SVERIGES RIKSBANK
WORKING PAPER SERIES

157

SVERIGES
RIKSBANK

Indicator Accuracy and Monetary Policy: Is Ignorance Bliss?

Kristoffer P. Nimark

DECEMBER 2003

WORKING PAPERS ARE OBTAINABLE FROM

Sveriges Riksbank • Information Riksbank • SE-103 37 Stockholm
Fax international: +46 8 787 05 26
Telephone international: +46 8 787 01 00
E-mail: info@riksbank.se

Indicator Accuracy and Monetary Policy: Is Ignorance Bliss?

Kristoffer P Nimark*

Sveriges Riksbank Working Paper Series
No. 157
December 2003

Abstract

This paper argues that assuming a common information set shared by the public and the central bank may be inappropriate when one is concerned with the value of information itself. Specifically, we argue that it may lead one to draw the conclusion that monetary policy do not benefit from accurate real time data. This paper sets up a New-Keynesian model with optimal discretionary monetary policy, where we allow for partial and diverse information. The model is used to show that monetary policy do benefit from private and accurate real time data, where 'private' is the crucial assumption. The representative household is better of with less accurate information since this reduces the relative price distortions due to inflation and staggered prices. An implication of the negative welfare consequences of a well informed public is that central banks should be restrictive with publishing their real time data.

Keywords: Monetary policy; Information; Kalman filter; Higher order beliefs; Real time data.
JEL classification numbers: E37, E47, E52, E58

Author correspondence: European University Institute, Department of Economics, Villa San Paolo, Via della Piazzuola 43, 50133 Florence, Italy. *E-mail*: kristoffer.nimark@iue.it. The paper has benefitted from comments and suggestions from Giuseppe Bertola, Fabrice Collard and Gerry Mueller at the EUI, Ulf Söderström and Malin Adolfson at Sveriges Riksbank, Francesco Lippi at the Bank of Italy and Mirko Wiederholt at the Ente Einaudi in Rome. The views expressed in this paper are solely the responsibility of the author and should not be interpreted as reflecting the views of the Executive Board of Sveriges Riksbank.

1 Introduction

Recent advances in modelling monetary policy have allowed economists to study how monetary policy is affected by noisy indicators. Building on earlier work on control with partial information by Pearlman (1986) and Currie, Levine and Pearlman (1986), Svensson and Woodford (2003, 2004) provide general procedures to solve a class of monetary policy models where some variables are unobservable and some are only observable with noise. Their framework has been utilized by for instance Ehrmann and Smets (2002) who investigate the performance of different policy rules in a calibrated forward/backward looking model while Lippi and Neri (2003) contributes substantially to related empirical methods by showing how the indicator accuracy and the structural parameters of a model can be estimated simultaneously. As noted by Lippi and Neri (2003) and Nimark (2003) noisy indicators often improve macroeconomic outcomes in this class of models, and thus suggests that central bankers do not benefit from the availability of accurate real time data. This paper will show that the crucial, and as we will argue, unrealistic, assumption underlying this result is the assumption of a common information set shared by the central bank and the representative household of the economy. In the model presented below, the central bank and the representative household will be endowed with diverse (but intersecting) information sets. This will allow us to show that both the policymaker and the representative household is better off with a well informed central bank. However, we will also show that staggered prices implies that welfare is decreasing in the accuracy of the information available to the representative household.

We will derive some analytical results on the effects of increasing the noise in the information set of one agent, while holding the actions of the other (class of) agent(s) fixed. While this method ignores some of the equilibrium behavior of the model, it allows us to build intuition for the main result of the paper: Monetary policy performs better the more the central bank and the less the households know about the state of the economy. We show that this effect is stronger the more important the future is for the determination of inflation today. We also calculate expected losses for the equilibrium dynamics of the model, to find out whether the equilibrium behavior left out of the analytical part are qualitatively important. This numerical exercise confirms that the behavior suggested by the partial analysis carries over to the general equilibrium dynamics of the model. We end the paper by arguing that less accurate indicators work as a de facto coordination mechanism between pricesetters that decreases relative price externalities caused by inflation and staggered prices.

Is the assumption of diverse information realistic? Romer and Romer (2000) and Ellingsen and Söderström (2003) provide empirical evidence of private sector responses to monetary policy consistent with some private central bank information. Romer and Romer argue that the private information of the central bank is probably due to superior information processing, i.e. more and better economists, rather than earlier or more accurate access to data. They thus argue that the Federal Reserve in general has better information than the market. This does not imply that the central bank knows everything the market knows. It is hard to imagine that the Fed is *never* surprised by a market reaction. We do not believe that many will find the assumption of some private household information controversial.

When agents in an economic model have partial and diverse information, a problem of how to handle higher order beliefs emerges. Papers by Sargent (1991) and Pearlman and Sargent (2002) develop modelling methods to deal with this problem, and apply them to the previously unsolvable two sector growth model of Section VII in Townsend (1983). We believe this paper to be the first to apply a technique similar to that of Sargent (1991) to solve a model of monetary policy, though higher order beliefs in models of monetary policy have been treated before by Amato and Shin (2003). Amato and Shin focus on partial and diverse information *within* a symmetric class of agents, i.e. firms, while in the present paper non-symmetric agents, i.e. the policymaker and households, hold diverse information with respect to each other. Svensson and Woodford (2003), like the present paper, sets up a model of diversity of information between the central bank and the public. However, in their framework the public is perfectly informed which avoids the problem of higher order beliefs and simplifies the filtering problem facing the central bank.

The next section presents the structural model. This is followed in Section 3 by a description of the nature of diversity of information and the signal extraction problems facing the agents. Section 4 contains the analytical (but partial) results while Section 5 explains how a diverse information equilibrium can be found. Section 6 confirms that the partial results of Section 4 carries over to the full equilibrium dynamics of the model. Section 7 returns to the microfoundations of the model and the implications of inflation and staggered prices on the welfare of the representative household. Section 8 concludes by summarizing the results and discussing policy implications and future directions of research.

2 The Model

We will use a standard new Keynesian model with Calvo pricing for the formal analysis.[1] Readers already familiar with the model can skip to the IS curve (14) and the New-Keynesian Phillips curve (19) to confirm their priors. They may also want to have a look on the exogenous processes (20),(21) and (22).

In what follows, lower case letters denote the natural logarithm of the variables denoted with the corresponding capital letters.

2.1 Consumers

The representative consumer maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \tag{1}$$

where N_t is labor supply in period t and β is the discount rate. C_t is a CES aggregator

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{2}$$

Let

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}. \tag{3}$$

be the aggregate price index. This yields three first order conditions for the representative consumer. A demand schedule for each good

$$c_t(j) = -\epsilon(p_t(j) - p_t) + c_t \tag{4}$$

a labor supply condition

$$w_t - p_t = \gamma c_t - \varphi n_t \tag{5}$$

where w_t is the nominal wage and an Euler equation for the intertemporal consumption choice

$$c_t = -\frac{1}{\gamma}\left(i_t - E_t\left[\pi_{t+1}\right] - \beta\right) + E_t\left[c_{t+1}\right] \tag{6}$$

where i_t is the short term interest rate.

[1] For a recent and clear derivation, see Gali (2002).

2.2 Production

Firms produce differentiated goods indexed by j with technology

$$Y_t(j) = A_t N_t(j) \tag{7}$$

where A_t is technology at time t. The log of A_t follows

$$a_t = \widehat{a} + \rho_a a_{t-1} + \xi_{\overline{a}t} \tag{8}$$

Aggregate output Y_t is

$$Y_t = \left(\int_0^1 Y_t\,(j)^{\frac{\epsilon-1}{\epsilon}}\, dj \right)^{\frac{\epsilon}{\epsilon-1}} \tag{9}$$

and since there is no storage technology, clearing of the goods market implies

$$Y_t = C_t \tag{10}$$

Substituting (10) into the Euler equation (6) yields the equilibrium condition

$$y_t = -\frac{1}{\gamma}\left(i_t - E_t\left[\pi_{t+1}\right] - \beta\right) + E_t\left[y_{t+1}\right] \tag{11}$$

2.3 Equilibrium output

We can assume without loss of generality that both output and inflation have zero means. Potential output can then be written as

$$\overline{y}_t = \psi \rho_a a_{t-1} + \psi \xi_{\overline{a}t} \tag{12}$$

where $\psi = \frac{1+\varphi}{\gamma+\varphi}$. This is the level of output that would prevail under flexible prices. Redefine this process as

$$\overline{y}_t = \rho_{\overline{y}} \overline{y}_{t-1} + \xi_{\overline{y}t} \tag{13}$$

The IS curve will then be

$$(y_t - \overline{y}_t) = E_t[y_{t+1} - \overline{y}_{t+1}] - \frac{1}{\gamma}(i_t - E_t \pi_{t+1}) + \varepsilon_{yt} \tag{14}$$

where we have added an exogenous demand shock, ε_{yt}. Demand shocks are usually motivated by either shocks to preferences or as shocks to government spending. For this paper it will suffice to say that the demand shocks do not stem from shocks to preferences. This would complicate the welfare analysis without any clear benefit to the argument of this paper. However, shocks to preferences can be handled in a similar framework, but the derivation of the potential level of output would then be more cumbersome.

