SVERIGES RIKSBANK

Sveriges Riksbank Working Paper Series

Capital Charges under Basel II: Corporate Credit Risk Modelling and the Macro Economy

SEPTEMBER 2002

No. 142

Kenneth Carling, Tor Jacobson, Jesper Lindé, Kasper Roszbach

WORKING PAPERS ARE OBTAINABLE FROM

Sveriges Riksbank • Information Riksbank • SE-103 37 Stockholm
Fax international: +46 8 787 05 26
Telephone international: +46 8 787 01 00
E-mail: info@riksbank.se

Capital Charges under Basel II: Corporate Credit Risk Modelling and the Macro Economy*

Kenneth Carling Tor Jacobson Jesper Lindé Kasper Roszbach

Sveriges Riksbank Working Paper Series
No 142
September 2002

Abstract

The Internal Ratings Based (IRB) approach for capital determination is one of the cornerstones in the proposed revision of the Basel Committee rules for bank regulation. We evaluate the IRB approach using historical business loan portfolio data from a major Swedish bank for the period 1994 to 2000. First, we estimate a duration model that takes into account both company, loan related and macroeconomic variables. Next, we obtain a Value-at-Risk-type (VaR) credit risk measure, by model-based simulations. Moreover, we study how both the bank's credit risk and buffer capital changes over time (had the bank been subject to the proposed rules). This approach allows us to (i) make individual forecasts of default risk conditional on company, loan and macro variables, (ii) study portfolio credit risk over time, (iii) assess to what extent the new Accord will achieve its main objective of increasing credit risk sensitivity in minimal capital charges, and (iv) compare current capital requirements to those under the proposed system. Our results show that macro conditions have great explanatory power in predicting default risk and calculating credit risk. The IRB approach, although sensitive to the choice of some horizon parameters, is an achievement in the intended direction.

Keywords: Internal Ratings Based approach, relative risk weights, Value-at-Risk, credit risk models

JEL: C41, G21, G33, G38

*Research Department, Sveriges Riksbank, 103 37 Stockholm. Thanks to David Cesarini for excellent research assistance. Preliminary and incomplete versions of this paper have been circulated under the title "The Internal Ratings Based Approach for Capital Adequacy Determination: Empirical Evidence from Sweden". We are grateful for comments and suggestions from seminar participants in the Workshop on Applied Banking organized by the Research Task Force of the Basel Committee in Oslo, 2001, Sveriges Riksbank, the Society for Economic Dynamics meeting in Stockholm, 2001, the Federal Reserve Bank of Philadelphia, and the conference on "The Theory and Practice of Credit Risk Modeling", organized by the Stockholm Institute for Financial Research, 2002. Special thanks to Sonja Daltung, Frank Heid, Gunnar Isacsson, Olivier Renault, and Anders Vredin. All remaining shortcomings are the authors responsibility. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Executive Board of Sveriges Riksbank.

1 Introduction

Banks play an important role in the economy as savings institutions and as providers of credit and capital. Beside government supervision, deposit insurance and other regulatory conditions, capital requirements limit risks for depositors, and reduce insolvency and systemic risks. However, they also form important restrictions on the workings of banks. Unnecessary capital requirements restrain credit provision needlessly, whereas inadequate capital requirements may lead to undesirable levels of systemic risk.

In 1988, the Basel Committee on Banking Supervision introduced the so-called Basel Accord that dictates regulation for banks' minimal capital requirements.[1] The purpose of the Accord is to promote safety and soundness in the banking industry. The Accord, initially intended for internationally active G-10 banks, has by now been adopted by more than 100 countries around the world. Over time there has been a growing concern that the effectiveness of the Accord has eroded as banks have devised ways to engage in regulatory capital arbitrage introducing mismatching of risks taken on and buffer capital held.[2] In response to these concerns, the Basel Committee released its proposal for the future capital adequacy rules, i.e., the new Accord known as Basel II.[3]

One of the guiding principles of the new Accord is that the size of the required buffer capital is made contingent on counterpart risk instead of being constant per credit type as under the current accord. A portfolio is characterized by a relatively small number of risk categories and each risk or 'rating' category will be associated with a specific risk weight. Summing over the loans yields the risk weighted estimate of the bank's assets that determines the required minimum buffer capital. The size of the risk weights can be based on either an external rating institution's evaluation of counterpart risk, or on information collected and processed within the bank, that is, the bank's internal ratings.

This paper employs a unique and rich data set from a leading Swedish international investment bank to address a number of questions. First, we examine the properties of the bank's internal rating system. Is credit risk really monotonically increasing over the rating classes? Are the classes consistent over time? Are the transitions from relatively risky classes to relatively safe ones in accordance with the general development of Swedish economic conditions and the reduction in bankruptcy incidents for this time period?

Secondly, we use the bank data to estimate a statistical duration model of the survival time of bank loans until default. The reduced form model that we employ enables us to enter

[1] This committee works, although not hierarchically, under the Bank for International Settlements (BIS).

[2] Jackson et al.[28] provides an overview of the extensive empirical literature on the impact of the 1988 Accord on banks' behaviour.

[3] The second, revised, proposal can be found on the homepage of Bank for International Settlements at: www.bis.org/publ/bcbsca.htm.

factors that drive default behavior in the corporate sector, and to quantify to what extent they contribute to predicting default realizations. Our approach differs from the 'structural' approach that a number of institutions have applied to develop portfolio credit risk models, like KMV, with PortfolioManager (www.portfoliomanager.com) and J.P. Morgan with CreditMetrics (www.riskmetrics.com). These are based on Merton's [36] model of firms' capital structure. We will briefly discuss how the two approaches differ in Section 1.2, but one important reason for using a reduced form duration model is that, unlike all previous studies of corporate default risk, we dispose of a large longitudinal data set with observations on loans to more than 50,000 firms during 24 quarters between 1994 and 2000.[4] The bank data has been augmented with real time information on the characteristics of the firms from Upplysningscentralen AB, a leading credit bureau in Sweden. As a result of this, we can separate idiosyncratic and macroeconomic effects - the latter represented by, i.a., the output gap and the yield curve - by exploring the time-series and the cross-sectional dimensions of the data.

Next, we calculate estimated probabilities of default for each counterpart in each of the 24 quarters using the credit risk model. By combining these estimates with exposure data we can derive total expected losses per counterpart, and also for the entire portfolio. Hence, the estimated credit risk model serves as a basis to simulate a time series of a portfolio credit risk measure of Value-at-Risk type.

Fourth, we examine the magnitude of variation in probability of default (PD) estimates based on both historical frequencies and on model-estimated probabilities.[5] Since the new Accord opens up for several alternatives for the important calculation of the average PD, which is meant to characterize a rating class, investigating PD's sensitivity to these alternative methods will provide us with important information for policy.

Fifth, we evaluate the risk-sensitiveness in the internal ratings based (IRB) capital requirement using the above mentioned VaR-measure. At this stage we also investigate if IRB capital requirements adequately capture changes in portfolio credit risk over the business cycle.

Finally, we study how the IRB capital requirement varies with probability-of-default estimation method. How does it behave over the business cycle? And how do the number of rating classes to be used in the IRB approach affect IRB capital requirements?

[4] An exception is McKinsey's portfolio credit risk model, described by Wilson [40]. CreditPortfolioView is a multifactor country/sector logit model that contains a set of macroeconomic indices, each modeled as an ARIMA process. No information is published on the coefficient estimates and other model properties, however.

[5] The most obvious approach is to base the estimate on the bank's own long-term default experience. Alternatively the bank can use default predictions from external ratings, or from a statistical default risk model.

1.1 Policy background

The 1988 Basel Accord, that dictates regulation for banks' minimal capital requirements, aims to promote safety and soundness in the banking industry. In recent years, however, concerns about erosion of the Accord's effectiveness have grown. In response to the Accord's rules, banks have devised ways and created means to evade the regulation's restrictions. For example, the securitization by banks of certain types of debt, like mortgage loans and low risk corporate loans, reflect the inability of Basel I to discern any difference in risk profile within a product category. As a result, it was perceived that banks had been increasing their portfolios' riskiness (and expected return) given a more or less predetermined size of the buffer capital.

In response to these concerns, the Basel Committee released a consultative document containing a proposal for new, adjusted, capital adequacy rules in June 1999. After extensive interaction with the banking industry, a second, revised, proposal for a New Basel Capital Accord was issued in January 2001. This proposal, once finalized, will replace the 1988 regulation.

The revised proposal is organized around three so-called pillars. The first one describes the rules for determination of banks' required buffer capital, intended to cover for incurred credit-losses. The second pillar concerns the supervisory review process of banks' internal procedures for capital determination with respect to risk profile. The purpose of the third pillar is to increase the transparency of bank's risk profiles for market participants through disclosure requirements, i.e., to promote market disciplinary effects towards sound banking practice.

The first pillar proposes two main routes for banks to follow when determining risk weights. First, a "standardized approach" designed to be applicable for every bank. In this approach a portfolio of bank loans will be characterized by a relatively small number of risk categories, and the risk weight associated with a given category is based on an external rating institution's evaluation of counterpart risk. Second, a more elaborate model: the so-called Internal Ratings Based (IRB) approach. The underlying idea of the IRB approach is to make further use of the information collected and processed in the bank's internal counterpart rating operation. Since banks make it a business to evaluate risks, these evaluations ought to be a reasonable basis for risk-contingent capital adequacy determination. Each internal rating category in a loan portfolio is characterized by an estimate of its average probability of default, calculated by the bank itself. By means of an estimated function, the supervisory authority provides a mapping from the estimated probability of default to a relative risk weight. The products of relative risk weight, exposure at the time of default (usually taken as the face value of the loan), and the 8 percent absolute capital requirement, summed over the loans of the portfolio give the bank's required buffer capital. The current proposal suggests that the banks may choose to apply the IRB-approach at either of two levels of sophistication. The more advanced requires bank internally generated inputs on loss given default and exposure at default, whereas the simpler only requires the bank to provide estimates of probability of default. Altman and Saunders [6] extensively criticize the Basel II proposal. Among other things, they find that relying on traditional agency ratings may produce cyclically lagging rather than leading capital requirements. They also conclude that the current risk based bucketing proposal lacks a sufficient degree of granularity. They advise to propose a weighting system that more closely resembles the actual loss experience on loans.

The Basel II proposal thus contains some elements of choice. In addition, information about the implications of these choices remains scarce. In Section 4 we will attempt to clarify a number of properties of the IRB approach that may prove to be important for these choices, but have hitherto remained more or less un-investigated with respect to corporate loans. Among other things, we will look at a number of important properties of the bank's internal rating system; the sensitivity of PD estimates to alternative estimation methods and the sensitivity of the IRB capital requirement to changes in risk and to variations over the business cycle. First, however, we will discuss some aspects of the current state of credit risk modeling.

1.2 Credit risk modeling

The literature on credit risk modeling is extensive and starts in the 1960's with research by Altman [2]. Following Altman, a number authors have estimated various types of default risk models on cross-sectional data sets. See for example Altman [3], Altman [4], Frydman, Altman and Kao [19], Li [34], and Shumway [38]. These papers all have a single focus on the analysis of (credit) risk and the prediction of bankruptcy at the firm level.

In the last decade, a whole range of modeling techniques has been developed to analyze portfolio credit risk. Broadly viewed, there are three groups of portfolio credit risk models. The first group is 'structural' and based on Merton's [36] model of firm capital structure: individual firms default when their assets' value fall below the value of their liabilities. Examples of such a microeconomic causal model are CreditMetrics and KMV's PortfolioManager. The second group consists of econometric factor risk models, like McKinsey's CreditPortfolioView. McKinsey's model is basically a logistic model where default risk in 'homogeneous' subgroups is determined by a macroeconomic index and a number of idiosyncratic factors. These two model types apply similar Monte Carlo simulations to calculate portfolio risk, as both are 'bottom-up' models that compute default rates at either the individual firm level or at sub-portfolio level. Both thus require a similar kind of aggregation. The third group contains 'top-down' actuarial models, like Credit Suisse's CreditRisk+, that make no assumptions with regard to causality.

Koyluoglu and Hickman [31] provide an elaborate description of the above mentioned types of portfolio credit risk models. They note that all model types, despite their differences, are built on three more or less general components to calculate portfolio loss distributions. First, they contain some process that generates conditional default rates for each borrower in each state of nature and a measure of co-variation between borrowers in different states of nature. Second, their set-up allows for the calculation of conditional default rate distributions for sets of homogeneous sub-portfolios (e.g., rating classes) as if individual borrower defaults are independent, since all joint behavior is accounted for in generating conditional default rates. Third, unconditional portfolio default distributions are obtained by aggregating homogeneous sub-portfolios' conditional distributions in each state of nature; then conditional distributions are averaged using the probability of a state of nature as the weighting factor. In Sections 3 and 4 we will see that the duration model approach followed in this paper is contained by this general description of portfolio credit risk models.

Gordy [20] confirms the general insights of Koyluoglu and Hickman in a thorough comparison of two influential benchmarks for credit risk models, CreditMetrics and CreditRisk+. He concludes that they have very similar mathematical structures and that the prime sources of discrepancies in predictions are differences in distributional assumptions and functional forms. Gordy's findings suggest some general insights into the workings of these credit risk models. Among other things, he concludes that the models are highly sensitive to both the average default correlations in the model - that in turn determine default rate volatility - and the shape of the implied distribution of default probabilities.

Since the work on the reform of the Basel Accord started, a number of efforts have been made to apply credit risk models to the ultimate goal of calculating capital requirements under a variety of alternative systems. Estrella [15], for example, contains a theoretical model of optimal bank capital. He finds that a regulatory minimum capital requirement based on VaR is likely to be procyclical and suggests some ways to remedy this procyclicality. Gordy [21] examines the relation between portfolio models of credit VaR and ratings-based bucket models. He concludes that the latter can be reconciled with the general class of credit VaR models and that even portfolio credit VaR models imply marginal capital charges that depend only on an asset's own characteristics under some very general assumptions. Carey [10] contains a new non-parametric methodology to estimate loss rates in the bad tail of the credit loss distribution. Calem and LaCour-Little [9] estimate a survival time model for mortgage loan data and apply Carey's method to simulate PD distributions. They find that capital charges vary substantially with loan or borrower characteristics. They also conclude that capital charges are generally below the current standard - thereby providing some empirical support for the occurrence of securitization. Hamerle et al. [23] follow another approach and model the (unconditional) PD's by means of a non-linear random effects probit and logit model. Carey and Hrycay [12] empirically examine the properties of the most commonly used methods to estimate average PD's by rating class. They find that the mapping and scoring-model methods are potentially subject to bias, instability and gaming.

As a result of the interest that the reform of the Basel rules has generated, a number of authors have also examined the design of banks' internal ratings systems and the consequences that their design have for the functioning of the Basel II. Treacy and Carey [39], for example, describe the ratings systems of large U.S. banks and collect some statistics on the distribution of loans over rating classes and the related loss rates and risk profiles. Carey [11] finds, based on simulated data, that the success of the IRB approach will depend on the extent to which it will take into account differences in assets and portfolio properties, such as granularity, risk properties and remaining maturities.

So far few studies contain a rigorous analysis of the effects from macroeconomic conditions on credit risk. Wilson [40], who describes the principles behind McKinsey's portfolio credit risk model CreditPortfolioView, is an exception. CreditPortfolioView incorporates a set of macroeconomic variables in a multifactor country/sector logit model. Each 'factor' is modeled as an ARIMA process. No information is published, however, on the coefficient estimates and other model properties. The duration model of the survival times of bank loans until default that we present in Section 3 includes both idiosyncratic and macroeconomic explanatory variables.

2 Data

This section is entirely devoted to a rather detailed description of the data set. It is probably not an exaggeration to state that these data are unique. Not only do we have detailed bank information on all counterparts in the bank's business loan portfolio, but also extensive information on the counterparts themselves, and, perhaps most interesting, we have this information for 24 consecutive quarters. The latter allows for dynamic analyses of, e.g., portfolio credit risk with respect to the macroeconomic development.

The final data set is a panel consisting of 576,768 observations on bank counterparts, covering six years of quarterly data on all 54,603 Swedish *aktiebolag* companies that had one or several loans outstanding at the bank on the last day of at least one quarter between March 31, 1994, and March 31, 2000. *Aktiebolag* are by approximation the Swedish equivalent of US corporations and UK limited businesses. Swedish law requires every *aktiebolag* to have at least SEK 100,000 (approximately US$ 10,000) of equity to be eligible for registration at the Swedish Patent and Registration Office (PRV). Firms are also required to submit an annual report to PRV. Although we have annual report data on small firms such as general partnerships, limited partnerships and sole proprietors, these will be disregarded because we do not dispose of the relevant credit histories. This means that we have deleted approximately 20% of all firms in the portfolio.[6] Observe, however, that a large part of the sample still consists of relatively small enterprises: 65% of the observations concern businesses with 5 or fewer employees.

The data on these firms come from two sources: from the bank itself and from Upplysnings-

[6] However, these 20% of firms only represent approximately 6% of the portfolio loan value.

centralen AB (UC), a major credit bureau in Sweden. The bank supplied a full history of internal credit related data, including variables like the amount of credit granted, actual exposure, the types of credit, the amount granted per credit type, collateral, payment status, and an internal risk classification. These data were available at a quarterly frequency. Upplysningscentralen provided us with non-bank specific data for each company in the bank's portfolio, which it collects from the PRV annual report data. For example, balance sheet and income statement data from the annual report were provided, but also historical data on events related to payment remarks and payment behavior for the company and for its principals. These data were available at different frequencies, varying from daily for payment remarks to annually for accounting data. We will discuss the specifics of both data sources in greater detail below.

