

# Housing over Time and over the Life Cycle: A Structural Estimation

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## ABSTRACT

This paper estimates a structural model of optimal life-cycle housing and non-housing consumption in the presence of realistic labor income and house price uncertainty. The model incorporates housing adjustment costs, and credit constraints from down payment requirements. It also postulates constant elasticity of substitution between housing service and nonhousing consumption. The model fits the cross-sectional and time-series household wealth and housing profiles generated from the Panel Study of Income Dynamics quite well. Our estimate suggests an intra-temporal elasticity of substitution between housing and nonhousing consumption of 0.33 and a housing adjustment cost that amounts to about 15 percent of house value for married households. We then use the estimated model to conduct policy experiments and find that households respond nonlinearly to house price changes with large house price declines leading to sizable decreases in both the aggregate homeownership rate and the aggregate non-housing consumption. When lending conditions are tighter in the form of higher down payments, interestingly, large house price declines result in more severe drops in the aggregate homeownership rate but milder decreases in non-housing consumption.

**KEY WORDS:** Life-cycle, Housing, Adjustment Costs, intratemporal substitution, methods of simulated moments

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# 1. Introduction

The U.S. housing market has experienced dramatic price movements in recent years. These movements, accompanied by substantial increases in household indebtedness, have drawn the attention of policy makers and academics to housing market developments and their impact on general economic activities. Calibrated life-cycle housing models are now increasingly used in studying the implications of housing on consumption and savings (Campbell and Cocco 2005, Fernandez-Villaverde and Krueger 2005, Li and Yao 2007, Stokey 2007, Kiyotaki, Michaelides, and Nikolov 2007), on stock market participation and asset allocation (Cocco 2005, Flavin and Yamashita 2002, and Zhang and Yao 2005), on asset pricing (Davis and Martin 2005, Siegel 2005, Piazzesi, Schneider, and Tuzel 2005, Lustig and Van Nieuwerburgh 2006, and Flavin and Nakagawa 2007), and on the effectiveness and transmission channel of monetary policy (Iacoviello 2005).

Despite the growing interest in housing models, there has been a relatively small amount of econometric work aimed at identifying the relevant preference parameters and assessing model performance. As a consequence, theoretical models are often calibrated with little econometric guidance as to the value of key preference parameters and the extent to which the model explains the data. The choice of a period utility function and its parameterization is largely out of convenience.<sup>1</sup> This paper fills in this gap by augmenting a standard dynamic stochastic model of life-cycle consumption and savings behavior with housing choices, and jointly identifying the intertemporal as well as intratemporal preference parameters between housing and other consumption by matching average wealth and housing profiles of the data with simulated moments from our model.

Among the small literature of econometric studies on housing preference, there has been little consensus on the magnitudes of housing preference parameters. Specifically, studies based on macro-level aggregate consumption or asset price data frequently suggest a value larger than one for the intratemporal elasticity of substitution between housing and non-housing

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<sup>1</sup>Many theoretical studies using numerical calibrations adopt a Cobb-Douglas utility function for its simplicity and often ignore housing adjustment cost. Researchers cite the relative constant share of aggregate housing expenditure in the National Income and Product Account as supporting evidence of the Cobb-Douglas preference. The Consumer Expenditure Survey, however, indicates that expenditure shares at aggregate as well as the Metropolitan Statistical Area (MSA) level have fluctuated over time with the aggregate share increasing over the last two decades. The movements at the MSA level are mixed with many experiencing upward movement and some downward. See Stokey (2007) and Kahn (2008) for additional evidence.

consumption — implying that economic agents reduce expenditure on housing substantially when house prices move up relative to prices of non-housing consumption (Davis and Martin 2005, and Piazzesi, Schneider, and Tuzel 2007), despite strong evidence of household heterogeneity and housing adjustment cost as documented in the literature (Eberly 1994, Caballero 1993, Carroll and Dunn 1997, and Attanasio 2000). These studies have typically assumed the existence of a representative agent and abstracted from housing adjustment cost.

In contrast, investigations using household-level data recover much lower values for the elasticity parameter, often in the range of 0.15 and 0.50 (See, for example, Flavin and Nakagawa 2008, Hanushek and Quigley 1980, Siegel 2004, and Stokey 2007.) These studies, however, often suffer from selection bias in that households make the renting versus owning as well as moving decisions endogenously. As a result, these analyses cannot separate the effects of elasticity of substitution from the effects of housing transaction costs.<sup>2</sup> Furthermore, the identification in many of the studies requires households having unlimited access to credit, which contradicts the practice in reality. The lack of robustness to market friction and incompleteness, thus, complicates the interpretation of these empirical estimates.<sup>3</sup>

Our model introduces housing choices and house price uncertainties to an otherwise standard life-cycle model of consumption and saving with labor income risks. Specifically, we postulate Constant Elasticity of Substitution (CES) preferences over housing and nonhousing consumption and allow households to make housing decisions along both the extensive margin of home ownership and the intensive margin of housing service flows and house value. Housing transaction costs and a collateral borrowing constraint are directly incorporated in the model. The model, thus, builds on a growing literature examining tenure choice and housing consumption within a life-cycle framework (Ortalo-Magne and Rady 1999, Fernandez-Villaverde and Krueger 2002, Gervais 2002, Campbell and Cocco 2003, Chambers, Garriga, and Schlagenhaut 2005, Yao and Zhang 2005, Li and Yao 2007, and Bajari, Benkard, and Krainer 2005).

Our estimation of the structural parameters of the model is achieved through Method of Simulated Moments (MSM). Specifically, we first construct the average wealth, homeownership rate, house value, and rents profiles from the Panel Study of Income Dynamics (PSID) data

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<sup>2</sup>See Cooper, Haltiwanger, and Willis (2007) for a discussion on the bias that arises in estimation of ex-post Euler equations.

<sup>3</sup>For example, a household with a large elasticity of substitution may not be able to adjust its house and consumption after significant house price appreciation, because it is expensive to gain access to appreciated housing assets.

set across three age groups for all years between 1984 and 2005. For homeownership rate, house values and rent values, we further differentiate households basing on the expensiveness of their state of residence. We then numerically solve our model for optimal behavior and simulate our model to generate life-cycle housing and wealth paths. Simulated model moments are computed in the same manner as the data moments to eliminate potential bias caused by cohort and time effects. By minimizing the weighted difference between the simulated model profiles to their empirical counterparts, we identify the parameters of our structural model.

Our simulated wealth and housing profiles offer a good match to the data over the sample period. Our estimation also reveals that after explicitly accounting for housing adjustment cost, the intratemporal elasticity of substitution between housing services and nondurables is around 0.33. This value is much lower than estimations based on aggregated time series, and captures the variations of households' homeownership and house value profiles with respect to house price distributions both over time and in a cross-section. Our estimate of the housing transaction costs for married couples amounts to 15 percent of house value, consistent with the low mobility rate in the data. Our estimated values of the coefficient of relative risk aversion and the discount factor are 6.19 and 0.96, in line with those provided by the previous literature. They are identified by the average wealth profile in the sample.

Finally, we use our estimated model to conduct policy experiments. In particular, we investigate how households respond to changes in house prices together with changes in financial conditions. We find that households respond nonlinearly to changes in house prices. Large house price declines do lead to significant decreases in both homeownership rate and non-housing consumption. The average marginal propensity to consume out of housing wealth ranges from 0.4 percent to 3.2 percent. Interestingly, a tighter borrowing constraint will exacerbate the negative effect of house price declines on homeownership rate, but it alleviates some of the negative impact on non-housing consumption.

To the best of our knowledge, our paper represents one of the first structural estimation of housing preference parameters that is consistent with both time series and cross-sectional evidence on households' housing consumption and savings decisions.<sup>4</sup>

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<sup>4</sup>The recent paper by Bajari, Chan, Krueger, and Miller (2008) is the closest in spirit to our paper. There are, however, important differences. First, our paper uses simulated method of moments, which is computationally more manageable than Bajari, et. al (2008)'s structural estimation that strives to match the whole distribution. As a result, we are able to solve the decision rules endogenously instead of imposing reduced forms. Second, we explicitly model and estimate households' tenure decision. Finally, we jointly

The rest of the paper proceeds as follows. In Section 2, we present the model of housing with adjustment cost. In Section 3, we lay out our estimation strategy and describe the data sources. Section 4 discusses our main findings and implications. We perform policy experiment in Section 5. Finally, we conclude and point to future extensions in Section 6.

## 2. The Model Economy

Our modeling strategy extends that of Yao and Zhang (2005) and Li and Yao (2007) by admitting a wider range of substitution elasticities between housing and other consumption. We consider an economy where a household lives for at most  $T$  ( $T > 0$ ) periods. The probability that the household lives up to period  $t$  is given by the following survival function,

$$F(t) = \prod_{j=0}^t \lambda_j, \quad 0 \leq t \leq T, \quad (1)$$

where  $\lambda_j$  is the probability that the household is alive at time  $j$  conditional on being alive at time  $j - 1$ ,  $j = 0, \dots, T$ . We set  $\lambda_0 = 1$ ,  $\lambda_T = 0$ , and  $0 < \lambda_j < 1$  for all  $0 < j < T$ .

The household derives utility from consuming a numeraire good  $C_t$  and housing services  $H_t$ , as well as from bequeathing wealth  $Q_t$ . The within-period utility demonstrates a constant elasticity of substitution between the two goods (CES), modified to incorporate a demographic effect:

$$U(C_t, H_t; N_t) = N_t \left[ (1 - \omega) \left( \frac{C_t}{N_t} \right)^{1 - \frac{1}{\zeta}} + \omega \left( \frac{H_t}{N_t} \right)^{1 - \frac{1}{\zeta}} \right]^{\frac{1}{1 - \frac{1}{\zeta}}}, \quad (2)$$

where  $N_t$  denotes the exogenously given effective family size, which captures the economies of scale in household consumption. The parameter  $\omega$  controls the consumption share of housing services; and  $\zeta$  governs the degree of intratemporal substitutability between housing and nondurable consumption goods. We denote the bequest function as  $B(Q_t)$ .

In each period, the household receives income  $Y_t$ . Prior to the retirement age, which is set exogenously at  $t = J$  ( $0 < J < T$ ),  $Y_t$  represents labor income and is given by

$$Y_t = P_t^Y \varepsilon_t, \quad (3)$$

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estimate housing adjustment costs with the intratemporal elasticity parameter as there are important tradeoffs between the two parameters.

where

$$P_t^Y = \exp\{f(t, Z_t)\}P_{t-1}^Y\nu_t \quad (4)$$

is the permanent labor income at time  $t$ .  $P_t^Y$  has a deterministic component  $f(t, Z_t)$ , which is a function of age and household characteristics  $Z_t$ .  $\nu_t$  represents the shock to permanent labor income.  $\varepsilon_t$  is the transitory shock to  $Y_t$ . We assume that  $\{\ln \varepsilon_t, \ln \nu_t\}$  are independently and identically normally distributed with mean  $\{-0.5\sigma_\varepsilon^2, -0.5\sigma_\nu^2\}$ , and variance  $\{\sigma_\varepsilon^2, \sigma_\nu^2\}$ , respectively. Thus,  $\ln P_t^Y$  follows a random walk with a deterministic drift  $f(t, Z_t)$ .