All firms face a common real marginal cost

$$mc_t = w_t - p_t - a_t - v + \varepsilon_{\pi t} \tag{15}$$

where v is a production subsidy given to achieve an efficient level of output in equilibrium and $\varepsilon_{\pi t}$ is an exogenous marginal cost (cost-push) shock . Use the intratemporal labor supply condition (5), the production function (7) and the market clearing condition (10) to write equilibrium marginal cost as a function of output, technology, the production subsidy and the cost-push shock

$$mc_t = (\gamma + \varphi)y_t - (1 + \varphi)a_t - v + \varepsilon_{\pi t}. \tag{16}$$

2.4 Price setting

If a fraction $(1 - \theta)$ of firms adjust prices each period, then the price level will follow

$$p_t = \theta p_{t-1} + (1 - \theta)p_t^* \tag{17}$$

where

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[p_{t+k} mc_{t+k} \right] \tag{18}$$

is the price set by firms adjusting prices in period t where μ is the steady state mark-up. It can then be showed that inflation will follow

$$\pi_t = \beta E_t \pi_{t+1} + \delta(\phi(y_t - \overline{y}_t) + \varepsilon_{\pi t}) \tag{19}$$

where $\{\pi_t, y_t, \overline{y}_t\}$ is inflation, output, and potential output in period t. E_t is the expectations operator based on information at time t. The nature of the information available to different agents in the economy will be made precise in the next section. δ and ϕ are defined as $\delta \equiv \frac{1-\theta}{\theta}(1 - \beta\theta)$ and $\phi \equiv (\gamma + \varphi)$. Together δ and ϕ determine the slope of the (short run) Phillips curve.

2.5 The exogenous variables

The exogenous variables in the model, $\{\varepsilon_{\pi t}, \varepsilon_{yt}, \overline{y}_t\}$ all follow AR(1) processes

$$\varepsilon_{\pi t} = \rho_\pi \varepsilon_{\pi t-1} + \xi_{\pi t} \tag{20}$$

$$\varepsilon_{yt} = \rho_y \varepsilon_{yt-1} + \xi_{yt} \tag{21}$$

$$\overline{y}_t = \rho_{\overline{y}} \overline{y}_{t-1} + \xi_{\overline{y}t} \tag{22}$$

or

$$X_t = \rho X_{t-1} + u_t \tag{23}$$

where $X_t = [\varepsilon_{\pi t}\ \varepsilon_{yt}\ \overline{y}_t]'$ and ρ is a matrix with the autoregressive parameters $\{\rho_\pi, \rho_y, \rho_{\overline{y}}\}$ on the diagonal. The vector of disturbances, $u_t = \left[\xi_{\pi t}\ \xi_{yt}\ \xi_{\overline{y}t}\right]'$ has mean zero and covariance Σ_{uu}.

Equations (14) and (19)-(22) describes the dynamics of the endogenous variables inflation and output, and can be put in compact form as

$$x_t = GX_t + Tx^h_{t+1|t} + Bi_t, \tag{24}$$

$$X_t = \rho X_{t-1} + u_t \tag{25}$$

$$G = G_0^{-1} G_1, T = G_0^{-1} T_1, B = G_0^{-1} B_1. \tag{26}$$

$$G_0 = \begin{bmatrix} 1 & -\delta\phi \\ 0 & 1 \end{bmatrix}, G_1 = \begin{bmatrix} \delta & 0 & -\delta\phi \\ 0 & 1 & 1 \end{bmatrix}, \tag{27}$$

$$T_1 = \begin{bmatrix} \beta & 0 \\ \frac{1}{\gamma} & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ -\frac{1}{\gamma} \end{bmatrix} \tag{28}$$

where $x_t = [\pi_t\ y_t]'$.

2.6 The Policymaker

The policymaker will set the interest rate to minimize the loss function

$$\Lambda_t = E_t \left[\sum_{k=0}^{\infty} \beta^k \left[\lambda (y_{t+k} - \overline{y}_{t+k})^2 + \pi^2_{t+k} \right] \right] \tag{29}$$

where λ is a preference parameter of the policymaker. In the later analysis we will assume that λ is such that there is no conflict of interests between the central bank and the households of the economy. This is implemented by setting $\lambda = \frac{\delta\phi}{\epsilon}$.[2] This formulation of the loss function implies that the policymaker's target level of output is state dependent and equal to potential output, and that the target level of inflation is zero. The linear-quadratic framework allows the optimal setting of the policy instrument, i.e. the interest rate to be expressed as a linear function F^f of the exogenous state vector

$$i = F^f X_t.$$

[2] See Woodford (2001) for a derivation of the lossfunction as a second order approximation of the representative household's utilityfunction.

Optimal policy under full information will be characterized by the policymaker's first order condition

$$y_t - \overline{y}_t = -\frac{\delta\phi}{\lambda}\pi_t.^3 \tag{30}$$

The New-Keynesian Phillips curve (19) and a $\lambda > 0$ in (29) implies that the policymaker will face a trade off between inflation and output gap stabilization in the presence of cost-push shocks. Optimal policy can offset demand and technology shocks completely and a perfectly informed policymaker thus only suffer losses from the cost-push shocks.

This completes the description of the economic model.

3 Diversity of Information

In full information models of monetary policy it is assumed that all agents know the complete structure of the economy and can observe all relevant variables perfectly. The full information set at time T shared by all agents, I_T^f, thus is

$$I_T^f = \{G, T, B, \lambda, X_t, x_t, u_t, \Sigma_{\mathsf{u}u} \mid t \leq T\} \tag{31}$$

We will make a slight departure from this setting by making the exogenous shocks, X_t, unobservable and the endogenous variables x_t observable only with measurement error. We will also make a distinction between observations available to the central bank and those available to the representative household. The information set of the partially informed agent i at time T, I_T^i, will be defined by (32) -(34)

$$I_T^i = \{G, T, D, D^i, \lambda, \Sigma_{\mathsf{u}u}, \Sigma_{vv}^i, Z_t^i \mid t \leq T\}, i \in \{cb, h\} \tag{32}$$

$$Z_t = D \begin{bmatrix} X_t \\ x_t \end{bmatrix} + \mathsf{v}_t \tag{33}$$

$$Z_t^i = D^i Z \tag{34}$$

where D is a matrix that picks out and scales the variables Z_t that are observable in principle, while D^i picks out what variables that are observable to agent i. The superscript cb denotes the central bank's information set

[3] See Clarida, Gali and Gertler (1999) for a derivation of the first order condition.

while h denotes that of households. Specifically,

$$D = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad Z_t = \begin{bmatrix} \pi_t \\ y_t \\ y_t \\ y_t \end{bmatrix} + \begin{bmatrix} v_{t\pi} \\ v_{ty}^1 \\ v_{ty}^2 \\ v_{ty}^3 \end{bmatrix}$$

The vector of measurement errors, $\mathsf{v}_t = \begin{bmatrix} v_{t\pi} & v_{ty}^1 & v_{ty}^2 & v_{ty}^3 \end{bmatrix}'$, has covariance Σ_{vv} with all off-diagonal elements equal to zero. Z_t thus consists of one measure of inflation and three measures of output. The diversity of information will be modelled by allowing the central bank to observe the inflation measure and the first two measures of output, while households will observe the inflation measure and the last two measures of output. Formally

$$\begin{aligned} D^{cb} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \; Z_t^{cb} = \begin{bmatrix} \pi_t \\ y_t \\ y_t \end{bmatrix} + \mathsf{v}_t^{cb}, \\ \mathsf{v}_t^{cb} &= \begin{bmatrix} v_{t\pi} \\ v_{ty}^1 \\ v_{ty}^2 \end{bmatrix}, \quad covar\left(\mathsf{v}_t^{cb}\right) = \Sigma_{vv}^{cb} \end{aligned}$$

$$\begin{aligned} D^{h} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; Z_t^{h} = \begin{bmatrix} \pi_t \\ y_t \\ y_t \end{bmatrix} + \mathsf{v}_t^{h} \\ \mathsf{v}_t^{h} &= \begin{bmatrix} v_{t\pi} \\ v_{ty}^2 \\ v_{ty}^3 \end{bmatrix}, \; covar\left(\mathsf{v}_t^{h}\right) = \Sigma_{vv}^{h}. \end{aligned}$$

Varying the accuracy of the private indicator of the central bank can be implemented by changing the variance of v_{ty}^1. Similarly, the accuracy of households private information can be varied by changing the variance of v_{ty}^3. It then follows that the accuracy of the common information is captured by the variance of $v_{t\pi}$ and v_{ty}^2.

