Liquidity standards and the value of an informed lender of last resort*

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Abstract

This paper provides a novel rationale for bank liquidity standards. We consider a dynamic model in which receiving liquidity support from the lender of last resort (LLR) may help banks to weather investor runs induced by shocks to banks' financial condition. In our setting, liquidity standards are costly because they force banks to forgo valuable investment opportunities. They can nonetheless be efficient. The reason is that, when a run happens, liquidity standards increase the time available before the LLR must decide on supporting the bank. This facilitates the arrival of information on the bank's financial condition and improves the efficiency of the decision taken by the LLR. We show the need for regulatory (vs. voluntarily adopted) liquidity standards when the underlying social trade-offs make the uninformed LLR inclined to support the troubled bank.

Keywords: Liquidity standards, lender of last resort, constructive ambiguity
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1 Introduction

Prior to the Great Recession, the focus of bank regulation was on bank capital. However, the liquidity problems that banks experienced since the onset of the financial crisis in 2007 brought to the forefront a debate about the potential value of regulating banks' liquidity.[1] Those problems also reignited the debate on the challenges that uncertainty about the financial condition of banks pose to the lender of last resort (LLR).[2] In this paper, we contribute to these debates by presenting a novel theory of banks' liquidity standards.

Our theory builds on what we believe is a distinctive feature of an instrument such as the liquidity coverage ratio of Basel III: Once a crisis starts, liquidity buffers provide banks the capability to autonomously accommodate potential debt withdrawals for some time. Having time to resist without LLR support is valuable; it allows for the release of information on the bank's financial condition that is useful for the LLR's decision on whether to grant support or not. This generally improves the efficiency of the decisions regarding the continuation of the bank as a going concern or its liquidation and, on occasions, allows for a resolution of the crisis without explicit intervention of the LLR. Moreover, there are situations in which the presence of liquidity may be sufficiently reassuring to debtholders for them to wait for further news on the bank's condition before starting to run on the bank.

We consider a model in which a bank ex ante decides how to allocate its funds across liquid and illiquid assets. Illiquid assets are ex ante more profitable than liquid assets but their quality is vulnerable to the realization of an interim shock to the bank's financial condition. If assets get damaged by the shock, the bank turns fundamentally insolvent and its early liquidation is efficient. In contrast, if assets do not get damaged, the bank remains fundamentally solvent and its early liquidation is inefficient. A crucial problem is that discerning whether the assets are damaged or not may take time.

The bank is funded with equity and debt, and faces roll over risk because each period

[1] See Gorton (2009) and Shin (2010) for a discussion on the role of banks' liquidity problems during the most recent financial crisis.

[2] Bagehot (1873) advocates that central banks should extend liquidity support to banks experiencing liquidity problems provided they are solvent. However Goodhart (1999) argues that the feasibility of establishing a clear-cut distinction between illiquidity and insolvency on the spot is a myth.

a portion of investors are entitled to decide whether to roll over their debt. Under these conditions, the shock to the bank's financial condition can trigger a run by investors, which if sustained for long enough, may lead the bank into failure, unless it can borrow from the LLR. In making its lending decision, the LLR faces the classical problem that the bank seeking liquidity support might be fundamentally insolvent. While it is optimal to grant liquidity to solvent banks, in the case of an insolvent bank early liquidation would be preferable.

In general, assessing the financial condition of the bank in real time is quite difficult. Following this view, we assume that the LLR is initially uncertain about the financial condition of the bank (the quality of its illiquid assets) but may obtain the relevant information over time. Thus, liquidity standards, which lengthen the time a bank can sustain a liquidity shock without outside support, allow for more information on the bank's financial condition to come out prior to the LLR to decide whether to extend its liquidity support to the bank. Such information is valuable because it improves the efficiency of the implied continuation versus liquidation decision regarding the bank's illiquid assets.[3] Our model, therefore, shows that postponing the time at which the bank needs liquidity support from the LLR may be conducive to a more efficient resolution of the crisis.

Our model also shows that, due to implicit subsidies associated with the potential support received from the LLR, the liquidity standards voluntarily adopted by bank owners may be lower than those that a regulator might like to set. Specifically, if bank owners expect support to be granted if the LLR remains uninformed about the quality of the assets once the bank exhausts its cash, they may prefer to opportunistically hold less liquidity than it would be socially optimal. By doing this, they shorten the spell over which the bank can resist the run without support and, thus, the chances of being supported by the LLR. In this case, introducing a minimal regulatory liquidity standard can increase overall efficiency relative to the laissez faire benchmark.

[3]For some parameter values, liquidity standards may help sustain what we define below as a *late run equilibrium*, in which investors do not start running right after the shock to the bank's financial condition, but only when further news confirm its illiquid assets to be bad. Intuitively, by increasing investors' prospect of recovering value out of their debt claims if the bank's assets turn out to be damaged, liquidity standards reduce investors' incentives to run. Under these circumstances, it is more likely that the crisis self-resolves without the intervention of the LLR and in the most efficient terms regarding the continuation versus liquidation of the bank's illiquid assets.

Until recently, there was no consensus among policy makers about the need for liquidity regulation. This was in contrast with an existing body of academic research that pointed to the existence of inefficiencies in worlds with a strictly private provision of liquidity, via either interbank markets (Bhattacharya and Gale 1987) or credit line agreements (Holmström and Tirole 1998). A common view was that liquidity regulation was costly for banks in spite of results pointing to its welfare enhancing effects, e.g. by reducing fire-sale effects in crises (Allen and Gale 2004) or the risk of panics due to coordination failure (Rochet and Vives 2004). Another view was that the effective action by the LLR rendered liquidity standards unnecessary.[4] There was also the view that although the financial system was vulnerable to panics (Allen and Gale 2000), there were positive incentive effects of the implied liquidation threat (Calomiris and Kahn 1991, Chen and Hasan 2006, Diamond and Rajan 2005).

The severity of banks' liquidity problems during the recent crisis led to a consensus among policy makers about the need to introduce some form of liquidity regulation for banks.[5] Those problems also motivated new academic papers analyzing bank liquidity standards. Perotti and Suarez (2011), for example, rationalize liquidity regulation as a response to the existence of systemic externalities and analyze the relative advantages of price-based vs. quantity-based instruments. Calomiris, Heider, and Hoerova (2012), in turn, show that liquidity requirements may substitute for capital requirements in a moral hazard setup. These studies, however, are unable to explain the differential marginal contribution of liquidity standards *a la* Basel III over relevant alternatives such as capital standards or the effective provision of emergency liquidity by the LLR.[6]

We contribute to close this gap in the literature with a theory that relies on a novel way of thinking about liquidity requirements – an instrument that, by making banks better able to withstand the initial phases of a crisis, allows the LLR to be better informed when it

[4] See Flannery (1996), Freixas, Parigi and Rochet (2000), and Rochet and Vives (2004).

[5] Banks' liquidity problems appear to have started in the summer of 2007 following the collapse of the asset-backed commercial paper (ABCP) market. These problems grew larger with the collapse or near collapse of several other markets, including the repo and the financial commercial paper markets, and even several segments of the interbank market, and with banks' shortages of collateral in part due to downward spirals in market and funding liquidity (Brunnermeier and Pedersen 2009).

[6] See Basel Committee on Banking Supervision (2010) for a description of Basel's proposed liquidity standards.

gets called into action.[7] Our paper is also related to papers about moral hazard and the potential value of commitment to be tough in the context of lending of last resort or bank rescue policies, including Mailath and Mester (1994), Perotti and Suarez (2002), Repullo (2005), Acharya and Yorulmazer (2007, 2008), Ratnovski (2009), Farhi and Tirole (2012), and Chari and Kehoe (2013). We add to this literature by showing the implications for these issues of having a LLR who is more or less informed about troubled banks at the time of deciding whether to support them or not.

Finally, our paper is related to the literature that attempts to explain investors' incentives to run on banks, including Diamond and Dybvig (1983), Jacklin and Bhattacharya (1988), and He and Xiong (2012). We contribute to this literature by discussing the effects of the potential arrival of information (and the time available for such information to arrive) on investors' incentives to run and the potential resolution of a run.

The rest of the paper is organized as follows. Section 2 introduces our dynamic model of runs. Section 3 analyzes several issues relevant for solving the model: the effects of information on crisis resolution, the time a bank can resist a run, and debtholders' expected payoffs in case of early liquidation. Section 4 characterizes the *early run equilibrium*: the situation in which investors start canceling their debt immediately after the shock to the bank's financial condition. Section 5 considers social welfare and the rationale for liquidity standards when such an equilibrium is anticipated. Section 6 discusses potential ramifications and extensions of the analysis, including the potential role of liquidity standards in inducing equilibria different from the early run equilibrium. Section 7 concludes the paper. All proofs are in Appendix A. Appendix B analyzes the possibility of sustaining other equilibria in greater detail.

[7] Nosal and Ordoñez (2013) describe a setup in which a government delays intervention in order to learn more about the systemic dimension of a crisis. Their analysis focuses on the strategic interaction between banks, which can restrain from risk taking in order to avoid getting into trouble earlier than their peers, i.e. at a time in which the government is still not supporting the banks in trouble.

2 The model

Consider a continuous time model of an individual bank in which time is indexed by $t \in \mathbb{R}$. There are three classes of agents: bank owners, investors, and a lender of last resort (LLR). All agents are risk neutral and discount future payoffs at a zero rate. Bank owners and investors care about the net present value of their own payoffs. The LLR is a benevolent maximizer of total net present value. The model focuses on what happens to the bank after some shock arriving at $t = 0$ weakens its perceived solvency.

The bank exists from a foundation date, say $t = -1$, at which its initial owners invest in assets of total size one, issue debt and equity among competitive investors, and, hence, appropriate as a surplus the difference between the value of the securities sold to the investors and the unit of funds needed to start up the bank.[8]

2.1 Assets and liabilities of the bank

The assets of the bank consist of an amount C of a liquid asset (*cash*) and an amount $1 - C$ of *illiquid assets*. The illiquid assets pay some potentially risky per-unit final returns equal to $\tilde{a}$ at termination and to $\tilde{q}$ in case of early liquidation. Early liquidation is feasible at any date prior to termination but cannot be partial: it must affect all the illiquid assets at once.

The debt issued by the bank at t=–1 is uniformly distributed among a measure-one continuum of investors. Each investor is promised a repayment of B at termination and is simultaneously given the option to "put" her debt back to the bank in exchange for an earlier repayment of $D < B$ at some exercise dates over the life of her contract. Debt putability is a convenient way to make investors face roll over decisions and banks face roll over risk similar to those that would emerge in a more complex environment with overlapping issues of short-term debt with fixed maturity.

To facilitate tractability, we assume that both the illiquid assets and the uncanceled debt of the bank mature at $T \to \infty$, which is a practical way to capture "the long run" in this

[8] Obviously, such a difference will have to be non-negative for the initial owners to be at all interested in founding the bank at $t = -1$.

model.[9] We also assume that debtholders' chances to put their debt arrive according to independent Poisson processes with intensity δ.[10]

2.2 Sequence of events after $t = 0$

To focus the analysis of the model on the possibility of bank runs, and the way the bank and the LLR cope with them, we assume that nothing affects the bank up to date $t = 0$, with no debt canceled prior to that date. At that date, there is a probability $1 - \varepsilon$ that its life continues tranquil forever, and a probability ε that the bank suffers a shock that impairs the quality of its illiquid assets.