After retirement, the household receives a constant income which constitutes a fraction  $\theta$  ( $0 < \theta < 1$ ) of its preretirement permanent labor income,

$$Y_t = \theta P_J^Y, \quad \text{for } t = J, \dots, T. \quad (5)$$

## 2.1. Housing and Mortgage Contracts

A household can acquire housing services through either renting or owning. A renter has a house tenure  $D_t^o = 0$ , and a homeowner has a house tenure  $D_t^o = 1$ . To rent, the household pays a fraction  $\alpha$  ( $0 < \alpha < 1$ ) of the market value of the rental house. The house price appreciation rate  $\tilde{r}_t^H$  follows an i.i.d. normal process with mean  $\mu_H$  and variance  $\sigma_H^2$ . The shock to house prices is thus permanent and exogenous.<sup>5</sup>

A household can finance home purchases with a mortgage. The mortgage balance denoted by  $M_t$  needs to satisfy the following collateral constraint at all times,

$$0 \leq M_t \leq (1 - \delta)P_t^H H_t, \quad (6)$$

where  $0 \leq \delta \leq 1$ , and  $P_t^H H_t$  denotes house price at time  $t$ .<sup>6</sup> The borrowing rate  $r$  is time-invariant and the same as lending rate. A homeowner is required to spend a fraction  $\psi$

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<sup>5</sup>Flavin and Yamashita (2002), Campbell and Cocco (2003), and Yao and Zhang (2005) also assume that house price shocks are i.i.d. and permanent. Case, Quigley, and Shiller (2003) explore home price dynamics using data between 1982 and 2003. They find that home buyers' expectations are substantially affected by recent experience. Even after a long boom, home buyers typically have expectations that prices over the next 10 years will show double-digit annual price growth.

<sup>6</sup>By applying collateral constraints to both newly initiated mortgages and ongoing loans, we effectively rule out default. Default on mortgages is, until recently, relatively rare in reality. According to the Mortgage Bankers Association, the seasonally adjusted three-month default rate for a prime fixed-rate mortgage loans is around 2 percent prior to 2007.

( $0 \leq \psi \leq 1$ ) of the house value on repair and maintenance in order to keep the housing quality constant.

At the beginning of each period, the household receives a moving shock,  $D_t^m$ , that takes a value of 1 if the household has to move for reasons that are exogenous to our model, and 0 otherwise. The moving shock does not affect a renter's housing choice since moving does not incur any costs for him. When a homeowner receives a moving shock ( $D_t^m = 1$ ), he is forced to sell his house.<sup>7</sup> A homeowner who does not have to move for exogenous reasons can choose to liquidate his house voluntarily. The selling decision,  $D_t^s$ , is 1 if the homeowner sells and 0 otherwise. Selling a house incurs a transaction cost that is a fraction  $\phi$  ( $0 \leq \phi \leq 1$ ) of the market value of the existing house. Additionally, the full mortgage balance becomes due upon the sale of the home. Following a home sale—for either exogenous or endogenous reason—a homeowner faces the same decisions as a renter coming into period  $t$ , and is free to buy or rent for the new period.

## 2.2. Liquid Assets

In addition to holding home equity, a household can save in liquid assets which earn the same constant risk-free rate  $r$  as the borrowing rate.<sup>8</sup> We denote the liquid savings as  $S_t$  and assume that households cannot borrow non-collateralized debt, i.e.,

$$S_t \geq 0, \quad \text{for } t = 0, \dots, T. \quad (7)$$

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<sup>7</sup>We assume that house prices in the old and new locations are the same. Hence in our model households cannot move for differential house prices.

<sup>8</sup>Under the assumption of costless refinancing, the household will never simultaneously hold both liquid savings and a mortgage if different lending and borrowing rates are allowed. When the lending and borrowing rates are the same, there is an indeterminacy with respect to liquid saving and mortgage holdings. From the household perspective, paying down the mortgage by \$1 is equivalent to increasing his liquid savings by the same amount as long as the collateral constraint is satisfied (equation (6)). To pin down the investors bond holding, in our subsequent analysis, we assume that the household always carries the maximum mortgage balance allowed, i.e.,  $M_t = (1 - \delta)P_t^H H_t$ .

## 2.3. Wealth Accumulation and Budget Constraints

We denote the household's spendable resources or "wealth" upon home sale by  $Q_t$ .<sup>9</sup> It follows that

$$Q_t = S_{t-1}(1+r) + P_{t-1}^Y \exp\{f(t, Z_t)\} \nu_t \varepsilon_t + D_{t-1}^o P_{t-1}^H H_{t-1} [(1 + \tilde{r}_t^H)(1 - \phi) - (1 - \delta)(1 + r)]. \quad (8)$$

The intertemporal budget constraint, therefore, can be written as follows:

$$\begin{aligned} Q_t = C_t + S_t + [(1 - D_{t-1}^o)(1 - D_t^o) + D_{t-1}^o D_t^s (1 - D_t^o)] \alpha P_t^H H_t \\ + [(1 - D_{t-1}^o) D_t^o + D_{t-1}^o D_t^s D_t^o] (\delta + \psi) P_t^H H_t \\ + D_{t-1}^o D_t^o (1 - D_t^s) (\delta + \psi - \phi) P_t^H H_{t-1} \end{aligned} \quad (9)$$

The third term on the right-hand-side of the budget constraint represents housing expenditure by those who decide to be renters in the current period; the fourth term represents housing expenditure by households who decide to buy houses; and the fifth term represents housing expenditure of households who reside in their old houses.<sup>10</sup>

## 2.4. The Optimization Problem

We assume that upon death, a household distributes its spendable resources  $Q_t$  among "L" beneficiaries to finance their numeraire good and housing services consumption for one period, the latter through renting. Parameter "L" thus controls the strength of bequest motives. Under CES utility, this assumption results in the beneficiary's expenditure on numeraire good and housing service consumption at a proportion that is a function of house price:

$$\frac{C_t}{C_t + \alpha P_t^H H_t} = \frac{(1 - \omega)^\zeta}{(1 - \omega)^\zeta + \omega^\zeta (\alpha P_t^H)^{1-\zeta}} \quad (10)$$

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<sup>9</sup>Under this definition, conditional on selling his house, a homeowner's problem is identical to that of the renter and depends only on his age  $t$ , permanent income  $P_t^Y$ , house price per unit of housing services  $P_t^H$ , and liquidated wealth  $Q_t$ .

<sup>10</sup>For the last group of households, we need to subtract from their expenditure housing selling cost that was subtracted from wealth in hand on the left-hand-side.



Then the bequest function is defined by

$$B(Q_t) = LQ_t \left[ (1 - \omega) \left( \frac{(1 - \omega)^\zeta}{(1 - \omega)^\zeta + \omega^\zeta (\alpha P_t^H)^{1-\zeta}} \right)^{1-\frac{1}{\zeta}} + \omega \left( \frac{\omega^\zeta (\alpha P_t^H)^{-\zeta}}{(1 - \omega)^\zeta + \omega^\zeta (\alpha P_t^H)^{1-\zeta}} \right)^{1-\frac{1}{\zeta}} \right]^{\frac{1}{1-\frac{1}{\zeta}}}. \quad (11)$$

The household solves the following optimization problem at time  $t = 0$ , given its house tenure status ( $D_{-1}^0$ ), after-labor income wealth ( $Q_0$ ), permanent labor income ( $P_0^Y$ ), house price ( $P_0^H$ ), housing stock ( $H_{-1}$ ), and mortgage balance ( $M_{-1}(1 + r)$ ):

$$\max_{\{C_t, H_t, S_t, D_t^o, D_t^s\}} E \sum_{t=0}^T \beta^t \left\{ F(t) U(C_t, H_t; N_t) + [F(t-1) - F(t)] B(Q_t) \right\}, \quad (12)$$

subject to the mortgage collateral borrowing constraint (equation 6), the borrowing constraint on liquid assets (equation 7), wealth processes (equation 8), and the intertemporal budget constraints (equation 9). The parameter  $\beta$  is the time discount factor.

## 2.5. Characterization of Individual Housing and Consumption Behavior

We simplify the household's optimization problem by exploiting the problem's scale independence, and normalize level variables by households' permanent income. After normalization, the household's vector of choice variables become  $a_t = \{c_t, h_t, s_t, D_t^o, D_t^s\}$ , where  $c_t = \frac{C_t}{P_t^Y}$  is the consumption-permanent income ratio,  $h_t = \frac{P_t^H H_t}{P_t^Y}$  is the house value-permanent income ratio, and  $s_t = \frac{S_t}{P_t^Y}$  is the liquid asset-permanent income ratio. We can characterize a household's decision rule by his location  $x_t = \{D_{t-1}^o, q_t, \bar{h}_t, P_t^H\}$ , where  $q_t = \frac{Q_t}{P_t^Y}$  is the household's wealth-permanent labor income ratio, and  $\bar{h}_t = \frac{P_t^H H_{t-1}}{P_t^Y}$  is the beginning-of-period house value to permanent income ratio.

An analytical solution for our problem does not exist. We thus derive numerical solutions through value function iterations. Appendix A provides details of our numerical method.

Qualitatively, for a given household age, the household's optimal decision rules are similar to those reported in Li and Yao (2007). A renter's house tenure decision is largely determined by his wealth-income ratio. The more wealth a renter has relative to his income, the more likely he will buy as more wealth on hand enables the renter to afford the down payment for a house of desired value. The wealth-income ratio that triggers homeownership is U-shaped,

reflecting the high mobility rates of young households and short expected duration of an older household. Once becoming a homeowner, a household will stay in the house so long as his house value-income ratio is not too far from the optimal level he would have chosen as a renter given his wealth-income ratio, in order to avoid incurring transaction costs.

A renter's consumption and savings functions are similar to those identified in the precautionary savings literature with liquidity constraints. At low wealth levels, a renter continues to rent and spends all his wealth on numeraire good and rent payment. At relatively higher wealth levels, a renter starts saving for intertemporal consumption smoothing and housing down payment. For a homeowner who stays in his existing house, the value of his house also affects his nonhousing consumption, reflecting the effect of substitution between the two goods.

If the utility function takes the form of Cobb-Douglas in our setup as in most macro studies, the household's choice of homeownership and house value is invariant to house price changes. In other words, in solving households' problems, we do not need to separate  $P_t^H$  from  $H_t$  (see Li and Yao 2007). Under the more flexible CES utility, however, things are different. In particular, when the intratemporal elasticity of substitution parameter  $\zeta$  is smaller than one, when facing a higher house price a household will spend a larger share of his expenditure on housing. This leads to a higher house value-income ratio for the desired house. The more expensive house in turn requires a larger down payment, which slows down a household's transition to homeownership.

### 3. Data and Estimation Procedure

We implement a two-stage Method of Simulated Moments (MSM) to estimate our theoretical model. This methodology was first introduced by Pakes and Pollard (1989), and Duffie and Singleton (1993) to estimate structural economic models without close-form solutions. Since then, MSM has been successfully applied to estimations of preference parameters in Gourinchas and Parker (2002), Cagetti (2003), and Laibson, Repetto, and Tobacman (2007), labor supply decisions by French (2005), and medical expenses and the savings of elderly singles by De Nardi, French, and Jones (2006), among many others.

The mechanics of our MSM approach is standard. In the first stage, we estimate or calibrate the parameters that can be cleanly identified without explicit use of our model. In the second stage, we take as given the parameters obtained in the first stage and estimate the rest of the model parameters by minimizing the distance between the simulated moments derived from household decision profiles and those observed in the data. We provide detailed discussions of first and second stage estimation after describing our sources of data.

### 3.1. Data Sources

The main data we use in this study are taken from The University of Michigan Panel Study of Income Dynamics (PSID). The PSID is a longitudinal survey that has followed a nationally representative, random sample of families and their extensions since 1968. The survey detailed economic and demographic information for a sample of households annually from 1968 to 1997 and biannually after 1997. From 1984 through 1999, a wealth supplement to the PSID surveyed the assets and liabilities of each household at five-year intervals. The supplemental survey becomes biennial after 1999, coinciding with the main survey frequency.

For households to be included in our data sample, they have to be present in the 1984 survey but not in the 1968 sample of low income families. Observations were further deleted for the following reasons:

- The age of the household head is younger than 25 or older than 54 in the 1984 survey.
- The state of residence is missing. Households obtained housing as a gift, or live in housing paid by someone outside of the family unit, or owned by relatives.
- Households live in public housing project owned by local housing authority or public agency. Households neither own nor rent.
- The head of the household is female. The head of the household is a farmer or rancher.
- The head of the household does not have a valid age variable.
- The household head is unmarried in any wave of survey.
- The real household labor income is less than 10,000 or more than 1 million dollars.
- Information on households' net worth, income, or house value for home owners is missing.

