Winners and Losers in Housing Markets*

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September 2007

*We would like to thank Francesco Caselli, Joao Cocco, Morris Davis, Marco Del Negro, Wouter den Haan, Xavier Gabaix, Bernardo Guimaraes, Matteo Iacoviello, Dirk Krueger, Victor Rios-Rull, Nicholas Souleles, Noah Williams and participants of presentations at the Atlanta Fed, the Bank of England, the CDMA conference at St Andrews, the New York Fed, the Philadelphia Fed, Princeton, Tokyo, ESSIM, SED and the NBER Summer Institute for helpful comments. An older version of this paper was circulated with the title "From Shirtsleeves to Shirtsleeves in a Long Lifetime". Financial support from the ESRC "World Finance and the Economy" program under grant RES-165-25-0025 is gratefully acknowledged.

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Abstract

This paper is a quantitatively-oriented theoretical study into the interaction between housing prices, aggregate production, and household behavior over a lifetime. We develop a life-cycle model of a production economy in which land and capital are used to build residential and commercial structures. We find that, in an economy where the share of land in the value of structures is large, housing prices react more to an exogenous change in expected productivity or the world interest rate, causing large redistribution effects between net buyers and net sellers of houses. Changing the financing constraint, however, has limited effects on housing prices.

JEL Classification: E20, R20, R30.

Key Words: Real estates, Land, Housing Prices, Life cycle, Collateral constraints.

1 Introduction

Over the last few decades, we observe considerable fluctuations in real estate values and aggregate economic activities in many economies. In Japan, both the real capital gains on real estate during the prosperous decade of the 1980s and the losses during the depressed decades of the 1990s and the early 2000s are in the order of multiple years worth of GDP. Recent fluctuations in housing prices in many countries raise concerns. To what extent are these housing price fluctuations consistent with fundamental conditions? How do the fluctuations affect the welfare of different groups of households? In this paper, we develop a life-cycle model to investigate how housing prices, aggregate production and the wealth distribution react to changes in technology and financial conditions. After checking whether the model is broadly consistent with the life-cycle of home ownership and consumption, we use the model to assess which groups of households gain and which groups lose from changes in fundamentals.

To develop a theoretical framework, we take into account the limitation on the supply of land and the limitation on the enforcement of contracts in real estate and credit markets. Land (or location) is an important input for producing residential and commercial structures. Because the supply of land is largely inelastic and because the price of structures includes the value of land, the price of structures is sensitive to a change in the expected productivity growth rate and the real interest rate in equilibrium. We also consider incomplete contract enforcement to be an essential feature of an economy with real estate. Often, because landlords are afraid that the tenant may modify the property against their interests (or disputes may arise over splitting the modification costs), landlords restrict tenants' discretion over the use and modification of the house, and tenants enjoy lower utility from renting the house compared to owning and controlling the same house. If there were no other frictions, then the household would buy the house straight away. The household, however, may face a financing constraint, because the creditor fears that the borrowing household may default. The creditor demands the borrower to put his house as collateral for a loan and asks him to provide a downpayment from his own net worth.

In this paper, we take the importance of land for production of structures, the loss of utility from rented housing and the tightness of collateral constraints as exogenous parameters, and examine how they affect household consumption and housing over the lifetime, housing prices and aggregate quantities.[1] For this purpose, we develop an overlapping generations model of a production economy in which land and capital are used to produce residential and commercial structures. We are also interested in the way households cope with idiosyncratic and uninsurable shocks to their labor income, which helps to generate realistic housing choices and wealth distribution.

The interaction between the collateral constraint and the loss of utility from renting a house turns out to generate a typical pattern of consumption and housing over a life-cycle. When the household is born (or becomes independent) without any inheritance, it cannot afford a sufficiently high downpayment for buying a house; the household rents and consumes modestly to save for a downpayment. When some net worth has been accumulated, the household buys a house subject to the collateral constraint, which is smaller than a house that would be bought without the collateral constraint. As net worth further rises, the household upgrades along the housing ladder with the collateral constraint continuing to be binding. At some stage, the household finds it better to start repaying the debt rather than maximizing the size of the house. When the time comes for retirement, possibly with idiosyncratic risk attached, the household moves to a smaller house, anticipating a lower income in the future.

In equilibrium, due to the limitation of land supply, the supply of structures tends to grow more slowly than final output causing an upward trend in the real rental price and the purchase price of structures. The more important is land for producing structures compared to capital (other parameters being equal), the higher is the expected growth rate of the rental price, and therefore the higher is the housing price-rental ratio.[2] This is true for a country like Japan or a metropolitan area. In such an economy, the household needs a larger downpayment relative to wage income in order to buy a house, and tends to buy a

[1] Here, the importance of land for production of structures is defined as the elasticity of structure supply with respect to land for a fixed level of the other input. See equation (2) later.

[2] The price-rental ratio is an increasing function of the importance of land, also because the effective depreciation rate of structures is a decreasing function of the importance of land (which does not depreciate).

house later in life, resulting in a lower home-ownership rate.

Moreover, in an economy where land is more important for producing structures compared to capital, we find the housing price to be more sensitive to exogenous changes in fundamentals such as the expected growth rate of labor productivity or the world interest rate, along the perfect foresight path from one steady state to another. Del Negro and Otrok (2007) find empirical evidence that is consistent with this prediction: using a factor decomposition of recent house price changes in the U.S., they attribute a higher percentage change to local factors in states where the share of land in the real estate value is larger.[3]

In contrast to the change in productivity growth and the world interest rate, we find that financial innovation which permanently relaxes the collateral constraint has a surprisingly small effect on housing prices, despite increasing the home-ownership rate substantially both in the transition and in the steady state. In our economy, tenants or credit-constrained home owners are relatively poor and own a small share of aggregate wealth as a group. As a result, the effect of relaxing the collateral constraint on housing prices is largely absorbed by a modest conversion from rented to owned units.[4]

In addition to the effect on the housing price and aggregate output, the exogenous changes in the productivity growth rate and the interest rate affect the welfare of various households differently, causing winners and losers in housing markets. This distribution effect on wealth and welfare is substantial, since housing wealth forms the largest component of nonhuman wealth for most households. As a general rule of thumb, net house buyers (such as young

[3] Davis and Palumbo (2007) find that the share of land in the value of houses has risen in U.S. metropolitan areas and they argue that this contributes to faster housing price appreciation and, possibly, larger swings in housing prices. Glaeser et. al. (2005) find that land use restrictions are needed to explain recent high housing prices in Manhattan. van Nieuwerburgh and Weill (2006) also argue that the increase in the dispersion of housing prices across regions can be quantitatively generated from an increase in the dispersion of earnings in the presence of planning restrictions. We ignore the restrictions on land use and planning, even though they further increase the natural limitation of land in supplying structures. Other factors that might be empirically relevant for house price determination (such as owner-occupied housing as a hedge against rent risk, the effects of inflation and money illusion) are not considered in our framework; see Sinai and Souleles (2005) and Brunnermeier and Julliard (2007).

[4] This is different from Ortalo-Magne and Rady (2006) who show that relaxing the collateral constraint increases housing prices substantially. We will later discuss further why our results differ from theirs.

worker-tenants) lose while net house sellers (such as retiree-home owners) gain from the house price hike.[5] The gap in welfare changes between winners and losers in the housing market is larger in an economy where land is more important for producing structures compared to capital, since in such an economy housing prices react more in response to an identical shock. Because the welfare effect depends on the underlying shocks causing house price changes, we need a general equilibrium framework with heterogeneous agents to analyze the effect of the shock on the welfare of different households.

Our work broadly follows two strands of the literature. One is the literature on consumption and saving of a household facing idiosyncratic and uninsurable earnings shock and a borrowing constraint, which includes Bewley (1977, 1983), Deaton (1991), Carroll (1997), Attanasio et. al. (1999) and Gourinchas and Parker (2002). Huggett (1993), Aiyagari (1994), and Krusell and Smith (1998) have examined the general equilibrium implications of such models. The second strand is the literature on the investment behavior of firms under liquidity constraints. In particular, Kiyotaki and Moore (1997) is closely related since they study the dynamic interaction between asset prices, credit limits and aggregate economic activity for an economy with credit constrained entrepreneurs. When many households borrow substantially against their housing collateral and move up and down the housing ladder, these households are more like small entrepreneurs rather than simple consumers.

Our attention to housing collateral is in line with substantial micro evidence in the UK (Campbell and Cocco (2004)) and the US (Hurst and Stafford (2004)) which suggests that dwellings are an important source of collateral for households. Given the empirical findings that connect housing prices, home equity and aggregate consumption, there has been sub-

[5]The household is a net house buyer if the expected present value of housing services consumption over the lifetime exceeds the value of the house currently owned. The present living population as a whole is a net seller of the existing houses to the future population (that is not born yet). But, the value of this aggregate net selling position is quantitatively very small, because the discounted value of selling the existing houses to the next unborn population in 70 to 80 years from now is negligible. In comparison, the redistribution within the present population between young and old, or between tenants and home owners, is much larger. Thus, unlike popular arguments, the wealth effect of housing prices on aggregate consumption is negligible (aside from the liquidity effect), because the positive wealth effect of the net house sellers is largely offset by the negative wealth effect of the net house buyers.

stantial research on building models that capture these relationships, either with a representative agent (Aoki et. al. (2004), Davis and Heathcote (2005), Lustig and van Nieuwerburgh (2005), Kahn (2007), Piazessi et. al. (2007)), or with heterogeneous agents (Ortalo-Magne and Rady (2006), Fernandez-Villaverde and Krueger (2001, 2006), Chambers, Garriga and Schlagenhauf (2004), Iacoviello (2005), Iacoviello and Neri (2007), Nakajima (2005), Rios-Rull and Sanchez (2005) and Silos (2007)). Distinguishing features of our analysis include an investigation of the interaction between household life-cycle choices and the aggregate economy, an explicit account of the role of land as a limiting factor in a production economy and evaluating welfare changes across heterogeneous households stemming from shocks to fundamentals.

Section 2 lays out the model and section 3 presents long-run observations relevant for housing markets. Section 4 investigates the individual and aggregate predictions of the model using calibration and Section 5 performs the welfare evaluations.

2 The Model

2.1 Framework

We consider an economy with homogeneous product, structures, labor, reproducible capital stock, and non-reproducible land. There is a continuum of heterogeneous households of population size $\overline{N}_t$ in period t, a representative foreigner, and a representative firm.

The representative firm has a constant returns to scale technology to produce output (Y_t) from labor (N_t) and productive structures (Z_{Yt}) as:

$$Y_t = F(A_t N_t, Z_{Yt}) = (A_t N_t)^{1-\eta} Z_{Yt}^{\eta}, \quad 0 < \eta < 1, \tag{1}$$

where A_t is aggregate labor productivity which grows at a constant rate, $A_{t+1}/A_t = G_A$. Structures (Z_t) are produced according to a constant returns to scale production function using aggregate capital (K_t) and land (L):

$$Z_t = L^{1-\gamma} K_t^{\gamma}, \quad 0 < \gamma < 1. \tag{2}$$

The structures are fully equipped or furnished, and can be used as productive structures (such as offices and factories) or houses interchangeably:

$$Z_t = Z_{Yt} + \int_0^{\overline{N}_t} h_t(i)\,di, \tag{3}$$

where $h_t(i)$ is housing used by household i in period t. With this technological specification of structures, the firm can continuously adjust the way in which the entire stock of land and capital are combined and can convert between productive structures and housing without any friction.[6] The parameter $(1-\gamma)$ measures the importance of land for the production of structures compared to capital, which would be equal to the share of land in property income if there were separate competitive rental markets for land and capital. Thus, we often call $(1-\gamma)$ as "the share of land in the production of structures" hereafter. Typically, the share of land in the production of structures is higher in urban than in rural areas, because land (or location) is more important for production with the agglomeration of economic activities.[7] We assume that the aggregate supply of land L is fixed. The capital stock depreciates at a constant rate $1-\lambda \in (0,1)$ every period, but can be accumulated through investment of goods (I_t) as:

$$K_t = \lambda K_{t-1} + I_t. \tag{4}$$

Structures built this period can be used immediately.

The representative firm owns and controls land and capital from last period and issues equity to finance investment. As the firm increases the size of structures with capital accumulation, it will be convenient in subsequent analysis to assume that the firm maintains the

[6] Davis and Heathcote (2005) use a production function in which only a fixed flow of new vacant land can be used for building new houses. Perhaps, in reality, the allocation of land and capital is not as flexible as in our model but not as inflexible as in Davis and Heathcote (2005). We also assume there is no productivity growth in the production of structures, because Davis and Heathcote (2005) calculate the growth rate of productivity in the US construction sector to be close to zero (-0.27 percent per annum). We ignore labor used in this sector for simplicity.

[7] We will not attempt to explain why agglomeration arises. We should not confuse the share of land $(1-\gamma)$ with the scarcity of land (or marginal product of land), because scarcity not only depends upon the share of land, but also on labor productivity, the capital-land ratio and the capital-labor ratio. We will later discuss how the share of land in the production of structures is related to the share of land in the value of structures in footnote 16.

number of shares to be equal to the stock of structures.[8] Let q_t be the price of equity before investment takes place and let p_t be the price of equity after investment takes place in this period. Let w_t be the real wage rate, and r_t be the rental price of structures. The firm then faces the following flow-of-funds constraint:

$$Y_t - w_t N_t - r_t Z_{Yt} - I_t + p_t Z_t = q_t Z_{t-1} \tag{5}$$

The left hand side (LHS) is the sum of the net cash flow from output production, minus investment costs and the value of equities after investment. The right hand side (RHS) equals the value of equity at the beginning of the period (before investment has taken place).

The owners of equity pay p_t to acquire one unit and immediately receive r_t as a rental payment (including imputed rents). Next period, the owner earns q_{t+1} before investment takes place. Therefore, the rate of return equals

$$R_t = \frac{q_{t+1}}{p_t - r_t}. \tag{6}$$

There are no aggregate shocks in this economy except for unanticipated, initial shocks. As a result, we assume that agents have perfect foresight for all aggregate variables, including the rate of return.

