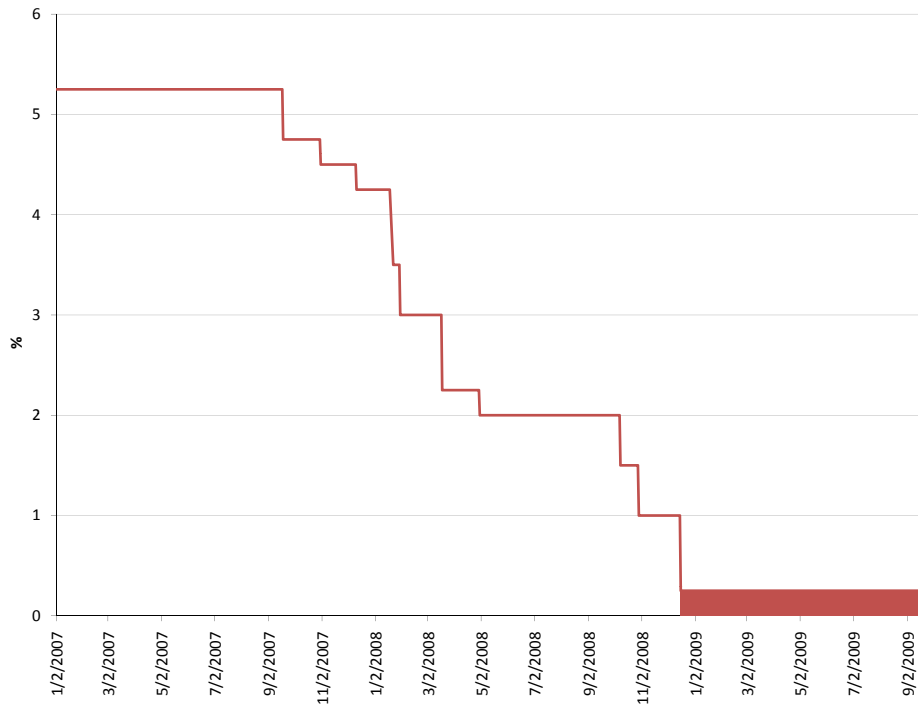


Figure 3: The FOMC's operating target for the federal funds rate. Since December 2008, the target rate has been replaced by a target band, from 0 to 25 basis points. (Source: Federal Reserve Board.)

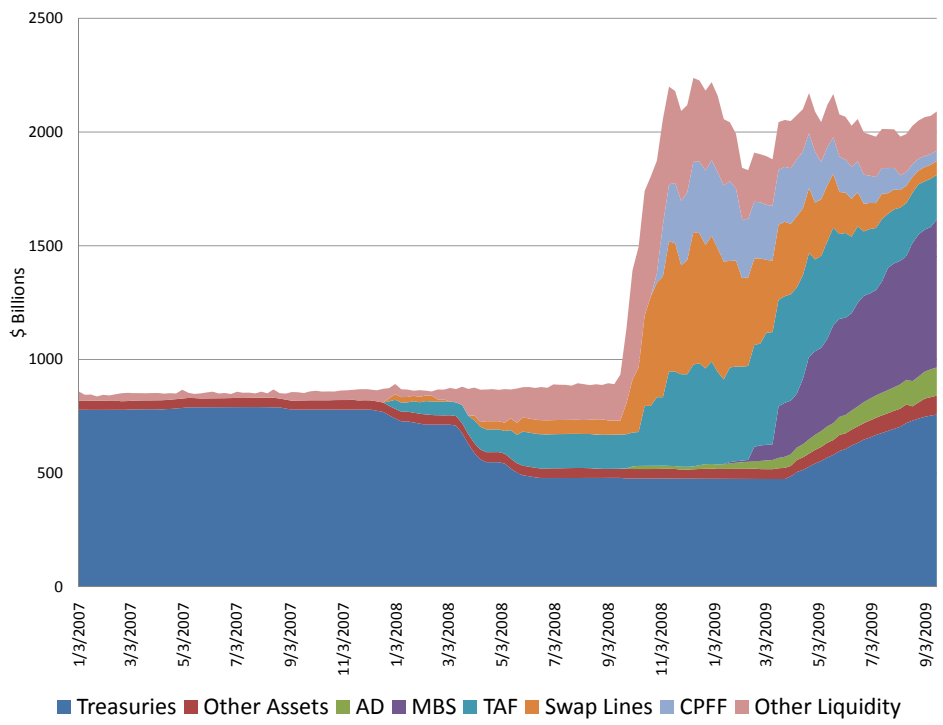


Figure 4: Assets of the Federal Reserve. (Source: Federal Reserve Board.)