The final sample consists of 17,396 observations of 1,069 households. We use this sample to estimate life-cycle income profile, as well as sample moments. We supplement PSID data with information from American House Survey (AHS) and the Office of Federal Housing Enterprise (OFHEO) for house price information, and Current Population Survey (CPS) for mobility and life expectancy information.

## 3.2. First-Stage Estimation and Calibration

We discuss our parameter choices in the first stage calibration in this subsection.

### 3.2.1. Life Cycle Income Profiles

The income in our model corresponds to after-tax non-financial income empirically. We calibrate the stochastic income process (equations 3 – 5) in the following manner. We first compute before-tax income as the total reported wages and salaries, social security income, unemployment compensation, workers compensation, supplemental social security, other welfare, child support, and transfers from relatives from both the head of household and his spouse.<sup>11</sup>

We then subtract from the households' pre-tax income defined above federal and state income tax liabilities as estimated by the National Bureau of Economic Research (NBER)'s TAXSIM program (Feenberg and Coutts 1993), which calculates taxes under the US Federal and State income tax laws from individual data, including marital status, wage and salary of household head and his or her spouse, and number of dependents.

The after-tax income is further deflated using non-shelter Consumer Price Index (CPI-NS) provided by Bureau of labor Statistics with year 2004-2005 normalized to 100. We refer to this deflated disposable income as household labor income in the paper.

Finally, we apply an approach similar to the one used in Cocco, Gomes, and Maenhout (2005) to estimate coefficients for a sixth-order polynomial function of age and retirement income replacement ratio, as well as standard deviation for permanent and transitory shocks to income. We estimate the standard deviation for the permanent income shock,  $\sigma_\varepsilon$ , to be

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<sup>11</sup>Recall that we only use married households from the PSID in our sample.

0.11, and for the transitory income shock,  $\sigma_\nu$ , to be 0.22. The income replacement ratio after retirement  $\theta$  is estimated to be 0.96. The technical details are provided in Appendix C.

### 3.2.2. Mortality, Mobility, and Household Composition

The conditional survival rates ( $\{\lambda_j\}_{j=0}^{j=T}$ ) are taken from the 1998 life tables of the National Center for Health Statistics (Anderson 2001). The exogenous moving rates are obtained by fitting a fifth-order polynomial of age to the CPS households moving across county in year 2005. The life-cycle profile of family equivalent size for all married couples in the PSID is computed following Scholz et al. (2006). These life-cycle profiles, along with the average after-tax labor income profile are presented in Figure 1.

### 3.2.3. The House Price Process

When solving for decision rules of our theoretical model, we assume that the rate of appreciation for house price  $r_t^H$  is serially uncorrelated. We set the mean rate of return to housing to 0 and the standard deviation  $\sigma_H$  to 0.10, similar to estimates in Campbell and Cocco (2003) and Flavin and Yamashita (2002). We further assume that there is no correlation between housing returns and shocks to labor income.

The house price demonstrates a strong positive trend over the sample period in 1984-2005. To better capture this trend, in simulating the model, we feed in the realized real housing return based on households' state of residence. Appendix D provides details on the construction of state level house price index over time.

### 3.2.4. Other Parameters

Other parameters in the first stage are largely chosen according to the literature. The decision frequency is annual. Households enter the economy at age 25 and lives to a maximum age of 100, i.e.,  $T = 75$ . The mandatory retirement age is 65 ( $J = 40$ ).

The annual real interest rate is set at 2.7 percent, approximately the average annualized post-WWII real return available on T-bills. The mortgage collateral constraint is set at 80

percent.<sup>12</sup> The consumption floor  $\eta$  is picked at a low 0.10 of permanent labor income. This number is within the range of those used in the literature (for example, De Nardi, French, and Jones 2006) and rarely binds in our simulation.

Table 2 summarizes parameters from our first-stage calibrations.

### 3.3. Second-Stage Estimation

In this subsection, we describe our choices of moment conditions and how they help to identify the structure parameters of our model. One major advantage of structural estimation of a rich life-cycle model is that it allows us to address potential biases directly, by replicating them in the simulation. For example, we account for the endogeneity of home ownership status, market frictions and incompleteness (for example, borrowing constraints and housing adjustment costs), by incorporating these features in our theoretical model. To mitigate potential biases caused by cohort and time effects, we group the households in our simulation to cohorts and subject them to the same house price shocks as in the data.

#### 3.3.1. Sample Moment Conditions

We estimate the following eight structure parameters in the second stage estimation:  $\beta$  – subjective time discount factor,  $\gamma$  – curvature measure for the utility function,  $L$  – bequest strength measure,  $\omega$  – housing expenditure share measure,  $\zeta$  – elasticity of intratemporal substitution,  $\phi$  – house selling costs,  $\psi$  – house maintenance costs, and  $\alpha$  – rental costs.<sup>13</sup>

To identify our structural parameters, we choose to match the average wealth, mobility rate, home ownership rate, rent–income ratio, and house value–income ratio profiles for three age-cohorts and for each year between 1985 and 2005.<sup>14</sup> The three age cohorts are constructed according to birth year. The first cohort consists of households whose heads were born between 1950 and 1959; the second cohort consists of households whose heads were born between 1940 and 1949; and the third cohort is made up of households with heads born between 1930 and

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<sup>12</sup>Using the 1995 American Housing Survey, Chambers, Garriga, and Schlagenhauf (2004) calculate that the down payment fraction for first time home purchases is 0.1979 while the fraction for households who previously owned a home is 0.2462.

<sup>13</sup>For  $\alpha$ , the estimation is performed in terms of renting premium, i.e.  $\alpha - r - \psi$ .

<sup>14</sup>We drop year 1984 in the moment matching since we initiate our simulation by randomly drawing households from the 1984 PSID data.

1939. Therefore, at the beginning of our sample year 1984, the three cohorts' age ranges are 25–34, 35–44, and 45–54, respectively.

In addition, to exploit the cross-sectional heterogeneity in house prices, for each age cohort–calendar year cell, we also match the average home ownership rate, rents, and house value profile for households residing in the most and least expensive states.<sup>15</sup>

Thus we have at most 11 moments for each age cohort–year cell, for a potential maximum of  $11 \times 21 \times 3 = 693$  moments. We lost 45 moments since wealth variables are only available for years 1984, 1989, 1994, 1999, 2001, 2003, and 2005. Further, we lost additional 18 moments since the rent variable is missing for 1988 and 1989. The number of total matched moments ends up as 630.<sup>16</sup>

The cell sizes are 434, 393, and 242 respectively at the start of the sample. These cell sizes declined to 277, 196, and 34 over time as some households dropped out of the survey.<sup>17</sup>

### 3.3.2. Construction of Simulated Moments

In the second stage estimation, we first choose a vector of structure parameters and solve the optimal decision rules as described in the previous section, taking the first stage parameters as given. We then simulate households' behavior to construct our simulated moments under the given choice of parameters.

To initialize our simulation, we randomly draw 1,000 households from each age group between 25 and 54 in the 1984 PSID data, for an initial simulated sample of 30,000 households.

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<sup>15</sup>We define the most expensive states as the 18 states with the highest house price level in 1995, and least expensive states as the 19 states with the lowest house price level. According to this definition, we have roughly equal number of households residing in the most expensive, least expensive, and medium price range states in 1984. We rank states based on house price in 1995 since it is the middle-year in our sample. The choice of 1995 is inconsequential since the ranking of house prices hardly changed during our sample period.

<sup>16</sup>We defer description about sample households' housing and wealth profile to the next section, where we discuss them in comparison to predictions from our model.

<sup>17</sup>We did not drop cell with a small size. In our weighting matrix, the cell with small sample count has a very low weight in the distance measure.

We then assign a series of moving, income, and house price shocks to each simulated path.<sup>18</sup> We update the simulated sample each time period based on the optimal decision rules.

Once all simulated paths are complete, we compute the average profiles for our target variables in the same way that we compute them from the real data, i.e. grouping by a calendar year  $\times$  age cohort cell. Finally we construct a model fitness measure by weighting the differences between average profile in the simulated model economy and the data with a weight matrix.<sup>19</sup>

The procedure is repeated until the weighted difference between the data and simulated profiles is minimized.<sup>20</sup> Appendix B provides more details on our MSM estimation technique.

## 4. Results

We present the estimation results in this section.

### 4.1. Housing and Wealth Profiles over Time and over the Life Cycle

Figures 3 to 10 show the fit of our baseline model to the empirical data profiles. The green solid line with solid dots depicts the empirical data profile, while the red dashed line marked with crosses represents the mean profile from our model.

Households become richer as they age for all cohorts. At the same time, older cohorts are also richer than young cohorts. The youngest cohort demonstrates a relatively slower rate of wealth accumulation for the first ten years in the sample, a behavior consistent with the existence of the borrowing constraint and precautionary savings motive.

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<sup>18</sup>While moving and income shocks come from computer random number generators governed by their respective stochastic process, the house price path comes from the actual realized house price in the household's state of residence in order to capture the aggregate trend in house price in the sample. By doing so, we allowed the ex post sample average house price appreciations over the short time period, which is used in simulation, to deviate significantly from the ex ante assumption of zero mean house price appreciation, which is used in the solution of optimal decision rules.

<sup>19</sup>The theoretically most efficient weighting matrix is the inverse of sample variance-covariance matrix. We use a diagonal matrix for weighting given our small sample size. Our weighting matrix takes the diagonal terms of the optimal weighting matrix for scaling purpose, while setting the off-diagonal term to be zero. A similar approach is adopted in De Nardi, French, and Jones (2006). According to Altonji and Segal (1996), the optimal weighting matrix, though asymptotically efficient, can be severely biased in small samples.

<sup>20</sup>The minimization of weighted moment distance is achieved through a combination of global population-based optimization (differential evolution method) and location non-gradient based search (simplex method).



Overall, the homeownership rate starts at around 70 percent for the youngest cohort, and quickly goes up to 90 percent in 10 years.<sup>21</sup> The other two older cohorts also demonstrate slight increases in homeownership rates over the sample period. By the end of the sample period, most households have achieved home ownership. This reflects the fact that most households make the transition to home ownership after they have accumulated enough wealth.

As expected, the homeownership rate of the youngest cohorts in the most expensive states is much lower than those in the least expensive states. For all three cohorts, the average house value–income ratios for those in the most expensive states are much higher than those in the least expensive states. The ratios also grow much faster over the sample period as well. While renters in the most expensive states also spend a larger share of their income in housing services, the trend over time is less clear since we have few renters in the sample, especially for the later years in the sample.

The moving rates are low in the sample, and are only over 10 percent for the youngest group in the part of earlier sample. The rates decrease slightly over time as households settle down, and are in the single digits over most of the sample years for the two older cohorts. The lack of moving points to large fixed costs associated with changing one’s residence.

Overall, our model captures the trend in data profiles reasonably well. We miss along some dimensions, though. The model generates lower wealth accumulation and higher rent expenditure than the data for the most senior cohorts. We have relatively fewer households in the old cohort in the data, especially renters. We suspect that the mismatch is largely caused by data idiosyncracies. Additionally, we abstract from considerations of more volatile equity market that plays a bigger role in the savings of old households.

## 4.2. Parameter Estimates and Identification

According to our estimation, the annual discount factor  $\beta$  is 0.96, and the risk aversion parameter is 6.19, both within the range viewed as plausible by most economists. The bequest strength  $L$  is estimated to be 1.00. While the time discount factor and risk aversion are largely

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<sup>21</sup>The overall homeownership rate in our sample is much higher than the country as a whole. This is due to our sample selection criterion in order to maintain household stability. Recall we only admit married couple with income above \$10,000 in our sample.

determined by the wealth accumulation earlier in life, the bequest strength is mostly driven by households' wealth profiles later in life.