We assume that the illiquid assets can be good (g) or bad (b). The final per-unit returns of good and bad assets are a_g or a_b, and their per-unit liquidation returns are q_g and q_b, respectively. In the absence of the shock, assets are good with probability one. But when the shock hits, assets are good with probability μ and bad with probability $1 - \mu$. To focus the analysis on the interesting case in which the efficient continuation decision depends on the quality of the assets and gets compromised by the possibility of runs, we assume:

$$a_b < q_b \leq q_g < 1 < D < B < a_g. \tag{1}$$

This configuration of parameters implies that a good bank that invests only in risky assets ($C = 0$) is *fundamentally solvent* at termination ($a_g > B$), and its assets are worth more if continued than if early liquidated ($a_g > q_g$). In contrast, a bad bank that invests only in risky assets is *fundamentally insolvent* at termination (since $B > 1 > a_b$), and its assets are worth more if early liquidated ($q_b > a_b$). This configuration also implies that the bank (irrespectively of C and of its type) will turn out *de facto insolvent* if sufficiently many debtholders exercise their puts, forcing the liquidation of the illiquid assets (since $D > 1 \geq C + (1 - C)q_i$ for $i = g, b$).

[9] Equivalently, we can think of both assets and debt maturing randomly according to Poisson arrival processes with intensities going to zero, which effectively means that their expected life-spans go to infinity and any other arrival process with positive intensity will arrive earlier on with probability one.

[10] One can think of the putability of bank debt as a feature that under "normal circumstances" allows investors to cease their investment in the bank for idiosyncratic reasons (that the model abstracts from). In those circumstances, the bank would have no problem in simply replacing the exiting debtholders with new debtholders who would buy debt identical to the one canceled. See Segura and Suarez (2014) for a model with this type of recursive refinancing structure.

Now, consistently with (1), guaranteeing that a bank with good assets is fundamentally solvent if it arrives to termination with its liquidity buffers untouched requires having:

$$a_g(1 - C) + C \geq B. \tag{2}$$

This condition imposes an upper bound on C,

$$C \leq \bar{C} = (a_g - B)/(a_g - 1) < 1, \tag{3}$$

which we assume to hold.[11]

Once the shock arrives at $t = 0$, debtholders' decisions regarding the exercise of their put options become non-trivial. If it is optimal for them to start exercising their put options, the bank will begin consuming its cash holdings. Once the bank runs out of cash, the LLR decides whether to support it ($\xi = 1$) or not ($\xi = 0$). If the bank gets supported, all the residual debtholders are paid D.[12] Otherwise, the bank is forced into liquidation as soon as it runs out of cash and its liquidation value gets proportionally divided between the residual debtholders.

When the bank is hit by the shock at $t = 0$, a process of potential revelation of the true quality of its illiquid assets starts. We assume that the arrival of news publicly revealing such quality follows a Poisson process with intensity λ. Under certain conditions (clarified below), learning that the illiquid assets are good at any time before the cash gets exhausted leads the crisis to self-resolve. In particular, an equilibrium can be sustained in which debtholders no longer exercise their puts and the illiquid assets remain (efficiently) continued up to termination. In contrast, when the news is bad, the debtholders find it optimal to continue or to start exercising their puts. The bank eventually runs out of cash, the LLR does not support it (since $a_b < q_b$), and the illiquid assets end up (efficiently) liquidated.

[11] Quite intuitively, a bank that invests too much in cash cannot promise $B > D > 1$ and remain solvent up to termination.

[12] In equilibrium, LLR support is only given when the bank's type remains unknown at the time of the intervention. To justify why debtholders recover just the early recovery D (rather than the termination payoff B), we can assume that market signals about the quality of the illiquid assets become uninformative after the LLR intervenes and that debtholders, afraid of ending up getting less than D at maturity, keep exercising their put options until getting rid of all their debt.

The decision of the LLR is less trivial if the quality of the illiquid assets remains uncertain when the bank runs out of cash. In this scenario, the LLR has to decide by comparing the *expected* continuation value of the illiquid assets, $\bar{a} \equiv \mu a_g + (1-\mu)a_b$, and their *expected* liquidation value, $\bar{q} \equiv \mu q_g + (1-\mu)q_b$. If $\bar{a} > \bar{q}$ (*strong bank case*), the bank is supported, while if $\bar{a} \leq \bar{q}$ (*weak bank case*), it is not. In any of these cases, the implied continuation vs. liquidation decision is, with some probability, less efficient than the one attained if the news on asset quality arrive on time.

2.3 Strategy for the analysis

To simplify the exposition, the core of the analysis is going to focus on the case in which the realization of the shock at $t = 0$ gives rise to an *early run (ER) equilibrium*, a situation in which debtholders start exercising their puts from $t = 0$. After establishing conditions that guarantee the existence of this equilibrium, we will discuss the impact of the bank's liquidity C and the expected intervention of the LLR on equilibrium outcomes.[13]

We will then move backwards, to discuss the trade-offs regarding the choice of the liquidity holdings C at $t = -1$ from the perspective of both the LLR (ex ante social welfare) and the initial owners (ex ante total market value of the bank). To keep the analysis simple, we will treat the capital structure of the bank, as defined by B, D, and δ, as exogenously given throughout the analysis. Yet, when discussing bank owners' decision on C we will take into account the impact of this variable on the issuance value of debt and the residual value of equity.[14]

3 Solving the model

It is convenient to start analyzing what happens when news reveal the type of the bank's illiquid assets during a run (i.e. prior to the exhaustion of the bank's cash). It is also conve-

[13] An alternative equilibrium configuration in which debtholders only start exercising their put options when further news confirm that the illiquid assets of the bank are bad is discussed in Appendix B.

[14] Endogenizing B, D, and δ would require attributing some value to debt financing as well as to the put options represented by $D > 0$ and $\delta > 0$. The literature offers abundant rationales for each of these features, but capturing them here in a fully structural way would blur the essence of our contribution.

nient to get familiar with the role of the Poisson processes in helping us obtain expressions for the time span during which the bank can resist a run that starts at $t = 0$ and for the probability that news arrive prior to the point in which the cash gets exhausted.

3.1 News and ex post efficiency

We want to show that the arrival of good news during a run stops the run, whereas the arrival of bad news implies that the bank gets liquidated once it fully consumes its cash. This implies that the arrival of news induces ex post efficient outcomes regarding the continuation vs. liquidation of the bank's illiquid assets.

Let the good news arrive at some date $t > 0$ in which the residual fraction of bank debtholders is n_t and the available cash is $C_t = C - (1 - n_t)D \geq 0$ (which reflects that a fraction $1 - n_t$ of the initial debt has been canceled using cash). Then, if the run stops at t, the terminal value of assets is $a_g(1 - C) + [C - (1 - n_t)D]$, while the residual debt promises to pay $n_t B$ at termination. Now, we can establish the following chain of inequalities:

$$a_g(1 - C) + [C - (1 - n_t)D] > B - (1 - n_t)D = n_t B + (1 - n_t)(B - D) > n_t B, \quad (4)$$

where the first inequality follows from (2) (we are just subtracting the consumed cash from both sides of it) and the second inequality derives from having $B > D$, as established in (1).

Equation (4) means that, insofar as the bank can accommodate the run using its cash, the bank with good assets remains fundamentally solvent and a Nash equilibrium in which residual debtholders do not exercise any further put is sustainable after the good news. Specifically, waiting to be paid B at termination rather than recovering D prior to termination is a best response for any individual debtholder who expects no other debtholder to exercise her put.[15]

[15] In the absence of a LLR, a second subgame perfect Nash equilibrium might also exist, based on the self-fulfilling prophecy that debtholders' run continues and the bank is forced to liquidate its assets. This is because, as in e.g. Diamond and Dybvig (1983), liquidating the illiquid assets produces insolvency. However, in our setup the possibility of such an equilibrium is removed by the expectation that, if the occasion arrived, the LLR would support the bank whose assets are known to be good. Eventually, then, the run stops as soon as the good news arrive, the LLR intervention is unneeded on the equilibrium path, and the bank can preserve any cash available when the news arrive.

Upon the arrival of bad news, the situation is more straightforward. The inequalities contained in (1) imply that the bank with bad assets is insolvent both if early liquidated and if continued, and irrespective of the available cash or the fraction of residual debtholders. Moreover, all agents anticipate that the LLR will not support the bank. Debtholders with the opportunity to recover D before the bank exhausts its cash find it optimal to do so because, as shown in detail in subsection 3.3, the payoff to residual debtholders at liquidation will be lower than D.

In sum,

Proposition 1 *When (3) holds, the arrival of good news during the early run stops the run, allowing the illiquid assets of the bank to continue up to termination. In contrast, the arrival of bad news does not stop the run and leads to the full liquidation of the bank once its cash gets exhausted.*

3.2 How long will the bank resist a run?

Suppose debtholders start exercising their puts immediately after the shock realizes at date 0 and assume that no good news arrive that interrupt the run. Let n_t denote as before the fraction of debtholders who have not exercised their put options by an arbitrary date $t \geq 0$. Since the opportunities to exercise the puts arrive among debtholders as independent Poisson processes with intensity δ, the dynamics of n_t is driven by

$$\dot{n}_t = -\delta n_t, \tag{5}$$

with the initial condition $n_0 = 1$. Integrating in (5) implies $n_t = \exp(-\delta t)$. So the bank will exhaust its cash at the date τ such that $(1 - n_\tau)D = C$, that is, when

$$[1 - \exp(-\delta\tau)]D = C. \tag{6}$$

Solving for τ yields the following result:

Proposition 2 *Once a run starts, the bank can resist it without assistance for a maximum time span of length*

$$\tau \equiv -\frac{1}{\delta}\ln(\frac{D-C}{D}), \tag{7}$$

which is greater than zero for $C > 0$, increasing in C, and decreasing in δ and D.

3.3 How much is recovered when the bank gets liquidated?

The bank is liquidated when its cash gets exhausted and the LLR does not support it ($\xi = 0$). At liquidation, the value of the bank's assets is $q_i(1-C)$, where $i = g, b$ denotes their quality, and the fraction of residual debtholders is $n_\tau = \exp(-\delta\tau) = (D-C)/D$ as already explained above. So the amount recovered by each residual debtholder, conditional on asset quality i, can be written as

$$Q_i = \frac{q_i\,(1-C)}{n_\tau} = \frac{q_i\,(1-C)\,D}{D-C} < D, \tag{8}$$

where the last inequality follows from having $q_i < 1$ and $D > 1$, by (1).

Thus the payoff received by the fraction $1 - n_\tau = 1 - \exp(-\delta\tau) = C/D$ of debtholders who manage to recover D prior to liquidation is strictly larger than the payoffs of those trapped at the bank when liquidated. This explains why the former will prefer to exercise their put options whenever the probability that the bank ends up liquidated is sufficiently large.

In the context of a run, whether debtholders manage to get paid D or Q_i is just a matter of luck. But from the perspective of the date at which the run starts, the expected payoffs accruing to debtholders, conditional on the quality of the illiquid assets being i and the bank ending up liquidated, can be computed as the weighted average:

$$[1 - \exp(-\delta\tau)]D + \exp(-\delta\tau)Q_i = C + q_i\,(1-C)\,, \tag{9}$$

which, quite intuitively, equals the total value of bank assets conditional on liquidation.[16]

4 The early run equilibrium

We define the *early run equilibrium* as the subgame perfect Nash equilibrium of the game that starts after the economy gets hit by a shock at $t = 0$ in which all debtholders exercise their put options as soon as they have the occasion to do so. In this equilibrium, the logic

[16] (9) obtains directly from (7) and (8).

pushing debtholders to take D whenever possible is that D is higher than the corresponding value of waiting for the next occasion, if any, to get back D (and deciding optimally again in such a case).

Let $V_t^{ER}(C)$ denote a residual debtholder's value of not exercising the put option at date $t \in [0, \tau]$ when the bank's *initial* cash holding is C, when no news have yet revealed the quality of the illiquid assets, and when in all subsequent opportunities residual debtholders (including the debtholder deviating at this point) are assumed to exercise their puts unless good news stop the run. Sustaining an early run equilibrium requires having $V_t^{ER}(C) \leq D$ for all $t \in [0, \tau]$, so that recovering D if having the occasion to do so is a debtholder's best response to the strategies followed by the subsequent players in the game (debtholders who have not yet canceled their debt and the LLR).