2.1 Bank data

As mentioned earlier, as part of its risk management system the bank maintains an internal credit rating scheme; this requires each business customer to be assigned to one of 15 credit rating classes. Rating class 1 represents the highest credit quality and Rating class 15 stands for the lowest credit quality, factual default, with the intermediate credit rating classes intended to imply a monotonically increasing risk profile. At a minimum, the bank updates the credit rating of each firm in its portfolio every 12 months. We refer to Section 4.1 for a more elaborate description of the rating scheme. For the purpose of this study we will use the bank's definition of a default: a loan that is assigned to Rating class 15. The necessary criteria for such an assignment is that principal or interest payments are 60 days overdue, but moreover, a bank official has to make a judgement and reach the conclusion that any such payment is unlikely to occur in the future. A comparison with data from the credit bureau shows that Rating class 15 is highly correlated with (the officially registered) bankruptcy. Generally the rating class leads the latter by one or more quarters, most likely due to the length of legal procedures that have to be completed before bankruptcy is officially invoked.

Figure 1 shows that there is quite some movement over time in the average default rate of the bank's portfolio. The maximum rate of default within the sample period is reached in the second quarter of 1995, at a level of 2%, although its overall peak in all likelihood occurred in 1991-92 in connection with the Swedish banking crisis. Over the whole of 1995, 4% of the bank's counterparts defaulted, compared with an average annual rate over the whole sample period of 2.6%. After 1995 the default rate declines, reaches two smaller peaks; in the second quarter of both 1996 and 1997, of 1.0% and 1.5%, and then steadily falls to an almost zero level in first quarter of 2000.

The bank provided us with the complete credit history of each business customer. The more important credit variables are: the size of the loan, actual exposure, the rating class, the industry code, and a number of variables splitting up total credit in different types of loans.

Appendix A contains a full list of the variables provided to us by the bank. We reduced a total of 19 types of credit to 5 broader groups, also used by the bank for certain analytical purposes: short term lending, long term lending, mortgages, guarantee loans and the remainder, mixed loans. Of all counterpart observations, 67% involve short term loans while 32% concern long run loans, 5% mortgages, 17% guarantee loans and 20% mixed loans (the remaining credit types). More than 40% of the observations involve at least two types of credit, and in 21%, businesses have both a short and a long term loan, implying that about two thirds of the businesses that borrow long also borrow short term. Other credit type combinations that have a frequency of 5% or more are: short term and guarantee loans, short term and mixed loans and guarantee and mixed loans. The average loan duration for a firm is 10.8 quarters. If split up according to credit type, the average duration for a short term loans is 9.9 quarters, whereas long term loans have an average duration of 10.7 quarters.

Figures 2 and 3 show that the default rate varies significantly, not only over time, but also across loan types and between (and even within) industries. For most of the sample period, short term loans are associated with the highest default rates. One exception is the last quarter of 1994, when mortgage defaults reach a peak of 3.1% and the period 1997 Q3 - 1998 Q3, when the long-term loan default rates slightly exceed short-term rates. The four largest industries in terms of total average exposure are multi-family real estate, manufacturing of machinery & equipment, commercial real estate and wholesale. Together they account for 48% of the bank's loan portfolio on average. The quarterly default rates in three of these industries peak simultaneously in the third quarter of 1995, although at highly varying levels. The fourth industry, multi-family real estate, reaches its peak default rate in the same quarter as the mortgage default rate: 1994 Q4. Commercial and multi-family real estate have the highest quarterly shares of defaults, 6.0% and 4.1% respectively. Wholesale and machinery & equipment default rates only reach top levels of 1.6% and 1.4%. After 1995, all four industries more or less follow the economy-wide pattern, their peaks in 1996 Q2 and 1997 Q2 ranging from .9% to 1.6%. Most other industries display a similar pattern over time. Two exceptions are the services industry, where the default rate appears to be more persistent, and the financial services industry, which displays a more erratic behavior, probably because of the smaller number of loans. In terms of average default rates, the commercial and multi-family real estate sectors rank first and second with quarterly rates of over 1%, followed at short distance by mining & quarrying, wood, pulp & paper and hotel & restaurants with rates between .8 and .9%. The best performing industries are electricity/gas and banking, with average rates of .2 and 0%. Chemicals, machinery & equipment and transport are the only other sectors with average default rates below 0.5%.

We suffice here with noticing that the main sources of the trend and fluctuations in the default rates were the Swedish real estate crisis in the early 1990's, the following recession which struck the Swedish economy during the first half and middle of the 1990's, and the accompanying banking crisis. Against this background, the zero average default rate for the bank's counterparts in the banking sector may appear somewhat surprising. This can be explained, however, by the fact that the Swedish government granted a non-bankruptcy guarantee to all banks and founded a national banking emergency authority in the fall of 1992. Bad loan portfolios of banks that were in risk of collapse were taken over and managed by this authority. For an extensive discussion of the macroeconomic background of the Swedish banking crisis, we refer to Englund [14].

The last variable of interest to be discussed here is the rating class. Figure 4 displays the default rates over time among companies in the rating classes for three different horizons: 1, 4 and 8 quarters ahead.[7] The general picture that is brought forward by these graphs is that default risk is not constant within rating classes over time. The upper left window shows, for example, that the one quarter default rate in Rating class 5 follows the movements of the business cycle and varies between 0 and .5% - even within short time intervals of 2 years. The same cyclical default pattern applies for Rating classes 6-14. Roughly, default rates appear to increase groupwise, with companies in Classes 6-10 exhibiting higher default risk than Classes 1-5, and Classes 11-14 representing the riskiest counterparts. Observe, however, that no risk monotonicity in any strict sense exists between the 14 classes. For example, Class 10 counterparts are clearly less risky than those in Rating classes 8 and 9. In Section 4.1 we discuss the rating classes more extensively.

2.2 Credit bureau data

The data set from the credit bureau contains information on most standard balance sheet and income statement variables. Some examples of balance sheet entries are cash, accounts receivable and payable, current assets and liabilities, fixed and total assets, total liabilities and total equity. Some examples of the income statement entries that were available are total turnover, earnings before interest, depreciation and amortization, depreciation, financial income, extraordinary income and taxes. Appendix B contains a complete list of all annual report variables. In addition to the annual report data collected by the Swedish Patent and Registration Office, we have information on the firms' track records regarding payment behavior as recorded by remarks for 61 different credit and tax related events. Two types of remarks exist. The first type is non-payment remarks, the storage and usage of which are regulated by the Credit Information Act, the Personal Data Act and overseen by the Swedish Data Inspection Board. Examples of events that are registered are: delays in tax payments, the repossession of delivered goods, the seizure of property, the resettlement of loans and actual bankruptcy. In practice, with a record of non-payment remarks individuals will not be granted any new loans and businesses will find it very difficult to open new lines of credit. The second type is bank remarks, which give an image of a firm's payment behavior at banks. All Swedish banks participate in this scheme and report any

[7] Zero default rates for some rating classes, like 11 and 14, in the first quarters are due to missing values.

abuse of a bank account or a credit card and slow loans (loans of which repayment is considered questionable) to the credit bureau that maintains these records. Their storage and usage is only regulated by the Personal Data Act. Whereas a bank remark may have the same consequences as a non-payment remark, this is not generally the case. Their effect on individual applications for credit presumably works mainly through the accumulation of negative indicators. Appendix C contains the complete list of non-payment and bank remarks.

As can be seen in Table 1, all descriptive statistics for accounting ratios and other credit bureau variables, such as non-payment and bank remarks and sales, were calculated based on different numbers of observations. For various reasons and depending on the specific variable, as many as 28,000 observations per variable could not be used directly in the estimation of the model. This could be due to incorrect entering of data by the credit bureau (unreasonable or negative values for non-negative balance sheet and income statement variables like total liabilities, total assets, inventories and sales), because of the nature of the ratio (a zero in the denominator), or simply the absence of any value. In all, this would have implied the deletion of approximately 10% of the sample. To avoid such reduction of our sample size, we replaced missing data on any variable by the mean value calculated on the basis of the available sample. As a result, the final estimation could be done with the full sample of 54,603 firms observed on 576,768 occasions.[8]

As annual reports typically become available with a significant time lag, it cannot in general be assumed that accounting data for year τ were available during or even at the end of year τ to forecast default risk in year $\tau + 1$. To account for this, we have lagged all accounting data by 4 quarters in the estimations. For most companies, who report balance sheet and income data over calendar years, this means that data for year τ is assumed to have been available in the first quarter of year $\tau + 2$. For a number of firms some transformation had to be applied to the accounting variables to adjust for reporting periods that did not coincide with the calendar year, to assure that each variable was measured in identical units for all companies. Some companies, for example, report accounting information referring to three-month or four-month periods for one or several years. In such cases, annual balance sheet figures were calculated as weighted averages of the multiple period values. In other cases companies did report numbers for a 12-month period, but the period did not coincide with the calendar year. The 1995 figures, for example, could refer to the period 1995-04-01 until 1996-03-31. In these cases, such 'deviations' were accounted for by adjusting the 'four quarter lag' (and thus the date at which the information is assumed to have been available) correspondingly.

[8]Imputing the mean for missing values may lead to underestimation of standard errors. Little and Rubin [35] propose use of multiple imputations to overcome this problem in a situation where values are missing in a non-systematic manner. Since statistical significance is hardly a matter of concern with well over half a million observations, we have chosen not to apply their technique in the analysis.

From the set of balance sheet and income statement variables in Appendix B, a number of commonly used accounting ratios was constructed. We selected 17 ratios that were employed in frequently cited articles studying bankruptcy risk. See Altman [1], [2], [3], and [4], Frydman, Altman and Kao [19], Li [34], and Shumway [38]. Most of the accounting ratios are closely related liquidity measures, two are leverage ratios and the remainder are profitability ratios. In our empirical model, we employ three accounting ratios: earnings before interest, depreciation, taxes and amortization over total assets (earnings ratio); total liabilities over total assets (debt ratio); and inventories over total sales (the inverse of inventory turnover). These three ratios were selected from the original list of 17 variables following a two-step procedure. First, the univariate relationship between the ratio and default risk was investigated. By visual inspection, ratios that displayed a clearly non-monotonic relation or lacked any correlation with default risk were deleted from the set of candidate explanatory variables. The left-hand column in Figure 5 illustrates this for the three selected ratios and for total sales (a proxy for firm size) by comparing default rates (solid line) and the cumulative distributions of the variables (dotted line). Default rates are calculated as averages over an interval of +/- 2,500 observations. There is a positive relationship between default risk and both the leverage ratio and the inverse of inventory turnover, while the figure suggests a negative relationship with both sales and the earnings ratio. We also checked if any significant differences in the average and median ratios existed between healthy and defaulting firms. Table 1 and the right-hand column in Figure 5 contain some additional information on the distribution and the development over time for the financial ratios and non-payment and bank remarks. Table 1 shows that defaulting firms in general have lower earnings, lower sales, higher inventories and a higher level of indebtedness. Figure 5 confirms this picture and suggests that these differences between (the median financial ratios of) healthy and defaulting firms are persistent, although possibly varying, over time. The median earnings of healthy enterprises, for example, are consistently more than twice as high as for defaulting ones. The difference in leverage ratio varies from approximately 15 percentage points in the mid-nineties to 25% in early 2000. On average, inventory turnover seems to be higher for defaulting firms, although there is some variation over time. Total sales differ in two respects between the two groups of businesses: they are strictly lower and vary more for defaulting firms than for healthy ones.

The above process led to the selection of six candidate variables: the three described above and three liquidity measures (cash over total assets, current assets over current liabilities, and accounts payable over sales). In a second step, their multivariate properties were studied by estimating a number of permutations of the empirical model. Neither of three liquidity measures turned out to make any significant contribution in the empirical model.

Table 1: Descriptive statistics for the credit bureau data.

Spell type	Statistic N	μ	σ	min	1%	50%	99%	max
Performing	573170							
TS (mn SEK)	560540	61.8	765.00	0	0	2.87	912.00	82600
EBITDA/TA	559525	.06	16.52	-8041	-.79	.11	.70	2946
TL / TA	559678	2.77	439.83	0	.09	.76	2.36	154051
I / TS	548862	.45	67.88	0	0	.03	1.92	24844
AMTYP25 (%)	573170	.20	.20	0				1
NA_AM (%)	573170	.90	.90	0				1
Defaulted	3598							
TS (mn SEK)	3077	8.58	36.50	0	0	1.80	120.00	810
EBITDA/TA	3062	-.36	14.20	-663	-2.39	.04	1.03	184
TL / TA	3063	19.24	552.13	0	.05	.93	9.70	19783
I / TS	2971	4.87	248.57	0	0	.05	4.34	13549
AMTYP25 (%)	3598	9.90	8.90	0				1
NA_AM (%)	3598	20.3	16.2	0				1

Notes: The definition of variables are: TS = total sales; EBITDA = earnings before taxes, interest payments and depreciations; TA = total assets; TL = total liabilities; I = inventories; AMTYP25 = a dummy variable taking the value of 1 if the firm has a "non-performing loan" at a bank in the preceding four quarters; NA_AM = a dummy variable taking the value of 1 if the firm has a payment remark due to one or more of the following events in the preceding four quarters; a bankruptcy petition, issuance of a court order to pay a debt, seizure of property. All variables are in nominal terms and in million SEK.

For the non-payment and bank remark variables the same procedure was followed. An intuitively reasonable starting point was to find remark events that (i) lead default as much as possible and (ii) are highly correlated with default. As it turned out, many remark variables are either contemporaneously correlated with default or lack a significant correlation with default behavior. An example of the first category is the start or completion of a company reconstruction. The most likely cause of this is the existence of a reporting lag. Tax related variables are typical examples of the second category. Of the remaining variables, many create a multicollinearity problem. For our final model, we selected two explanatory remark variables. One is a composite dummy of three events: a bankruptcy petition, the issuance of a court order - because of absence during the court hearing - to pay a debt, and the seizure of property. The other variable is "having a non-performing loan".

Finally, going back to Figure 3 we can note an interesting characterization of the default behavior by firm size. Table 1 shows that 10 respectively 20 % of the defaulting firms have a slow loan or a record of non-payment, in sharp contrast with the less than 1% among companies with performing loans. The second window confirms the common perception that smaller firms, such as small businesses without employees, run a higher risk of defaulting. At nearly every bankruptcy peak, these companies fail at a higher rate than other businesses.

2.3 Macro data

The importance of macroeconomic effects for credit risk is a virtually non-existing topic in the empirical literature, in all likelihood due to a lack of suitable historical credit data. Intuitively, increased aggregate demand should drive down default risks in the business sector. But what is the quantitative importance of macro over and above idiosyncratic risk factors? We hope to contribute to this area using the bank data described above.

The last window in Figure 6 shows the developments of the growth rate in real GDP, in 1995 prices, and the output gap, given by the difference between actual and estimated potential GDP, for the period 1980 Q1 to 2000 Q2. The series for potential output is computed using an unobservable components method due to Apel and Jansson [7]. The deep recession in the beginning of the 1990's can be clearly seen in the figure, with negative growth figures (over 4 per cent at most) and a negative output gap of over 8 per cent. The strong economic improvement of 1994-1996 is also evident. In the first window, the yield curve and the output gap series are related to the average portfolio default rate. There is a strong downward trend in the default rate over the sample period, reflecting the general improvement of the macroeconomic environment. Finally, in the middle window we show the Swedish households' expectations of the future macroeconomic development, with a lag of 2 quarters, along with the average default rate of the portfolio.

A priori, we think that these three macroeconomic variables should have a measurable impact on the default risk of any given firm. Starting with the output gap, it may supposedly work as an indicator of demand conditions, i.e., increased demand in the economy reducing default risk. Figure 6 seems, at large, consistent with this view, although there are some big spikes in the default rate that clearly have to be attributed to other variables. Apart from firm-specific factors, we believe that the two other macroeconomic variables presented above are important. Recent research, see, e.g., Estrella and Hardouvelis [16] and Estrella and Mishkin [17], suggests that the yield curve can be an important indicator of future real activity; i.e., a positively sloping yield curve signaling higher future economic activity and vice versa. Therefore, we expect that an increase in the spread between a short- and long-term interest rate is associated with decreasing default rates, since banks and firms will act upon this information. Banks will have stronger incentives to renegotiate loan terms with firms at the brink of bankruptcy. Given prospects of increased future demand, firms will likewise have incentives to avoid defaulting. By similar arguments, we expect that higher household expectations about future economic activity also reduce the default rate today.

We use the difference between the nominal interest rates (annualized) on 10-year government bonds and 3-month treasury bills as the measure of the spread. The index of household expectations about the future stance of the macroeconomy is taken from the survey data produced by Statistics Sweden. In the credit risk model, we will enter the series for the output gap and the household expectations with a lag of two quarters, since they are available for forecasting purposes with approximately that time delay. However, we will not lag the series for the yield curve spread, since this variable is accessible in real time.