As for the intratemporal utility function, the share parameter  $\omega$  is estimated to be a  $2.56 \times 10^{-4}$ , while the intratemporal elasticity of substitution between housing and non-housing consumption is estimated to be 0.33. These two parameters are largely identified through the cross-sectional as well as time series variation of house value-income ratio and home ownership rates. Households in expensive states spend more relative to their income on housing, both when renting and when owning. The higher house value-income ratio requires a larger down payment, which takes longer to accumulate and delays transition to homeownership. To illustrate the implications of our estimated  $\omega$  and  $\zeta$  parameters on the cross-sectional house expenditure patterns, we compute the implied fraction of total expenditure allocated to housing for all 50 states based on the house price in year 2005, and present it in Figure 2. The share varies from 13.1 percent for the cheapest state (Kansas) at \$46.2 per square foot, to 42.8 percent in the most expensive state (Washington D.C.) at \$493.6 per square foot.

Our point estimate of intratemporal elasticity of substitution is much smaller than the macro estimates. The difference between our estimate and the macro estimates results largely from the fact that the macro literature has examined the aggregate consumption data in time series. Aggregation masks important cross-sectional heterogeneities among households in terms of their housing expenditure patterns.

Our estimate is closer to some of the micro estimates. Hanushek and Quigley (1980) look at data from the Housing Allowance Demand Experiment, which involved a sample of low-income renters in Pittsburgh and Phoenix. Households in each city were randomly assigned to treatment groups which received rent subsidies that varied from 20 percent to 60 percent and a control group that received no subsidy. The estimated price elasticities were 0.64 for Pittsburgh and 0.45 for Phoenix. Siegel (2004) estimates the elasticity from the PSID over the period 1978-1997. He limits the sample to homeowners, using total household food expenditure as a proxy for nondurable consumption and the self-reported value of the owner occupied house for housing. Siegel uses a household's first report as the value of a house and assumes durable consumption is constant until the household moves. Aggregating across households and using only the time series information, the estimated elasticity of substitution is about 0.53. Flavin and Nagazawa (2007), by contrast, also use PSID over the period 1975 to 1985. Instead of using households' self-reported house value, they construct a housing

service measure and derive Euler Equation conditions on consumption for households who do not move. Their estimate of the elasticity of substitution between housing and non-housing consumption is a very low 0.13.<sup>22</sup>

The house selling cost parameter  $\phi$  is estimated to be 15 percent of the house value. This number is identified by the (low) level of mobility rate in the data. While this number looks large since a realtor typically charges a 5-6 percent commission for selling a house, the cost measure should also take into account search costs, moving costs, mortgage closing costs, as well as possible psychological costs.<sup>23</sup> Since our sample is for married couples only, we expect the moving cost to be even higher.

The house maintenance cost is estimated to be 2.26 percent of the house value, which implies that the user cost of home ownership is  $\psi + r_f - \mu_h = 4.96$  percent. While the cross-section *variation* of the house value–income ratio helps to pin down the intratemporal preference parameters, the *average* level of the same ratio identifies the house maintenance parameter.

Renting is estimated to incur an extra cost close to 1.85 percent of the property value. The spread is identified through homeownership profiles. The implied  $\alpha$  parameter, which is the sum of the cost of capital, maintenance, depreciation, and rental premium, is therefore, at 6.81 percent. Our estimation of rental costs is within the range, albeit at the lower end, of the user cost for home ownership as calculated by Himmelberg, Meyer, and Sinai (2005) for 46 metro areas.

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<sup>22</sup>By focusing on non-movers, Flavin and Nagazawa (2007)'s methodology is robust to the existence of adjustment cost. However their estimate could be sensitive to assumptions about borrowing constraints and other market incompleteness.

<sup>23</sup>Closing fees generally include: 1) loan origination fee; 2) loan application fee; 3) title search; 4) title insurance; 5) inspection fee; 6) appraisal fee, 7) credit report fee; 8) attorney / settlement fee; and 9) government recording and transfer charges. Unlike realtors' fees, these fees vary substantially from state to state and often depend on the amount of the loan, the amount of the down payment, and the credit worthiness of the borrower. Woodward (2003) estimates total closing costs to be \$4,050 on a house with a value of \$162,500, or 2.5 percent of the house value. Regarding the search and psychic cost of moving, using the Housing Allowance Demand Experiment, Bartik, Butler, and Liu (1992) found that the average household was willing to pay 10 to 17 percent of their current income to stay in their current residence rather than move. If we use the industry lending standard that house value is about 4 times of annual income, this amounts to 2.5 to 4.3 percent of house value. Adding together the estimated realtors' fee, closing cost, and psychic cost of moving, we obtain number of over 10 percent of house value associated with selling a house.

## 5. Policy Analysis

Using our estimated model, we now conduct policy experiments. The goal is to investigate how households respond to changes in house prices in conjunction with changes in credit accessibility in the mortgage market.

To conduct our experiment, we first draw the initial population from the 2005 PSID data, and then simulate it forward using the optimal decision rules. Between year 2005 and 2007, shocks to house price and income follow their realized counterparts at the state and national levels, respectively.<sup>24</sup> We grow household permanent income from year 2008 to 2010 according to rates forecasted by Macroeconomic Advisors (MA). Specifically, MA forecasts that real per capita disposable household income will grow at rates -0.0266, 0.0055, and 0.0125, respectively, for years 2008 to 2010. We set the growth rate to 0 for year 2011 since MA does not provide forecast beyond 2010. The growth path for house prices between year 2008 and 2011 depends on the specific experiments.

In the benchmark case, labeled "Flat", we assume a zero house price growth rate. In the second experiment, labeled "MA", we grow house prices at the rates forecasted by MA, i.e., -0.08, -0.0275, and -0.02, respectively, for years 2008 to 2010. We set the growth rate to 0 for year 2011. In the third experiment, labeled "CS", we change the house price growth path to the more pessimistic Case-Shiller forecast for the 20 city composite index. In particular, house price changes are forecasted to be -0.15, -0.13, -0.05, and -0.02 for years 2008 to 2011.<sup>25</sup> In the last experiment, labeled "CS+70% LTV", we repeat the third experiment, but restricts mortgage credit availability by imposing a lower maximum possible mortgage loan to value ratio of 70 percent as opposed to 80 percent. We report our benchmark simulation results in levels in Table 4 and all four simulation results as changes relative to 2007 in Table 5.<sup>26</sup>

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<sup>24</sup>We supplement these aggregate shocks with idiosyncratic shocks from the computer random number generator governed by their respective stochastic process.

<sup>25</sup>We calculated the house price forecast based on the futures price from the Chicago Mercantile Exchange.

<sup>26</sup>The reported aggregate statistics is based on a sample constant in age distribution. We achieve so by admitting one new young age group into the sample each year while dropping the oldest households from the sample. The aggregate statistics is then computed using population weight for each age group from from the 2000 Census. Specifically, we only include households between the age 30 and 80 for the calculation. In other words, a household that is 29 in 2005 will not appear in the calculation of the aggregate statistics in 2005, but will enter the 2006 calculation as the household turns 30. Similarly, a household who is 80 in 2005 will appear in the 2005 sample but will drop out of the 2006 sample.

MA forecasts a large decline of 2.66 percent in income 2008, followed by a small recovery of 0.55 percent in 2009, and a fast recovery of 1.25 percent in 2010. In the benchmark simulation, as reported in both Tables 4 and 5, homeownership rate dropped sharply, by over 2 percentage points, in 2008. This reflects many homeowners' inability to maintain their mortgage loan to value ratio and/or their house maintenance cost. Homeowners may also exit homeownership in order to obtain liquidity to maintain their non-housing consumption. The decline in homeownership rate was much more muted as income started to recover in 2009. By 2011, homeownership rate began to rise again. By comparison, the average house value for homeowners and average non-housing consumption for all households tracked the income growth path much more closely. Both fell significantly in 2008, but rose immediately as income started to recover. Note the drop in average house value and non-housing consumption in 2011 came from our assumption that income growth rate declined from 1.25 percent in 2010 to 0 in 2011. The changes in average house value among homeowners come from the selection effect as poor households have exited homeownership.

When the income changes are accompanied with modest house price declines, as in experiment "MA", homeownership rates fell through all four years. Interestingly, the fall in the first year is smaller than the benchmark case. This is because with lower house prices, more renters could now afford to become homeowners. More importantly, existing homeowners had less incentive to sell their houses for liquidity reasons as their house value has also declined. As house prices declined further, however, more and more homeowners were forced to sell as they could no longer maintain the required mortgage loan to value ratio. The average house value for homeowners fell steadily through all four years, reflecting the net outcome of a selection effect resulting from poor households exiting homeownership, and the direct effect of house price declines for those homeowners keeping their residence. The non-housing consumption changes followed the same pattern as that of the average house value, consistent with the complementarity of these two goods under our estimation.

The fall in homeownership rates, average house value for homeowners, and non-housing consumption become much more evident under a more pessimistic assumption regarding housing growth as shown in experiment CS, with the homeownership rate suffering the most decline, more than twice as much as those in MA. By 2011, with a cumulative house price decline of 35 percent (as opposed to 15 percent in MA), relative to 2007, homeownership rate dropped 7.38

percent, average house value dropped 29.07 percent, and non-housing consumption dropped 4.02 percent.

When the severe house price declines occur in an environment with reduced access to the credit market as in experiment "CS+70%LTV", the homeownership rate dropped even further as households found it harder to obtain credit to purchase or maintain homeownership. Interestingly, the declines in average house value and average non-housing consumption, were milder than those under CS. The milder declines in house value arise because a tighter borrowing limit forced out even more relatively poor households of homeownership. The milder declines in consumption reflect several forces. First, with lower house prices, renters spend less resources on housing and as a result, were able to spend more on non-housing consumption. Second, those who exit homeownership now have more liquidity to fund non-housing consumption. Last, only relatively richer households now can stay as homeowners and these households were more capable of weathering declines in house prices.

## 6. Conclusions and Future Extensions

In this paper, we provide a structural estimation of a dynamic model of household consumption over the life cycle augmented with housing. We explicitly model housing adjustment along both the extensive margin of owning versus renting and the intensive margin of house size. The model also includes a credit constraint in the form of collateral mortgage borrowing. The paper, thus, contributes to the analysis and understanding of household housing demand and the impact of housing market on household consumption, both housing and non-housing.

Our estimation indicates that the intratemporal elasticity of substitution between housing and non-housing consumption is about 0.33 and the housing adjustment cost for married stable households amounts to 15 percent of the house value. Policy experiments using the estimated model further reveal that households respond nonlinearly to house price changes with large house price declines leading to sizable drops in total homeownership rate as well as in non-housing consumption. Interestingly, in an environment with tightened lending conditions, while households homeownership decision becomes more sensitive to house price changes, their non-housing consumption is less affected.

There are many natural extensions to our model. One is to allow for richer household portfolio decisions by differentiating further between stock and bond. Another is to model the mortgage contract more explicitly and realistically in order to capture more closely the effect of financial market conditions on housing demand. We leave those to future research.

## Appendix A: Model Simplifications and Numerical Solutions

Given the recursive nature of the problem, we can rewrite the intertemporal consumption and investment problem as follows:

$$V_t(X_t) = \max_{A_t} \{ \lambda_t [U(C_t, H_t; N_t) + \beta E_t[V_{t+1}(X_{t+1})]] + (1 - \lambda_t)B(Q_t) \}, \quad (13)$$

where  $X_t = \{D_{t-1}^o, Q_t, P_t^Y, P_t^H, H_{t-1}\}$  is the vector of endogenous state variables, and  $A_t = \{C_t, H_t, S_t, D_t^o, D_t^s\}$  is the vector of choice variables.