From (5) and (6) under perfect foresight, the value of the firm (V_t^F) to the equity holders from the previous period is equal to the present value of the net cash flow from production and the rental income of structures produced:

$$\begin{aligned} V_t^F &\equiv q_t Z_{t-1} = Y_t - w_t N_t - r_t Z_{Yt} - I_t + r_t Z_t + (p_t - r_t) Z_t \qquad (7) \\ &= Y_t - w_t N_t - r_t Z_{Yt} - I_t + r_t Z_t + \frac{1}{R_t} V_{t+1}^F \end{aligned}$$

The firm takes $\{w_t, r_t, R_t\}$ as given and chooses a production plan $\{N_t, Z_{Yt}, Y_t, I_t, K_t\}$ to maximize the value of the firm, subject to the constraints of technology (1), (2), (3) and (4).

[8] This means the firm follows a particular policy of equity issue and dividend payouts. However, alternative policies do not change allocations because the Modigliani-Miller Theorem holds in our economy under perfect foresight and would only complicate subsequent expressions.

Since the production function of output is constant returns to scale, there is no profit from output production. Therefore, the value of the firm equals the value of the structure stock. Given that the number of equities are maintained to equal the stock of structures (by assumption), the price of equities equals the price of structures. Hereafter, we refer to the shares of the firm as the shares of structures.

Households are heterogeneous in labor productivity, and can have either low, medium, or high productivity, or be retired. Every period, there is a flow of new households born with low productivity without any inheritance of the asset. Each low productivity household may switch to medium productivity in the next period with a constant probability δ^l. Each medium productivity household has a constant probability δ^m to become a high productivity one in the next period. Once a household has switched to high productivity it remains at this high productivity until retirement. All the households with low, medium and high productivity are called *workers*, and all the workers have a constant probability $1-\omega \in (0,1)$ of retiring next period. Once retired, each household has a constant probability $1-\sigma \in (0,1)$ of dying before the next period. (In other words, a worker continues to work with probability ω, and a retiree survives with probability σ in the next period). The flow of new born workers is $G_N - \omega$ fraction of the workforce in the previous period, where $G_N > \omega > \delta^i$ for $i = l, m$. The productivity level of the individual household is private information. All the transitions are i.i.d. across a continuum of households and over time, and thus there is no aggregate uncertainty on the distribution of individual labor productivity. Let N_t^l, N_t^m and N_t^h be populations of low, medium and high productivity workers, respectively, and let N_t^r be the population size of retired households in period t. Then, we have:

$$\begin{aligned}
N_t^l &= (G_N - \omega)\left(N_{t-1}^l + N_{t-1}^m + N_{t-1}^h\right) + (\omega - \delta^l) N_{t-1}^l, \\
N_t^m &= \delta^l N_{t-1}^l + (\omega - \delta^m) N_{t-1}^m, \\
N_t^h &= \delta^m N_{t-1}^m + \omega N_{t-1}^h, \\
N_t^r &= (1-\omega)(N_{t-1}^l + N_{t-1}^m + N_{t-1}^h) + \sigma N_{t-1}^r.
\end{aligned}$$

We choose to formulate the household's life-cycle in this stylized way, following Diaz-Gimenez, Prescott, Fitzgerald and Alvarez (1992) and Gertler (1999), because we are mainly interested in the interaction between the life-cycles of households and the aggregate economy. The three levels of labor productivity give us enough flexibility to mimic a typical life-cycle of wage income for our aggregate analysis.

Each household derives utility from the consumption of output (c_t) and housing services (h_t) of rented or owned housing, and suffers disutility from supplying labor (n_t). (We suppress the index of household i when we describe a typical household). We assume that, when the household rents a house rather than owning (as an owner-occupier) and controlling the same house, she enjoys smaller utility by a factor $\psi \in (0,1)$. This disadvantage of rented housing reflects the tenant's limited discretion over the way the house is used and modified according to her tastes. The preference of the household is given by the expected discounted utility as:

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \left[u\left(c_t, \left[1 - \psi I(rent_t)\right] h_t\right) - v(n_t, \varepsilon_t) \right] \right), \ 0 < \beta < 1, \tag{8}$$

where $I(rent_t)$ is an indicator function which takes the value of unity when the household rents the house in period t and zero when she owns it.[9] Disutility of labor $v(n_t, \varepsilon_t)$ is subject to idiosyncratic shocks to its labor productivity ε_t. The value of ε_t is either high (ε^h), medium (ε^m), low (ε^l), or 0, depending on whether the household has high, medium or low productivity, or is retired, and follows the stationary Markov process described above. $E_0(X_t)$ is the expected value of X_t conditional on survival at date t and conditional on information at date 0. For most of our computation, we choose a particular utility function with inelastic labor supply as:

$$u\left(c_t, h_t\right) = \frac{\left(\left(\frac{c_t}{\alpha}\right)^{\alpha} \left(\frac{\left[1 - \psi I(rent_t) h_t\right]}{1-\alpha}\right)^{1-\alpha} \right)^{1-\rho}}{1-\rho}, \tag{9}$$

and $v_t = 0$ if $n_t \leq \varepsilon_t$, and v_t becomes arbitrarily large if $n_t > \varepsilon_t$. The parameter $\rho > 0$ is the coefficient of relative risk aversion (as well as the inverse of the elasticity of intertemporal substitution) and $\alpha \in (0,1)$ reflects the share of consumption of goods (rather than housing services) in total expenditure. We normalize the labor productivity of the average worker to

[9] We assume that, in order to enjoy full utility of the house, the household must own and control the entire house used. If the household rents a fraction of the house used, then she will not enjoy full utility even for the fraction of the house owned.

unity as:

$$N_t^l \varepsilon^l + N_t^m \varepsilon^m + N_t^h \varepsilon^h = N_t^l + N_t^m + N_t^h. \tag{10}$$

We focus on the environment in which there are problems in enforcing contracts and there are constraints on trades in markets. There is no insurance market against the idiosyncratic shock to labor productivity of each household. The only asset that households hold and trade is the equity of structures (and the annuity contract upon this equity). An owner-occupier can issue equity on its own house to raise funds from the other agents. But the other agents only buy equity up to a fraction $1 - \theta \in [0, 1)$ of the house. Thus, to control the house and enjoy full utility of a house of size h_t, the owner-occupier must hold sufficient equity s_t to satisfy:

$$s_t \geqslant \theta h_t. \tag{11}$$

We can think of this constraint as a collateral constraint for a residential mortgage — even though in our economy the mortgage is financed by equity rather than debt — and we take θ as an exogenous parameter of the collateral constraint. Because the tenant household does not have a collateral asset, we assume the tenant cannot borrow (or issue equities):

$$s_t \geq 0. \tag{12}$$

We restrict tradeable assets to be the homogeneous equity of structures in order to abstract from the portfolio choice of heterogeneous households facing collateral constraints and uninsurable labor income risk. Because we analyze the economy under the assumption of perfect foresight about the aggregate states, this restriction on tradeable assets is not substantive (because all the tradeable assets would earn the same rate of return), except for the case of an unanticipated aggregate shock. Although we do not attempt to derive these restrictions on market transactions explicitly as the outcome of an optimal contract, the restrictions are broadly consistent with our environment in which agents can default on contracts, misrepresent their labor productivity, and can trade assets anonymously (if they wish).[10]

[10] The outside equity holders (creditors) ask the home owners to maintain some fraction of the housing equity to prevent default. There is no separate market for equities on land and capital upon it, because

The flow-of-funds constraint of the worker is given by:

$$c_t + r_t h_t + p_t s_t = (1-\tau) w_t \varepsilon_t + r_t s_t + q_t s_{t-1}, \tag{13}$$

where τ is a constant tax rate on wage income. The LHS is consumption, the rental cost of housing (or opportunity cost of using a house rather than renting it out), and purchases of equities. The RHS is gross receipts, which is the sum of after tax wage income, the rental income from equities purchased this period, and the pre-investment value of equity held from the previous period.[11]

For the retiree who only survives until the next period with probability σ, there is a competitive annuity market in which the owner of a unit annuity will receive the gross returns q_{t+1}/σ if and only if the owner survives, and receive nothing if dead.[12] The retiree also receives the benefit b_t per person from the government, which is financed by the tax revenue on wage income of the workers as

$$b_t N_t^r = \tau w_t (N_t^l + N_t^m + N_t^h). \tag{14}$$

Because the productivity of each household is private information and a low productivity worker can pretend to be retired, the viable retirement benefit does not exceed after-tax

people prefer to control land and capital together in order to avoid the complications. Cole and Kocherlakota (2001) show that, if agents can misrepresent their idiosyncratic income and can save privately, the optimal contract is a simple debt contract with a credit limit. See Lustig (2004) and Lustig and van Nieuwerburgh (2005b) for analysis of optimal contracts with tangible assets as collateral.

[11] When the worker is an owner-occupier of a house of size h_t and issues equity to the outside equity holders (creditors) by outstanding size of $(h_t - s_t)$ in period t, she faces the flow-of-funds constraint:

$$c_t + [p_t h_t - q_t h_{t-1}] + r_t (h_t - s_t) = (1-\tau) w_t \varepsilon_t + [p_t (h_t - s_t) - q_t (h_{t-1} - s_{t-1})].$$

The LHS is an outflow of funds: consumption, purchases of the owned house over the resale value of the house held from last period, and rental income paid to the outside equity holders of this period. The RHS is an inflow: after-tax wage income, and the value of new issues of outside equity above the value of outside equity from the previous period. By rearranging this, we find that both the home-owner and tenant face the same flow-of-funds constraint (13), in which only the net position of equity matters.

[12] When the retiree who owned the house dies, then the house is sold and the proceeds are split among the surviving annuity holders.

wage income of the low productivity worker[13], or:

$$b_t/w_t = \tau \frac{G_N - \sigma}{1 - \omega} \leq (1 - \tau)\varepsilon^l.$$

The flow-of-funds constraint for the retiree is

$$c_t + r_t h_t + p_t s_t = b_t + r_t s_t + \frac{q_t}{\sigma} s_{t-1}. \tag{15}$$

Each household takes the equity from the previous period (s_{t-1}) and the joint process of prices, and idiosyncratic labor productivity shocks $\{w_t, r_t, p_t, q_t, \varepsilon_t\}$ as given, and chooses the plan of consumption of goods and housing, and the equity holding $\{c_t, h_t, s_t\}$ to maximize the expected discounted utility subject to the constraints of flow-of-funds and collateral.

The representative foreigner makes purchases of goods C_t^* and equities of structures S_t^* in the home country (both C_t^* and S_t^* can be negative), subject to the international flow-of-funds constraint against home agents as:

$$C_t^* + p_t S_t^* = r_t S_t^* + q_t S_{t-1}^*. \tag{16}$$

The LHS is gross expenditure of foreigners on home goods and equities, and the RHS is the gross receipts of foreigners. Instead of explicitly deriving the foreigners' behavior, we posit the reduced form demand function for home equities of the representative foreigner as an increasing function of the gap between the rate of return on home equities and the rate of return on the foreign asset, R_t^*, as:

$$S_t^* = S^*(R_t,\ R_t^*) = \overline{S}^* + \xi(R_t - R_t^*),$$

where $\xi > 0$ is the sensitivity of demand with respect to the gap in the rates of return, and $\overline{S}^*$ is the parameter which summarizes the other determinants of their demand. One special case is a small open economy in which $\xi \to \infty$, and another special case is a closed economy in which $\overline{S}^* = \xi = 0$.

[13] The firm observes each worker's labor contribution to its production, but it does not observe whether the worker works elsewhere as well. The firm pays uniform payroll taxes before paying wages to the workers. Although the government does not observe the productivity of each household, it observes whether the household works or not, at least with some probability by random monitoring. We assume that the penalty of getting caught for cheating is sufficiently high, so that no worker receives the benefit while working.

Given the above choices of households, the representative firm and the foreigner, the competitive equilibrium of our economy is characterized by the prices $\{w_t, r_t, p_t\}$ which clear the markets for labor, output, equity and the use of structures as:

$$N_t = \int_0^{\bar{N}_t} n_t(i)\,di = \varepsilon^l N_t^l + \varepsilon^m N_t^m + \varepsilon^h N_t^h = N_t^l + N_t^m + N_t^h, \tag{17}$$

$$Y_t = \int_0^{\bar{N}_t} c_t(i)\,di + I_t + C_t^*, \tag{18}$$

$$Z_t = \int_0^{\bar{N}_t} s_t(i)\,di + S_t^*. \tag{19}$$

and (3)[14]. Because of Walras' Law, only three out of four market clearing conditions are independent.

2.2 Behavior of Representative Firm

The first order conditions for the value maximization of the representative firm are:

$$w_t = (1-\eta)Y_t/N_t, \tag{20}$$

$$r_t = \eta Y_t/Z_{Yt} = \eta\left(\frac{M_t}{f_t Z_t}\right)^{1-\eta}, \text{ where } M_t \equiv A_t N_t \text{ and } f_t \equiv Z_{Yt}/Z_t, \tag{21}$$

$$1 - \frac{\lambda}{R_t} = r_t\gamma\left(\frac{L}{K_t}\right)^{1-\gamma} = \gamma\eta L^{(1-\gamma)\eta}\left(\frac{M_t}{f_t}\right)^{1-\eta} K_t^{\gamma\eta-1}. \tag{22}$$

The first two equations are the familiar equality of price and marginal products of factors of production. The value of M_t is the labor in efficiency unit, and f_t is a fraction of structures used for production. The last equation says that the opportunity cost of holding capital for one period – the cost of capital – should be equal to the marginal value product of capital. Thus we have

$$K_t = \left[\frac{\gamma\eta}{1-\frac{\lambda}{R_t}} L^{(1-\gamma)\eta}\left(\frac{M_t}{f_t}\right)^{1-\eta}\right]^{1/(1-\gamma\eta)}, \tag{23}$$

[14] The name of individual household i is such that a fraction of new-born households named after the names of the deceased households and the remaining fraction of newborns are given new names for $i \in \left(\overline{N}_{t-1}, \overline{N}_t\right]$. In this way, the name of households are always distributed uniformly in $\left[0, \overline{N}_t\right]$ at date t.

$$Y_t = f_t \left[\left(\frac{\gamma\eta}{1 - \frac{\lambda}{R_t}} \right)^{\gamma\eta} L^{(1-\gamma)\eta} \left(\frac{M_t}{f_t} \right)^{1-\eta} \right]^{1/(1-\gamma\eta)}. \tag{24}$$

Because the production function of output is constant returns to scale, there is no profit associated with regular production. The resulting value of the firm is:

$$\begin{aligned} V_t^F &= r_t Z_t - (K_t - \lambda K_{t-1}) + \frac{1}{R_t}\left[r_{t+1}Z_{t+1} - (K_{t+1} - \lambda K_t)\right] + ... \\ &= \lambda K_{t-1} + \eta(1-\gamma)\left(\frac{Y_t}{f_t} + \frac{1}{R_t}\frac{Y_{t+1}}{f_{t+1}} + \frac{1}{R_t R_{t+1}}\frac{Y_{t+2}}{f_{t+2}} + ... \right). \end{aligned} \tag{25}$$

The first term of the RHS is the capital stock inherited from the previous period, and the second term is the value of land, which is proportional to the present value of the return to land which comes from output and housing service production. Thus, the equity holders as a whole receive returns from capital and land through their holdings of equities of the entire structure.

