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The Volatility of Swedish Treasury Bonds: Testing the Expectations Model of the Term Structure using Variance Bounds

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Abstract

This paper investigates the interest rate volatility of Swedish long term treasury bonds for the period 1984 – 1994. According to the expectations model of the term structure, the long term interest rate may be expressed as a weighted average of expected future short rates. This makes it possible to derive theoretical bounds on the variance of long term interest rates (Shiller (1979), LeRoy and Porter (1980)). The empirical investigation of the long term yield variance and the implied variance bounds shows that the long term rate is too volatile to accord with the model. Moreover, it is found that these violations are statistically significant. The conclusion is therefore that the expectations model of the term structure of interest rates is rejected.

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Contents

1 Introduction

A question commonly asked is whether the volatility of long term interest rates is too high to be explained by changes in expectations made by rational investors. Specifically, if interest rates are assumed to be formed in agreement with the expectations model of the term structure of interest rates, are long term rates too volatile to accord with this model? Since the expectations model allows the long term interest rate to be written as a weighted average of expected future short rates, the long rate should be a very smooth series. However, the observed realized long term interest rate is, in fact, not very different from short term rates.

Hence, this seems to reflect the same kind of puzzle as the observed excess volatility in stock prices compared with future dividends. Methods for testing whether the variance of stocks or bonds indeed is too high to be justified by some asset pricing model have been developed by Shiller (1979) and LeRoy and Porter (1981). These so-called variance bounds tests provide upper and lower limits on the variance of the price or yield of some asset. When applied to empirical data, the variance bounds tests have, almost without exception, resulted in the rejection of the models under consideration.[1,2]

Shiller (1979) found that the variances of one-period holding yields of U.S. long term bonds exceeded the upper bound implied by the expectations model. Shiller did not, however, perform any tests of whether or not the violations were statistically significant. Singleton (1980) extended the results on variance bounds by Shiller and LeRoy and Porter (1981), by developing testing procedures and distributional properties of the estimators, in a spectral analysis framework. Like Shiller, Singleton found that the variances of the one-period holding returns of long term bonds exceeded their upper bounds. He also found that the upper bound of the variance of the long term interest rate was exceeded by the observed sample variance. Furthermore, he concluded that the violations of the variance bounds were statistically significant.

The results of Shiller and Singleton add to the list of studies performed on U.S. and U.K. data which have resulted in the rejection of the expectations model; see Shiller (1990) for a survey of empirical work. To my knowledge, no variance bounds tests have been carried out on Swedish interest rate data. However, the expectations model of the term structure has been tested using

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1 See e.g. Shiller(1979), Singleton (1980), Shiller (1981b), and LeRoy and Porter (1981).

2 In a recent review article, Cochrane (1991) shows that variance bounds tests are equivalent to Euler equation tests of discount-rate models, and argues that these have econometric advantages over the traditional volatility tests. Cochrane (1991) is, however, mainly concerned with present value models for the stock market, which imply an infinite time horizon.

other methods. Hörngren (1986), using quotations from the short term end of the Swedish bank certificate market during 1980–1985, rejected the pure expectations theory. Bergman (1988) found evidence of a time varying risk premium in the market for Swedish t-bills during the period 1985–1986. The expectations hypothesis was thus rejected also in Bergman's study. However, neither of these studies included long term bonds in the investigations. Ekdahl and Warne (1990), on the other hand, using 5-year bonds and 30-day treasury bills during 1983–1988 were not able to reject a rational expectations model of the term structure. Finally, for the period 1987–1992 Hördahl (1994) found no evidence of any time-varying risk premium for 10-year treasury bonds in an ARCH-M framework.

The purpose of this study is to apply the variance bounds tests proposed by Shiller (1979) and LeRoy and Porter (1981) to Swedish long term treasury bond data, in order to find out whether the bounds implied by the expectations model are supported by the data. The paper is organized as follows. Section 2 gives a short description of a linearized version of the expectations model of the term structure of interest rates. Section 3 summarizes the variance bounds under consideration, while section 4 describes the methods of estimation used in the study. The results are found in section 5, and an alternative test is considered in section 6. Finally, section 7 concludes the paper with a summary and a short discussion.

2 The Linearized Expectations Model of the Term Structure

The linearized version of the rational expectations model of the term structure (Shiller, 1979) states the relation between the long term n-period bond yield (R_t^n) and expected future short (one-period) interest rates (r_t) in the following manner:

$$R_t^n = \alpha \sum_{s=0}^{n-1} \gamma^s {}_t r_{t+s}^e + \Phi_n, \tag{1}$$

where

$$\alpha = \frac{1-\gamma}{1-\gamma^n}, \quad 0 < \gamma < 1, \tag{2}$$

and

$${}_t r_{t+s}^e = E_t\left[r_{t+s} \middle| \Psi_t\right]. \tag{3}$$

The symbol Φ_n denotes a liquidity premium which may be dependent on the maturity n, but not on time, while Ψ_t represents all information available at time t.[3] The expression in (1) states the n-period interest rate as a weighted average of the expected future short rates, where the weights are constructed as a truncated exponential distribution, and scaled so that the weights sum to one. This construction assigns greater weight to the expected future short rates which are close in time, than to rates further ahead in the future.

The linearized expression in (1) was derived by Shiller for coupon-bonds selling near par, or for bonds with infinite maturity. By linearizing the rational expectations model around the one-period coupon yield, $C = \overline{R}$, (1) is obtained with $\gamma = 1/(1+\overline{R})$. Thus, the long term interest rate is expressed as the present value of the future expected short rates, discounted by the factor $\overline{R}$, plus a possible liquidity premium. Note that since the premium is assumed constant, it will not affect the variance of the long term rate.

[3] The relationship in (1) is a linearization of a number of versions of the rational expectations model; see Shiller (1979), pp. 1195–1199. Particularly, if the liquidity premium is zero, we have the the pure expectations model, while if it is nonzero (but constant over time), we have the ordinary expectations model.

3 Variance Bounds

The expectations model of the term structure may be tested by examining the bounds on the variance of the long term interest rate which are implied by the model. Long term interest rates may, according to the expectations model, be expressed as a weighted moving average of expected future short term rates plus possibly a constant premium. Consequently, the variance of the long term rate should be quite low compared to the variance of the short rate. The question, therefore, is whether the actual realized long term rates are more volatile than implied by the expectations model.

Bounds on the volatility of asset prices were first considered by Shiller (1979) and by LeRoy and Porter (1981). LeRoy and Porter presented three theorems on the relation between the variance of the process of a dependent variable and the variance of the independent variable process. These theorems give successively sharper bounds on the variance of the dependent variable. A brief summary of the derivation of LeRoy and Porters theorems of variance bounds is given below.[4]

The starting point of the analysis is a general present-value relation, which relates a dependent time series R_t to an independent series r_t:

$$R_t = \sum_{s=0}^{\infty} \beta^s \, {}_t r_{t+s}^e, \quad {}_t r_{t+s}^e = E_t\left[r_{t+s} \middle| \Psi_t\right], \tag{4}$$

where β is a discount factor, assumed to be less than one. Clearly, (4) could represent the relation between the yield of a perpetuity and the expected future short term interest rates, i.e. relation (1) with $n=\infty$.

Next, the series R_t^* is defined as the series generated by the present-value relation (4) when r_t is perfectly forecastable. The series R_t^* is called the "perfect foresight" or "ex post rational" dependent series:

$$R_t^* = \sum_{s=0}^{\infty} \beta^s r_{t+s}. \tag{5}$$

Furthermore, $\hat{R}_t$ is defined as the series generated by relation (4) when the future $\hat{r}_t$ are *not* perfectly forecastable, and when it is assumed that the only relevant variables for forecasting $\hat{R}_t$ are lagged r_t:

4 From LeRoy and Porter (1981), pp. 555–564. The theorems derived and proven in LeRoy and Porter are of a general nature. The summary given here will concentrate on the specific term structure relation given by (1). Furthermore, since only the first two theorems will be used in the subsequent tests, theorem 3 is omitted here.

$$\hat{R}_t = \sum_{s=0}^{\infty} \beta^s \, {}_t\hat{r}_{t+s}, \qquad {}_t\hat{r}_{t+s} = E_t\left[r_{t+s} \middle| r_t, r_{t-1}, \ldots\right]. \tag{6}$$

The following relations constraining the variance of the series R_t are then proposed by LeRoy and Porter:

$$\mathrm{var}\left(R_t\right) < \mathrm{var}\left(r_t\right), \tag{7}$$

and

$$\mathrm{var}\left(\hat{R}_t\right) \le \mathrm{var}\left(R_t\right) < \mathrm{var}\left(R_t^*\right). \tag{8}$$

Since the relation (8) is of greatest interest in the empirical tests, a brief outline of the proof of this relation is given here.[5] Recall that R_t^* may be expressed as

$$R_t^* = r_t + \beta r_{t+1} + \beta^2 r_{t+2} + \ldots \tag{9}$$

Let η_t be defined as the discounted values of the forecast errors:

$$\eta_t = \sum_{s=1}^{\infty} \beta^s \left(r_{t+s} - {}_t r_{t+s}^e\right), \tag{10}$$

where ${}_t r_{t+s}^e$ are general (not perfect) forecasts, as in (4). Given this definition, R_t^* may be expressed as

$$R_t^* = R_t + \eta_t, \tag{11}$$

where by construction R_t and η_t are independent. Thus, the relationship between the variances of these variables may be written in the following way:

$$\mathrm{var}(R_t^*) = \mathrm{var}(R_t) + \mathrm{var}(\eta_t). \tag{12}$$

Therefore, $\mathrm{var}(R_t^*)$ provides an upper bound on $\mathrm{var}(R_t)$. The strict inequality of the upper bound in (8) follows from the assumption that uncertainty cannot be entirely eliminated in the model. The lower bound in (8) is given by the fact that the information available to investors always includes past and present realizations of r_t. Including any additional information available at time t in the forecasting process, means that the variance of R_t can never be greater than that of $\hat{R}_t$.