We simplify the household's optimization problem by exploiting the scale-independence of the problem and normalize the household's continuous state and choice variables by its permanent income  $P_t^Y$ . The vector of endogenous state variables is transformed to  $x_t = \{D_{t-1}^o, q_t, \bar{h}_t, P_t^H\}$ , where  $q_t = \frac{Q_t}{P_t^Y}$  is the household's wealth-permanent labor income ratio, and  $\bar{h}_t = \frac{P_t^H H_{t-1}}{P_t^Y}$  is the beginning-of-period house value to permanent income ratio. Let  $c_t = \frac{C_t}{P_t^Y}$  be the consumption-permanent income ratio,  $h_t = \frac{P_t^H H_t}{P_t^Y}$  be the house value-permanent income ratio, and  $s_t = \frac{S_t}{P_t^Y}$  be the liquid asset-permanent income ratio. The evolution of normalized endogenous state variables is then governed by:

$$q_{t+1} = \frac{s_t(1+r) + D_t^o h_t [(1 + \tilde{r}_{t+1}^H)(1 - \phi) - (1 - \delta)(1 + r)]}{\exp\{f(t+1, Z_{t+1})\} \nu_{t+1}} + \varepsilon_{t+1}, \quad (14)$$

$$\bar{h}_{t+1} = \left[ \frac{1 + \tilde{r}_{t+1}^H}{\exp\{f(t+1, Z_{t+1})\} \nu_{t+1}} \right], \quad (15)$$

$$P_{t+1}^H = P_t^H (1 + \tilde{r}_{t+1}^H). \quad (16)$$

The household's budget constraint can then be written as

$$\begin{aligned} q_t = c_t + s_t + & [(1 - D_{t-1}^o)(1 - D_t^o) + D_{t-1}^o D_t^s](1 - D_t^o) \alpha h_t \\ & + [(1 - D_{t-1}^o) D_t^o + D_{t-1}^o D_t^s D_t^o](\delta + \psi) h_t \\ & + D_{t-1}^o D_t^o (1 - D_t^s)(\delta + \psi - \phi) \bar{h}_t + \eta. \end{aligned} \quad (17)$$



Define  $v_t(x_t) = \frac{V_t(X_t)}{(P_t^Y)^{1-\gamma}}$  to be the normalized value function, then the recursive optimization problem (13) can be rewritten as:

$$v_t(x_t) = \max_{a_t} \left\{ \lambda_t \left[ \frac{N_t^\gamma}{1-\gamma} \left( (1-\omega)c_t^{1-\frac{1}{\zeta}} + \omega(h_t/P_t^H)^{1-\frac{1}{\zeta}} \right)^{\frac{1-\gamma}{1-\frac{1}{\zeta}}} \right. \right. \\ \left. \left. + \beta E_t \left( v_{t+1}(x_{t+1}) (\exp\{f(t+1, Z_{t+1})\} \nu_{t+1})^{1-\gamma} \right) \right] \right. \\ \left. + (1-\lambda_t) \frac{L^\gamma q_t^{1-\gamma}}{1-\gamma} \left[ (1-\omega) \left( \frac{(1-\omega)^\zeta}{(1-\omega)^\zeta + \omega^\zeta (\alpha P_t^H)^{1-\zeta}} \right)^{1-\frac{1}{\zeta}} \right. \right. \\ \left. \left. + \omega \left( \frac{\omega^\zeta (\alpha P_t^H)^{-\zeta}}{(1-\omega)^\zeta + \omega^\zeta (\alpha P_t^H)^{1-\zeta}} \right)^{1-\frac{1}{\zeta}} \right]^{\frac{1-\gamma}{1-\frac{1}{\zeta}}} \right\},$$

subject to

$$c_t > 0, \quad h_t > 0, \quad s_t \geq 0, \quad l_t \leq 1 - \delta,$$

and equations (15) to (17), where  $a_t = \{c_t, h_t, s_t, D_t^o, D_t^s\}$  is the normalized vector of choice variables. Hence the normalization reduces the number of continuous state variables to three with  $P_t^Y$  no longer serving as a state variable.

We discretize the wealth–labor-income ratio ( $q_t$ ) into 160 grids equally-spaced in the logarithm of the ratio, the house value-labor income ratio ( $\bar{h}_t$ ) into equally-spaced grids of 80, and the house price ( $P_t^H$ ) into 80 grids equally-spaced in the logarithm of the price. The boundaries for the grids are chosen to be wide enough so that our simulated time series path always falls within the defined state space.

Under the assumption that only liquidated wealth will be passed along to beneficiaries, the household's house tenure status and housing positions do not enter the bequest function. At the terminal date  $T$ ,  $\lambda_T = 0$ , and the household's value function coincides with the bequest function,

$$v_T(x_T) = \frac{L^\gamma q_t^{1-\gamma}}{1-\gamma} \left[ (1-\omega) \left( \frac{(1-\omega)^\zeta}{(1-\omega)^\zeta + \omega^\zeta (\alpha P_t^H)^{1-\zeta}} \right)^{1-\frac{1}{\zeta}} + \omega \left( \frac{\omega^\zeta (\alpha P_t^H)^{-\zeta}}{(1-\omega)^\zeta + \omega^\zeta (\alpha P_t^H)^{1-\zeta}} \right)^{1-\frac{1}{\zeta}} \right]^{\frac{1-\gamma}{1-\frac{1}{\zeta}}}. \quad (18)$$

The value function at date  $T$  is then used to solve for the optimal decision rules for all admissible points on the state space at date  $T - 1$ .

For a household coming into period  $t$  as a renter ( $D_{t-1} = 0$ ), we perform two separate optimizations conditional on house tenure choices – renting or owning – for the current period. A renter's optimal house tenure choice for the current period is then determined by comparing

the contingent value functions of renting and owning. To calculate the expected next period's value function, we use two discrete states to approximate the realizations of each of the three continuous exogenous state variables ( $\ln \varepsilon$ ,  $\ln \nu$ , and  $\tilde{r}_t^H$ ) by Gaussian quadrature (Taughen and Hussey 1991). Together with two states for the realizations of moving shocks, the procedure results in sixteen discrete exogenous states for numerical integration. For points that lie between grid points in the state space, depending on the household's current period house tenure choice, we use either a two-dimension or a three-dimension cubic spline interpolation to approximate the value function.

For a household coming into period  $t$  as a homeowner, we perform an optimization conditional on staying in the existing house for the current period. In this case, the household cannot adjust its house value-income ratio, i.e.  $h_t = \bar{h}_t$ , but can adjust its numeraire consumption. The value function contingent on moving – either endogenously or exogenously – is the same as the value function of a renter who is endowed with the same wealth-income ratio ( $q_t$ ) and house price ( $P_t^H$ ). We compare the value functions contingent on moving and staying to determine the optimal house liquidation decision. Under our assumption and parameterization, a homeowner always has positive amount of equity in his house after home sales and thus has no incentive to default. A homeowner who cannot satisfy the mortgage collateral constraint or afford the house maintenance cost has to sell his home. This procedure is repeated recursively for each period until the solution for date  $t = 0$  is found.

## Appendix B: Estimation Mechanics in the MSM Estimator

We assume that the “true” parameter vector

$$\theta^* = \{\beta, \gamma, L, \omega, \zeta, \phi, \psi, \alpha\}$$

lies in the interior of the compact set  $\Theta \subset \mathbb{R}^{11}$ . Our estimator,  $\hat{\theta}$ , is the value of  $\theta$  that minimizes the weighted distance between the estimated life cycle profiles for life cycle profiles for wealth, mobility rate, home ownership rate, house value, and rent observed from the data and the simulated profiles generated by the model. We choose to match these five variables, which are interacted with age cohort ( $T = 3$ ) and calendar year ( $C = 3$ ). Additional interactions are used for last three house related variables, which are further interacted with three house price levels in the state where a household resided. This interaction results in additional six moments. The moment count per year and cohort is therefore equal to  $11(5+6)$ . The overall count of moments is  $11 \times C \times T = 33T$ . We combine all these moment conditions by stacking them and solving the optimal problems jointly.

Given a data set of  $I_c$  independent individuals within a given age cohort  $c$  who are observed repeatedly for  $T$  periods, let  $\delta(\theta)$  denote a vector of moment conditions with  $11T$  elements, with  $\hat{\delta}$  representing its sample counterpart. The MSM estimator  $\hat{\theta}$  is given by

$$\operatorname{argmin}_{\theta} \sum_{c=1}^C \frac{I_c}{1 + \tau_c} \hat{\delta}_{I_c}(\theta)' \widehat{W}_{I_c} \hat{\delta}_{I_c}(\theta), \quad (19)$$

where  $\widehat{W}$  is a  $11T \times 11T$  weighting matrix, and  $\tau_c$  is the ratio of the number of observations in data for cohort  $c$  to the number of simulated observations. If the regularity conditions presented in Pakes and Pollard (1989) are met, our MSM estimator  $\hat{\theta}$  is both consistent and asymptotically normally distributed:

$$\sqrt{I}(\hat{\theta} - \theta^*) \rightsquigarrow N(0, V),$$

with the variance-covariance matrix  $V$  given by

$$V = (1 + \tau)(D'WD)^{-1}D'WSWD(D'WD)^{-1},$$

where  $S$  is the variance-covariance matrix of the data, and

$$D = \left. \frac{\partial \delta(\theta)}{\partial \theta'} \right|_{\theta=\theta^*}, \quad (20)$$

which is the  $33T \times 11$  Jacobian matrix of the population moment vector; and  $W = \text{plim}_{T \rightarrow \infty} \rightsquigarrow \{\widehat{W}_I\}$ . Newey (1985) presents the following  $\chi^2$  statistic for specification testing the moment estimator.

$$\frac{I}{1 + \tau} \widehat{\delta}_I(\widehat{\theta})' Q^{-1} \widehat{\delta}_I(\widehat{\theta}) \rightsquigarrow \chi_{33T-11}^2,$$

where  $Q^{-1}$  is the generalized inverse of

$$\begin{aligned} Q &= PSP \\ P &= I - D(D'WD)^{-1}D'W. \end{aligned}$$

Analogous to the optimal weighting matrix in a GMM model, the efficiency of our SMM estimator improves as  $\widehat{W}_I$  converges to  $S^{-1}$ , which is the inverse of the sample variance-covariance matrix. If  $W = S^{-1}$ , then  $V$  is reduced to  $(1 + \tau)(D'S^{-1}D)^{-1}$ , and  $Q$  is equivalent to  $S$ . According to Altonji and Segal (1996), the optimal weighting matrix, though asymptotically efficient, can be severely biased in small samples. We use a diagonal matrix for weighting given our small sample size. Our weighting matrix takes the diagonal terms of the optimal weighting matrix for scaling, while setting the off-diagonal term to be zero. A similar approach is adopted in De Nardi, French, and Jones (2006).

## Appendix C: Constructing Labor Income Process

Using PSID households from 1984 to 2005, we eliminate the Survey of Economic Opportunities subsample and households live in public housing project owned by local housing authority or public agency. We further exclude households that neither own nor rent or whose head is female, a farmer or rancher. We use only households whose heads were married and between 20 and 70 years of age. As described in Section 4, the federal and state income tax liabilities are obtained from the TAXSIM simulation program. We regress the logarithm of after-tax household labor income on dummy variables for age, education, and household composition, using a household fixed effect model. A fifth-order polynomial is used to fit the age dummies in order to obtain the labor income profile, which is presented in Figure 1. Furthermore, the replacement ratio  $\theta$  in equation (5), which determines the amount of retirement income, was approximated as the ratio of the average of our labor income variable defined above for retiree-headed households to the average of labor income in the last working year.

Following the variance decomposition procedure described by Carroll and Samwick (1997), we first define a  $d$ -year income difference as follows:

$$r_d = [\log(Y_{t+d}) - \log(P_{t+d}^Y)] - [\log(Y_t) - \log(P_t^Y)].$$

Thus,

$$\text{Var}(r_d) = d * \sigma_\varepsilon^2 + 2 * \sigma_\nu^2.$$

We then regress  $\text{Var}(r_d)$ 's calculated from the data on  $d$ 's to obtain estimates on  $\sigma_\varepsilon^2$  and  $\sigma_\nu^2$ . We choose  $d$  to be 1,2,...,22.