2.3 Household Behavior

The household chooses one among three modes of housing - becoming a tenant, a credit constrained owner-occupier, and an unconstrained owner-occupier. The flow-of-funds constraint of the worker and retiree can be rewritten as

$$\begin{aligned} c_t + r_t h_t + (p_t - r_t)s_t &= (1-\tau)w_t\varepsilon_t + q_t s_{t-1} \equiv x_t, \\ c_t + r_t h_t + (p_t - r_t)s_t &= b_t + \left[q_t/\sigma\right] s_{t-1} \equiv x_t, \end{aligned}$$

where x_t is the liquid wealth of the household. Liquid wealth is the wealth of the household, excluding illiquid human capital (the expected discounted value of future wages and pension income). We call liquid wealth "net worth" hereafter.

2.3.1 The tenant

The tenant chooses consumption of goods and housing services to maximize the utility, which leads to:

$$\frac{c_t}{r_t h_t} = \frac{\alpha}{1-\alpha}.$$

Using the flow-of-funds constraint we can express housing and consumption as functions of current expenditure:

$$c_t = \alpha[x_t - (p_t - r_t)s_t],$$

and

$$h_t = \frac{(1-\alpha)\,[x_t - (p_t - r_t)s_t]}{r_t}.$$

Substituting these into the utility function we get the following indirect utility function:

$$u^T(s_t, x_t; r_t, p_t) = \frac{1}{1-\rho}\left[\frac{x_t - (p_t - r_t)s_t}{[r_t/(1-\psi)]^{1-\alpha}}\right]^{1-\rho}.$$

Due to the lower utility from living in a rented house, the tenant effectively faces a higher rental price than the owner-occupier for the same utility, i.e., $[r_t/(1-\psi)]$ rather than r_t.

2.3.2 The constrained owner-occupier

The constrained owner-occupier faces a binding collateral constraint as:

$$s_t = \theta h_t.$$

Thus he consumes $h_t = s_t/\theta$ amount of housing services, and spends the remaining on goods as:

$$c_t = x_t - \left(p_t - r_t + \frac{r_t}{\theta}\right) s_t.$$

The indirect period utility of the constrained home owner is now:

$$u^C(s_t, x_t; r_t, p_t) = \frac{1}{1-\rho}\left\{\left[\frac{x_t - \left(p_t - r_t + \frac{r_t}{\theta}\right) s_t}{\alpha}\right]^{\alpha}\left[\frac{s_t/\theta}{1-\alpha}\right]^{1-\alpha}\right\}^{1-\rho}.$$

2.3.3 The unconstrained owner-occupier

The collateral constraint is not binding for the unconstrained owner-occupier. Her intra-temporal choice is identical to the tenant's but she does not suffer from the limited discretion associated with renting a house.

$$u^U(s_t, x_t; r_t, p_t) = \frac{1}{1-\rho}\left[\frac{x_t - (p_t - r_t)s_t}{{r_t}^{1-\alpha}}\right]^{1-\rho}$$

2.3.4 Value functions

Let $\overline{A}_t$ be the vector of variables and a function that characterizes the aggregate state of the economy at the beginning of period t :

$$\overline{A}_t = (A_t,\ N_t^l,\ N_t^m, N_t^h,\ N_t^r,\ K_{t-1},\ S_{t-1}^*,\ \Phi_t(\varepsilon_t(i), s_{t-1}(i)))',$$

where $\Phi_t(\varepsilon_t(i), s_{t-1}(i))$ is the date t joint distribution function of present productivity and equity holdings from the previous period across households. Each household has perfect foresight about the future evolution of this aggregate state, even if each faces idiosyncratic risks on her labor productivity. The prices (w_t, r_t, p_t, q_t) would be a function of this aggregate state in equilibrium. We can express the value functions of the retiree, high, medium and the low productivity worker by $V^r(x_t, \overline{A}_t)$, $V^h(x_t, \overline{A}_t)$, $V^m(x_t, \overline{A}_t)$, and $V^l(x_t, \overline{A}_t)$ as functions of the individual net worth and the aggregate state.

The retiree chooses the mode of housing and an annuity contract on equities, s_t, subject to the flow-of-funds constraint. Then, the retiree's value function satisfies the Bellman equation:

$$V^r(x_t, \overline{A}_t) = \underset{j=T,C,U}{Max} \left(\max_{s_t} \left\{ u^j\left(s_t, x_t; r_t, p_t\right) + \beta\sigma V^r\left(b_{t+1} + \left[q_{t+1}/\sigma\right] s_t, \overline{A}_{t+1}\right) \right\} \right),$$

where $u^j\left(s_t, x_t; r_t, p_t\right)$ is the indirect utility function of present consumption and housing services when the mode of housing is tenant $(j = T)$, constrained owner-occupier $(j = C)$, or unconstrained owner-occupier $(j = U)$.

The worker chooses the mode of housing and saving in equities. The value function of a high-productivity worker satisfies the Bellman equation:

$$V^h(x_t, \overline{A}_t) = \underset{j=T,C,U}{Max} \left(\max_{s_t} \left\{ \begin{array}{c} u^j\left(s_t, x_t; r_t, p_t\right) + \beta[\omega V^h((1-\tau)\varepsilon^h w_{t+1} + q_{t+1}s_t, \overline{A}_{t+1}) \\ +(1-\omega)V^r(b_{t+1} + q_{t+1}s_t, \overline{A}_{t+1})] \end{array} \right\} \right).$$

The high productivity worker may retire with probability $1-\omega$ next period, and continues to work with probability ω.

The value function of a medium productivity worker satisfies:

$$V^m(x_t, \overline{A}_t) =$$
$$\underset{j=T,C,U}{Max} \left(\max_{s_t} \left\{ \begin{array}{c} u^j\left(s_t, x_t; r_t, p_t\right) + \beta[(\omega-\delta^m)V^m((1-\tau)\varepsilon^m w_{t+1} + q_{t+1}s_t, \overline{A}_{t+1}) \\ +\delta^m V^h((1-\tau)\varepsilon^h w_{t+1} + q_{t+1}s_t, \overline{A}_{t+1}) + (1-\omega)V^r(b_{t+1} + q_{t+1}s_t, \overline{A}_{t+1})] \end{array} \right\} \right).$$

Next period, the medium productivity worker switches to high productivity with probability δ^m, retires with probability $1-\omega$, and remains with medium productivity with probability $\omega - \delta^m$. The value function of a low productivity worker is similar to the value function of a medium productivity worker, except for m being replaced by l and h being replaced by m.

Growth in the economy with land presents a unique problem for the solution of the individual agent problem because wages grow at different rates from the rental price and the equity price even in the steady state. This means that we need to transform the non-stationary per capita variables in the model into stationary per capita units. In Appendix B, we describe how to convert the value functions of the household into a stationary representation.

2.4 Steady State Growth

Before calibrating, it is useful to examine the steady state growth properties of our economy. Let $G_X = X_{t+1}/X_t$ be the steady state growth factor of variable X_t. In the following we simply call the growth factor as the "growth rate". In steady state, the growth rate of aggregate output variables should be equal:

$$\frac{Y_{t+1}}{Y_t} = \frac{I_{t+1}}{I_t} = \frac{K_{t+1}}{K_t} = G_Y.$$

The growth rate of structures need not be equal the growth rate of output, but it should be equal to the growth rate of productive structures:

$$\frac{Z_{t+1}}{Z_t} = \frac{Z_{Yt+1}}{Z_{Yt}} = G_Z.$$

Then, from the production functions, these growth rates depend upon the growth rates of aggregate labor productivity and population as $G_Y = (G_A G_N)^{1-\eta}\, G_Z^{\eta}$, and $G_Z = G_Y^{\gamma}$. Thus

$$G_Y = (G_A G_N)^{(1-\eta)/(1-\gamma\eta)}, \qquad (26)$$

$$G_Z = (G_A G_N)^{\gamma(1-\eta)/(1-\gamma\eta)}.$$

Because the supply of land is fixed, to the extent that land is an important input for producing structures, the growth rates of output and structures are both smaller than the growth rate of labor in efficiency units. Moreover, because structures are more directly affected by the limitation of land than output, the growth rate of structures is lower than the growth rate of output, when labor in efficiency units is growing.

In the steady state of the competitive economy, the growth rate of the real rental price and the purchase price of structures is equal to the ratio of the growth rate of output and the growth rate of structures:

$$G_r \equiv \frac{r_{t+1}}{r_t} = \frac{p_t}{p_t} = \frac{G_Y}{G_Z} = G_Y^{1-\gamma}. \tag{27}$$

To the extent that land is important for the production of structures ($\gamma < 1$), the rate of increase of the rental price and the purchase price of structures is an increasing function of the growth rate of workers in efficiency units in steady state. The wage rate grows in the steady state with the same rate as the per capita output as

$$G_w = \frac{G_Y}{G_N} = \left[G_A^{1-\eta} G_N^{-\eta(1-\gamma)} \right]^{1/(1-\gamma\eta)}.$$

Because the per capita supply of land decreases with population growth, the growth rate of the wage rate is a decreasing function of the population growth rate.

3 Observations

Here, we gather some observations, which give us some guidance for our calibrations.

3.1 Features of U.S. Economy

Table 1 summarizes the features of the US. economy, relevant for our aggregate economy.

Table 1: Long run aggregate features of the U.S. economy

	1900	1939	1958	Average
Nonfarm private tangible assets/GDP	3.01	3.00	2.66	3.3
Fraction of productive structures	0.56	0.50	0.51	0.53
Land/GDP	1.61	0.96	0.66	-
Residential Land/GDP	-	0.28	0.18	0.39
Market Value of Homes/GDP	-	1.30	1.10	1.28

Notes to Table 1: Nonfarm private tangible assets, the fraction of productive structures and total land from 1900 to 1958 are from Goldsmith (1962), Tables A35-A43. GDP is from GDP - Millennial Edition Series of Table Ca9-19 of Volume 3 of Carter et. al. (2006). The nonfarm private tangible assets is the sum of nonfarm private residential and nonresidential structures and land, producer and consumer durables, and inventory. The fraction of productive structures is defined as (nonfarm private nonresidential structures and land+producer durables+inventory)/(nonfarm private tangible assets). Average for the first two rows refers to the average quarterly estimates between 1952:Q1 and 2005:Q4 for the US economy based on Flow of Funds data (see Appendix C for the construction). The numbers for the last two rows are from Davis and Heathcote (2007) and the average is the annual average between 1930 and 2000.

We observe that tangible assets of nonfarm private sector (which include the value of land) is about 3 years worth of GDP, and that the ratio is fairly stable. The fraction of productive structures (Z_{Yt}/Z_t) shows only small change over the long period of time. On the other hand, the ratio of land value to annual GDP falls from 1.61 in 1900 to 0.66 in 1958. This is largely due to a decline of the share of agricultural land. If we look at the ratio of private nonfarm land to GDP, it only falls from 0.57 in 1900 to 0.36 in 1958 (according to Goldsmith (1962) Table A-5). This remaining decline suggests that the elasticity of substitution between land and reproducible capital in production of fully equipped structures may exceed unity in the United States, because the share of land value decreases as capital accumulates. (Roughly speaking, the scarcity of land is relatively easily mitigated by using technology with higher capital-land ratio). Thus, our assumption of a Cobb-Douglas production function is a rough approximation of the production of structures.[15]

3.2 Evolution of home-ownership rates

There exists considerable variation in home ownership rates across countries and over time. *Table* 2 shows the home ownership rates (fraction of households that own and control houses as owner-occupiers) of selected developed countries between 1970 and 2003 taken from IMF

[15] For Japan, Kiyotaki and West (2006) provide evidence that the elasticity of substitution between land and capital is not significantly larger than unity for the period 1961-1995.

World Economic Outlook (Terrones (2004)). The table shows a general upward trend and considerable variation in home-ownership rates across countries since 1970. These variations may reflect differences in financing constraints, and utility losses from renting, factors that we analyze in the theoretical model.

Table 2: Home ownership rates in %	1970	1980	1990	2003
United States	64.2	65.6	64.0	68.3
Germany	-	41.0	39.0	43.6
Italy	-	59.0	68.0	80.0
United Kingdom	50.0	55.0	66.0	70.0
Japan	-	60.0	61.0	62.0

Notes to Table 2: See Table 2.1 in page 73 of World Economic Outlook (September 2004).

Focussing on the U.S., *Table* 3 shows the evolution of home ownership rates for white and black households for the 1900-1990 period derived from Collins and Margo (2001).

Table 3: U.S. Home ownership rates in %						
	1900	1920	1940	1960	1980	1990
whites	48.5	47.1	42.1	64.0	68.6	66.5
blacks	24.1	24.6	20.5	35.8	43.8	40.9

We observe that there is a substantial gap between white and black households. Moreover, the home ownership rates for both whites and blacks declined during the Great Depression, before increasing after WWII. During the 1980s, average home ownership rate declined for both groups. These within-country differences across race and time might possibly be reflecting differences in income and access to credit markets, factors that are prominent in our model.

4 Calibrations

4.1 Parameters for Calibration

All but two parameters are chosen to be roughly consistent with aggregate or individual data. The two free parameters are the discount factor (β) that is chosen to generate a reasonable structures to output ratio (3.3), and the fraction of utility loss from renting a house (ψ) that is chosen to generate a sufficient number of tenants (around 25%). All the parameters for the baseline calibration are given in $Table$ 4.

