Policy Rules for Inflation Targeting*

Glenn D. Rudebusch† Lars E.O. Svensson‡

February 1998

Abstract

Policy rules that are consistent with inflation targeting are examined in a small macroeconometric model of the US economy. We compare the properties and outcomes of explicit "instrument rules" as well as "targeting rules." The latter, which imply implicit instrument rules, may be closer to actual operating procedures of inflation-targeting central banks. We find that inflation forecasts are central for good policy rules under inflation targeting. Some simple instrument and targeting rules do remarkably well relative to the optimal rule; others, including some that are often used as representing inflation targeting, do less well.

1. Introduction

In this paper, we use a small empirical model of the U.S. economy to examine the performance of policy rules that are consistent with a monetary policy regime of inflation targeting. In the real world, explicit inflation targeting is currently pursued in New Zealand, Canada, the U.K., Sweden, Australia, and arguably also in Finland and Spain (although the participation of the last two in the Exchange Rate Mechanism in the European Monetary System raises some questions). Inflation targeting in these countries is characterized by (1) a publicly-announced numerical inflation target (either in the form of a target range, a point target, or a point target with a tolerance interval), (2) a framework for policy decisions which involves comparing

*This paper was prepared for the NBER Conference on Monetary Policy Rules, January 16–17, 1998. We thank James Stock, Frederic Mishkin, and other conference participants for comments. The paper was also presented at the Federal Reserve Bank of San Francisco-Center for Economic Policy Research, Stanford University, conference on Central Bank Inflation Targeting, March 6–7, 1998. We thank Ben Bernanke, Carl Walsh, and other conference participants for comments. Part of the paper was written when Lars Svensson visited Reserve Bank of New Zealand and Victoria University of Wellington. He thanks these institutions for their hospitality. The views expressed in the paper do not necessarily reflect the views of the Federal Reserve Bank of San Francisco, the Federal Reserve System, or the Reserve Bank of New Zealand. We thank Charlotta Groth and Heather Royer for research assistance, and Christina Lönnblad for secretarial and editorial assistance.

†Glenn.Rudebusch@sf.frb.org, http://www.frbsf.org/econrsrch/, Federal Reserve Bank of San Francisco.

‡Lars.Svensson@iies.su.se, http://www.iies.su.se/leosven/, Institute for International Economic Studies, Stockholm University; CEPR; and NBER.

an inflation forecast to the announced target, thus, providing an "inflation-forecast targeting" regime for policy where the forecast serves as an intermediate target (cf. Haldane [38], King [45], and Svensson [71]), and (3) a higher-than-average degree of transparency and accountability.[1]

We model an inflation-targeting policy regime using loss functions over policy goals. In our loss functions, inflation targeting always involves an attempt to minimize deviations of inflation from the explicit inflation target. In addition, however, our inflation targeting loss functions also allow concerns about real output (or more precisely about the variability of output because the natural rate hypothesis is assumed). That is, we would argue there is no necessary connection between the specification of the loss function (other than that inflation variability must enter with a non-negligible weight) and the specification of an inflation-targeting policy regime.[2] For support of this view, see, for example, the recent discussion by Fischer [29], King [46], Taylor [78] and Svensson [70] in Federal Reserve Bank of Kansas City [28].[3] Thus, we interpret inflation targeting as consistent with a conventional quadratic loss function, where in addition to the variability of inflation around the inflation target there is some weight on the variability of the output gap.[4]

In examining policy rules that are consistent with inflation targeting, we consider two broad classes of rules: instrument rules and targeting rules. An explicit instrument rule expresses the monetary policy instrument as an explicit function of available information. We examine both optimal unrestricted instrument rules (a tradition that goes back at least to Taylor [76]; recent contributions include Blake and Westaway [8]) as well as optimal simple or restricted instrument rules, which involve only a few parameters or arguments (for instance, current inflation and output as in Taylor's [77] rule). However, no central bank, whether inflation-targeting or not, follows an explicit instrument rule (unrestricted or simple). Every central bank uses more information than the simple rules are based on, and no central bank would voluntarily restrict itself to react mechanically in a predescribed way to new information. The role of unrestricted

[1] The rapidly increasing literature on inflation targeting includes the conference volumes Leiderman and Svensson [48], Haldane [36], Federal Reserve Bank of Kansas City [28], and Lowe [50]. See also the survey by Bernanke and Mishkin [4].

[2] One may argue, though, that the high degree of transparency and accountability serves to increase the commitment to minimizing the loss function, and to ensure that any concern about the real economy is consistent with the natural rate hypotheses and therefore reduces, or eliminates, any inflation bias.

[3] As discussed in Svensson [73], concerns about the stability of the real economy, model uncertainty, and interest rate smoothing all have similar effects under inflation targeting, namely a more gradualist policy. Thus, if inflation is away from the inflation target, it is brought back to target more gradually ("flexible" rather than "strict" inflation targeting, the inflation forecast hits the target at a horizon that is longer than the shortest possible). Svensson [72] argues that all inflation-targeting central banks in practice behave in this way, possibly with differing weights on the different reasons for doing so.

[4] Because inflation-targeting central banks, like other central banks, also seem to smooth interest rates, our loss function also includes some weight on the variability of interest rate changes.

or simple explicit instrument rules is at best to provide a baseline and comparison to the policy actually followed.

A targeting rule may be closer to the actual decision framework under inflation targeting. It is represented by the assignment of a loss function over deviations of a goal variable from a target level, or deviations of an intermediate target variable from an intermediate target level (cf. Rogoff [62], Walsh [79] and Svensson [71] and [73]). A targeting rule, combined with a particular model, is only an *implicit* instrument rule; typically, the equivalent of a first-order condition has to be solved in order to find the corresponding *explicit* instrument rule. (For an intermediate target variable that the central bank has complete control over, the first-order condition is trivial: equality between the intermediate target variable and the target level.) As an example, note that one interpretation of "inflation-forecast targeting" is that the policy instrument is adjusted such that a conditional inflation forecast (the intermediate target variable) hits the inflation target at an appropriate horizon. Combined with a particular model, the instrument then becomes an implicit function of current information; when the corresponding system of equations is solved for the instrument, the explicit instrument rule results. We shall examine several such targeting rules below.

Our analysis proceeds as follows. Section 2 presents the empirical model we use, which is a simple two-equation model of U.S. output and inflation, somewhat similar to the theoretical model in Svensson [71]. The model captures some realistic dynamics (for example, monetary policy actions affect output before inflation) in a very simple but tractable form. Section 3 first attempts to reduce the confusion caused by the literature's use of two different meanings of "targeting," and then presents the different instrument and targeting rules we examine. Section 4 reports our results, with focus on output and inflation variability under a large set of various policy rules. We find that some simple instrument and targeting rules involving inflation forecasts do remarkably well in minimizing the loss function (relative to the optimal rule). Other policy rules, some of which are frequently used in the literature as representing inflation targeting, do less well. Finally, section 5 concludes.

2. An Empirical Model of U.S. Output and Inflation

2.1. Motivation

Our choice of an empirical model of output and inflation is motivated by three considerations. First, we choose a simple linear model (as well as quadratic preferences below), so our analysis will be tractable and our results transparent. Our model consists of an aggregate supply equation (or "Phillips curve") that relates inflation to an output gap and an aggregate demand equation (or "IS curve") that relates output to a short-term interest rate. Obviously, our model glosses over many important and contentious features of the monetary transmission mechanism. Still, we feel that the model has enough richness—for example, in dynamics—to be of interest, especially when judged relative to some of the models used in previous theoretical discussions.