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5 See LeRoy and Porter (1981), pp. 560–564 for complete proofs of both (7) and (8).

In addition to the variance bounds in (7) and (8), a bound proposed by Shiller (1979) will be considered. This bound restricts the variance of the linearized one-period holding yield of a long-term bond, H_t:[6]

$$H_t = \frac{\left(R_t^n - \bar{\gamma}R_{t+1}^{(n-1)}\right)}{(1-\bar{\gamma})}, \tag{13}$$

where

$$\bar{\gamma} = \frac{\gamma\left(1-\gamma^{n-1}\right)}{\left(1-\gamma^n\right)}. \tag{14}$$

Shiller shows that the following relation between the variance of the holding yield and the one period interest rate is implied by the expectations model:[7]

$$var\left(H_t\right) \leq \frac{var(r_t)}{\left(1-\bar{\gamma}^2\right)}. \tag{15}$$

Singleton (1980) notes that (15) provides a more demanding test of the expectations model of the term structure than the bounds on the variance of the long term rate in (7) and (8).

The relations given in (7), (8) and (15) will be used to test the rational expectations model of the term structure of interest rates. If the variance of the investigated long term interest rate violates any one of the relations, the model will be rejected. The next section describes how the variances will be estimated.

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6 Shiller (1979) showed that the approximation error induced by the linearization of the holding period yield is quite small. Ekdahl and Warne (1990) showed the same thing for Swedish data.

7 See Shiller (1979), p. 1203 for a proof.

4 Method of Estimation

In an empirical application of the variance bounds tests, LeRoy and Porter (1981) examined the volatility of stock returns using multivariate ARMA models. Singleton (1980), on the other hand, estimated the variances and their bounds with the use of spectral analysis methods in an empirical investigation of U.S. interest rates. He extended the results of LeRoy and Porter to generalized present-value relations for the expected future values of several variables. Furthermore, Singleton developed an asymptotic distribution theory for testing the variance bounds in the frequency domain framework. This methodology will constitute the basis of the main empirical investigation in this paper, and thus a summary of the methods used by Singleton is given in appendix A.

According to the linearized version of the expectations model of the term structure, the n-period yield to maturity may be expressed in terms of one-period expected future spot rates in accordance with (1). Thus, the results in the previous section are conveniently transferred to the linearized expectations model for bonds maturing in n periods. The "perfect foresight" or "ex post rational" long term interest rate is then given by (ignoring the term premium):

$$R_t^* \equiv \alpha \sum_{s=0}^{n-1} \gamma^s r_{t+s}, \tag{16}$$

where

$$\alpha \equiv \frac{(1-\gamma)}{(1-\gamma^n)}, \tag{17}$$

and, following Shiller (1979), $\gamma = 1/(1+\bar{R})$.

Similarly, the long term rate generated when past and present short term rates constitute the entire information set of agents is

$$\hat{R}_t = \alpha \sum_{s=0}^{n-1} \gamma^s \hat{r}_{t+s}, \quad \hat{r}_{t+s} = E_t\left[r_{t+s} \middle| r_t, r_{t-1}, \ldots\right]. \tag{18}$$

A useful result is provided by Singleton (1980), who show that the variance bounds relations may be extended to situations where the period of expectation is longer than the interval of the sampled data. Thus, for example monthly data can be used to test the expectations hypothesis, while the short rate selected for the test may be a 6-month rate. In this case, the term structure relationship can be rewritten as

$$R_t^{6n} = \alpha \sum_{s=0}^{n-1} \gamma^s r_{t\,t+6s}^e, \qquad r_{t\,t+6s}^e = E_t\left[x_{t+6s} \middle| \Psi_t\right], \tag{19}$$

where R_t^{6n} is the yield of a $6n$-month bond. The forecasted yields in (16) and (18) are rewritten as

$$R_t^* \equiv \alpha \sum_{s=0}^{n-1} \gamma^s r_{t+6s} \tag{20}$$

and

$$\hat{R}_t = \alpha \sum_{s=0}^{n-1} \gamma^s \hat{r}_{t+6s}, \qquad \hat{r}_{t+6s} = E_t\left[r_{t+6s} \middle| r_t, r_{t-1}, \ldots\right]. \tag{21}$$

The expression for the one-period holding yield (13) is adjusted in a similar fashion. The parameters α and γ are not changed by the altered sampling frequency, but in the estimation of the variances a few adjustments are necessary.[8]

With the use of the variance bounds presented in (7), (8) and (15), the expectations hypothesis will be tested for Swedish data. Specifically, the yields to maturity of 5- and 10-year treasury bonds are chosen as the long term rates, while the 1- or 3-month treasury bill rate is selected as the one-period interest rate. The data consists of daily bid and ask rates set at the end of each day.[9] The yields used in the tests are the averages of the bid and the ask rates of the selected day of each month. The investigated period is February 17, 1984 through November 23, 1994 for the 5-year bond, and January 2, 1987 up to and including November 23, 1994 for the 10-year bond.

Following Singleton (1980), the variances of the series $\hat{R}_t$, R_t and R_t^* are calculated using the spectral methods described in Appendix A. Specifically, the variance of a series is given by the integral of the spectral density of this series over $[-\pi, \pi]$. The spectral density function, or power spectrum, gives a representation of the variance as a function of the frequency, or period of oscillation. Thus, in order to obtain estimates of the variances, the spectral densities of the series involved have to be estimated.

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8 In particular, the sequences $a(F)$ and $b(L)$ used in (A.4) and (A.8) of appendix A will have to be adjusted to reflect the forecast horizon of six months. This means that when estimating the variance of the "perfect forecast" long term yield (20), $a(F)$ should be written as $a(F) = \alpha(1+\gamma F^6 + \gamma^2 F^{12} + \ldots)$, and $b(L)$ should be adjusted in a similar fashion when estimating the variance of (21).

9 The data was kindly provided by the Riksbank.

If any linear trends were to be present in the data, the spectral estimators will be distorted over the low frequency part of the spectrum.[10] Therefore, the series are first detrended using ordinary trend regression. Another problem is that the data is very likely to be serially correlated, which leads to a problem called "leakage". By this it is meant that variance is concentrated in particular frequency bands in a way which leads to a significant loss of efficiency in the estimation procedure.[11] It is therefore necessary to flatten the spectrum, which can be done by prewhitening the data. Following Singleton, this is accomplished with the quasi-difference filter $(1-0.95L)$.

The actual estimation of the spectral density of the variable in question, say r, is then carried out, noting that the associated periodogram, $P_r(\lambda)$, is an asymptotically unbiased estimator of the spectrum.[12] The periodogram is given by the squared modulus of the Fourier transform of the series r:

$$P_r(\lambda) = \frac{1}{2\pi T}\left|\tilde{r}(\lambda)\right|^2, \tag{22}$$

where

$$\tilde{r}(\lambda) = \sum_{t=1}^{T} r_t e^{-i\lambda t}, \qquad -\pi \leq \lambda \leq \pi.$$

Unfortunately, the periodogram is not a consistent estimator of the spectral density.[13] Therefore, the periodogram is commonly averaged over adjacent frequencies using for example a rectangular window of width L. This gives a smoothed spectral estimator, and provides a consistent estimate of the spectrum:

$$\hat{S}_r(\lambda_j) = L^{-1} \sum_{i=-(m-1)/2}^{(m-1)/2} P_r\left(\lambda_{j-i}\right). \tag{23}$$

In the actual calculations of the sample variances, the nearest odd integer larger than $0.75\sqrt{T}$ is used as the width of the window. With the methods described above, and padding the series to the nearest higher power of 2, estimates of the prewhitened spectra are obtained.[14] We arrive at the final

10 See e.g. Shumway (1988), p. 125.
11 See e.g. Fishman (1969), pp. 112–118.
12 See e.g. Dhrymes (1970), pp. 422–423.
13 Ibid. pp. 424–430.