3 The credit risk model

In this section we present a reduced form statistical model for estimation of counterparty probability of default for the bank's corporate sector loan portfolio. The general idea is to enter factors that determine the probability of default and quantify how these contribute towards predicting default realizations. With such estimated probabilities we may proceed to calculate expected losses per counterpart exposure, i.e., the product of exposure size and estimated default probability. In a second step, expected losses per exposure are used to derive total expected losses for the portfolio, and, thus, enable a calculation of the loss-ratio, i.e., the total expected losses in relation to the total value of the outstanding loans. In a third step, the estimated model serves as a basis for simulating an estimate of the distribution of losses, which, in turn, will allow for a Value-at-Risk-type measure of portfolio credit risk. Moreover, under various assumptions about the future development of, e.g., the macro economy, or the bank's credit policy in terms of portfolio composition, the estimated model can be used for stress-testing experiments where conditional VaR is calculated. Stress-testing is something we propose to do in future work. To conclude, a well-specified model ought to be informative about various aspects of portfolio credit risk.

3.1 Outline of the statistical model

As discussed earlier, we will, due to data limitations, model counterpart default risk, and not the risk for the individual loan. Nevertheless, we will in this section discuss the model in terms of loans. Default risk will be modelled by use of an econometric duration model, which contains the logistic regression model as a special case, see, e.g., Lancaster [33]. By choosing a duration model, rather than the more common logistic model, we avoid having to assume that the risk of default is constant over the life-time of the loan. The implication is that we focus on the time it takes for a loan to default, rather than simply whether a loan will default or not.

Define the discrete random variable T to be the number of quarters it takes for a loan to default. $\Pr[T = 1]$ is the probability that a loan default in the same quarter the loan was granted by the bank. $\Pr[T = 2 \mid T > 1]$ is the conditional probability of default in the second quarter, given that the loan had not defaulted in the first quarter. In general, we are interested in the quantity $\Pr[T = k \mid T > k-1]$, k being a positive integer, which is usually referred to as the discrete time hazard rate. If $\Pr[T = k \mid T > k-1] = \Pr[T = k+l \mid T > k-1+l]$ for all positive, integer values of k and l, then default risk will be constant over time. In that case, default or no default could be considered a binary random variable and, e.g., a logistic regression would be a reasonable credit risk model. For the data at hand we see no justification for stipulating that the default risk for a loan will not change over time and hence we intend to estimate $\Pr[T = k \mid T > k - 1]$ for $k = 1, ..., K$, where K equals the number of quarters the loans are followed. In the analysis we observe the loans for at most 24 quarters and thus $K = 24$.

Estimation of the hazard rate is in principle straightforward. The Maximum Likelihood estimator of $\Pr[T = k \mid T > k - 1]$ is given by the number of loans that defaulted in the kth quarter divided by the number of loans that had not defaulted prior to this quarter. There are however two minor technical problems involved. The first concerns the question whether the definition of T is meaningful for this application. According to the definition, the outcome of T may be any integer greater than zero, and, as a consequence, the model will assign a probability of default greater than zero to any positive integer value of k. This has the bizarre consequence that the model assigns a positive probability to a default occurring after, say, 10,000 years. Not many loans are issued with an intended duration exceeding this number of years. One solution is to redefine T such that the maximum equals the intended duration (expressed in quarters) of the loan. However, in the analysis we consider counterpart default risk and we find it reasonable to stick to the definition of T since the intention of the counterpart most likely is to survive eternally (or, at least, the survival of the counterpart is not predetermined). The second problem concerns the question whether default is the only potential end state. In reality, most loans that drop out of the portfolio will not be observed to default, rather they drop out because the counterparts decide to clear them. Thus, a loan can end in one of several absorbing states, in other words, there exist competing risks, c.f. Lagakos [32]. We will assume that the competing states are independent of the state of default and hence treat loans subject to such events as being censored in the last quarter that they were active.

In Section 2 we discussed various factors that may affect default risk. These will be introduced in the model using the following notation: x refers to factors specific to the loan, and z to factors specific to the general operating environment of all firms. Hence, x may represent variables like loan size, firm size, and various performance measures based on accounting data, as well as historical payment records on the payment behavior of the firm. The purpose of x is to describe idiosyncratic risks, whereas z is supposed to capture business cycle effects and may be represented by variables such as measures of the yield curve and the output gap, and the rates of inflation and unemployment. We also make a distinction between variables that are time-constant and those that vary over time. The latter is indicated by τ, where τ denotes calendar time. The conditional probability of default, taking the heterogeneity of the loans into account, is

$$\Pr\left[T = k \mid T > k-1, x, z, x(\tau), z(\tau)\right], \tag{1}$$

where $x(\tau)$ and $z(\tau)$ are the factors evaluated at calendar time τ.[9] The effects of x and z are identified by cross-sectional variation in the probability of default and the effects of $x(\tau)$ and $z(\tau)$ are identified by cross-sectional variation in the default probability at *different calendar times.*

Identification of the parameters implied by (1) is theoretically straightforward and can be done non-parametrically. However, this is not particularly practical and it is useful to impose some additional structure on the model. In doing so we denote $\Pr\left[T = k \mid T > k-1, x, z, x(\tau), z(\tau)\right]$ by $h(k)$ and $\Pr\left[T = k \mid T > k-1\right]$ by $h_0(k)$. In a seminal paper Cox [13] suggested a multiplicative relationship between the variables and the hazard rate. This yields a model of the following form

$$h(k) = h_0(k) \exp\left(m\left[x, z, x(\tau), z(\tau); \beta, \gamma, \beta^\tau, \gamma^\tau\right]\right). \tag{2}$$

Thus, the model postulates a base-line hazard, $h_0(k)$, common to all loans and a multiplicative component which depends on the values of the variables for the loan. β and γ denote the parameters that pertain to the x and z variables (the superscript τ indicates that the parameter is associated with time-varying variables), $m[\ \ ]$ is some real-valued function whereas the exponential function is introduced for the purpose of ensuring a non-negative hazard. In many empirical analyses, m is taken to be linear in the variables in the unit originally measured, perhaps because it is a non-trivial matter to determine m for continuous regressors. In order to improve on this common strategy, we make use of a version of regression smoothers for censored or discrete response variables. We start by defining the response variable as the logarithm of the ODDS of default conditional on non-default in the previous quarter and then use a regression smoother for the relation between logODDS and the regressor (see in particular Hastie and Loader [24], as well as Härdle [25], Gray [22] and Kooperberg, Stone, and Troung [26]). Thereafter, we sequentially add regressors and determine the functional form of the linking function m.

Once m is fixed, we may proceed to estimation of the model. The data comprise a total of 54,603 firms and 69,249 loan spells, which means that some firms are recorded with multiple loan spells.[10] Thus, there are 69,249 potential observations of T. k_i will denote the ith loan's

[9] The time-varying variables are assumed to be constant within each quarter.

[10] A few words should be said about the construction of the credit spells. We observe each firm for 24 quarters and are thus able to determine whether a credit was held in a given quarter, or not. It is stipulated that the spell starts in the first quarter a credit is observed and continues until the first quarter for which the credit no longer is hold. An interruption of one, or several quarters, is taken to indicate the end of a first spell and the beginning of a second one. The ending of a spell is due to only one of three reasons; default (which is identified by a counterpart classification in Rating class 15), the loan was redeemed, or, the credit was active in the last quarter of observation (2000:1). The latter two reasons imply that the spell was incompletely observed, and it

duration, i.e., the number of quarters of survival prior to default, or exit into some other state. Only 3,598 spells were observed to default; remaining spells departed to other exits, or survived throughout the 24 quarters they were followed. Let the censoring indicator, c_i, indicate with unity if the loan was observed to default and zero otherwise. The set of variables pertaining to the ith loan will be indexed by i, and $h_i(k)$ is the hazard rate evaluated for a loan with characteristics x_i and z_i and a time path of $x_i(\tau - k : \tau)$ and $z_i(\tau - k : \tau)$. The maximum likelihood estimates of the parameters are obtained by maximizing

$$\ln L\left(h_0(k), \beta, \gamma, \beta^\tau, \gamma^\tau\right) = \sum_{i=1}^{n} \left(c_i \ln h_i(k) - \sum_{s=1}^{k_i - 1} h_i(s) \right). \tag{3}$$

where n is the number of spells, i.e., 69,249.

Suitable explanatory variables have already been discussed in the data section, and clearly our choice is influenced by previously published work. However, the uniqueness and richness of the data has permitted us to explore, rather freely, an additional number of potentially important variables. Naturally, multicollinearity often restricts a too opulent set of variables. The guiding principles are (in order of importance); (i) previously proposed and theoretically justified variables, (ii) stability of the model - both in terms of predictions and in the estimates, (iii) simplicity of the model, (iv) statistical significance of the estimated parameters, and, finally, (v) the fit of the model in terms of a pseudo-R^2. The first point (i) is discussed in the data section, whereas the fourth point (iv) is a conventional principle, although in part less meaningful for very large data sets like the present. Nevertheless, we have been reluctant to include variables with t-ratios smaller than two, unless the non-significance of the variable is of interest *per se*. The pseudo-R^2 measure (v) is designed to resemble the conventional R^2 measure of linear regression models. It can be interpreted as the degree to which the distribution of predicted probabilities of default for performing loans does not overlap the distribution of predicted probabilities of default for loans that actually defaulted. The smaller the degree of overlap, the better the model discriminates between defaulted loans and non-defaulted ones, and hence, the better the predictive power of the model, c.f. Fienberg [18]. The third principle (iii) means that we have avoided complicated transformations or interactions of various variables, unless a substantial improvement has been achieved. Finally, the stability criterion (ii) has been checked by excluding, in turn, the following subsets; a 90 % fraction of the performing loans, loans after the second quarter of 1997, loans with missing values on at least one of the variables, and loans having values of the variables outside the 10 to 90 percentile range. Moreover, the stability of the estimates has been checked, in addition to above mentioned checks, by including competing

is therefore viewed as censored. Likewise, many firms were observed to hold a credit in 1994:2 (the first quarter we observe). It is reasonable to believe that in many cases the beginning of those spells occurred before 1994, nonetheless the starting time has been set to 1994:2. This is a way to overcome the length-bias sampling problem which arises in stock-samples (our sampling scheme is a mixture of stock and flow sample, see [33]).

variables.

3.2 The empirical model

In Table 2 we present coefficients and standard errors for the estimated model.[11] First, there is very weak evidence of a duration dependence. For instance, the estimate implies that the risk of default increases by roughly three percent in the second year of the loan compared with the first year, though the difference is far from being significant.[12] Second, the risk of default is markedly higher for short-term credits compared with long-term ones, the risk is about twice as high for the short-term credits.

The strongest determinant of default is registered payment remarks for a firm in the preceding four quarters. Any such remark implies that the risk of default increases by 14 times, i.e., 1,400 per cent. Add to this a remark of category 25 and the risk increases by about 34 times. In contrast, the predictive power of the accounting data is more modest, although the liability-to-assets ratio (TL/TA) is quite useful. It should be noted, however, that the accounting data provide decent predictions of default occurrences whenever remark data are excluded from the model: it is the inclusion of the remark data in the model that reduces the importance of the accounting data.

We have evaluated a number of macroeconomic variables and we find that the output gap and the yield curve indeed are reliable indicators of the evolution of default risk over time. Additional improvement in the fit is achieved by using the households' expectations of the Swedish economy. One way of appreciating the importance of the macro-indicators is to consider the output gap. It varies from a low -7 per cent in the early part of the observation period, to a zero gap between actual and potential GDP in the later part. This implies that the change in the output gap yields predicted default rates for the later part of the sample that are roughly 10 times smaller than those for the early part of the observation period. The estimated parameters for the macroeconomic variables have the expected signs and enter significantly in the model. Presumably, the big spikes in the average default rate that occur during the years 1995 and 1997 are very helpful in distinguishing the effects of firm-specific and macroeconomic variables on default risk in the model. Among the macroeconomic variables, current real economic activity seems most important for the default rate. So, although we do not have an estimation period

[11] The coefficients in Table 2 can be interpreted in the following way: $100 * (exp(\text{coefficient}) - 1)$ gives the increase in the hazard (in percent) due to a one unit increase in the variable. For instance, an increase in sales by one million SEK would lead to an increase of the hazard by -7.6 percent (i.e., the hazard would actually decrease). In general, there is no simple way of interpreting the coefficients in terms of expected duration. However, if the hazard is constant (which is approximately the case here), then the expected duration increases by $100 * (exp(-\text{coefficient}) - 1)$ percent for a one unit increase in the variable.

[12] One interpretation of this finding is that the explanatory variables capture dynamic aspects of firms' default behaviour. Moreover, a logit approach may very well suffice.

that covers a complete business cycle (unless one chooses to label the low economic growth during 1996 as a recession period, see Figure 6), the estimation results are encouraging and confirm that macro is important for explaining firms default behavior.

Table 2: The estimated coefficients in the credit risk model.

	Coefficient	Standard error
Duration		
1:st year	0	–
2:nd year	.026	.051
3:rd year	−.194	.057
4:th year	.236	.070
5:th year	−.104	.110
6:th year	.274	.175
Credit type[a]		
Long-term	0	–
Mix of short- and long-term	.511	.057
Short-term	.761	.049
Remarks with credit bureau[b]		
No remarks	0	–
Category 25 remarks	.903	.137
Category 8, 11, 16, 25, 31 remarks	2.638	.080
Accounting data[c]		
TS (mn SEK)	−.079	.023
EBITDA/TA	.038	.026
I/TS	.436	.124
TL/TA	2.829	.114
Macroeconomic variables[b]		
Output-gap (lagged 2 quarters)	−.341	.018
Household expectations (first differences lagged 2 quarters)	−.163	.016
Yield curve, $(10Y - 3M)$	−.232	.025

Notes: [a]variables taken to be constant over time, [b]variables taken to be time-varying, with quarterly variation, [c]variables taken to be time-varying, with yearly variation, although there is some variation between quarters due to the variation in reporting period that individual firms apply.

The lack of transparency for the non-linear model we apply is a serious drawback. Model checking is therefore of critical importance. We find a pseudo-R^2 of 63 %, which is quite respectable considering the predominantly cross-sectional nature of the data. A model specification conditioning only on rating class yields a pseudo-R^2 of 50 %, hence our model is a substantial improvement of the bank's internal rating procedure. The model is able to accurately distinguish between contributions from firm-specific variables on the one hand, and macroeconomic variables on the other. This is illustrated in Figure 7. The figure depicts the actual and two predicted default rates quarter by quarter, one based on the complete model as presented in Table 2, whereas the other one is based on a model with the output gap as sole explanatory variable. The actual default rate is quite erratic, whereas it is obvious that the output gap captures only the smooth changes in the default rate over time. The full-model predicted rate follows quite accurately the short-term variation in the actual default rate, although it fails somewhat to capture two of the later peaks. Simple linear regression models that explain the time series variation in the actual average default rate yield R^2-values of 41 and 85 per cent, when using output gap-predicted and the full-model-predicted default rates, respectively, as explanatory variables. To sum up, the estimated model demonstrates the need to take account of both idiosyncratic risk factors, as captured by payment remark data and accounting data, as well as macroeconomic effects.

For clarity and for future reference note that the predicted default probability, $\widehat{p}_{i,\tau}$ say, for loan i at quarter τ is given by (1) where the determinants are set at the value corresponding to the ith loan. The predicted quarterly average default rate is simply the mean of all $\widehat{p}_{i,\tau}$.

3.3 Value-at-Risk

An estimated credit risk model can serve many purposes. However, two immediately spring to mind. First, the model can quantify the sub-portfolio risk at each time-point, e.g., a portfolio of loans as defined by a particular internal rating category. Such estimates of loss distributions could then provide estimates of required capital for given estimated probability of default, and hence admit, e.g., estimation of a relative-risk-weight mapping function for use in the IRB-approach. Second, the model may provide answers to questions like; what happens to portfolio risk and relative risk weights if the bundle of loans in the portfolio is changed? And what happens to the portfolio risk and relative risk weights if, e.g., the interest rate spread increases? In other words, the model can be used to simulate the consequences of a hypothetical future change in the environment or a hypothetical change in the portfolio strategy. For the purposes of this study, an evaluation of the IRB-approach, we will use the estimated model for simulating credit risk measures, both for the portfolio and for the individual rating classes.[13] These risk measures can function as a standard, or basis, when evaluating the outcomes of calculated buffer capital under the proposed Accord.

Consider, as a first step in a derivation of a Value-at-Risk-measure, the following simple observation. As stated in the previous subsection, the model-predicted probability of default is denoted by $\widehat{p}_{i,\tau}$. If $S_{i,\tau}$ denotes the exposure (utilized amount of credit minus expected recovery in case of default) for loan i in quarter τ, it follows that the expected loss for that particular loan in the quarter of interest equals $\widehat{p}_{i,\tau} \times S_{i,\tau}$. Summing over all loans would readily yield

[13] The same underlying idea has been used in an analysis of credit risk in a consumer credit portfolio, see [30].

the expected losses for that quarter. Value-at-Risk requires a somewhat more sophisticated procedure, as it refers to the potential loss in a worst case scenario.

Calculation of Value-at-Risk, $VaR(\tau)$, will be done for each quarter τ separately, $\tau = 1994Q2, ..., 2000Q1$. We suggest the following algorithm:

i) Draw a uniform random variate, u_i, and define $D_{i,\tau} = I(\widehat{p}_{i,\tau} > u_i)$ for all loans i in the portfolio in quarter τ, such that $D_{i,\tau} = 1$ if $\widehat{p}_{i,\tau} > u_i$ and $D_{i,\tau} = 0$ otherwise. $D_{i,\tau}$ is thus simply an indicator variable for default.
ii) Define $VaR_{r,\tau} = \sum_{i=1}^{N_\tau} D_{i,\tau} \times S_{i,\tau}$ where N_τ is the number of firms in the portfolio in quarter τ.
iii) Repeat R times.
iv) Form an empirical distribution of the $r = 1, ..., R$ outcomes of $VaR_{r,\tau}$.
v) Let $VaR(\tau)$ equal some percentile, e.g., the 99th, of the distribution of $VaR_{r,\tau}$.

