A theory-consistent system approach for estimating potential output and the NAIRU

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Abstract

A new approach is proposed for estimating potential output and the NAIRU. Identification is achieved using Okun's law and a Phillips curve. The performance of the methodology is exemplified using data from Canada, the UK, and the US.

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1. Introduction

Although there is some dissension about issues concerning the "potential" levels of output and unemployment and the "gaps" between potential and actual levels, most economist would probably agree on the following, rather general, propositions: (i) the unemployment gap and the output gap are closely related and mutually dependent concepts; (ii) since the gaps measure how far the economy is from its long-run sustainable level, they are likely to affect the development of inflation. Thus, inflation is likely to contain information about the size of the gaps. Yet, commonly used estimation methods seem to disregard at least one of these two conditions. On the one hand there are methods that make use of inflation information but do not take into account the mutual dependence between the unemployment gap and the output gap. For example, in the frequently used so called production-function approach, potential output is generated by inserting an exogenous NAIRU (or NAWRU) series (based on price (or wage) information) into an assumed production function together with other input components (see, e.g., Giorno et al. (1995)). On the other hand there are methods that recognise the mutual dependence between cyclical unemployment and cyclical output but do not make use of the information contained in inflation (see, e.g., Clark (1989) and Moosa (1997)). In addition, of course, there are methods that consider neither (i) nor (ii), e.g., purely statistical smoothing techniques such as the HP filter.

In this paper we propose a new approach for estimating potential output and the NAIRU which takes both conditions (i) and (ii) explicitly into account. Identification is achieved by using an Okun's law relation and a Phillips curve as restrictions in an unobserved-components model. Together, these equations yield an empirical system that is consistent with the common definitions of potential output and the NAIRU. It can thus be used to derive mutually dependent and economically interpretable estimates of the two key unobservable variables. The performance of the approach is illustrated using data from Canada, the UK, and the US.[1]

2. The model

The model consists of the following equations (here directly presented in their empirical format):

[1] For an application to Swedish data, see Apel and Jansson (1999).

$$\Delta\pi_t = \sum_{i=1}^{3} \rho_i \Delta\pi_{t-i} + \sum_{j=0}^{1} \eta_j (u_{t-j} - u_{t-j}^n) + \sum_{k=0}^{4} \omega_k z_{t-k} + \varepsilon_t^{pc} \ , \tag{1}$$

$$y_t - y_t^p = \sum_{l=0}^{1} \phi_l (u_{t-l} - u_{t-l}^n) + \varepsilon_t^{ol} \, , \tag{2}$$

$$u_t^n = u_{t-1}^n + \varepsilon_t^n \, , \tag{3}$$

$$y_t^p = \alpha + y_{t-1}^p + \varepsilon_t^p \, , \tag{4}$$

and

$$u_t - u_t^n = \sum_{m=1}^{2} \delta_m (u_{t-m} - u_{t-m}^n) + \varepsilon_t^c \, , \tag{5}$$

where π_t is the log difference of the CPI, u_t the unemployment rate, u_t^n the NAIRU, z_t supply-shock proxies (normalised so that $z_t = 0$ means that supply shocks are absent), y_t the log of real output, and y_t^p the log of potential output. The error terms ε_t^{pc}, ε_t^{ol}, ε_t , ε_t^p, and ε_t^c are assumed to be IID and mutually uncorrelated with constant variances. The lag lengths are set on the basis of standard mis-specification tests.

Equation (1) is a version of Gordon's so-called "triangle" Phillips model.[2] Usually, Phillips curves are estimated using either exogenous measures of the unemployment (or output) gap, or measures of cyclical activity that can be directly observed. By introducing the Phillips-curve mechanism as an identifying restriction within the unobserved-components system, we are able to consistently acknowledge the unobservability of the NAIRU, while explicitly imposing restrictions suggested by theory. Thus, as implied by theory, the estimated position of the NAIRU will depend on the development of actual inflation.[3] Equation (2) is an Okun's law relation, associating cyclical unemployment fluctuations with cyclical output fluctuations. This equation introduces potential output as an endogenous variable and ensures that the

[2] See, e.g., Gordon (1997) and Apel and Jansson (1999) for further discussions.
[3] There are, of course, alternative definitions of potential output and potential unemployment. One definition, brought to the fore by the introduction of inflation-targeting strategies is the level of output and unemployment

estimates of the NAIRU and potential output will be mutually consistent. Equations (3) and (4) imply that both the NAIRU and potential output are assumed to be characterised by stochastic trends. More specifically, the NAIRU is assumed to follow a pure random walk whereas potential output is assumed to follow a random walk with drift. The random-walk assumption is a standard one in this type of framework, but it should be emphasised that the methodology as such does not restrict the processes for potential output and the NAIRU to random walks. Other processes are also feasible within this framework and one may also incorporate possible structural determinants of potential output and the NAIRU. Equation (5) specifies the assumed evolution of cyclical unemployment, in this case a purely autoregressive process. Alternative specifications are of course possible here as well. For purposes of estimation, it is convenient to re-write the model (1)-(5) in state-space form. One may then apply the Kalman filter and maximum likelihood to obtain estimates of the unknown parameters and of the time series of the unobserved components, i.e. potential output and the NAIRU. Briefly, the Kalman filter generates, for a given set of model parameters and starting values, a sequence of optimal conditional predictions of the observable variables. The prediction errors are then used in a maximum-likelihood routine to find the optimal set of parameters and the corresponding estimates of the unobserved components.[4]