14 The padding involves extending a series of length T to $T' = 2^n$, where the integer n is chosen so that $T' > T$, by adding zeros to the original series. This is done because the fast Fourier algorithm used works best for series where the number of points is a power of 2. The values of the original

spectral estimates of the original series by recoloring the estimated spectra in (23). This is done by dividing $\hat{S}_r(\lambda)$ by the frequency response function of the filter, in this case by $(1-2\cdot 0.95\cdot \cos\lambda + 0.95^2)$.[15] The estimated variances of the series are given by the appropriately scaled sums of the spectral densities, while the variance-covariance matrices of these estimated variances are given by (A.10), with the integrals replaced by sums.

series are not changed, only the frequencies at which the Fourier transform is evaluated. (Shumway (1988), pp. 66–67).

15 See Fishman (1969), pp. 112–118.

5 Empirical Results

The results for the 5-year bond, using daily data, are presented in table 1, where 1- and 3-month yields are used as the short rate. Table 2 displays the results for the 10-year bond. In addition to the estimated variances and associated variance-covariance matrices, test statistics for the inequalities in (7), (8) and (15) are reported. These test statistics are simply given by

$$\frac{\hat{\sigma}^2_{R_1} - \hat{\sigma}^2_{R_2}}{SD\left(\hat{\sigma}^2_{R_1} - \hat{\sigma}^2_{R_2}\right)} \qquad (24)$$

where $\hat{\sigma}^2_{R_1}$ denotes the estimated variance of R_1, and $SD(\cdot)$ is the standard deviation of the argument. The null hypothesis, i.e. the expectations model of the term structure, is rejected if any of the test statistics are negative and significant.

From tables 1 and 2 it is clear that the expectations hypothesis is rejected for all four combinations of bonds and bills. The test statistics pertaining to the variance of the one-period holding yield $\left(\hat{\sigma}^2_H\right)$ are significantly negative in all cases. On average, the variance of the holding yield is about five times higher than the upper bound implied by the expectations hypothesis. Furthermore, the variance of the long term interest rate $\left(\hat{\sigma}^2_R\right)$ on average exceeds the upper bound set by the "perfect foresight" rate $\left(\hat{\sigma}^2_{R^*}\right)$ by a factor of 2.8, and the violations are statistically significant in three of the four cases.

Table 1: Estimated variances and test statistics for variance bounds
Swedish 5-year bonds

5-year bonds /1-month bills — Period: February 17, 1984–November 23, 1994

Estimated variances: $\hat{\sigma}^2_r = 23.906$ $\quad \hat{\sigma}^2_R = 1.951$ $\quad \hat{\sigma}^2_{R^*} = 0.825$

$\hat{\sigma}^2_R = 0.024$ $\quad \hat{\sigma}^2_H = 1044.015$ $\quad \hat{\sigma}^2_B = 560.136$

$$\mathrm{var}\begin{bmatrix}\hat{\sigma}^2_r \\ \hat{\sigma}^2_R\end{bmatrix} = \begin{bmatrix}6.568 & 1.115 \\ 1.115 & 0.232\end{bmatrix}$$

Test statistic $[\mathrm{var}(R_t) < \mathrm{var}(r_t)]$: 10.271

$$\mathrm{var}\begin{bmatrix}\hat{\sigma}^2_{R^*} \\ \hat{\sigma}^2_R\end{bmatrix} = \begin{bmatrix}0.317 & 0.108 \\ 0.108 & 0.232\end{bmatrix}$$

Test statistic $[\mathrm{var}(R_t) < \mathrm{var}(R^*_t)]$: -1.954 *

$$\mathrm{var}\begin{bmatrix}\hat{\sigma}^2_R \\ \hat{\sigma}^2_R\end{bmatrix} = \begin{bmatrix}0.232 & 0.001 \\ 0.001 & 0.00001\end{bmatrix}$$

Test statistic $[\mathrm{var}(R_t) \geq \mathrm{var}(\hat{R}_t)]$: 4.025

$$\mathrm{var}\begin{bmatrix}\hat{\sigma}^2_B \\ \hat{\sigma}^2_H\end{bmatrix} = 10^4 \cdot \begin{bmatrix}0.361 & 0.817 \\ 0.817 & 3.225\end{bmatrix}$$

Test statistic $[\mathrm{var}(H_t) \leq \mathrm{var}(B_t)]$: -3.464 **

5-year bonds / 3-month bills — Period: February 17, 1984–November 23, 1994

Estimated variances: $\hat{\sigma}^2_r = 6.031$ $\quad \hat{\sigma}^2_R = 1.950$ $\quad \hat{\sigma}^2_{R^*} = 0.461$

$\hat{\sigma}^2_R = 0.024$ $\quad \hat{\sigma}^2_H = 541.905$ $\quad \hat{\sigma}^2_B = 48.681$

$$\mathrm{var}\begin{bmatrix}\hat{\sigma}^2_r \\ \hat{\sigma}^2_R\end{bmatrix} = \begin{bmatrix}1.736 & 0.632 \\ 0.632 & 0.231\end{bmatrix}$$

Test statistic $[\mathrm{var}(R_t) < \mathrm{var}(r_t)]$: 4.865

$$\mathrm{var}\begin{bmatrix}\hat{\sigma}^2_{R^*} \\ \hat{\sigma}^2_R\end{bmatrix} = \begin{bmatrix}0.117 & 0.066 \\ 0.066 & 0.231\end{bmatrix}$$

Test statistic $[\mathrm{var}(R_t) < \mathrm{var}(R^*_t)]$: -3.209 **

$$\mathrm{var}\begin{bmatrix}\hat{\sigma}^2_R \\ \hat{\sigma}^2_R\end{bmatrix} = \begin{bmatrix}0.231 & 0.003 \\ 0.003 & 0.00003\end{bmatrix}$$

Test statistic $[\mathrm{var}(\mathrm{R}_t) \geq \mathrm{var}(\hat{\mathrm{R}}_t)]$: 4.050

$$\mathrm{var}\begin{bmatrix}\hat{\sigma}^2_B \\ \hat{\sigma}^2_H\end{bmatrix} = 10^3 \cdot \begin{bmatrix}0.113 & 0.981 \\ 0.981 & 16.310\end{bmatrix}$$

Test statistic $[\mathrm{var}(H_t) \leq \mathrm{var}(B_t)]$: -4.102 **

$\hat{\sigma}^2_B$ is the estimated variance of $\mathrm{var}(r_t) / (1-\bar{\gamma}^2)$, as in (15). * denotes that the tested difference is significantly smaller than zero at the 5% level, and ** at the 1% level. The test statistic is given by (24).

Table 2: Estimated variances and test statistics for variance bounds Swedish 10-year bonds

10-year bonds / 1-month bills		Period:	January 2, 1987 – November 23, 1994
Estimated variances:	$\hat\sigma_r^2 = 30.569$	$\hat\sigma_R^2 = 1.273$	$\hat\sigma_{R^*}^2 = 0.604$
	$\hat\sigma_{\hat R}^2 = 0.006$	$\hat\sigma_H^2 = 3085.304$	$\hat\sigma_B^2 = 1138.032$

$$\text{var}\begin{bmatrix}\hat\sigma_r^2\\ \hat\sigma_R^2\end{bmatrix} = \begin{bmatrix}11.630 & 0.917\\ 0.917 & 0.090\end{bmatrix} \qquad \text{Test statistic } [\text{var}(R_t) < \text{var}(r_t)]:\ 9.317$$

$$\text{var}\begin{bmatrix}\hat\sigma_{R^*}^2\\ \hat\sigma_R^2\end{bmatrix} = \begin{bmatrix}0.383 & 0.047\\ 0.047 & 0.090\end{bmatrix} \qquad \text{Test statistic } [\text{var}(R_t) < \text{var}(R_t^*)]:\ -1.085$$

$$\text{var}\begin{bmatrix}\hat\sigma_R^2\\ \hat\sigma_{\hat R}^2\end{bmatrix} = \begin{bmatrix}0.090 & 0.0002\\ 0.0002 & 0.000\end{bmatrix} \qquad \text{Test statistic } [\text{var}(R_t) \geq \text{var}(\hat R_t)]:\ 4.231$$

$$\text{var}\begin{bmatrix}\hat\sigma_B^2\\ \hat\sigma_H^2\end{bmatrix} = 10^4 \cdot \begin{bmatrix}1.612 & 5.858\\ 5.858 & 39.709\end{bmatrix} \qquad \text{Test statistic } [\text{var}(H_t) \leq \text{var}(B_t)]:\ -3.579^{**}$$

10-year bonds / 3-month bills		Period:	January 2, 1987– November 23, 1994
Estimated variances:	$\hat\sigma_r^2 = 5.887$	$\hat\sigma_R^2 = 1.273$	$\hat\sigma_{R^*}^2 = 0.458$
	$\hat\sigma_{\hat R}^2 = 0.009$	$\hat\sigma_H^2 = 343.292$	$\hat\sigma_B^2 = 74.992$

$$\text{var}\begin{bmatrix}\hat\sigma_r^2\\ \hat\sigma_R^2\end{bmatrix} = \begin{bmatrix}1.498 & 0.366\\ 0.366 & 0.090\end{bmatrix} \qquad \text{Test statistic } [\text{var}(R_t) < \text{var}(r_t)]:\ 4.986$$