The reasoning that may lead to having $V_t^{ER}(C) \leq D$ is a combination of what explains why a debtholder might find it profitable to recover D even if no other debtholders were trying to subsequently recover D (a *fundamental run*), the logic of a *dynamic run* a la He and Xiong (2012) (where each debtholder's incentive to run is reinforced by the fear that, if subsequent debtholders are also early runners, the bank will be consuming its cash and the chances to recover D at a later date will be declining), and distortions to that logic that come from the potential support received from the LLR.

In the weak bank case ($\bar{a} \leq \bar{q}$), debtholders anticipate that the uninformed LLR will not support the bank and the He and Xiong (2012) effect unambiguously reinforces debtholders' incentives to run. However, in the strong bank case ($\bar{a} > \bar{q}$), the expectation of support from the uninformed LLR creates a countervailing effect: the bank is more likely to be supported the closer the bank is to exhaust its cash (since this makes less likely the potential revelation that its assets are bad). So the expectation of being supported may increase as time passes and the bank's residual cash approaches zero. Yet, there are parameter configurations for which $V_t^{ER}(C) \leq D$ for all $t \in [0, \tau]$, so that the ER equilibrium exists. And we will focus on them.

As shown in detail in the proof of the following proposition, in order to find out the expression for $V_t^{ER}(C)$, it is convenient to think of it as the weighted average, using weights

μ and $1-\mu$, of the expected payoffs that a debtholder not exercising her put option at date t would obtain conditional on the illiquid assets of the bank being good and bad, respectively. The result is the following:

Proposition 3 *A residual debtholder's value of not exercising the put option at some date $t \in [0, \tau]$ during an early run can be written as follows*

$$\begin{aligned} V_t^{ER}(C) &= D + \mu[1 - \exp(-(\delta+\lambda)(\tau - t))]\frac{\lambda}{\delta+\lambda}(B - D) \\ &\quad - \exp(\delta t)\{\mu \exp(-\lambda(\tau - t))[D - C - q_g(1 - C)] + (1 - \mu)[D - C - q_b(1 - C)]\} \\ &\quad + \xi \exp(-\lambda(\tau - t)) \exp(\delta t)[D - C - \bar{q}(1 - C)]. \end{aligned} \tag{10}$$

Equation (10) reflects that the holder of one unit of debt during an early run does not always recover D. Specifically, its second term says that if the assets are good and the news come on time, the debtholder recovers B instead of D. The third term says that, if the debtholder gets trapped at the bank and the illiquid assets end up liquidated, her payment is lower than D. The sub-term multiplied by μ reflects that, if the illiquid assets are good, liquidation only happens if no news arrive prior to date τ (and no LLR support is received at τ). The sub-term multiplied by $1-\mu$ reflects that, in contrast, a bad bank not supported by the LLR will get liquidated irrespectively of the possible arrival of news prior to date τ. The last term in (10) captures the gains, relative to the liquidation payoffs that we have just described, associated with receiving LLR support ($\xi = 1$) at date τ.

It is easy to check that, in the weak bank case ($\xi = 0$), $V_t^{ER}(C)$ is decreasing in t. So having $V_t^{ER}(C) \leq D$ for all $t \in [0, \tau]$ only requires having $V_0^{ER}(C) \leq D$. But in the strong bank case ($\xi = 1$), the subsidy associated with LLR support introduces a countervailing effect: the subsidy gets more likely to materialize the closer t gets to τ. This can potentially generate situations in with $V_0^{ER}(C) \leq D$ but $V_t^{ER}(C) > D$ at some later $t < \tau$. For simplicity we will focus the core of our analysis on parameters for which $V_t^{ER}(C) \leq D$ for all $t \in [0, \tau]$, so that an ER equilibrium exists. Alternative configurations of equilibrium are further discussed in Appendix B.

5 Welfare and optimal liquidity holdings

Assessing ex ante welfare in the early run equilibrium, $W_{-1}^{ER}(C)$, is equivalent to properly accounting for the returns that the bank's initial assets end up producing over the various possible courses of events that the bank can follow. Building on the analysis that led us to obtain an expression for $V_t^{ER}(C)$ in Proposition 3, we obtain the following result:

Proposition 4 *The ex ante welfare associated with the early run equilibrium is*

$$\begin{aligned} W_{-1}^{ER}(C) &= C + \{[1-\varepsilon(1-\mu)]a_g + \varepsilon(1-\mu)q_b\}(1-C) \\ &\quad -\varepsilon\exp(-\lambda\tau)[\mu(1-\xi)(a_g-q_g) + (1-\mu)\xi(q_b-a_b)](1-C), \end{aligned} \tag{11}$$

where $\exp(-\lambda\tau) = ((D-C)/D)^{\lambda/\delta}$ *by (7).*

The first two terms in (11) represent the returns that the bank would generate in a full information scenario in which its illiquid assets were continued or liquidated according to the ex post most efficient rule (that is, depending on whether they are good or bad, respectively). The third term represents the deadweight losses due to the uninformed nature of the decision made by the LLR when the bank exhausts its cash at date τ and no news on the quality of the illiquid assets has been received. In our model, consistent with Bagehot's doctrine, LLR support ($\xi = 1$) is welfare enhancing if the illiquid assets are good and welfare reducing if they are bad. But, in the absence of news about asset quality by date τ, the LLR decision involves either type I error (good assets are liquidated) or type II error (bad assets are not liquidated). As reflected in (11), type I error occurs, with a cost proportional to $a_g - q_g > 0$, in the weak bank case ($\xi = 0$), while type II error occurs, with cost proportional to $q_b - a_b > 0$, in the strong bank case ($\xi = 1$).

Is there a social value to postponing the LLR support decision? The quick answer is yes. To see this, consider a notional ceteris paribus increase τ. Such change would reduce the absolute size of the third term of $W_{-1}^{ER}(C)$ (which is negative) and, thus, be good for welfare. Intuitively, it would increase the probability that news arrive prior to date τ and reduce the type I or II errors potentially associated with the otherwise uninformed decision of the LLR.

The right answer, however, requires an important qualification. In our setup, τ can only be increased by increasing C, which implies forgoing part of the bank's investment in illiquid assets, which is its only potential source of strictly positive net present value.[17]

To formally analyze the dependence of $W_{-1}^{ER}(C)$ with respect to C, it is convenient to rewrite it as

$$W_{-1}^{ER}(C) = C + A_H(1-C) - A_L\left(\frac{D-C}{D}\right)^{\lambda/\delta}(1-C) \tag{12}$$

where $A_H = [1-\varepsilon(1-\mu)]a_g + \varepsilon(1-\mu)q_b$, $A_L = \varepsilon[\mu(1-\xi)(a_g - q_g) + (1-\mu)\xi(q_b - a_b)]$, and $((D-C)/D)^{\lambda/\delta}$ replaces $\exp(-\lambda\tau)$. Intuitively, $A_H - 1$ can be interpreted as the *fundamental net present value* of illiquid assets at $t = -1$ (the net present value that they would generate under efficient full-information decisions on continuation vs. liquidation), which must be positive for the investment in the bank to be a source of social surplus.[18]

We can prove the following result:

Proposition 5 *The ex ante welfare associated with the early run equilibrium, $W_{-1}^{ER}(C)$, is a strictly concave function of C, which, depending on parameters, may be increasing or decreasing at $C = 0$. If it is decreasing at $C = 0$, $W_{-1}^{ER}(C)$ is maximized with $C^* = 0$. If it is strictly increasing at $C = 0$, $W_{-1}^{ER}(C)$ is maximized with some unique $C^* \in (0, \bar{C}]$.*

As shown in the proof of Proposition 5, having strictly positive optimal cash holdings, $C^* > 0$, requires the net present value of the assets of the bank under the liquidation policy induced with $C = 0$, which is $A_H - A_L - 1$, to be small relative to the losses, A_L, that can be avoided by having enough time to obtain the relevant information during a run. The effectiveness of cash holdings as a means for gaining the relevant information is, at $C = 0$, directly proportional to $\lambda/\delta D$. This suggests that, ceteris paribus, liquidity holdings make more sense in situations in which the rate of arrival of information during a run is high relative

[17] Mathematically, τ could also be reduced, without affecting other terms in (11), by reducing D or δ, which we are treating as exogenously fixed parameters. As already mentioned in Footnote 14 such treatment is justified by the fact that the model does not attribute, for simplicity, any social value to the putability of bank debt. So, mathematically speaking, setting $C = D = 0$ or $C = \delta = 0$ would trivially maximize $W_{-1}^{ER}(C)$ but it would be inadequate to conclude that our model really justifies prescribing that D or δ should be zero.

[18] Otherwise, $W_{-1}^{ER}(C)$ would be trivially maximized at $C = 1$, where $W_{-1}^{ER}(1) = 1$.

to the rate at which debt gets canceled. While liquidity holdings do not necessarily enhance the social surplus generated by the bank in our model, there are parameters configurations under which they definitely do so.

5.1 Total market value and the need for liquidity standards

Before turning to numerical examples that illustrate the possibility of having an interior welfare maximizing value of C, it is worth clarifying the relationship between ex ante welfare $W_{-1}^{ER}(C)$ and the ex ante total market value of the bank, $TMV_{-1}^{ER}(C)$, which will be the driver of the decision on C of the bank's initial owners in the absence of regulation. As further described below, $TMV_{-1}^{ER}(C)$ is made up of the market value of the debt and the equity issued by the bank at $t = -1$. Bank debt is assumed to be competitively priced by the risk neutral investors under each choice of C by the bank, the (given) values of the parameters B, D and δ that describe the putable debt contract, and the anticipated course of events in subsequent stages of the game. Bank equity is also competitively priced by investors at $t = -1$ and is a residual claim that entitles its holders to receive the part of the total expected cash flows generated by bank assets which is not paid out to the debtholders.

In the weak bank case ($\xi = 0$), the LLR never intervenes on the equilibrium path and, thus, there are no subsidies from its potential support of the bank. So all the returns valued in $W_{-1}^{ER}(C)$, and nothing more than them, get distributed to the bank security holders either through payoffs to debtholders (as discussed in prior sections) or as residual payoffs to equity holders. So, by standard corporate finance arguments, the bank's initial owners appropriate the full expected value of all these payoffs when selling debt and equity to investors, implying $TMV_{-1}^{ER}(C) = W_{-1}^{ER}(C)$. Therefore, in the weak bank case, the initial owners fully internalize any potential net social gains associated with their choice of C at $t = -1$ and there is no obvious rationale for imposing C^* by means of regulation.

In the strong bank case ($\xi = 1$), the only (but crucial!) difference is that LLR support, which occurs when the early run takes place and no news arrive prior to date τ, leaves a net subsidy of value $(D - C) - a_b(1 - C) > 0$ if the illiquid assets of the bank are bad.[19] So the

[19] When the assets are good, the implicit assumption is that the LLR advances $D - C$ to facilitate the

total market value of the bank can be generally written as

$$TMV_{-1}^{ER}(C) = W_{-1}^{ER}(C) + \xi\varepsilon \exp(-\lambda\tau)(1-\mu)[(D-C) - a_b(1-C)], \tag{13}$$

where the term multiplied by ξ is decreasing C, both because τ increases with C (more cash prolongs the time over which the strong bank can resist a run) and because the net subsidy received when the illiquid assets are bad is also decreasing in C (since $a_b < 1$). Hence, the marginal value of liquidity holdings is lower for the owners of the strong bank than for an ex ante social welfare maximizer.

In fact, as shown in the proof of the following proposition, the value of the subsidy term (which is decreasing in C) under $\xi = 1$ turns out to fully offset the information gains that might make $W_{-1}^{ER}(C)$ increasing in C. So in the strong bank case $TMV_{-1}^{ER}(C)$ is strictly decreasing in C.