Obviously, this procedure does not take explicit account of risk dependencies between firms in the portfolio, i.e., in effect we make an implicit assumption of zero correlations between individual probabilities of default. If invalid, such an assumption will result in under-estimation of the portfolio VaR (given positive correlations). However, there are two reasons that speak in favor of the assumption being approximately correct. First, since the credit risk model is estimated conditional on business cycle effects, common drivers of credit risk have been eliminated. To some extent, such macro effects are also indirectly captured by the accounting data; a downturn in a particular industry should manifest in, e.g., total sales for firms in that industry. Second, the portfolio comprises credits from all regions and almost all industries in the Swedish economy. On average, about half of the firms in the portfolio are small businesses with 5 or fewer employees. Hence, we believe that the portfolio can be characterized as being well-diversified and therefore less susceptible to non-zero correlations.[14]

Figure 8 shows the expected and actual losses on the bank's loan portfolio and three (the 90th, the 95th, and the 99th) Value-at-Risk percentiles for the entire sample period.[15] For this purpose, we have defined the credit loss in case of a predicted default as the utilized amount of credit (not the granted amount of credit) times one minus the recovery rate. The recovery rate that we use here was calculated by the bank as a non-time-varying sample average for each loan type. Although this loss rate is implicitly affected by collateral that businesses provide, any individual differences in loss rates between firms due to variations in the available collateral have not (yet) been taken into account. The expected loss rate has a weak trend similar to

[14] In future work we hope to be able to elaborate on the idea of introducing industry- and region-specific measures of performance.

[15] The x-th percentile Value-at-Risk can be interpreted as the amount in SEK (alternatively the share of the portfolio) that will be lost by the bank with a maximum probability of x percent. Another way to interpret this is: with a probability of (100-x) percent, the loss by the bank will not be greater than some SEK amount (alternatively some share of the portfolio).

that in the actual loss rate, with the expected quarterly loss declining from approximately 2% in 1994-Q2 to 0.4% in 1995-Q4. From 1996 and onward the expected loss rate remains below 0.5%. Although the *expected* loss series appears to capture the general trend in the actual loss rate and the macro series, it does not indicate if the portfolio *risk* increases at any given point in time. For instance, the peak in the actual loss rate in 1995 is missed completely. Therefore, we have also calculated three series of Value-at-Risk for the bank's loan portfolio over time. The upper three lines in Figure 8 are the 90th, the 95th and the 99th VaR percentiles. These clearly show that an expected credit loss measure fails to capture any variations in downward risk that the bank is exposed to. In fact, the same appears to be the case for the 90th and the 95th VaR percentiles, as they move more or less parallel with the expected loss rate, although at a somewhat higher level. The 99th percentile however, shows much more variation than any of the other three credit risk measures. In the second and third quarter of 1995, for example, the 99th VaR percentile rises to 3.4% of the portfolio, an increase of .6% compared to the preceding quarter. Expected credit losses as well as the 90th and 95th percentiles remain unchanged over the same period, however. Between the first and fourth quarter of 1997, the relative growth in VaR is even bigger as the 99th percentile rises from 1.6% to 2.9%, while the expected loss merely increases from .3% to .5%. In general we can conclude from Figure 8 that any given level of expected credit losses is associated with widely varying levels of risk. For expected loss rates between .3% and .5%, 99th percentile Value-at-Risk actually ranges from 1.6% to 2.9%. For loss rates between .2% and .5% the VaR interval widens by another .5%. Consequently, a risk-weight-mapping function that maps expected loss rates into relative risk weights may fail to account fully for variations in portfolio risk.

Figure 9 contains the outcomes of similar calculations as those underlying Figure 8 for each separate rating class.[16] Expected loss rates (ELR) and VaR-values are displayed only for those quarters in which at least 10 companies were assigned to the rating class in question. A general property that the rating classes appear to share with the portfolio is that the volatility of the Value-at-Risk measures is much greater than the volatility of the expected loss rate. Whereas it can be seen in the last window in Figure 9 that the expected loss rate varies between 0% and 25% for Rating class 14, its 99th percentile ranges from 2% to 61%. Although such fluctuations may be expected for companies that are close to default, similar movements would appear more surprising for the 'safest' debtors. The window for Rating class 2 shows, however, that even the

[16]For the purpose of calculating Value-at-Risk, we made one change to the data material. First, we merged Rating classes 14 and 15. As we already mentioned in Section 2, firms that were assigned to Rating class 15 in two or more subsequent quarters were assumed to have defaulted and exited from the sample in the first of this series of quarters. The only exception we made to this rule was for firms that had different types of loans in two subsequent quarters. This occurred for only a very small number of firms. For the purpose of VaR, we considered it more useful to treat classes 14 and 15 jointly, both because of the small number of firms with subsequent spells in Rating class 15 and because of the uncertainty about the causes of such multiple spells.

low risk part of the portfolio displays much variation in VaR and little variation in the expected loss rate. While the ELR ranges from 0 to 3% over all 15 rating classes, VaR at the same time takes values between 0 and 36%. Over all rating classes, the ratio between the ELR and 99% VaR varies from a factor 1 to a factor 46 for Rating class 14. Although a strictly monotonic relationship between class size and variance in VaR does not exist, it is worthwhile to observe that the 99th percentile VaR's for Rating class 9, the largest group, is quite smooth over time and has a maximum value of merely 6.2%. This compares more or less to the maximum of 4% for the whole portfolio. By comparison, of the 'safer' Rating classes 1-8, only number 1 and four have lower VaR maxima. This reveals one of the less attractive features of an internal rating system with a large number of 'finer' rating classes. In general, given a predetermined average default rate, small rating classes (in the sense of number of companies) will tend to have higher Value-at-Risk peaks than big rating classes. If buffer capital is to reflect not only the first moment but also the second moment of portfolio credit risk, attempts in a finitely sized portfolio to refine risk estimates may actually lead to higher capital requirements. When designing an internal rating system, group size will consequently have important consequences for the corresponding risk weight mapping function.

Finally, Figure 10 displays the ELR and VaR measures for the same three groups of internal rating classes as in Figure 4. When drawing some preliminary conclusions from them, we should keep in mind that the above mentioned group size effect may distort the 'risk monotonicity' properties between groups. Although for example Rating class 4 appears to display a strictly lower expected loss rate than Rating class 2 for the second half of the sample period, and Rating class 10 has lower risk than 8 and 9, no major or persistent inconsistencies with the monotonicity property are found. For the VaR measures, the outcome is somewhat different. Rating class 2, for example, despite the relatively large number of spells, displays a steep increase in 99% VaR in 1997 (35%), whereas classes 3-5 experience only minor rises (below 10%). Similarly, Rating classes 6 and 8 show large increases in VaR (15-25%) while Classes 9 and 10 stay at levels below 10%. Thus although monotonicity roughly appears to hold between groups for expected losses, this is much less likely for a Value-at-Risk measure.

4 The bank's internal ratings

In this section we will analyze the bank's internal ratings with a focus on IRB capital charges and their changes over the sample period. We begin by providing a set of stylized facts for the ratings and then carry on with the issue of estimating the average probability of default characteristics for each rating class. Thereafter we calculate the risk-weights for the rating classes and examine their consistency. Equipped with the estimated credit risk model and associated VaR-measures, we will then check if IRB capital reflects changes in portfolio risk over time, i.e., are capital charges and VaR reasonably correlated. Finally, we try to shed some light on how many rating classes to use in the IRB approach.

4.1 Stylized facts

The bank's internal rating system comprises 15 classes. Table 3 shows the bank's appreciation of how these 15 rating classes relate to the well-known rating categories of Moody's and Standard & Poor's. We have also included estimates of long-run average default rates for the ratings from Moody's and Standard & Poor's.

Table 3: Rating classes and corresponding ratings from Moody's and Standard & Poor's.

Bank rating	Moody's	Def.rate	S& P's	Def.rate
1	Aaa, Aa1	0.00	AAA, AA+	0.00
2	Aa2, Aa3	0.03	AA, AA-	0.00
3	A1, A2	0.01	A+, A	0.04
4	A3	0.01	A-	0.04
5	Baa1	0.15	BBB+	0.22
6	Baa2	0.15	BBB	0.22
7	Baa3	0.15	BBB-	0.22
8	Ba1	1.34	BB+	0.92
9	Ba2	1.34	BB	0.92
10	Ba3	1.34	BB-	0.92
11	B1	6.50	B+	4.82
12	B2, B3	6.50	B, B-	4.82
13	Caa, Ca	26.16	CCC, CC	20.39
14	C		C	
15	D		D	

Remark: Default rates, in per cent, are given by average one-year transitions for the periods 1980-1998, Moody's, and 1981-1998, S&P's, as reported by BIS (2000) on p. 149.

The assignment of an internal rating class to a new loan, or the re-evaluation of a counterparty rating in connection with the annual review process of all counterparts, is performed according to a set of quantitative and qualitative criteria. There are two quantitative measures. First, an external rating performed by the credit bureau UC, for details and an evaluation of their model based approach, see Jacobson and Lindé [29]. UC provides an assessment of counterparty bankruptcy risk for the next 8 quarters. Second, the bank runs a calibrated risk model where one input is the rating from UC and other inputs consist of internal information. Unfortunately, we have no information on the details of this model. The qualitative criteria are summarized in a counterparty rating classification handbook. The handbook provides verbal descriptions of the properties of firms in a given rating class along a number of dimensions. Tables 4a-b attempt to capture the essentials of the handbook's characterization of the rating classes. It should be noted that the three criteria are not weighted according to some formal "scoring" procedure for the rating decision, they are used as independent inputs.

Table 4a: Characterization of a selection of rating classes.

Risk rating	Ownership	Industry	Management
1	listed shares, easy access to additional capital	industry leader, recession resistant counter-cyclical industry	highly respected and experienced
6	acceptable structure, may have difficulty to raise new capital	well-established in cyclical industry, small market shares	adequate to above average
9	structure just adequate, doubts whether new capital can be raised	in cyclical industry recovering from recession, or newly established	adequate
14	weak owners, cannot access new capital shares trading suspended	negligible market shares in a troubled industry, small chances of continued operation	little experience in tough decision-making, significant management turnover, no plan for financial crisis

To get a better understanding of the dynamics in the bank's internal rating system, we have calculated the empirical transition frequencies between the 15 rating classes. The frequencies after 1, 4 and 8 quarters in Tables 5a, 5b and 5c, respectively, have been obtained in the following way. For any transition horizon h, we compared and counted the internal rating of each company that was included in the bank's portfolio at both time τ and $\tau + h$, for $\tau = 1,\ 2,\ ...,\ 24 - h$. Companies that defaulted, i.e., ended up in the absorbing state of Rating class 15 between τ and $\tau + h$, were also taken into account.

By and large these transition matrices display properties that one would expect from a reasonably functioning rating system. There are many zero elements in the upper-right corners reflecting that initially high-rated counterparts are not downgraded very far or very often. Likewise, the transitions of poorly rated counterparts are not in the direction of the better rating classes. The diagonal elements reveal a high degree of persistence in the short run, i.e. counterparts tend to remain in their rating class from one quarter to another. For longer horizons there is clearly less persistence, in particular for middle ground and poor rating classes.

Table 4b: Characterization of a selection of rating classes.

Risk rating	Financial status	General
1	steady sales growth, very conservative balance sheet ratios, very solid cash flow, excellent debt service capacity	only a handful of large firms make it to this class
6	moderate potential growth in sales, adequate balance sheet ratios, volatile cash flow, at times thin debt service coverage	unlikely that well established firms in solid markets fall beyond this class
9	little or no potential to change mediocre sales growth, possible over-capacity problems, great volatility in cash flow	–
14	negative sales growth outlook, balance sheet ratios give rise to serious concern, cash flow shows extreme volatility, may be in process of distressed selling of critical assets	marked increase or unacceptable level of delinquency in payment to trade creditors

The second column in each table contains the average relative share of each rating class in the whole portfolio, in terms of numbers of loans. Rating class 9 has by far the largest share, with around 30% of all firms, at quite some distance followed by Classes 5, 8, and 11 with shares around 10%. From the columns under heading 15 we can also see which rating classes are the most important sources of defaulting firms. Out of those ranked in Class 14, 7.2% default after one quarter, compared with 3.9%, 1.2% and 0.5% for Classes 13, 12 and 11. Default frequencies after four quarters in these classes are 12.5%, 5.5%, 2.5%, and 0.3%, respectively, and after yet another four quarters frequencies are up to 40.7%, 22.7%, 10.9%, and 6.0%. Hence, for these, the four poorest rating classes, default risk increases consistently with rating class. For Rating classes 1-10 we find a slightly blurred picture, for instance Classes 5 and 8 have too high default frequencies given their rank. Our estimated default frequencies, at least for the poor rating classes, agree with the estimates for Moody's and Standard and Poor's ratings.

Table 5a: Internal ratings' transition matrix, average 1 quarter forward movements, in per cent.

		To														
From	%	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0.1	92.1	1.4	0.0	0.0	0.5	2.3	0.0	0.9	2.3	0.9	0.0	0.0	0.0	0.0	0.0
2	3.2	0.0	62.2	0.1	4.2	0.2	0.4	3.9	5.8	16.9	3.7	0.0	1.7	0.6	0.1	0.1
3	0.3	0.1	1.5	91.0	2.8	0.4	1.6	0.4	0.4	1.6	0.1	0.2	0.0	0.0	0.0	0.0
4	5.6	0.0	13.7	0.2	60.6	0.1	0.3	2.0	6.5	12.6	0.9	0.0	2.1	0.6	0.1	0.2
5	8.1	0.0	0.0	0.1	0.1	74.2	7.7	2.1	3.1	5.3	3.3	2.9	0.8	0.2	0.1	0.1
6	6.9	0.0	0.0	0.0	0.2	7.5	81.0	2.1	1.4	4.3	1.8	1.3	0.2	0.0	0.0	0.0
7	5.9	0.0	0.0	0.1	0.2	5.3	10.7	71.4	2.3	5.0	2.2	1.9	0.6	0.2	0.0	0.1
8	9.6	0.0	0.0	0.0	0.0	4.0	0.7	1.3	78.5	7.4	1.1	3.3	2.2	0.8	0.2	0.4
9	30.5	0.0	0.8	0.0	3.9	2.6	1.1	1.9	3.9	77.0	2.0	2.1	3.0	1.0	0.2	0.4
10	5.6	0.0	0.0	0.0	0.1	3.5	0.6	1.6	2.8	5.2	84.2	1.2	0.6	0.1	0.0	0.1
11	9.0	0.0	0.0	0.0	0.0	0.7	0.4	0.4	3.4	2.1	0.5	88.8	2.1	0.8	0.3	0.5
12	8.4	0.0	0.8	0.0	3.2	0.5	0.2	0.4	2.7	14.9	1.5	3.7	65.9	4.2	0.7	1.2
13	5.9	0.0	0.2	0.0	0.5	0.2	0.1	0.1	1.0	5.6	0.2	2.4	5.0	79.0	2.6	3.1
14	0.9	0.0	0.0	0.0	0.0	0.1	0.0	0.1	0.4	1.2	0.1	2.1	2.9	9.8	76.2	7.2

Table 5b: Internal ratings' transition matrix, average 4 quarters forward movements, in per cent.

		To														
From	%	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0.1	77.6	4.2	0.0	4.9	0.7	7.0	0.0	0.0	4.2	1.4	0.0	0.0	0.0	0.0	0.0
2	0.2	0.7	76.0	5.7	2.6	0.0	4.8	2.6	0.2	5.8	0.9	0.9	0.0	0.0	0.0	0.0
3	0.4	1.1	4.6	74.9	7.0	1.0	5.6	0.9	0.1	3.0	1.0	0.7	0.0	0.0	0.0	0.0
4	0.8	0.5	0.4	7.0	75.9	0.9	6.3	3.0	1.2	3.7	0.5	0.4	0.1	0.0	0.0	0.0
5	12.6	0.1	0.1	0.1	0.3	23.8	23.7	3.6	8.5	18.1	11.0	7.5	1.9	0.4	0.2	0.7
6	9.9	0.0	0.1	0.1	0.5	24.8	37.2	8.4	5.3	12.0	6.2	4.3	0.7	0.1	0.0	0.3
7	8.5	0.0	0.1	0.2	0.6	9.9	39.6	13.7	5.4	15.5	6.8	5.8	1.4	0.3	0.1	0.6
8	11.5	0.0	0.0	0.1	0.1	6.4	3.6	1.4	49.2	14.1	3.7	12.0	4.9	1.7	0.7	2.2
9	27.4	0.0	0.1	0.1	0.3	5.7	5.0	3.1	10.1	55.7	6.2	7.7	3.0	1.3	0.6	1.2
10	6.7	0.0	0.0	0.1	0.2	7.0	2.4	2.4	6.6	15.7	58.5	4.6	1.5	0.5	0.2	0.3
11	11.6	0.0	0.0	0.0	0.1	1.5	1.9	0.4	10.3	7.7	1.9	64.7	6.2	1.9	0.8	2.5
12	5.8	0.0	0.0	0.0	0.0	2.9	0.9	0.3	7.6	6.2	1.5	19.6	46.0	6.9	2.5	5.5
13	3.5	0.0	0.0	0.0	0.0	1.4	0.6	0.2	3.8	3.8	1.5	15.5	13.9	41.7	5.0	12.5
14	1.1	0.0	0.0	0.0	0.0	0.4	0.2	0.0	1.4	1.1	0.2	5.4	7.9	9.4	49.1	24.9

Table 5c: Internal ratings' transition matrix, average 8 quarters forward movements, in per cent.