Second, our model captures the spirit of many practical policy-oriented macroeconometric models. Some (e.g., McCallum [56]) have argued that because there is no academic consensus on the structure of the economy, any proposed monetary policy rule should perform well in a variety of models. We are completely sympathetic to this argument. We believe that robustness to plausible model variation is a crucial issue and one that this conference volume, taken as a whole, should provide some insight into. However, we also believe that monetary policy analysis will be most convincing to central bankers (who are, of course, among the most important ultimate consumers of this research) if it is conducted using models that are similar in structure to the ones actually employed by central bankers. Thus, for example, at this stage of analysis, we focus our attention on a model that (1) uses a short-term interest rate as the policy instrument with no direct role for monetary aggregates, (2) is specified in terms of output gaps from trend instead of output growth rates, and (3) includes a Phillips curve with adaptive or autoregressive expectations that is consistent with the natural rate hypothesis. Such a structure is typical of many central bank policy models (including, for example, the 11 models described in the central bank model comparison project for the Bank for International Settlements [3]) and because our empirical analysis uses U.S. data, we will be keen to match the properties of the Federal Reserve's venerable MPS macroeconometric model.[5] Of course, the appropriate way to model expectations for policy analysis remains particularly contentious (see, for example, the early

[5] In 1996, the FRB/US model replaced the MPS model as the Federal Reserve Board's main quarterly macroeconometric model. The major innovation of this model is its ability to explicitly model various types of expectations including model-consistent ones (see Brayton and Tinsley [13]). Still, across a range of expectations processes, the properties of the new model are broadly similar to those of our model. For example, the FRB/US model exhibits an output sacrifice ratio of between two and five, which, as noted below, brackets our model's sacrifice ratio of about three.

discussion by Lucas [52] and Sims [67]). We are persuaded that the importance of the Lucas Critique is in large measure an empirical issue as in, for example, Oliner, Rudebusch, and Sichel [60]. In this regard, Fuhrer [33] tests an autoregressive Phillips curve like ours against a forward-looking version and cannot reject it. Moreover, many policymakers appear more comfortable with the backward-looking version, including Federal Reserve Governor Meyer [58] and former Vice-Chairman Blinder [9]. Finally, in this regard, it should be noted our backward-looking expectations may be particularly appropriate during the introduction of a new rule for inflation targeting. As stressed by Taylor [77] and Bomfim and Rudebusch [10], rational expectations may be unrealistic during the transition period when learning about the new policy rule is taking place.

Our third consideration in model selection is empirical fit to the data. To judge whether our model is able to reproduce the salient features of the data, we compare its fit and dynamics to an unrestricted VAR. VARs have become a very popular tool recently for describing the dynamics of monetary transmission, and they are a natural benchmark for model evaluation. Indeed, if one dislikes the structural interpretation that we attach to our model, one can simply consider it a reduced-form VAR and so our analysis is similar in spirit to Feldstein and Stock [30] or Cecchetti [16].

2.2. Model Estimates

The two equations of our model are

$$\pi_{t+1} = \alpha_{\pi 1}\pi_t + \alpha_{\pi 2}\pi_{t-1} + \alpha_{\pi 3}\pi_{t-2} + \alpha_{\pi 4}\pi_{t-3} + \alpha_y y_t + \varepsilon_{t+1} \tag{2.1}$$

$$y_{t+1} = \beta_{y1} y_t + \beta_{y2} y_{t-1} - \beta_r \left(\bar{i}_t - \bar{\pi}_t\right) + \eta_{t+1}, \tag{2.2}$$

where π_t is quarterly inflation in the GDP chain-weighted price index (p_t) in percentage points at an annual rate, i.e., $400(\ln p - \ln p_{t-1})$; $\bar{\pi}_t$ is four-quarter inflation in the GDP chain-weighted price index, i.e., $\frac{1}{4}\Sigma_{j=0}^{3}\pi_{t-j}$; i_t is quarterly average federal funds rate in percentage points at an annual rate; $\bar{\imath}_t$ is four-quarter average federal funds rate, i.e., $\frac{1}{4}\Sigma_{j=0}^{3} i_{t-j}$; y_t is the relative gap between actual real GDP (q_t) and potential GDP (q_t^*) in percentage points, i.e., $100(q_t - q_t^*)/q_t^*$. These five variables were de-meaned prior to estimation, so no constants appear in the equations.

The first equation relates inflation to a lagged output gap and to lags of inflation.[6] The

[6] Our series on the output gap is essentially identical to those that have been used in a variety of Federal Reserve and other government studies including, for example, Congressional Budget Office [22] and Hallman, Porter, and Small [40]. Our estimation results were little changed by using a flexible trend for potential output such as a quadratic trend.

lags of inflation are an autoregressive or adaptive representation of inflation expectations, which is consistent with the form of the Phillips curve in the MPS model described in Brayton and Mauskopf [12]. In our empirical analysis below, we will not reject the hypothesis that the coefficients of the four inflation lags sum to one; thus, we will use an accelerationist form of the Phillips curve, which implies a long-run vertical Phillips curve. The second equation relates the output gap to its own lags and to the difference between the average funds rate and average inflation over the previous four quarters—an approximate ex post real rate. The third term is a simple representation of the monetary transmission mechanism, which, in the view of many central banks, likely involves nominal interest rate (e.g., mortgage rates), ex ante real short and long rates, exchange rates, and possibly direct credit quantities as well. Equation (2.2) appears to be a workable approximation of these various intermediate transmission mechanisms.

The estimated equations, using the sample period 1961:1 to 1996:2, are shown below. (Coefficient standard errors are given in parentheses, and the standard error of the residuals and Durbin-Watson statistics also are reported.)

$$\pi_{t+1} = \underset{(.08)}{.70}\,\pi_t - \underset{(.10)}{.10}\,\pi_{t-1} + \underset{(.10)}{.28}\,\pi_{t-2} + \underset{(.08)}{.12}\,\pi_{t-3} + \underset{(.03)}{.14}\,y_t + \varepsilon_{t+1},$$

$$\mathrm{SE} = 1.009, \quad \mathrm{DW} = 1.99,$$

$$y_{t+1} = \underset{(.08)}{1.16}\,y_t - \underset{(.08)}{.25}\,y_{t-1} - \underset{(.03)}{.10}\,(\bar{i}_t - \bar{\pi}_t) + \eta_{t+1},$$

$$\mathrm{SE} = 0.819, \quad \mathrm{DW} = 2.05.$$

The equations were estimated individually by OLS.[7] The hypothesis that the sum of the lag coefficients of inflation equals one had a p-value of .16, so this restriction was imposed in estimation.[8]

The subsample stability of our estimated equations is an important condition for drawing inference from our model—whether it is given a structural or reduced form (VAR) interpretation. In particular, because ours is a backward-looking model, the Lucas Critique may apply with particular force. The historical empirical importance of this Critique can be gauged by econometric stability tests (again, see Oliner, Rudebusch, and Sichel [60]). Our estimated equations appear to easily pass these tests. For example, consider a stability test from Andrews [1]:

[7] Almost identical parameter estimates were obtained by the SUR and by system ML methods because the cross-correlation of the errors is essentially zero.

[8] This p-value was obtained by simulating the above inflation equation 1000 times and ranking the sum of coefficients from the unrestricted Phillips curve estimated from the actual data (i.e., .969) in the set of unrestricted sums estimated from the simulated data. This is in the spirit of Rudebusch [63]. For comparison, the simple t-test gives a p-value of .42.

the *maximum* value of the likelihood-ratio test statistic for structural stability over all possible breakpoints in the middle 70 percent of the sample. For our estimated inflation equation, the maximum likelihood-ratio test statistic is 9.77 (in 1972:3), while the 10 percent critical value is 14.31 (from table 1 in Andrews [1]). Similarly, for the output equation, the maximum statistic 7.87 (in 1982:4), while the 10 percent critical value is 12.27.

2.3. Comparison to other empirical estimates

It is useful to compare our model with other empirical estimates in order to gauge its plausibility and its conformity to central bank models. From the perspective of monetary policy, there are two features of particular interest: (1) the sensitivity of real activity to movements in the policy instrument, and (2) the responsiveness of inflation to slack in the economy. *Table 1* provides some evidence on both of these issues with a comparison of simulations from our model (2.1)–(2.2) and the MPS model, which was used regularly in the Federal Reserve's forecasting process for over 25 years. The experiment considered (as outlined in Smets [68] and Mauskopf [54]), assumes that the Federal Reserve raises the federal funds rate by one percentage point for two years and then returns the funds rate to its original level thereafter. Table 1 reports for output and inflation the average difference between this simulation and a constant funds rate alternative in each of the first three years after the funds rate increase. The responses of the MPS model and our model to this temporary tightening of monetary policy are quite similar. In both models, output averages almost 0.5 percentage point lower in year two and between two-thirds and 1 percentage point lower in year three, while inflation falls by about a quarter of a percentage point by the third year. Both models require about 3.3 years of a one percentage point output gap in order to induce a one percentage point change in the inflation rate–that is, they exhibit an output sacrifice ratio of just over three.[9] Most importantly, the magnitude of the link between the funds rate and inflation, which will be crucial for our inflation-targeting analysis, is essentially the same across the two models.[10]

Finally, it is also useful to compare the fit and impulse responses of our model to those of a VAR. While one may be deeply skeptical of the use of VARs for certain structural investigations

[9] For comparison, with a rough back-of-the-envelope calculation, Ball (1994) reports an output sacrifice ratio for the U.S. of 2.4.