Proposition 6 *The ex ante total market value of the bank associated with the early run equilibrium, $TMV_{-1}^{ER}(C)$, coincides with $W_{-1}^{ER}(C)$ in the weak bank case ($\xi = 0$), while it is strictly larger than $W_{-1}^{ER}(C)$ and strictly decreasing in C in the strong bank case ($\xi = 1$).*

The fact that $TMV_{-1}^{ER}(C)$ is strictly decreasing in C when $\xi = 1$ has the important implication that if $C^* > 0$, it will be socially optimal to impose a liquidity requirement of the form $C \geq C^*$, which will be binding in equilibrium.[20] Intuitively, the owners of a strong bank anticipate that LLR support will be granted if the bank exhausts its cash prior to the revelation of the quality of its illiquid assets. And they foresee that the payoffs to security holders in such a situation are better than in the alternative situation in which the quality of the illiquid assets is discovered on time, so they choose the lowest possible liquidity.[21]

recovery of D by all the residual debtholders. The LLR eventually recovers just $D-C$ from the continuation value of the good illiquidity assets, which satisfies $a_g(1-C) > D-C$, and the difference $a_g(1-C)-(D-C) > 0$ accrues to the bank equityholders at termination. The size of the implicit subsidy and some of the distortions associated with it would be reduced if the LLR could charge on its lending an interest rate higher than the risk free rate.

[20] In fact any liquidity requirement imposing $C \geq \hat{C}$ with $\hat{C} \in (0, C^*]$ in such a situation, would be binding and would increase ex ante welfare relative to a laissez faire scenario without liquidity regulation.

[21] Notice that, if the assets were discovered to be bad, liquidation would occur and the residual debtholders would be paid less than D.

While not needed for the core of our discussion, the total market value of the bank at $t = -1$ can be easily broken down into the issuance value of debt and the issuance value of equity. In particular, by first principles, the value of debt at $t = -1$ can be written as

$$V_{-1}^{ER}(C) = (1 - \varepsilon)B + \varepsilon V_0^{ER}(C),$$

which uses the fact that debtholders get the full repayment B at termination if the bank is not hit by a shock at $t = 0$, while they obtain expected payments equal to $V_0^{ER}(C)$ otherwise. In turn, the value of equity at $t = -1$, $E_{-1}^{ER}(C)$, can be simply found as a residual:

$$E_{-1}^{ER}(C) = TMV_{-1}^{ER}(C) - V_{-1}^{ER}(C).$$

5.2 Determinants of optimal liquidity holdings

To further understand the key forces influencing the value of C^* in our model, we are going to explore several numerical examples. All examples are generated as variations from a baseline example whose parameters are described in the following table.

Table 1
Parameter values in our baseline example
(one unit of time = one month)

a_g	a_b	q_g	q_b	ε	μ	B	D	δ	λ
1.2	0.5	0.8	0.8	0.2	0.5	1.10	1.05	0.167	2.5

In the baseline example, the bank qualifies as strong (conditional on being hit by the shock at $t = 0$, illiquid assets have an expected continuation value of 0.85, which is larger than their expected liquidation value of 0.80). The value of δ implies a relatively conservative maturity structure, with an average debt maturity of 6 months, while the value of λ implies a relatively rapid revelation of information, with an expected span of 0.4 months (about 12 days).[22]

All panels in Figure 1 depict, as functions of the initial cash holdings C, the ex ante welfare generated by the bank, $W_{-1}^{ER}(C)$, its ex ante total market value, $TMV_{-1}^{ER}(C)$, and the

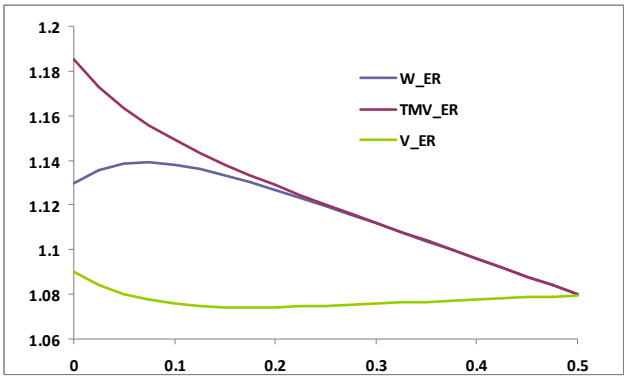
[22] The corresponding expected times intervals can be computed as $1/\delta$ and $1/\lambda$, respectively.

issuance value of its debt, $V_{-1}^{ER}(C)$. By definition, the vertical distance between $TMV_{-1}^{ER}(C)$ and $W_{-1}^{ER}(C)$ reflects the expected value at $t = -1$ of the subsidy associated with LLR support. And the vertical distance between $TMV_{-1}^{ER}(C)$ and $V_{-1}^{ER}(C)$ reflects the issuance value of its equity.

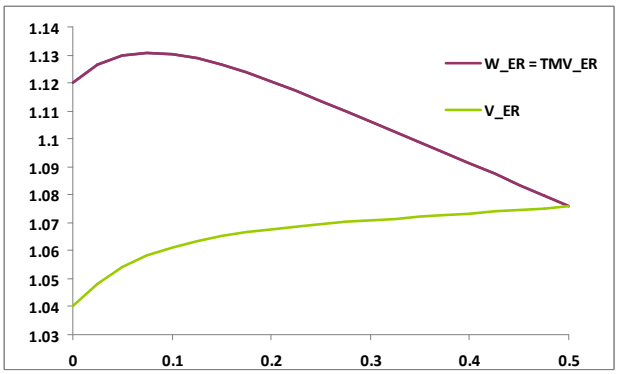
Panel 1 of Figure 1 corresponds to the baseline parameterization, where the socially optimal liquidity holdings C^* are positive (equal to about 7.5% of total assets, which implies a capacity to resist the run without support for about 0.44 months). The curve representing $TMV_{-1}^{ER}(C)$ confirms that bank owners would like to set $C = 0$, which, relative to the situation with $C = C^*$, would allow them to appropriate additional market value (due to the LLR subsidy) equivalent to over 2% of the bank's initial assets. If a regulatory liquidity standard imposes $C = C^*$, the issuance values of equity and debt decline, relative to the laissez faire situation, but net social welfare increases.

Panel 2 of Figure 1 describes what happens if the probability of bank assets being good after the shock, μ, is 0.4 rather than 0.5. Such variation is sufficient to make the bank weak, so that the LLR does not find it optimal to blindly support the bank when it runs out of cash. In this case, there is no subsidy associated with LLR support and $TMV_{-1}^{ER}(C)$ and $V_{-1}^{ER}(C)$ coincide. The socially optimal liquidity holdings C^* are very similar to those in Panel 1 but in this case there is no need to impose them through regulation. Notice also the dependence of the ex ante value of bank debt with respect to C is different than in Panel 1. The time bought by C during the run increases the chances that the assets are reveled to be good and the run self resolves instead of eventually leading to the liquidation of the bank.

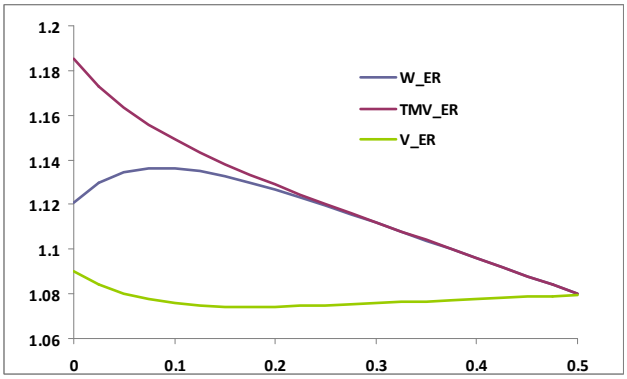
Back to the strong bank case, Panel 3 shows what happens when the continuation return of the bad assets, a_b, is 0.41 rather than 0.5. This change increases the inefficiency associated with granting blind liquidity support to the bank once it exhausts its cash and, hence, the value of expanding the time available for the LLR to become informed about the quality of the illiquid assets. Consistent with this, the optimal liquidity holdings C^* increase (to a value close to 10% of assets, which implies a capacity to resist the run of about 0.6 months).



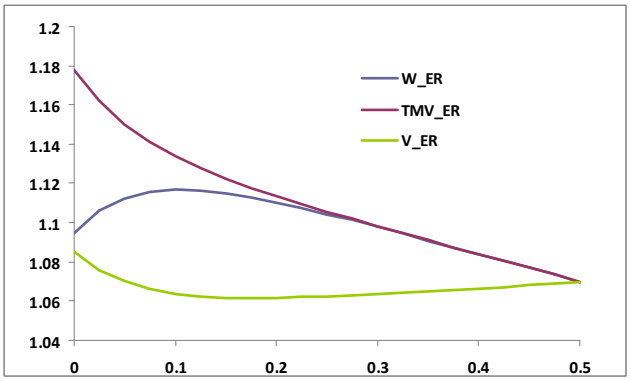
1.Baseline example, strong bank, interior C^*



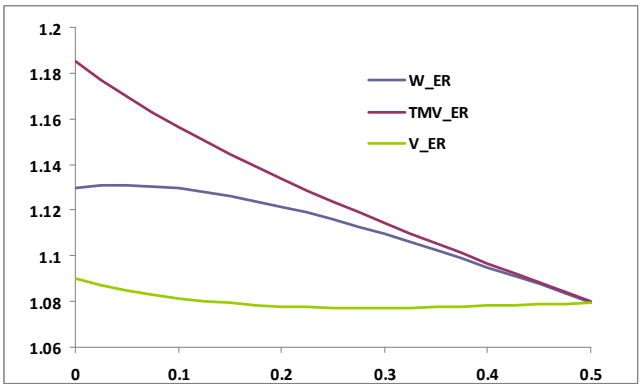
2. Weak bank (μ=0.4), aligned incentives



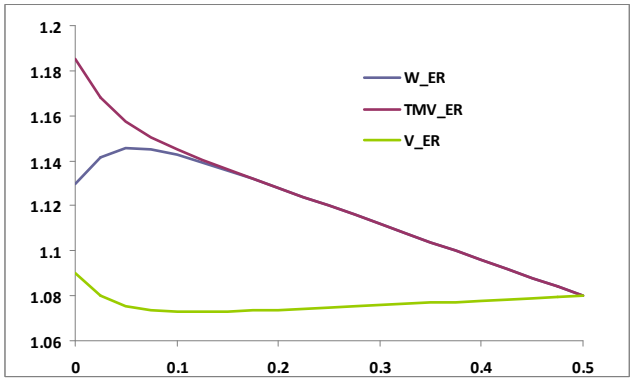
3. Lower a_b (0.41) , higher C^*



4. Higher ε (0.3), higher C^*



5. Higher δ (0.4), $1/\delta = 2.5$, lower C^*



6. Lower δ (0.083), $1/\delta = 12$, lower C^*

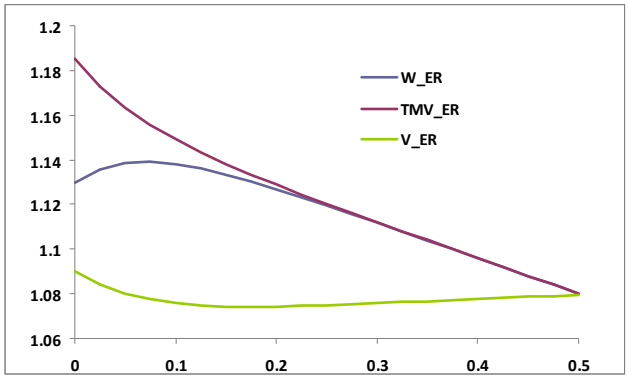
Figure 1 Examples exploring the determinants of optimal liquidity holdings C^*

In Panel 4 we increase the probability ε of the shock that impairs the bank's financial condition from 0.2 to 0.3. This increases the expected losses associated with inefficiencies regarding the continuation versus liquidation of the illiquid assets and increases the value of C^*. Other effects are self-explanatory.