		To														
From	%	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0.0	63.8	5.2	0.0	17.2	0.0	10.3	0.0	0.0	3.4	0.0	0.0	0.0	0.0	0.0	0.0
2	0.2	2.1	61.0	12.6	5.2	0.0	7.7	2.8	0.0	8.0	0.0	0.6	0.0	0.0	0.0	0.0
3	0.3	1.7	8.1	60.5	15.0	2.7	5.6	0.4	0.0	4.0	1.9	0.2	0.0	0.0	0.0	0.0
4	0.8	0.7	0.9	13.5	61.8	2.0	8.3	3.7	1.9	6.2	0.2	0.2	0.3	0.3	0.0	0.0
5	12.7	0.2	0.2	0.2	0.5	22.7	12.4	2.6	11.1	23.9	9.9	11.2	2.5	0.7	0.2	1.7
6	10.2	0.0	0.2	0.1	0.7	9.1	21.3	12.4	10.8	22.6	11.6	8.5	1.5	0.3	0.1	0.7
7	12.3	0.0	0.1	0.2	0.6	17.1	18.9	7.9	8.9	22.8	11.2	8.9	1.7	0.4	0.1	1.0
8	11.6	0.0	0.0	0.1	0.2	4.9	4.0	0.6	30.6	21.6	7.5	17.3	5.4	1.9	1.0	5.0
9	23.5	0.0	0.1	0.2	0.5	5.4	6.1	1.1	11.1	47.9	8.9	11.1	3.2	1.0	0.6	3.0
10	5.7	0.0	0.0	0.3	0.3	4.2	6.4	0.7	9.4	20.8	45.1	8.2	2.0	1.0	0.4	1.1
11	10.6	0.0	0.0	0.1	0.1	2.5	1.3	2.1	14.2	12.3	7.7	43.2	7.1	2.3	1.1	6.0
12	6.5	0.0	0.0	0.1	0.0	1.4	1.3	0.3	9.9	9.9	4.4	25.1	28.6	5.9	2.2	10.9
13	4.3	0.0	0.0	0.0	0.1	0.8	0.7	0.1	5.9	6.4	2.2	20.4	13.9	22.5	4.2	22.7
14	1.2	0.0	0.0	0.0	0.0	0.5	0.2	0.1	2.3	2.2	0.6	7.5	9.2	8.4	28.4	40.7

4.2 The bank's risk-weighted capital requirements

As noted in the introduction, a key input for IRB-determined risk-exposed assets is the average probability of default (PD) associated with a given rating class. According to the proposal the estimated PDs can be determined in three ways; by historical default frequencies, by external ratings, or by means of a credit risk model. Below we will present PD estimates for the 14 non-defaulting rating classes. The purpose is to examine this potential source of variation in IRB capital determination, prior to the study of the IRB risk-weighted charges for the portfolio over time. We consider two approaches for the PD estimates. First, and naturally, the method that most likely will be the one used by most banks: historical default frequencies. In Figure 11, first column, we present 4-quarter and 12-quarter moving average PD estimates based on the estimated probability of defaults given by historical default frequencies. Likewise, the second column of Figure 11 shows corresponding average PD estimates using predictions from the credit risk model. Both columns refer to the Rating classes 1, 6, 9, and 14. It is clear that the estimated PDs are not stable over time. Hence, the rating classes cannot be characterized by a fixed, long-run PD, reflecting that changes in risk for the portfolio is only manifested in transitions between classes. It is therefore worthwhile to consider how much information to make use of when estimating the PD for a rating class, recognizing the trade-off between on the one hand wanting the PD to accurately reflect risk and, on the other hand, avoid short-run, erratic instability in the estimate. Judging by Figure 11, a 4-quarter moving average estimate seems to be a reasonable compromise. Moreover, the model-based PD estimates are *per se* smoother over time, cf., e.g., the windows that show the two estimation approaches for Rating class 14.

Another apparent benefit of model-based PD estimates is the ensured existence of estimates despite the lack of defaults in a particular rating class. This effect is highlighted for Rating class 1.

The next natural step is to study how the calculated IRB risk-weights behave for the bank's rating classes. Figure 12 shows risk-weights using the six probability-of-default estimates discussed above, evaluated for Rating classes 11, 12, 13, and 14. The weights $RW_{c,\tau}$ for a rating class c in quarter τ have been calculated using the following formula, as given by the January 16, 2001, Basel Committee proposal:

$$RW_{c,\tau} = \left(\frac{LGD_{c,\tau}}{50}\right) \times BRW_{c,\tau} \text{ or } 12.5 \times LGD_{c,\tau}, \text{ whichever is smaller,} \tag{4}$$

where $LGD_{c,\tau}$ is an estimated loss given default for a rating class in quarter τ and $BRW_{c,\tau}$ is the so called corporate benchmark risk weight given by

$$\begin{aligned} BRW_{c,\tau} &= 976.5 \times N\left(1.118 \times N^{-1}\left(\widehat{PD}_{c,\tau}\right) + 1.288\right) \times \\ &\times \left(1 + 0.0470 \times \frac{\left(1 - \widehat{PD}_{c,\tau}\right)}{\widehat{PD}_{c,\tau}^{.44}}\right), \end{aligned} \tag{5}$$

where N is a standard Normal c.d.f. and $\widehat{PD}_{c,\tau}$ is the estimated probability of default. First, we see that the weights do reflect the general trend of declining risk in the portfolio. Second, the weights for the different rating classes are, with a few exceptions, distinct from each other. Hence, the differences in credit risk for the rating classes are preserved reasonably well in the risk weights, irrespectively of PD estimation method. However, model-based PDs *per se* yield smoother and also somewhat smaller risk weights. Risk weight monotonicity increases with window length in the moving average estimates, but the differences between a 12-quarter and a 4-quarter window are small.

Now, to the heart of the matter, Figure 13 and Table 6 present the IRB capital charges for the entire portfolio over time, with PDs given by 1, 4, and 12 quarters moving averages of historical default frequencies and credit risk model estimates, respectively. In order to convey a risk characterization of the portfolio, we have also included the estimated VaR-percentiles. First, the results seem quite reasonable overall. The turbulent, risky beginning of our sample period is associated with relatively high charges in the range of 10 to 20 per cent. As time progresses, and risks decline, so do the capital charges. It is interesting to note that the charges are actually raised in 1997, thus capturing the temporary worsening in macroeconomic conditions. The agreement of capital charges and portfolio credit risk, as measured by the estimated VaR, is remarkable. The correlations over time between the six capital charges and the 99% VaR is given by the last row of Table 6. Not surprisingly we find that smoothing the PD estimate will somewhat distort the risk-sensitiveness of charges.

Although the six columns with capital charges in Table 6 are highly correlated over time, they obviously imply dramatically different levels. Contrasting PD estimation methods, we find that model based charges are on average 30% (Q1), 23% (Q4), and 3% (Q12) higher than charges based on historical default frequencies. Likewise, the size of the smoothing window matters. Charges based on four-quarter moving averages in relation to one-quarter based ones are on average 24% higher when the PDs have been estimated using historical default frequencies. The corresponding number for model-based PD charges is somewhat smaller, 16%, suggesting that the window length matters less in this case. It seems reasonable to strike a compromise between risk sensitivity and erratic movements in charges by considering a 4 period moving average PD estimate.

Table 6: IRB capital charges and VaR-estimates for the portfolio (%).

	Capital charges						VaR-estimates		
Quarter	Hist.Q1	Hist.Q4	Hist.Q12	Mod.Q1	Mod.Q4	Mod.Q12	99%	95%	90%
94:2	12.33	-	-	22.73	-	-	3.95	3.13	2.75
94:3	12.32	-	-	21.87	-	-	3.33	2.59	2.28
94:4	17.41	-	-	23.41	-	-	3.22	2.42	2.09
95:1	9.70	11.48	-	17.91	19.15	-	2.76	1.90	1.52
95:2	11.87	11.66	-	21.00	20.44	-	3.42	1.97	1.54
95:3	15.46	9.70	-	11.54	11.47	-	3.42	1.90	1.48
95:4	7.86	8.84	-	10.46	9.16	-	2.19	1.13	0.81
96:1	8.06	8.84	-	9.04	8.76	-	2.06	1.24	0.84
96:2	7.11	6.45	-	8.13	6.54	-	2.52	1.35	0.94
96:3	4.27	5.70	-	5.92	6.73	-	1.85	0.87	0.59
96:4	2.40	5.03	-	5.38	5.64	-	1.85	0.98	0.66
97:1	6.34	5.44	5.72	5.72	5.69	5.92	1.63	0.93	0.69
97:2	10.13	6.48	6.45	6.58	5.87	6.12	1.98	1.15	0.78
97:3	4.44	6.59	6.77	6.49	6.39	6.62	1.95	1.11	0.73
97:4	4.04	5.69	5.63	6.90	6.35	6.17	2.92	1.32	0.88
98:1	4.56	5.57	5.61	6.80	6.75	6.38	2.29	1.16	0.81
98:2	2.55	3.50	5.81	6.05	6.38	6.15	2.05	0.96	0.71
98:3	3.58	3.30	4.61	4.55	6.04	6.01	1.24	0.68	0.43
98:4	2.09	2.76	4.26	4.10	4.99	5.44	1.13	0.65	0.43
99:1	6.41	3.94	4.65	4.89	5.47	6.53	1.36	0.76	0.47
99:2	1.76	3.04	3.40	2.17	2.94	3.88	1.15	0.46	0.28
99:3	1.24	2.41	3.05	1.37	2.16	3.11	0.86	0.28	0.18
99:4	1.49	2.00	2.62	1.00	1.54	2.54	0.75	0.22	0.15
00:1	0.23	0.25	0.37	0.06	0.07	0.13	0.82	0.20	0.12
corr	0.83	0.86	0.77	0.89	0.80	0.79			

Finally, in spite of charges falling rapidly, so does VaR, to the effect that the portfolio is at all times, but for the last quarter, fully protected. In that quarter, 2000:1, capital charges are extremely small; on average 0.28% using historical default frequency based PDs and 0.09% using model based PDs. The 99% VaR-estimate is larger than all estimated capital charges. In fact, the model-based charges are inadequate even for the 90% VaR. Presumably this effect suggests that risk-weight mapping function needs some further development.

Although the Basel Committee's proposal for internal ratings based capital charges is very specific on certain points, it leaves quite some room for interpretation on others. One of those 'open' points is the number of rating classes. As equations (4) and (5) show, variations in the capital charges are the product of (i) transitions between rating classes, (ii) changes in the PD for each rating class and (iii) differences in the loss rate given default (LGD) for each rating class. As a result, changing the number of rating classes may well affect the IRB capital charges. A smaller number of rating classes will, for example, change the distribution of loans over classes and is likely to reduce the transition activity between classes. But it will also change the PD's associated with risk classes as the riskiest classes will merge with less risky classes and the least risky classes will merge with riskier classes. What the net effect of these changes is, and if they are quantitatively important, is an empirical issue.

To investigate this matter, we have transformed and reduced the number of risk classes from 15 to 6 (including the default class). Figure 14 displays the capital charges using the original 15 rating classes versus using only 6 classes. Clearly, capital charges under both 'regimes' follow the same broad movements, with more or less equal slopes over the whole sample period and both regimes peaking at the same dates. Correlations over time with the 99% VaR-percentile are 0.82, 0.84, and 0.79 for charges based on historical frequency estimated PDs using 1, 4, and 12 quarters moving averages respectively. Apart for these similarities, there is also an important difference: the difference in capital charge levels. For example, in 1997$Q2$, using 6 rating classes would have required 33% more capital compared with 15 classes (with PD based on 1 lag of information). This (qualitative) pattern is consistently seen at other peaks and also for the cases with PD($Q4$) and PD($Q12$). On average, for the period 1997$Q1$-1999$Q4$, capital charges are 27% ($Q1$), 32% ($Q4$), and 36% ($Q12$) higher using 6 classes versus 15 classes. Thus, it seems that having fewer rating classes actually reinforces the effect noted above: PDs based on a longer window in the moving average calculation, yield higher capital charges.

IRB capital charges thus appear, as may be expected, to be sensitive to the number of risk classes. By and large, both a smaller and a larger number of rating classes can capture the variation in portfolio risk that is generated by macro and micro variables. However, the number of rating classes will matter, just like the choice of PD estimation method, for the level of capital charges.

5 Conclusions

In this paper we have analyzed a unique data set covering the corporate sector credit portfolio of a large Swedish bank, as taken on the last day of each of 24 quarters from 1994 to 2000.

The paper is an attempt to evaluate the Internal Ratings Based approach for capital adequacy determination. In particular, we study how the bank's capital charges vary over time (had the bank been subject to the proposed rules). One fundamental purpose of the new Accord is to increase credit risk sensitivity in minimal capital charges. Finding capital time-varying does not necessarily imply an increased credit risk coherence. We also need to have a measure of portfolio credit risk in order to assess the performance of IRB capital.

To this end we estimate a dynamic credit risk model of duration type, i.e., the dependent variable is given by the counterparts survival until default. Apart from the frequently used default behavior predictors such as firms' total sales, earnings before taxes and dividends, and total liabilities over total assets, we find that information on firms past payment - or rather non-payment - behavior to be at least as predictive as accounting data. Thanks to the time series dimension in our data we can also condition on macroeconomic variables; an estimated output gap, a measure of the yield curve, and household's expectations of future economic conditions. The results suggest that both micro and macroeconomic variables are important. A Value-at-Risk-type credit risk measure, derived by simulation from the estimated credit risk model allows us to asses portfolio credit risk, which in turn allows for an assessment of IRB capital. The overall impression is that the new Basel Accord is an achievement in the intended direction, although the capital charges for the last evaluated quarter are extremely small (also in relation to the portfolio VaR in that quarter). Presumably the mapping function between the estimated probability of default and the risk weight for a given rating class needs further adjustment. Also, a risk-weight-mapping function that maps expected loss rates into relative risk weights cannot be expected to fully account for variations in portfolio risk.

The paper also addresses two IRB-related methodological questions. First, how can the important probability of default characterization of a rating class be estimated? We examine two approaches; the historical default frequencies and the credit risk model estimated probabilities. The simpler default frequency approach is found to be adequate, which is reassuring since it will presumably be the method used by most banks in practise. We also take a look at the amount of smoothing one should apply when calculating these average default probabilities. Comparing moving average estimates based on 1, 4, and 12 quarters, we conclude that using 4 quarters strikes a reasonable compromise between risk coherence on the one hand and erratic movements in capital charges on the other hand. Second, what number of rating classes should be used? We compare capital charges based on an aggregated set of 6 classes with those obtained for the original 15 bank classes. The results show that both sets of rating classes yield capital charges that are highly correlated with portfolio VaR, but using 6 classes results in consistently higher charges, on average by as much as 30-35 per cent.

References

[1] Altman, Edward I., (1968), Financial ratios, discriminant analysis and the prediction of corporate bankruptcy, *Journal of Finance*, Vol. XXIII No. 4, pp. 589-611.

[2] Altman, Edward I., (1971), Railroad bankruptcy propensity, *Journal of Finance*, Vol. XXVI No. 2, pp. 333-345.

[3] Altman, Edward I., (1973), Predicting Railroad bankruptcies in America, *Bell Journal of Economics*, No. 4 (1), pp. 184-211.

[4] Altman, Edward I., (1984), The success of business failure prediction models, *Journal of Banking and Finance*, No. 4, pp. 171-198.

[5] Altman, Edward I., and Anthony Saunders, (1997), Credit risk measurement: developments over the last twenty years, *Journal of Banking and Finance*, No. 21 (11-12), pp. 1721-42.

[6] Altman, Edward I., and Anthony Saunders, (2001), An analysis and critique of the BIS proposal on capital adequacy and ratings, *Journal of Banking and Finance*, No. 25 (1), pp. 25-46.

[7] Apel, Mikael and Per Jansson, (1999), "System estimates of potential output and the NAIRU", *Empirical Economics*, Vol. 24, No. 3, pp. 373-388.

[8] Blume, Marshall E., Felix Lim, and A. Craig Mackinley, (1998), The declining credit quality of U.S. corporate debt: myth or reality?, *Journal of Finance*, Vol. LIII No. 4, pp. 1389-1413.

[9] Calem, Paul and Michael LaCour-Little, (2001), Risk-based capital requirements for mortgage loans, Finance and economics discussion series no. 2001-60, Federal Reserve Board, Washington D.C.

[10] Carey, Mark, (1998), Credit risk in private debt portfolios, *Journal of Finance*, Vol. LIII, No. 4, pp. 1363-1387.

[11] Carey, Mark, (2000), Dimensions of credit risk and their relationship to economic capital requirements, in: *Prudential Supervision: What Works and What Doesn't*, Fredric S. Mishkin (ed.), NBER and UC Press.

[12] Carey, Mark and Mark Hrycay, (2001), Parameterizing credit risk models with rating data, *Journal of Banking and Finance,* 25, pp. 197-270.