Some comments on this information structure is in order. Assuming only one common measure of inflation is intended to capture the fact that inflation is measured quickly and accurately and that the numbers are publicly available almost immediately. Letting the agents have diverse but intersecting indicator sets for output is meant to capture that a lot of the information about the level of activity in the economy is publicly available, but that the central bank is likely to have some private information, perhaps from survey data etc., that is not published. Also, it is intellectually appealing (at least for non-central bankers) to allow for the possibility that there is some information around in the economy unknown to the central bank. This is captured by the private information of the households.

3.1 Estimating the state

The Kalman filter offers to the partially informed a method to estimate the unobservable state variables. Since the partially informed agents know the effect of the unobservable variables on the imperfectly measured but observable variables through the structural equations, they will be able to estimate the state of all the variables in the economy. Let $X^i_{t|t}$ denote agent i's estimate of the state X_t at time t. If observations were independent of the endogenous variables, the estimate $X^i_{t|t}$ would be given by

$$X^i_{t|t} = X^i_{t|t-1} + K(Z^i_t - LX^i_{t|t-1}) \tag{35}$$

where K is the Kalman gain matrix and L a matrix that maps a state into an expected observation. The term inside brackets can be interpreted as the surprise part (the 'innovation') of the observation Z^i_t. When Z^i_t is a function of the endogenous variables in the model, the contemporaneous effect of the estimate should be subtracted from the term inside brackets to preserve the interpretation as the surprise part of the observation. Intuitively for our model, the part of the observation on inflation and output that can be attributed to the central bank's setting of the interest rate contains no information about the exogenous state variables. In the same way, the contemporaneous effect from the partially informed households' estimate should be subtracted. When households and the central bank share the same information set, their estimate of the exogenous state will be identical for all time periods. It is then straightforward to augment (35) to

$$X_{t|t} = X_{t|t-1} + K(Z_t - LX_{t|t-1} - MX_{t|t}), \quad X_{t|t} = X^{cb}_{t|t} \equiv X^h_{t|t} \tag{36}$$

where L and M are matrices mapping the exogenous state and the common contemporaneous estimate into an expected observation. L and M will depend on the structural parameters of the model.

3.1.1 The signal extraction problem of the central bank

When the agents of the economy have partial and diverse information, matters become more complicated. To form an efficient estimate of the exogenous state, it is no longer enough for an agent to subtract the contemporaneous effect of his own estimate. Consider the problem facing the central banker of our model. He needs to estimate the exogenous variable in the economy to set the interest rate such that the first order condition holds in expectations. But what he observes is only the endogenous variables, inflation and output, which depend on the expectations of the households. The households' expectations of inflation and output tomorrow in turn depend on the households' estimate of the central bank's estimate of the state tomorrow, which depends on the central bank's estimate of households' estimate of the central bank's estimate of the expectation..... Continuing with this logic, one enters the infinite recursion of estimating the estimates of the estimates and so on, or with the words of Townsend who, in parallel with Phelps, formulated the problem in 1983, 'to forecast the forecasts of forecasts...of others'. This is a general problem that emerges whenever forward looking and interacting agents hold diverse and hidden expectations of future or current states. This structure makes it optimal for the agents to fit a VAR-process of both infinite dimension and of infinite order, i.e. not only should they define the state of the system to include the 'forecasts of forecast of forecasts' to infinity, but it would also be optimal to condition on the complete history of observations. Sargent (1991) proposes a way to deal with this problem. Instead of letting the agents fit an infinite order VAR of infinite dimension, they can efficiently extract the information in the equilibrium outcome by fitting individual mixed vector ARMA processes of finite dimension, where the MA part captures the interaction of the higher order estimates. This approach will be followed to solve the signal extraction problem of the central bank. The updating equation for the central bank's estimate of the state at time t will then be

$$X_{t|t}^{cb} = X_{t|t-1}^{cb} + K^{cb}(Z_t^{cb} - LX_{t|t-1}^{cb} - M^{cb}X_{t|t}^{cb} - \zeta_{t|t}^{cb} - H\zeta_{t-1|t}^{cb}) \qquad (37)$$

where $\zeta_{t|t}^{cb}$ has mean zero and covariance $\Sigma_{\zeta\zeta}^{i}$.The details of how to set up the Kalman filter for this process is given in the Appendix. Intuitively, this method uses that the central bank don't need to know the higher order estimates of the households as long as the implied behavior of the households do not cause the variance of the endogenous variables to tend to infinity. The influence of the households higher order estimates is modelled as an additional stochastic process shocking the endogenous variables. The best the central bank can do is to exploit the stochastic properties of this process when it estimates the exogenous state.

3.1.2 The signal extraction problem of the households

Part of the filtering problem of the central bank consists of not being able to observe the action of the households directly, and therefore it cannot filter out movements in the endogenous variables caused by households' (potentially inaccurate) expectations. Here households have an informational advantage by being able to observe the action of the central bank, i.e. the interest rate. The following updating equation of the households estimate exploits this fact.

$$X^h_{t|t} = X^h_{t|t-1} + K^h(Z^h_t - L^h X^h_{t|t-1} - M^h X^h_{t|t} - Bi_t) \tag{38}$$

The interpretation of L^h and M^h is the same as before. L^h captures the expected effect of the exogenous state on inflation and output while M^h maps the contemporaneous household estimate into its effect on inflation and output.

3.2 Partial information in the economic model

We will follow previous papers on monetary policy with partial information when we adjust the structural model (24) to allow for the central bank's and households' partial information. Policy, i.e. the interest rate, in period t will no longer be a function of the exogenous state, X_t, but be a linear function F of the estimate $\widehat{X}^{cb}_{t|t}$, where $\widehat{X}^{cb}_{t|t} \equiv \begin{bmatrix} X^{cb}_{t|t} & \zeta^{cb}_{t|t} & \zeta^{cb}_{t-1|t-1} \end{bmatrix}'$. Similarly, the mathematical expectation of the endogenous variables $E_t[x_{t+1}]$ will be replaced by the household forecast $x^h_{t+1|t}$. However, we do differ from the common information set literature by letting the central bank and households condition their estimates on diverse observations.

The way actual shocks enter directly into (14) and (19) are not affected by partial information. Inflation and output will then follow

$$x_t = GX_t + Tx^h_{t+1|t} + BF\widehat{X}^{cb}_{t|t}. \tag{39}$$

This completes the description of how partial information is implemented in the structural model.

4 Indicator accuracy and the endogenous variables

In this section we will derive some of the effects of varying the accuracy of the information set of one agent, while holding the actions of the other (class of) agent(s) fixed. Admittedly, this approach ignores some of the interactions between the two classes of agents behavior that occurs when we change the information set of one of them. But bear with us. In Section 6 below we will show that the interactions left out are not qualitatively important.