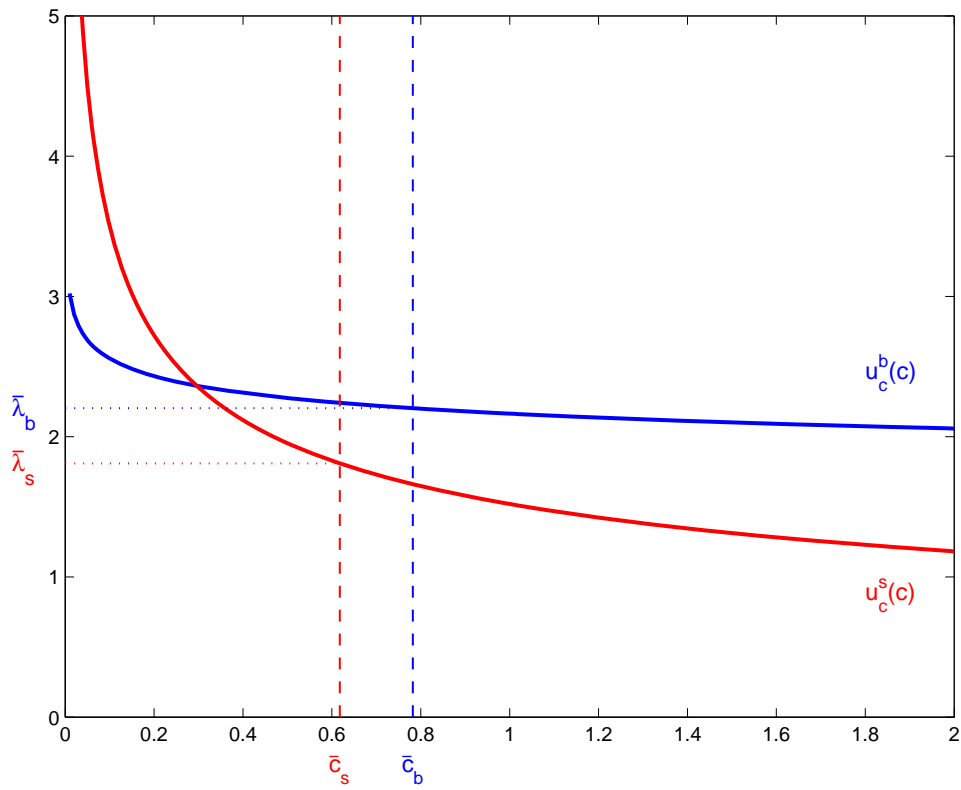


Figure 5: Marginal utilities of consumption for households of the two types. The values \bar{c}^s and \bar{c}^b indicate steady-state consumption levels of the two types, and $\bar{\lambda}^s$ and $\bar{\lambda}^b$ their corresponding steady-state marginal utilities.

