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Deposit Insurance, Capital Constraints, and Risk Taking by Banks

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Abstract

The paper examines the impact of deposit insurance on bank risk-taking when claim holders cannot observe bank portfolio choice. Due to the informational asymmetry the debt financed bank has incentive to take too much risk, though diversification of the bank portfolio reduces the risk incentive. A fixed price deposit insurance *does not* necessarily aggravate the incentive problem. It is shown that when the marginal source of funding of the bank is not deposits, but funds raised in the capital market, deposit insurance may induce the bank to take *less* risk. If deposit insurance is fairly priced, there is no scope for capital constraints, but, with a constant insurance premium, a capital requirement may reduce risk-taking by the bank.

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1 Introduction

The recent financial distress experienced by banks in many countries has fueled debate about bank regulation. The common view is that non-risk-rated deposit insurance encourages risk-taking by banks. While a risk-based insurance scheme could eliminate the risk-incentives, many believe that it is not enforceable as banks have private information about the quality of their assets. The view on the effect of capital constraints on bank risk-taking is not equally unanimous. It has been argued that, quite contrary to their purpose, capital constraints may induce banks to take larger risks, but these results are not generally accepted.[1]

A deficiency of the existing literature on bank regulation is that it does not take the functions of the bank explicitly into account. This is a drawback because the functions of the bank determine both the need for bank regulation and the banks' response to regulation. The purpose of this paper is to analyze the portfolio choice of a bank, which fulfills two functions. First, the bank evaluates and provides credit to projects, which due to information problems cannot be funded in the capital markets. Secondly, the bank invests the means of risk-averse savers, who find it too risky to invest in the capital markets. I will analyze the impact of deposit insurance on the risk-taking behavior of the bank, taking into account the information problem as well as the reason for deposit insurance. I will also examine the scope for capital constraints.

Because this paper takes the functions of the bank explicitly into account, its benchmark case (the situation without regulation) differs from the benchmark used in the previous literature on bank regulation. In order to clarify this, I will discuss the key assumptions in the model before presenting the results.

The first assumption is that outside agents cannot observe the portfolio choice of the bank. This assumption is motivated by the evaluating function of the bank. In the literature, the motivation given for why a non-risk-rated deposit insurance encourages risk-taking by the bank is that it gives the bank incentive to increase the variance in the portfolio return in order to maximize the value of deposit insurance (Merton (1977)). In the current paper, the deposit rate does not respond to the risk-taking behavior of the bank even if deposits are not insured, as depositors cannot observe the investment policy of the bank. It is well-known from the agency literature in corporate finance that this gives the owners of the bank an incentive to invest in assets which promise very

[1]For contributions to the debate about the impact of tighter capital constraints on bank risk-taking see for instance Gennotte and Pyle (1991) and Rochet (1992a).

high payoffs if successful, even if they have a low probability of success. Such "risk-shifting" strategy yields high profits if successful, and if it is not successful, the creditors bear most of the costs (Jensen and Meckling (1976)). Hence, even if deposits are uninsured the bank has incentive to take risk, and it is not obvious that deposit insurance would aggravate the risk-incentive of debt.[2]

It has also been argued that deposit insurance aggravates the moral hazard problem of the bank, because it removes the incentives of depositors to discipline the bank. Ronn and Verma (1986) argue that in the absence of deposit insurance, riskier banks would be able to attract deposits only at higher rates. Boot and Greenbaum (1993) claim that deposit insurance ruins the incentive of banks to build a reputation as safe banks. However, compared to the insurance agency, depositors typically have less information about the bank portfolio, and less capabilities to monitor the bank. Therefore, the insurance agency should have at least as large possibilities as depositors to discipline the bank.[3] In this paper, all outside agents have the same information. They cannot observe the bank portfolio, but they have rational expectations about bank behavior.

The second assumption is that the bank portfolio consists of loans to projects, which due to information problems cannot be funded in the capital markets. For the bank to be able to raise debt, it must hold a diversified portfolio, as diversification of the bank portfolio mitigates the risk-incentive of debt. The importance of diversification for bank risk-taking has not been well analyzed in the literature. The traditional banking literature treats the bank as merely one of many investors in the capital market, choosing from a range of assets which ones to invest their deposits in. In this case, the bank maximizes the value of the deposit insurance by choosing a non-diversified portfolio (Merton (1977)). A recent literature recognizes that the bank is not only another investor in the capital market, but the bank has private information about its assets. This literature generally assumes that the bank invests in only one project (Boot and Thakor (1991), Boot and Greenbaum (1993), Campbell, Chan, and Marino (1992), Chan, Greenbaum and Thakor (1992), John, John, Senbet (1991), and Rochet (1992b)).[4]

[2]That the incentive of banks to take risk does not emanate from deposit insurance *per se*, but from the fixed claim property of the deposit contract and from the fact that the investments of the bank are imperfectly observed by its debt holders, has been pointed out by John, John, and Senbet (1991).

[3]Another question, which is not addressed in this paper, is whether the insurance agency has the incentive to discipline the bank.

[4]A notable exception is Besanko and Thakor (1993), who assume that the bank invests in several projects, but they assume that projects within the same risk class

The third assumption is that the marginal source of funding of the bank is the capital markets. Many believe that it is important that deposit insurance is fairly priced. They argue that an under-priced deposit insurance aggravates the risk-taking incentives of the bank as the bank then can borrow from depositors at a subsidized rate. This argument is based on the assumption that deposits are the marginal source of funding. However, most theories of the *raison d'être* of banks imply that there are economies of scale in financial intermediation.[5] Furthermore, in most countries, except in the US, there are relatively few banks. This suggests that banks face an inelastic supply of deposits. In the capital markets, on the other hand, each bank is just one of many agents. This implies that the bank faces a more elastic supply of funds in the capital markets than in the deposit market, which suggests that the marginal funding cost is mainly determined in the capital markets.[6] In this paper the bank has a (regional) monopoly in providing credit and deposit services. There is an inelastic supply of deposits. As deposits are not sufficient to finance all profitable projects, the bank raises additional funds in the capital markets, where the bank faces a perfectly elastic supply of funds.

Because the debt holders of the bank cannot observe the portfolio choice of the bank, the uninsured bank in this paper takes too much risk in the sense of financing projects with expected negative net return. I will show that introducing a fairly priced deposit insurance scheme, where deposits have first priority in case of bank failure, would induce the bank to take less risk. The reasons are twofold. First, by making other debt subordinated to deposits, the bank will be less able to exploit the marginal investor. Secondly, the total funding cost of the bank will be lowered as the bank does not have to pay any risk-premium to its risk-averse depositors. This reduces the probability of bank failure, and the safer the bank is, the less incentive it has to engage in risk-shifting.

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have perfectly correlated outcomes, and that there are only two risk classes. In contrast, I assume that project outcomes are independent across entrepreneurs.

[5] Functions of the bank that give rise to economies of scale include investing on behalf of investors that face transaction costs (Klein(1973)), monitoring (Diamond (1984)), project evaluation (Boyd and Prescott (1986)), and liquidity provision (Diamond and Dybvig (1983)).

[6] Furthermore, the marginal funding cost is determined in the capital market independently of whether or not deposits are the only funding source. The reason is that deposits are not merely funds; the bank also provides services to its depositors. Thus, if the demand for deposit services (supply of deposits) is larger than the demand for loans, the bank will invest the excess means in securities, and the bank's alternative to extend the marginal loan is to invest in the capital markets, and not to borrow one unit less from depositors. See section *V* for further discussion.

If the insurance is fairly priced, that is if the insurer expects to make zero profits on the insurance given the available information, there is no scope for capital constraints. In this case the bank will bear the full agency cost of debt. Therefore, if an issue of equity reduces the risk-incentive of debt, the bank will have the incentive to raise equity capital.