4.1 Noise and inaccurate policy

The most intuitive consequence of noisy indicators is that policy will be less precise. Optimal discretionary policy is characterized by the first order condition of the central bank

$$y_t - \overline{y}_t + \frac{\delta\phi}{\lambda}\pi_t = 0 \tag{40}$$

In a full information model, this proportional relationship between the output gap and inflation will hold for all periods since the central bank can observe $y_t, \overline{y}_t$ and π_t perfectly. In a partial information model, the central banks estimate of $y_t, \overline{y}_t$ and π_t will in general be different from the actual values and the first order condition holds only in expectations. The deviation of the target variables from the optimal proportionality can be written as a function of the deviation of the central bank estimate of the state, $\widehat{X}^{cb}_{t|t}$, and the actual state $\widehat{X}_t$

$$\begin{aligned} \left(y_t - y^{cb}_{t|t}\right) - \left(\overline{y}_t - \overline{y}^{cb}_{t|t}\right) + \frac{\delta\phi}{\lambda}\left(\pi_t - \pi^{cb}_{t|t}\right) &= \\ = \left[\Delta\widehat{X}_t + BF\widehat{X}^{cb}_{t|t}\right] - \left[\Delta\widehat{X}_{t|t} + BF\widehat{X}^{cb}_{t|t}\right] &= \\ = \Delta\left[\widehat{X}_t - \widehat{X}^{cb}_{t|t}\right], & \end{aligned} \tag{41}$$

$$\Delta = \left[\begin{bmatrix} \frac{\delta\phi}{\lambda} & 1 \end{bmatrix}\begin{bmatrix} G & I & H^{cb} \end{bmatrix} - \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}\right]$$

The variance of this term is given by

$$var\left(\Delta\left[\widehat{X}_t - \widehat{X}^{cb}_{t|t}\right]\right) = \Delta P^{cb}\Delta' \tag{42}$$

where P^{cb} is the intratemporal forecast error covariance matrix of the central bank. P^{cb} is given by the Riccati equation

$$P^{cb} = \widehat{\rho}(P^{cb} - P^{cb}L^{cb\prime}(L^{cb}P^{cb}L^{cb\prime} + \Sigma^{cb}_{vv})^{-1}L^{cb}P^{cb})\widehat{\rho}' + \widehat{\Sigma}^{cb}_{\mathsf{uu}}.$$

All the diagonal elements of P^{cb} is increasing in all the diagonal elements of Σ^{cb}_{vv},keeping $\widehat{\Sigma}^{cb}_{\mathsf{uu}}$constant. The quadratic expression of the variance of the policy deviation (42) is thus also increasing in the elements of Σ^{cb}_{vv}. Noisier central bank indicators thus tend to increase the variance of the deviations from an optimal policy.

4.2 Noise and muted responses to shocks

In this section we will show that increasing the noisiness of the private household indicator will reduce the variance of inflation and the output gap, and thus improves the welfare of the representative agent. We will proceed in two steps. In the first we will derive the positive relation between the variances of inflation and the output gap and the variance of the household estimate of the cost-push shock. In the second step, we will show that the variance of the estimate is decreasing in the variance of the noise, and thus that the variance of inflation and the output gap is also decreasing in the variance of the noise.

4.2.1 Household estimates and the target variables

Since we want isolate the effects of the accuracy of the households private indicators we can temporarily assume a perfectly informed central bank. This allows us substitute the first order condition (30) into the New-Keynesian Phillips (19) curve to get

$$\pi_t = \beta E_t \pi_{t+1} + \delta(\phi(-\frac{\delta\phi}{\lambda}\pi_t) + \varepsilon_{\pi t}). \tag{43}$$

Rearrange this to get inflation as a function of the cost-push shock and expected future inflation

$$\pi_t = \frac{\beta}{1 + \frac{1}{\lambda}\delta^2\phi^2} E_t \pi_{t+1} + \frac{\delta}{1 + \frac{1}{\lambda}\delta^2\phi^2} \varepsilon_{\pi t}. \tag{44}$$

Iterate forward

$$\pi_t = \frac{\delta}{1 + \frac{1}{\lambda}\delta^2\phi^2} \sum_{k=1}^{\infty} \left(\frac{\beta}{1 + \frac{1}{\lambda}\delta^2\phi^2} \rho_\pi \right)^k \varepsilon_{\pi t} + \frac{\delta}{1 + \frac{1}{\lambda}\delta^2\phi^2} \varepsilon_{\pi t} \tag{45}$$

where we used that $E_t\left[\varepsilon_{\pi t+k} \mid \varepsilon_{\pi t}\right] = \rho_\pi^k \varepsilon_{\pi t}$. $0 < \frac{\beta}{1+\frac{1}{\lambda}\delta^2\phi^2}\rho_\pi < 1$ implies that the sum is finite. Replace the the cost push shock in the forward looking term with the household estimate and redefine the coefficients

$$\pi_t = \eta \varepsilon^h_{\pi t|t} + \tau \varepsilon_{\pi t}. \tag{46}$$

The variance of inflation, σ^2_π, will then be

$$\sigma^2_\pi = \eta^2 \sigma^{2h}_{\varepsilon\pi} + \tau^2 \sigma^2_{\varepsilon\pi} + 2covar(\tau\varepsilon_{\pi t}, \eta\varepsilon^h_{\pi t|t}) \tag{47}$$

where $\sigma_{\varepsilon\pi}^{2}$ and $\sigma_{\varepsilon\pi}^{2h}$ are the variances of the actual and estimated cost-push shocks. The covariance of the cost-push shock and the estimate of the cost-push shock is positive and it is thus sufficient to show that the variance of the estimate is decreasing in the magnitude of noise for also the variance of inflation to be decreasing. With a perfectly informed policymaker, this implies that the variance of the output gap is also decreasing since

$$\sigma_{y-\overline{y}}^{2} = \left(\frac{\delta\phi}{\lambda}\right)^{2} \sigma_{\pi}^{2}. \tag{48}$$

One should note that the variance of both inflation and output is positively dependant on ρ and β. If either today tells us nothing about tomorrow, i.e. if ρ is small, or if households don't care much for tomorrow, i.e. if β is small, the variance of the estimates will have small consequences for the variance of inflation and the output gap. We now turn to the question of how the variance of the estimate, $\sigma_{\varepsilon\pi}^{2h}$, is affected by less accurate household indicators.

4.2.2 Noisy indicators and the variance of the estimates

In the figure below, simulated estimates of a cost push shock are plotted for different measurement errors together with the actual cost-push shocks hitting the economy.

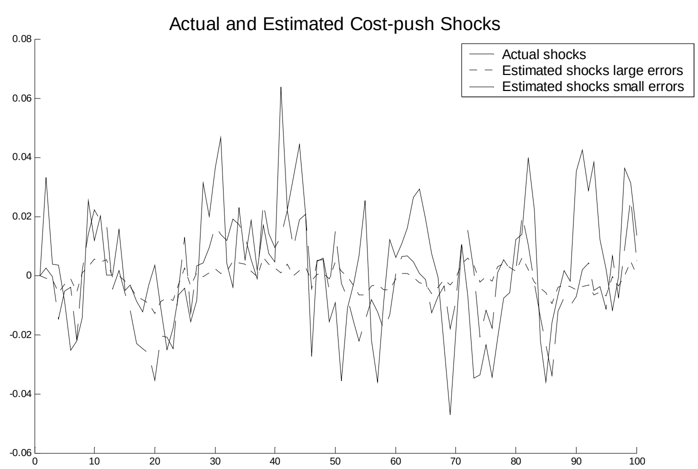

Figure 1

We can see that the larger the measurement errors, the closer to the (zero) mean are the estimates.[4] To understand the mechanics of this, it is instructive to look at the limit case of infinitely noisy indicators. In this case the observation holds no information at all about any of the variables and the best estimate of a variable is its unconditional mean. This does not amount to systematic errors, since the true value of the shock is never revealed to the partially informed agents. The same point can be made by using the Kalman filter updating equation

$$\begin{aligned} X_{t|t}^{h} &= \rho X_{t-1|t-1}^{h} + K^{h}(Z_{t}^{h} - LX_{t|t-1}^{h} - MX_{t|t}^{h} - Bi_{t}) && (49) \\ \varepsilon_{\pi t|t}^{h} &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} X_{t|t}^{h} && (50) \end{aligned}$$

and the formula for the Kalman gain matrix

$$K^{h} = P^{h}L^{h\prime}(L^{h}P^{h}L^{h\prime} + \Sigma_{vv}^{h})^{-1}. \quad (51)$$

The variance of the estimates $X_{t|t}^{h}$ is given by the discrete Lyaponov equation

$$\begin{aligned} \Sigma_{XX}^{h} &= \rho \Sigma_{XX}^{h} \rho' + K^{h} J K^{h\prime} && (52) \\ J &= covar(Z_{t}^{h} - LX_{t|t-1}^{h} - MX_{t|t}^{h} - Bi_{t}) \end{aligned}$$

Since ρ is a diagonal matrix, we can write the variance of the cost-push shock estimate as

$$\sigma_{\varepsilon\pi}^{2h} = \left(1 - \rho_{\pi}^{2}\right)^{-1} K_{1,i}^{h} J K_{1,i}^{h\prime} \quad (53)$$

where $K_{1,i}^{h}$ denotes the first row of the Kalman gain matrix. When the variance of the measurement errors, Σ_{vv}^{h}, becomes infinitely large the elements of K^{h} will approach zero. With $K = \mathbf{0}$, the updating equation (49) becomes a deterministic difference equation converging to the zero mean from any initial condition and the variance of the estimate will tend to zero.

The reduced variance of the cost-push estimate from an increase in noise completes the link from household indicator noise to the variance of the target variables. Less informed households thus tend to reduce the losses suffered from inflation and output gap volatility.