Table 4: Parameters for Baseline Calibration
$\eta = 0.258$: share of productive structures in production of output
$\gamma = 0.9$: share of capital in the production of structures
$1 - \lambda = 0.1$: depreciation rate
$\overline{S}^* = 0$: exogenous foreign demand for domestic equities
$\xi = 0$: elasticity of foreign demand with respect to return gap
$\beta = 0.96$: utility discount factor
$\alpha = 0.75$: share of nondurables in total expenditure
$\rho = 2$: coefficient of relative risk aversion
$\psi = 0.09$: fraction of utility loss from renting a house
$\theta = 0.3$: fraction of house that needs downpayment
$\delta^l = 0.08$, $\delta^m = 0.014$: probability of switching to a higher wage
$\varepsilon^l = 0.331$, $\varepsilon^m = 0.663$ and $\varepsilon^h = 2.650$: labor productivities
$\frac{b}{w} = 0.2$: ratio of retirement benefit to pre-tax wages of average worker
$\omega = 0.978$: probability of continuing working
$\sigma = 0.945$: probability of the retiree to survive
$G_A = 1.02$: labor productivity growth
$G_N = 1.01$: population growth

We consider one period of our model to be roughly one year and think of the baseline economy as the United States. The share of productive structures in the production of non-housing final output ($\eta = 0.258$) is a bit lower than the one used in other studies ($0.3 - 0.4$ range), because we treat the production of housing services separately (and this is a capital intensive sector). Using the Cooley and Prescott (1995) methodology of aligning the data to their theoretical counterparts, Appendix C outlines how we estimate η from the U.S. Flow of Funds and NIPA data for the period 1952:Q1 to 2005:Q4.

A key parameter in our model is the share of land in the production of structures $(1-\gamma)$. Thinking of the U.S. economy as our baseline, we set $\gamma = 0.9$ since Haughwout and Inman (2001) calculate the share of land in property income between 1987 and 2005 to be about 10.9%, while Davis and Heathcote (2005) also use $\gamma = 0.9$. Davis and Heathcote (2007) note that the share of land in residential housing values has risen recently in the U.S., and it is close to 50% in major metropolitan areas like Boston and San Francisco. For comparisons, we will use $\gamma = 0.5$ to highlight the influence of the share of land on the allocations in the steady state as well as in the transition.[16]

The depreciation rate of the capital stock $(1-\lambda)$ is set at 10 percent per annum, while the annual discount factor is set at 0.96 and the coefficient of relative risk aversion at 2, all standard parameter choices. For the baseline, we consider a closed economy so that both $\overline{S}^*$ and ξ are set to be zero. Recent papers have calibrated α (the share of non-durables in total expenditure) at around 0.8 (Diaz and Luengo-Prado (2007) use 0.83 and Li and Yao (2006) use 0.8 based on the average share of housing expenditure found in the 2001 Consumer Expenditure Survey). We use a slightly lower number since we think of housing as inclusive of other durables.

The utility loss from renting a house (ψ) is set to generate reasonable implications for the aggregate home-ownership rate: a small value for ψ at around 0.09 worked well to generate the observed fraction of renters in the data. The fraction of a house that needs a downpayment (θ) is set at 30% but we perform extensive comparative statics relative to this parameter since one of our goals is to better understand the role of collateral constraints on home-ownership rates, house prices and allocations.

[16] Caselli and Freyer (2007) note that, in recent World Bank data, the share of land value in total tangible assets ranges between 12 and 27 percent for a range of countries but rises to 51 percent for Japan. In the steady state, the share of land in the value of structures is $\left[\frac{1-\gamma}{1-(G_Y/R)}\right] / \left[\frac{\gamma}{1-(\lambda/R)} + \frac{1-\gamma}{1-(G_Y/R)}\right]$ as a function of the share of land in property income $(1-\gamma)$, because capital depreciates while the imputed rental income of land grows at the rate of output growth. Thus in the baseline economy in which $R = 1.0683$ and $G_Y = 1.0292$, when the share of land income is $1-\gamma$ =10%, the share of land in the value of structures is equal to 32%. When $\gamma = 0.5$, we have $G_Y = 1.0257$, and the share of land in the value of structures is 80% for the same real rate of return. Even though the income share of land at 50% is perhaps too high, it helps to understand the metropolitan area of Tokyo, New York or London.

The probability (δ^l, δ^m) of switching earnings states is set so that population ratio of low, medium and high productive workers is approximately equal to 30%, 50%, and 20%. The ratio of the earnings shocks are calibrated to have mean normalized to one and the relative shares are chosen to reflect substantial earnings heterogeneity. We use the levels used by Castaneda, Diaz-Gimenez and Rios-Rull (2003), while ignoring their fourth state that captures the wealth distribution of the super wealthy for simplicity. The probability of continuing to work (ω) is set so that the expected duration of working life is 45.5 years, while the probability of the retiree to survive (σ) implies an expected retirement duration of 18.2 years. The replacement ratio (b) implies that the ratio of the government retirement benefit to the after-tax wage is equal to $b/\left[w\left(1-\tau\right)\varepsilon^{l}\right]=0.647$ for a low productivity worker, and is equal to $b/\left[w\left(1-\tau\right)\varepsilon^{h}\right]=0.081$ for a high productivity worker. This generates the intended redistribution of the pension system. We set the growth rate of labor productivity (G_A) to two percent, and the population growth rate (G_N) to one percent.

4.2 General Features of Household Behavior

The household chooses present consumption, saving, and mode of housing, taking into account its net worth and its expectations of future income. *Figure* 1*A* illustrates the consumption of goods, housing services and the mode of housing of the worker with low productivity as a function of net worth. In order to explore the stable relationship between the household choice and the state variable, we detrend all variables using their own theoretical trend as in *Appendix B*. When the worker does not have much net worth, $x < x_{1l}$, he does not have enough to pay for a downpayment of even a tiny house. He chooses to rent a modest house and consume a modest amount. Hoping to become more productive in the future, the low productivity worker hardly saves. In *Figure* 1*B*, the locus $s' = s(s, q, yl)$ shows the equity-holding at the end of the present period as a function of the equity-holding at the end of the last period for the low productivity worker. Everyone enters the labor market with low productivity and no inheritance ($s_0 = 0$). As long as the worker continues to be with low productivity, he does not save, and continues to live in a rented house.[17]

Figure 2A shows the choice of a worker in the medium productivity state. When she does not have much net worth to pay for a downpayment to buy a house, $x < x_{1m}$, she chooses to rent a place, a similar behavior with the low productivity worker. The main difference is that the medium productivity worker saves vigorously to accumulate the downpayment to buy a house in the future. In *Figure 2B*, the $s' = s(s, q, ym)$ locus (the transition of equity-holdings of the medium productive worker from this to the next period) lies above the 45-degree line for $s < sm^*$, so that the equity holding at the end of this period is larger than the last period. When the medium productivity worker accumulates modest net worth, $x \in [x_{1m}, x_{2m}]$ in *Figure 2A*, she buys her own house subject to the binding collateral constraint. Here, the size of an owned house is a sharply increasing function of net worth, because the worker maximizes the size of the house subject to the downpayment constraint.[18] When the medium productivity worker has substantial net worth $x > x_{2m}$, she becomes an unconstrained home owner, using her saving partly to repay the debt (or increase the housing equity ownership). In *Figure 2B*, the medium productivity worker continues to accumulate her equity holding until she reaches the level of equity-holding at sm^*, the intersection of $s(s, q, ym)$ and the 45-degree line.

The behavior of the high productivity worker is similar to the medium productivity one, except that she accumulates more equities: $s' = s(s, q, yh)$ lies above $s' = s(s, q, ym)$ and her converging equity-holding sh^* is larger than that of medium productive worker sm^*.

[17] No saving by a low productivity worker is not always true. If the income gap between low productivity and higher productivity workers is small, the transition probability from less to more productive states is small, or the pension is very limited, then the low productivity worker saves to buy a house or for retirement.

[18] The size of the house at net worth $x = x_{1m}$ is smaller than the house rented at net worth slightly below x_{1m}, because she can only afford to pay downpayment on a smaller house. (Nonetheless, she is happier than before, because she derives more utility from the owned home than a rented place). The worker moves to a bigger house every period in our model because there are no transaction costs. If there were transaction costs, the worker would move infrequently, and change housing consumption by discrete amounts, rather than continuously. (The housing ladder would become a true ladder, instead of having a continual upward slope). She may even buy first a larger house than the house rented before, anticipating the future transaction cost. But the basic features remain the same.

Therefore, the equity holding of all the workers is distributed in $s \in [0, sh^*]$, with mass of workers at both $s = 0$, $s = sm^*$ and $s = sh^*$.

$Figure$ 3A illustrates the consumption and housing choices of the retiree. $Figure$ 3B illustrates the transition of equity-holding of the retiree. Because in our economy the productive workers have strong incentives to save for retirement and mitigate the collateral constraint, the equilibrium level of the capital stock and structures tend to be fairly large. Then, for a large set of parameters, the rate of return on equity-holding (in terms of utility) is not high relative to the time preference rate, taking into account the effect of growth. (Note that the real rate of return should be sufficiently higher than the time preference rate in a growing economy for the retiree to maintain their relative equity holding). Thus, the transition of equity-holding of the retirees, the locus $s' = s(s, q, b)$, lies below the 45-degree line for $s > sr^*$. Thus the retiree slowly decreases his equity-holding along the locus $s(s, q, b)$ until $s = sr^*$. The relative decumulation of equity holding of the retiree stops at $s = sr^*$, the threshold for him to become a constrained home owner, and his holding stays at sr^* afterwards.[19]

Putting together these arguments, we can draw a picture of a typical life-cycle in $Figure$ 4. The horizontal axis counts years from the beginning of work-life, and the vertical axis measures housing consumption (h) and equity-holding (s). Starting from no inheritance, he chooses to live in a rented house without saving during the young and low wage periods until the 19th year. When he becomes a medium productivity wage worker at the 20th year, he

[19] In the baseline economy, there is a small population of the retirees who never had medium productivity during the working period and thus retire without any net worth. Because they give up hope of becoming more productive at the time of retirement and their pension is not much lower than their after-tax wage income in the Baseline economy, they save to become a constrained home owner by accumulating equity holding along the locus of $s' = s(s, q, b)$ (which lies above 45-degree line for $s < sr^*$). This saving pattern of the retiree will not arise in an economy in which there are sufficient incentives for low wage workers to save (because of a small pension, for example). In such an economy, the equilibrium real interest rate is low and the retiree's equity-holding rule $s' = s(s, q, b)$ lies below the 45-degree line for all $s > 0$. Then, the retiree will become a constrained home owner and then become a tenant as he gets older. Eventually, the shareholding of the retiree will stop when he eats up all the equities at point $s = sr^* = 0$. After that, the retiree will rely entirely on the benefit to pay for rent and consumption.

starts saving vigorously. Quickly, he buys a house subject to the collateral constraint. Then he moves up fast the housing ladder to become a unconstrained home owner at the $22nd$ year. Afterwards, he starts increasing the fraction of his own equity of the house (similar to repaying the debt). By the time of retirement, he has repaid all the mortgage and has accumulated equities higher than the value of his own house.[20] When the worker hits the wall of retirement (with the arrival of a retirement shock) at the $51st$ year, his permanent income drops, and he moves to a smaller house. He also sells all the equities to buy an annuity contract on the equities, because the annuity earns the gross rate of return which is $(1/\sigma) > 1$ times as much as straightforward equity-holding. But his effective utility discount factor shrinks by a factor σ too. Thus as the rate of return on the annuity is not sufficiently high to induce the retiree to save enough, he decumulates slowly the relative equity-holding, downsizing his consumption of goods and housing services relative to the working population as he gets older. When he dies, his assets drop to zero, according to the annuity contract (which pays zero if the contract holder dies).

4.3 Comparison of Steady States

4.3.1 Closed Economy

We present our results from comparing steady states in $Table$ 5. In the baseline calibration in the first column, the fraction of tenants in the population is about 25%, which is substantial but a bit lower than the number from Collins and Margo (2001). The fraction of constrained home owners is 8.3%. The fraction of houses lived in by tenants and constrained home owners is smaller than the fraction of their population, because they live in smaller houses than the unconstrained home owners, on average. The average size of a tenant's house is about 34.6% (= 8.69/25.16) of the average house size of the economy, and the average house size of constrained home owners is about 29% of the economy average. The tenants and the constrained home owners live in smaller houses than the unconstrained home owners, mainly

[20]Remember that the aggregate equity-holding of structures of all the households is the sum of all the houses and productive structures in equilibrium.

because the former have lower permanent income.[21] The distribution of equity-holding is even more unequal among the groups of households in different modes of housing. The fraction of total equities held by tenants is negligible (0.05%), the fraction of total equities held by constrained home owners is 0.33%, and the remainder is held by unconstrained home owners. This is consistent with the conventional wisdom that the distribution of wealth is much more skewed than the distribution of income. Perhaps a new insight would be that, when the distribution of wealth and income are difficult to observe, we can infer inequality by looking at the home ownership rates across different groups of people, as Collins and Margo (2001) did.

Turning to prices and aggregate variables, the gross rate of return on equity-holding is 1.068 in terms of goods, and is equal to $1.068 \div G_r^{1-\alpha} = 1.067$ in terms of the consumption basket. The latter is smaller than the inverse of the discount factor, which, adjusted for growth effects, equals $(1/\beta)\left(G_w/G_r^{1-\alpha}\right)^{\rho} = 1.080$. This is not because people are impatient, but because people tend to save substantially during the working period to cope with idiosyncratic shocks to labor productivity and to mitigate the collateral constraint. Many general equilibrium models with uninsurable idiosyncratic risk have such a feature, including Bewley (1983) and Aiyagari (1994). Even though some aggregate variables are not the same as the numbers in *Table* 3, they are broadly consistent with the main features of the US economy. The ratio of average housing value to the average wage is 2.6 years, while the housing price to rental ratio is 8.5 years in the baseline economy. The ratio of value of total structures to GDP is 3.3 years, while the share of housing in total structures is 46%.