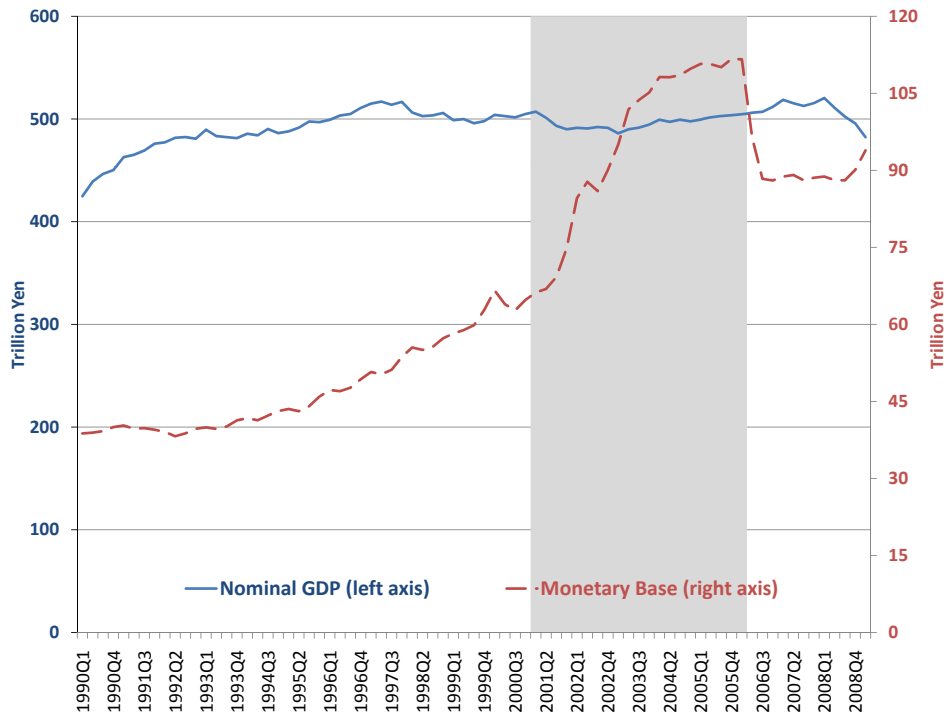


Figure 6: The monetary base and nominal GDP for Japan (both seasonally adjusted), 1990-2009. The shaded region shows the period of “quantitative easing,” from March 2001 through March 2006. (Source: IMF International Financial Statistics and Bank of Japan.)

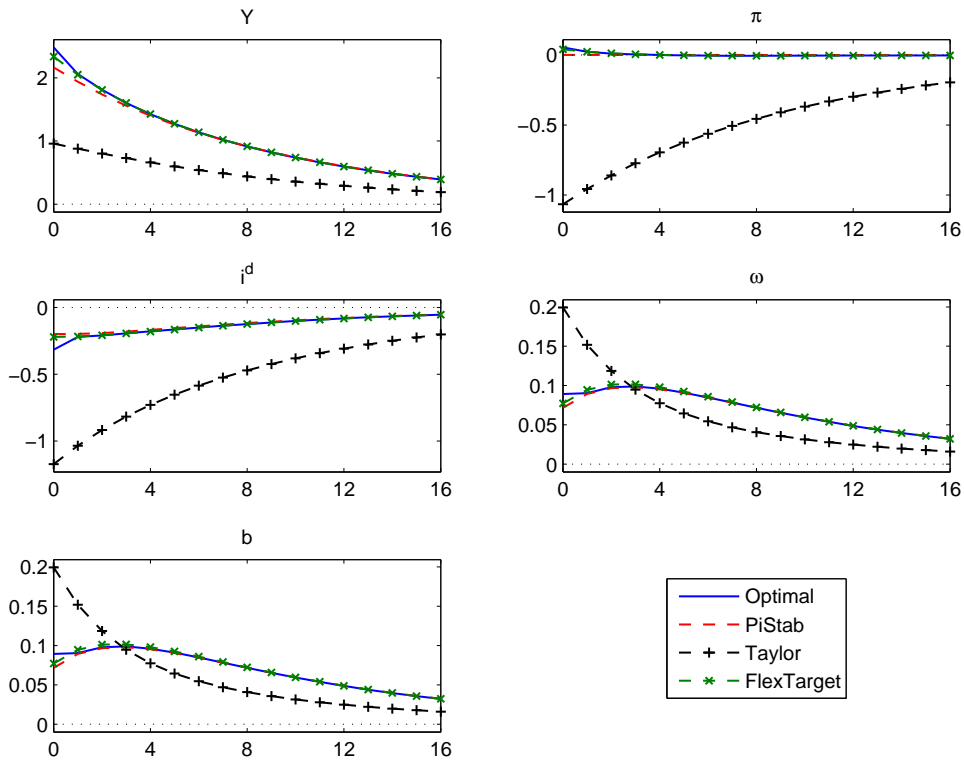


Figure 7: Impulse responses to a 1 percent increase in total-factor productivity, under four alternative monetary policies. (Source: Cúrdia and Woodford (2009a).)