An actuarially fair premium is lower for a bank that has more equity capital. If the premium instead is constant, independent of the amount of bank equity, the bank would have incentive to increase leverage in order to increase the value of deposit insurance. This is true, although the marginal source of funding is debt raised in the capital market, and the interest rate on this debt actually responds to capital structure. This implies that, if the insurance premium is constant, capital constraints may reduce risk-taking by the bank.

Many of the results of this paper relate to results in corporate finance. For instance, the result that giving first priority to deposits reduces risk-taking by the bank relates to the result of Berkovitch and Kim (1990) that decreasing seniority of new debt decreases the incidence of over-investment. This is not surprising, since the lending decision of the bank, in the same way as the investment decision of a firm, is closely related to the financing decision.[7]

The paper is organized in seven sections. The model is introduced in the next section. In section 3, the risk-taking behavior of an uninsured bank is analyzed as a benchmark case. Deposit insurance is analyzed in section 4, and capital constraints in section 5. In section 6 the results are shown to hold for some alternative model assumptions. Finally, some concluding remarks are made in section 7. Part of the formal analysis is relegated to the appendix.

[7]The paper only considers the static issues connected to financing. Thus, it does not consider the allocation of control rights connected to capital structure. For an analysis of how the allocation of control rights affects bank performance see Dewatripont and Tirole (1993).

2 The Model

Consider an economy, which consists of m entrepreneurs and d savers, and is served by a single bank. The economy has access to international capital markets and can be thought of as a town or a rural district. The competitive expected return of an investment in the capital markets is y.

Each entrepreneur has the opportunity to carry out a one-period investment project with stochastic return. Each project requires one unit of capital, and if it succeeds it will return x units at the end of the period, where $x > y$. If it fails, it will return nothing. Project outcomes are independent across entrepreneurs. The probability that the project succeeds, p, differs among entrepreneurs. For each entrepreneur, p is drawn from a uniform distribution on [0, 1].

An entrepreneur has no capital of his own, and thus has to borrow in order to carry out his project. The type of an individual entrepreneur (the value of p) is not public information, only the distribution of types is publicly known. The expected average return on the projects in the economy is less than the alternative cost of the capital, y. Therefore, the entrepreneur cannot fund his project in the capital markets, but has to turn to the bank for funding.

The bank evaluates the project before determining whether to grant credit. The evaluation gives the bank perfect information about p. The evaluation outcome is not observed by outside agents. If it grants credit, the bank will charge the monopoly loan rate, giving the entrepreneur an expected utility equal to his reservation utility, which is assumed to be equal to zero for all entrepreneurs.[8] Thus, if the bank grants credit, it will charge a loan rate that is equal to x (interest rates include the repayment of the principal).

Each saver has one unit that she wishes to save for one period. Denote aggregate savings by D. Savers are risk-averse, and they have identical von Neumann-Morgenstern utility functions, $U(w)$, $U'(w) > 0$, $U''(w) < 0$, where w is the wealth at the end of the period and primes denote derivatives in the usual way. Savers have access to a perfectly safe storing technology with zero return. There are some indivisibilities in the capital markets, so that the riskiness of a small investment in the capital markets is sufficiently large for the saver to prefer the storing technology. Due to the information problem, the saver is not willing to lend directly to

[8]The results do not depend on how the bank and the entrepreneur share the surplus. The monopoly assumption is made for simplicity.

entrepreneurs, so that the best alternative to depositing the unit at the bank is to store it. The saver will choose to deposit her unit at the bank if the deposit rate is sufficiently large for the saver's expected utility to be at least equal $U(1)$.

3 Diversification and risk-taking

Let us first examine the lending policy of the bank in an unregulated economy in order to constitute a benchmark case. Table 1 illustrates the balance sheet of the bank. The bank assets consist of loans to entrepreneurs in the economy, while bank liabilities consist of deposits and public debt, the latter referred to as bank securities. The alternative return in the capital markets is sufficiently high for the bank to prefer to borrow from savers, although the uninsured bank must compensate savers for bank deposits not being a perfectly safe investment. However, in the economy aggregate savings are not sufficient to finance all the profitable projects so the bank raises additional funds in the capital markets. In section 6 will argue that the results also apply to the case in which deposits are larger than the total requirements of the profitable projects.

Table 1 Balance sheet of a bank with public debt

Assets	**Liabilities**
Loans	Deposits
	Bank securities

As seen from Table 1, the bank does not hold securities. Given that investments in the capital markets are observable, the bank has no incentive to raise funds in the capital markets to invest in these markets. Such operations would be correctly priced and give zero expected net return, and they would not affect the lending policy of the bank.[9] Furthermore, the bank has no equity or subordinated debt. I assume that investors cannot observe the total amount of ordinary debt issued by the bank, and that the return on subordinated debt cannot be contingent on the amount of ordinary debt. As the ordinary debt contract allows the bank to exploit the investor more than what for instance the subordinated debt contract does, the bank prefers to issue ordinary debt, and it cannot credibly restrict the amount of ordinary debt. Since the results in this and in the next section depend on ordinary debt being the marginal source of funding, and not on how much equity and subordinated debt the bank has, the latter are for simplicity set equal to zero. The choice of capital structure is analyzed in section 5.

[9]This requires that the return on an investment in the capital market is independently distributed of the returns on bank loans, or that the correlations are known. As discussed in section 6, an alternative assumption that leads to the bank not holding securities is that the bank is prohibited from investing in anything else than safe securities.

Since there is a finite number of projects in the economy, the bank portfolio return, denoted by z, is a random variable, and the liabilities of the bank are risky. The distribution of the portfolio return depends on the lending policy of the bank.[10] The lending policy is fully characterized by the lowest repayment probability for which the bank grants credit, denoted by $\bar{p}$. Loans with a repayment probability of $\bar{p}$ are referred to as marginal loans and the corresponding projects as marginal projects. For a given $\bar{p}$, the expectation of the number of loans in the portfolio (the investment volume) Q, the mean of the portfolio return $\bar{Z}$, and the variance of the portfolio return σ^2 are given by (see the appendix for derivation):

$$Q(\bar{p}) = m(1-\bar{p}), \tag{1}$$

$$\bar{Z}(\bar{p}) = m\frac{1-\bar{p}^2}{2}x (= m\int_{\bar{p}}^{1} px\,dp),$$

$$\sigma^2(\bar{p}) = m\frac{1-\bar{p}^4}{4}x^2 (= m\int_{\bar{p}}^{1} p(1-p)x^2\,dp + V(\bar{p})).$$

Of the m entrepreneurs, $m(1-\bar{p})$ are expected to have projects with expected return equal to, or larger than, $\bar{p}x$, since p is uniformly distributed on [0, 1]. If the bank grants credit, it charges the monopoly loan rate x, so that the expected return on a loan with success probability p is equal to px, and the variance is $p(1-p)x^2$. The term $V(\bar{p})$ in the expression for the variance, which is positive, arises because the m projects are drawn from a population of projects.

I assume that all agents treat the standardized portfolio return, $\tilde{z}$, defined by

$$\tilde{z}(z) \equiv \frac{z-\bar{Z}}{\sqrt{\sigma^2}}, \tag{2}$$

as having a standard normal distribution.[11]

[10]The bank determines its lending policy (what types to grant credit to) before it has evaluated the projects. This implies that the bank bases its choice on its expected portfolio. If the bank instead would determine its lending policy after it had evaluated all loan applicants, the lending policy of the bank would depend on its actual portfolio. This would complicate the analysis, but should not affect results.