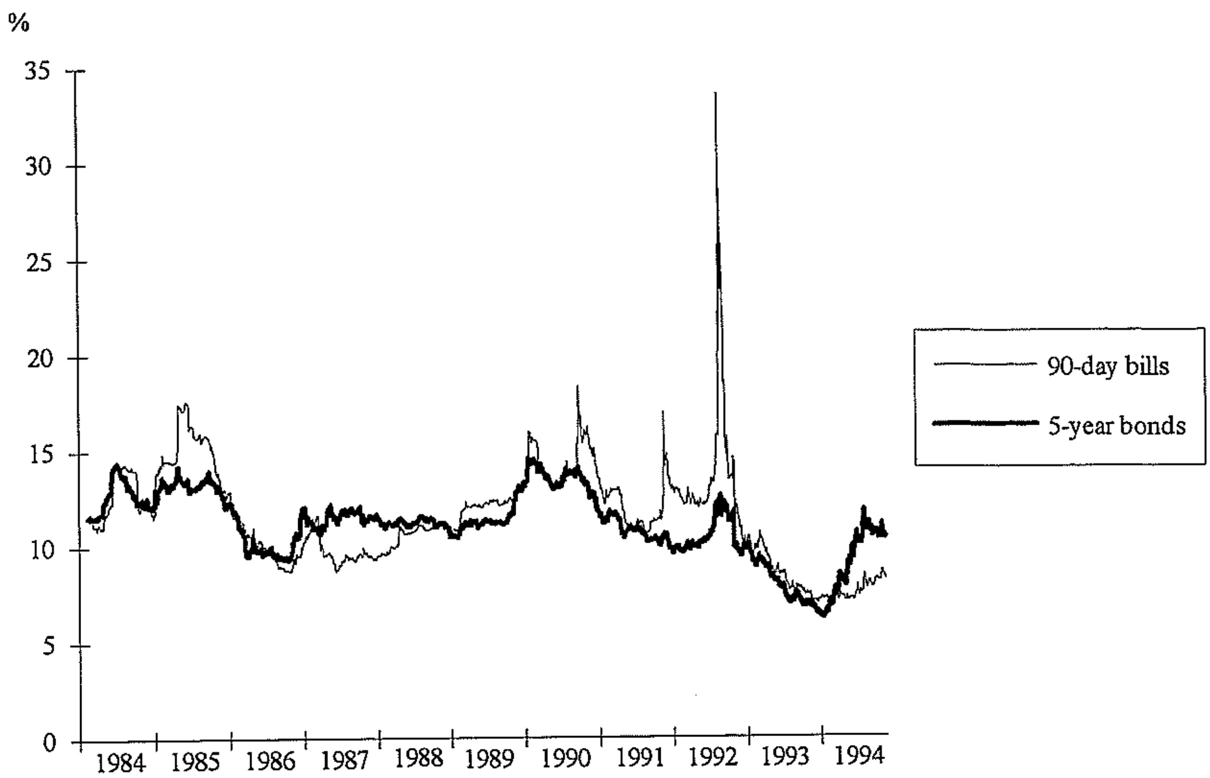
$$\text{var}\begin{bmatrix}\hat\sigma_{R^*}^2\\ \hat\sigma_R^2\end{bmatrix} = \begin{bmatrix}0.112 & 0.042\\ 0.042 & 0.090\end{bmatrix} \qquad \text{Test statistic } [\text{var}(R_t) < \text{var}(R_t^*)]:\ -2.377^{**}$$

$$\text{var}\begin{bmatrix}\hat\sigma_R^2\\ \hat\sigma_{\hat R}^2\end{bmatrix} = \begin{bmatrix}0.090 & 0.0006\\ 0.0006 & 0.000\end{bmatrix} \qquad \text{Test statistic } [\text{var}(R_t) \geq \text{var}(\hat R_t)]:\ 4.239$$

$$\text{var}\begin{bmatrix}\hat\sigma_B^2\\ \hat\sigma_H^2\end{bmatrix} = 10^3 \cdot \begin{bmatrix}0.243 & 0.743\\ 0.743 & 4.908\end{bmatrix} \qquad \text{Test statistic } [\text{var}(H_t) \leq \text{var}(B_t)]:\ -4.431^{**}$$

$\hat\sigma_B^2$ is the estimated variance of $\text{var}(r_t)/(1-\bar\gamma^2)$, as in (15). * denotes that the tested difference is significantly smaller than zero at the 5% level, and ** at the 1% level. The test statistic is given by (24).

Figure 1: 5-year bond and 90-day bill rates, February 1984–November 1994



Daily, annualized yields for the period February 17, 1984 to November 23, 1994.

As displayed by figure 1, there was great turbulence surrounding the speculative attacks against, and subsequent fall of, the Swedish fixed currency regime in the autumn of 1992. Undoubtedly, this influences the estimated variances and implied variance bounds. The variances and their bounds are therefore reestimated for the period up to and including August of 1992. The test statistics are shown in table 3; the estimates may be found in tables B1 and B2 in appendix B. The exclusion of the turbulent period in late 1992 leads to a significant reduction in the estimated variances and the associated covariances. In general, the negative test statistics of the perfect foresight bound are more significant, and all eight negative statistics are now significant at the 1% level. Hence, the conclusion that the expectations hypothesis does not hold is strengthened.

Table 3: Variance bounds test statistics for the period up to August 1992

Bound	5 year / 1 month (840217–920831)	5 year / 3 month (840217–920831)	10 year / 1 month (870102–920831)	10 year / 3 month (870102–920831)
$\text{var}(R_t) < \text{var}(r_t)$	3.977	3.635	5.099	4.412
$\text{var}(R_t) < \text{var}(R_t^*)$	-2.737 **	-2.821 **	-3.133 **	-3.111 **
$\text{var}(R_t) \geq \text{var}(\hat{R}_t)$	3.543	3.544	3.369	3.368
$\text{var}(H_t) \leq \text{var}(B_t)$	-5.388 **	-3.738 **	-4.096 **	-4.018 **

B is the upper variance bound $\text{var}(r_t)/(1-\bar{\gamma}^2)$ in (15). * denotes that the tested difference is significantly smaller than zero at the 5% level, and ** at the 1% level (one-sided test).

Table 4: Variance bounds test statistics for the period after November 1992

Bound	5 year / 1 month (921201–941123)	5 year / 3 month (921201–941123)	10 year / 1 month (921201–941123)	10 year / 3 month (921201–941123)
$\text{var}(R_t) < \text{var}(r_t)$	-2.574 **	-2.646 **	-2.556 **	-2.629 **
$\text{var}(R_t) < \text{var}(R_t^*)$	-2.724 **	-2.711 **	-2.714 **	-2.717 **
$\text{var}(R_t) \geq \text{var}(\hat{R}_t)$	2.707	2.707	2.697	2.697
$\text{var}(H_t) \leq \text{var}(B_t)$	-2.922 **	-2.297 *	-2.908 **	-2.904 **

B is the upper variance bound $\text{var}(r_t)/(1-\bar{\gamma}^2)$ in (15). * denotes that the difference in the test is significantly different from zero at the 5% level, and ** at the 1% level (one-sided test).

An interesting aspect to look at is whether the magnitude of the variance bounds violations have increased or decreased in recent times. A natural way of examining this is to investigate the period after the floating of the Swedish krona. The test statistics for the period December 1992 to November 1994 are shown in table 4; the actual estimates are found in appendix C.

We again note that the test statistics of the second and fourth inequality are negative and highly significant. Furthermore, during this two-year period, the variance of the long term bond is in fact higher than the variance of the short term bill used in the comparisons. Hence, the least demanding variance bound (relation (7)) is violated by the data. Thus, during this period, all three upper bounds are violated at very high levels of significance, and the rejection of the expectations hypothesis still stands firmly.

6 An Alternative Test

In this section, a slightly different volatility test than the ones used in the previous sections is considered. The reason behind this is that the variance bounds tests proposed by Shiller (1979) and LeRoy and Porter (1981) have received some criticism concerning possible statistical problems. Specifically, Flavin (1983) pointed out that in small samples the variance bounds tests are likely to be biased towards rejection of the hypothesis of no excess volatility, if the fundamental series is serially correlated. Marsh and Merton (1983) focused on the practice of detrending series prior to conducting variance bounds tests. They argued that detrending could induce biases in the test results in a way which would increase the probability of finding excess volatility.