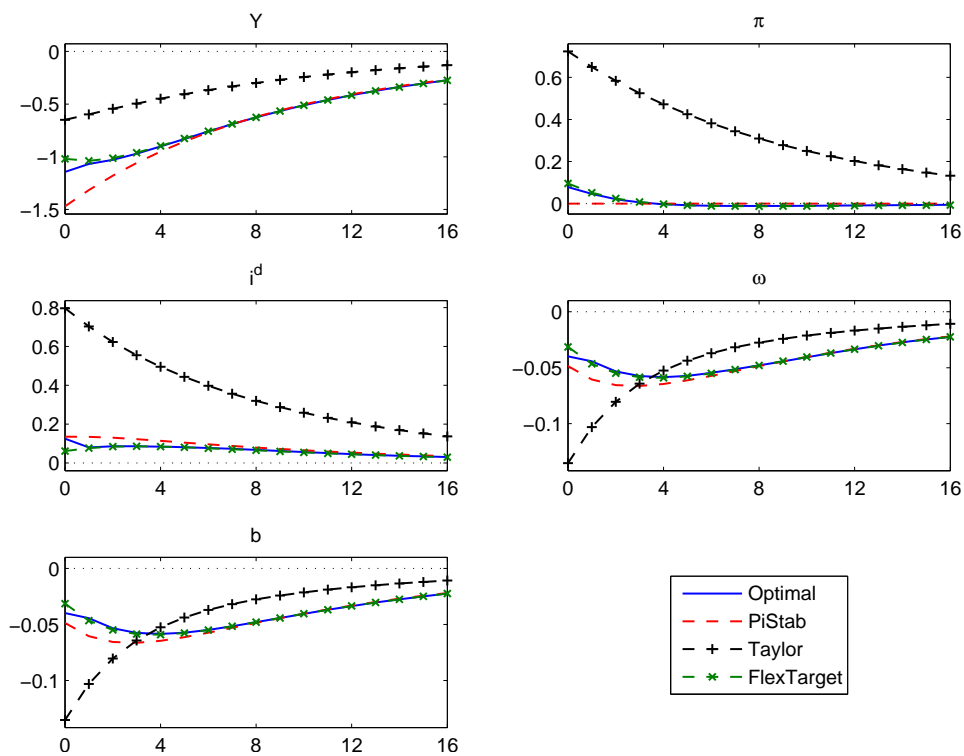


Figure 8: Impulse responses to a 1 percent increase in the wage markup, under four alternative monetary policies. (Source: Cúrdia and Woodford (2009a).)