[10] Our model estimates appear comparable to other recent small empirical structural models of the United States, including Fuhrer and Moore [35], Clark, Laxton, and Rose [20], and Fair and Howrey [27]. This is true even though the models use different interest rates in the IS curve: Fuhrer uses an ex ante real long rate, Clark, Laxton, and Rose use an ex ante real short rate, and Fair and Howrey use a nominal short rate. In fact, over the postwar historical sample, the four measured rates used appear to have moved together fairly closely.

(see Rudebusch [65]), they can provide simple atheoretical summaries of the general dynamics of the data and thus can provide a useful benchmark for the overall fit of a model. Our model can be viewed as two restricted equations from a trivariate VAR with four lags. The VAR output equation regresses the gap on four lags of π, y, and i. The VAR inflation equation regresses inflation on the same lags as well as the contemporaneous value of the gap.[11] *Table 2* compares the Schwarz and Akaike Information Criteria (SIC and AIC, respectively) for each VAR equation with those of our structural model. These two model selection criteria, which are functions of the residual sum of squares, are differentiated by their degrees-of-freedom penalty for the number of parameters estimated. As shown in table 2, the structural model's inflation equation is favored over the VAR's inflation equation by both the SIC and the AIC. For the output equation, there is a split decision. The SIC, which more heavily penalizes extra parameters, favors the structural model, while the AIC favors the VAR. Overall, the information criteria do not appear to view our structural model restrictions unfavorably.

As a final comparison of our structural model to the VAR, *Figure 1* shows their responses to various shocks. This exercise completes the VAR with the usual VAR funds rate equation that regresses the funds rate on four lags of the three variables as well as contemporaneous values of the output gap and inflation. This VAR funds rate equation–with its interpretation as a Federal Reserve reaction function–is also added as a third equation to our model. The impulse responses of this structural system are shown as solid lines in figure 1, while the usual VAR impulse responses are shown as long-dashed lines along with their 95 percent confidence intervals as short-dashed lines. Because the funds rate reaction function equation is identical across the two systems, any differences in dynamics are attributable to the structural model restrictions on the output and inflation equations.

Figure 1 suggests that these restrictions do not greatly alter the dynamics of the model relative to an unrestricted VAR. In response to a positive funds rate shock, output and inflation decline in a similar manner in each system.[12] Also, a positive output shock persists over time and boosts inflation in a like fashion in both models. Only for an inflation shock (the left column of figure 1), does our model's responses edge outside the VAR's confidence intervals. This discrepancy reflects our model's output sensitivity to the real interest rate, which falls after an inflation shock because the VAR funds rate reaction function has such an extremely weak

[11] Thus, our VAR has a Cholesky factorization with a causal order of output, inflation, and, finally, the funds rate.

[12] There is a modest, insignificant "price puzzle" exhibited by the VAR but not the structural model.

interest-rate response to inflation. The implausibility of such VAR reaction functions, which mix several decades of very different Federal Reserve behavior, is highlighted in Rudebusch [65] and Judd and Rudebusch [44]. As shown below, with more plausible reaction functions where the Fed raises the funds rate by more than inflation shock (so the real rate raises, as in the Taylor rule), output will fall following an inflation shock in the structural model.

3. Monetary Policy Rules

3.1. Instrument Rules and Targeting Rules

As noted in the introduction, by an (explicit) *instrument rule,* we mean that the monetary policy instrument is expressed as an explicit function of available information. Classic examples of instrument rules are the McCallum [56] rule for the monetary base, or the Taylor [77] rule for the federal funds rate. By a *targeting rule*, we mean that the central bank is assigned to minimize a loss function that is increasing in the deviation between a target variable and the target level for this variable. The targeting rule will, as we shall see, imply an *implicit* instrument rule.

In the literature, the expressions "targeting variable x_t," or "having a target level x^* for variable x_t," have two meanings. According to the first meaning, the expressions above are used in the sense of "*setting a target* for variable x."[13] Thus, "having a target" means "*using all relevant available information* to bring the target variable in line with the target," or more precisely to minimize some loss function over expected future deviations of the target variable from the target level; for instance, the quadratic loss function

$$\min_{i_t} \mathrm{E}_t \sum_{\tau=0}^{\infty} \delta^{\tau}(x_{t+\tau} - x^*)^2,$$

where δ, $0 < \delta < 1$, is a discount factor and E_t denotes the expectations operator conditional on information available in period t. We will use "targeting" according to this first meaning, following, for instance, Rogoff [62], Walsh [79] and Svensson [71] and [73].

According to the second meaning, "targeting" and "targets" imply a particular *information restriction* for the instrument rule, namely that the instrument must only depend on the gap between the target variable and the target level (and lags of this gap, and/or lags of itself).[14]

[13] This is in line with Merriam-Webster [59]: **target** *vt* (1837) **1:** to make a target of; *esp* to set as a goal **2:** to direct or use toward a target.

[14] See, for instance, Judd and Motley [43], McCallum [57] and Bernanke and Woodford [5]. Bernanke and Woodford's criticism of Svensson's [71] use of the term "inflation-forecast targeting" seems to take the second meaning of "targeting" for granted and disregard the first meaning (which indeed is the one used in [71]).

Thus, the instrument rule is typically restricted to be

$$A(\mathrm{L})i_t = B(\mathrm{L})(x_t - x^*),$$

where $A(\mathrm{L})$ and $B(\mathrm{L})$ are polynomials in the lag operator L. To convey the second meaning, "responding only to $x_t - x^*$" seems more precise. Note that "inflation targeting" according to this second meaning, but *not* according to the first meaning, might correspond to an instrument rule like

$$i_t = hi_{t-1} + \varphi(\pi_t - \pi^*).$$

This instrument rule turns out to perform much worse than other instrument rules. Note also that "inflation-forecast targeting" according to the second meaning (as in, e.g., Haldane [37]), but generally *not* according to the first meaning, might be an instrument rule like

$$i_t = hi_{t-1} + \varphi(\pi_{t+T|t} - \pi^*),$$

where $\pi_{t+T|t}$ denotes some conditional inflation forecast of inflation T quarters ahead (more on this below).

A targeting rule for a *goal* variable is hence equivalent to having an objective for this variable. Examples of such rules are "annual inflation shall fall within the interval 1-3 percent per year on average at least 3 years out of 4," or "minimize the expected value of a discounted sum of future weighted squared deviations of annual inflation from 2 percent per year and squared output gaps," etc. We shall assume an objective of the last kind.

Similarly, a targeting rule for an *intermediate target* variable is equivalent to having a loss function for this intermediate target variable (an intermediate loss function), where the target level sometimes is not constant but depends on current information. The targeting rule can also be expressed as an equation that the target variable shall fulfill, for instance that the target level for the intermediate target is an explicit function of available information. The equation for the intermediate target variable may be interpreted as a first-order condition of an explicit or implicit loss function for the goal variable (see Svensson [71] and [73] for examples). Thus, a targeting rule in the end expresses the intermediate target level as a function of current information. Examples of intermediate target rules are "minimize the expected future deviation of M3 growth from the sum of a given inflation target, a forecast of potential output growth, and a velocity trend," "keep the exchange rate within $\pm$ 2.25 percent band around a given central parity," or "adjust the instrument such that the forecast for inflation 4-8 quarters ahead, conditional on the current state of the economy and on holding the instrument at constant level for the next 8 quarters, is 2 percent per year." We shall consider some targeting rules of this last kind.

A targeting rule in a given model implies a particular instrument rule, but this instrument rule is *implicit* rather than explicit. That is, the targeting rule has to be solved for the instrument rule in order to express it as a function of current information.