5 Finding the partial information equilibrium dynamics

In the analysis above we found it useful to hold the responses of households to shocks fixed when we discussed how noise affected the accuracy of policy,

[4] See Appendix C for the parameter values used in the graph.

and vice versa we assumed that the central bank was perfectly informed when we analyzed the impact of noise on the behavior of households. This approach leaves out some of the interactions between one agent's actions and the informational content of the observation of the other (class of) agent(s). For the agents of the model to solve their filtering problem efficiently, these interactions must be taken into account. This section describes the agents' perceived laws of motion of the observable variables and defines the equilibrium dynamics as a fixed point in the mapping from perceived to actual laws of motion.

5.1 The central bank's perceived law of motion

The central bank perceives the endogenous variables to follow the process

$$x_t = L^{cb} X_t + B i_t + \zeta^{cb}_{t|t} + H^{cb} \zeta^{cb}_{t-1|t-1}. \tag{54}$$

The properties of the exogenous variables X_t and the coefficients in L^{cb} and B are derivable from the structural model and are independent of the measurement errors, but the knowledge of these are not enough to efficiently estimate the state of the economy. The central bank also need to know the stochastic properties of the MA process capturing the hidden actions of households, and these depend on the measurement errors of the indicators observable by households. There are thus three objects in the central bank's perceived law of motion that cannot be determined independently of the accuracy of the households' indicator: The variance of $\zeta^{cb}_{t|t}$, $\Sigma^{cb}_{\zeta\zeta}$, the covariance of the actions of households and the exogenous variables, $\Sigma^{cb}_{\zeta X}$, and the coefficients on the lagged shock vector in the MA process, H^{cb}.

5.2 Households' perceived law of motion

Households need to estimate X_t and forecast policy tomorrow to form an expectation of inflation and output. If the central bank were perfectly informed, households expectations of inflation and output next period would be given by

$$x^h_{t+1|t} = T^{-1} M^h X^h_{t|t}. \tag{55}$$

We now want to find the optimal household forecast of inflation and output when the central bank has only partial information.

The optimal proportionality between inflation and the output gap, as described by the first order condition of the central bank, is attainable in each period. Any deviation of the actual outcome from these proportions must stem from inaccurate information on behalf of the central bank. Similarly, any deviation between households' estimates of the target variables and the optimal proportions must be due to informational discrepancies between households and the central bank. From the perspective of the household, this discrepancy can be viewed as an additional state variable, holding information about next periods inflation and output gap.[5] The predictive power of this additional state variable depends on the accuracy of the information of both the central bank and households. Households expectations of next periods inflation and output will be of the form

$$x^h_{t+1|t} = T^{-1} M^h X^h_{t|t} + H^h \zeta^h_{t|t} \tag{56}$$

$$\zeta^h_{t|t} = \left(y^h_{t|t} - \overline{y}^h_{t|t} + \frac{\delta\phi}{\lambda} \pi^h_{t|t} \right) \tag{57}$$

where the coefficients in H^h have to be determined together with the accuracy of all agents indicators.

5.3 Finding the equilibrium dynamics

We have now introduced all the principal ingredients necessary to solve for the dynamics of the whole system, i.e. the dynamics of not only the estimates $X^i_{t|t}$ but also the endogenous variables x_t as functions of the exogenous shocks and the measurement errors. What remains is to determine the four objects $\left\{ \Sigma^{cb}_{\zeta\zeta}, \Sigma^{cb}_{\zeta X}, H^{cb}, H^h \right\}$, and this have to be done numerically. We need to find what Sargent (1991) calls the 'limited information rational expectations equilibrium', defined as a fixed point on the mapping from perceived to actual laws of motion. The fixed point can be found by iterating on the following algorithm.

1. Start with an initial guess of $\Sigma^{cb}_{\xi\xi}, \Sigma^{cb}_{\zeta X}, H^{cb}$ and H^h.
2. Calculate the optimal policy F given the central banks perceived laws of motion.
3. Use the structure of the economy to map the perceived laws of motion of all classes of agents into actual laws of motion.
4. Replace the initial guess of $\Sigma^{cb}_{\xi\xi}$, $\Sigma^{cb}_{\zeta X}, H^{cb}$ and H^h with their actual counterparts.

[5] One should note that this is true irrespectively of whether households or the central bank are closer to knowing the true state.

5. Repeat 1-4 until perceived and actual laws of motion have converged for all agents.

This will imply that K^i, P^i also have converged and that we have a system of the form

$$\widetilde{X}_t = \Pi \widetilde{X}_{t-1} + \mathsf{e}_t \tag{58}$$

where $\widetilde{X}_t = \begin{bmatrix} X_t & \widehat{X}_t^{cb} & X_t^h & Z_t & x_t & \zeta_t^{cb} & \zeta_{t|t}^h & i_t \end{bmatrix}'$. The equilibrium implies that the agent's perception of $\left\{\Sigma_{\zeta\zeta}^{cb}, \Sigma_{\zeta X}^{cb}, H^{cb}, H^h\right\}$ are consistent with their observations. The set up of the complete system and the solution procedure is detailed in Appendix A.

6 Equilibrium dynamics, welfare and indicator accuracy

With a process in the form of (58) it is straightforward to calculate the expected value of the loss function

$$\Lambda_t = E_t \left[\sum_{k=0}^{\infty} \beta^k \left[\lambda (y_{t+k} - \overline{y}_{t+k})^2 + \pi_{t+k}^2 \right] \right].$$

In this section we will see how the expected losses are affected when we change the accuracy of the central bank's and households private indicators, as well as the accuracy of the indicator observable to both. The technical details of calculating the expected loss and the parameters that were used in the computation of the graphs of this section are given in Appendix B and C.

In Figure 2 we have plotted expected losses (on the vertical axis) when we vary the accuracy of the central banks' private indicator (along the horizontal axis).

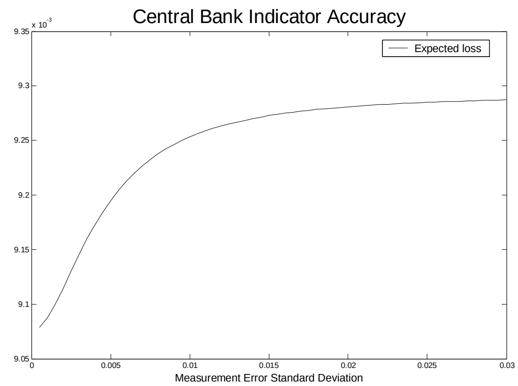

Figure 2

Here we can see that losses are increasing in the magnitude of the measurement errors of the central banks private indicator, and that the result from the previous section carries over to the full equilibrium case. Monetary policy do benefit from increased accuracy of its private indicator. Where the graph flattens out, the magnitude of noise has become so large in the private indicator that the central bank relies entirely on the commonly observable measures of output and inflation to set policy.

In the opposite sloping graph in Figure 3 below, we can confirm that also the suggested effects of inaccurate household indicators carries over to the full equilibrium dynamics.

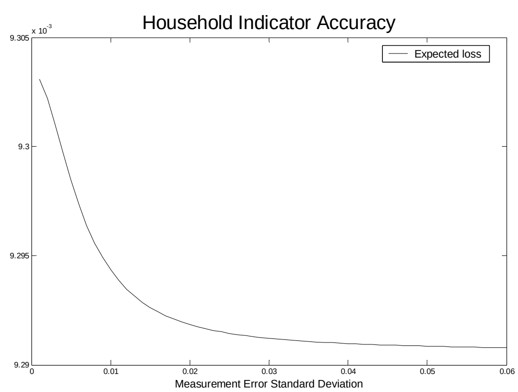

Figure 3

Losses are decreasing in the magnitude of the measurement errors and the next section discusses how this can be interpreted at the micro level.

Will the positive or negative effect of measurement errors dominate when we vary the accuracy of the common information set? The derivations of Section 5 suggested that the more persistent the shocks were, the more likely was the positive effects of noise to dominate.