## Appendix D: Constructing House Price Series at State Level

Our state-level house price index (HPI) comes from the Office of Federal Housing Enterprise Oversight (OFHEO). The HPI is a time series price index that is set to 100 for every state for the base year 1980. This price index is thus not comparable cross-sectionally. To create a series of state-level price index that is also cross-sectionally comparable, we utilize the housing price information from the PSID. In particular, we define house prices as prices per square footage of living space. Unfortunately, PSID does not provide information on living space and we have to impute the square footage of homes for our data. Following Flavin and Nakagawa (2007), we first use data from the American Housing Survey (AHS) (1985-2005) to estimate a model of square footage as a function of the number of rooms and other housing characteristics common to both the AHS and the PSID, such as dummy variables representing whether the household was 1) located in a suburb, 2) located in a non-SMA region, 3) living in a mobile home, and a third order polynomial in the number of rooms. Separate models were estimated for each of the four regions (Northeast, Mideast, South, and West). The regional models estimated from the AHS data, reported in Table 1, were then used to generate estimated square footage data for each PSID household. Using these estimates, we predict house sizes for all homeowners in our PSID sample. The nominal house prices per square foot are then obtained by dividing the house value reported from the PSID by the predicted house size. The nominal house prices for individual households are then collapsed by state and year to obtain average house prices. For each state, we can use the imputed nominal price in any year, along with the HPI from OFHEO to calculate the nominal house price for a benchmark year, 1993, which is the midpoint of the time frame of our data. Given the fact that OFHEO and PSID surveyed different random sample of American households, we anticipate that the nominal prices for 1993 converted from different years might vary. We therefore choose to use the median of these converted values. Once the median nominal price is determined for each state in the benchmark year, we can scale the HPI from OFHEO so that the new HPI for each state  $i$  in year  $t$  as follows,  $HPI_{i,t}^{new} = HPI_{i,t}^{OFHEO} * NominalPrice_{i,1993} / HPI_{i,1993}^{OFHEO}$ .

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**Table 1**  
**Relationship Between House Size and Housing Characteristics**  
**(Dependent variable: House size in square feet)**

Independent Variables	Northeast	Midwest	South	West
Constant	-69.40 (51.47)	89.45 (45.40)	456.46 (34.71)	221.22 (32.85)
Urban	-75.44 (11.55)	-94.50 (8.05)	-91.32 (5.70)	-113.10 (8.47)
MSA	27.62 (14.07)	67.48 (8.09)	41.41 (5.84)	9.76 (8.88)
Mobile home	-492.63 (25.44)	-467.63 (15.46)	-299.46 (8.87)	-236.33 (12.53)
# rooms	282.68 (21.92)	204.28 (19.98)	-40.10 (15.01)	107.60 (13.86)
(# rooms) <sup>2</sup>	20.88 (3.12)	27.39 (2.87)	55.90 (2.12)	34.55 (1.97)
(# rooms) <sup>3</sup>	-1.55 (0.14)	-1.71 (0.13)	-2.50 (0.09)	-1.70 (0.86)
Adjusted R <sup>2</sup>	0.23	0.25	0.30	0.33
Number of observations	77,126	108,727	159,671	94,800

Notes: Data is from 1987 to 2005 biannual American Housing Survey. Robust standard errors are reported in parentheses. We don't report estimates of survey year dummies.

**Table 2**  
**Calibrated Parameters**

Parameter	Symbol	Value
<i>Demographics</i>		
Maximum life-cycle period	$T$	75
Mandatory retirement period	$J$	40
<i>Labor Income and House Price Processes</i>		
Standard deviation of permanent income shock	$\sigma_v$	0.10
Standard deviation of temporary income shock	$\sigma_\varepsilon$	0.22
Income replacement ratio after retirement	$\theta$	0.96
Standard deviation of housing return	$\sigma_H$	0.100
<i>Liquid Savings</i>		
Risk-free interest rate	$r$	0.027
<i>Housing and Mortgage</i>		
Down payment requirement	$\delta$	0.200

**Table 3**  
**Estimated Structural Parameters**

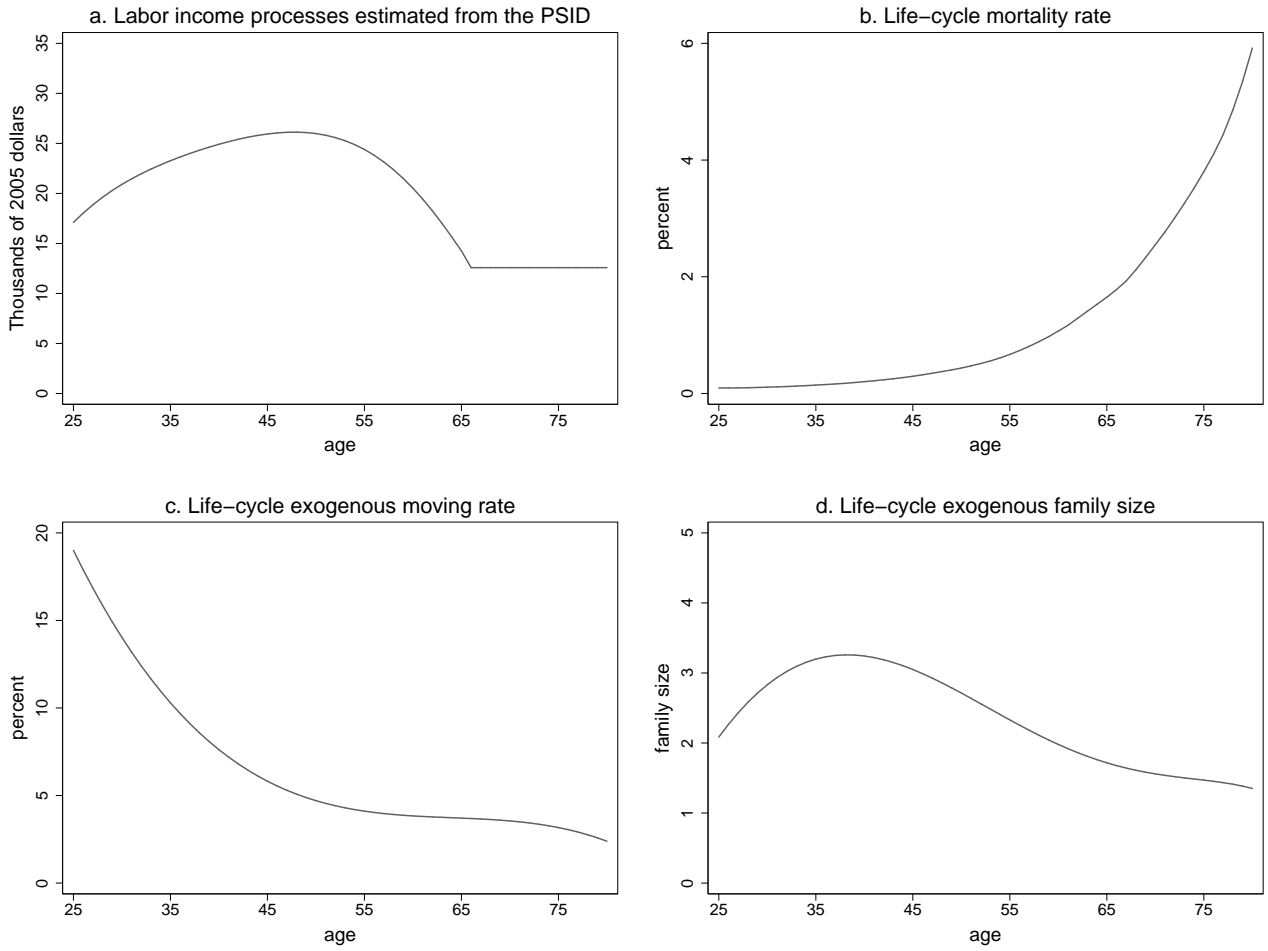
Parameter	Symbol	Value	Std Err
Discount rate	$\beta$	0.961	0.003
Curvature parameter	$\gamma$	6.186	0.059
Bequest strength	$L$	1.001	0.046
Housing service share	$\omega$	2.557e-04	9.207e-06
Intra-temporal elasticity of substitution	$\zeta$	0.323	9.379e-04
Housing selling cost	$\phi$	0.149	6.546e-05
Housing maintenance cost	$\psi$	0.026	1.491e-05
Rental premium		0.018	1.491e-05

**Table 4**  
**Policy Analysis: The Benchmark Simulation**

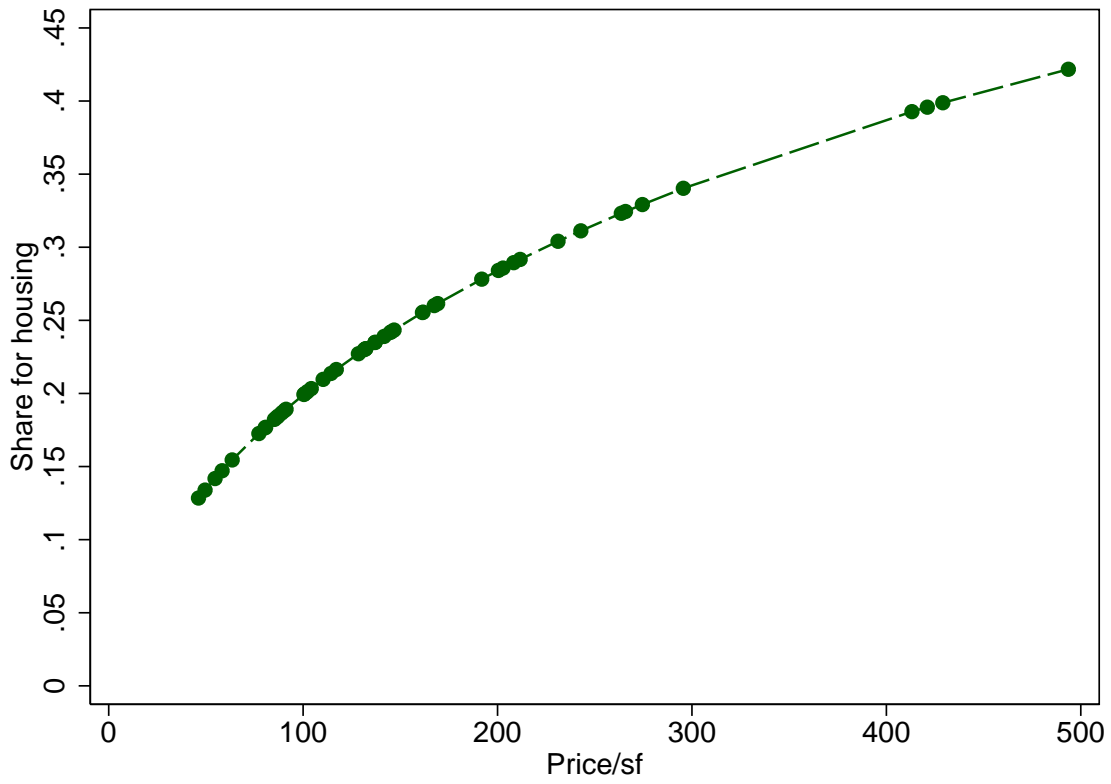
Year	Homeownership rate (%)	Average house value (2004\$)	Non-housing consumption (2004\$)
2007	90.63	340,084	59,485
2008	87.77	262,867	57,183
2009	87.11	285,812	58,111
2010	87.07	309,739	58,637
2011	88.04	320,724	58,536

Note. In the benchmark simulation, we let the mean growth rate of house price changes be flat and households' permanent income grow at rates forecasted by Macroeconomic Advisors (MA), i.e., -2.66 percent, 0.55 percent, 1.25 percent and 0 percent, respectively, for years 2008 to 2011.