3.2. The Model

Let the model be given by (2.1) and (2.2), and let ϵ_t and η_t be i.i.d. zero-mean disturbances with variances σ_ε^2 and σ_η^2 and covariance $\sigma_{\varepsilon\eta}$. The coefficients of the lagged inflation terms in (2.1) are restricted to sum to one,

$$\sum_{j=1}^{4} \alpha_{\pi j} = 1.$$

In our analysis, we will interpret "inflation targeting" as having a loss function for monetary policy where deviations of inflation from an explicit inflation target are always given some weight, but not necessarily all the weight. In particular, for a discount factor δ, $0 < \delta < 1$, we consider the intertemporal loss function in quarter t,

$$\mathrm{E}_t \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau}, \tag{3.1}$$

where the period loss function is

$$L_t = \bar{\pi}_t^2 + \lambda y_t^2 + \nu \left(i_t - i_{t-1}\right)^2, \tag{3.2}$$

(π_t and $\bar{\pi}_t$ are now interpreted as the deviation from a constant given inflation target), and $\lambda \geq 0$ and $\nu \geq 0$ are the weights on output stabilization and interest-rate smoothing, respectively.[15] We will refer to the variables $\bar{\pi}_t$, y_t, and $i_t - i_{t-1}$ as the goal variables. As defined in Svensson [73], "strict" inflation targeting refers to the situation where only inflation enters the loss function ($\lambda = \nu = 0$), while "flexible" inflation targeting allows other goal variables (nonzero λ or ν).

When $\delta \to 1$, the sum in (3.1) becomes unbounded. It consists of two components, however; one corresponding to the deterministic optimization problem when all shocks are zero, and one proportional to the variances of the shocks (see appendix B). The former component converges for $\delta = 1$ (because the terms approach zero quickly enough), and the decision problem is actually

[15] Then i_t can be interpreted as the deviation of the federal funds rate from the sum of the inflation target and the natural real interest rate (the unconditional mean of the real interest rate).

well-defined also for that case. For $\delta \to 1$, the value of the intertemporal loss function approaches the infinite sum of unconditional means of the period loss function, $\mathrm{E}[L_t]$. Then, the scaled loss function $(1-\delta)\mathrm{E}_t \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau}$ approaches the unconditional mean $\mathrm{E}[L_t]$. It follows that we can also define the optimization problem for $\delta = 1$ and then interpret the intertemporal loss function as the unconditional mean of the period loss function, which equals the weighted sum of the unconditional variances of the goal variables,

$$\mathrm{E}\left[L_t\right] = \mathrm{Var}\left[\bar{\pi}_t\right] + \lambda \mathrm{Var}\left[y_t\right] + \nu \mathrm{Var}\left[i_t - i_{t-1}\right]. \tag{3.3}$$

We shall use (3.3) as our standard loss function, hence assuming the limiting case $\delta = 1$.

3.3. State-space Representation

The model (2.1) and (2.2) has a convenient state-space representation,

$$X_{t+1} = AX_t + Bi_t + v_{t+1}. \tag{3.4}$$

The 9×1 vector X_t of state variables, the 9×9 matrix A, the 9×1 column vector B, and the 9×1 column disturbance vector v_t are given by

$$X_t = \begin{bmatrix} \pi_t \\ \pi_{t-1} \\ \pi_{t-2} \\ \pi_{t-3} \\ y_t \\ y_{t-1} \\ i_{t-1} \\ i_{t-2} \\ i_{t-3} \end{bmatrix}, \; A = \begin{bmatrix} \sum_{j=1}^{4} \alpha_{\pi j} e_j + \alpha_y e_5 \\ e_1 \\ e_2 \\ e_3 \\ \beta_r e_{1:4} + \beta_{y1} e_5 + \beta_{y2} e_6 - \beta_r e_{7:9} \\ e_5 \\ e_0 \\ e_7 \\ e_8 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\frac{\beta_r}{4} \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \; v_t = \begin{bmatrix} \varepsilon_t \\ 0 \\ 0 \\ 0 \\ \eta_t \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix};$$

where e_j $(j = 0, 1, ..., 9)$ denotes a 1×9 row vector, for $j = 0$ with all elements equal to zero, for $j = 1, ..., 9$ with element j equal to unity and all other elements equal to zero; and where $e_{j:k}$ $(j < k)$ denotes a 1×9 row vector with elements $j, j+1, ..., k$ equal to $\frac{1}{4}$ and all other elements equal to zero.

Furthermore, it is convenient to define the 3×1 vector Y_t of goal variables. It fulfills

$$Y_t = C_X X_t + C_i i_t, \tag{3.5}$$

where the vector Y_t, the 3×9 matrix C_X and the 3×1 column vector C_i are given by

$$Y_t = \begin{bmatrix} \bar{\pi}_t \\ y_t \\ i_t - i_{t-1} \end{bmatrix}, \; C_X = \begin{bmatrix} e_{1:4} \\ e_5 \\ -e_7 \end{bmatrix}, \; C_i = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Then the period loss function can be written

$$L_t = Y_t' K Y_t, \tag{3.6}$$

where the 3×3 matrix K has the diagonal $(1, \lambda, \nu)$ and all its off-diagonal elements are equal to zero.

3.4. Linear Feedback Instrument Rules

We will consider the class of linear feedback instruments rules, that is, rules of the form

$$i_t = f X_t, \tag{3.7}$$

where f is a 1×9 row vector. This class of rules includes the optimal instrument rule (see below).

For any given instrument rule of the form (3.7), the dynamics of the model follows

$$\begin{aligned} X_{t+1} &= M X_t + v_{t+1} \\ Y_t &= C X_t, \end{aligned}$$

where the matrices M and C are given by

$$M = A + Bf \tag{3.8}$$

$$C = C_X + C_i f. \tag{3.9}$$

For any given rule f that results in finite unconditional variances of the goal variables, the unconditional loss (3.3) fulfills[16]

$$\mathrm{E}\left[L_t\right] = \mathrm{E}\left[Y_t' K Y_t\right] = \operatorname{trace}\left(K \Sigma_{YY}\right), \tag{3.10}$$

where Σ_{YY} is the unconditional covariance matrix of the goal variables (see appendix A).

3.5. The Optimal Instrument Rule

With (3.4) and (3.6), the problem is written in a form convenient for the standard stochastic linear regulator problem (cf. Chow [17] and Sargent [66]). Minimizing (3.1) in each quarter, subject to (3.4) and the current state of the economy, X_t, results in a linear feedback rule for the instrument of the form (3.7). In the limit when $\delta = 1$, the optimal rule converges to the one minimizing (3.3). The expression for the optimal instrument rule is given in appendix B.[17]

[16] The trace of a matrix A, trace(A), is the sum of the diagonal elements of A.

[17] Since there are no forward-looking variables, we need not distinguish between the commitment and discretion solutions, since they are the same.

3.6. Inflation Forecasts

Given the lags in the monetary transmission mechanism, inflation-targeting central banks focus on inflation forecasts. Indeed, several of these banks have started to publish inflation reports that are completely devoted to describing the recent history and future prospects for inflation. The actual inflation forecasts that have been reported have fallen into two broad categories depending on how monetary policy is projected forward: constant-interest-rate inflation forecasts and rule-consistent inflation forecasts.

3.6.1. Constant-Interest-Rate Inflation Forecasts

Inflation-targeting central banks often refer to, and report, inflation forecasts conditional upon a given constant interest rate. We will call such forecasts constant-interest-rate inflation forecasts. Such inflation forecasts are frequently used in the following way. If a constant-interest-rate inflation forecast for the current interest rate is above (below) target for a given horizon, monetary policy has to be tightened (eased) and the interest rate increased (decreased). If the inflation forecast is on target, the current interest-rate setting is deemed appropriate. (see, for instance, Mayes and Riches [55] and Svensson [71]). Such forecasts, based on a fixed nominal rate, may seem overly simplistic,[18] but they have been widely used at central banks, perhaps most notably at the Bank of England, where (before operational independence in 1997) the Bank produced such forecasts because it could not presuppose policy changes by the government.[19]

In an attempt to represent this, it is convenient to define the "T-quarter-ahead constant-interest-rate inflation forecast". By this we mean a forecast of 4-quarter inflation $T \geq 2$ quarters ahead, conditional on a given constant current and future interest rate (and on the current state variables X_t). Denote this conditional forecast by $\bar{\pi}_{t+T|t}(i)$, for the given constant current and future interest rate i. It is given by

$$\bar{\pi}_{t+T|t}(i) \equiv e_{1:4}\tilde{M}^{T-1}\left(AX_t + Bi\right), \tag{3.11}$$

where $\tilde{M}$ is a 9×9 matrix given by

$$\tilde{M} = A + Be_7 \tag{3.12}$$

(we note that $e_7 X_{t+1} = i_t$).