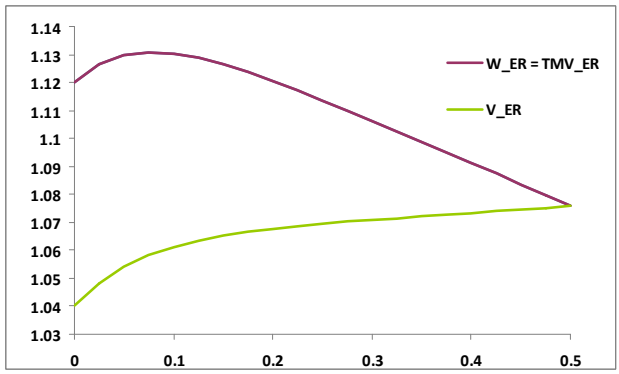
Panels 5 and 6 show the effects of increasing and decreasing, respectively, the size of the rate δ at which debtholders receive the option to put their debt, which in turn determines the speed at which the bank consumes its cash holdings during a run. The results reveal a non-monotonicity. The two illustrated changes lead to a lower value of C^*. In the case with higher δ the result can be rationalized as an implication of the lower cost-effectiveness of C in "buying time" for information on asset quality to arrive before the LLR intervenes (which shows up in the form of a very flat $W_{-1}^{ER}(C)$ around zero). In the case with lower δ, the larger cost-effectiveness brings additional curvature to $W_{-1}^{ER}(C)$ and higher welfare at C^* but also allows C^* to decline relative to Panel 1.

Panels 5 and 6 altogether suggest the existence of a non-trivial relationship between the two regulatory tools with which Basel III proposes to deal with liquidity risk, since its liquidity coverage ratio (LCR) can be interpreted as a requirement in terms of C, while its net stable funding ratio (NSFR) can be interpreted as an attempt to extend the maturity of banks' liabilities (i.e. to induce a lower δ). From a social welfare maximization perspective, they may effectively work as complementary or as substitutes at the margin depending on the level at which they are set.

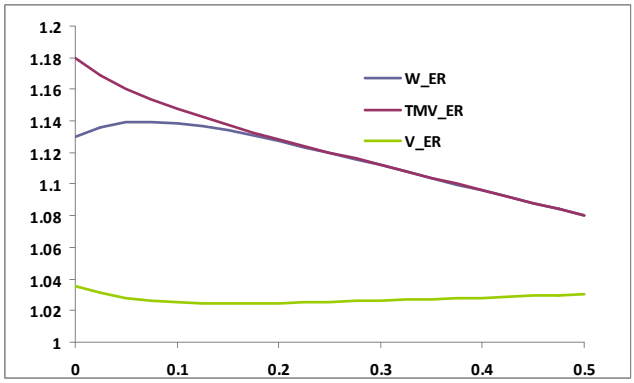
Figure 2 provides a tentative exploration of the substitutability between liquidity standards and capital standards. As previously argued in footnote 14, our model abstracts from attributing a rationale to bank leverage and the putability of bank debt and, hence, is unsuitable for a welfare assessment of the effects of modifying B, D and δ, which we treat as exogenous parameters. Yet we can address the analysis of the extent to which the optimal liquidity standards C^* depend on those parameters (as we already did with δ in the last two panels of Figure 1).



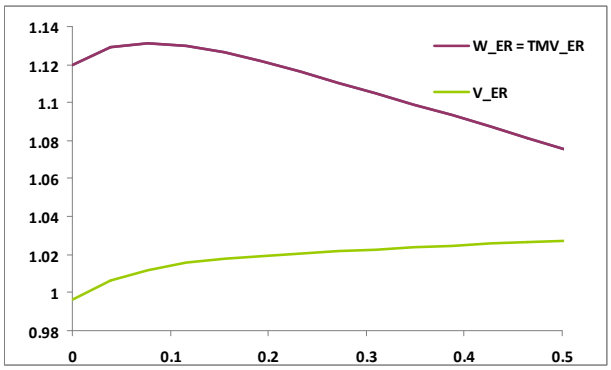
1.Baseline example, strong bank, interior C^*



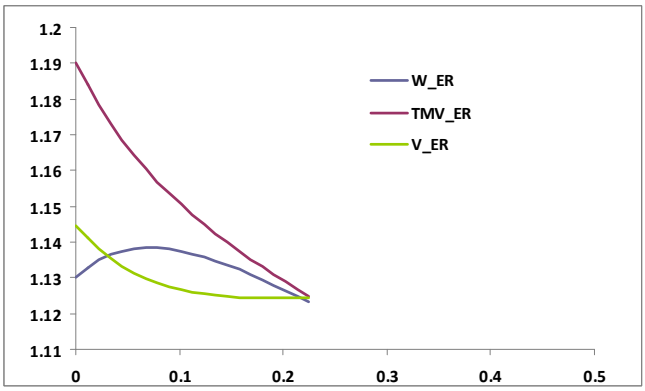
2. Weak bank (μ=0.4), aligned incentives



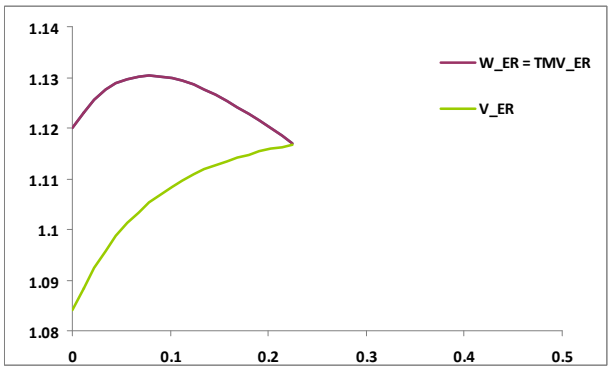
3. Lower leverage (D'/D=B'/B=0.95), $\simeq$same C^*



4. Weak bank, lower leverage, $\simeq$same C^*



5. Higher leverage (D'/D=B'/B=1.05), $\simeq$same C^*



6. Weak bank, higher leverage, $\simeq$same C^*

Figure 2 Examples exploring the interaction with capital standards

Panels 3 and 5 in Figure 2 explore variations in leverage relative to our baseline example, which reproduced again in Panel 1. In Panel 3, we reduce B and D to 95% of their baseline values (D'/D=B'/B=0.95). As one might predict, the change reduces the contribution of debt to $TMV_{-1}^{ER}(C)$ as well as the subsidy associated with LLR support (the gap between $TMV_{-1}^{ER}(C)$ and $W_{-1}^{ER}(C)$). However, the value of the optimal liquidity holdings C^* does not get visibly modified.

In Panel 5, we increase leverage to 105% of its baseline value (D'/D=B'/B=1.05) and obtain changes which are very much the mirror image of those in Panel 3. Again the value of C^* does not get visibly modified.[23]

Panels 4 and 6 repeat the exercise taking the weak bank case depicted in Panel 2 (which is the same as in Figure 1) as a benchmark. Once again, leverage does not seem to have a first order effect on C^*. So, all in all, the message from these examples is that, in the context of our model, the discussion on liquidity standards based on their role in "buying time" for the LLR to get better informed during an early run is somewhat separable from the discussion on capital standards.

However, there are features of our model that call for taking this conclusion with caution. Specifically, the model considers, for simplicity, a dichotomic distribution of asset qualities after the shock that occurs at $t = 0$. And our assumptions guarantee that the bank is fundamentally solvent when the illiquid assets are good and fundamentally insolvent when they are bad. So marginal changes in leverage (or capital standards) do not have a material impact on solvency.

With a continuum of asset qualities after the shock, capital standards would modify the range of realizations of such quality over which the bank remains fundamentally solvent, would modify the value of getting further information about asset quality during a run, and interact less trivially with liquidity standards.

[23] The curves in Panels 5 and 6 cover a lower range of values of C due to the impact of B on the upper limit $\bar{C}$ described in (3).

6 Further discussion

6.1 Can liquidity standards prevent early runs?

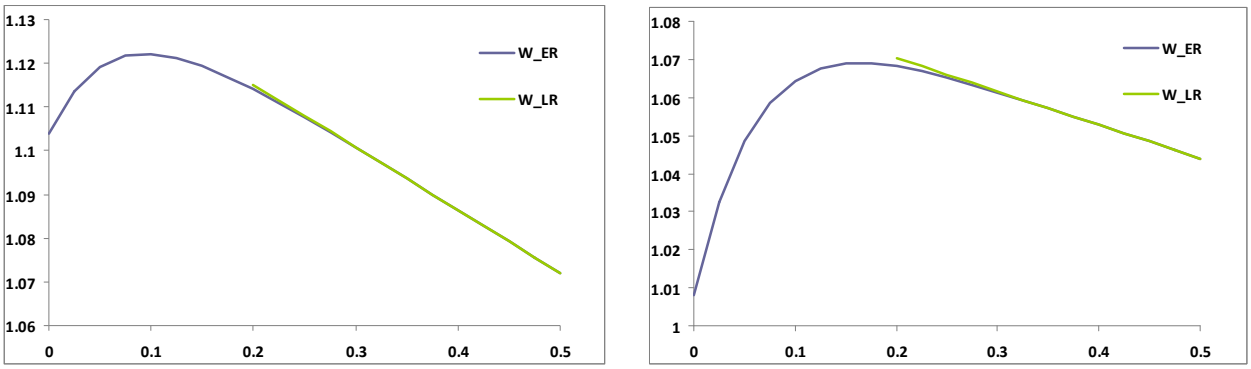
In Appendix B we explore the possibility of sustaining equilibria different from the early run equilibrium analyzed so far. Specifically, we provide the conditions under which the model may sustain a *late run equilibrium* (LR equilibrium) in which debtholders do not start exercising their put options immediately after the shock at $t = 0$ but only if further news reveal that the illiquid assets of the bank are bad. It also shows that, under some parameter configurations, the LR equilibrium is not sustainable with $C = 0$ but only with $C \geq \hat{C}$ (specifically, if the value of $\hat{C}$ defined in (20) is lower than the value of $\bar{C}$ defined in (3)).

Following a discussion that would be long to reproduce here, Panels 3 and 4 in Figure A1 provide an example in which increasing the bank's liquidity from 15% to 30% would lead from a situation in which only the ER equilibrium is sustainable to another in which only the LR equilibrium is sustainable. At first sight this might suggest the desirability of inducing the LR equilibrium by setting C at a sufficiently large value. However, this is not necessarily the case, since large values of C imply forgoing the profitability of the investment in illiquid assets and may generate ex ante welfare lower than the best level attainable with the ER equilibrium, $W_{-1}^{ER}(C^*)$. In fact, with the parameters behind Panels 3 and 4 in Figure A1 and $\varepsilon = 0.20$ (as in our baseline example of prior sections), we have $C^* = 0$ and $W_{-1}^{ER}(C^*) = 1.173$, while even under the best value of C that makes the LR equilibrium sustainable, which is $\hat{C} = 0.184$, we have $W_{-1}^{LR}(\hat{C}) = 1.151$, so inducing the LR equilibrium is not socially optimal.

Further, under the baseline parameterization behind Figure 1, we have $\hat{C} = 1 > \bar{C} = 0.5$ so sustaining the LR equilibrium is unfeasible. However, we can modify several parameters in order to provide an example in which sustaining the LR equilibrium is feasible and possibly superior to the ER equilibrium. The new parameters imply $\hat{C} = 0.2 < \bar{C} = 0.5$ so the LR equilibrium can be sustained with $C \in [0.2, 0.5]$.[24] Panel 1 of Figure 3 depicts $W_{-1}^{ER}(C)$ and

[24] Specifically, we set $a_b = 0$, $q_b = 0.5$, $\mu = 0.8$ and $D = 1$. Other parameters are as further indicated in the text or as in Table 1.

$W_{-1}^{LR}(C)$ for a first variation of this example with $\varepsilon = 0.4$. In this case, we have $C^* = 0.075$ and $W_{-1}^{ER}(C^*) = 1.122 > W_{-1}^{LR}(\hat{C}) = 1.115$, so trying to induce the LR equilibrium by setting $C = \hat{C}$ would actually not be ex ante optimal.