[11]In the appendix, I show that z is a binomial random variable with parameters $(m, \frac{1-\bar{p}^2}{2})$. By the Central Limit Theorem, the distribution of $\tilde{z}$ approaches the standard normal distribution as the number of projects m goes to infinity (see *e.g.* Ross (1976)). Since I assume that m is large, but not infinite, $\tilde{z}$ is only approximately

Denote the fixed repayment per unit of deposit, henceforth the deposit rate, by r_d. For future use, denote the insurance premium per unit of deposit by t.[12] Denote the fixed repayment on the debt raised in the capital market, the security rate, by r. Then, denoting the debt obligations of the bank by ω, we have that

$$\omega(\bar{p}, r, r_d, t, D) = (r_d + t)D + r(Q(\bar{p}) - D). \tag{3}$$

In this section $t = 0$.

If the bank cannot meet its obligations to its debt holders, it goes bankrupt. This implies that the bank goes bankrupt whenever $z < \omega$, so that the probability of bank failure is equal to $\Phi(\tilde{z}(\omega))$, where $\tilde{z}(\cdot)$ is defined by (2) and Φ is the cumulative distribution function of a standard normal random variable.

The expected profit of the bank, which is equal to the excess return in the non-default states, can be written as

$$\pi = \bar{Z} - \omega + \int_{-\infty}^{\omega} \Phi(\tilde{z}(z))dz, \tag{4}$$

where the integral term is the expected shortfalls of the debt.

What lending policy does the bank choose? Savers and investors cannot observe the lending policy of the bank, as they do not know the types of the borrowers and cannot observe the evaluation outcomes. Thus, their interest requirements do not depend on the actual lending policy of the bank, but on their expectations about the the lending policy. This implies that the bank chooses the lending policy which maximizes its profit for given interest rates. From (4) it is seen that, for a given net return, the profit of the bank increases with the spread in the portfolio return. This is the familiar result of Stiglitz and Weiss (1981), and it is related to the point made by Merton (1977) that with a fixed price on deposit insurance, the bank has incentive to maximize the variance of

normal. However, by increasing the number of projects and simultaneously decreasing x, the approximation can be improved at a non-negligible probability of bank failure. Figure 2 in the appendix shows the true distribution and the normal distribution in a numerical example.

[12]The price of the deposit insurance is determined at the same time as the interest rates. For simplicity, the premium is assumed to be paid out of profits at the end of the period. In the literature, a point has been made of the timing of the payment of the insurance premium, but then one assumes that up front costs are paid out of equity. If the bank pays the premium with means raised in the capital markets, and the insurer can invest the proceeds in the capital markets and get an expected return of y, the equilibrium results do not depend on whether the premium is paid in the beginning of the period or at the end, only on the expected price of the insurance.

the portfolio to maximize the value of deposit insurance. As is shown in the appendix, the derivative of (4) with respect to $\bar{p}$ can be written

$$\frac{\partial \pi}{\partial \bar{p}} = -m[(\bar{p}x - r)(1 - \Phi(\tilde{z}(\omega))) + \bar{p}^3 x^2 \frac{1}{2\sqrt{\sigma^2}} \Phi'(\tilde{z}(\omega))], \qquad (5)$$

where $\Phi'(\cdot)$ is the density function of a standard normal variable. Marginal loans are financed in the capital market at the interest rate r. The bank gets the net return of the loan only if it does not fail, but on the other hand it does not have to carry any loss if it goes bankrupt. Hence, there is a positive effect of an increase in the portfolio variance, as captured by the second term within the square brackets in (5).

It is true that outside agents cannot observe the lending policy, but they have rational expectations about bank behavior, which the bank takes into account. It is easily seen that the profit of the bank, given by (4), decreases with r and r_d. This implies that the bank sets the lowest values of r_d and r for which rational savers and investors accept the contracts, and chooses the lending policy that maximizes profits given the interest rates. Hence, the rational expectation equilibrium is given by the vector $(\bar{p}^u, r^u, r_d^u)$ (u for uninsured) that solves the following equation system:

$$(\bar{p}x - r)(1 - \Phi(\tilde{z}(\omega))) + \bar{p}^3 x^2 \frac{1}{2\sqrt{\sigma^2}} \Phi'(\tilde{z}(\omega)) = 0, \qquad (6)$$

$$r(1 - \Phi(\tilde{z}(\omega))) + \int_{-\infty}^{\omega} \frac{r}{\omega} z d\Phi = y, \qquad (7)$$

$$U(r_d)(1 - \Phi(\tilde{z}(\omega))) + \int_{-\infty}^{\omega} U(\frac{r_D}{\omega} z) d\Phi = U(1). \qquad (8)$$

The promised return on bank securities is r. However, the investor gets this return only if the bank does not fail, the probability of which is $(1 - \Phi(\tilde{z}(\omega)))$. If the bank goes bankrupt, the assets of the bank are distributed among its debt holders in proportion to their claims on the bank. Without deposit insurance, depositors are just ordinary debt holders, with a promised return of r_d.

In the appendix, it is shown that an equilibrium exists provided that the economy is sufficiently large. If the economy is small an equilibrium may not exist, because a bank, which is very risky, will choose to finance all projects independent of success probability, and then there will be no interest rates at which savers and investors are willing to buy the claims of the bank. Hereafter, I assume that the economy is sufficiently large for there to exist an equilibrium.

From (6) follows that the expected return on a marginal loan is lower than the security rate in equilibrium. This means that granting credit to the marginal project increases the probability that the bank will go bankrupt. Hence, if the bank chooses a lower $\bar{p}$ it takes more risk in the sense that the probability of bank failure is higher than it would be with a higher value of $\bar{p}$. Therefore, I will refer to the choice of $\bar{p}$ as the risk-taking behavior of the bank, and I will say that the bank takes too much risk, if it finances projects with an expected return lower than the alternative return, y.

Will the risk-incentive arising from the debt contract induce the bank to take too much risk, despite the fact that investors and savers take this incentive into account when determining whether to accept the offer of the bank? As shown in the appendix, solving the integral in (7) and substituting the result into (6) gives

$$\bar{p}x - y + (\frac{rQ}{\omega}\frac{\bar{Z}}{Q} - \bar{p}x)\Phi(\tilde{z}(\omega)) + (\frac{\bar{p}^3x^2}{2} - \frac{rQ}{\omega}\frac{\sigma^2}{Q})\frac{1}{\sqrt{\sigma^2}}\Phi'(\tilde{z}(\omega)) = 0, \quad (9)$$

The bank will take too much or too little risk dependent on whether the sum of the last two terms in (9) is positive or negative. These terms represent the impact of the marginal project on the return to the security holder. The bank does not take this into account, since it treats the security rate as constant when determining its lending policy. Thus, we have

Proposition 1 *If the marginal loan reduces the expected return to the security holder for given interest rates, the uninsured bank will take too much risk.*

Proof: See the appendix.

Will the marginal loan reduce the expected return of the security holder? The third term in (9) is positive. Adding the marginal loan to the portfolio implies that the average expected return of the portfolio decreases, $\bar{p}x < \frac{\bar{Z}}{Q}$. Furthermore, the given portfolio is partly financed by cheap deposits, while the marginal loan must be fully financed with securities, $\frac{rQ}{\omega} > 1$. Both these effects tend to reduce the expected return of the security holder. However, the last term in (9) is negative, because in this model the average variance is reduced by adding the marginal project. It depends on the parameters of the model which effect dominates. Because what worries regulators is that banks take too much risk, I will focus on the cases in which the first effect dominates, and

$$(\frac{rQ}{\omega}\frac{\bar{Z}}{Q} - \bar{p}x)\Phi(\tilde{z}(\omega)) + (\frac{\bar{p}^3x^2}{2} - \frac{rQ}{\omega}\frac{\sigma^2}{Q})\frac{1}{\sqrt{\sigma^2}}\Phi'(\tilde{z}(\omega)) > 0, \qquad (10)$$

in equilibrium. This means that the uninsured bank finances projects with an expected return lower than the alternative return.

4 Deposit insurance

The risk-aversion of savers is a potential motive for introducing deposit insurance. Guaranteeing savers a certain return gives rise to a potential welfare gain. The question analyzed in this section is whether deposit insurance induces the bank to take more risk, which then possibly could eliminate the welfare gain of the insurance.