Columns 2 and 3 of *Table* 5 report the results for a different level of financial development. Column 2 is the case of a more advanced financial system, where the fraction of house that needs downpayment is 0.1 instead of 0.3 (the baseline number). The main difference relative to the baseline economy is that now there are more constrained home owners instead of tenants. Intuitively, because borrowing becomes easier, relatively poor households buy a house with high leverage (outside equity ownership) instead of renting. Column 3, by comparison, is the case of no housing mortgage ($\theta = 1$) so that the household must buy the

[21] The constrained home owners, in addition, tend to choose smaller housing in order to meet the collateral constraint. This feature is due to the absense of transaction costs. See footnote (18).

house from its own net worth. In this economy, the fraction of tenants is significantly larger. Financial development affects substantially the home-ownership rate. On the other hand, financial development by itself has limited effects on prices and aggregate quantities in steady state. This result arises because the equity holding of tenants and constrained households (who are directly influenced by the financing constraint) is a small fraction of aggregate wealth, and because the required adjustment is mostly achieved through the conversion of houses from rental to owner-occupied units.

In column 4, we consider an economy in which the growth rate of labor productivity is three percent instead of two percent. A higher growth rate of productivity leads to a substantially higher rate of return on equity in the closed economy, given the low elasticity of intertemporal substitution. The housing price-rental ratio is lower because of the higher real interest rate which dominates the effect of a larger expected growth rate of the rental price. The effects on workers' saving rate and home-ownership rate are limited, because the higher rate of return encourages saving while higher expected wages in the future discourage saving.

In Column 5, we decrease the ratio of the retirement benefit to average pre-tax wage to 0.1 from 0.2 (the baseline). This has significant overall effects on both the distribution of the mode of housing and aggregate allocations because households save more in preparing for the retirement shock. As a result of the more vigorous saving among workers, the rate of return on equities is lower than the rate of time preference, and the home ownership rate increases in the new steady state.

In Columns 6, we consider an economy in which share of land in production of structures is larger ($\gamma = 0.5$) than in the baseline. Because the share of land is larger, the housing price-rental ratio is substantially higher (11.1 years instead of 8.5 years in the baseline), reflecting the higher expected growth rate of future rental rates. The rate of return in terms of output is substantially higher, even though the rate of return on the consumption basket is muted by the growth in rents – $R/G_r^{1-\alpha} = 1.088$ instead of 1.092. The home-ownership rate is higher because a higher real rate of return on equity encourages saving, which outweighs the negative effect of a higher housing price-rental ratio.

4.3.2 Small Open Economy

We can conduct the above comparative steady state for the case of a small open economy, i.e., $\xi = \infty$ instead of $\xi = 0$, by keeping the real interest rate at the exogenous level of the world interest rate $R^* = 1.0683$ in *Table* 6. This exercise is useful to examine a regional economy within a country (like London in the U.K. or New York in the U.S.), as well as to prepare the analysis of the transition in the next section.

The baseline results (column 1) in *Table* 6 are identical to their closed economy counterparts (column 1 of *Table* 5) since the world real return is chosen to be the same as the baseline of the closed economy. The main difference from the closed economy arises in the effect of higher labor productivity in column 4. Faced with higher productivity growth, and with the real return not adjusting, there is a pronounced increase in the housing price-rental ratio and a substantial decrease in the home-ownership rate. This contrasts sharply with the closed economy, and suggests that the level of international capital market integration may be important in assessing the way in which fundamentals affect housing prices, home-ownership rates and aggregate allocations. A one percent reduction in the world real rate in column 6 also leads to a substantial increase in house price-rental ratio. The other experiments generate similar effects as in the closed economy model.

5 Winners and Losers in Housing Markets

We now examine how the small open economy reacts to a once-for-all change in different fundamental conditions in technology and the financial environment. We change a parameter once-and-for-all unexpectedly and solve for the path of prices and quantities that lead the economy to the new steady state. Here, we assume perfect foresight except for the initial surprise. Details of the numerical procedure can be found in Appendix A, but the basic procedure is: first guess a set of rental rates over the next (say) 50 years, which converges to the new steady state; then solve backwards the household problem based on these prices; and finally update this price vector until the market for use of structures clears in all periods. To highlight the importance of land, we compare the reaction of the economy with a larger share of land in the production of structures ($\gamma = 0.5$) with the baseline economy ($\gamma = 0.9$). This gives us a sense of how an economy like Japan or the UK might respond differently to shocks, relative to the U.S. baseline.

5.1 Welfare Evaluations

We are particularly interested in how an unanticipated change in fundamentals affects the welfare of various groups of households differently. Here, using the joint distribution of current productivity and equity holdings from the previous period $\Phi(\varepsilon_t(i), s_{-1}(i))$ in the steady state before the shock hits, we define the group as the set I_g of individual households of a particular labor productivity (low, medium, high, and retired (l, m, h, r)), and a particular range of equity holdings of the previous period which corresponds to a particular home-ownership mode (tenant, constrained owner or unconstrained owner) in the old steady state. For example, the low-wage worker tenant group is a group of agents with low labor productivity who choose to be tenants under the old steady state.

One simple measure of the distribution effect is the average rate of change of net worth. Let $j(i)$ be present labor productivity of ($j(i) = h, m, l$ and r) of individual i. Then the net worth of individual i depends upon the wage rate and equity price as:

$$x(i) = w\epsilon^{j(i)} + q\widetilde{s}_{-1}(i),$$

where $\epsilon^j = (1-\tau)\varepsilon^j$ for worker of productivity j and $\epsilon^j = (b/w)$ for $j = r$, retired, $\widetilde{s}_{-1}(i) = s_{-1}(i)$ if i was a worker and $\widetilde{s}_{-1}(i) = s_{-1}(i)/\sigma$ if i was a retiree in the previous period. Then, the average rate of change in net worth (non-human wealth) of group I_g is:

$$\text{average of } \left(\frac{[w_n\epsilon^{j(i)} + q_n\widetilde{s}_{-1}(i)]}{[w_o\epsilon^{j(i)} + q_o\widetilde{s}_{-1}(i)]} - 1 \right) \text{ for all } i \in I_g \tag{28}$$

where (w_o, q_o) are the wage rate and equity price in the old steady state, and (w_n, q_n) are those immediately after the shock.

Alternatively, we can use the value functions. Given that we have solved for the prices and value functions for all the periods in the transition, we know that the value functions at the period when the change in fundamentals takes place is a sufficient statistic for the welfare effect of the shock. Let $V_o^{j(i)}(x(i))$ be the value function at the old steady state and $V_n^{j(i)}(x(i))$ be the value function in the period of the shock's arrival as a function of net worth $x(i)$ and labor productivity.[22] We compute a measure of welfare change for the group I_g as:

$$\overline{\mu}_g = \text{average of } \left[\left(\frac{V_n^{j(i)}([w_n \epsilon^{j(i)} + q_n \widetilde{s}_{-1}(i)])}{V_o^{j(i)}([w_o \epsilon^{j(i)} + q_o \widetilde{s}_{-1}(i)])} \right)^{\frac{1}{1-\rho}} - 1 \right] \text{ for all } i \in I_g. \tag{29}$$

We call this measure as the certainty expenditure equivalent, because we convert the change of the value into the dimension of expenditure before taking the average.[23]

5.2 Transition of Small Open Economy following a Change in Fundamentals

Figure 5 shows the responses to a once-for-all increase in the growth rate of labor productivity from 2% to 3%. Because the economy is growing, all the following figures show the percentage difference from the steady state growth path of the baseline economy. In both economies the housing price increases substantially initially and continues to increase afterwards. In the economy with a larger share of land ($\gamma = 0.5$), the increase in house prices is larger, and real house price inflation afterwards is higher. The housing price-rental ratio is going to be higher, anticipating the increase in the rental price in the future. The home-

[22] Note that V_n is the value function that has been derived after the full perfect foresight transition has been solved for and therefore includes all this information about the transition to the new steady state.

[23] We also computed the net worth equivalent that would make a household indifferent between the period before and after the shock as the value of $\lambda(i)$ such that

$$V_o^{j(i)}([w_o \epsilon^{j(i)} + q_o \widetilde{s}_{-1}(i)]) = V_n^{j(i)}(\lambda(i)[w_n \epsilon^{j(i)} + q_n \widetilde{s}_{-1}(i)])$$

The value of $\lambda(i)$ measures how much the initial net worth must be multiplied immediately after the shock in order to maintain the same level of the expected discounted utility as the old steady state. We can find the net worth equivalent uniquely, because the value functions are monotonically increasing. We can then compute the average of individual $\lambda(i) - 1$ for a particular group g of agents as $\widetilde{\mu}_g$. This welfare measure suffers from the drawback that net worth does not include the value of human capital. Thus, if two groups have different ratios of net worth (liquid wealth) to human capital, a difference in $\widetilde{\mu}_g$ may reflect the difference of the ratio of human to non-human wealth rather than the difference in the welfare effect.

ownership rate gradually declines because young workers take a longer time to accumulate a sufficient downpayment to buy a house. Consumption of goods and housing services increase initially as well as afterwards, reflecting higher permanent income. The share of productive structures (Z_{Yt}/Z_t) falls initially, to accommodate larger demand for residential structures by converting productive to residential structures.

Table 7 reports the average rate of change of welfare (29) in Panel A and the average rate of change of non-human net worth (28) in Panel B for each group against changes in the fundamentals, for the baseline economy ($\gamma = 0.9$) and the economy with a larger share of land ($\gamma = 0.5$). The first and second columns report the average rate of changes from an increase in the growth rate of labor productivity from 2% to 3%. Given the higher productivity growth, households are on average better off with a higher permanent income. (Remember the retiree's benefit is proportional to the wage rate of present workers). The higher housing price, however, affects the welfare of different groups of households differently. Those who buy (or expand) houses in the future lose from the housing price hike, while those who sell houses in the future gain. This redistribution effect is larger in the economy with the larger share of land since the house price hike is bigger in this economy. We can observe the change in non-human net worth in Panel B. The net worth of tenant workers falls because the wage dips with the initial conversion from productive to residential structures. On the other hand, workers with higher holdings of shares (constrained and unconstrained homeowners) and retirees experience an increase in net worth with the house price rise, and the increase is more pronounced where land is more important.

Figure 6 shows how these two economies react to a once-for-all fall in the world real interest rate by 1%. In both economies, housing prices and output increase with large inflows of capital, and the adjustment of housing prices is fast. In the economy with a larger share of land, the swing of net exports and consumption is larger, output takes a longer time to increase despite the large increase in the capital stock, because a large amount of structures gets allocated to housing in the early stages of the transition. The home-ownership rate declines gradually because the lower real interest rate discourages saving, delaying the age of switching from renting to owning a house over the life cycle. The third and fourth columns of *Table* 7 report the reaction of welfare to this decrease in the world real interest rate for the two economies with different shares of land. Because the real interest rate is lower than the time preference rate in the old steady state, a further decrease in the real interest rate hurts a majority of people who save for retirement. The higher house prices also have important redistribution effects between net buyers and net sellers of houses. Low-wage workers as a group lose more in the economy with a larger share of land. Retirees (particularly unconstrained homeowner retirees) who tend to be net sellers of houses, gain more with a larger share of land, because they benefit more from the house price hike. Looking at the value of net worth in Panel B, all groups are wealthier from a higher house price, and the wealth increase is larger for each group (except for low income workers) in the economy with a larger share of land than the baseline economy.

The fifth and sixth columns of *Table* 7 report the welfare effects from down-sizing the pension system. Given the constant world interest rate, there is a small increase in house prices reflecting the higher private saving in the economy. Because now people have to save more privately for retirement with a lower real interest rate, welfare tends to fall significantly, and the fall is more dramatic for the currently retired households (whose present value of pension falls dramatically). Workers are affected but less than retirees, and most of the fall appears in the tenant workers who have to save more for retirement. The revaluation effects are very small in panel B, illustrating that most of the welfare change is coming from a change in savings behavior in this instance.

We have also done the experiment of lowering the downpayment requirement from 30% to 10% permanently. This provides extra liquidity for households, especially for constrained home owners, and encourages consumption initially. At the same time, with a less stringent collateral constraint, some low wage workers and tenants from the previous period buy houses. Overall, however, relaxing the financing constraint has a very limited effect on housing price and aggregate production in the transition, a result similar to the comparisons of the steady states, because the necessary adjustment is mostly achieved by the modest conversion of rented to owned units rather than by the housing price. This contrasts Ortalo-Magne and Rady (2006), who show that relaxing the collateral constraint increases the housing price substantially by increasing the housing demand of credit constrained households. In their model, the individual household faces a discrete choice of whether to live with parents, own a flat or a house of fixed size. Thus, relaxing the collateral constraint will generate a large inflow of new owners of flats and houses, which is not offset by conversion from rented to owned units.[24] The welfare effects in our economy are also relatively modest even for the tenants: for the baseline economy, the welfare gains in the certainty expenditure equivalent is 0.75% for the former tenant-worker and 1% for the tenant-retiree. Because there are virtually no wealth effects through the redistribution of net worth, the welfare gain does not exceed the maximum direct gains of switching from renting to owning a house. Roughly, the maximum potential gain is equal to the utility loss from renting times the expenditure share of housing services (9% times 25% = 2.25%), if the fraction of the expected lifetime enjoying homeownership rises from zero to 100%.

5.3 A Scenario for House Price Changes?

Putting together the simulation results from these experiments, we can conclude that, if we were to explain the large increase in housing prices in many developed countries in the last decades, we could look for increases in the expected growth rate of labor productivity and for decreases in the real interest rate. Suppose that the expected growth rate of labor productivity rises from 2% to 3%. Then in the baseline economy, housing prices would increase initially by 9% and the housing inflation rate would afterwards increase from 0.22% to 0.29% in terms of the consumption basket.[25] In an economy with a larger share of land ($\gamma = 0.5$), the housing price would initially increase by 20%, and the real housing price inflation increases from 0.96% to 1.27%.

Suppose next that the world real rate of return on assets falls from 6.83% to 5.83%, in addition to the above 1% increase in the growth rate of labor productivity. Then, in the baseline economy, the housing price would increase initially by approximately 18%, followed

[24] Also, in Ortalo-Magne and Rady (2006), the net worth of home-owners with outstanding mortgage is sensitive to the housing price due to the leverage effect, which magnifies the effect of any shock to fundamentals, while there is no leverage effect in our equity financing economy. A comprehensive analysis of leverage effect and the portfolio decision in the presence of uninsurable earnings and aggregate risk is a topic for future research.