1. The ER equilibrium is preferable (ε=0.4)

2. The LR equilibrium is preferable (ε=0.8)

Figure 3 Examples in which the late run equilibrium can be sustained

In Panel 2 of Figure 3 we explore a second variation with $\varepsilon = 0.8$. In this case, we obtain $C^* = 0.175$ and $W_{-1}^{ER}(C^*) = 1.0689 < W_{-1}^{LR}(\hat{C}) = 1.0704$, which implies that the LR equilibrium induced by $C = \hat{C}$ is socially slightly better than the ER equilibrium sustainable with $C = C^*$.[25]

6.2 Banks' incentives to produce information

In our model, we assume that the arrival of information on the bank's financial condition is exogenous and follows a Poisson process. As we discussed, the rate at which information comes out and the nature of it (whether it is good or bad news) have important implications. For example, the arrival of good news at any time before the bank's cash gets exhausted during an early run will eliminate debtholders' incentives to continue exercising their puts, leading the run to an end. In contrast, when the news is bad, debtholders continue exercising

[25] Under these parameters, however, both the ER and the LR equilibria coexist for $C = \hat{C}$, so its is unclear whether $C = \hat{C}$ suffices to induce the latter (and discussing on equilibrium selection in the presence of multiple equilibria exceeds the scope of this subsection).

their puts and it becomes clear that the LLR will not support the bank once it exhausts its cash. In this context, bank owners are not going to be generally indifferent about whether information gets disclosed, or the rate at which it is disclosed.

To see this, suppose bank owners have the ability to affect the speed at which information gets disclosed during an early run. In the weak bank case, since by default LLR support will not be blindly granted, bank owners will find it advantageous to disclose information about the bank's assets. If the bank is bad, things will not be worse than without the information. But if the bank is good, some extra value can be generated. By contrast, in the strong bank case, bank owners are not interested in accelerating the production of information. In fact, they will try to delay it, since keeping the LLR "blind" is a way to guarantee its support, and appropriate the corresponding implicit subsidy.

These examples show that it would not be efficient to assign banks the responsibility to produce (and disclose) information about their financial condition on real time.

6.3 Systemically important banks

One key feature of systemically important banks (SIBs), especially in the absence of a fully effective regime for the recovery and resolution of too-big-to-fail institutions, is the possibility that their early (and disorderly) liquidation causes significant damage to the rest of the financial system or the wider economy (e.g. in the form of fire sale externalities, contagion, etc.). This suggests that for a LLR dealing with a SIB, the trade-offs relevant for deciding whether to grant liquidity support or not might be driven by considerations beyond the fundamental solvency of the bank (or, in model terms, the intrinsic quality of its illiquid assets). One important consideration is the size of the *systemic externalities* that might be avoided by supporting the bank. These externalities increase the social value of allowing the bank to continue in operation after it exhausts its cash, as opposed to pushing it into liquidation.

From the perspective of the LLR, variation in the size of these systemic externalities can play the same role as variation in the quality of the illiquid assets in our model. And, from this viewpoint, we could also assimilate non-SIBs to our weak banks (i.e. banks that, in the

absence of further information, would not be supported by the LLR) and SIBs to our strong banks (i.e. banks that, in the absence of further information, would be supported). Hence, it is natural to establish a parallel between the model analyzed in prior sections and a model in which banks in trouble (say, to simplify, with bad illiquid assets) can generate small or large systemic externalities if they fail. In such a setup, liquidity standards would give the LLR time to receive information on the size of the externalities.

A full formal analysis of this alternative framework would require more than a pure re-labeling of the objects present in the current one. Parallel to the current setup, the size of the systemic externalities is relevant for the LLR decision and, through it, for debtholders' expectations on whether the bank will be supported or not. But one important difference is that systemic externalities do not directly affect debtholders' payoffs contingent on continuation so their size being large or small cannot be fully assimilated to the value of illiquid assets being high or low in our model. Hence, the details of several equations would change.

Yet, it is safe to conjecture that non-SIBs will have greater incentives than SIBs to choose liquidity holdings close to those that maximize social welfare, since, by a logic similar to the one explored in our model, the subsidies that they will expect to obtain through the support granted by a blind LLR are lower (if any) than the subsidies that SIBs will expect to obtain. As in our analysis above, the socially optimal liquidity standards would have to trade-off gains from increasing the likelihood that the LLR gets informed about the true systemic importance of the bank and the losses from forcing banks to ex ante forgo potentially more profitable uses of funds.

7 Conclusions

We provided in this paper a novel rationale for banks' liquidity standards, one which builds on the idea that liquidity buffers provide banks with the capability to deal with debt withdrawals for some time before they have to seek support from the LLR. This ability to wait before seeking LLR support is valuable because it allows for the release of information on the bank's financial condition that is useful for the LLR's decision on whether to grant support. Specifically, it generally improves the efficiency of the decisions regarding the continuation

of the bank as a going concern or its liquidation. Liquidity standards can be important for another reason: they reduce investors' incentives to run on the bank following an adverse shock, thereby lowering the bank's roll over risk and the need to seek liquidity support prior to the release of further information about the quality of bank assets.

Our paper also provides some ideas for future research. For example, we have shown that the aforementioned benefits of liquidity standards cannot be trivially mimicked with capital standards. However, out setup is not currently suitable for the analysis of the desirability of capital standards. Given the current coexistence of capital and liquidity regulation, a potential interesting area for future research would be to expand our model in order to explicitly capture the rationale for banks' leverage and maturity transformation and, thus, the trade-offs relevant to investigate the interplay between both regulations.

We have assumed in our model that the arrival of information on the bank's financial condition following a shock is exogenous. However, in general the nature and the speed at which information on the bank's financial condition is produced and disclosed is endogenous and depends on the entity responsible for this activity. Further, as we discussed in the last section, the bank may not have the proper incentives to disclose that information in a timely manner. This provides a rationale for entrusting an agency with the authority to produce information about the bank's financial condition. Importantly, this information would have to be made available not only to the LLR but also to the bank's investors, as it is key for their decision to roll over their debt. Since the disclosure of information affects the LLR's incentives and those of investors differently, an interesting question for future research would be to investigate which agency or agencies should have authority to gather and disclose information on banks' financial condition in real time.[26]

[26] See Kahn and Santos (2006) for a model in which differences in regulatory agencies' mandates induce agencies to hold information from their counterparts.

Appendices

A Proofs

Proof of Proposition 1 The proof is provided by the arguments that precede the statement of the proposition in the main text.■

Proof of Proposition 2 The proof is provided by the arguments that precede the statement of the proposition in the main text and basic calculus.■

Proof of Proposition 3 We structure this proof in three parts. First we find expressions for a debtholder's value of not exercising the put option at some $t \in [0, \tau]$ conditional on bank assets being bad and good, respectively. Then, we put together the corresponding unconditional value of not exercising the put at t so as to arrive to (10).

Part I. Value of not exercising the put conditional on assets being bad We can compute this value as the weighted average over two possible courses of events:

1. News arrive prior to date τ. Since news arrival is a Poisson process with intensity λ, the time span to the arrival of (the next) news, say x, follows and exponential distribution with parameter λ. Thus, the probability that news arrive prior to date τ can be computed as $\Pr(x \leq \tau - t)) = 1 - \exp(-\lambda(\tau - t))$. If news about the bad quality of the illiquid assets arrive prior to date τ, the bank ends up liquidated at date τ. Some lucky debtholders will recover D prior to τ and the remaining ones will obtain $Q_b < D$ at liquidation. Since the arrival of the chance to recover D follows a Poisson process with intensity δ, the probability of having a chance to recover D prior to liquidation is $1 - \exp(-\delta(\tau - t))$, so the expected payoff over this course of events can be written as

$$\begin{aligned} [1 - \exp(-\delta(\tau - t))]\, D + \exp(-\delta(\tau - t))Q_b &= D - \exp(\delta t)\exp(-\delta\tau)(D - Q_b) \\ &= D - \exp(\delta t)[D - C - q_b(1 - C)], \quad (14) \end{aligned}$$

where the last equality is obtained using (9) for $i = b$.

2. News do not arrive prior to date τ. This happens with probability $\exp(-\lambda(\tau - t))$. When the bank runs out of cash and the quality of its assets remains unknown, the LLR decides to support the bank ($\xi = 1$) if the bank is strong ($\bar{a} > \bar{q}$) and not to support it ($\xi = 0$) if it is weak ($\bar{a} \leq \bar{q}$). So debtholders with the opportunity to exercise their puts prior to date τ will obtain D, while the remaining ones will obtain $\xi D + (1 - \xi)Q_b$, and the

expected payoffs over this course of events can be written as

$$\begin{aligned}
&[1-\exp(-\delta(\tau-t))]\,D+\exp(-\delta(\tau-t))\,[\xi D+(1-\xi)Q_L] \\
&= D-(1-\xi)\exp(-\delta(\tau-t))(D-Q_L) \\
&= D-(1-\xi)\exp(\delta t)[D-C-q_b(1-C)], \qquad (15)
\end{aligned}$$

where $\exp(-\delta(\tau-t))$ is, as above, the probability of not having the chance to recover D prior to date τ, and we also use (9) to reexpress the term in $(D-Q_L)$ in the last equality.

Putting together these results, the value of not exercising the put for a residual debtholder at date t conditional on the illiquid assets being bad can be written as

$$V_t^{ER}(C)_{|i=b} = D-[1-\xi\exp(-\lambda(\tau-t))]\exp(\delta t)\,[D-C-q_b(1-C)]\,, \qquad (16)$$

where the term multiplied by ξ captures the contribution of the *subsidy* associated with LLR support in the strong bank case.

Part II. Value of not exercising the put conditional on assets being good The simplest way to obtain an expression for $V_t^{ER}(C)$ conditional on assets being good is also to look at how events may unfold for a typical debtholder who retains her debt at $t=0$. We can distinguish three mutually exclusive courses of events:

1. The debtholder gets the chance to put her debt and obtain D prior to the arrival of news and prior to the exhaustion of the bank's cash. So the debtholder receives D.

2. The news arrive prior to the debtholder having the opportunity to put her debt and prior to the exhaustion of the bank's cash. So the debtholder obtains B by waiting up to termination, since the crisis self-resolves.

3. The bank runs out of cash prior to the debtholder having the opportunity to put her debt and prior to the arrival of news. So the debtholder obtains $\xi D+(1-\xi)Q_g$.

Thus, using the fact that the payment associated with the exhaustion of cash will occur at date τ if none of the other relevant events occurs before that date, and the independent nature of the Poisson processes driving the arrival of these events, we can write:

$$\begin{aligned}
V_t^{ER}(C)_{|i=g} \;=\; & [1-\exp(-(\delta+\lambda)(\tau-t))]\left(\frac{\delta}{\delta+\lambda}D+\frac{\lambda}{\delta+\lambda}B\right)+ \\
& \exp(-(\delta+\lambda)(\tau-t))\,[\xi D+(1-\xi)Q_g]\,.
\end{aligned}$$

The factors 1–$\exp(-(\delta+\lambda)\tau)$ and $\exp(-(\delta+\lambda)\tau)$ are explained by the fact that if two Poisson processes arrive independently with intensities δ and λ, the arrival of the first of them is a

Poisson process with intensity $\delta + \lambda$, and the corresponding span to such an arrival follows an exponential distribution with parameter $\delta + \lambda$. So $\exp(-(\delta + \lambda)\tau)$ is the probability that no first event occurs by date τ and $1 - \exp(-(\delta + \lambda)\tau)$ is the probability that at least one event arrives. The factors $\delta/(\delta + \lambda)$ and $\lambda/(\delta + \lambda)$ describe the probabilities with which the first event is the option to exercise the put and the arrival of (good) news, respectively.