[13] Cox, David, (1972), Regression models and life tables, *Journal of the Royal Statistical Society*, Series B, 34, pp. 187-220.

[14] Englund, Peter, (1999), The Swedish banking crisis: roots and consequences, *Oxford Review of Economic Policy*, 15 (3) Autumn , pp. 80-97.

[15] Estrella, Arturo, (2001), *The cyclical behavior of optimal bank capital*, mimeo, Federal Reserve Bank of New York.

[16] Estrella, Arturo and Gikas A. Hardouvelis, (1991), The term structure as a predictor of real economic activity, *Journal of Finance*, Vol. 46, No. 2, pp. 555-576.

[17] Estrella, Arturo and Frederic S. Mishkin, (1998), Predicting U.S. recessions: financial variables as leading indicators, *Review of Economics and Statistics*, Vol. 80, No. 1, pp. 45-61.

[18] Fienberg, S.E., (1987), *The Analysis of Cross-Classified Categorical Data*, The MIT Press.

[19] Frydman, Halina, Edward I. Altman, and Duen-Li Kao, (1985), Introducing recursive partitioning for financial classification: the case of financial distress, *Journal of Finance*, Vol. XL, No. 1, pp. 269-291.

[20] Gordy Michael B., (2000a), A comparative anatomy of credit risk models, *Journal of Banking and Finance*, 24, No 1-2, pp. 119-149.

[21] Gordy, Michael B., (2000b), *A risk-factor model foundation for ratings-based bank capital rules*, mimeo Board of Governors of the Federal Reserve System, Washington D.C.

[22] Gray, R.J., (1996), 'Hazard rate regression using ordinary nonparametric regression smoothers', *Journal of Computational and Graphical Statistics*, Vol 5, No 2, pp. 190-207.

[23] Hamerle, Alfred, Thilo Liebig and Daneil Rösch, (2002), *Credit risk factor modeling and the Basel II IRB approach*, mimeo, Deutsche Bundesbank.

[24] Hastie, T., and Loader, C., (1993), Local regression: automatic kernel carpentry, *Statistical Science*, Vol 8, pp. 120-143.

[25] Härdle, W., (1991), *Smoothing techniques: with implementation in S*, New York, Springer Verlag.

[26] Kooperberg, C., Stone, C.J., and Troung, Y.K., (1995), Hazard regression, *Journal of the American Statistical Association*, Vol 90, No 429, pp. 78-94.

[27] Honjo, Yuji, (2000), Business failure of new firms: an empirical analysis using a multiplicative hazards model, *International Journal of Industrial Organization,* No. *18*, pp. 557-574.

[28] Jackson, Patricia, Craig Furfine, Hans Groeneveld, Diana Hancock, David Jones, William Perraudin, Lawrence Radecki, and Masao Yoneyama, (1999), *Capital requirements and bank behaviour: the impact of the Basle Accord*, Basle Committee on Banking Supervision Working Papers, No.1.

[29] Jacobson, Tor and Jesper Lindé, (2000), Credit rating and the business cycle: can bankruptcies be forecast?, *Sveriges Riksbank Economic Review*, 2000, No 4, pp. 11-33.

[30] Jacobson, Tor and Kasper Roszbach, (2002), Bank lending policy, credit scoring and value-at-risk, forthcoming in the *Journal of Banking and Finance.*

[31] Koyluoglu, H. Ugur, and Andrew Hickman, (1998), A generalized framework for credit risk portfolio models, forthcoming in *Risk.*

[32] Lagakos, S.W., (1979), General right censoring and its impact on the analysis of survival data, *Biometrics*, Vol 35, pp. 135-156.

[33] Lancaster, T., (1990), *The econometric analysis of transition data*, Cambridge University Press, Cambridge.

[34] Li, Kai, (1999), Bayesian analysis of duration models: an application to Chapter 11 bankruptcy, *Economics Letters*, No. 63, pp. 305-312.

[35] Little, R.J.A. and Rubin, D.B., (1987), *Statistical Analysis with Missing Data*, Wiley, New York.

[36] Merton, Robert, (1974), On the pricing of corporate debt: the risk structure of interest rates, *Journal of Finance,* Vol 29, pp. 449-470.

[37] Platt, Harlan D., and Marjorie B. Platt, (1991), A note on the use of industry-relative ratios in bankruptcy prediction , *Journal of Banking and Finance*, Vol 15, pp. 1183-1194.

[38] Shumway, Tyler, (2001), Forecasting bankruptcy more accurately: a simple hazard model, *Journal of Business*, Vol 74, No 1, pp. 101-124.

[39] Treacy, William and Mark Carey, (1998), *Internal credit risk rating systems at large U.S. banks*, Federal Reserve Bulletin, November 1998, Board of Governors of the Federal Reserve System, Washington D.C.

[40] Wilson, Thomas, (1997), Portfolio credit risk (I), *Risk*,Vol. 10, No. 9, pp. Xx-xx+5.

A Bank variables

EX_DAT = Measurement date
BR_K = 4 figure industry classification (by bank)
KO_KU_NR =
KU_KAT =
BR_GR_K = 2 figure industry classification (by bank)
UT_KRED = Amount of credit utilized
BE_KRED = Granted credit
RI_K = Rating class
KONK = Bankruptcy dummy
SAK_BEL =
SEBRRKAP = Collateral 1
ENARRKAP = Collateral 2
KSKVPRGR = Dummy, 1 if short term credit is granted
KSKVBEKR = Amount of short term credit granted
LSLVPRGR = Dummy, 1 if long term credit is granted
LSLVBEKR = Amount of long term credit granted
S1S4PRGR = Dummy, 1 if mortgage is granted
S1S4BEKR = Amount of mortgage granted
GUARPRGR = Dummy, 1 if guarantee loan is granted
GUARBEKR = Amount of guarantee loan granted
MIXTPRGR = Dummy, 1 if other mixed credit is granted
MIXTBEKR = Amount of other mixed credit granted

B Credit bureau variables

SAOMSTIL = Current Assets
SAKORTSK = Current Liabilities
SATILLG = Total Assets
LIKVID = Cash
VARULAG = Inventories
SALONGSK = Long Liabilities
LEVSKULD = Accounts Payable
SAANLTIL = Fixed Assets
SAEGETKA = Total Equity
OMSAETT = Total Sales
RESFOEAV = Earnings bef. Interest, Depreciation and Amortizations
ANTANS = No. employees
LOENER = Wages
AVSKRIVN = Depreciation
FININT = Financial income
FINKOST = Financial costs
EXTORDIN = Extraordinary costs
EXTORDKO = Extraordinary income
SKATT = Taxes
KUNDFORD = Accounts receivable
OVOMSTIL = Other liquid assets
SPAERRKO = Blocked accounts (e.g escrows)
GOODWILL = Goodwill
INVENT = Machinery etc
OBESRES = Untaxed reserves
AKTIEKAP = Nominal equity
OVREGBUN = Other Equity
SASKOEGE = Sum of taxes and equity (equals total assets)
SCBSNIKO = Statistics Sweden industry code
SCBSTKL = Statistics Sweden company size code
ORGNR = Company's 10 figure identification number
PANTER = Total of property pledges for non-mortgage loans)
ANSVAR = Total guaranties assumed for third party loans
SAFTGINT = Total of property pledges for mortgages in public register

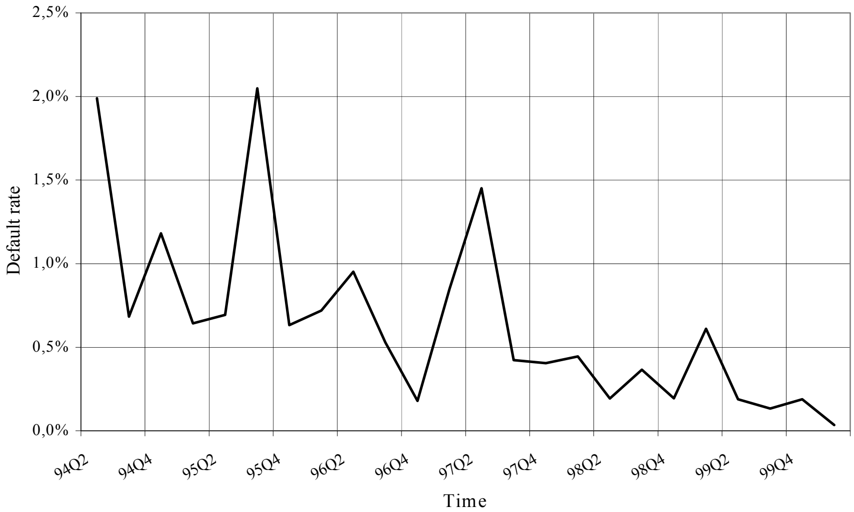

Figure 1: Default rates for the entire portfolio

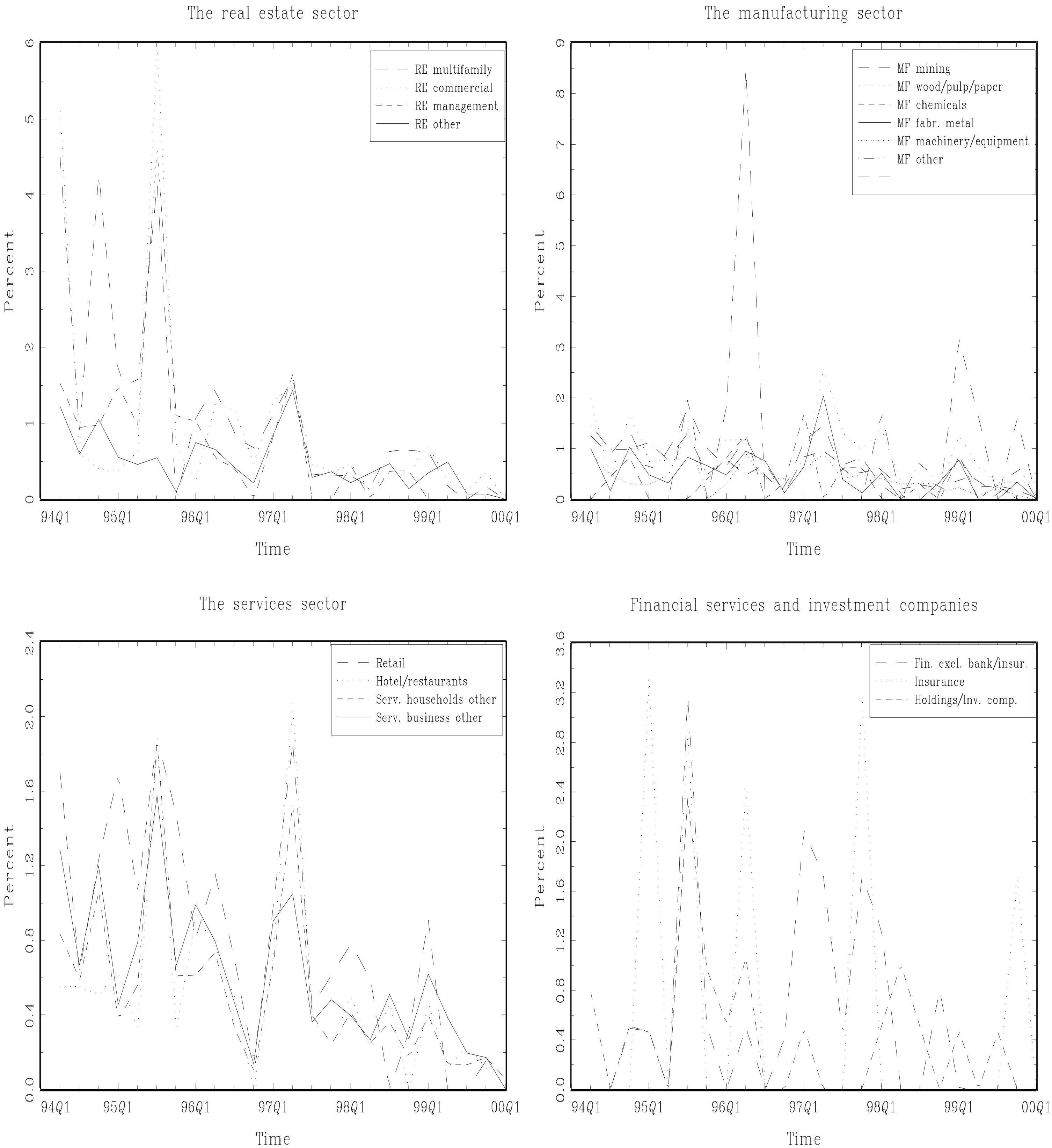

Figure 2: Default rates by industry.

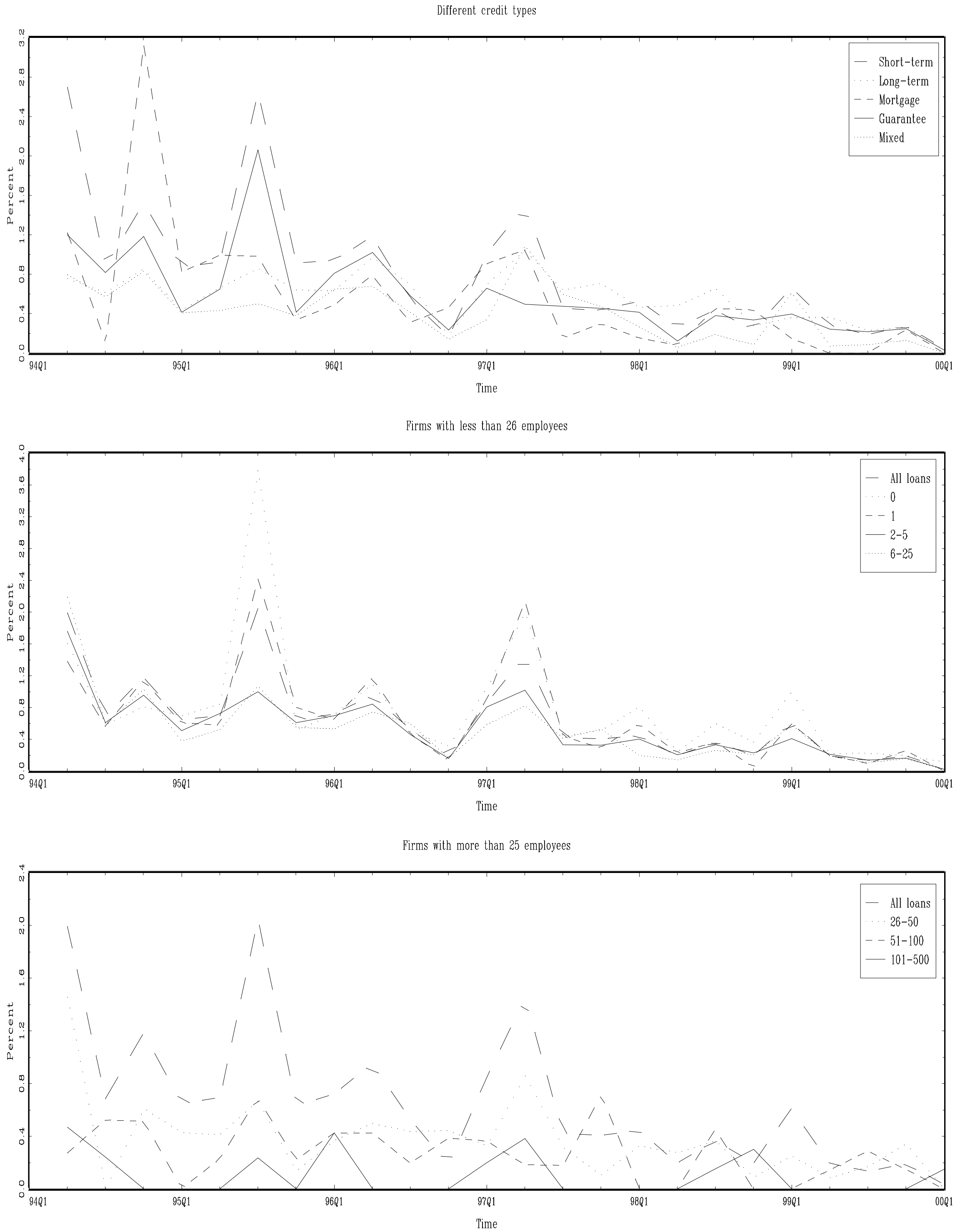

Figure 3: Default rates by loan type and number of employees.