[25] Here we use equation (27) for computing the growth rate of housing prices in the steady state.

by an annual housing price inflation of 0.29% in terms of the consumption basket. In the economy with a larger share of land, the initial increase in the real housing price would be 44%, followed by the real housing price inflation of 1.27% annually. In 10 years, the cumulative increase in housing prices in terms of the consumption basket would be about 21% in the baseline economy and would be 57% in the economy with the larger share of land. Thus, if half the population lives and works in the area with the 50% land share and another half lives and works in the area with the 10% land share, *and* labor does not move while capital moves freely between the areas, then the cumulative housing price increase would be roughly $(21 + 57)/2 = 39\%$ in terms of the consumption basket in 10 years. Of course, this is a very crude calculation, ignoring how regional agglomeration takes place. Nonetheless, it gives us some guidance that a significant fraction of the increase in real housing prices may be explained by a combination of an increase in the growth rate of labor productivity, a decrease in the real interest rate, and the fact that a large fraction of economic activity is taking place in the area in which the share of land in property income is large.[26]

6 Conclusions

This paper develops an aggregate life-cycle model to investigate the interaction between housing prices, aggregate production, and household behavior over a lifetime. We take into account land as a fixed factor for producing residential and commercial structures in order to analyze the implications for the aggregate time series and the cross section of household choices. Comparing two small open economies with different shares of land in the production of structures, the economy with a larger share of land has a higher housing price-rental ratio and a lower homeownership rate in the steady state. The transitions of the small open economy along the perfect foresight path illustrate that, where the share of land is larger, once-for-all shocks to the growth rate of labor productivity or the world interest rate generate a greater movement in housing prices. A permanent change in the collateral constraint, however, has a limited impact on housing prices and aggregate production, even

[26] At the same time our model predicts that if the expected growth rate of labor productivity falls and/or the world interest rate rises, housing prices will fall substantially.

though it affects the home-ownership rate substantially.

We also find that the permanent increase in the growth rate of labor productivity and the decrease in the world real interest rate substantially redistribute wealth from the net buyers of houses (relatively poor tenants) to the net sellers (relatively rich unconstrained homeowners) with the house price hike. On average, households gain from the increase in the growth rate of labor productivity and lose from the decrease in the world interest rate. Because the welfare change gap between winners and losers in the housing market is substantial, especially where land is more important for producing structures compared to capital, we think that a credible welfare evaluation should take into account household heterogeneity and contract enforcement limitations in housing and credit markets that generate realistic life-cycles of consumption and homeownership.

Appendix A: Solving the model

Solving the household's decision problem

We discretize net worth (x_t^i) using 200 grid points, with denser grids closer to zero to take into account the higher curvature of the value function in this region. The grid range for the continuous state variable is verified ex-post by comparing it with the values obtained in the simulations. For points which do not lie on the state space grid, we evaluate the value function using cubic spline interpolation along net worth. We simulate the idiosyncratic exogenous productivity shock from its three-point distribution. The realizations of these exogenous random variables are held constant when searching for the market clearing prices (p and r). We use the policy functions to simulate the behavior of 10000 agents over 600 (the exact number depends on the probability of exiting working life and the survival probability) periods and aggregate the individual housing and equity demands to determine the market clearing rental and housing price and the equilibrium household allocations.

Solving the perfect foresight model

We guess a sequence of structure rental rates $\{r_t\}_{t=1}^T$ such that the rental rate has converged to the new steady state. For an exogenous real interest rate R in the small open economy, use (22) to calculate a sequence of capital stocks $\{K_t\}_{t=1}^T$ and then use (2) to compute the sequence of $\{Z_t\}$. Then we get structure prices $\{q_t, p_t\}_{t=1}^T$ from (25) and

$V_t^F = q_t Z_{t-1} = p_t Z_t - I_t$ (which follows from the firm flow-of-funds and the zero profit condition). Given these guessed prices, we solve the household's problem backwards from period T when the economy is assumed to have converged to the new steady state. Households are assumed to know the realization of the entire path of structure prices and rental rates. The value function in period T is the value function for the new steady state. Then the value function in period T-1 is computed as follows:

$$V_{T-1}\left(x_{T-1}|r_{T-1}, p_{T-1}\right) = \max_{c_T, h_T}\left[u\left(c_{T-1}, h_{T-1}\right) + \beta V_T\left(x_T|r_T, p_T\right)\right]$$

We simulate the model forward, starting from the capital stock and the joint distribution of labor productivity and equity of the original steady state. In each period, we simulate a cross-section of 10000 agents over 600 periods and aggregate their individual housing choices, computing the excess demand for structures in each period. We increase the rental rate in periods with an excess demand in the market for structures use, and decrease the rental rate in periods with an excess supply, generating a new path $\{r_t\}_{t=1}^T$ of the rental rate. We repeat this until successive paths of the rental rate are less than 0.0001% from each other.

Appendix B: Stationary Representation of Value Functions

The stationary representation of the household's problem

Using the property of the steady state equilibrium of Section 2.4, we normalize the quantities and prices using the power function of labor in efficiency units $M_t \equiv A_t N_t$ and population N_t. Both variables are exogenous state variables, and there can be a jump or a kink in the trend if labor productivity experiences a once-for-all change in its level or growth rate. Let us denote the normalized variable X_t as $\widetilde{X}_t$. Then we have:

$$\begin{aligned}
\widetilde{K}_t &= K_t / M_t^{\frac{1-\eta}{1-\gamma\eta}}, \quad \widetilde{S}_t^* = S_t^* / M_t^{\gamma\frac{1-\eta}{1-\gamma\eta}} \\
\left(\widetilde{w}_t, \widetilde{x}_t\right) &= \left(w_t, x_t\right) / \left(M_t^{\frac{1-\eta}{1-\gamma\eta}} / N_t\right) \\
\left(\widetilde{h}_t, \widetilde{s}_t\right) &= \left(h_t, s_t\right) / \left(M_t^{\gamma\frac{1-\eta}{1-\gamma\eta}} / N_t\right) \\
\left(\widetilde{r}_t, \widetilde{p}_t, \widetilde{q}_t\right) &= \left(r_t, p_t, q_t\right) / M_t^{(1-\gamma)\frac{1-\eta}{1-\gamma\eta}} \\
\widetilde{V}_t^i &= V_t^i / \left[\frac{M_t^{\frac{1-\eta}{1-\gamma\eta}} / N_t}{M_t^{(1-\alpha)(1-\gamma)\frac{1-\eta}{1-\gamma\eta}}}\right]^{1-\rho}, \text{ for } i = l, m, h, \text{ or } r
\end{aligned}$$

We also define the normalized discount factor as:

$$\widetilde{\beta} = \beta \left(\frac{G_w}{G_r^{1-\alpha}} \right)^{1-\rho}.$$

Let us assume population grows along the steady state path. Let $\widetilde{A}_t$ be deviation of labor productivity from the trend. Then the vector of normalized state variables adjusted by the productivity change are:

$$\overline{\widetilde{A}}_t = \left(\widetilde{A}_t, \widetilde{K}_{t-1}, \widetilde{S}^*_{t-1}, \widetilde{\Phi}_t \left(\varepsilon_t, \widetilde{s}_{t-1}(i) \right) \right)'.$$

Using these normalized variables, we can define the normalized value function. For an example, the stationary representation of the retiree's problem is

$$\widetilde{V}^r \left(\widetilde{x}, \overline{\widetilde{A}}_t \right) = Max($$

$$\max_{\widetilde{s}} \left\{ \begin{array}{c} \frac{1}{1-\rho} \left[\frac{\widetilde{x} - (\widetilde{p}_t - \widetilde{r}_t)\widetilde{s}}{[\widetilde{r}_t/(1-\psi)]^{1-\alpha}} \right]^{1-\rho} \\ + \widetilde{\beta} \sigma \widetilde{V}^r \left(\widetilde{b}_{t+1} + \frac{\widetilde{q}_{t+1}}{\sigma} G_Z \widetilde{s}, \overline{\widetilde{A}}_{t+1} \right) \end{array} \right\},$$

$$\max_{\widetilde{s}} \left\{ \begin{array}{c} \left\{ \left[\frac{\widetilde{x} - \left(\widetilde{p}_t - \widetilde{r}_t + \frac{\widetilde{r}_t}{\theta} \right) \widetilde{s}}{\alpha} \right]^{\alpha} \left[\frac{\widetilde{s}\widetilde{r}_t/\theta}{1-\alpha} \right]^{1-\alpha} \right\}^{1-\rho} / (1-\rho) \\ + \widetilde{\beta} \sigma \widetilde{V}^r \left(\widetilde{b}_{t+1} + \frac{\widetilde{q}_{t+1}}{\sigma} G_Z \widetilde{s}, \overline{\widetilde{A}}_{t+1} \right) \end{array} \right\}, .$$

$$\max_{\widetilde{s}} \left\{ \begin{array}{c} \frac{1}{1-\rho} \left[\frac{\widetilde{x} - (\widetilde{p}_t - \widetilde{r}_t)\widetilde{s}}{\widetilde{r}_t{}^{1-\alpha}} \right]^{1-\rho} \\ + \widetilde{\beta} \sigma \widetilde{V}^r \left(\widetilde{b}_{t+1} + \frac{\widetilde{q}_{t+1}}{\sigma} G_Z \widetilde{s}, \overline{\widetilde{A}}_{t+1} \right) \end{array} \right\})$$

Appendix C: Data sources and definitions

To compute the share of income of productive structures (η), we use quarterly data from the US Flow of Funds accounts and from the NIPA for the period of 1952 Q1 - 2005Q4. We follow Cooley and Prescott (1995). We define unambiguous capital income as the sum of corporate profits (π), net interest (i), non-housing rental income (r) from the NIPA (table 1.12)[27]. We also measure the depreciation of capital (DEP) by the consumption of fixed

[27] We use the average share of residential to total structures to compute non-housing rental income from the total rental payments of all persons reported in NIPA table 1.12.

capital (NIPA, table 1.14). We allocate η fraction of proprietors' income (Y_P, NIPA, Table 1.12) to the income from productive structures. Then, the income from productive structures, Y_{ZP}, can be computed as the sum of unambiguous capital income, depreciation, and η fraction of proprietors' income:

$$Y_{ZP} = \pi + i + r + DEP + \eta Y_P = \eta Y$$

where Y is GDP excluding explicit and implicit rents from housing. Solving this for η, we have

$$\eta = \frac{\pi + i + r + DEP}{Y - Y_P}$$

This is a similar expression for the share of capital in output found in Cooley and Prescott (1995, p.19).

Averaging the quarterly data for the U.S. from 1952 to 2005, we obtain a value of η equal to 0.26. This is lower than the share of capital in output in the real business cycle literature (estimates there range between 0.3 and 0.4) because our η excludes the capital intensive production of housing services. We can decompose economy-wide tangible assets between the household and the firm. The exact definitions in the data and their counterparts in the theoretical model are given in the following table:

Economic concept	**Flow of Funds concept**
pZ_y	Non-farm, non-financial tangible assets (Non-residential structures+Equipment+software+Inventories) Flow of funds, Tables B.102 and B.103 FL102010005.Q+FL112010005
$p\int h(i)di = pH$	Household tangible assets (Residential structures+Equipment+software+Consumer durables) Flow of funds, Table B.100 FL152010005.Q

Using these definitions, we compute the average numbers of $Z_Y/(Z_Y + H) = 0.47$ between 1952:Q1 and 2005:Q4. The ratio of total tangible assets to GDP $(p(Z_y + H)/Y)$

is 3.3. If farm corporate and non-corporate tangible assets (FL132010005.Q in the Flow of Funds)[28] are added to the non-farm tangible assets, then the ratio of household tangible assets to total tangible assets falls from 0.47 to 0.44, while the ratio of total tangible assets to GDP rises from 3.3 to 3.6.

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[28] Thanks to Michael Palumbo (Board of Governors) of kindly sending us this series in private correspondence.

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FIGURE 1A: Policy functions for a low productivity worker