Isolating D and using (9) to write $\exp(-\delta(\tau - t))(D - Q_g)$ as $\exp(\delta t)[D - C - q_b(1 - C)]$, we obtain

$$\begin{aligned} V_t^{ER}(C)_{|i=g} &= D + [1 - \exp(-(\delta + \lambda)(\tau - t))]\frac{\lambda}{\delta + \lambda}(B - D) \\ &\quad -(1 - \xi)\exp(-\lambda(\tau - t))\exp(\delta t)[D - C - q_g(1 - C)], \end{aligned} \tag{17}$$

which reflects that, conditional on bank assets being good, the residual debtholders at time t do not always end up recovering D during the early run. They gain the additional amount $B - D > 0$ if the good news arrive on time (so that they can wait until termination) and they incur an additional expected loss $\exp(\delta t)[D - C - q_g(1 - C)]$ if the bank is weak ($\xi = 0$) and runs out of cash prior to the revelation of the quality of its assets.

Part III. Unconditional value of not exercising the put in an early run Putting together expressions (16) and (17), we obtain the unconditional value of one unit of residual bank debt during an early run as reported in (10).■

Proof of Proposition 4 Ex ante welfare can be calculated as the expected value of the overall asset returns that the bank generates over all the possible courses of events, which can be described as follows:

1. No shock occurs at $t = 0$. This occurs with probability $1 - \varepsilon$. The bank assets are good and never liquidated. The bank generates returns $C + a_g(1 - C)$.

2. The shock occurs at $t = 0$ and the run starts. This occurs with probability ε.

   (a) The illiquid assets are bad. This happens with (conditional) probability $1 - \mu$.

      i. News arrive prior to date τ. This occurs with (conditional) probability $1 - \exp(-\lambda\tau)$. The bank ends up liquidated, so its overall asset returns are $C + q_b(1 - C)$.

      ii. News do not arrive prior to date τ. This occurs with (conditional) probability $\exp(-\lambda\tau)$. The bank ends up liquidated in the weak bank case ($\xi = 0$) and

continued in the strong bank case ($\xi = 1$), so its overall asset returns are $C + [q_b - \xi(q_b - a_b)](1 - C)$.

(b) The illiquid assets are good. This happens with (unconditional) probability μ.

i. News arrive prior to date τ. This occurs with (conditional) probability $1 - \exp(-\lambda\tau)$. The bank continues up to termination, so its overall asset returns are $C + a_g(1 - C)$.

ii. News do not arrive prior to date τ. This occurs with (conditional) probability $\exp(-\lambda\tau)$. The bank ends up liquidated in the weak bank case ($\xi = 0$) and continued in the strong bank case ($\xi = 1$), so its overall asset returns are $C + [q_g + \xi(a_g - q_g)](1 - C)$.

Putting together these payoffs and after some algebra, we obtain the expression reported in (11).■

Proof of Proposition 5 From (12), it is a matter of algebra to check that the first and second derivatives of $W_{-1}^{ER}(C)$ with respect to C can expressed as

$$\frac{dW_{-1}^{ER}(C)}{dC} = -(A_H - 1) + A_L \left(\frac{D - C}{D}\right)^{\lambda/\delta} \left[1 + \frac{\lambda(1 - C)}{\delta(D - C)}\right],$$

$$\frac{d^2W_{-1}^{ER}(C)}{dC^2} = -A_L \left(\frac{D - C}{D}\right)^{\lambda/\delta} \frac{\lambda}{\delta^2(D - C)^2}[\delta(D - 1) + \delta(D - C) + \lambda(1 - C)],$$

where the sign of the first is ambiguous, while the sign of the second is strictly negative. So $W_{-1}^{ER}(C)$ is strictly concave in C. If it is strictly increasing at $C = 0$, i.e.

$$\frac{\lambda}{\delta D} A_L > A_H - A_L - 1, \tag{18}$$

then $W_{-1}^{ER}(C)$ must reach a maximum over the interval $[0, \bar{C}]$ at some point $C^* > 0$. Such point must be unique because $W_{-1}^{ER}(C)$ is strictly concave in C. By the same token, if (18) does not hold, $W_{-1}^{ER}(C)$ reaches its maximum at $C^* = 0$.■

Proof of Proposition 6 Most of the results in this proposition are proven by the arguments already included in the main text, prior to the proposition. It remains to be proven that $TMV_{-1}^{ER}(C)$ is strictly decreasing in C when $\xi = 1$. To see this, let us rewrite the expression

in (13) using (12) and $\exp(-\lambda\tau) = ((D-C)/D)^{\lambda/\delta}$:

$$\begin{aligned} TMV_{-1}^{ER}(C) &= C + A_H(1-C) - A_L\left(\frac{D-C}{D}\right)^{\lambda/\delta}(1-C) \\ &\quad +\xi\varepsilon\left(\frac{D-C}{D}\right)^{\lambda/\delta}(1-\mu)[(D-C) - a_b(1-C)]. \end{aligned}$$

But with $\xi = 1$, we have $A_L = \varepsilon(1-\mu)(q_b - a_b)$, so the last two terms of the above expression can be grouped together, yielding

$$\begin{aligned} TMV_{-1}^{ER}(C) &= C + A_H(1-C) + \varepsilon(1-\mu)\left(\frac{D-C}{D}\right)^{\lambda/\delta}[(D-C) - a_b(1-C) \\ &\quad -(q_b - a_b)(1-C)] \\ &= C + A_H(1-C) + \varepsilon(1-\mu)\left(\frac{D-C}{D}\right)^{\lambda/\delta}[(D-C) - q_b(1-C)], \end{aligned}$$

which is strictly decreasing in C since $A_H > 1$ and $q_b < 1$.■

Proof of Proposition 7 The proof is provided by the arguments that precede the statement of the proposition in the main text.■

Proof of Proposition 8 Proposition 7 implies that sustaining the LR equilibrium requires a minimal C of either 0, if $V_0^{LR}(0) \geq D$, or some $\hat{C} \in (0, \bar{C})$, if $V_0^{LR}(0) < D \leq V_0^{LR}(\bar{C})$. Assume then that the LR equilibrium arises whenever $C \geq \max\{\hat{C}, 0\}$, where $\hat{C}$ is given by (20). However, under $A_H - 1 > 0$, $W_{-1}^{LR}(C)$ is decreasing in C. So setting C strictly larger than $\max\{\hat{C}, 0\}$ would be detrimental to welfare.■

Proof of Proposition 9 Given the absence of subsidies associated with LLR support, we have $TMV_{-1}^{LR}(C) = W_{-1}^{LR}(C)$ and the result follows trivially from the arguments provided in the proof of Proposition 8.■

B The late run equilibrium

We denote a *late run equilibrium* (or LR equilibrium) the subgame perfect Nash equilibrium that begins after the shock arrives at $t = 0$ in which debtholders only start exercising their puts if further news confirm that the bank's illiquid assets are bad. In this equilibrium, the arrival of good news allows the bank to end the crisis with its liquidity untouched.

In the LR equilibrium the situation of the bank only changes when the news come. So, if it is not a profitable deviation for an individual debtholder to exercise her put at $t = 0$, then it will not be a profitable deviation either at any other point before news arrive. But news, on the other hand, arrive in finite time with probability one, revealing the bank to be good with probability μ and bad with probability $1 - \mu$. So debtholders' value of not exercising their put in the late run equilibrium can be written as

$$V_0^{LR}(C) = \mu B + (1 - \mu)\left[C + q_b(1 - C)\right], \tag{19}$$

reflecting that debtholders are eventually paid B if the bank is good (recall that there is no discounting) and receive an expected payoff $C + q_b(1 - C)$ if the bank is bad (recall (9)).

Sustaining an equilibrium with late runs requires having $V_0^{LR}(C) \geq D$, so the following proposition can be proven by direct inspection of the relevant expressions.

Proposition 7 *A LR equilibrium exists if and only if $V_0^{LR}(C) \geq D$. Such condition holds when the bank is sufficiently likely to be good. When $V_0^{LR}(0) \geq D$, the LR equilibrium exists even with $C = 0$. When $V_0^{LR}(0) < D \leq V(\bar{C})$, there exist a minimum liquidity standard*

$$\hat{C} = \frac{D - \mu B - (1 - \mu)q_b}{(1 - \mu)(1 - q_b)} \in (0, \bar{C}] \tag{20}$$

such that the LR equilibrium is only sustainable if $C \geq \hat{C}$.

Hence the holding of (moderate amounts of) liquidity can facilitate the sustainability of the late run equilibrium. It can do so by enhancing the value of the bank when its illiquid assets are bad, which in turn increases debtholders's payoff from waiting for news. In other words, cash reassures debtholders about the value of their stake at the bank and makes them willing to delay the exercise of their option to run.

From an allocational perspective, making the debtholders effectively more patient during a crisis contributes to "buying" the time needed for the arrival of news that, eventually,

facilitate an efficient resolution of the crisis, in that the bank with good assets continues and the bank with bad assets is liquidated.[27]

What is the connection between liquidity and LLR support in the LR equilibrium? On the one hand, by facilitating the sustainability of the LR equilibrium, liquidity may contribute to actually make LLR support unneeded on the equilibrium path. On the other, the LLR's willingness to support the bank when its assets are known to be good rules out the possibility of self-fulfilling prophecies that might precipitate the start of a run at date $t = 0$ and lead it not to stop even after news indicating that assets are good.

B.1 Welfare and firm value in the late run equilibrium

Let us first consider the case in which $D \leq V_0^{LR}(\bar{C})$, which means that the LR equilibrium can be sustained by choosing a suitable value of C. And suppose that C is set at a value that indeed sustains the LR equilibrium. How large is the welfare generated by the bank in these circumstances?

We measure the ex ante welfare associated with this equilibrium, $W_{-1}^{LR}(C)$, as the expected value of the overall payoffs generated by the bank from t=–1 onwards, that is, the returns produced by its initial assets across possible states. Using the fact that, in the LR equilibrium, good illiquid assets get continued up to termination, while bad assets get early liquidated, we obtain

$$W_{-1}^{LR}(C) = C + \{[(1-\varepsilon) + \varepsilon\mu]a_g + \varepsilon(1-\mu)q_b\}(1-C)$$

or, in terms of the notation introduced in (12),

$$W_{-1}^{LR}(C) = C + A_H(1-C),$$

where $A_H - 1$ was referred to as the fundamental net present value potentially associated with the bank's investment in illiquid assets. Importantly, $W_{-1}^{LR}(C)$ is linear in C, and strictly decreasing in C if an only if $A_H - 1 > 0$. Therefore:

Proposition 8 *If the illiquid assets have strictly positive fundamental net present value at t=–1, and the LR equilibrium can be sustained, it is not socially optimal to set C strictly larger than* $\max\{\hat{C}, 0\}$, *where $\hat{C}$ is given by (20).*

[27] Strictly speaking, the bank with bad assets continues up to the exhaustion of its cash. Alternatively, we could assume that a resolution authority forces the bank into liquidation as soon as it is learned to be bad. Given the absence of discounting, none of our equations and results would change in such an alternative scenario.

In the LR equilibrium, the LLR never supports the bank, so the full value reflected in $W_{-1}^{LR}(C)$ also constitutes the ex ante *total market value* of the bank in this equilibrium, $TMV_{-1}^{LR}(C)$. This is the object that bank owners aim to maximize when choosing C and selling the bank's debt and equity to investors. Therefore:

Proposition 9 *If the illiquid assets have positive fundamental net present value at t=–1 and the LR equilibrium can be sustained, it is not privately optimal for bank owners to set C strictly larger than* $\max\{\hat{C}, 0\}$, *where $\hat{C}$ is given by (20).*

So, conditional on inducing a LR equilibrium, there appears to be no discrepancy between the private and the social incentives for the choice of C and, hence, no clear rationale for regulatory liquidity standards. However, there might be situations where, even if a LR equilibrium can be sustained via a proper choice of, say, $C = \hat{C} > 0$, bank owners find it privately optimal to set $C < \hat{C}$ and induce the emergence of a different equilibrium (e.g. the ER equilibrium) were the total market value of the bank happens to be larger than $TMV_{-1}^{LR}(\hat{C})$. To discuss this, we first need to analyze more systematically the possible coexistence of the ER and LR equilibria in our economy.