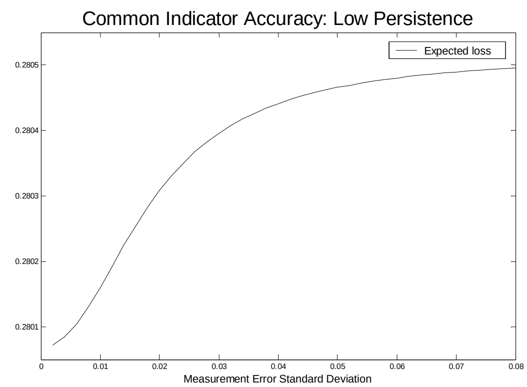

Figure 4

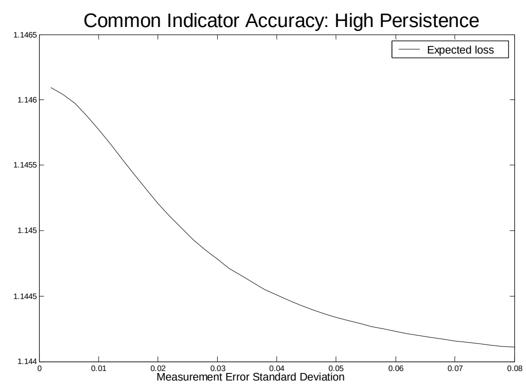

Figure 5

That persistence of shocks can matter qualitatively is confirmed by in Figure 4 and 5, where we have negative welfare effects of noise with small shock persistence, and positive welfare effects with large shock persistence. However, this is not a general result. We can get negative effects of noise for any degree of persistence, by setting the private household indicator to be accurate enough. Then the common indicator have little informational value to the households, but may still be important for the filtering problem of the central bank. The negative effect of less accurate policy then dominates. The reverse results, but with positive effects for all degrees of persistence, can be obtained by having a very accurate private central bank indicator.

7 Why is household ignorance (micro) bliss?

Even when the mechanics of how less accurate household indicators leads to less volatile output gaps and inflation, one may legitimately ask the question 'What prevents the agents from achieving this outcome under full information?' To answer this question, it will be useful to quickly rehash the argument in Woodford (2001) about the welfare implications of inflation stabilization when prices are staggered.

Remember the CES aggregator of goods

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{59}$$

and note that the differentiated goods enter the index symmetrically and with decreasing marginal weight. The optimal demand schedule for each good

$$c_t(j) = -\epsilon(p_t(j) - p_t) + c_t, \tag{60}$$

shows that if prices were the same for all goods, it would be optimal to consume the same amount of each good. From an aggregate perspective, the 'price' of the good is the amount of resources spent on producing it. When the individual firms' technology is symmetric and displays non-increasing returns to scale, it will also be optimal for the aggregate economy to produce the same amount of each good. In a decentralized equilibrium this will be achieved when the price for all goods are the same.[6] In a model with staggered prices, i.e. a model where firms change prices infrequently and in a non-synchronized fashion, inflation implies changes in the relative prices of goods and $p_t(j) = p_t \ \forall \ j$ only in the steady state. Inflation thus leads to an inefficient composition of production and consumption. In a similar fashion one can argue that changes in the output gap corresponds to non-efficient variations in the relative price between goods today and goods tomorrow. An intriguing feature of this type of models is thus that the first best outcome can be achieved either through perfect price flexibility or perfect price rigidity, given that a benevolent and perfectly informed policymaker set the interest rate such that the economy always operate at the efficient potential output level.

[6] This argument (and these types of models in general) rests on the assumption that the economy consists of Yeoman farmers with perfect income insurance, or that all agents symmetrically own and supply labor to all firms.

So what prevents the agents from achieving the superior outcome under full information? The pricing behavior of the structural model assumes that each firms maximizes its own profit. The less desirable outcome stems from a failure of the individual firms to internalize the aggregate effects of changing prices. Less accurate information thus works as a coordinating mechanism, that is detrimental to the individual firm, but beneficial to society as a whole, by inducing the firms that do adjust prices in one period to adjust them less aggressively. This can be contrasted to the results of Bomfim (2001), who finds that even though less accurate information leads to less volatile cycles also in a standard RBC framework, they do not represent welfare improvements. In his model, the perfect information responses are optimal responses to the shocks hitting the economy.

8 Some concluding remarks

In the analysis above, we have argued that assuming a common information set shared by the public and the central bank may be inappropriate when one is concerned with the value of information it self. Specifically, we argued that it may lead one to draw the conclusion that monetary policy do not benefit from accurate real time data. However, when a model of diverse information was set up, we could show that the central bank and the representative household do benefit from a well informed monetary policy. However, we also showed that monetary policy improved when the accuracy of the private household information set was decreased, due to weaker responses to shocks. We argued that this decreases the relative price distortions due to staggered prices, and that less precise household indicators improves the welfare of the representative consumer.

We cannot a priori say which one of these effects that will dominate when the accuracy of the common information set is changed, but our analysis pointed out that the positive effects of more noise are likely to be smaller, either when the households do not care much about the future, or when today's shock have little predictive power of future shocks, i.e. with low shock persistence. Indeed, we could produce opposite welfare effects of increased accuracy by varying the persistence of cost push shocks, holding all other parameters fixed.

One should note the difference between the forces at work in the model presented here, and those at work in models where households are perfectly informed. As Pearlman (1991) and Nimark (2003) point out, partially informed discretionary policy tends to display more inertia, and thus mimics the behavior of optimal policy under commitment. In the absence of a commitment technology, noise in the central bank's indicators can improve welfare. Under the assumption of a perfectly informed public, the positive effects of noise disappears when a commitment technology exists. However, the positive welfare effects of a badly informed public should be robust to the existence of a commitment technology.

An implication of the negative welfare consequences of a well informed public is that central banks should be restrictive with publishing their real time data. This result holds regardless of whether the central bank is better or worse informed than the public, since any additional information released to the public will improve the accuracy of the households' filtering problem. This is both in line with and contrary to previous results by Amato and Shin (2003), who also find that a central bank should not release its real time data to the public. The mechanics behind their result is different from the one in the present paper though, and relies on the (realistic) assumption of diversity of information within households. In their model, it is the assumption that the released information is common among households that makes it detrimental to welfare, not its accuracy. Indeed, they find that the release of indicator data to the public is less detrimental to welfare the more accurate it is.

There are of course other reasons why transparency may be desirable that are not covered in the present paper. One often cited such reason is accountability. Access to the real time data that past decisions were based on is vital to a fair judgement of those decisions. In practice there should thus exist a trade off between the negative effects of transparency put forward in this paper, and the need to hold decision makers accountable.

The present paper points out some directions of possible future research. The quantitative importance of the welfare effects of indicator accuracy are largely unknown. To address this issue we need to develop convincing methods of characterizing present indicator uncertainty. The paper by Lippi and Neri (2003) is a first important step in this direction. This may also lead us to revise estimates of the structural parameters of the model, since these are not independent of the size of measurement errors or of the information structures assumed.

A The Partial and Diverse Information Model: Set up and solution.

Write the structural model of Section 2 as

$$\begin{aligned} x_t &= GX_t + Tx^h_{t+1|t} + Bi_t, & (61)\\ X_t &= \rho X_{t-1} + u_t & (62)\\ G &= G_0^{-1}G_1, T = G_0^{-1}T_1, B = G_0^{-1}B_1. & (63)\\ G_0 &= \begin{bmatrix} 1 & -\delta\phi \\ 0 & 1 \end{bmatrix}, G_1 = \begin{bmatrix} \delta & 0 & -\delta\phi \\ 0 & 1 & 1 \end{bmatrix}, & (64)\\ T_1 &= \begin{bmatrix} \beta & 0 \\ \frac{1}{\gamma} & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ -\frac{1}{\gamma} \end{bmatrix} & (65) \end{aligned}$$

A.1 Agents' perceived laws of motion

The central bank will fit the following process

$$x_t = L^{cb}X_t + Bi_t + \zeta^{cb}_{t|t} + H^{cb}\zeta^{cb}_{t-1|t-1} \tag{66}$$

while households will fit

$$x_t = GX_t + Tx^h_{t+1|t} + Bi_t \tag{67}$$

$$\begin{aligned} x^h_{t+1|t} &= \left(G + M^h\right)\rho X^h_{t|t} + H^h\zeta^h_{t|t} & (68)\\ \zeta^h_{t|t} &= \left(y^h_{t|t} - \overline{y}^h_{t|t} + \frac{\delta\phi}{\lambda}\pi^h_{t|t}\right) & (69) \end{aligned}$$

(66) to (69) are the central bank's and households' perceived laws of motion of the endogenous variables.