The impact of deposit insurance on the risk-taking behavior of the bank depends on the design of the insurance scheme. The maintained assumptions are that the insurer is rational and has the same information as savers and investors, and that the insurer compensates the depositors with certainty. The deposit insurance premium per unit of deposit, t, is determined at the same time as the interest rates, but for simplicity it is assumed to be paid out of profits at the end of the period.[13]

Of cource, the price of the insurance will affect the risk-taking behavior of the bank. However, since the insurer has the same information as savers, he can set the insurance premium so that the deposit insurance would not affect the risk-taking behavior of the bank. The deposit insurance implies that savers are willing to deposit at the bank if the deposit rate is at least equal to the return on the storage technology. Thus, the insured bank sets $r_d = 1$. Then there is an insurance premium which is such that $1 + t = r_d^y$. At that premium the risk-taking behavior of the bank is not affected at all by the introduction of deposit insurance.

It is not necessarily the case that a higher premium leads to less risk-taking. The reason is that deposits are not the marginal source of funding, so that an increased deposit cost does not directly affect the marginal funding cost of the bank. There are two counteracting forces on the risk-incentive of the bank, which are captured by the derivative of the left hand side of (9) with respect to t:[14]

[13] I do not analyze the incentive of the insurer to charge a certain premium. I merely assume that the insurer enforces a given insurance scheme as best he can. Nor do I analyze wherefrom the insurer gets the means to compensate depositors. One could think of it as the insurer being perfectly diversified across banks. In actuality tax means are sometimes used to bail out depositors, which is costly. It is obviously important to include these costs when determining whether deposit insurance is beneficial or not, but, as I argue in the concluding section, it is also important to include other costs of bank failure. See also footnote 12.

[14] This derivative being positive is a sufficient, but not necessary, condition for a reduction of the premium to *reduce* risk-taking by the bank. Morover, if the supply of funds in the deposit market is not perfectly inelastic, *and* the supply in the capital market is not perfectly elastic, the reduction of the deposit cost would affect the marginal funding cost, and there would be an additional force towards increased risk-taking. However, as long as the supply of funds is much more elastic in the

$$-\frac{D}{m}\frac{d\tilde{z}(\omega)}{d\bar{p}}\Phi'(\tilde{z}(\omega)) - \frac{rD}{\omega^2}[\bar{Z}\Phi(\tilde{z}(\omega)) - \sqrt{\sigma^2}\Phi'(\tilde{z}(\omega))]. \qquad (11)$$

Since the marginal projects increase the probability that the bank will go bankrupt, $\frac{d\tilde{z}(\omega)}{d\bar{p}} < 0$ in equilibrium, the first effect is positive. The intuition is that a less profitable bank will have more incentive to engage in risk-shifting, since it is more likely to fail. There is a counteracting effect in that the return to the security holder in the default states decreases when the insurance premium is increased. That is an increase in the deposit cost decreases the bank's possibility of exploiting the marginal investor, which mitigates the moral hazard problem of the bank. We cannot generally tell which effect dominates.

What effect would an actuarially fair deposit insurance scheme have on the risk-incentive of the bank? An actuarially fair premium is such that the expected payments of the bank to the insurer are equal to the expected shortfalls of the bank's payments to savers. The insurer has the same information and form the same expectation as savers. However, savers have risk-aversion (which here is the motive for deposit insurance). This implies that the introduction of a fairly priced deposit insurance reduces the deposit cost of the bank. Hence, we cannot generally tell whether a fairly priced insurance induces the bank to take more or less risk.

The second effect in (9) suggests that reducing the return of the marginal investor in the default states would reduce risk-taking by the bank. The obvious way to reduce the return of the marginal investor in the default states is to reduce the priority of his claim. Thus, if the bank could commit to finance marginal investments with subordinated debt, ex ante it would like to do so, since with competitive capital markets the (owner of the) bank has to bear the costs of excessive risk-taking. However, investors, who cannot observe the amount of ordinary debt, will require a high interest rate on subordinated debt, as they expect all other debt to have higher priority. Thus, ex post it will be less expensive for the bank to raise ordinary debt.

A way to help the bank out of this commitment problem is to combine deposit insurance with a covenant, which states that insured deposits have higher priority than other bank debt in case of bank failure. Consider a fairly priced insurance with such a covenant. In that case the fair premium is given by

capital market than in the deposit market, this force should be weak.

$$t(1-\Phi(\tilde{z}((r_d+t)D)))+\int_{-\infty}^{(r_D+t)D}(\frac{z}{D}-r_d)d\Phi=0. \qquad (12)$$

If the bank can, it pays t per unit of deposits to the insurer (and r_d to the depositors). If the bank fails to meet its deposit obligations, the insurer will appropriate the bank portfolio return and pay the deposit rate to the depositors. The participation constraint of the security holder then is

$$r(1-\Phi(\tilde{z}(\omega)))+\frac{1}{Q-D}\int_{(r_D+t)D}^{\omega}(z-(r_d+t)D)d\Phi=y. \qquad (13)$$

The investor gets the promised return only if the bank can meet its total debt obligations. If the bank cannot meet the total obligations, but it can meet the deposit obligations, the investors recover the rest. If the bank cannot even meet its obligations towards depositors, the investor gets nothing.

The bank's problem is choosing the lending policy and the interest rates given the insurance premium t, so as to maximize profits. Due to the deposit insurance, the bank does not have to pay interest on deposits, so that $r_d = 1$. Hence, the rational expectation equilibrium is now given by the vector $(\bar{p}^i, r^i, t^i)$ (i for insured) that solves the equation system consisting of (6), (12), and (13), for $r_d = 1$. Comparing this equilibrium with the equilibrium for the uninsured bank $(\bar{p}^u, r^u, r_d^u)$ gives the following result:

Proposition 2 *Introducing an actuarially priced deposit insurance scheme, where deposits have higher priority than other bank debt in case of bank failure, induces the bank to take less risk.*

Proof: See the appendix.

The reasons for the bank to take less risk are twofold. First, the deposit insurance lowers the total funding cost of the bank, as the bank does not have to pay any risk-premium to its risk-averse depositors. This reduces the probability of bank failure, and a safer bank has less incentive to engage in risk-shifting. Secondly, by making other debt subordinated to deposits, the possibility of the bank to exploit the marginal investor is reduced.

It should be emphasized that the result of Proposition 2 is based on the assumption that the insurer will, or at least is expected to, appropriate the bank assets when the bank fails to meet its deposit obligations. If

security holders were expecting to have higher priority than the insurer, the capital market would provide less discipline. For the same reason, the bank would take even more risk if investors expected that the government would not allow the bank to go bankrupt. Such a bank guarantee implies that investors do not worry about bank failure, and the marginal funding cost of the bank is equal to y. If the government cannot credibly commit to not bailing out all claim holders of the bank, the insurance has to cover all bank debt in order for it to reduce risk-taking by the bank. However, this commitment problem of the government might not be connected to deposit insurance. Many countries without explicit deposit insurance have practiced an implicit insurance or a bank guarantee. With this as benchmark, the introduction of an explicit deposit insurance covering all bank debt would reduce risk-taking by the bank, as long as the premium is positive.

To conclude: If it is possible to combine deposit insurance with a covenant which states that insured deposits have higher priority than other bank debt in case of bank failure, the introduction of such a deposit insurance would unambiguously reduce risk-taking by the bank, given that investors believe that the insurance contract will be enforced. If it is not possible to change the priority rules, we will not generally be able to tell whether deposit insurance increases or reduces the risk-incentive of debt. However, even if deposit insurance would reduce risk-taking by the bank, the bank will take too much risk, given that (10) is fulfilled in equilibrium. Then, the question is whether deposit insurance should be coupled with capital constraints in order to reduce the risk-incentive of debt.