B.2 Early run vs. late run equilibria

To analyze the possible coexistence of the ER and LR equilibria, it is useful to start comparing $V_t^{ER}(C)$ with $V_0^{LR}(C)$. To this effect, it is convenient to re-express (10) as

$$\begin{aligned} V_t^{ER}(C) &= \mu\{D + [1 - \exp(-(\delta+\lambda)(\tau - t))]\frac{\lambda}{\delta+\lambda}(B - D)\} \\ &\quad +(1-\mu)\{D - \exp(\delta t)[D - C - q_b(1 - C)]\} \\ &\quad -\mu \exp(-\lambda(\tau - t)) \exp(\delta t)[D - C - q_g(1 - C)] \\ &\quad +\xi \exp(-\lambda(\tau - t)) \exp(\delta t)[D - C - \bar{q}(1 - C)]. \end{aligned} \tag{21}$$

1. The first term can be compared to the first term in (19): it is smaller. What appears multiplied by μ is lower than B because all the factors that multiply the term $B-D > 0$ within the curly brackets are lower than one.

2. The second term can be compared to the second term in (19): it is weakly smaller. Specifically, it is identical for $t = 0$ and decreasing in t, so it is strictly smaller for $t \in (0, \tau]$.

3. The third term is negative, while there are no further terms in (19).

4. The fourth term is zero in the weak bank case ($\xi = 0$) and positive (and equal to the expected subsidy associated with LLR support) in the strong bank case ($\xi = 1$).

Therefore:

1. In the weak bank case ($\xi = 0$), we necessarily have $V_t^{ER}(C) < V_0^{LR}(C)$ for all $t \in [0, \tau]$, and hence $V_t^{ER}(C) < D$ for all $t \in [0, \tau]$ whenever $V_0^{LR}(C) < D$. Hence either the ER or the LR equilibrium always exists. In fact, in situations with $V_0^{ER}(C) \leq D \leq V_0^{LR}(C)$, the LR and the ER equilibria coexist, due to the self-fulfilling potential of the prophecies (on likelihood that the bank ends up liquidated) attached to the ER equilibrium.

2. In the strong bank case ($\xi = 1$), the fourth term in (21) is a source of ambiguity for the comparison between $V_t^{ER}(C)$ and $V_0^{LR}(C)$. In fact, in this case, the third and fourth terms in (21) can be consolidated into a net positive term:

$$+(1-\mu)\exp(-\lambda(\tau - t))\exp(\delta t)[D - C - q_b(1 - C)],$$

whose comparison with the positive gap between $V_0^{LR}(C)$ and the first two terms of $V_t^{ER}(C)$ is generally ambiguous. In this case, analytical conditions guaranteeing $V_t^{ER}(C) \leq D$ for all $t \in [0, \tau]$ whenever $V_0^{LR}(C) < D$ are convoluted. Yet, numerical examples show that there are parameter values under which this property is preserved, as well as cases in which it is not.

B.3 Taxonomy of equilibria in the strong bank case

To further understand the taxonomy of situations that we may find in the strong bank case, Figure A1 depicts the values of D, $V_0^{LR}(C)$, and $V_t^{ER}(C)$ for all $t \in [0, \tau]$ for a number of examples (t appears on the horizontal axes, D, $V_0^{LR}(C)$, and $V_t^{ER}(C)$ on the horizontal ones). The examples rely on the common parameter values described in the following table and the values of (μ, C) described under each of the panels of Figure A1.

Table A1
Common parameters behind Figure A1
(one unit of time = one month)

a_g	a_b	q_g	q_b	B	D	δ	λ
1.2	0.5	0.8	0.8	1.10	1.05	0.167	2

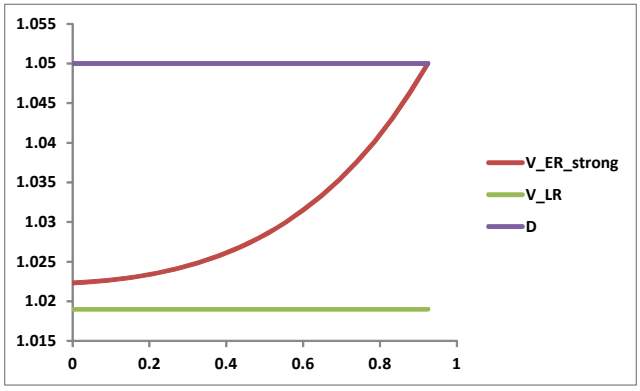
Under all combinations of parameters explored in Figure A1, the bank is strong, i.e. the expected value of its illiquid assets is higher, unconditionally, if continued than if early liquidated, so the previously described complexity regarding the potential taxonomy of equilibria arises. The values of δ and λ imply expected time spans to the arrival of the occasion to put bank debt and to the arrival of news on asset quality equal to 6 and 0.5 months, respectively.[28] We generate the various panels of Figure A1 by varying the probability of the asset being good, μ, by rows, and the bank cash holdings, C, by columns, as indicated under each panel.

Panel 1 describes a benchmark case in which, with a strong bank, the ER equilibrium is sustainable, while the LR equilibrium is not. Interestingly, in this case the subsidy linked to LLR support makes $V_t^{ER}(C) > V_0^{LR}(C)$ for all t. Panel 2 shows that larger liquidity lengthens the potential duration of the run and modifies the time-profile of $V_t^{ER}(C)$, which now starts below $V_0^{LR}(C)$ but eventually becomes larger than it, but never larger than D. So the ER equilibrium is sustainable, while the LR equilibrium is not.

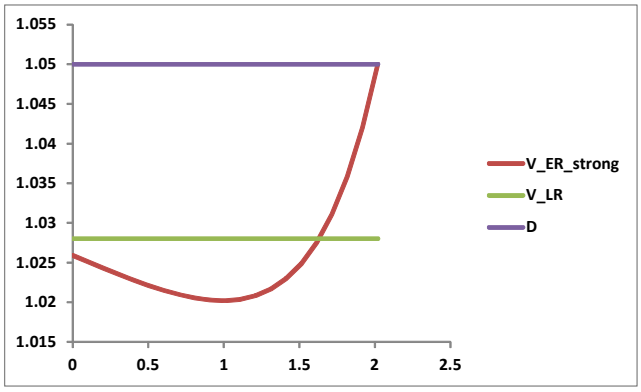
Panels 3 and 4 illustrate what happens when μ is larger, very close to the bound above which the LR equilibrium would become sustainable even with $C = 0.15$. In Panel 3, the ER equilibrium is sustainable while the LR equilibrium is not. In this case, increasing C to 0.30 makes the LR equilibrium sustainable (because $V_0^{LR}(C) > D$), and turns the ER equilibrium unsustainable (because $V_t^{ER}(C)$ is larger than D at low values of t).

In panels in the bottom row, μ is large enough for the LR equilibrium to be sustainable even with $C = 0.15$ (Panel 5) but with those liquidity holdings the ER equilibrium is also sustainable. In this case, increasing C to 0.30 (Panel 6) makes the ER equilibrium unsustainable. Intuitively, the additional liquidity holdings reduce the effective net subsidy associated with LLR support in a way that makes $V_t^{ER}(C)$ larger than D at some values of t. This means that a debtholder's best response to anticipating that subsequently debtholders will exercise their put options is no longer to exercise her own option, so the logic sustaining the ER equilibrium unfolds.

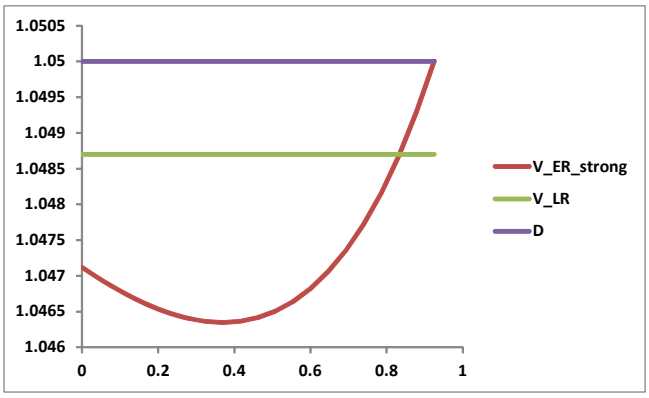
[28] The corresponding expected time spans can be computed as $1/\delta$ and $1/\lambda$, respectively. Having $1/\delta$ large relative to $1/\lambda$ is necessary for $V_t^{ER}(C)$ to vary significantly with t. Otherwise, $V_t^{ER}(C)$ is very close to D from the very start of the run.



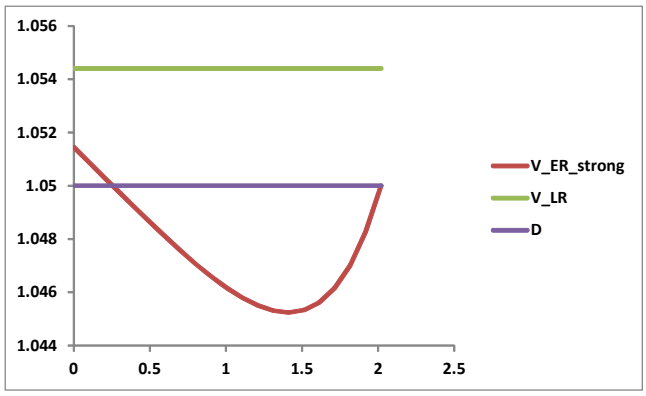
1. (μ, C)=(0.70,0.15). Only ER is an equilibrium



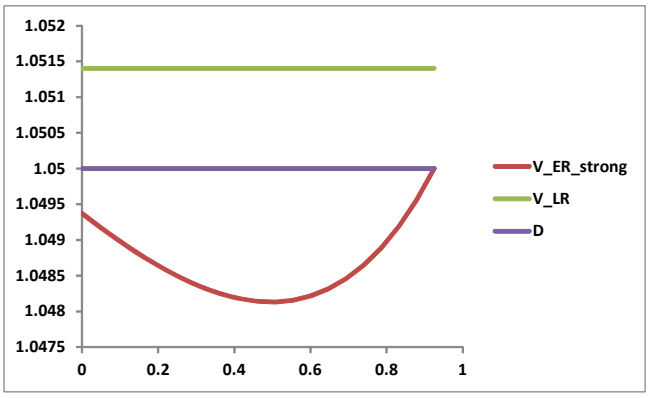
2. (μ, C)=(0.70,0.30). Only ER is an equilibrium



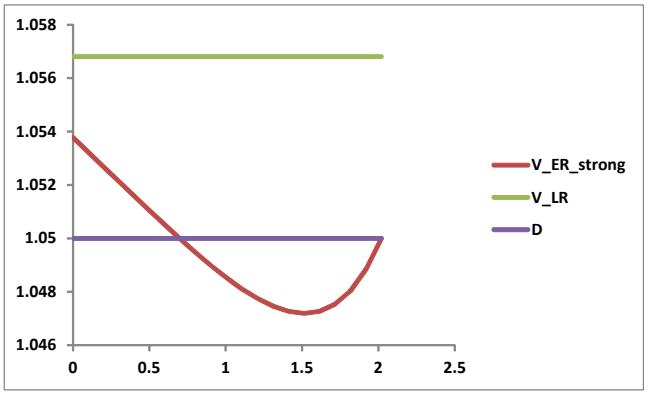
3. (μ, C)=(0.81,0.15). Only ER is an equilibrium



4. (μ, C)=(0.81,0.30). Only LR is an equilibrium



5. (μ, C)=(0.82,0.15). Both LR and ER are equilibria



6. (μ, C)=(0.82,0.15). Only LR is an equilibrium

Figure A1 Early vs. late run equilibria in the strong bank case

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