A.2 Defining the observables Z_t

$$\begin{aligned} Z_t &= D\begin{bmatrix} X_t \\ x_t \end{bmatrix} + \mathsf{v}_t, & (70)\\ Z^i_t &= D^iZ & (71) \end{aligned}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$D^{cb} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, D^{h} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A.3 Estimating the state

A.3.1 The signal extraction problem of the central bank

The central bank need to estimate the state vector $\widehat{X}_t^{cb}$ defined as

$$\widehat{X}_t^{cb} = \left[X_t, \zeta_t^{cb}, \zeta_{t-1}^{cb}\right]' \tag{72}$$

where the estimate $\widehat{X}_{t|t}^{cb}$ will be given by

$$\widehat{X}_{t|t}^{cb} = \widehat{X}_{t|t-1}^{cb} + K^{cb}\left(Z_t^{cb} - L^{cb}\widehat{X}_{t|t-1}^{cb} - M^{cb}\widehat{X}_{t|t}^{cb}\right) \tag{73}$$

where

$$L^{cb} = D^{cb}\begin{bmatrix} G & I & H^{cb} \end{bmatrix} \tag{74}$$

$$M^{cb} = D^{cb}BF \tag{75}$$

$$K^{cb} = P^{cb}L^{cb\prime}(L^{cb}P^{cb}L^{cb\prime} + \Sigma_{vv}^{cb})^{-1} \tag{76}$$

$$P^{cb} = \widehat{\boldsymbol{\rho}}(P^{cb} - P^{cb}L^{cb\prime}(L^{cb}P^{cb}L^{cb\prime} + \Sigma_{vv}^{cb})^{-1}L^{cb}P^{cb})\widehat{\boldsymbol{\rho}}' + \widehat{\Sigma}_{\mathsf{u}u}^{cb} \tag{77}$$

$$\widehat{\rho} = \begin{bmatrix} \boldsymbol{\rho} & 0 & 0 \\ \Sigma_{\zeta X}^{cb}\left(\Sigma_{\zeta\zeta}^{cb}\right)^{-1}\boldsymbol{\rho} & 0 & 0 \\ 0 & I & 0 \end{bmatrix}, \widehat{\Sigma}_{\mathsf{u}u}^{cb} = \begin{bmatrix} \Sigma_{\mathsf{u}u} & 0 & 0 \\ 0 & \Sigma_{\zeta\zeta}^{cb} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{78}$$

Note that

$$\widehat{X}_{t|t-1}^{cb} = \widehat{\boldsymbol{\rho}}\widehat{X}_{t-1|t-1}^{cb}. \tag{79}$$

A.3.2 The optimal interest rate under discretion

The optimal interest rate under discretion is set such that

$$\frac{\partial \Lambda_{t|t}^{cb}}{\partial i_t} = 0$$

subject to the central bank's perceived law of motion

$$x_t = GX_t + Bi_t + \zeta_{t|t}^{cb} + H^{cb}\zeta_{t-1|t-1}^{cb}$$

The linear-quadratic framework implies that the optimal interest rate can be written as linear function F of the estimated extended state $\widehat{X}^{cb}_{t|t}$

$$i_t = F\widehat{X}^{cb}_{t|t} \tag{80}$$

where

$$\begin{aligned} F \;=\; [\; & -\tfrac{b_1 g_{11}+\lambda b_2 g_{21}}{b_1^2+\lambda b_2^2} \quad -\tfrac{b_1 g_{12}+\lambda b_2 g_{22}}{b_1^2+\lambda b_2^2} \quad 0 \\ & -\tfrac{b_1}{b_1^2+\lambda b_2^2} \quad -\tfrac{\lambda b_2}{b_1^2+\lambda b_2^2} \quad -\tfrac{b_1 h^{cb}_{11}+\lambda b_2 h^{cb}_{21}}{b_1^2+\lambda b_2^2} \quad -\tfrac{b_1 h^{cb}_{12}+\lambda b_2 h^{cb}_{22}}{b_1^2+\lambda b_2^2} \;] \end{aligned} \tag{81}$$

Lower case letters denote element$_{i,j}$ of the coefficient matrices denoted by the corresponding capital letters.

Households' estimate $X^h_{t|t}$ will be

$$X^h_{t|t} = X^h_{t|t-1} + K^h\left(Z^h_t - L^h X^h_{t|t-1} - M^h X^h_{t|t} - Bi_t\right) \tag{82}$$

where

$$K^h = P^h L^{h\prime}(L^h P^h L^{h\prime} + \Sigma^h_{vv})^{-1} \tag{83}$$

$$P^h = \rho(P^h - P^h L^{h\prime}(L^h P^h L^{h\prime} + \Sigma^h_{vv})^{-1} L^h P^h)\rho' + \Sigma^h_{uu}; \tag{84}$$

$$L^h = D^h G \tag{85}$$

$$M^h = D^h \begin{bmatrix} \frac{\delta}{1+\frac{1}{\lambda}\delta^2\phi^2}\sum_{k=1}^{\infty}\left(\frac{\beta}{1+\frac{1}{\lambda}\delta^2\phi^2}\rho_\pi\right)^k & 0 & 0 \\ \left(-\frac{\delta\phi}{\lambda}\right)\frac{\delta}{1+\frac{1}{\lambda}\delta^2\phi^2}\sum_{k=1}^{\infty}\left(\frac{\beta}{1+\frac{1}{\lambda}\delta^2\phi^2}\rho_\pi\right)^k & 0 & \rho_{\overline{y}} \end{bmatrix} + D^h T H^h \Delta^h \tag{86}$$

$$\Delta^h = [[\begin{matrix} \frac{\delta\phi}{\lambda} & 1 \end{matrix}]\, G - [\begin{matrix} 0 & 0 & 1 \end{matrix}]] \tag{87}$$

A.4 The actual law of motion

The perceived laws of motion can be mapped into an AR(1) actual law of motion

$$\Pi_0 \begin{bmatrix} X_t \\ \widehat{X}^{cb}_{t|t} \\ X^h_{t|t} \\ Z_t \\ x_t \\ \zeta^{cb}_t \\ \zeta^h_{t|t} \\ i_t \end{bmatrix} = \Pi_1 \begin{bmatrix} X_{t-1} \\ \widehat{X}^{cb}_{t-1|t-1} \\ X^h_{t-1|t-1} \\ Z_{t-1} \\ x_{t-1} \\ \zeta^{cb}_{t-1} \\ \zeta^h_{t-1|t-1} \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} \mathsf{u}_t \\ 0 \\ 0 \\ \mathsf{v}_t \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{88}$$

Where

$$\Pi_0 = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & \Gamma^{cb} D^{cb} & 0 & 0 & 0 & 0 \\ 0 & 0 & I & \Gamma^h D^h & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & -I & 0 & 0 & 0 \\ -G & -M^{cb} & -T\left(G + M^h\right)\rho & 0 & I & 0 & -TH^h & -B \\ -L^{cb} & 0 & 0 & 0 & -I & I & 0 & B \\ 0 & 0 & -\Delta^h & 0 & 0 & 0 & I & 0 \\ 0 & -F & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix} \tag{89}$$

$$\Pi_1 = \begin{bmatrix} \rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Psi^{cb} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Psi^h & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -H^{cb} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{90}$$

$\Gamma^{cb} = -\left[I + K^{cb} M^{cb}\right]^{-1} K^{cb}$
$\Gamma^h = -\left[I + K^h M^h\right]^{-1} K^h$
$\Psi^{cb} = \left[\left[I + K^{cb} M^{cb}\right]^{-1} \widehat{\rho} - \left[I + K^{cb} M^{cb}\right]^{-1} K^{cb} L^{cb} \widehat{\rho}\right]$
$\Psi^h = \left[\left[I + K^h\right]^{-1} \widehat{\rho} - \left[I + K^h T\right]^{-1} K^h L^h \widehat{\rho}\right]$
or

$$\widetilde{X}_t = \Pi \widetilde{X}_{t-1} + \mathsf{e}_t \tag{91}$$

$$\Pi = \Pi_0^{-1} \Pi_1 \tag{92}$$

where e_t has covariance

$$\Sigma_{ee} = \Pi_0^{-1\prime} \begin{bmatrix} \Sigma_{uu} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Sigma_{vv} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Pi_0^{-1} \tag{93}$$

A.5 Finding the implied actual laws

To find the actual $\left\{\Sigma^{cb}_{\zeta\zeta}, \Sigma^{cb}_{X\zeta}, H^{cb}, H^{h}\right\}$, given the system (91) we first need to find the covariance matrix of $\widetilde{X}_t$.This is given by the discrete Lyaponov equation

$$\Sigma_{\widetilde{x}\widetilde{x}} = \Sigma_{ee} + \Pi'\Sigma_{\widetilde{x}\widetilde{x}}\Pi \tag{94}$$