[18] Indeed, given a long enough forecast horizon, the forecasted inflation path will normally be explosive.

[19] However, even after operational independence, the Bank's forecasts have assumed unchanged short-term interest rates (see Britton, Fisher, and Whitley [14]). Similarly, it is our impression that internal staff forecasts at the Federal Reserve Board are often conditioned on a constant federal funds path. Thus, constant-interest-rate forecasts may have some general advantages—perhaps, in ease of communication, as noted by Rudebusch [64].

Consider also the T-quarter-ahead constant-interest-rate inflation forecast in quarter t, when the interest rate is held constant at a level equal to that of the previous quarter, i_{t-1}. This conditional inflation forecast, the "T-quarter-ahead unchanged-interest-rate inflation forecast," $\bar{\pi}_{t+T|t}(i_{t-1})$, fulfills

$$\begin{aligned} \bar{\pi}_{t+T|t}(i_{t-1}) &\equiv e_{1:4}\tilde{M}^{T-1}(AX_t + Bi_{t-1}) \\ &= e_{1:4}\tilde{M}^{T-1}(AX_t + Be_7X_t) \\ &= e_{1:4}\tilde{M}^{T}X_t. \end{aligned} \tag{3.13}$$

3.6.2. Rule-Consistent Inflation Forecasts

There are of course many other assumptions that one could make about monetary policy in order to produce inflation forecasts. For example, one could condition on a constant *real* interest rate, or one could set the rate in each future period according to a given reaction function for policy. Recently, the Reserve Bank of New Zealand (see [61]) has moved beyond constant-interest-rate forecasts, and started to report official inflation forecasts conditional upon a particular reaction function. (This results in inflation forecasts always returning to the target.) Below, we shall also consider a rule that employs such forecasts.

3.7. Simple Instrument Rules

By a simple instrument rule we mean an instrument rule of the form (3.7), where the vector f is restricted in some way. We will distinguish no fewer than 9 types of simple instrument rules by characterizing them in terms of three forms and three arguments.[20]

3.7.1. Three forms

We consider three forms: smoothing, level and difference; the latter two are special cases of the first form. The *smoothing* form, denoted S, is given by

$$\begin{aligned} i_t &= hi_{t-1} + gX_t \\ f &= he_7 + g, \end{aligned} \tag{3.14}$$

where h is a coefficient and g is a 1×9 row vector of response coefficients. When the coefficient h fulfills $0 < h \leq 1$, this form of instrument rule is characterized by "partial adjustment," or

[20] The theory and practice of simple policy rules is examined in Currie and Levin [23].

"smoothing" of the instrument. The larger the coefficient h, the more smoothing (the more partial the adjustment).

Recall that i_t is the deviation from the average nominal interest rate, which in our model equals the sum of the inflation target (the average inflation rate) and the natural real interest rate (the average real interest rate). If we, temporarily in this paragraph, let all variables denote absolute levels, and denote the average level of variable x_t by x^0, we can write (3.14) as

$$\begin{aligned} i_t &= hi_{t-1} + (1-h)i^0 + g(X_t - X^0) \\ &= hi_{t-1} + (1-h)(r^0 + \pi^*) + g(X_t - X^0) \\ &= hi_{t-1} + (1-h)(r^0 + \bar{\pi}_t) + \tilde{g}(X_t - X^0), \end{aligned} \tag{3.15}$$

where $\tilde{g} \equiv g - (1-h)e_{1:4}$ and we have used $i^0 = r^0 + \pi^*$. Thus, (3.14) is equivalent to (3.15), which is a frequent way of writing instrument rules.[21]

The *level* form, denoted L, is the special case of the autoregressive form when $h = 0$, whereas the *difference* form, denoted D, is the special case when $h = 1$.[22]

3.7.2. Three arguments (restrictions on g)

We consider three combinations of arguments (variables that the instrument responds to). That is, we consider three different restrictions on the vector g of response coefficients. First, we consider a response to $\bar{\pi}_t$ and y_t, denoted $(\bar{\pi}_t, y_t)$, which implies[23]

$$\begin{aligned} gX_t &= g_\pi \bar{\pi}_t + g_y y_t \\ g &= g_\pi e_{1:4} + g_y e_5, \end{aligned}$$

where g_π and g_y are the two response coefficients. Second, we consider a response to the T-quarter-ahead unchanged-interest-rate inflation forecast only, denoted $(\bar{\pi}_{t+T|t}(i_{t-1}))$. This implies

$$\begin{aligned} gX_t &= g_\pi \bar{\pi}_{t+T|t}(i_{t-1}) \\ g &= g_\pi e_{1:4} \tilde{M}^T, \end{aligned}$$

[21] Clarida, Gali and Gertler [19] and [18] model interest-rate smoothing as

$$i_t = hi_{t-1} + (1-h)(\bar{\pi}_t + \tilde{g}X_t),$$

which is obviously consistent with (3.15) (as long as $h \neq 1$) since we can identify $(1-h)\tilde{g}$ above with $\tilde{g}$ in (3.15).

[22] Note that, since $i_{t-1} = X_{7t} = e_7 X_t$, we can always write $i_t = fX_t$ as $i_t = i_{t-1} + (f - e_7)X_t$. Thus, unless g_7 is restricted to fulfill $g_7 = 0$, the difference form does not imply any restriction.

[23] Note that responding to $\bar{\pi}_t$ means responding to the discrepancy between inflation and the inflation target, since $\bar{\pi}_t$ is the deviation from the mean, and the mean coincides with the inflation target, since there is no inflation bias in our model.

where we have used (3.13). Finally, we consider a response to both the T-quarter-ahead unchanged-interest-rate inflation forecast and the output gap, denoted $(\bar{\pi}_{t+T|t}(i_{t-1}), y_t)$, which implies

$$\begin{aligned} gX_t &= g_\pi \bar{\pi}_{t+T|t}(i_{t-1}) + g_y y_t \\ g &= g_\pi e_{1:4} \tilde{M}^T + g_y e_5. \end{aligned}$$

A particular instrument rule is denoted Ta, with the type T = S, L, or D, and the argument a = $(\bar{\pi}_t, y_t)$, $(\bar{\pi}_{t+T|t}(i_{t-1}))$ or $(\bar{\pi}_{t+T|t}(i_{t-1}), y_t)$. By a *Taylor-type* rule we mean a simple instrument rule of the form L$(\bar{\pi}_t, y_t)$,

$$i_t = g_\pi \bar{\pi}_t + g_y y_t.$$

The classic *Taylor Rule* (Taylor [77]) is a Taylor-type rule with $g_\pi = 1.5$ and $g_y = 0.5$.[24]

We do not include the case of a response to only $\bar{\pi}_t$, $gX_t = g_\pi \bar{\pi}_t$, since it consistently performed very badly.

3.7.3. An information lag

McCallum has in several papers, for instance [57], argued that it is more realistic from an information point of view, to restrict the instrument in quarter t to depend on the state variables in quarter $t-1$,

$$i_t = f X_{t-1}.$$

On the other hand, it can be argued that the central bank has much more information about the current state in the economy than captured by the few state variables in the model. Then, assuming that the state variables in quarter t are known in quarter t is an implicit way of acknowledging this extra information.[25] This is the main reason why our baseline case has the instrument depending on the state variables in the same quarter.

For comparability with results of other authors, we nevertheless would like to be able to restrict the instrument to depend on state variables one quarter earlier. Thus, we consider the case when there is response to $\bar{\pi}_{t-1}$ and y_{t-1}, denoted $(\bar{\pi}_{t-1}, y_{t-1})$, with and without interest-rate smoothing,

$$i_t = h i_{t-1} + g_\pi \bar{\pi}_{t-1} + g_y y_{t-1}. \tag{3.16}$$

[24] See McCallum [56], Bryant, Hooper and Mann [15], Judd and Motley [43] and Henderson and McKibbin [41] for other examples of explicit instrument rules.

[25] In fact, obtaining a good description of the real-time information set of policymakers is a complicated assignment (see Rudebusch [65]). For example, simply lagging variables ignores data revisions (see Diebold and Rudebusch [26]).

This requires some technical modifications in our state-space setup, which are detailed in appendix C.