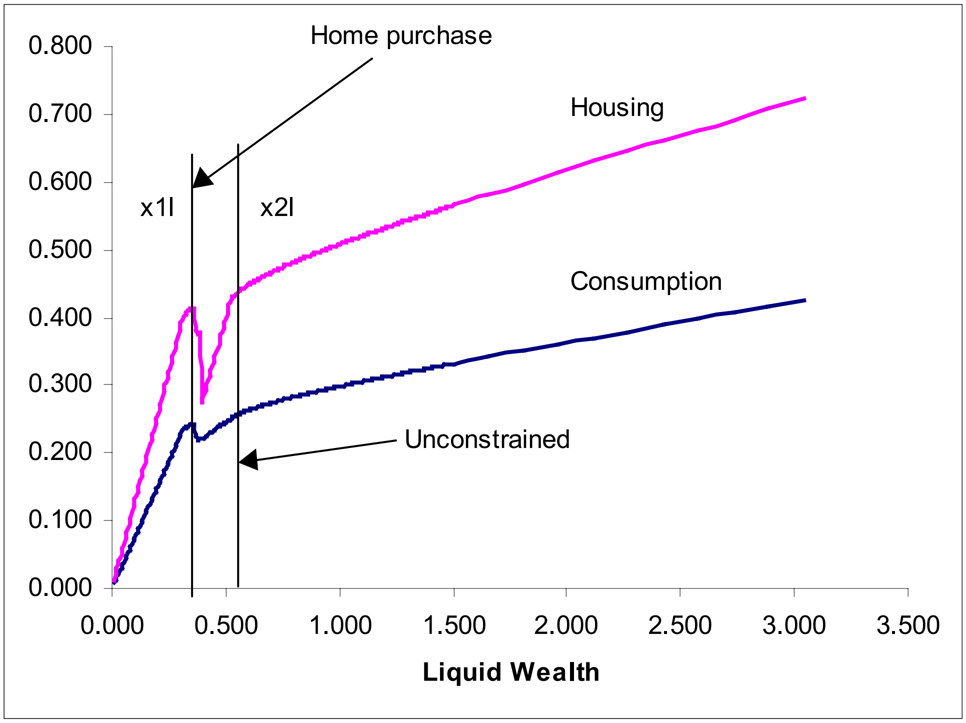

FIGURE 1B: Evolution of savings for a low productivity household

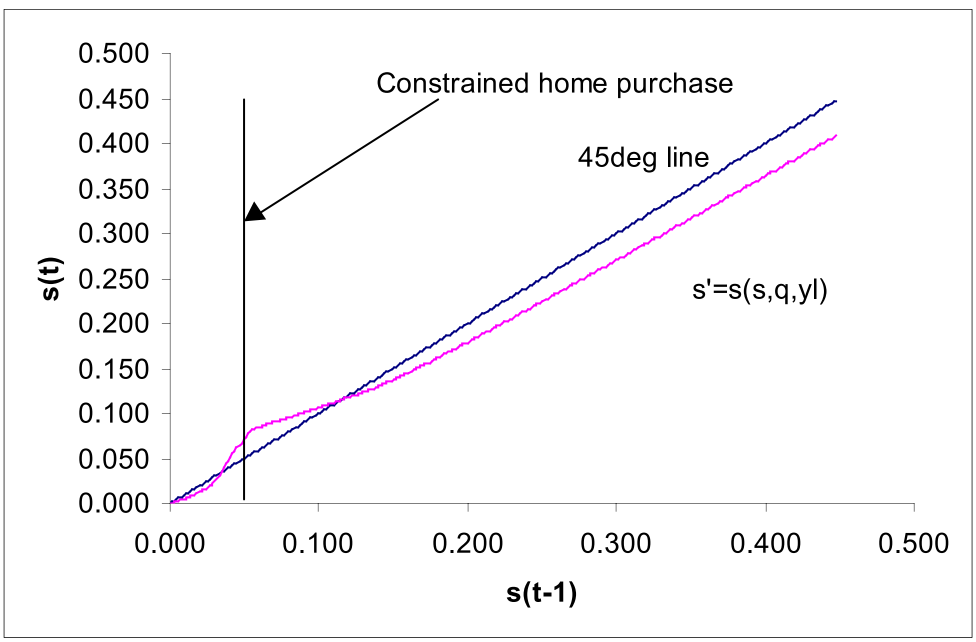

FIGURE 2A: Policy functions for a medium productivity worker

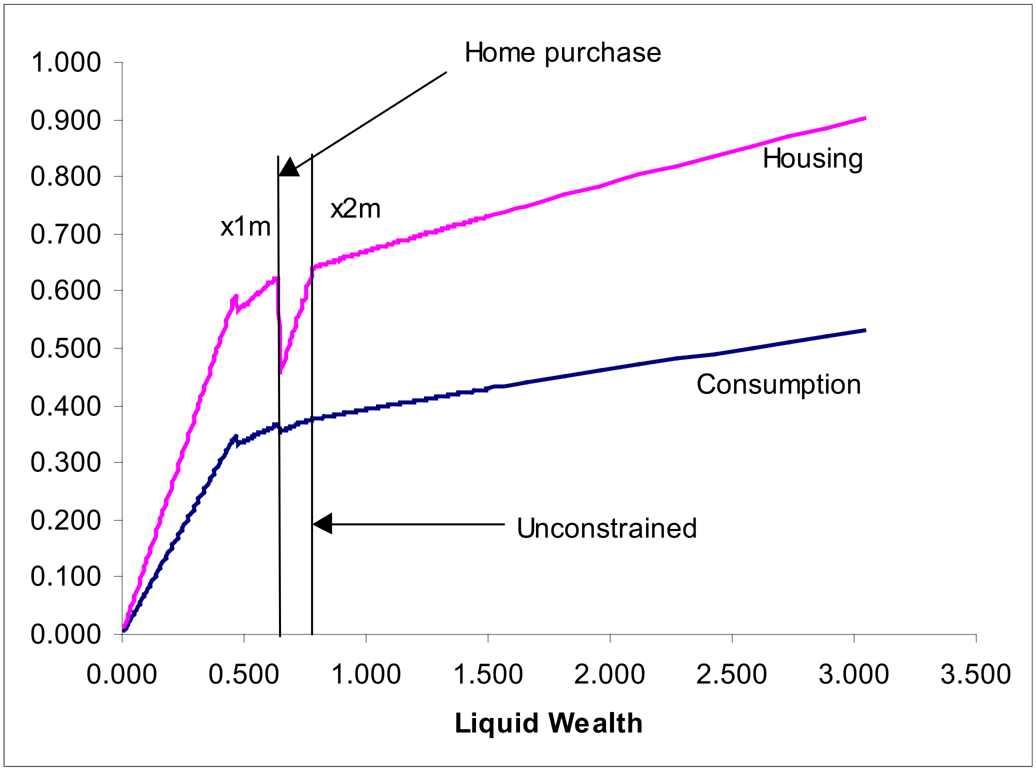

FIGURE 2B: Evolution of savings for a medium productivity household

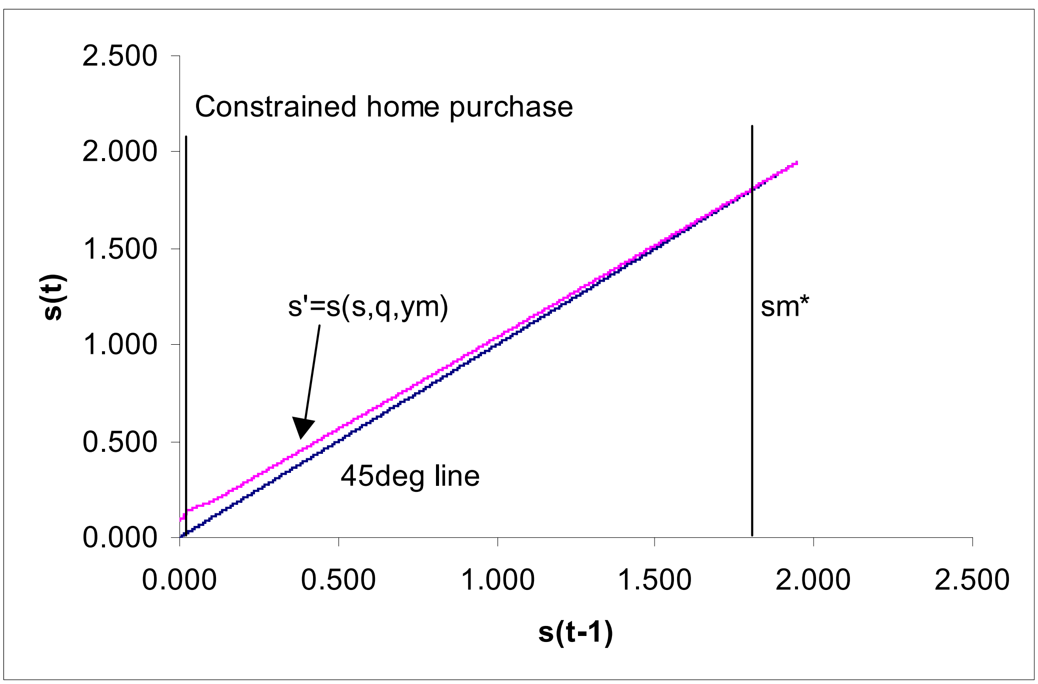

FIGURE 3A: Policy functions for the Retiree

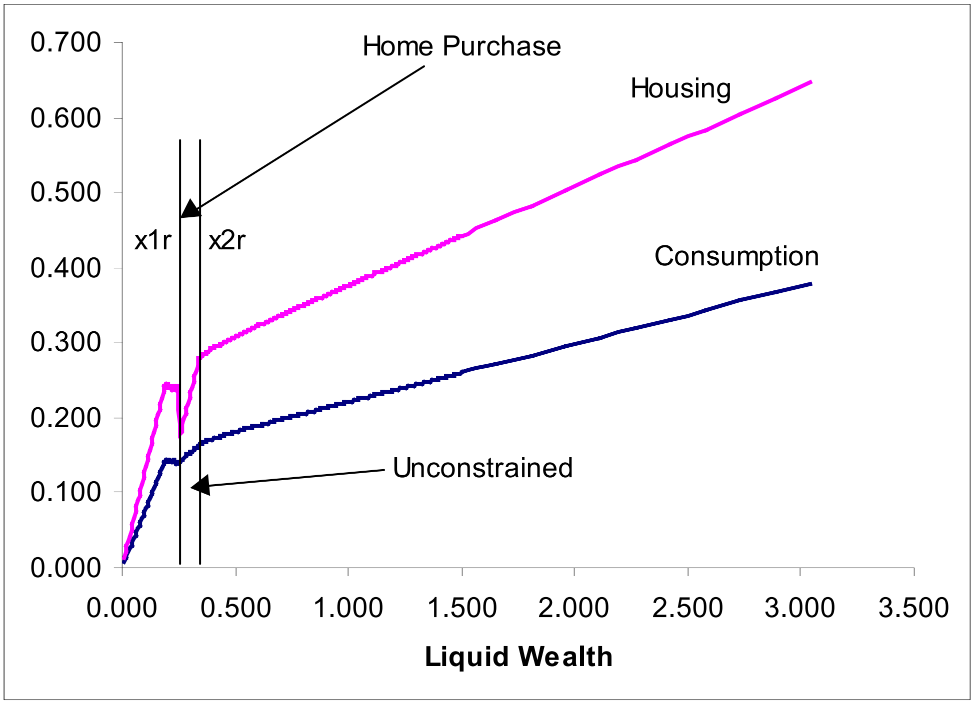

FIGURE 3B: Evolution of savings for the retiree

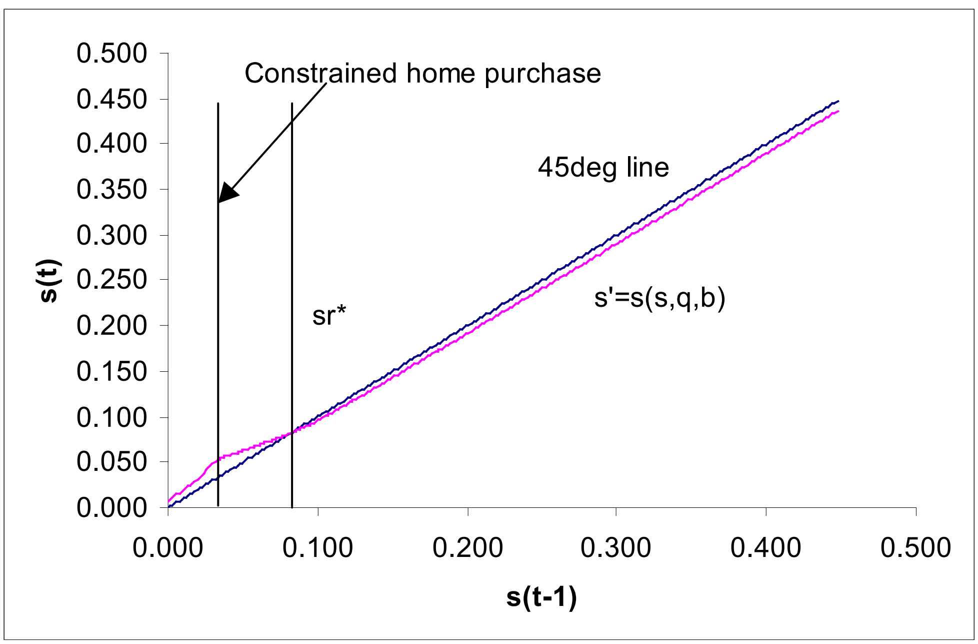

FIGURE 4:An example life time

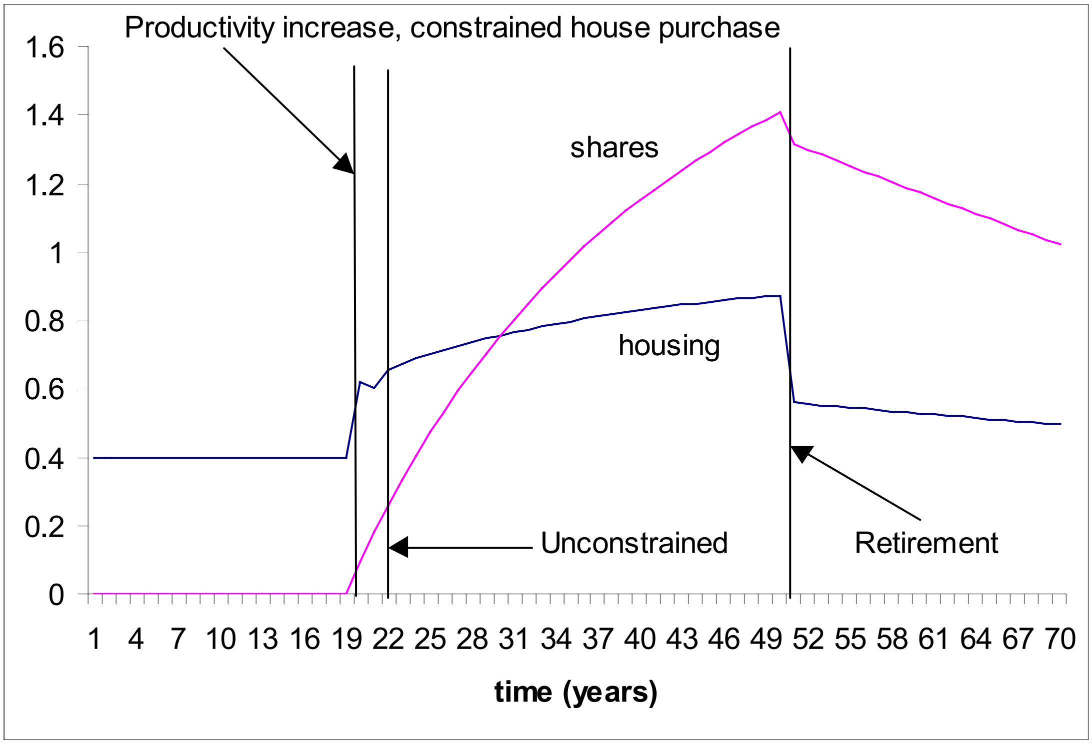

Figure 5
Transition dynamics from a 1% increase in labour productivity growth
(solid line: γ=0.9, dotted line: γ=0.5)