With this criticism in mind, Mankiw, Romer and Shapiro (MRS) (1985) proposed new, unbiased volatility tests. They showed that these tests are not influenced by serial correlation, and do not require detrending or any assumption of stationarity. The tests are based on some "naive forecast", R_t^0, of the long term interest rate:[16]

$$R_t^0 \equiv \alpha \sum_{s=0}^{n-1} \gamma^s F_t[r_{t+s}], \tag{25}$$

where $F_t[r_{t+s}]$ is a naive forecast of the short term rate r_{t+s}, made at time t. This naive forecast is not required to be rational or efficient in any way. The only prerequisite is that all information used for the forecast is available at the time it is made. The following identity is then formed:

$$R_t^* - R_t^0 = \left(R_t^* - R_t\right) + \left(R_t - R_t^0\right), \tag{26}$$

where R_t^* is given by (16). From (11), we know that the forecast error, η_t, is defined as

$$\eta_t = R_t^* - R_t, \tag{27}$$

and that it is uncorrelated with any information available at time t. Thus, it must be true that

$$E_t\left[\left(R_t^* - R_t\right)\left(R_t - R_t^0\right)\right] = 0. \tag{28}$$

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16 MRS used a portfolio of stocks in their volatility tests.

By squaring both sides of (26) and taking the expected value we obtain

$$E\left[\left(R_t^* - R_t^0\right)^2\right] = E\left[\left(R_t^* - R_t\right)^2\right] + E\left[\left(R_t - R_t^0\right)^2\right], \tag{29}$$

which gives the following volatility relations:

$$E\left[\left(R_t^* - R_t^0\right)^2\right] \geq E\left[\left(R_t^* - R_t\right)^2\right], \tag{30}$$

$$E\left[\left(R_t^* - R_t^0\right)^2\right] \geq E\left[\left(R_t - R_t^0\right)^2\right]. \tag{31}$$

Hence, the relations above provide bounds on the volatility of the long term yield by using mean squared errors. The inequality in (30) says that the actual realized rate is a better predictor of the "ex-post rational" yield than the naive forecast is. Relation (31) states that the volatility of R_t^* is greater around the naive forecast than the volatility of R_t around R_t^0. As pointed out by Mankiw et al., (31) is equivalent to the upper bound of (8), where the variances are measured around the naive forecast instead of the sample mean.

The inequalities proposed above are used to test the expectations hypothesis for Swedish 5-year bonds and 1-month treasury bills. By using monthly data, the problem of overlapping data is eliminated. From the time series of daily observations, the rates which are quoted as close as possible to the middle of each month are selected. This is due to the fact that Swedish treasury bills usually mature at the middle of the month, and therefore the reported maturities of the instruments will be closest to the actual at this time of the month.

The first step in applying the tests empirically is the question of choosing the method of obtaining the naive forecast. As mentioned before, the only requirement of the naive forecast is that it is based on information which is available at the time it is made. The forecast considered here will simply be an unweighted average of the monthly short term yield for the five years prior to the date of the forecast.

Data for the long term yield is available from February 1984, and four different final dates are used for the investigation; November 1994, November 1991, November 1989, and August 1987.[17] The first date

[17] The 1-month t-bill rate is available from January 1983. For the computation of the naive forecasts, the prevailing rate on January 1983 was used in the place of the short term rates prior to this date.

corresponds to the end of the available sample. However, utilizing the entire sample means that the true "perfect forecast" cannot be constructed for all time periods without using a terminal short rate, r_T, at the end of the sample.[18] If November 1991 is used as the final date, then the bias attributable to r_T is reduced considerably. At most, the terminal short rate is then used for the last two fifths of the future short rates in the construction of R_t^*, and thus receive relatively low weights. November 1989 is the latest available date which allows the true "perfect forecast" rate series to be constructed. Finally, using August 1987 as the final date in the tests removes any influence on R_t^* caused by the turbulence in late 1992. Thus, the various samples reflect different trade-offs between the sample size and the possible bias introduced by the terminal rate r_T, or the events in the autumn of 1992.

Figure 2 exhibits the two constructed forecast series, together with the actual 5-year market rate, for the period up to November 1991. Table 5 gives the results of the MRS volatility tests for the different subsamples. The second column of the estimated variances in table 5 should contain smaller values than the first column in order for relation (30) to be valid, and similarly the values in column three should be lower than the ones in column one for (31) to hold. Clearly, the implied inequalities are violated for all investigated periods. In fact, the bounds in the first column are on average exceeded by a factor of three.

Table 5: MRS Volatility tests: 5-year bonds and 1-month bills

Period	$E\left[\left(R_t^* - R_t^0\right)^2\right]$	$E\left[\left(R_t^* - R_t\right)^2\right]$	$E\left[\left(R_t - R_t^0\right)^2\right]$
Feb. '84–Nov. '94	1.649	3.195	6.088
Feb. '84–Nov. '91	0.758	2.081	2.056
Feb. '84–Nov. '89	0.877	2.253	1.845
Feb. '84–Aug. '87	0.513	1.677	2.756

Mankiw et al. do not provide any distributional theory for testing the statistical significance of the violations of relations (30) and (31). However, by using a Monte Carlo technique called the bootstrap, the distribution of the mean squared errors in table 5 may be approximated from the available data set, and the statistical significance of the violations can be tested. First, a number of artificial samples are obtained by resampling from the observed

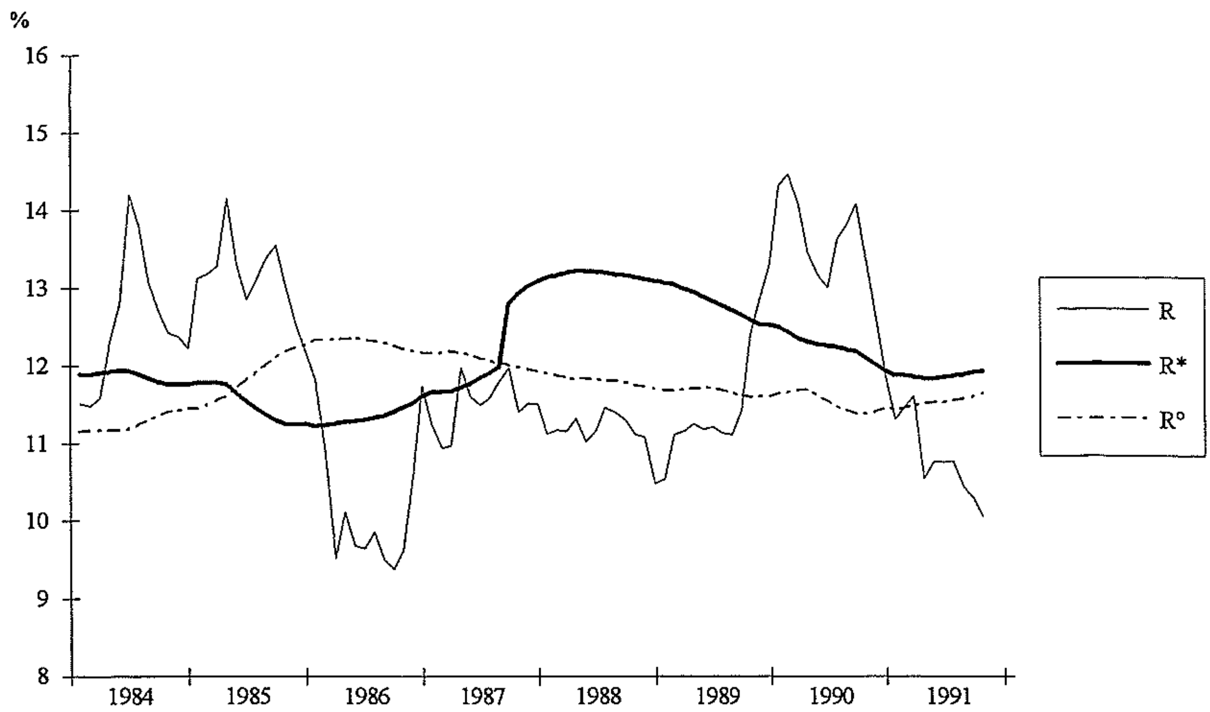
18 Following Shiller (1979), the terminal short rate used in the tests is the sample mean of the short rate.

data. If we are interested in testing for example relation (30), a natural way is to test whether the difference

$$E\left[\left(R_t^* - R_t^0\right)^2\right] - E\left[\left(R_t^* - R_t\right)^2\right] \tag{32}$$

is greater than or equal to zero. This is done by drawing 10.000 bootstrap samples with replacement from each of the two series of squared interest rates, and then computing the difference of the means for each sample. From these 10.000 bootstrap means, a 99% confidence limit is obtained by the simple percentile method, i.e. the limit is chosen so that 1% of the bootstrap replications exceed the limit. If this limit is negative, the hypothesis that (32) is greater than or equal to zero is rejected at the 1% level of significance, and thus relation (30) is found to be violated. The same test is performed for relation (31). Not surprisingly, in all four samples the violations of relations (30) and (31) are found to be statistically significant at the 1% level.

Figure 2: Realized 5-year market rate (R), the constructed "ex-post rational" rate (R), and the naive forecast rate (R^0) for February 1984 to November 1991*



These results suggest that the naive predictor performs better than the realized long term market rate in forecasting the "ex-post rational" rate in a mean squared error sense, and that the market rate is more volatile around R_t^0 than the "ex-post rational" rate is. Hence, as in the previous section, the conclusion is that the behaviour of the volatility of the long term interest rate is inconsistent with the expectations hypothesis of the term structure.