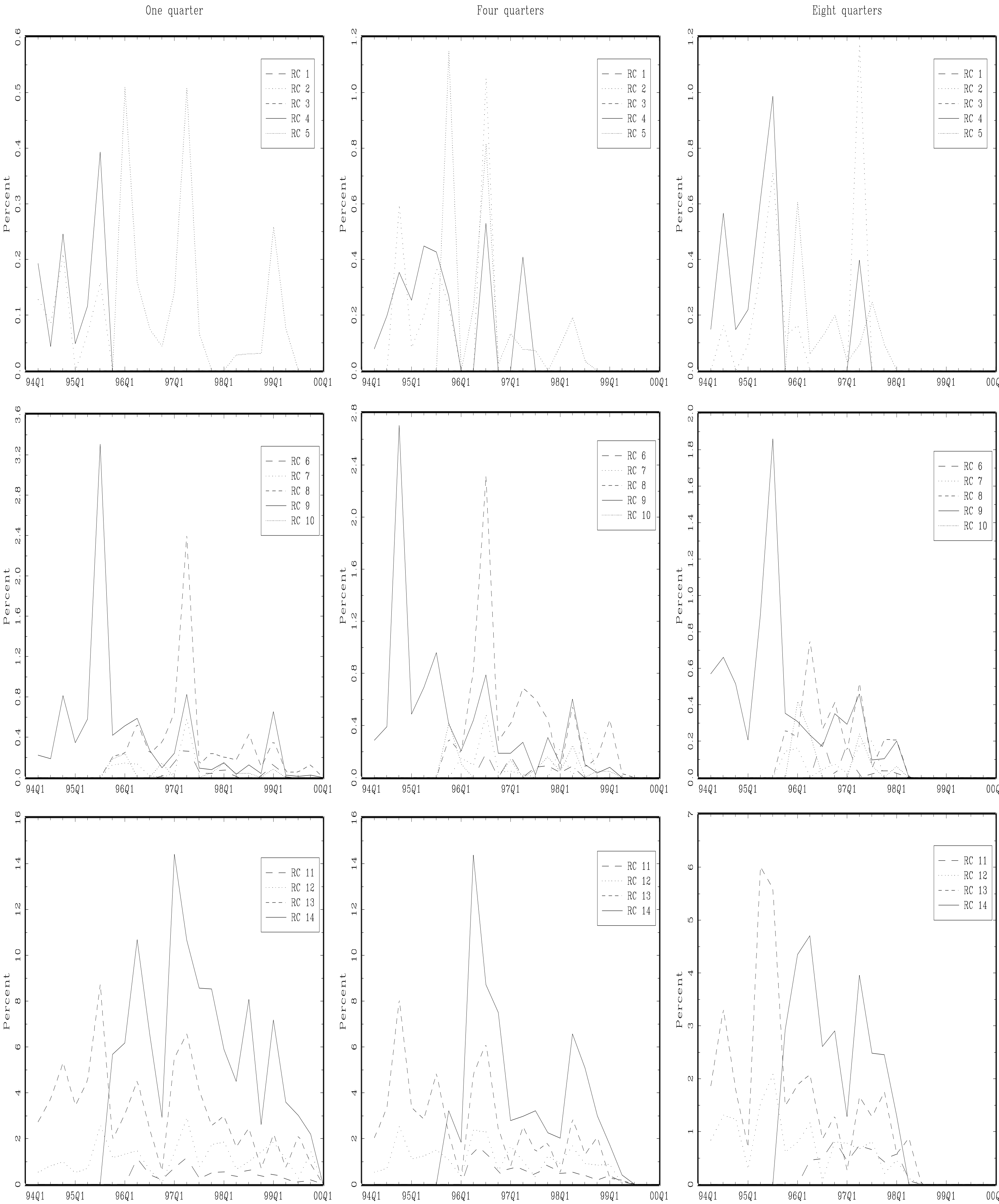

Figure 4: Default rates by rating class; one, four and eight quarters ahead.

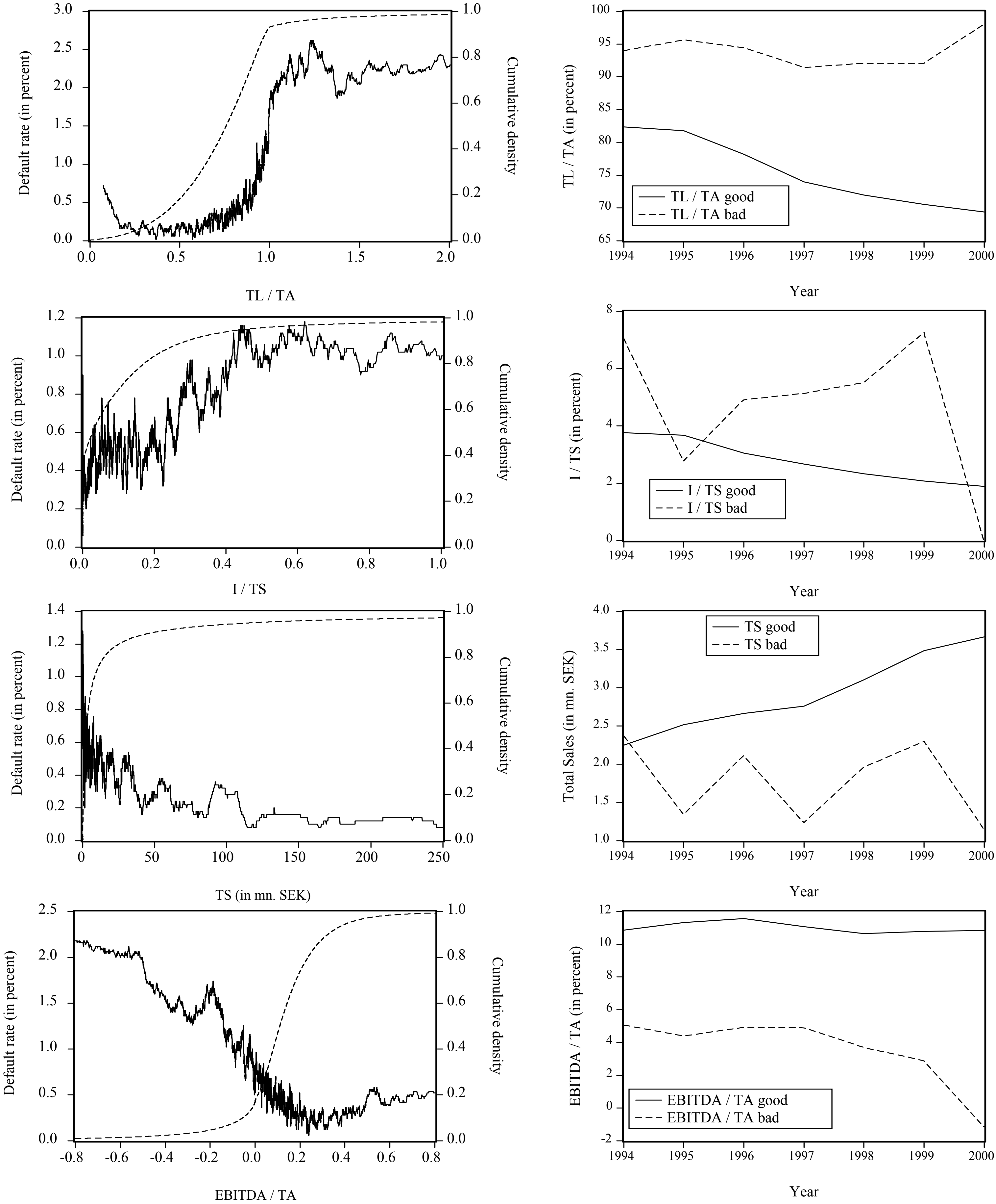

Figure 5: Default rates and cumulative distribution functions for the accounting data.

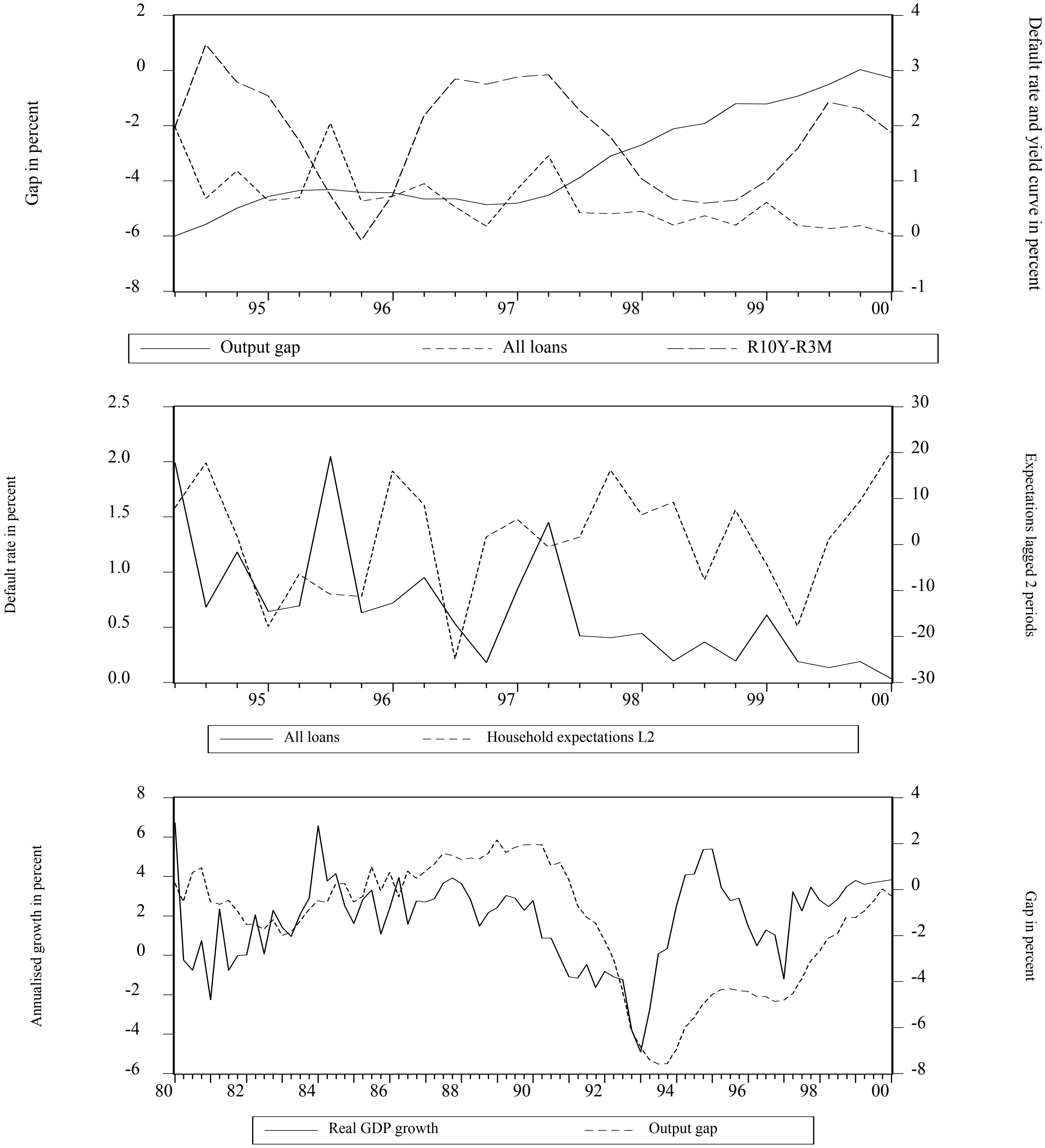

Figure 6: Macroeconomic variables and the average portfolio default rate.

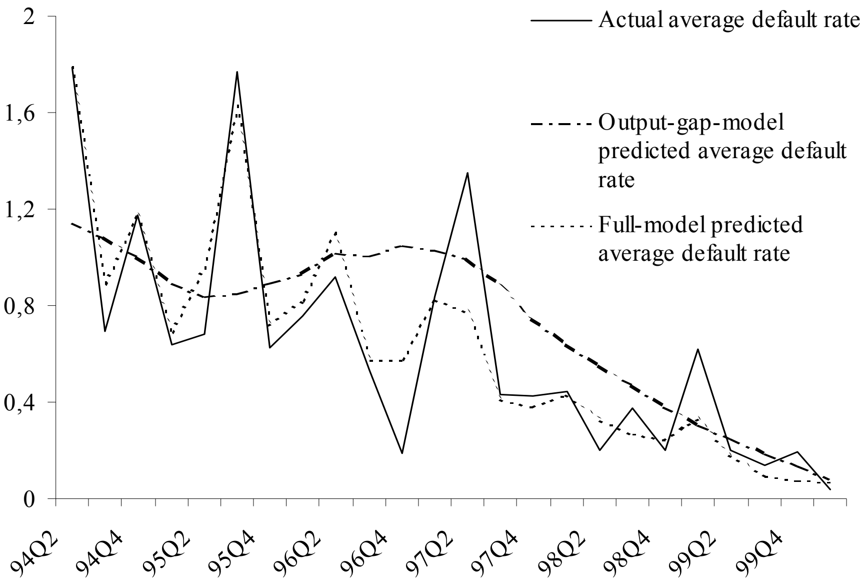

Figure 7: Actual and model predicted average default rates for the portfolio.

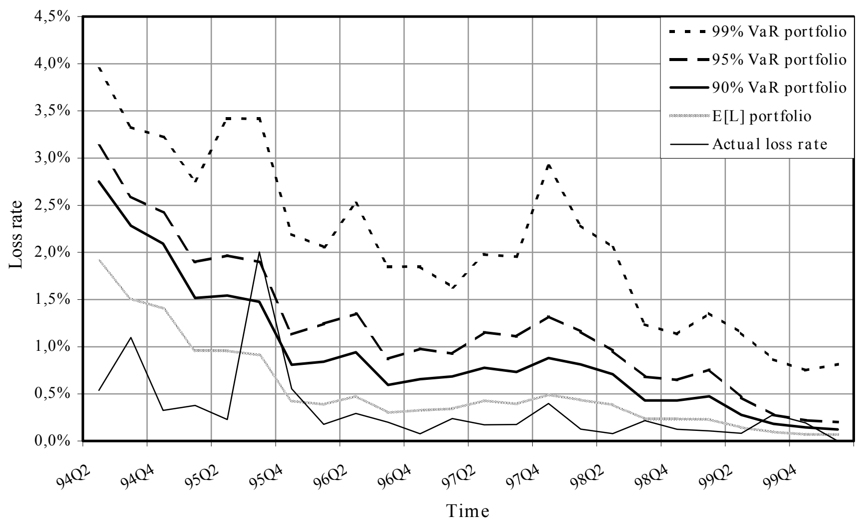

Figure 8: Actual and Expected loss rates and Value-at-Risk for entire portfolio

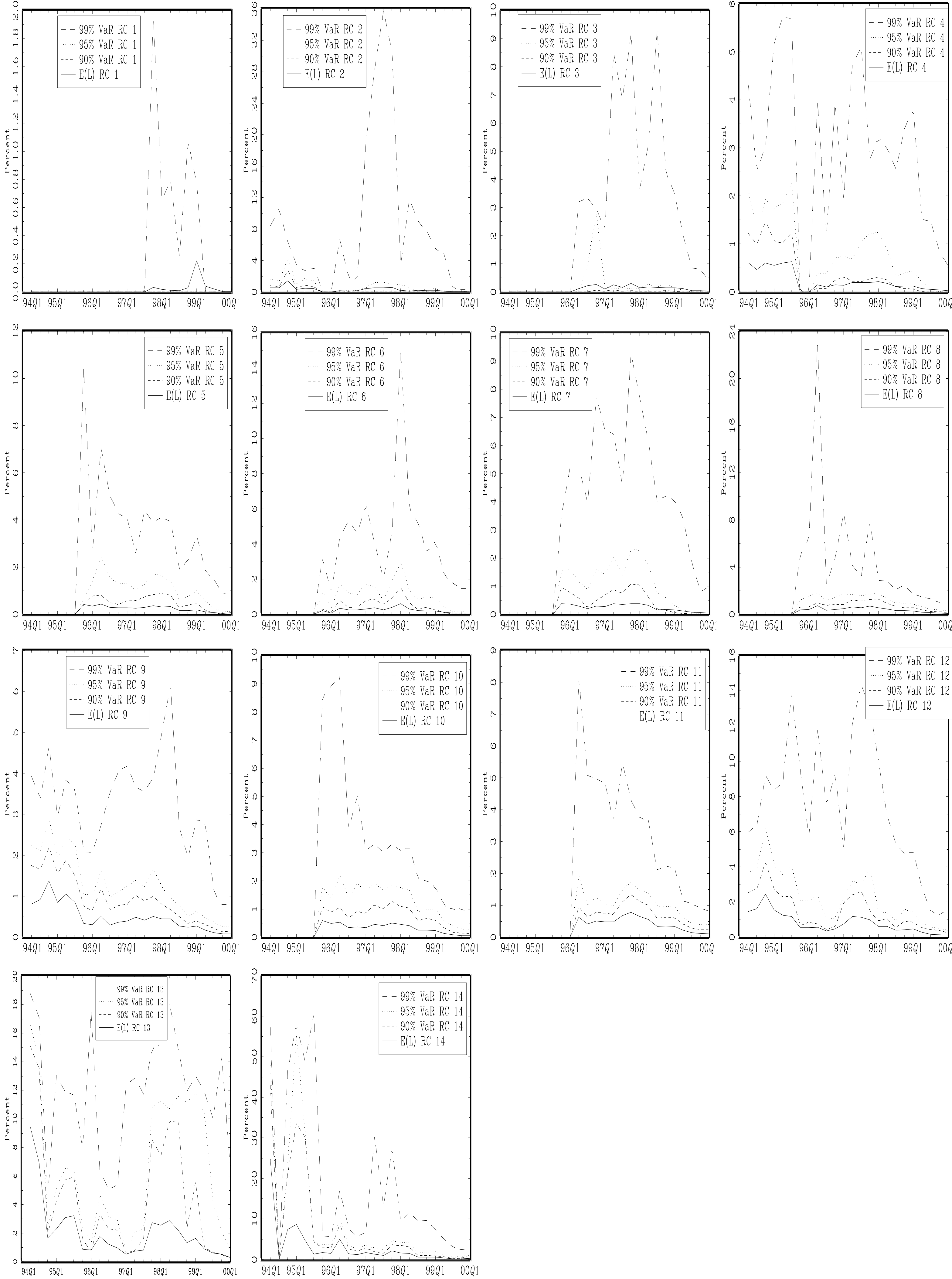

Figure 9: Expected loss rates and 90, 95 and 99-percent Value-at-Risk ratios per rating class.