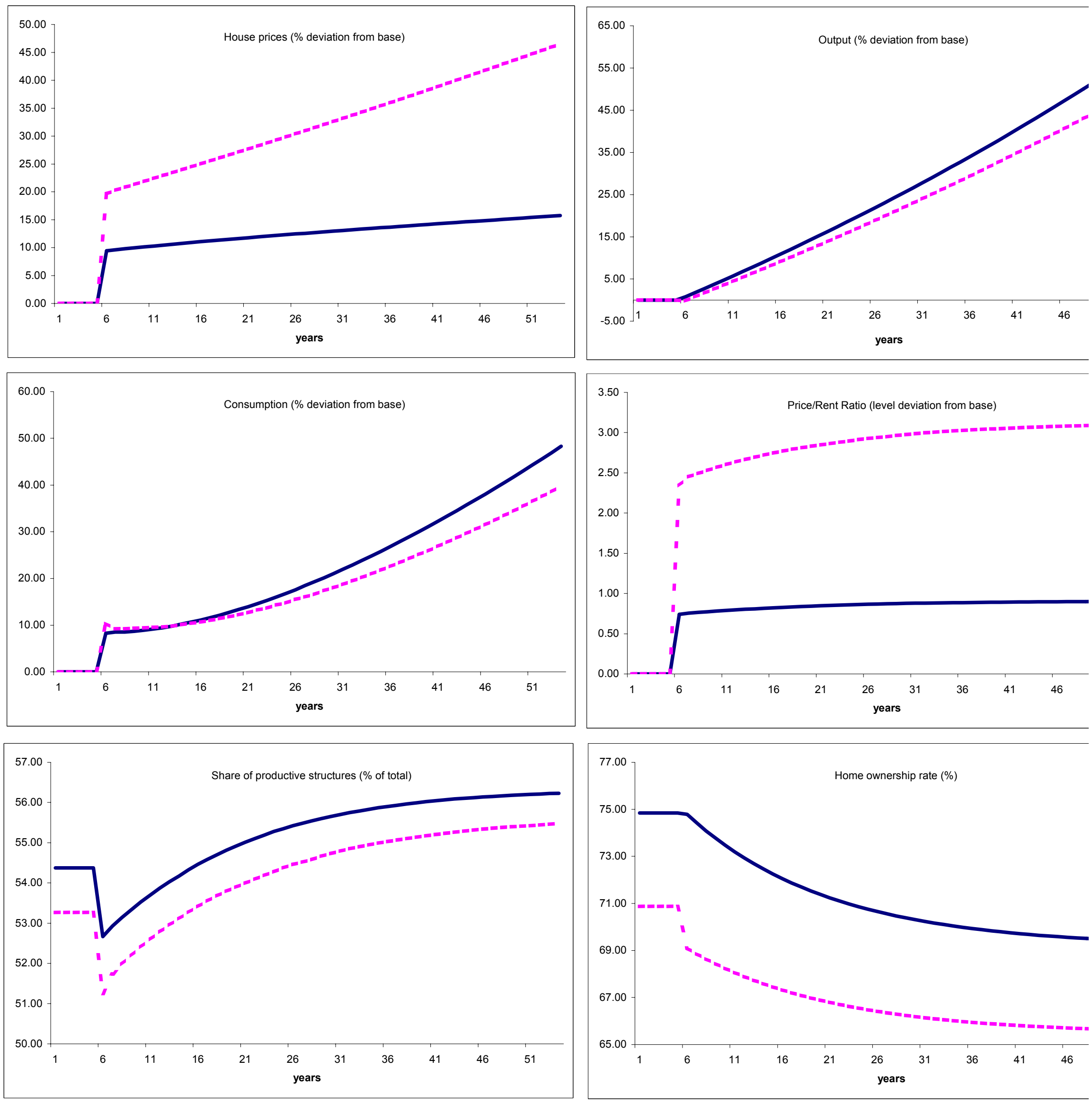

Figure 6
Transition dynamics from a 1% reduction in the world real interest rate
(solid line: γ=0.9, dotted line: γ=0.5)

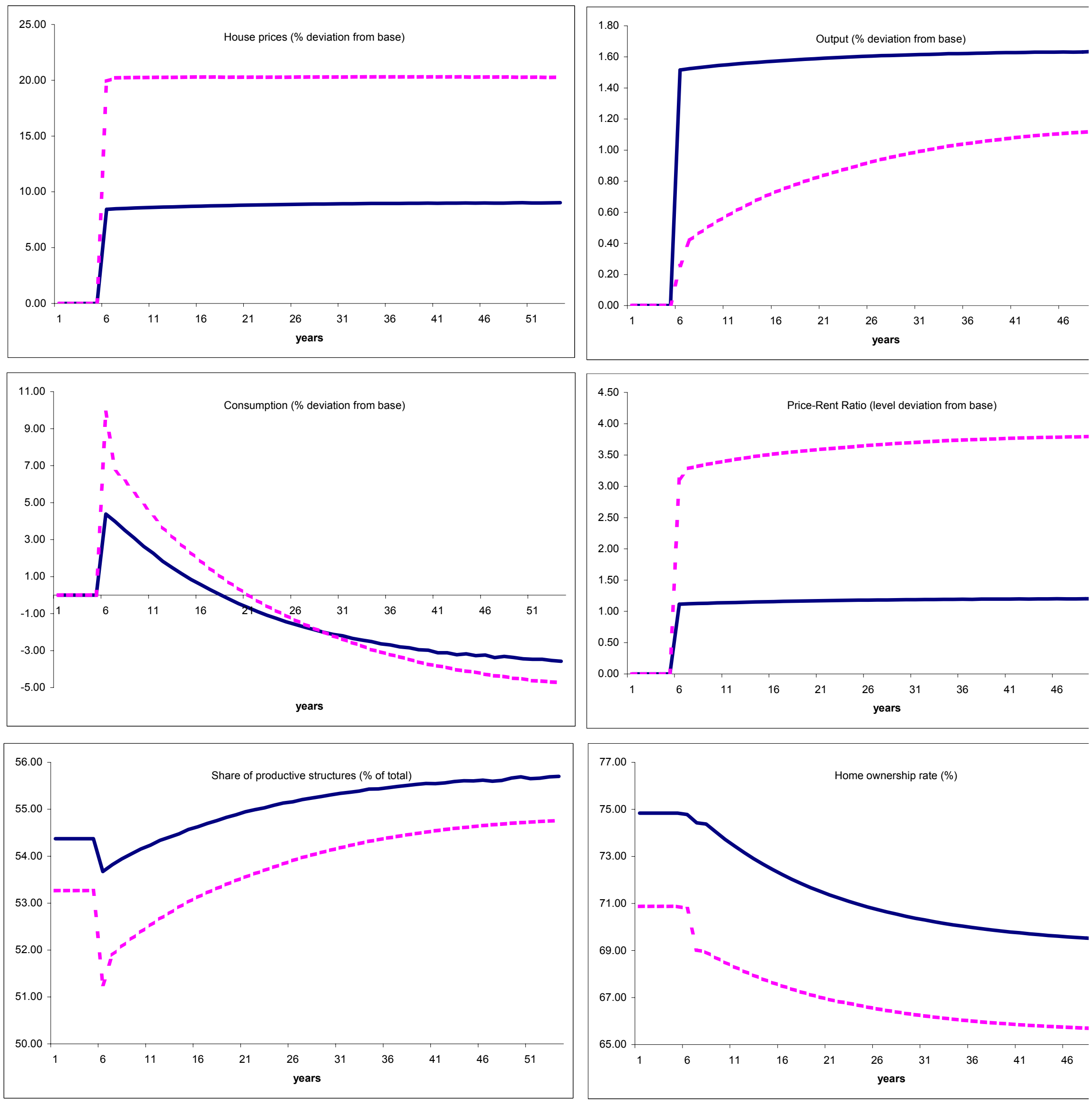

Table 5

	baseline	*θ=0.1*	*θ=1.0*	*ga=1.03*	*b=0.1*	*γ=0.5*
Column	*1*	*2*	*3*	*4*	*5*	*6*
% of tenants	25.16	2.95	37.48	25.16	5.07	10.30
% of constrained households	8.34	25.71	10.94	8.56	4.65	8.50
% of unconstrained homeowners	66.50	71.35	51.58	66.28	90.28	81.20
% of housing used by tenants	8.69	0.57	13.17	8.67	1.34	2.45
% of housing used by constrained	2.42	8.11	6.48	2.49	0.90	2.22
% of shares owned by tenants	0.05	0.02	0.82	0.08	0.10	0.12
% of shares owned by constrained	0.33	0.37	2.95	0.34	0.12	0.32
Current account as % of GDP	0.00	0.00	0.00	0.00	0.00	0.00
Net foreign Assets as % of GDP	0.00	0.00	0.00	0.00	0.00	0.00
Value of total structures to GDP	3.29	3.30	3.30	3.25	3.01	3.70
Housing structures to total structures	0.46	0.46	0.46	0.46	0.45	0.52
Value of housing to wages	2.57	2.46	2.46	2.27	2.72	4.16
Housing price to rental rate	8.45	8.46	8.48	7.70	9.54	11.08
Real return	6.83	6.81	6.79	8.40	5.90	9.20

Notes to Table 5: Results from the closed economy with zero demand for domestic shares by the representative foreigner. In the baseline economy, the collateral constraint is denoted by θ and is equal to 0.3, gn denotes population growth and is equal to 1.01 (one percent per annum), ga=1.02 denotes a two percent annual productivity growth, and b=0.2 denotes a twenty percent gross replacement rate during retirement. The results from reducing γ from its baseline value of 0.9 to 0.5 are reported in column (6) labeled {γ=0.5, (6)}.

Table 6

	baseline	*θ=0.1*	*θ=1.0*	*ga=1.03*	*b=0.1*	*R*=5.83*	*γ=0.5*
Column	*1*	*2*	*3*	*4*	*5*	*6*	*7*
% of tenants	25.16	2.95	37.48	30.81	2.95	30.81	29.12
% of constrained households	8.34	25.70	9.87	6.61	3.66	3.71	5.79
% of unconstrained homeowners	66.50	71.36	52.65	62.58	93.39	65.48	65.09
% of housing used by tenants	8.69	0.75	13.76	10.83	0.65	10.60	9.06
% of housing used by constrained	2.42	8.10	5.69	2.68	0.69	1.33	1.70
% of shares owned by tenants	0.05	0.10	0.89	0.20	0.02	-0.01	1.08
% of shares owned by constrained	0.33	0.37	2.54	1.03	0.07	0.34	1.22
Current account as % of GDP	0.00	-0.07	-0.16	4.79	-4.18	3.56	13.05
Net foreign Assets as % of GDP	0.00	1.53	3.40	-124.34	86.37	-92.34	-251.26
Value of total structures to GDP	3.29	3.30	3.30	3.25	3.01	3.70	4.65
Housing structures to total structures	0.46	0.46	0.46	0.44	0.47	0.44	0.47
Value of housing to wages	2.57	2.57	2.57	2.68	2.68	2.78	4.82
Housing price to rental rate	8.45	8.45	8.45	9.35	8.45	9.65	15.71
Real return	6.83	6.83	6.83	6.83	6.83	5.83	6.83

Notes to Table 6: Results from the small open economy with a given demand for domestic shares by a representative foreigner (world interest rate is 6.83% and γ=0.9). In the baseline economy, the collateral constraint is denoted by θ and is equal to 0.3, gn denotes population growth and is equal to 1.01 (one percent per annum), ga=1.02 denotes a two percent annual productivity growth, and b=0.2 denotes a twenty percent gross replacement rate during retirement. R* is the world real return. γ=0.5 (column (8)) reports the results from setting γ equal to 0.5 at this given world interest rate.

Table 7

Scarcity of Land Parameter	γ=0.9	γ=0.5	γ=0.9	γ=0.5	γ=0.9	γ=0.5
Column	1	2	3	4	5	6
Panel A: Certainty expenditure equivalent	*ga+1%*	*ga+1%*	*R*-1%*	*R*-1%*	*b=0.15*	*b=0.1*
Workers	9.69	8.14	-0.89	-0.21	-5.06	-4.16
Tenant Workers	9.67	7.33	-0.45	-1.22	-11.38	-10.51
Constrained Homeowner Workers	9.64	7.14	-1.45	-1.87	-8.23	-6.55
Unconstrained Homeowner Workers	10.21	9.12	-1.53	-0.33	-2.78	-1.68
Low Income Workers	9.60	7.30	-0.53	-1.14	-11.40	-10.75
Middle Income Workers	10.36	8.73	-1.19	-0.36	-4.65	-3.49
High Income Workers	9.73	9.36	-1.79	0.00	0.90	1.62
Retirees	7.56	8.45	0.00	3.47	-29.86	-26.90
Tenant Retirees	6.93	5.60	-0.30	-0.33	-43.79	-44.06
Constrained Homeowner Retirees	7.42	5.80	-0.05	0.15	-42.60	-40.73
Unconstrained Homeowner Retirees	8.61	9.38	0.79	4.53	-22.02	-18.77
Panel B: Wealth change						
Workers	4.09	6.94	5.94	10.44	0.00	0.00
Tenant Workers	-0.16	-0.97	1.51	0.58	0.03	0.25
Constrained Homeowner Workers	1.51	4.13	3.18	6.93	0.21	0.45
Unconstrained Homeowner Workers	6.53	10.96	8.12	15.39	0.47	0.47
Low Income Workers	-0.15	-1.20	1.50	0.24	0.10	0.23
Middle Income Workers	5.81	9.54	7.23	13.77	0.42	0.58
High Income Workers	7.67	12.32	9.16	16.99	0.00	0.26
Retirees	5.66	9.92	7.22	14.25	0.00	0.00
Tenant Retirees	0.89	1.25	2.30	3.36	0.45	0.45
Constrained Homeowner Retirees	3.43	6.81	4.41	9.95	0.50	0.64
Unconstrained Homeowner Retirees	7.65	11.97	8.15	16.60	0.58	0.68

Notes to Table 7: Panel A reports the certainty expenditure equivalent changes (in percent) from shifts in the specified fundamentals relative to the baseline steady state in the small open economy (details of this computation are given in the text). In the baseline economy, the collateral constraint is denoted by θ and is equal to 0.3, gn denotes population growth and is equal to 1.01 (one percent per annum), ga=1.02 denotes a two percent annual productivity growth, and b=0.2 denotes a twenty percent gross replacement rate during retirement. R* is the world real return. Panel B reports the Wealth Change (in percent) right after the unexpected change in the specified fundamentals for the same cases (details for these calculations are given in the text).