7 Summary and Conclusions

This paper investigates the interest rate volatility of long term Swedish treasury bonds for the period 1984 – 1994. By examining the relation between the observed long term interest rate variance and the variance bounds which are implied by the expectations model of the term structure of interest rates, it is concluded that the long term rate is too volatile to accord with the model. On average, the variance of the one-period holding yield of a long term bond exceeds its theoretical upper bound by a factor of five, and it is found that the violations are statistically significant. Furthermore, the variance of the long term interest rate exceeds the upper bound provided by the perfect foresight rate by on average a factor of 2.8. This result is also significant in most cases. These results are in line with findings of U.S. studies (Shiller (1979), Singleton (1980)).

The result that the variance bounds are violated by the data is stable for the investigated subperiods before and after the floating of the Krona. In fact, the conclusion is strengthened for the latest period, December 1992 to November 1994, in which the variance of the long term bond rate exceeds the variance of the short term t-bill rate. Thus, in this period, all three upper variance bounds are violated.

There are a few possible theories which could explain the results in this study. The tests are conducted under the assumption of a constant (or zero) liquidity premium. A time-varying premium could, in theory, account for the outcome of the tests. However, studies by Pesando (1983) and Amsler (1984) show that the hypothesis of time varying liquidity premia is unlikely to account for the rejections of the variance bounds tests. This conclusion is due to the fact that the premia would have to be extremely large to bring the results in line with some rational expectations model. Furthermore, in a study of Swedish time varying risk premia, Hördahl (1994), using an ARCH-M model, found no evidence of any such premia for long term bonds.

Another possibility is that the market does not form its expectations rationally. This explanation would be in line with claims made that fads due to noise and liquidity trading are an important cause of movements in the market. The appearance in recent times of large international money-managing funds in the Swedish bond markets could support this view. These funds often have very short investment horizons, mainly due to the fact that the investments are financed by short term loans. Consequently, long term fundamentals could come to play a less important role for the decisions made by these institutions, and thus for the pricing of the bonds. The result would be a short term noise component in the long term interest rates which would be manifested by excess volatility.

Still another explanation of the rejection of the expectations hypothesis is simply that it is not the right model for explaining the term structure in Sweden. It might therefore be of interest to test the competing models such as the preferred habitat or market segmentation hypothesis more rigorously.

Appendix A: Summary of Singleton's (1980) Spectral Estimation Methods[19]

Singleton starts by proving that the variances of R_t^* and $\hat{R}_t$ can be estimated from the observed sample of r_t. First, let $S_r(\lambda)$ denote the spectral density function of r. $S_r(\lambda)$ is given by the Fourier transform of the autocovariance function of r, $C_r(s)$:

$$S_r(\lambda) = \frac{1}{2\pi} \sum_{s=-\infty}^{\infty} C_r(s) e^{-i\lambda s}, \qquad -\pi \le \lambda \le \pi \tag{A.1}$$

where

$$C_r(s) = E\left[\left(r_t - E\left[r_t\right]\right)\left(r_{t+s} - E\left[r_{t+s}\right]\right)\right]$$

and $i = \sqrt{-1}$. The spectral density, or power spectrum, gives a representation of the variance as a function of the frequency, or period of oscillation. Since the variance of a variable can be estimated by the sum of the estimated spectral density function of that series, we need to show that the spectral densities $S_{R^*}(\lambda)$ and $S_{\hat{R}}(\lambda)$ can be estimated from $S_r(\lambda)$.

If some variable v_t can be written as $v_t = \sum_{s=-\infty}^{\infty} a_s x_{t-s}$, then the spectral density function of this variable is given by[20]

$$S_v(\lambda) = \tilde{a}(\lambda)^* S_x(\lambda) \tilde{a}(\lambda), \qquad -\pi \le \lambda \le \pi, \tag{A.2}$$

where $\tilde{a}(\lambda)$ denotes the Fourier transform of a_s:

$$\tilde{a}(\lambda) = \sum_{s=-\infty}^{\infty} a_s e^{-i\lambda s}, \qquad -\pi \le \lambda \le \pi. \tag{A.3}$$

$\tilde{a}(\lambda)^*$ symbolises the complex conjugate of $\tilde{a}(\lambda)$. This result may be conveyed to the case of the perfect foresight series, R_t^*, by noting that this may be written as $R_t^* = (1-\beta F)^{-1} r_t \equiv a(F) r_t$, where F denotes the forward shift operator. Consequently, once the discount factor β is determined, the spectral density function of R_t^* is given by

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[19] In Singleton (1980), the proofs are stated for the general present-value relation involving the expected future value of K variables. Since the term structure model in question only involves one variable, the short term interest rate, the summary here is only given in the context of a one-variable present-value model.

[20] Singleton (1980), p. 1164, and Fishman (1969) p. 71.

$$S_{R^*}(\lambda) = \tilde{a}(\lambda)^* S_r(\lambda)\tilde{a}(\lambda), \qquad -\pi \le \lambda \le \pi, \tag{A.4}$$

where $\tilde{a}(\lambda)$ is the Fourier transform of the discount series. The variance of R_t^* is then estimated by taking the sum of the spectral density estimate of R_t^*.

Next, for the estimation of $S_{\hat{R}}(\lambda)$, Singleton assumes that r_t has a one-sided moving average representation:

$$r_t = \sum_{s=0}^{\infty} \alpha_s \varepsilon_{t-s} = \alpha(L)\varepsilon_t, \tag{A.5}$$

where L is the lag operator. Then, the optimal linear least squares predictor of future x based on the past and current values of x is given by[21]

$$\hat{r}_{t+s} = \left[\frac{\alpha(L)}{L^s}\right]_+ \alpha(L)^{-1} r_t. \tag{A.6}$$

The operator $[\ \ ]_+$ denotes that negative powers of the argument are ignored. Given this, $\hat{R}_t$ can be expressed by

$$\begin{aligned} \hat{R}_t &= \left[\sum_{s=0}^{\infty} \beta^s L^{-s} \alpha(L)\right]_+ \alpha(L)^{-1} r_t \\ &= \left[\sum_{s=0}^{\infty} \beta^s F^s \alpha(L)\right]_+ \alpha(L)^{-1} r_t \\ &= \left[a(F)\alpha(L)\right]_+ \alpha(L)^{-1} r_t \\ &\equiv b(L) r_t. \end{aligned} \tag{A.7}$$

Analogous to the result in (3.13), the spectral density function of $\hat{R}_t$ R_t is given by

$$S_{\hat{R}}(\lambda) = \tilde{b}(\lambda)^* S_r(\lambda)\tilde{b}(\lambda), \qquad -\pi \le \lambda \le \pi, \tag{A.8}$$

where $\tilde{b}(\lambda)$ is the Fourier transform of $b(L)$ in (A.7). Since $\tilde{b}(\lambda)$ can be expressed as

$$\tilde{b}(\lambda) = \left[\tilde{a}(\lambda)\tilde{\alpha}(\lambda)\right]_+ \tilde{\alpha}(\lambda)^{-1}, \tag{A.9}$$

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21 See Singleton (1980), p. 1165.

and $\tilde{a}(\lambda)$ is known (given the assumption that β is known), $\tilde{b}(\lambda)$ can be estimated if it is possible to estimate $\tilde{\alpha}(\lambda)$. Singleton points out that $\tilde{\alpha}(\lambda)$ can be estimated by the canonical factorization of $S_r(\lambda)$. An alternative method is given in Koopmans (1974), pp. 235-237, where $\tilde{\alpha}(\lambda)$ is estimated using the log spectral density of r_t. Once the estimate of $\tilde{\alpha}(\lambda)$ is obtained, the spectral density of $\hat{R}_t$ is estimated according to (A.8), and an estimate of the variance of $\hat{R}_t$ is given by the sum of $S_{\hat{R}}(\lambda)$. The sample variances needed for testing inequality (15) are estimated using similar methods as for the variances in (8).

Singleton then goes on to show that the sample variances of R_t^*, R_t, and $\hat{R}_t$ $(\hat{\sigma}^2_{R^*}, \hat{\sigma}^2_R, \hat{\sigma}^2_R)$ discussed above are consistent estimators of the true population variances, and that the asymptotic distribution of R_t^*, R_t, and $\hat{R}_t$ is multivariate normal.[22] Furthermore, the asymptotic variance-covariance matrix of $\hat{\sigma}^2_{R^*}, \hat{\sigma}^2_R, \hat{\sigma}^2_R$ is shown to be given by

$$\operatorname{var}\begin{bmatrix}\hat{\sigma}^2_{R^*}\\ \hat{\sigma}^2_{R}\\ \hat{\sigma}^2_{R}\end{bmatrix} = \frac{4\pi}{T}\begin{bmatrix}\delta^2_{R^*} & \delta_{R^*R} & \delta_{R^*\hat{R}}\\ \delta_{RR^*} & \delta^2_{R} & \delta_{R\hat{R}}\\ \delta_{\hat{R}R^*} & \delta_{\hat{R}R} & \delta^2_{\hat{R}}\end{bmatrix}, \tag{A.10}$$

where T denotes the sample size, and where

$$\delta^2_{R^*} = \int_{-\pi}^{\pi} S_{R^*}(\lambda)^2 d\lambda, \quad \delta_{R^*R} = \int_{-\pi}^{\pi} \left|S_{R^*R}(\lambda)\right|^2 d\lambda, \quad \text{etc.}$$

The expression $\left|S_{R^*R}(\lambda)\right|^2$ is the squared modulus of the cross-spectral density function of R_t^* and R_t. Given these results, the restrictions on the variance of the series R_t imposed by (8) may be tested empirically with an ordinary one-sided test.