5 Capital constraints

So far I have assumed that the bank issues no equity. In order to analyze whether capital constraints can reduce risk-taking by the bank, the question addressed in this section is whether the bank has incentive to issue equity itself. I will argue that this depends on whether or not the deposit insurance is fairly priced. Consider first the case with a fairly priced deposit insurance.

Assume that the bank finances its lending to entrepreneurs partly by issuing shares to investors. Denote the total stock of shares by S, the number of shares owned by the old owners (inside equity holders), who have full information and full control, by S^i, and the number of shares sold to outside investors by S^o. The price of one share is set equal to one.[15] The expected return to inside equity holders is

$$\frac{S^i}{S}\pi = \frac{S^i}{S}(\bar{Z} - \omega + \int_{-\infty}^{\omega} \Phi(\tilde{z}(z))dz), \tag{14}$$

where $\omega = (1+t)D + r(Q(\bar{p}) - S^o - D)$ (given deposit insurance, the bank sets the deposit rate equal to 1).

Assume that the inside equity holders act as one agent, referred to as the bank. I will first assume that the bank issues shares before it determines its lending policy, the interest rates, and the number of shares of inside equity holders. Afterwards I will examine whether the bank has incentive to change its capital structure after it has determined its lending policy. I assume that the bank also raises debt in the capital market, that is $S^o < Q - D$.

Investors observe the number of shares that the bank issues, but they cannot observe the total amount borrowed in the capital markets. They base their decisions about whether or not to buy the securities on their expectations of the lending policy of the bank. In the same way the insurer bases the premium on his expectation of the lending policy of the bank. Given the expectations of investors and the insurer and the number of shares sold on the market, the security rate and the number of shares of inside equity holders must be such that

$$r(1 - \Phi(\tilde{z}(\omega))) + \int_{-\infty}^{\omega} \frac{r}{\omega} z d\Phi = y, \tag{15}$$

$$\frac{1}{S}\pi = y, \tag{16}$$

[15] It turns out to be simpler to use the number of shares of inside equity owners as equilibrium variable than to have an endogenous equity price.

and the actuarially fair insurance premium is given by

$$(t+1)(1-\Phi(\tilde{z}(\omega)))+\int_{-\infty}^{\omega}\frac{t+1}{\omega}zd\Phi=1. \tag{17}$$

This means that for given S^o, the rational expectation equilibrium is given by the vector $(\bar{p}^e, r^e, (S^i)^e, t^e)$ (e for equity) that solves the equation system consisting of (6), (15), (16), and (17).

Now it is possible to answer whether the bank has incentive to issue equity itself, and we have the following result:

Proposition 3 *If the deposit insurance is fairly priced, the bank will have incentive to issue equity whenever equity financing would reduce risk-taking by the bank.*

Proof: Substituting (15), (16), and (17) into (14) gives the equilibrium return of inside equity holders:

$$\bar{Z}-y(Q-D)-D. \tag{18}$$

The bank chooses S^o so as to maximize (18). The first order condition is

$$-m(\bar{p}x-y)\frac{d\bar{p}}{dS^o} \tag{19}$$

Since $\bar{p}x-y<0$ for $S^o=0$, the bank will issue equity whenever $\frac{d\bar{p}}{dS^o}>0$. Q.E.D.

With an actuarially fair insurance premium, the bank bears the entire agency cost of debt financing, and therefore it has incentive to issue equity, if equity financing increases the equilibrium value of $\bar{p}$.

Will equity financing reduce risk-taking by the bank? There are two counteracting effects, familiar from the previous analysis, of a change in S^o on the risk-taking behavior of the bank. On the one hand, increased equity financing reduces the probability of bank failure. This tends to make the bank take less risk, since a safer bank has less incentive to engage in risk-shifting. On the other hand, increased equity financing increases the return of the debt holders in the default states. This tends to make the bank take more risk, as it increases the bank's possibilities to exploit its debt holders. Thus, these are the same forces as those arising from a reduction of the deposit cost, and we cannot generally tell which effect dominates.

Proposition 3 implies that, if deposit insurance is fairly priced, there is no scope for capital constraints. If the bank does not issue equity, it is because equity financing does not increase the equilibrium value of $\bar{p}$. This means that at the best capital constraints have no effect on the risk-taking behavior of the bank, at the worst they increase risk-taking by the bank.

However, if the deposit premium is not actuarially priced, there may be a reason for capital requirements. Particularly, an actuarially priced deposit insurance requires that the premium is dependent on the capital structure of the bank, as a larger amount of equity capital implies lower expected shortfalls of the bank's payments to depositors, and we have

Proposition 4 *With a constant deposit insurance premium the bank may not issue shares even though equity financing would reduce risk-taking by the bank.*

Proof: With a constant insurance premium the deposit cost of the bank is fixed as deposit insurance implies that the deposit rate is equal to 1 independently of the capital structure of the bank. Denote the fixed deposit cost by c_d. The equilibrium is given by equations (6), (15), and (16). Substituting (15) and (16) into (14) gives the equilibrium profit of the bank

$$\bar{Z} - y(Q - D) - \frac{c_D}{r} yD. \tag{20}$$

The derivative of (20) with respect to S^o is

$$-m(\bar{p}x - y)\frac{d\bar{p}}{dS^o} + \frac{c_D}{r^2} yD\frac{dr}{dS^o}. \tag{21}$$

From the Implicit Function theorem follows that $\frac{dr}{dS^o} < 0$. Hence, the first order condition may be negative even though equity financing reduces risk-taking by the bank. Q.E.D.

Proposition 4 implies that with a constant insurance premium there may be scope for capital constraints. The reason is that the bank has incentive to exploit the fact that the deposit insurance premium does not increase as leverage is increased.

So far I have assumed that the bank can commit to a certain level of equity financing before determining its lending policy and interest rates. I will now examine whether the bank, if it could, would have incentive to change its capital structure after it has determined its lending policy.

Proposition 5 *If the deposit insurance is fairly priced, the bank will be indifferent to how to finance a given investment volume.*

Proof: See the appendix.

The security rate and the actuarially fair insurance premium depend on how much equity the bank issues, because the more equity capital the bank has, the lower is ω and the higher is the expected return of each debt holder in the default states, given the expectations of the lending policy. As the bank issues one more share, the security rate and the insurance premium decrease just enough for the expected return of inside equity holders to be invariant to changes in S^o. Hence, if the deposit insurance is fairly priced, the bank will have no incentive to change its capital structure after it has determined its lending policy. The next result is that this is not the case with a constant insurance premium:

Proposition 6 *If the insurance premium is independent of the capital structure of the bank, the bank will have incentive to maximize leverage.*

Proof: See the appendix.

Thus, the familiar result of Merton (1977), that with a fixed price on deposits the bank has incentive to maximize leverage to maximize the value of deposit insurance, holds true although the bank also has uninsured debt, the cost of which does depend on the capital structure. Hence, with a constant insurance premium the bank has incentive to increase leverage after it has determined its lending policy, which increases the scope for capital constraints.

6 Extensions

In this section I examine the implications of changing two of the assumptions in the previous analysis.

First, the analysis is based on the assumption that investments in the capital markets are perfectly observable and correctly priced. This might seem to be a realistic assumption. Investments in the capital markets should be easier to evaluate for outsiders than loans to entrepreneurs, and even if depositors themselves might not have incentive to evaluate the bank portfolio, rating agencies should be able to do the evaluation for them. However, in reality the bank can quickly change its capital market portfolio. As a simple way to capture that the deposit rate cannot be continuously revised, consider the alternative model assumption that the depositors and the insurer cannot observe the investments made in the capital markets, and the deposit rate and the insurance premium cannot be contingent on these investments. This informational asymmetry gives the bank incentive to invest in capital market securities with high variance (or a high covariance with the loan portfolio). It also gives the bank incentive to expand its balance sheet by raising funds from investors and investing the proceeds in risky assets.