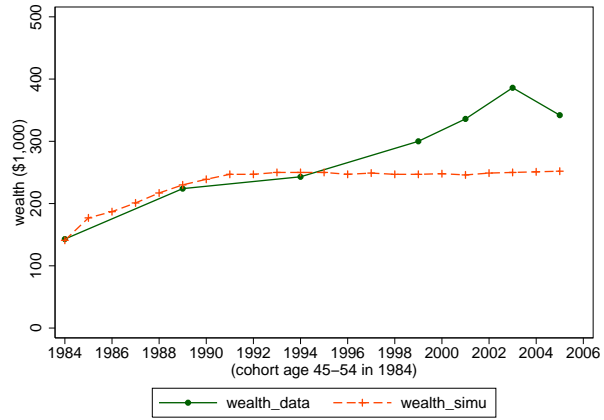
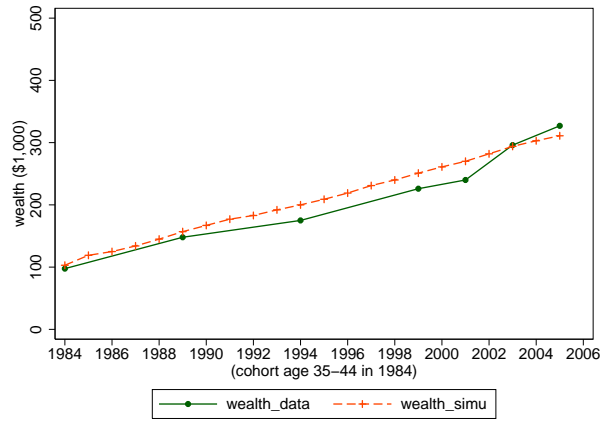
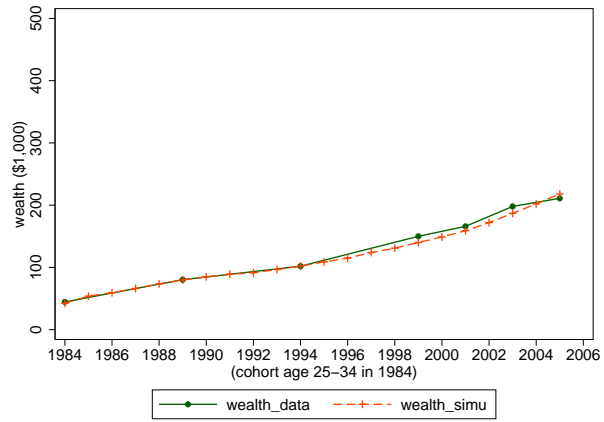




**Figure 1.** Exogenous processes in the model



**Figure 2.** Simulated housing expenditure shares



**Figure 3.** Wealth by cohorts in all states



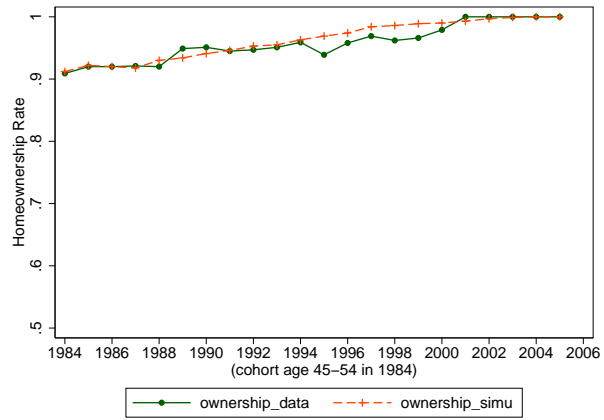
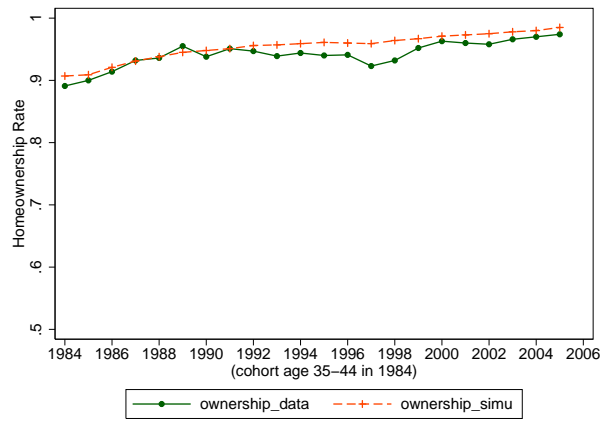
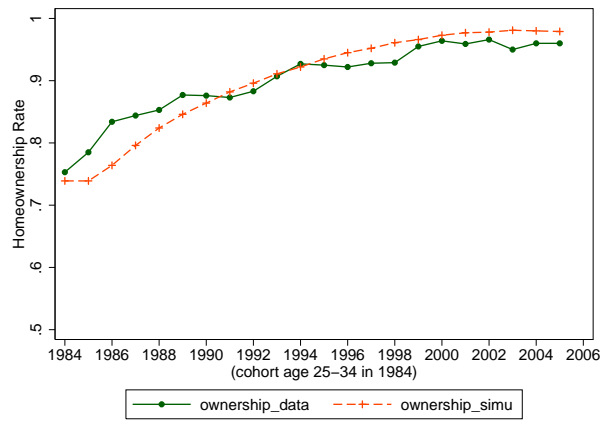
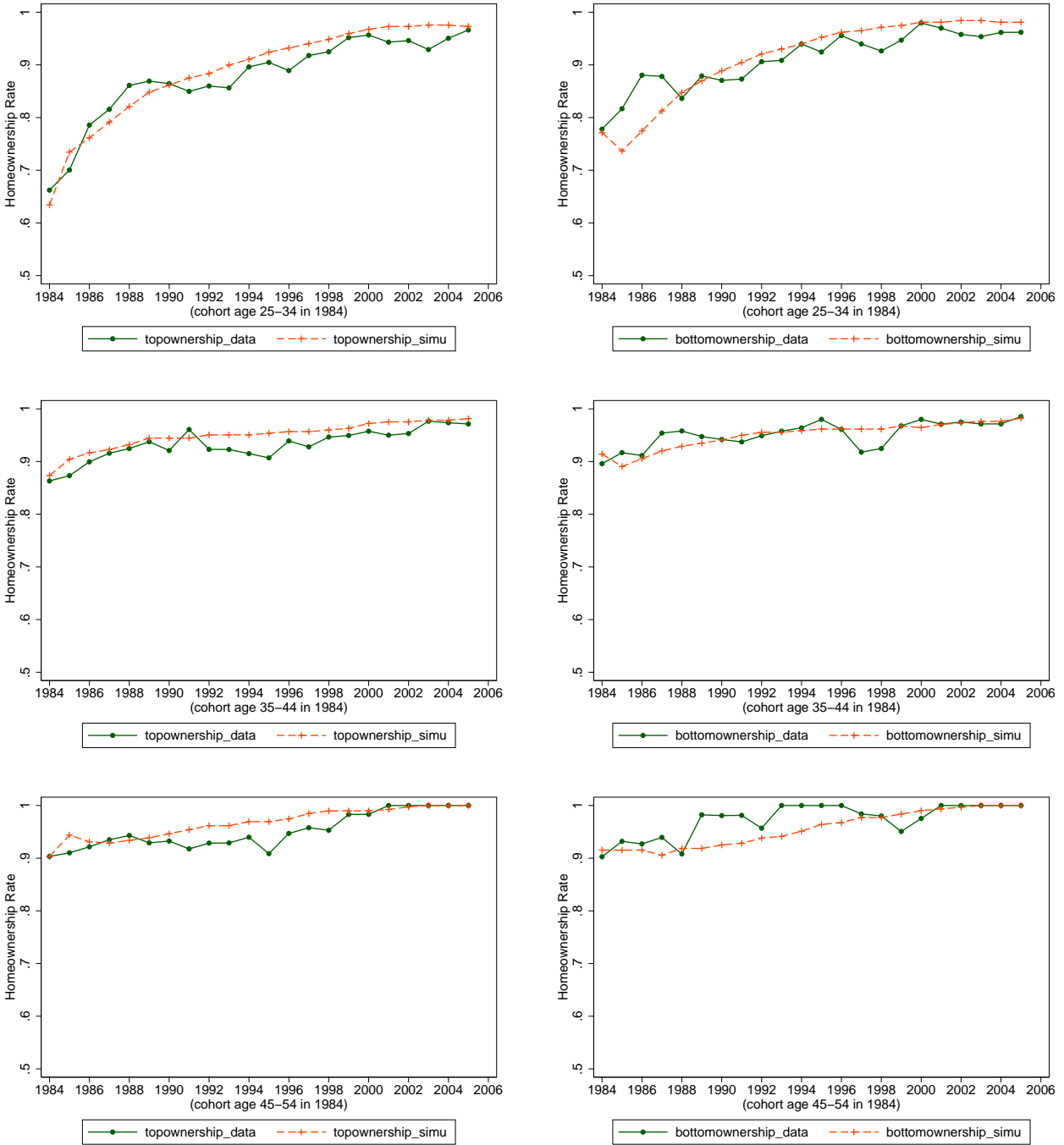
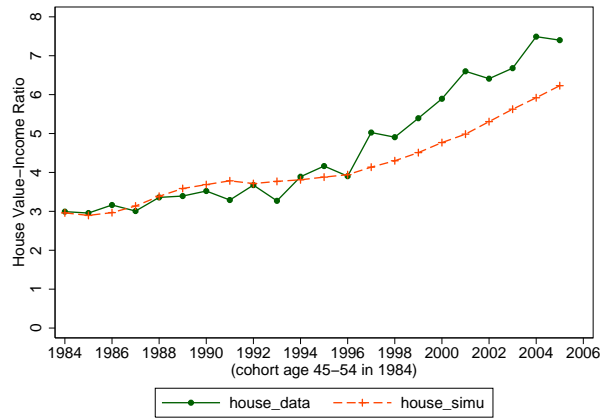
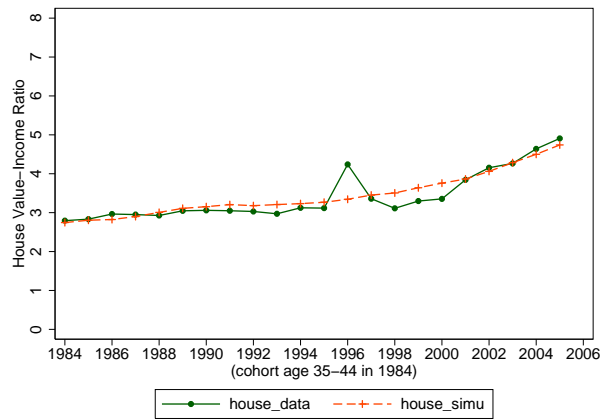
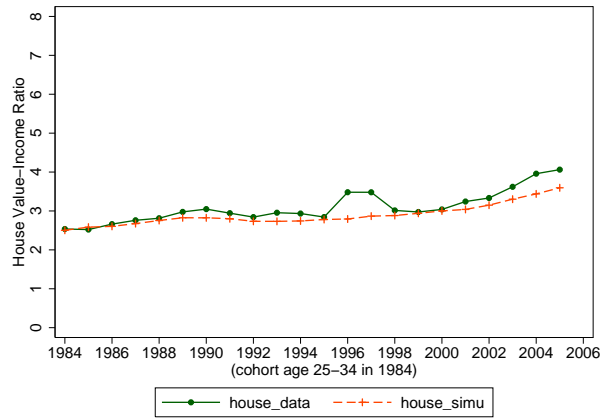