[22] Singleton (1980), pp. 1167–1168.

Appendix B: Estimates for the period up to August 1992

Table B1: Estimated variances and test statistics for variance bounds Swedish 5-year bonds

5-year bonds / 1-month bills — Period: February 17, 1984–August 31, 1992

Estimated variances:

$\hat{\sigma}_r^2 = 4.502$ $\quad \hat{\sigma}_R^2 = 1.428$ $\quad \hat{\sigma}_{R^*}^2 = 0.347$

$\hat{\sigma}_{\hat{R}}^2 = 0.003$ $\quad \hat{\sigma}_H^2 = 756.108$ $\quad \hat{\sigma}_B^2 = 104.134$

$$\operatorname{var}\begin{bmatrix}\hat{\sigma}_r^2\\ \hat{\sigma}_R^2\end{bmatrix} = \begin{bmatrix}1.378 & 0.472\\ 0.472 & 0.163\end{bmatrix}$$

Test statistic $[\operatorname{var}(R_t) < \operatorname{var}(r_t)]$: 3.977

$$\operatorname{var}\begin{bmatrix}\hat{\sigma}_{R^*}^2\\ \hat{\sigma}_R^2\end{bmatrix} = \begin{bmatrix}0.099 & 0.053\\ 0.053 & 0.163\end{bmatrix}$$

Test statistic $[\operatorname{var}(R_t) < \operatorname{var}(R_t^*)]$: -2.737 **

$$\operatorname{var}\begin{bmatrix}\hat{\sigma}_R^2\\ \hat{\sigma}_{\hat{R}}^2\end{bmatrix} = \begin{bmatrix}0.163 & 0.000\\ 0.000 & 0.0003\end{bmatrix}$$

Test statistic $[\operatorname{var}(R_t) \geq \operatorname{var}(\hat{R}_t)]$: 3.543

$$\operatorname{var}\begin{bmatrix}\hat{\sigma}_B^2\\ \hat{\sigma}_H^2\end{bmatrix} = 10^4 \cdot \begin{bmatrix}0.074 & 0.229\\ 0.229 & 1.848\end{bmatrix}$$

Test statistic $[\operatorname{var}(H_t) \leq \operatorname{var}(B_t)]$: -5.388 **

5-year bonds / 3-month bills — Period: February 17, 1984–August 31, 1992

Estimated variances:

$\hat{\sigma}_r^2 = 3.880$ $\quad \hat{\sigma}_R^2 = 1.427$ $\quad \hat{\sigma}_{R^*}^2 = 0.337$

$\hat{\sigma}_{\hat{R}}^2 = 0.021$ $\quad \hat{\sigma}_H^2 = 366.500$ $\quad \hat{\sigma}_B^2 = 30.947$

$$\operatorname{var}\begin{bmatrix}\hat{\sigma}_r^2\\ \hat{\sigma}_R^2\end{bmatrix} = \begin{bmatrix}1.158 & 0.432\\ 0.432 & 0.162\end{bmatrix}$$

Test statistic $[\operatorname{var}(R_t) < \operatorname{var}(r_t)]$: 3.635

$$\operatorname{var}\begin{bmatrix}\hat{\sigma}_{R^*}^2\\ \hat{\sigma}_R^2\end{bmatrix} = \begin{bmatrix}0.087 & 0.050\\ 0.050 & 0.162\end{bmatrix}$$

Test statistic $[\operatorname{var}(R_t) < \operatorname{var}(R_t^*)]$: -2.821 **

$$\operatorname{var}\begin{bmatrix}\hat{\sigma}_R^2\\ \hat{\sigma}_{\hat{R}}^2\end{bmatrix} = \begin{bmatrix}0.162 & 0.003\\ 0.003 & 0.00004\end{bmatrix}$$

Test statistic $[\operatorname{var}(R_t) \geq \operatorname{var}(\hat{R}_t)]$: 3.544

$$\operatorname{var}\begin{bmatrix}\hat{\sigma}_B^2\\ \hat{\sigma}_H^2\end{bmatrix} = 10^3 \cdot \begin{bmatrix}0.073 & 0.558\\ 0.558 & 9.103\end{bmatrix}$$

Test statistic $[\operatorname{var}(H_t) \leq \operatorname{var}(B_t)]$: -3.738 **

σ_B^2 is the estimated variance of $\operatorname{var}(r_t)/(1-\bar{\gamma}^2)$, as in (15). * denotes that the tested difference is significantly smaller than zero at the 5% level, and ** at the 1% level. The test statistic is given by (24).

Table B2: Estimated variances and test statistics for variance bounds
Swedish 10-year bonds

10-year bonds / 1-month bills — Period: January 2, 1987–August 31, 1992

Estimated variances:	$\hat{\sigma}_r^2 = 2.048$	$\hat{\sigma}_R^2 = 0.893$	$\hat{\sigma}_{R^*}^2 = 0.080$
	$\hat{\sigma}_{\hat{R}}^2 = 0.0004$	$\hat{\sigma}_H^2 = 1748.713$	$\hat{\sigma}_B^2 = 74.424$

$\mathrm{var}\begin{bmatrix}\hat{\sigma}_r^2 \\ \hat{\sigma}_R^2\end{bmatrix} = \begin{bmatrix}0.234 & 0.126 \\ 0.126 & 0.070\end{bmatrix}$	Test statistic $[\mathrm{var}(R_t) < \mathrm{var}(r_t)]$:	5.099
$\mathrm{var}\begin{bmatrix}\hat{\sigma}_{R^*}^2 \\ \hat{\sigma}_R^2\end{bmatrix} = \begin{bmatrix}0.013 & 0.008 \\ 0.008 & 0.070\end{bmatrix}$	Test statistic $[\mathrm{var}(R_t) < \mathrm{var}(R_t^*)]$:	-3.133 **
$\mathrm{var}\begin{bmatrix}\hat{\sigma}_R^2 \\ \hat{\sigma}_{\hat{R}}^2\end{bmatrix} = \begin{bmatrix}0.070 & 0.00002 \\ 0.00002 & 0.000\end{bmatrix}$	Test statistic $[\mathrm{var}(R_t) \geq \mathrm{var}(\hat{R}_t)]$:	3.369
$\mathrm{var}\begin{bmatrix}\hat{\sigma}_B^2 \\ \hat{\sigma}_H^2\end{bmatrix} = 10^4 \cdot \begin{bmatrix}0.031 & 0.523 \\ 0.523 & 17.724\end{bmatrix}$	Test statistic $[\mathrm{var}(H_t) \leq \mathrm{var}(B_t)]$:	-4.096 **

10-year bonds / 3-month bills. — Period: January 2, 1987–August 31, 1992

Estimated variances:	$\hat{\sigma}_r^2 = 1.659$	$\hat{\sigma}_R^2 = 0.893$	$\hat{\sigma}_{R^*}^2 = 0.151$
	$\hat{\sigma}_{\hat{R}}^2 = 0.003$	$\hat{\sigma}_H^2 = 194.941$	$\hat{\sigma}_B^2 = 20.656$

$\mathrm{var}\begin{bmatrix}\hat{\sigma}_r^2 \\ \hat{\sigma}_R^2\end{bmatrix} = \begin{bmatrix}0.190 & 0.115 \\ 0.115 & 0.070\end{bmatrix}$	Test statistic $[\mathrm{var}(R_t) < \mathrm{var}(r_t)]$:	4.412
$\mathrm{var}\begin{bmatrix}\hat{\sigma}_{R^*}^2 \\ \hat{\sigma}_R^2\end{bmatrix} = \begin{bmatrix}0.017 & 0.015 \\ 0.015 & 0.070\end{bmatrix}$	Test statistic $[\mathrm{var}(R_t) < \mathrm{var}(R_t^*)]$:	-3.111 **
$\mathrm{var}\begin{bmatrix}\hat{\sigma}_R^2 \\ \hat{\sigma}_{\hat{R}}^2\end{bmatrix} = \begin{bmatrix}0.070 & 0.0002 \\ 0.0002 & 0.000\end{bmatrix}$	Test statistic $[\mathrm{var}(R_t) \geq \mathrm{var}(\hat{R}_t)]$:	3.368
$\mathrm{var}\begin{bmatrix}\hat{\sigma}_B^2 \\ \hat{\sigma}_H^2\end{bmatrix} = 10^3 \cdot \begin{bmatrix}0.029 & 0.173 \\ 0.173 & 2.199\end{bmatrix}$	Test statistic $[\mathrm{var}(H_t) \leq \mathrm{var}(B_t)]$:	-4.018 **

σ_B^2 is the estimated variance of $\mathrm{var}(r_t)/(1-\bar{\gamma}^2)$, as in (15). * denotes that the tested difference is significantly smaller than zero at the 5% level, and ** at the 1% level. The test statistic is given by (24).