On the other hand, assuming that investments in the capital markets are not observable, but the bank can be forced through random checking to invest in safe securities only, would not change the analysis of the previous sections. To see why, note that all assets in the capital markets give an expected return of y. Safe assets give y in every state. This means that even in the good states the return of the safe asset is lower than the security rate. Hence, the bank has no incentive to raise funds from investors in order to invest the proceeds in relatively safe assets in the capital market.

Secondly, the analysis has been based on the assumption that aggregate savings are smaller than the input requirements of the profitable projects in the economy so that the bank raises additional funds in the capital market. Assume instead that deposits are larger than the total input requirement, and that the bank invests the difference $D-Q$ in the capital markets. Thus, the bank's balance sheet looks like in table 2.

Table 2 Balance sheet of a bank with no public debt

Assets	**Liabilities**
Loans	Deposits
Securities	

Because of the simple structure of the model, if investments in the capital markets are perfectly observable, depositors and the insurer can infer the lending policy of the bank, given that they know the total amount of deposits. This means that the bank will take the correct amount of risk and the introduction of a fairly priced deposit insurance has no effect on bank behavior. The more interesting case is when investments in the capital markets are not observable.

To avoid the bank expanding its balance sheet, assume that the bank can invest in safe securities only. It is easy to show that the bank will have the incentive to invest the difference, $D - Q$, in safe securities, if and only if the cost per unit of deposit, $r_d + t$, is less than y. Thus, given that y is sufficiently large, the alternative to extend the marginal loan is to invest the means in safe securities, and not to borrow one unit less from savers, and the first order condition for an optimal lending policy is given by

$$(\bar{p}x - y)(1 - \Phi(\tilde{z}(\omega))) + \bar{p}^3 x^2 \frac{1}{2\sqrt{\sigma^2}} \Phi'(\tilde{z}(\omega)) = 0, \qquad (22)$$

where $\omega = (r_d + t)D$.

The rational expectation equilibrium is given by the couple $(\bar{p}, r_d)$ that solves the equation system consisting of (22) and (8), and we see directly from condition (22) that the bank will take too much risk. Introduction of a fairly priced deposit insurance unambiguously reduces risk-taking by the bank, as the alternative return is not affected by the reduction of the deposit cost. For the same reason, equity financing unambiguously reduces risk-taking by the bank. However, if the insurance is fairly priced, there is no scope for capital constraints.

7 Conclusions

One main deficiency of the literature on the impact of bank regulation on the risk-taking behavior of banks is that it does not explicitly take into account the functions of the bank. In this paper, I have analyzed the risk-taking behavior of a monopoly bank, which fulfills two functions. First, the bank provides credit to entrepreneurs who cannot fund their projects in the capital markets because of information problems. Secondly, it invests the means of savers, who find it too risky to invest in the capital markets. These functions have two important implications. First, depositors and other creditors of the bank are not able to observe the lending policy of the bank. Secondly, the marginal source of funding of the bank is not deposits.

That the claim holders of the bank cannot observe its lending policy gives the bank incentive to take too much risk. This is well-known from corporate finance. What is less well-known is that diversification of the bank portfolio mitigates the moral hazard problem of the bank. Furthermore, because deposits is not the marginal funding source of the bank, deposit insurance does not necessarily increase the risk-incentive of the bank. The introduction of an actuarially fair deposit insurance scheme combined with a covenant, which states that insured deposits have higher priority than other bank debt in bankruptcy, unambiguously reduces risk-taking by the bank. If the deposit insurance is fairly priced, there is no scope for capital constraints. A fair premium depends on the amount of equity of the bank, and the bank has incentive to issue equity whenever equity financing reduces risk-taking by the bank. However, with a constant insurance premium, which among other things is independent of the amount of bank equity, the bank might not have the incentive to issue equity, and capital constraints may reduce risk-taking by the bank.

The paper provides by no means a comprehensive analysis of the pros and cons of deposit insurance and capital constraints. The aim is merely to emphasize the importance of the benchmark when making statements about the efficiency of bank regulation. Perhaps the most important limitation is that the analysis does not include bankruptcy costs. The perceived problems of controlling bank risk-taking has led to suggestions for radical changes of the banking structure. According to the so-called "narrow banking" proposal, deposit-taking institutions should be allowed to invest in tradeable assets only, while other banks should be financed in the capital markets. The emphasis of this proposal is on minimizing the probability that the insurer has to compensate de-

positors, because using tax means to compensate depositors is costly, as taxes distort real allocations. The idea of the proposal is that the deposit-taking bank can be induced to take the correct amount of risk, since its investment policy is observable. However, most agree that bank failures also involve other costs. Production is disrupted as the loan customers of the bank have to pay higher interest rates to lenders that know less about them than their bank. They might even have difficulties in finding new financiers due to adverse selection problems. These costs are probably higher with a narrow banking structure, as the pure credit bank is likely to have higher total funding costs (no access to cheap deposits), and therefore is less profitable and more likely to go bankrupt than the traditional bank. I believe that we need a better understanding of the costs of bank failure before making radical changes to the banking structure.

A Appendix

Derivation of the portfolio distribution: m projects are drawn from the population of projects. Let q be the number of projects with a probability of success equal to or larger than $\bar{p}$. Then, q is a binomial random variable with parameters m and $1-\bar{p}$, in short $q = Bin(m, 1-\bar{p})$. We have that

$$E(q) = m(1-\bar{p}),$$

where E is the expectation operator. Let P_i be the probability of success of project i, and Y_i be the return on a loan to project i. P_i is uniformly distributed on [0, 1] for $i = 1, 2, ..., m$. Let p_i be the realization of P_i. Given that $P_i = p_i$, $Y_i = xBin(1, p_i)$. Let $Z_i = Y_i 1\{P_i \geq \bar{p}\}$, where $1\{P_i \geq \bar{p}\} = 1$, if $P_i \geq \bar{p}$, and $1\{P_i \geq \bar{p}\} = 0$, if $P_i < \bar{p}$. Z_i takes one of two values, x and 0. What is the probability that Z_i is equal to x? Using that

$$P(Z_i = x) = E(P(Z_i = x|P_i = p_i)),$$

where P is the probability operator, and

$$P(Z_i = x|P_i = p_i) = p_i 1\{P_i \geq \bar{p}\},$$

we get,

$$P(Z_i = x) = \int_{\bar{p}}^{1} p \; dp = \frac{1-\bar{p}^2}{2}.$$

Hence,

$$E(Z_i) = \frac{1-\bar{p}^2}{2}x$$

$$Var(Z_i) = \frac{1-\bar{p}^2}{2}\frac{1+\bar{p}^2}{2}x^2 = \frac{1-\bar{p}^4}{4}x^2,$$

where Var is the variance operator. The portfolio outcome z is given by, $z = \sum_{i=1}^{m} Z_i$, and we have that $E(z) = m\ E(Z_i)$ and $Var(z) = m\ Var(Z_i)$, since Z_i are independent, which gives (1).

First order condition for the optimal lending policy of the debt financed bank: We have that

$$\int_{\omega}^{\infty}(z-\omega)d\Phi = \bar{Z} - \omega(1-\Phi(\tilde{z}(\omega))) - \int_{-\infty}^{\omega} z\Phi'(\tilde{z}(z))\frac{1}{\sqrt{\sigma^2}}dz.$$

The expected profit of the bank is equal to the expected portfolio return minus the expected payments to the debt holders. The integral term on the right hand side can be rewritten as

$$\int_{-\infty}^{\omega} \frac{z-\bar{Z}}{\sqrt{\sigma^2}}\Phi'(\tilde{z}(z))dz + \int_{-\infty}^{\omega} \frac{\bar{Z}}{\sqrt{\sigma^2}}\Phi'(\tilde{z}(z))dz.$$

Using that $\Phi''(x) = -x\Phi'(x)$ we get

$$\int_{-\infty}^{\omega} z\Phi'(\tilde{z}(z))\frac{1}{\sqrt{\sigma^2}}dz = -\sqrt{\sigma^2}\Phi'(\tilde{z}(\omega)) + \bar{Z}\Phi(\tilde{z}(\omega)). \tag{23}$$

Substituting 23 into the profit function gives

$$\pi = (\bar{Z}-\omega)(1-\Phi(\tilde{z}(\omega))) + \sqrt{\sigma^2}\Phi'(\tilde{z}(\omega)). \tag{24}$$

Differentiating 24 with respect to $\bar{p}$, remembering that Q, $\bar{Z}$, and σ^2 are functions of $\bar{p}$, and again using that $\Phi''(x) = -x\Phi'(x)$, gives (6).