3. Empirical illustrations

The model is estimated using quarterly data from Canada, the UK, and the US for the period 1970:1 to 1998:2. The main justification for illustrating the approach using data from these countries is that unemployment in these economies has developed in a way that can be expected to reasonably well fit our random-walk specification. In many other countries, unemployment appears to contain elements of deterministic trends and/or shifts that would call for an appropriate modification of equation (3). Even if the random-walk hypothesis seems reasonable for the countries at hand, the estimations should foremost be seen as an illustration of the approach and the identical, rather parsimonious, model specification that is used could certainly be improved upon by, e.g., taking into account more country-specific characteristics, regarding, e.g., dynamics and supply factors. In this simple specification, we

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corresponding to a specific *level* of inflation. Apel and Jansson (1999) discuss various alternative specifications of equation (1), involving both the level and the first difference of inflation.

[4] For full technical details, see, e.g., Hamilton (1994).

assume that z_t merely equals the percentage change in the relative price of oil. Except for the oil price, the data source for all variables is *OECD Main Economic Indicators*. The oil price series is taken from the *EcoWin* database. The key results are displayed in Table 1 and Fig. 1. For purposes of more clearly highlighting the covariation between the estimated cycles and inflation, the high-frequency noise of inflation has in Fig.1 been dampened using a HP filter (with $\lambda = 1600$).

Table 1. Estimation result for selected parameters

	CANADA	*UK*	*US*
η_0	-0.059	-0.700*	-0.602**
η_1	0.038	0.680*	0.487**
ϕ	-3.455**	-7.066**	-3.634**
ϕ_1	0.981**	5.883**	1.507**
α	0.669**	0.544**	0.664**

*: Significant at 5 % level; **: Significant at 1 % level.

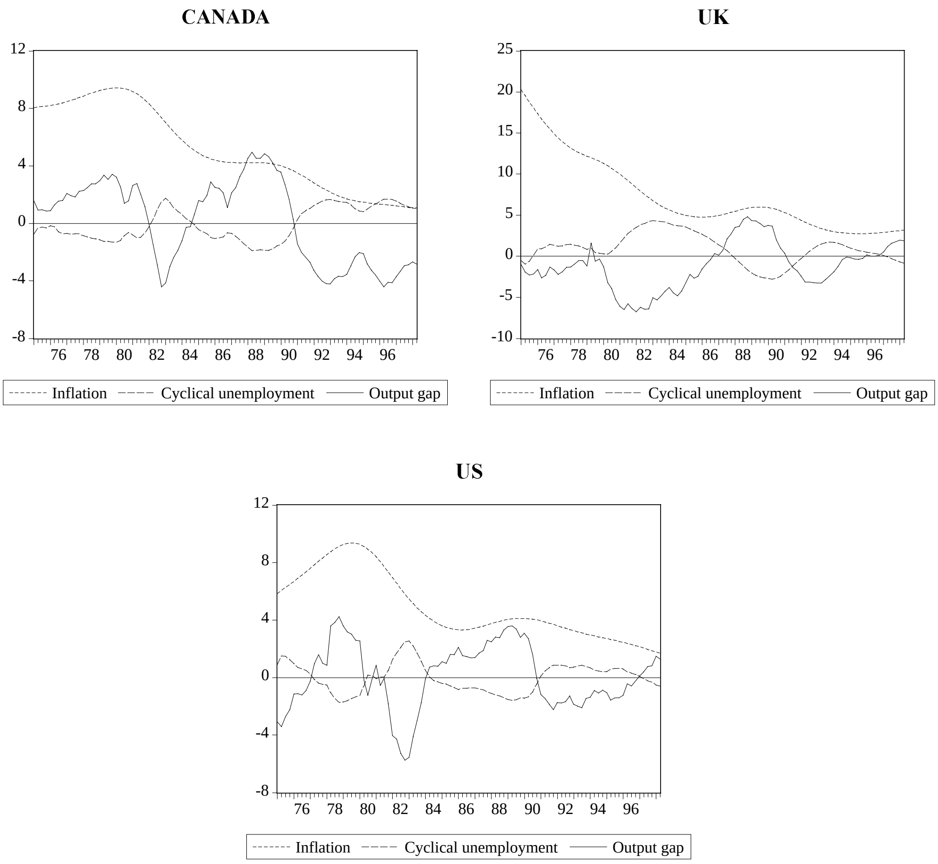

Fig. 1. Cyclical unemployment rates, output gaps, and smoothed annualised inflation in Canada, UK, and the US.

The parameter estimates of the Phillips curves and Okun's law relations have the expected signs and are highly significant in most cases. The Phillips-curve parameters in Canada are an exception, however. The negative relationship between cyclical output and cyclical unemployment, implied by Okun's law, is clearly visible in the plots. The Phillips-curve mechanism is reflected in Fig. 1 by the fact that negative output gaps and positive unemployment gaps are in general associated with falling "trend" inflation, and vice versa.

4. Concluding remarks

We have proposed an approach for estimating potential output and the NAIRU with two important properties that previously used methods do not simultaneously have or even lack completely: it generates mutually dependent estimates of potential output and the NAIRU, and it makes use of data on actual inflation in the estimations. This, together with the fact that the approach allows for appropriate modifications of the underlying model in order to fit the characteristics of different economies, makes it appear as an interesting and promising alternative or complement to methods that are presently used.

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