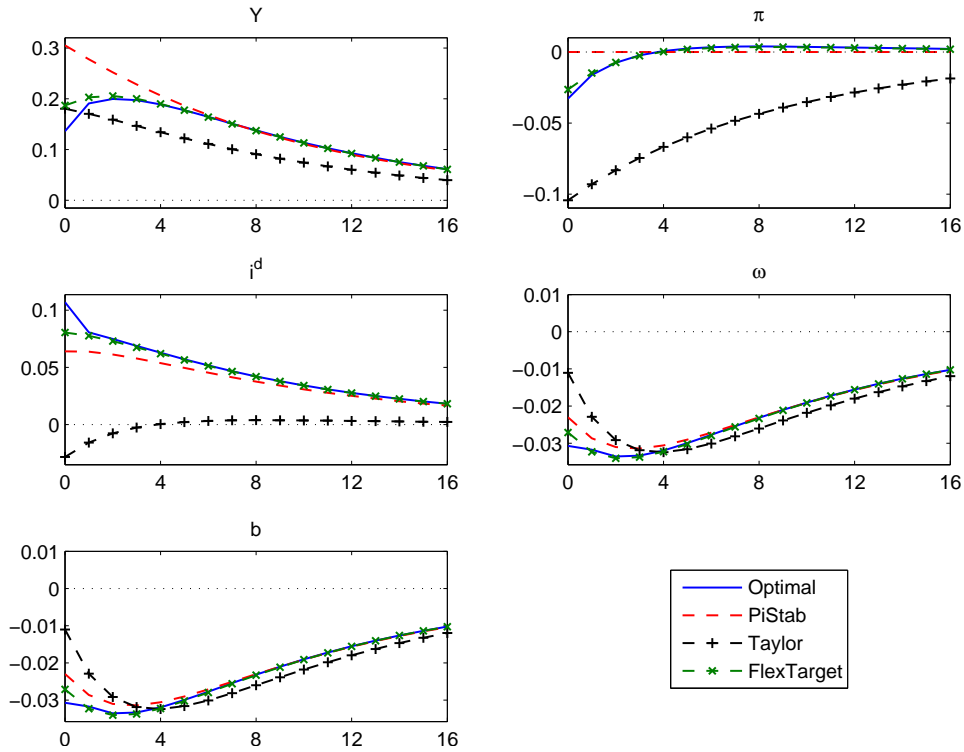


Figure 9: Impulse responses to an increase in G_t equal to 1 percent of steady-state output, under four alternative monetary policies. (Source: Cúrdia and Woodford (2009a).)

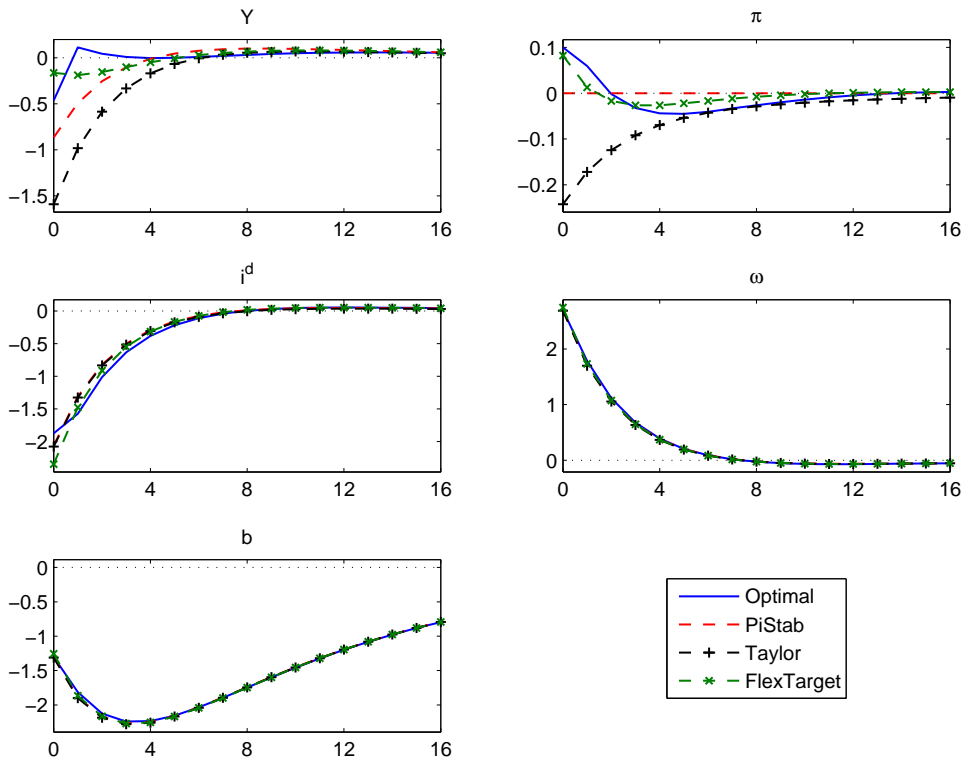


Figure 10: Impulse responses to a shift in the function $\chi_t(L)$ that triples the size of $\bar{\omega}_t(\bar{L})$ for each value of L , under four alternative monetary policies. (Source: Cúrdia and Woodford (2009a).)