Proof of Proposition 1: Substituting 23 into (10) gives

$$r(1-\Phi(\tilde{z}(\omega))) + \frac{r}{\omega}[\bar{Z}\Phi(\tilde{z}(\omega)) - \sqrt{\sigma^2}\Phi'(\tilde{z}(\omega))] = y. \tag{25}$$

The derivative of the left hand side of 25 with respect to $\bar{p}$ is

$$m\frac{r}{\omega}[(\frac{r}{\omega}\bar{Z} - \bar{p}x)\Phi(\tilde{z}(\omega)) + (\frac{\bar{p}^3x^2}{2} - \frac{r}{\omega}\sigma^2)\frac{1}{\sqrt{\sigma^2}}\Phi'(\tilde{z}(\omega))]. \tag{26}$$

By comparing 26 with (12) one sees that $\bar{p}x < y$, when the expected return of the security holder increases with $\bar{p}$ for given r and r_d. *Q.E.D.*

The existence of the benchmark equilibrium: Define

$$F^1(\bar{p},r) = (\bar{p}x - r)(1-\Phi(\tilde{z}(\omega))) + \bar{p}^3x^2\frac{1}{2\sqrt{\sigma^2}}\Phi'(\tilde{z}(\omega)),$$

$$F^2(\bar{p},r) = r(1-\Phi(\tilde{z}(\omega))) + \frac{r}{\omega}[\bar{Z}\Phi(\tilde{z}(\omega)) - \sqrt{\sigma^2}\Phi'(\tilde{z}(\omega))] - y.$$

where $\omega = r_d(\bar{p}, r)D + r(Q-D)$, and $r_d(\bar{p}, r)$ solves the participation constraint of the depositor for a given lending policy and security rate. We have that

$$F_1^1 = x(1-\Phi(\tilde{z}(\omega))) + (3\bar{p}^2x^2 + m(\bar{p}^3x^2)^2\frac{1}{2\sigma^2})\frac{1}{2\sqrt{\sigma^2}}\Phi'(\tilde{z}(\omega))$$

$$-(\bar{p}x - r - \bar{p}^3x^2\frac{\bar{Z}-\omega}{2\sigma^2})\Phi'(\tilde{z}(\omega))\frac{d\tilde{z}(\omega)}{d\bar{p}},$$

$$F_2^1 = -(1-\Phi(\tilde{z}(\omega))) - (\bar{p}x - r - \bar{p}^3x^2\frac{\bar{Z}-\omega}{2\sigma^2})\Phi'(\tilde{z}(\omega))\frac{d\tilde{z}(\omega)}{dr}$$

where F subindex t is equal to the partial derivative of F with respect to argument t. For $F^1 = 0$, $\bar{p}x \leq r$, and, including the effect on r_d, $\frac{d\tilde{z}(\omega)}{d\bar{p}}$ is negative and $\frac{d\tilde{z}(\omega)}{dr}$ is positive. Thus, the signs of the derivatives depend on the profitability and the diversification of the bank. For $\bar{p}$ and r such that $\bar{Z} > \omega$, there exists a size of the economy (m and d increased in proportion) so that F_1^1 is positive and F_2^1 is negative. Hence, for a sufficiently large economy the values of $(\bar{p},\, r)$ that fulfill $F^1(\bar{p},\, r) = 0$ represent an upward-sloping locus in the part of the $(\bar{p},\, r)$ plane where $\bar{Z} > \omega$, and as m approaches infinity the slope of the curve approaches x. Furthermore, we have that

$$F_1^2 = -m\frac{r}{\omega}[\bar{p}x\Phi(\tilde{z}(\omega)) - \bar{p}^3 x\frac{1}{2\sqrt{\sigma^2}}\Phi'(\tilde{z}(\omega))]$$

$$+\frac{r}{\omega^2}[\bar{Z}\Phi(\tilde{z}(\omega)) - \sqrt{\sigma^2}\Phi'(\tilde{z}(\omega))](rm - \frac{\partial r_d}{\partial \bar{p}}D),$$

$$F_2^2 = (1-\Phi(\tilde{z}(\omega)))+\frac{1}{\omega}(1-\frac{r}{\omega}(Q-D+\frac{\partial r_d}{\partial r}D))[\bar{Z}\Phi(\tilde{z}(\omega))-\sqrt{\sigma^2}\Phi'(\tilde{z}(\omega))],$$

For a sufficiently large bank, $F_2^2 > 0$ and the slope of the $F^2(\bar{p},\, r) = 0$ curve depends on the sign of F_1^2, but as m approaches infinity the slope approaches zero at $r = y$. Thus, as $x > y$, there is a size of m for which it exists an equilibrium.

Figure 1 illustrates the equilibrium for an economy consisting of 50 entrepreneurs and 20 risk-neutral savers, and where $y = 1.1$ and $x = 1.5$. In order to illustrate how well the normal distribution approximates the true distribution of z, Figure 2 shows both distributions in this numerical example. It is worth mentioning that the probability of bank failure in the equilibrium is equal to 0.16.

Figure 1 Illustration of the equilibrium

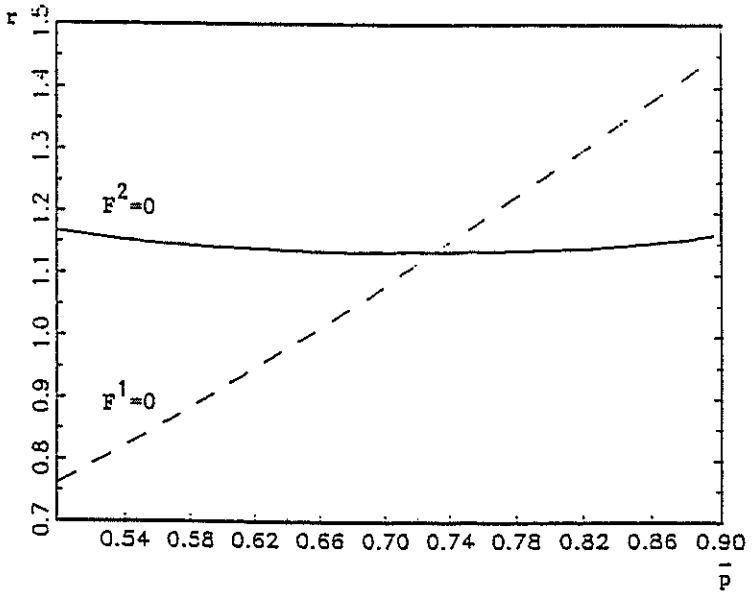
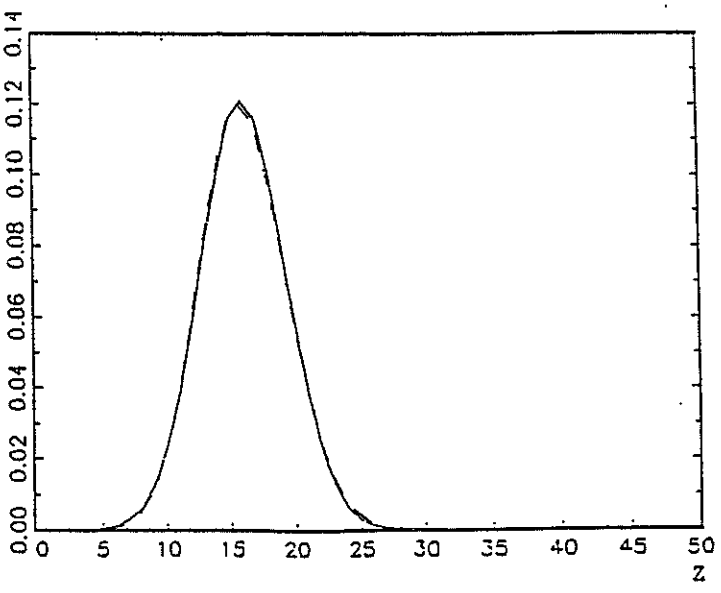