Appendix C: Estimates for the Period after November 1992

Table C1: Estimated variances and test statistics for variance bounds
Swedish 5-year bonds

5-year bonds / 1-month bills — Period: December 1, 1992– November 23, 1994

Estimated variances: $\hat{\sigma}^2_r = 0.346$ $\hat{\sigma}^2_R = 1.277$ $\hat{\sigma}^2_{R^*} = 0.030$

$\hat{\sigma}^2_R = 0.0001$ $\hat{\sigma}^2_H = 2807.130$ $\hat{\sigma}^2_B = 8.577$

$\text{var}\begin{bmatrix}\hat{\sigma}^2_r \\ \hat{\sigma}^2_R\end{bmatrix} = \begin{bmatrix}0.012 & 0.052 \\ 0.052 & 0.223\end{bmatrix}$	Test statistic $[\text{var}(R_t) < \text{var}(r_t)]$:	-2.574 **
$\text{var}\begin{bmatrix}\hat{\sigma}^2_{R^*} \\ \hat{\sigma}^2_R\end{bmatrix} = \begin{bmatrix}0.002 & 0.007 \\ 0.007 & 0.223\end{bmatrix}$	Test statistic $[\text{var}(R_t) < \text{var}(R_t^*)]$:	-2.724 **
$\text{var}\begin{bmatrix}\hat{\sigma}^2_R \\ \hat{\sigma}^2_R\end{bmatrix} = \begin{bmatrix}0.223 & 0.00002 \\ 0.00002 & 0.000\end{bmatrix}$	Test statistic $[\text{var}(R_t) \geq \text{var}(\hat{R}_t)]$:	2.707
$\text{var}\begin{bmatrix}\hat{\sigma}^2_B \\ \hat{\sigma}^2_H\end{bmatrix} = 10^4 \cdot \begin{bmatrix}0.0007 & 0.170 \\ 0.170 & 92.040\end{bmatrix}$	Test statistic $[\text{var}(H_t) \leq \text{var}(B_t)]$:	-2.922**

5-year bonds / 3-month bills — Period: December 1, 1992– November 23, 1994

Estimated variances: $\hat{\sigma}^2_r = 0.416$ $\hat{\sigma}^2_R = 1.277$ $\hat{\sigma}^2_{R^*} = 0.049$

$\hat{\sigma}^2_R = 0.002$ $\hat{\sigma}^2_H = 639.119$ $\hat{\sigma}^2_B = 3.541$

$\text{var}\begin{bmatrix}\hat{\sigma}^2_r \\ \hat{\sigma}^2_R\end{bmatrix} = \begin{bmatrix}0.022 & 0.069 \\ 0.069 & 0.223\end{bmatrix}$	Test statistic $[\text{var}(R_t) < \text{var}(r_t)]$:	-2.646 **
$\text{var}\begin{bmatrix}\hat{\sigma}^2_{R^*} \\ \hat{\sigma}^2_R\end{bmatrix} = \begin{bmatrix}0.003 & 0.010 \\ 0.010 & 0.223\end{bmatrix}$	Test statistic $[\text{var}(R_t) < \text{var}(R_t^*)]$:	-2.711 **
$\text{var}\begin{bmatrix}\hat{\sigma}^2_R \\ \hat{\sigma}^2_\sigma\end{bmatrix} = \begin{bmatrix}0.223 & 0.0003 \\ 0.0003 & 0.000\end{bmatrix}$	Test statistic $[\text{var}(R_t) \geq \text{var}(\hat{R}_t)]$:	2.707
$\text{var}\begin{bmatrix}\hat{\sigma}^2_B \\ \hat{\sigma}^2_H\end{bmatrix} = 10^3 \cdot \begin{bmatrix}0.002 & 0.273 \\ 0.273 & 77.099\end{bmatrix}$	Test statistic $[\text{var}(H_t) \leq \text{var}(B_t)]$:	-2.297 *

σ^2_B is the estimated variance of $\text{var}(r_t) / (1-\bar{\gamma}^2)$, as in (15). * denotes that the tested difference is significantly smaller than zero at the 5% level, and ** at the 1% level. The test statistic is given by (24).

Table C2: Estimated variances and test statistics for variance bounds Swedish 10-year bonds

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10-year bonds / 1-month bills — Period: December 1, 1992–November 23, 1994

Estimated variances:

$\hat{\sigma}^2_r = 0.346$	$\hat{\sigma}^2_R = 1.241$	$\hat{\sigma}^2_{R^*} = 0.029$
$\hat{\sigma}^2_R = 5 \cdot 10^{-5}$	$\hat{\sigma}^2_H = 7155.788$	$\hat{\sigma}^2_B = 13.830$

$$\mathrm{var}\begin{bmatrix}\hat{\sigma}^2_r \\ \hat{\sigma}^2_R\end{bmatrix} = \begin{bmatrix}0.012 & 0.051 \\ 0.051 & 0.212\end{bmatrix}$$

Test statistic $\left[\mathrm{var}(R_t) < \mathrm{var}(r_t)\right]$: -2.556 **

$$\mathrm{var}\begin{bmatrix}\hat{\sigma}^2_{R^*} \\ \hat{\sigma}^2_R\end{bmatrix} = \begin{bmatrix}0.002 & 0.007 \\ 0.007 & 0.212\end{bmatrix}$$

Test statistic $\left[\mathrm{var}(R_t) < \mathrm{var}(R_t^*)\right]$: -2.714 **

$$\mathrm{var}\begin{bmatrix}\hat{\sigma}^2_R \\ \hat{\sigma}^2_R\end{bmatrix} = \begin{bmatrix}0.212 & 0.00001 \\ 0.00001 & 0.000\end{bmatrix}$$

Test statistic $\left[\mathrm{var}(R_t) \geq \mathrm{var}(\hat{R}_t)\right]$: 2.697

$$\mathrm{var}\begin{bmatrix}\hat{\sigma}^2_B \\ \hat{\sigma}^2_H\end{bmatrix} = 10^4 \cdot \begin{bmatrix}0.002 & 0.703 \\ 0.703 & 604.563\end{bmatrix}$$

Test statistic $\left[\mathrm{var}(H_t) \leq \mathrm{var}(B_t)\right]$: -2.908 **

10-year bonds / 3-month bills — Period: December 1, 1992–November 23, 1994

Estimated variances:

$\hat{\sigma}^2_r = 0.416$	$\hat{\sigma}^2_R = 1.241$	$\hat{\sigma}^2_{R^*} = 0.042$
$\hat{\sigma}^2_R = 0.0006$	$\hat{\sigma}^2_H = 792.258$	$\hat{\sigma}^2_B = 5.675$

$$\mathrm{var}\begin{bmatrix}\hat{\sigma}^2_r \\ \hat{\sigma}^2_R\end{bmatrix} = \begin{bmatrix}0.022 & 0.010 \\ 0.010 & 0.212\end{bmatrix}$$

Test statistic $\left[\mathrm{var}(R_t) < \mathrm{var}(r_t)\right]$: -2.629 **

$$\mathrm{var}\begin{bmatrix}\hat{\sigma}^2_{R^*} \\ \hat{\sigma}^2_R\end{bmatrix} = \begin{bmatrix}0.003 & 0.0001 \\ 0.0001 & 0.212\end{bmatrix}$$

Test statistic $\left[\mathrm{var}(R_t) < \mathrm{var}(R_t^*)\right]$: -2.717 **

$$\mathrm{var}\begin{bmatrix}\hat{\sigma}^2_R \\ \hat{\sigma}^2_R\end{bmatrix} = \begin{bmatrix}0.212 & 0.0001 \\ 0.0001 & 0.000\end{bmatrix}$$

Test statistic $\left[\mathrm{var}(R_t) \geq \mathrm{var}(\hat{R}_t)\right]$: 2.697

$$\mathrm{var}\begin{bmatrix}\hat{\sigma}^2_B \\ \hat{\sigma}^2_H\end{bmatrix} = 10^3 \cdot \begin{bmatrix}0.004 & 0.338 \\ 0.338 & 74.040\end{bmatrix}$$

Test statistic $\left[\mathrm{var}(H_t) \leq \mathrm{var}(B_t)\right]$: -2.904 **

σ^2_B is the estimated variance of $\mathrm{var}(r_t)/(1-\bar{\gamma}^2)$, as in (15). * denotes that the tested difference is significantly smaller than zero at the 5% level, and ** at the 1% level. The test statistic is given by (24).

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