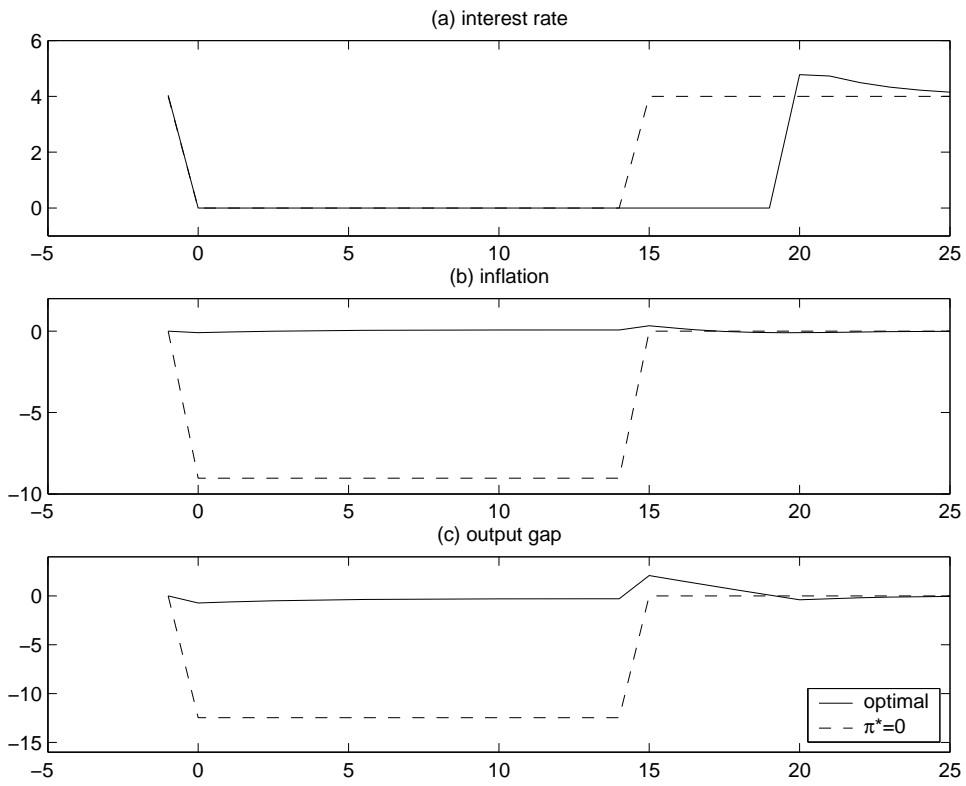


Figure 11: Equilibrium responses of the policy rate, inflation, and the output gap under two alternative monetary policies, when the expected probability of loan default exogenously increases for a period beginning in quarter zero and ending in quarter 15. (Source: Eggertsson and Woodford, 2003.)