Figure 2 True and approximated distribution



Proof of Proposition 2: I will show that $\bar{p}^i > \bar{p}^u$. Let c_d be the bank's total deposit cost per unit of deposits, that is $c_d = r_d + t$. From the definition of risk-aversion follows that

$$c_d(1 - \Phi(\tilde{z}(c_d D))) + \int_{-\infty}^{c_d D} \frac{z}{D} d\Phi - 1$$

is larger than

$$U(c_d)(1 - \Phi(\tilde{z}(c_d D))) + \int_{-\infty}^{c_d D} U(\frac{z}{D}) d\Phi - U(1),$$

for given $\bar{p}$ and r. This means that for a given lending policy and a given security rate the value of c_d that gives a fairly priced insurance is lower than the value that fulfills the participation constraint of the uninsured depositor. Thus, introducing a fairly priced deposit insurance reduces the bank's deposit cost for given $\bar{p}$ and r. Giving first priority to deposits then reduces the deposit cost even more for given $\bar{p}$ and r. (This implies that the first order condition for an optimal lending policy in figure 1, $F^1 = 0$, shifts outwards.)

How does the deposit insurance affect the IR-restriction of the security holder? Giving first priority to deposits reduces the expected return of the security holder for given $\bar{p}$ and r, while the reduction of the deposit cost increases the return. I will show that, when the insurance is fairly priced, the first effect dominates. The expected return to the security holder can be written

$$r(1 - \Phi(\tilde{z}(\omega))) + \frac{1}{Q - D}[\bar{Z}\Phi(\tilde{z}(\omega)) - \sqrt{\sigma^2}\Phi'(\tilde{z}(\omega)) - (\bar{Z}\Phi(\tilde{z}(c_d D))$$

$$- \sqrt{\sigma^2}\Phi'(\tilde{z}(c_d D)))] - \frac{c_D D}{Q - D}(\Phi(\tilde{z}(\omega)) - \Phi(\tilde{z}(c_d D)). \qquad (27)$$

For a fairly priced deposit insurance

$$\bar{Z}\Phi(\tilde{z}(c_d D)) - \sqrt{\sigma^2}\Phi'(\tilde{z}(c_d D)) = D - c_d D(1 - \Phi(\tilde{z}(c_d D))). \qquad (28)$$

Substituting 28 into 27 and multiplying by $\frac{\omega}{r}$ gives

$$\omega(1 - \Phi(\tilde{z}(\omega))) + [\bar{Z}\Phi(\tilde{z}(\omega)) - \sqrt{\sigma^2}\Phi'(\tilde{z}(\omega))]$$

$$+ \frac{\omega}{(Q - D)r}(c_d D(1 - \Phi(\tilde{z}(\omega))) + \frac{c_D D}{\omega}[\bar{Z}\Phi(\tilde{z}(\omega)) - \sqrt{\sigma^2}\Phi'(\tilde{z}(\omega))] - D). \qquad (29)$$

For an actuarially priced insurance, given r and $\bar{p}$, the last term of 29 is negative. The first two terms are equal to the return without insurance multiplied with $\frac{\omega}{r}$. Since these terms increase with r_d, the return must be higher without insurance than with, for given lending policy and security rate. Hence, for a given lending policy, the security holder requires a higher expected return with deposit insurance than without. (The IR-restriction of the security holder in Figure 1, $F^2 = 0$, shifts upwards.) This means that for a given lending policy the deposit rate decreases and the security rate increases, which both reduce the value of the first order condition for an optimal lending policy. Hence, $\bar{p}^i > \bar{p}^u$. *Q.E.D.*

Proof of Proposition 5: Define G^1 and G^2 in the following way:

$$G^1(r, S^i; S^o) \equiv \omega(1 - \Phi(\tilde{z}(\omega))) + \bar{Z}\Phi(\tilde{z}(\omega)) - \sqrt{\sigma^2}\Phi'(\tilde{z}(\omega)) - \frac{\omega}{r}y, \quad (30)$$

$$G^2(r, S^i; S^o) \equiv (\bar{Z} - \omega)(1 - \Phi(\tilde{z}(\omega))) + \sqrt{\sigma^2}\Phi'(\tilde{z}(\omega)) - Sy, \quad (31)$$

where $\omega = r(\frac{1+t}{r}D + Q - S^o - D)$. For a fairly priced deposit insurance, in equilibrium $\frac{1+t}{r} = \frac{1}{y}$. Hence, for given S^o, the equilibrium values of r, and S^i simultanously fulfill $G^1 = 0$, $G^2 = 0$, with $\frac{1+t}{r} = \frac{1}{y}$, given the expectations of the lending policy of the investors and the insurer. Denote these values by hat. Then, according to the Implicit Function Theorem, the induced changes in r and S^i by a change in S^o are

$$\begin{pmatrix} \frac{d\hat{r}}{dS^o} \\ \frac{d\hat{S}^i}{dS^o} \end{pmatrix} = -J^{-1} \begin{pmatrix} G^1_{S^o} \\ G^2_{S^o} \end{pmatrix}$$

where J^{-1} is the inverse of the Jacobian. Evaluating the expressions for the derivatives gives

$$\frac{dr}{dS^o} = \frac{r(1 - \Phi(\tilde{z}(\omega))) - y}{(\frac{1}{y}D + Q - S^o - D)(1 - \Phi(\tilde{z}(\omega)))}, \quad (32)$$

$$\frac{dS^i}{dS^o} = 0. \quad (33)$$

The change in the expected return of inside equity holders from a change in S^o is equal to the derivative of (19) with respect to S^o, taking into account the required change in r :

$$\frac{S^i}{S}(r(1 - \Phi(\tilde{z}(\omega))) - (\frac{1}{y}D + Q - S^o - D)(1 - \Phi(\tilde{z}(\omega)))\frac{d\hat{r}}{dS^o} - \frac{1}{S}\pi). \quad (34)$$

By substituting 32 into 34, and using that in equilibrium $\frac{1}{S}\pi = y$, one sees that 34 is equal to zero. Hence, the expected return of insiders is invariant to changes in S^o. $Q.E.D.$

Proof of Proposition 6: Let ω be given by $\omega = c_d D + r(Q(\bar{p}) - S^o - D)$ in 30 and 31, where c_d is the fixed cost per unit of deposit. Then

$$\frac{d\hat{r}}{dS^o} = \frac{r(1 - \Phi(\tilde{z}(\omega))) - y}{(Q - S^o - D)(1 - \Phi(\tilde{z}(\omega))) + \frac{c_D}{r^2} Dy},$$

$$\frac{dS^i}{dS^o} = \frac{(r(1 - \Phi(\tilde{z}(\omega))) - y)\frac{c_D}{r^2} D}{(Q - S^o - D)(1 - \Phi(\tilde{z}(\omega))) + \frac{c_D}{r^2} Dy}.$$

By using these expressions, one gets that for any given lending policy the derivative of the expected return of inside equity holders (19) with respect to S^o is negative for all values of S^o. $Q.E.D.$

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