3.7.4. An instrument rule with response to a rule-consistent inflation forecast

Consider the following rule,

$$i_t = hi_{t-1} + \varphi\pi_{t+T|t}, \tag{3.17}$$

where $\varphi > 0$ and $\pi_{t+T|t}$ $(T \geq 2)$ is the rational expectation of π_{t+T}, conditional upon X_t, (3.4) and (3.17). Thus, $\pi_{t+T|t}$ is a rule-consistent inflation forecast as described above, although in this case the rule being conditioned upon includes the forecast. This rule, where the instrument responds to a rule-consistent inflation forecast, is not an explicit instrument rule, because it does not express the instrument as an explicit function of current information (or, in the context of our model, of predetermined variables). It is not a targeting rule, in the sense we have used the term, since it is not explicitly related to some loss function. Nor does it express an intermediate target level as a function of current information. The rule is an *equilibrium condition* because the right side of (3.17) is endogenous and depends on the rule itself. Hence, it is an *implicit* instrument rule. The self-referential, rational expectations nature of the rule complicates its analytical derivation in terms of an explicit instrument rule.[26] However, the rule remains a simple instrument rule similar in form to the $\mathrm{S}(\bar{\pi}_{t+T|t}(i_{t-1}))$ rule described above, only that the instrument responds to an endogenous variable rather than a predetermined one. We consequently denote the rule in (3.17) by $\mathrm{S}(\pi_{t+T|t})$.

Like the $\mathrm{S}(\bar{\pi}_{t+T|t}(i_{t-1}))$ rule, the $\mathrm{S}(\pi_{t+T|t})$ rule has considerable intuitive appeal, inasmuch as it implies that if new information makes the inflation forecast at the horizon T increase, the interest rate should be increased, and vice versa. Even better, however, $\mathrm{S}(\pi_{t+T|t})$ rule uses an inflation forecast that can be conditioned on a non-constant interest-rate path. The $\mathrm{S}(\pi_{t+T|t})$ rule is similar to the reaction function used in Bank of Canada's Quarter Projection Model (QPM, see for instance [21]) and Reserve Bank of New Zealand's Forecasting and Policy System (FPS, see [6]), and identical to the rule considered by Haldane and Batini [39] at this conference.[27] Indeed, this rule appears to be a frequent reference rule among inflation-targeting central banks. It is (when $h = 1$) what Haldane [37] calls "the generic form of the feedback rule

[26] In equilibrium, the rational expectations inflation forecast becomes an endogenous linear function of the state variables (where the coefficients depend on the parameters T, φ and h), which by (3.17) results in (3.7). For $T = 2$, the explicit instrument rule is easy to derive. For $T \geq 3$, the derivation is more complex. The details are provided in appendix D.

[27] It is also used in Black, Macklem and Rose [7].

under an inflation target," which "encapsulates quite neatly the operational practice of most inflation targeters."

Nevertheless, the $S(\pi_{t+T|t})$ rule is not derived as a first-order condition of some loss function corresponding to inflation targeting.[28] The question then arises: How efficient is this rule in achieving an inflation target? This question is particularly relevant because of its use in the inflation projections by two prominent inflation-targeting central banks, and by its intuitive appeal to many as representing generic inflation targeting. Consequently we examine the performance of this rule within the framework of our model.

3.7.5. Optimal Simple Instrument Rules

In order to find the optimal simple instrument rule for a given type of rule and with a given combination of arguments, we optimize (3.3) over g, h, and φ taking the corresponding restrictions into account.

3.8. Targeting Rules

3.8.1. The Optimal Targeting Rule

Above we have noted the existence of an optimal instrument rule. Of course, the corresponding minimization problem defines an optimal targeting rule as well. Here, however, we show that the first-order condition for an optimum can be interpreted as an optimal intermediate-targeting rule.

Consider the first-order condition for minimizing (3.1) and (3.6) subject to (3.4) and (3.5),

$$\begin{aligned} 0 &= \sum_{\tau=0}^{\infty} \frac{\partial Y'_{t+\tau|t}}{\partial i_t} K Y_{t+\tau|t} \\ &= C'_i K Y_t + \sum_{\tau=1}^{\infty} B' \left(A^{\tau-1}\right)' C'_X K Y_{t+\tau|t}, \end{aligned} \tag{3.18}$$

where we have used that

$$\frac{\partial Y_t}{\partial i_t} = C_i, \quad \frac{\partial Y_{t+\tau|t}}{\partial i_t} = C_X \frac{\partial X_{t+\tau|t}}{\partial i_t} = C_X A^{\tau-1} B, \quad \tau = 1, 2, ...$$

[28] Because the rule is not derived as a first-order condition, its precise form is not obvious. As alternatives to (3.17) one can consider

$$i_t = h i_{t-1} + (1-h)\bar{\pi}_t + \varphi \pi_{t+T|t},$$

or even

$$i_t = h i_{t-1} + g_\pi \bar{\pi}_t + \varphi \pi_{t+T|t},$$

where g_π is unrestricted.

and let the discount factor fulfill $\delta = 1$. This is a linear relation between the current and conditionally forecasted future goal variables, $Y_{t+\tau|t}$, $\tau = 0, 1, 2, ...$, conditional upon the current instrument and the future policy. The task of the monetary authority can be described as setting an instrument in the current quarter so as to achieve the relation (3.18). This relation can then be interpreted as an intermediate target path for the forecast of future goal variables. That is, the forecasts of future goal variables are considered intermediate target variables. Then, the task of the monetary authority is to choose, conditional upon the current state variable X_t, a current instrument i_t and a plan $i_{t+\tau|t}$ $(\tau = 1, 2, ...)$ for future instruments, such that the resulting conditional forecast of future goal variables $Y_{t+\tau|t}$ fulfill the intermediate target (3.18), where

$$\begin{aligned} Y_t &= C_X X_t + C_i i_t \\ Y_{t+\tau|t} &= C_X X_{t+\tau|t} + C_i i_{t+\tau|t} \\ &= C_X A^\tau X_t + \sum_{j=0}^{\tau-1} C_X A^{\tau-1-j} B i_{t+j|t} + C_i i_{t+\tau|t}, \end{aligned}$$

$\tau = 1$, 2,..., where we have used that

$$\begin{aligned} X_{t+\tau+1|t} &= A X_{t+\tau|t} + B i_{t+\tau|t} \\ &= A^{\tau+1} X_t + \sum_{j=0}^{\tau} A^{\tau-j} B i_{t+j|t}. \end{aligned}$$

We note that the $Y_{t+\tau|t}$ $(\tau = 0, 1, 2, ...)$ that fulfill (3.18) can be seen as impulse responses of the goal variables for the optimal solution, for impulses that put the economy at its initial state. We can now imagine a Governor or a Board of Governors pondering over a set of alternative current and future instrument settings and alternative forecasts for the goal variables that have been provided for consideration by the central bank staff, in order to decide on the current instrument setting. When the Governor or Board of Governors end up selecting one instrument path and corresponding goal variable forecasts that they believe are best, their behavior (if rational) can be seen as implicitly selecting forecasts that fulfill (3.18) for some implicit weight matrix K in their loss function.

In general, (3.18) involves a relation between all the goal variables. The case when inflation and the output gap are the only goal variables is examined in Svensson [71] and [73]. Since, by the Phillips curve (2.1), the forecast of output can be written as a linear function of the forecast of inflation, this linear function can then be substituted for the output forecast in (3.18), which results in a relation for the forecast of future inflation only. That relation can be interpreted as an intermediate target for the inflation forecast. In the special case examined in Svensson [71] and [73], these relations for the inflation forecast are both simple and optimal. In the general case these relations need not be optimal. Here we will examine them as potential simple targeting rules, called inflation-forecast targeting rules.

3.8.2. Simple Targeting Rules

Consider targeting rules for the T-quarter-ahead constant-interest-rate inflation forecast. These rules imply implicit instrument rules which are normally not "simple," since they normally depend on most state variables. We will consider four kinds of simple targeting rules, namely strict and flexible inflation-forecast targeting, with and without smoothing.

In Svensson [71], the following first-order condition for the inflation forecast is derived, for the case of flexible inflation targeting with some non-negative weight on output stabilization, $\lambda \geq 0$, but zero weight on interest-rate smoothing, $\nu = 0$,

$$\pi_{t+2|t}(i_t) - \pi^* = c(\lambda)\left(\pi_{t+1|t} - \pi^*\right).$$

In the model in [71], $\pi_{t+1|t}$ is predetermined, $\pi_{t+2|t}(i_t)$ is the inflation forecast for the earliest horizon that can be affected, and $c(\lambda)$ is an increasing function of λ, fulfilling $0 \leq c(\lambda) < 1$, $c(0) = 0$, $c(\lambda) \to 1$ for $\lambda \to \infty$.