One should note that the 'true' ζ^{cb}_t is included as a state variable defined as

$$\zeta^{cb}_t = x_t - Bi_t - H^{cb}\zeta^{cb}_{t-1}.$$

This is necessary to find the implied actual $\Sigma^{cb}_{\zeta\zeta}$ and $\Sigma^{cb}_{X\zeta}$, given by the elements of $\Sigma_{\widetilde{x}\widetilde{x}}$ corresponding to the covariance of the 'true' ζ^{cb}_t with itself, and the exogenous process X_t. The implied actual coefficient matrices H^{cb} and H^h can be found by

$$H^{cb} = \Pi\Sigma_{x\zeta cb}\Sigma^{-1}_{\zeta\zeta cb} \tag{95}$$

$$H^{h} = \Pi\Sigma_{x\zeta h}\Sigma^{h-1}_{\zeta\zeta} \tag{96}$$

where

$$\begin{aligned} \Sigma_{x\zeta cb} &= covar\left(x_t, \zeta^{cb}_t\right), \Sigma_{\zeta\zeta cb} = covar(\zeta^{cb}_t), \\ \Sigma_{x\zeta h} &= covar\left(x_t, \zeta^{h}_{t|t}\right). \end{aligned}$$

A.6 Finding the limited information rational expectations equilibrium

1. Start with an initial guess of $H^{cb}, \Sigma^{cb}_{\zeta\zeta}, \Sigma^{cb}_{X\zeta}$ and H^h in (74), (78), and (86).
2. Calculate the optimal policy F given the central banks perceived laws of motion.
3. Use (88) -(96) to map the perceived laws of motion of all classes of agents into actual laws of motion.
4. Replace the initial guess of $H^{cb}, \Sigma^{cb}_{\zeta\zeta}, \Sigma^{cb}_{X\zeta}$ and H^h with their actual counterparts.
5. Repeat 1-4 until perceived and actual laws of motion have converged for all agents.

B Computing the loss function

The procedure is a slightly modified version of the one described in Söderlind (1999). Start by rewriting the loss function (29) in matrix form as

$$L_t = E_t \left[\sum_{i=0}^{\infty} \beta^i \widetilde{X}'_{t+i} Q' R Q \widetilde{X}_{t+i} \right]. \tag{97}$$

where

$$Q_{2x30} = \left[\begin{array}{ccccccccc} q_X & 0 & 0 & 0 & q_x & 0 & 0 & 0 & 0 \end{array} \right], \tag{98}$$

$$q_X = \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & -1 \end{array} \right], q_x = \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right] \tag{99}$$

$$R = \left[\begin{array}{cc} 1 & 0 \\ 0 & \lambda \end{array} \right] \tag{100}$$

The loss function can now be calculated as

$$L_t = \widetilde{X}'_t V \widetilde{X}_t + \frac{\beta}{1-\beta} trace\left(V \Sigma_{ee}\right) \tag{101}$$

V can be found by iterating (backwards in 'time') on

$$V_s = Q' R Q + \beta \Pi' V_{s+1} \Pi \tag{102}$$

C Parameter values used in figures

Figure	ρ_π	ρ_y	$\rho_{\overline{y}}$	σ_{vy1}	σ_{vy2}	σ_{vy3}
1	.5	.5	.9	0.1/1.0		
2	.5	.5	.9	-	0.01	0.2
3	.5	.5	.9	0.2	0.01	-
4	.1	.5	.5	0.04	-	0.2
5	.9	.5	.5	0.04	-	0.2

β	γ	φ	ϵ	θ	$\sigma_{\xi\pi}$	$\sigma_{\xi y}$	$\sigma_{\xi\overline{y}}$	$\sigma_{v\pi}$
.99	2	$1\frac{1}{2}$	10	$\frac{3}{4}$	0.035	0.015	0.01	0.001

References

[1] Amato, J. and Shin, H.S., 2003, Public and Private Information in Monetary Policy Models", mimeo Bank of International Settlements.

[2] Bomfim, Antulio N., 2001, "Measurement error in general equilibrium: the aggregate effects of noisy indicators", Journal of Monetary Economics, vol 48, pp.585-603.

[3] Clarida, R., Gali, J. and Gertler, M. , 1997, "Monetary Policy Rules in Practice: Some International Evidence", NBER working paper 6254.

[4] Clarida, R., Gali, J. and Gertler, M. 1999, "The Science of Monetary Policy: A New Keynesian Perspective",Journal of Economic Literature Vol XXXVII, pp 1661-1707.

[5] Diebold, F. and Rudebusch, G., 1991, "Forecasting output with a composite leading index: a real-time analysis.", Journal of the American Statistical Association 86, pp 603-610.

[6] Ehrmann, M. and Smets, F., 2001, "Uncertain Potential Output: Implications for Monetary Policy", ECB working paper series, no. 59.

[7] Ellingsen, T and Söderström, U., 2003, "Monetary Policy and the Bond Market", mimeo (?) Sveriges Riksbank.

[8] Gali, Jordi, 2002, "New Perspectives on Monetary Policy, Inflation and the Business Cycle", NBER working paper 8767.

[9] Harvey, Andrew C., 1989, Forecasting, Structural Time Series Models and the Kalman Filter, Cambridge University Press, Cambridge

[10] Kriz, Peter N., (2003), "Optimal Monetary Targeting Regimes Under Imperfect Information", unpublished draft, University of California at Santa Cruz.

[11] Krueger, Alan B., 1997, Comment on "How Precise Are Estimates of the Natural Rate of Unemployment?", in Romer, D. and Romer, C.(eds), Reducing inflation: motivation and strategy, University of Chicago Press.

[12] Kuttner, Kenneth N., 1994, "Estimating Potential Output as a Latent Variable", Journal of Business & Economic Statistics, Vol 12, Nr 3, pp361-368.

[13] Lippi, F. and Neri, S., 2003, "Information variables for monetary policy in a small structural model of the Euro area", mimeo Bank of Italy.

[14] Mankiw, N.G., Runkle, D. and Shapiro, M., 1984, "Are preliminary announcements of money stock rational forecasts?", Journal of Monetary Economics, vol 14, pp15-27.

[15] Nimark, Kristoffer P., 2003, "Monetary Policy and the Accuracy of Information", EUI working paper ECO #2003/8.

[16] Pearlman, Joseph G., 1986, "Diverse Information in Rational Expectations Models", Journal of Economic Dynamics and Control 10, pp333-338.

[17] Pearlman, Joseph G., 1991, "Reputational and nonreputational policies under partial information", Journal of Economic Dynamics and Control 16, pp339-357.

[18] Pearlman, J.G., Currie, D., and Levine, P.,1986, "Rational Expectations Models with Partial Information", Economic Modeling 3, pp 90-105.

[19] Pearlman, J.G. and Sargent. T.J. , 2002, "Knowing the Forecasts of Others", mimeo, New York University.

[20] Phelps, Edmund S., 1983, "The Trouble with 'Rational Expectations' and the Problem of Inflation Stabilization", in R. Frydman and E. Phelps, eds, Individual Forecasting and Aggregate Outcomes, Cambridge University Press, pp 31-41.

[21] Rotemberg, J. and Woodford, M., 1999, "Interest Rate Rules in an Estimated Sticky Price Model",in John B. Taylor (Ed.), Monetary Policy Rules, University of Chicago Press.

[22] Sargent, Thomas J., (1991) "Equilibrium with Signal Extraction from Endogenous Variables", Journal of Economic Dynamics and Control 15,pp245-273.

[23] Staiger, D., Stock, J.H., and Watson, M.W., 1997, "How Precise Are Estimates of the Natural Rate of Unemployment?" , in Romer, D. and Romer, C.(eds), Reducing inflation: motivation and strategy, University of Chicago Press.

[24] Svensson, L.E.O, and Woodford, M., 2003,"Indicator Variables for Optimal Policy", Journal of Monetary Economics, vol 50, pp 691-720.

[25] Svensson, L.E.O, and Woodford, M.,2004, "Indicator variables for optimal policy under asymmetric information", Journal of Economic Dynamics and Control, vol 28, Issue 4, pp 661-690

[26] Söderlind, Paul, 1999, "Solution and estimation of RE macromodels with optimal policy", European Economic Review 43, pp 813-823.

[27] Townsend, Robert M.,1983, "Forecasting the Forecasts of Others", Journal of Political Economy 91, pp 546-588.

[28] Woodford, Michael, 1999, "Inflation Stabilization and Welfare", NBER working paper no. 8071.

Earlier Working Papers:

Sveriges Riksbank
Visiting address: Brunkebergs torg 11
Mail address: se-103 37 Stockholm

Website: www.riksbank.se
Telephone: +46 8 787 00 00, Fax: +46 8 21 05 31
E-mail: registratorn@riksbank.se

ISSN 1402-9103 Riksbank Printing Office Stockholm 2003