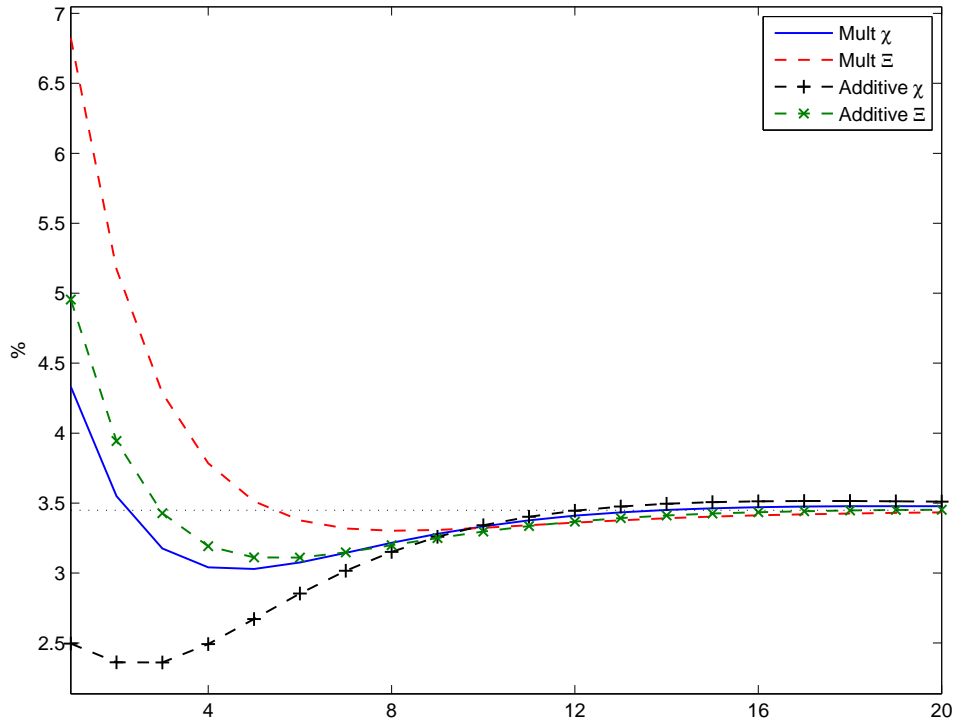


Figure 12: Response of the critical threshold value of $\Xi^{cb'}(0)$ for a corner solution, in the case of four different types of “purely financial” disturbances, each of which increases $\omega_t(\bar{L})$ by 4 percentage points.

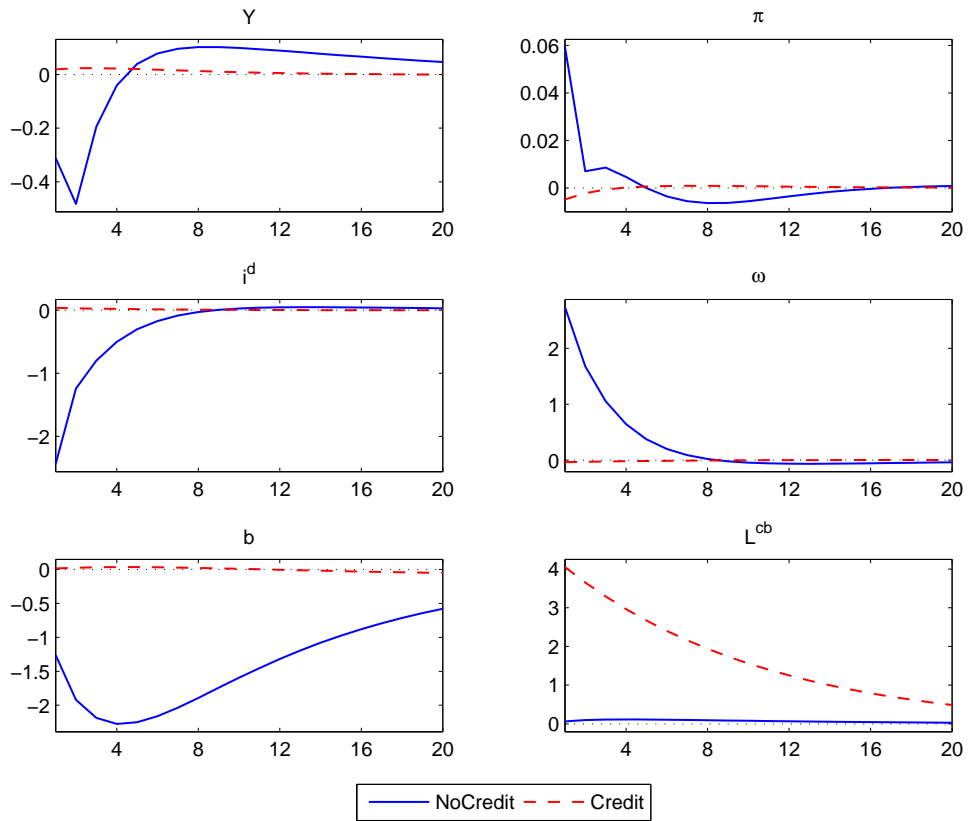


Figure 13: Impulse responses to a shift in the function $\bar{\Xi}_t(L)$ that triples the size of $\bar{\omega}_t(\bar{L})$ for each value of L , under optimal interest-rate policy and two alternative assumptions about credit policy.