In the present model, we can consider a generalization of this framework,

$$\bar{\pi}_{t+T|t}(i_t) = c\bar{\pi}_{t+1|t}, \tag{3.19}$$

where c and T fulfill $0 \leq c < 1$ and $T \geq 2$. This we refer to as *flexible T-quarter-ahead inflation-forecast targeting*, denoted FIFT(T).

The expression (3.19) denotes a targeting rule, where the corresponding instrument rule is *implicit*. In order to solve for the instrument rule, we use (3.11) to write (3.19) as

$$e_{1:4}\tilde{M}^{T-1}(AX_t + Bi_t) = ce_{1:4}AX_t.$$

Then the implicit instrument rule can be written

$$i_t = g(c, T)X_t,$$

where the row vector $g(c, T)$ is a function of c and T given by

$$g(c, T) \equiv \frac{e_{1:4}(cI - \tilde{M}^{T-1})A}{e_{1:4}\tilde{M}^{T-1}B}, \tag{3.20}$$

where I is the 9×9 identity matrix (note that $e_{1:4}\tilde{M}^{T-1}B$ is a scalar and $e_{1:4}(cI - \tilde{M}^{T-1})A$ is a 1×9 row vector).

Strict T-quarter-ahead inflation-forecast targeting, denoted SIFT(T), is the special case of (3.19) when $c = 0$,

$$\bar{\pi}_{t+T|t}(i_t) = 0. \tag{3.21}$$

The corresponding implicit instrument rule is

$$i_t = g(0, T)X_t, \tag{3.22}$$

where

$$g(0, T) \equiv -\frac{e_{1:4}\tilde{M}^{T-1}A}{e_{1:4}\tilde{M}^{T-1}B} \tag{3.23}$$

Note that the numerator in (3.23) equals the constant-interest-rate inflation forecast corresponding to a zero interest rate, $\bar{\pi}_{t+T|t}(0)$. The denominator, $e_{1:4}\tilde{M}^{T-1}B$, is the constant-interest-rate policy multiplier for the 4-quarter inflation T-quarters ahead, since by (3.11)

$$\frac{\partial\bar{\pi}_{t+T|t}(i)}{\partial i} = e_{1:4}\tilde{M}^{T-1}B. \tag{3.24}$$

Hence, very intuitively the instrument rule corresponding to strict inflation-forecast targeting can be written as

$$i_t = -\frac{\bar{\pi}_{t+T|t}(0)}{\partial\bar{\pi}_{t+T|t}(i)/\partial i_t},$$

the negative of the *zero-interest-rate* inflation forecast divided by the constant-interest-rate policy multiplier.

We can equivalently write this instrument rule in terms of *changes* in the interest rate. By (3.11) we have

$$\bar{\pi}_{t+T|t}(i_t) - \bar{\pi}_{t+T|t}(i_{t-1}) = e_{1:4}\tilde{M}^{T-1}B(i_t - i_{t-1}).$$

By (3.21) we can write

$$i_t - i_{t-1} = -\frac{\bar{\pi}_{t+T|t}(i_{t-1})}{e_{1:4}\tilde{M}^{T-1}B} = -\frac{e_{1:4}\tilde{M}^{T}X_t}{e_{1:4}\tilde{M}^{T-1}B} = (f(0, T) - e_7)X_t.$$

Very intuitively, the interest-rate adjustment equals the negative of *unchanged-interest-rate* inflation forecast for unchanged interest rate divided by the constant-interest-rate policy multiplier.

Note that strict inflation-forecast targeting implies that the inflation forecast conditional on the future instrument rule (3.22), rather than conditional on a constant interest rate, deviates from zero,

$$\mathrm{E}_t\bar{\pi}_{t+T} \neq 0,$$

and in practice reaches zero later than T quarters ahead. This is apparent from the impulse responses for $\bar{\pi}_{t+\tau|t}$ under strict inflation-forecast targeting.

Note that strict T_1-quarter inflation-forecast targeting may be approximately equal to flexible T_2-quarter flexible inflation-forecast targeting, when the horizon for strict inflation targeting exceeds that of flexible inflation targeting, $T_1 > T_2$.

The above targeting rules can be considered under *smoothing* (partial adjustment) of the interest rate,

$$\begin{aligned} i_t &= hi_{t-1} + (1-h)g(c,T)X_t \\ f &= he_7 + (1-h)g(c,T), \end{aligned}$$

where it may be reasonable to restrict the smoothing coefficient h to fulfill $0 \leq h < 1$. Note that under smoothing, h is not generally the "net" coefficient on i_{t-1}, since $g_7(c,T)$ is generally not zero. These targeting rules under smoothing are denoted FIFTS(T) and SIFTS(T) respectively.

The *optimal* inflation-forecast targeting rules are found by minimizing the loss function (3.3) over the parameters c, h and T, taking into account the restrictions on these and that $T \geq 2$ is an integer. For instance, under strict inflation targeting without smoothing, we have $c = h = 0$, and the only free parameter is T.

4. Results

4.1. Optimized Rules

In this subsection, we consider the performance of various rules for several illustrative cases of different preferences over goal variables. The rules we consider have been optimized in terms of their parameter settings for the given preferences and the given form of the rule assumed.

Tables 3–7 provide results for five different sets of preferences over goals. In each table, the volatility of the goal variables (measured as the unconditional standard deviations), the minimized loss, and the relative ranking in terms of loss are shown for 22 different rules. Loss is calculated under the assumption that output and inflation variability are equally distasteful ($\lambda = 1$) in table 3 and that output variability is much less costly ($\lambda = 0.2$) in table 4 and much more costly ($\lambda = 5$) in table 5. Variability of nominal interest-rate changes are also costly in these three tables ($\nu = 0.5$).[29] Variation in the costs of variability of interest-rate changes are

[29] Such costs are suggested, in part, by the concern central banks display for financial market fragility (see, e.g., Rudebusch [64]).

considered in tables 6 ($\nu = 0.1$) and 7 ($\nu = 1.0$) (both assuming $\lambda = 1$). The preferences in table 3 imply a concern not only about inflation stabilization but also about output stabilization and interest-rate smoothing, which we believe is realistic for many central banks, also inflation-targeting ones. Comparison with tables 4–7 allow us to note the consequences of relatively more or less emphasis on output stabilization and interest-rate smoothing.

The first rule at the top of each table is the unrestricted optimal control rule—the obvious benchmark. The optimal rule in table 3 produces volatility results not too far from our historical sample results, which are $\mathrm{Std}[\bar{\pi}_t] = 2.33$, $\mathrm{Std}[y_t] = 2.80$, and $\mathrm{Std}[i_t - i_{t-1}] = 1.09$. The next four lines consider level rules with current inflation and output, $\mathrm{L}(\bar{\pi}_t, y_t)$, future inflation $\mathrm{L}(\bar{\pi}_{t+8|t}(i_{t-1}))$, and future inflation and current output $\mathrm{L}(\bar{\pi}_{t+8|t}(i_{t-1}), y_t)$ as arguments (where the forecasts are the 8-quarter-ahead "unchanged-interest-rate" 4-quarter inflation forecast). The next three lines consider smoothing instrument rules with the same arguments. The following three rows are for the interest-rate-smoothing rule $\mathrm{S}(\pi_{t+T|t})$, using the 8-, 12-, and 16-quarter-ahead rule-consistent quarterly inflation forecasts. The final twelve rows of each table present various implicit inflation-forecast targeting rules at horizons of 8, 12, and 16 quarters. For all of the rules (except the optimal one), the relevant optimal rule parameters are given in the tables as well.

These tables suggest several conclusions:

First, simple instrument rules appear to be able to perform quite well in our model. Consistently across the tables, the top performing rule is the $\mathrm{S}(\bar{\pi}_{t+8|t}(i_{t-1}), y_t)$ one, which reacts to the constant-interest-rate inflation forecast and the current output gap. Indeed, these simple "forward-looking" Taylor-type rules are always extremely close to matching the optimal rule in terms of overall loss. This result is somewhat surprising given that the inflation forecast incorporated into these rules is simply a single 8-quarter-ahead inflation projection conditioned on an unchanged interest-rate path.