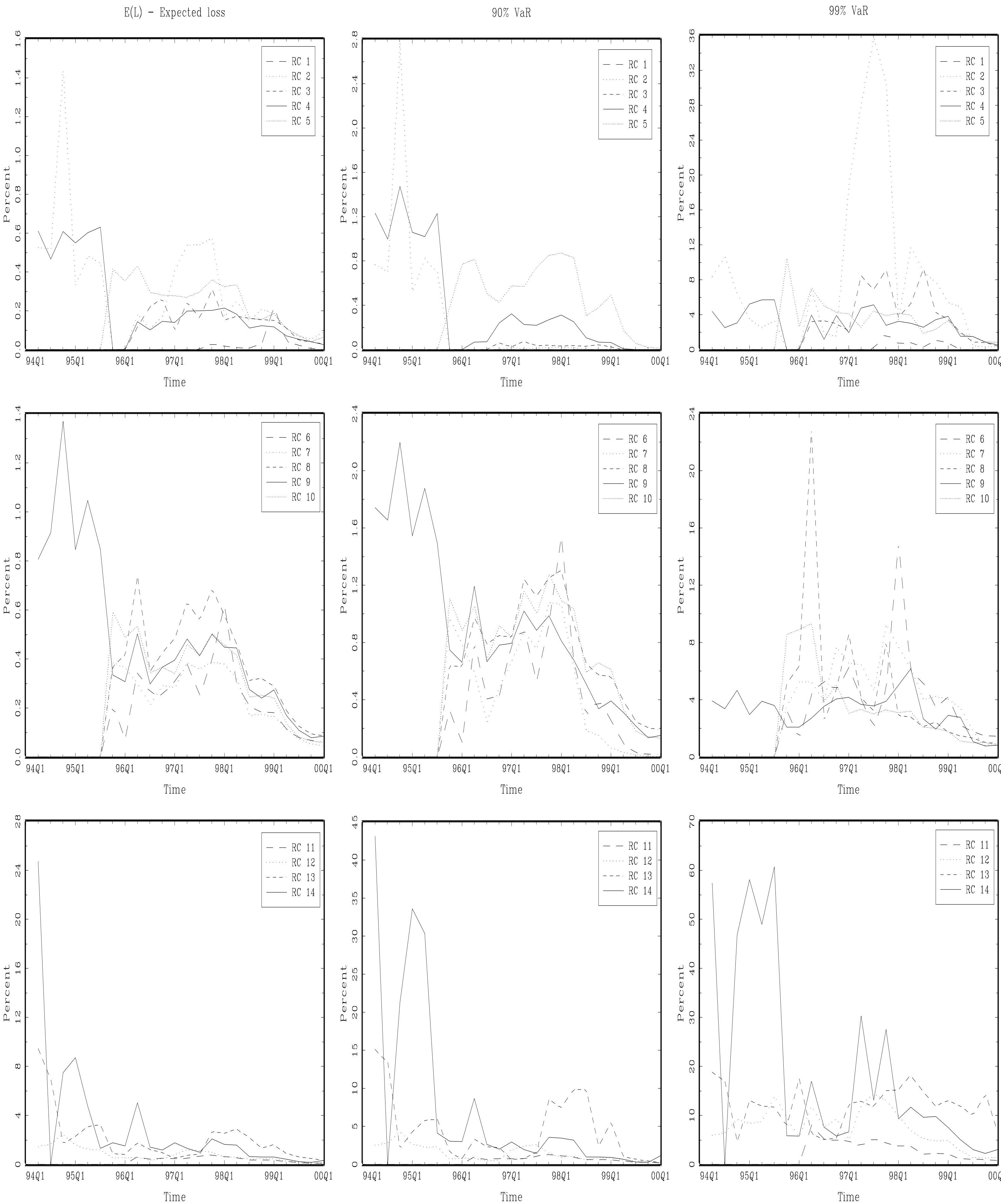

Figure 10: Expected loss rates and Value-at-Risk for Rating classes 1-14.

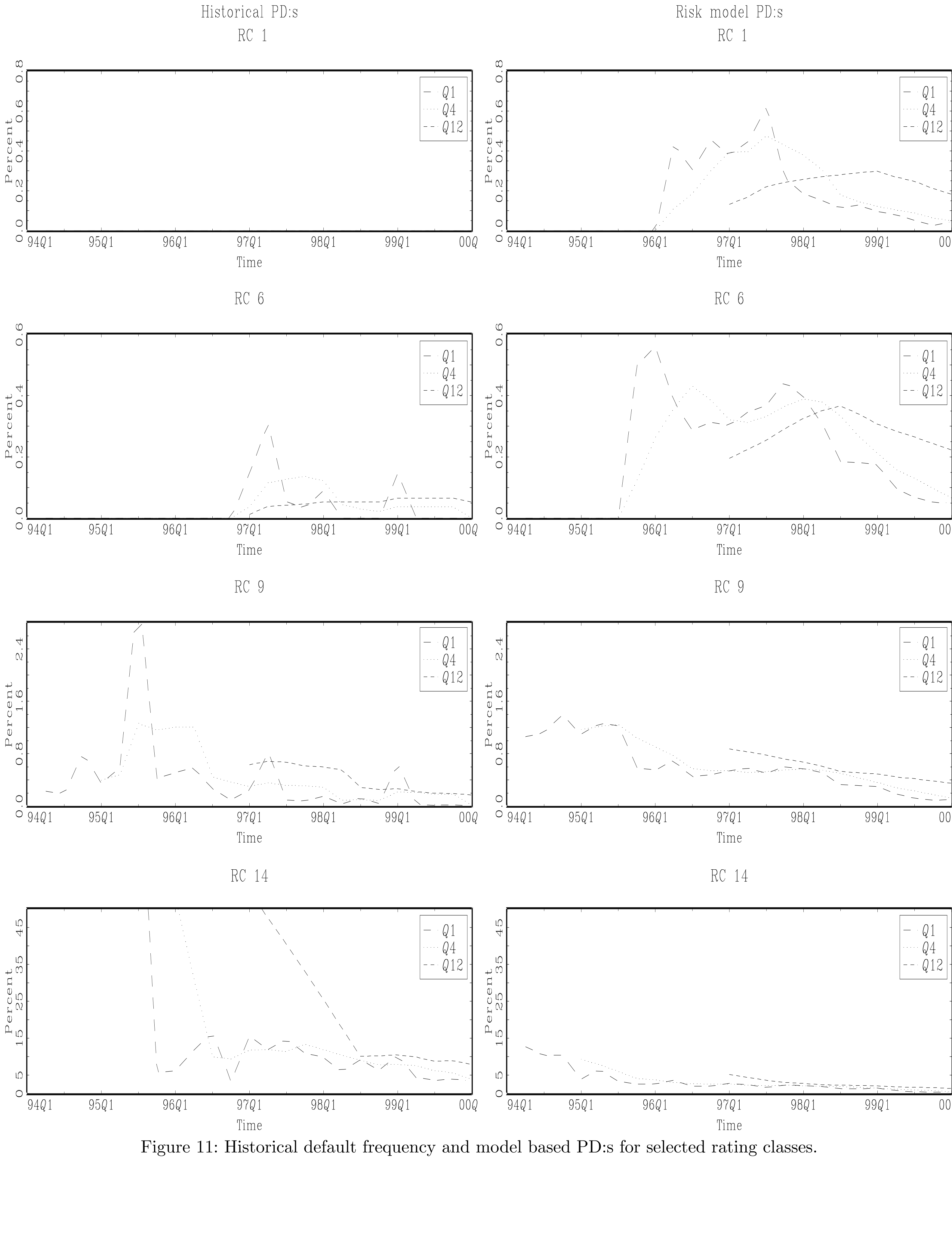

Figure 11: Historical default frequency and model based PD:s for selected rating classes.

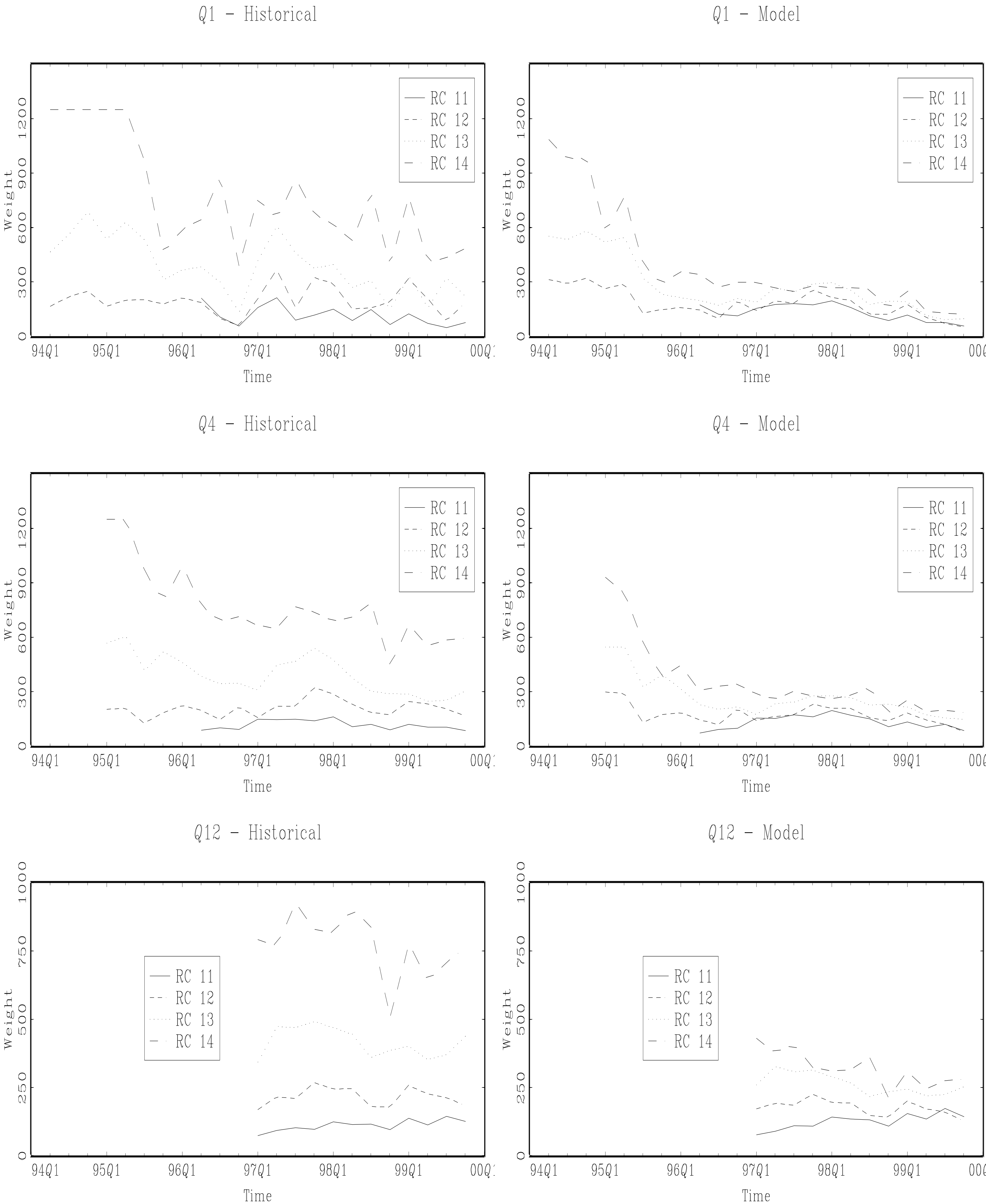

Figure 12: IRB riskweights using historical default frequency and model based PD:s.

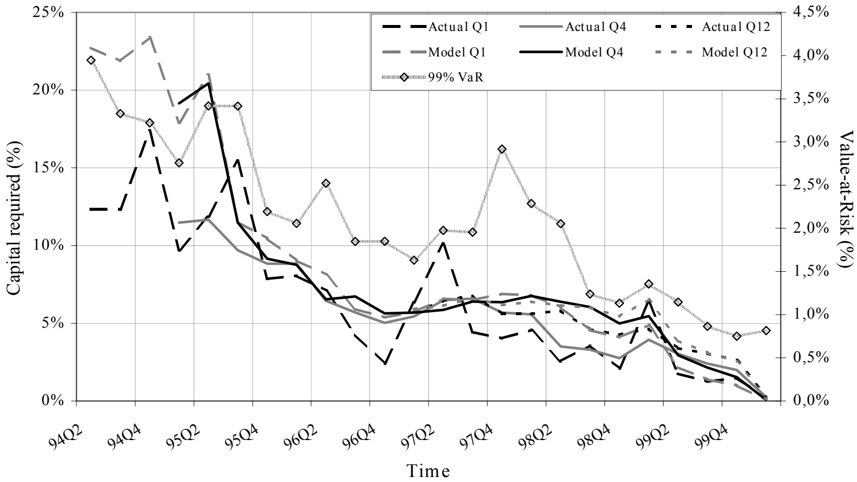

Figure 13: IRB determined capital requirement

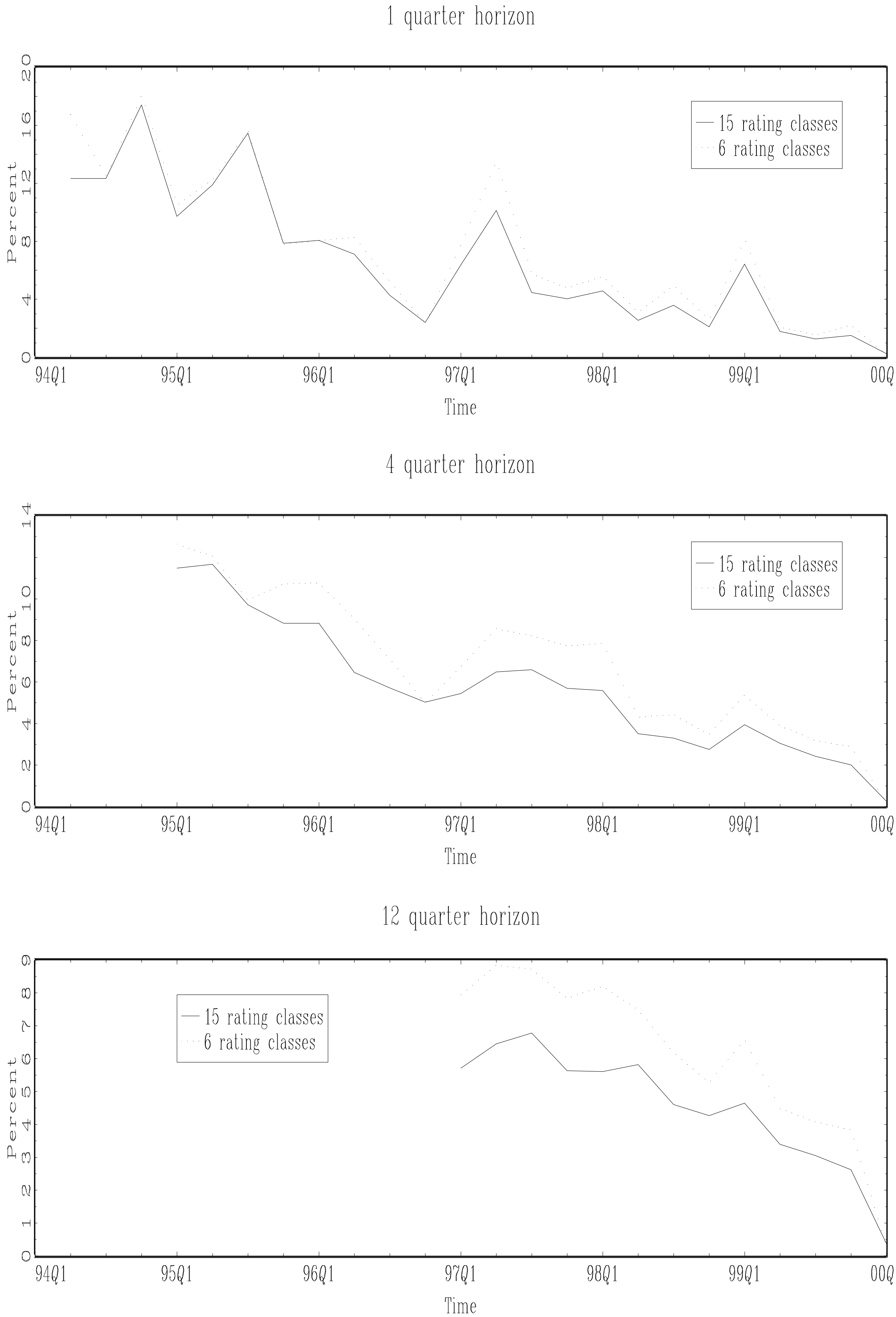

Figure 14: IRB capital charges for 15 versus 6 rating classes.

Earlier Working Papers:

SVERIGES
RIKSBANK

Sveriges Riksbank • Visiting address: Brunkebergs torg 11 • Mail address: se-103 37 Stockholm
Website: www.riksbank.se • E-mail: registratorn@riksbank.se • Telephone: +46 8 787 00 00 • Fax: +46 8 21 05 31

ISSN 1402-9103 Riksbank Printing Office Stockholm 2002