Figure 4. Home ownership by cohorts in all states



**Figure 5.** Home ownership by cohorts in high and low house price states



**Figure 6.** House value-income ratio by cohorts in all states

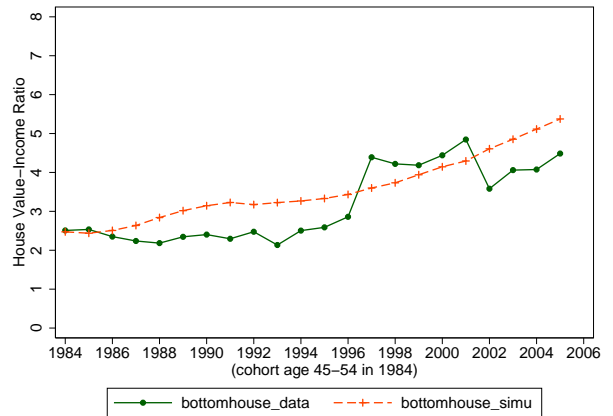
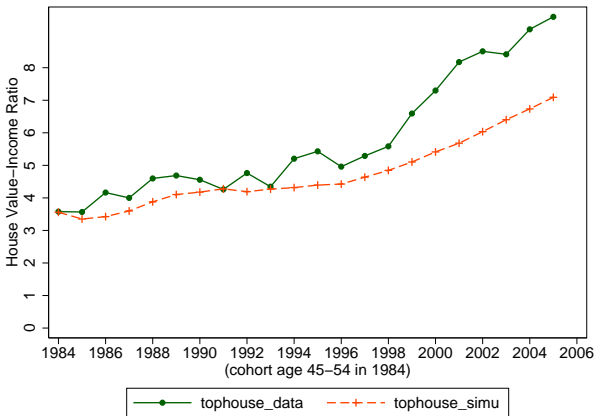
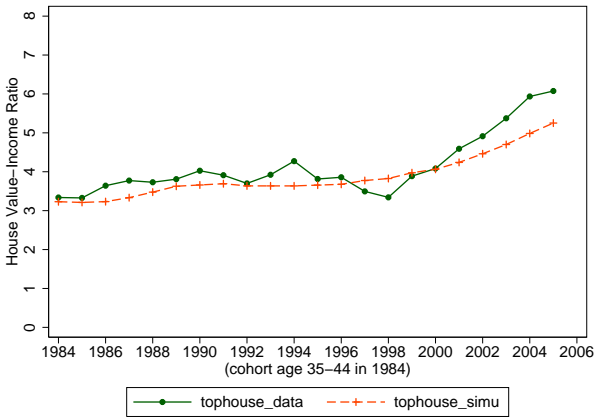
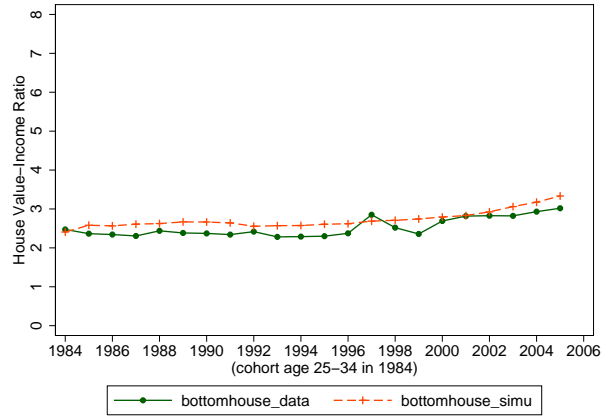
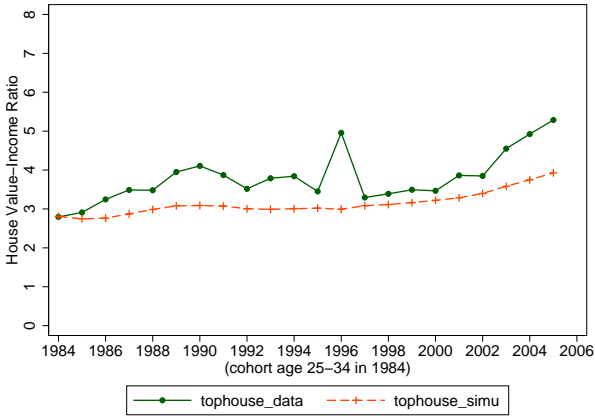


Figure 7. House value-income ratio by cohorts in high and low house price states

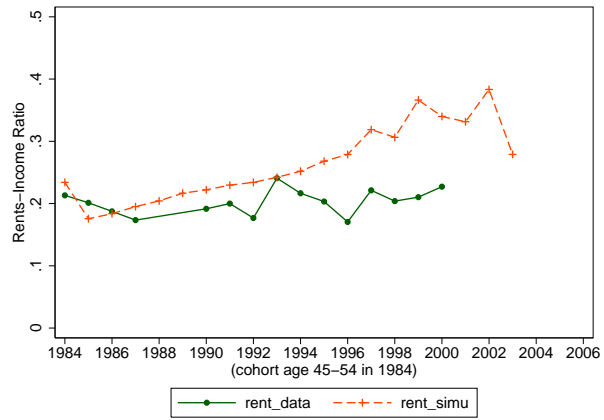
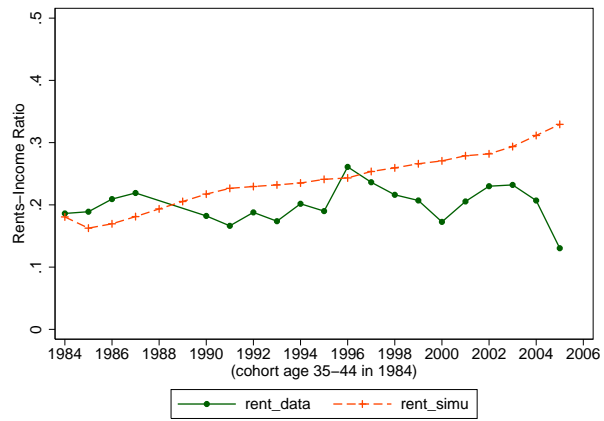
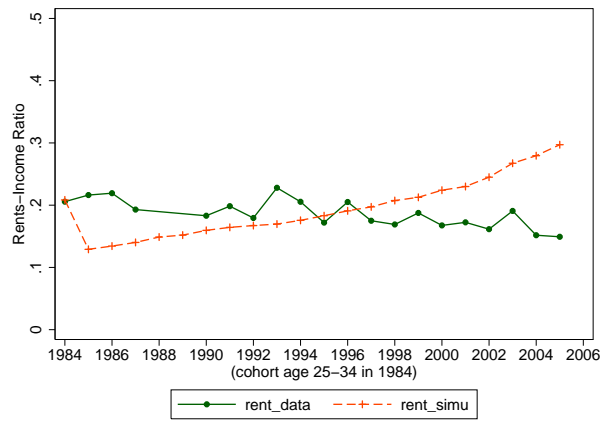
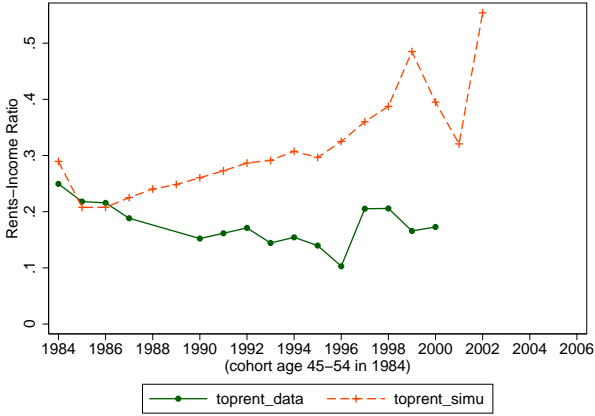
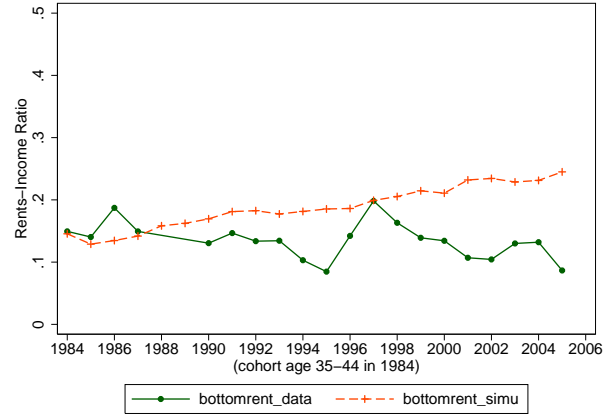
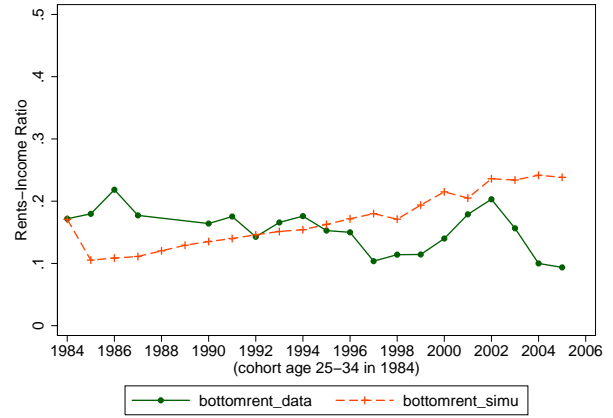
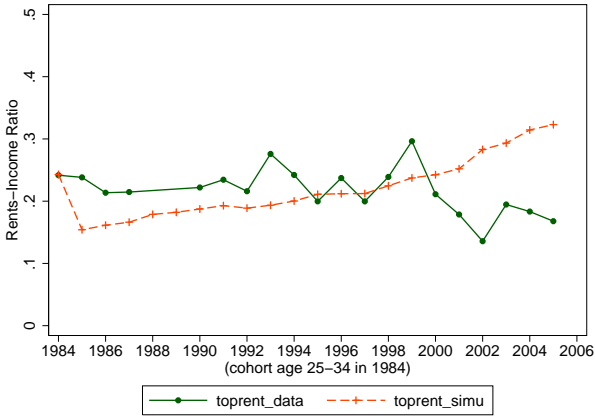


Figure 8. Rent-income ratio by cohorts in all states



**Figure 9.** Rent-income ratio by cohorts in high and low house price states

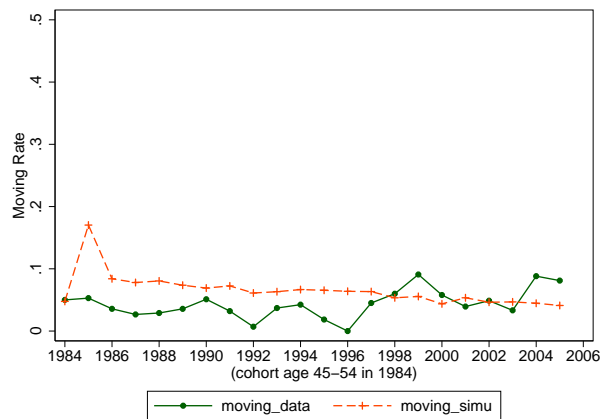
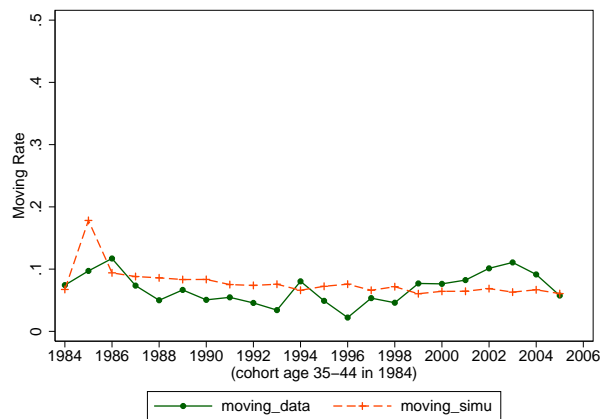
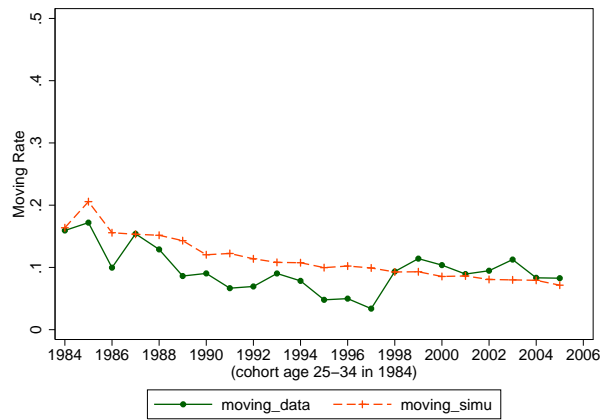


Figure 10. Homeowners' mobility by cohorts in all states