Perhaps even more surprising, the current inflation and output Taylor-type rules—$\mathrm{L}(\bar{\pi}_t, y_t)$ and $\mathrm{S}(\bar{\pi}_t, y_t)$—are nearly as good. Particularly, in table 3 (with $\lambda = 1$), these rules perform with output and inflation gap variances that are similar to those of the optimal rule. In order to understand the exceptional performance of these rules, it is instructive to compare the coefficients of these simple rules to those of the optimal rule. The optimal rule in table 3 (the optimal rules from the other tables have broadly similar parameterizations) has the form

$$i_t = .88\pi_t + .30\pi_{t-1} + .38\pi_{t-2} + .13\pi_{t-3} + 1.30y_t - .33y_{t-1} + .47i_{t-1} - .06i_{t-2} - .03i_{t-3}.$$

The $\mathrm{L}(\bar{\pi}_t, y_t)$ rule, for example, comes close to matching this by setting the first four parameters all equal to 0.68 (that is, $g_\pi/4$) , the y_t parameter equal to 1.57, and the other parameters equal to zero. Because the Taylor rule has received so much attention, it is also interesting to note that across all of the tables the parameters for our $\mathrm{L}(\bar{\pi}_t, y_t)$ Taylor-type rules are fairly high. Instead of the original Taylor rule parameters of 1.5 on inflation (g_π) and 0.5 on output (g_y), our optimal $\mathrm{L}(\bar{\pi}_t, y_t)$ rules sets these parameters above 2 and 1, respectively, in all of the tables.[30]

Second, in distinct contrast to the simple rules that include contemporaneous output gaps, the simple instrument rules that respond only to inflation forecasts do quite poorly—even when the weight on output stabilization is small, as in table 4. Of course, the optimal rule does include large coefficients on output, but presumably these reflect in large part the inflation-forecasting properties of output (especially for low λ). However, the simple instrument rules $\mathrm{L}(\bar{\pi}_{t+8|t}(i_{t-1}))$ and $\mathrm{S}(\bar{\pi}_{t+8|t}(i_{t-1}))$ that incorporate only future inflation do not fare very well. One might conjecture that these rules do poorly because of the mechanical nature of the forecasts used, which are simple projections assuming a constant nominal funds rate. However, the $\mathrm{S}(\pi_{t+8|t})$ rule, which conditions the inflation forecast on a time-varying, rule-consistent interest-rate path, does little better than the $\mathrm{S}(\bar{\pi}_{t+8|t}(i_{t-1}))$ rule. More likely, the restricted fashion in which the inflation forecasts enter the rule—the instrument responds only to the deviation between the forecast and the inflation target—is to blame. This illustrates what was emphasized in section 3.7.4, namely that these rules are not first-order conditions to our loss function. However, note that these rules do better for a smaller λ (table 4) and worse for a larger λ (table 5). This indicates that they are closer to a first-order condition of a loss function that only involves inflation stabilization and interest-rate smoothing.[31]

Third, the inflation-forecast targeting rules perform quite will given enough flexibility and interest-rate-smoothing ability. The FIFTS rule (flexible inflation-forecast targeting with smoothing) is essentially able to match the performance of the $\mathrm{S}(\bar{\pi}_{t+8|t}(i_{t-1}), y_t)$ rule—and hence the optimal rule—in all cases except when there is a very high weight on output stabilization (table 5). Across all of the tables, the best inflation-forecast horizon to use with this rule is usually 12 quarters but sometimes 8 quarters. The IFT rules without interest-rate smoothing are heavily penalized by the cost of large changes in the nominal interest-rate instrument. Note that this is

[30] Ball (1997), in a simple, calibrated theoretical model similar to our own, argues that the optimal Taylor-type rule should have higher coefficients than the original Taylor rule. However, Ball also argues that in the optimal rule the output parameter should be larger than the inflation parameter, which is generally contrary to our results.

[31] The length of the forecast horizon (T) in the $\mathrm{S}(\pi_{t+T|t})$ rule makes only a modest contribution. That is, the targeting horizon trade-off discussed in Haldane [37] is relatively modest in our model with this rule.

true even in table 6 when the cost of variability of interest-rate change is quite low.

To augment the tables, *figure 2* shows the trade-offs between inflation variability and output gap variability that result for varying the weight on output stabilization (λ) from 0 to 10 and assuming $\nu = 0.5$.[32] The trade-off resulting from the optimal rule is shown as a solid line. For increasing λ, the optimal rule corresponds to points further southeast on the curve. The dashed lines correspond to the smoothing rules $\mathrm{S}(\bar{\pi}_t, y_t)$, $\mathrm{S}(\bar{\pi}_{t+8|t}(i_{t-1}))$, $\mathrm{S}(\pi_{t+8|t})$, and $\mathrm{S}(\bar{\pi}_{t+8|t}(i_{t-1}), y_t)$. Only the last of these is consistently close to the optimal rule. Note that $\mathrm{S}(\pi_{t+8|t})$ is close to the optimal rule for small λ.

Also, the triangle shows the sample (1960:1–1996:2) standard deviation of inflation and the output gap. The circle shows the standard deviations that result from an estimated Taylor-type rule for the sample 1985:1–1996:2 (with $g_\pi = 1.76$ and $g_y = 0.74$). The square shows the standard deviations that result from the Taylor rule (with $g_\pi = 1.5$ and $g_y = 0.5$).

The trade-offs from flexible inflation-forecast targeting with smoothing (FIFTS) at 8, 12, and 16 quarter horizons are shown as the dashed-dotted lines. For $T = 8$ quarters, the trade-off is consistently close to that of the optimal rule.

The trade-offs from the flexible inflation-forecast targeting without smoothing (FIFT) shown as the dotted lines. A shorter horizon T is associated more with less output variability than with less inflation variability (cf. table 3).

Finally, *figures 3 and 4* give the dynamic impulse responses of the model under various optimal simple smoothing rules and targeting rules, respectively. All of the rules have broadly similar features, especially a large quick interest-rate rise in response to a positive inflation or output shock.[33] There are, however, some subtle but telling differences among the rules. In figure 3, the $\mathrm{S}(\pi_{t+8|t})$ rule, which considers only the inflation forecast, has the mildest response to an output shock, which allows inflation (through the Phillips curve) to get a bit more out of control, and requires a slightly longer slowdown in output to compensate. In figure 4, the inflation-targeting rules without smoothing show large initial interest-rate spikes in response to the shocks. With smoothing, however, the FIFTS rule is able to mimic the hump-shaped pattern of interest rates of the smoothing instrument rules.

[32] Although plots of such trade-offs are common in the literature, they sweep interest rate smoothing considerations under the rug, so we have some preference for the tabular results.

[33] Note the great contrast between figures 3 and 4 and the left two columns of figure 1. Again, the poor results in figure 1 can be traced to the misspecification of the VAR interest rate equation.

4.2. Common Conference Rules

In this subsection, we consider the five rules that are to be common across all of the investigations at this conference. These rules and our results on volatility and loss (assuming $\lambda = 1$ and $\nu = 0.5$) are summarized in *table 8*. The results with lagged information, which are shown in the lower half of table 8, are qualitatively the same as those with contemporaneous information, so we concentrate on the latter.

First, consider the two level rules (in our terminology) that are common. Rule III(1) and IV(1) have much weaker inflation and output response coefficients than our optimal $\mathrm{L}(\bar{\pi}_t, y_t)$ rule (in table 3), and inflation variability under the common rules is much larger than with the optimal ones, while output variability is slightly lower and variability of interest-rate changes is about the same. The parameters of the common conference rules could only be optimal for a very large λ (much greater than 10).

Second, the set of common conference rules included 2 difference rules and one smoothing rule with $h = 1.3$. None of these rules provided dynamically stable solutions in our model. Note that the optimal value of h for rule $\mathrm{S}(\bar{\pi}_t, y_t)$ equals 0.14 in table 3 and is hence not close to one. The optimal difference rule $\mathrm{D}(\bar{\pi}_t, y_t)$ that is shown in table 8 requires very low coefficients in order to ensure stability. Even so its performance is quite poor.[34]

4.3. A Non-Negative Nominal Interest-Rate Constraint

In this subsection, we consider the occurrence of negative nominal interest rates. Negative nominal interest rates, although highly implausible in practice, are almost never excluded in policy rule analyses and our study is no exception. As noted in section 2, our model has many much-debated simplifications; however, one of its least debated approximations is its completely linear nature with its symmetry with respect to zero for all quantities including nominal interest rates. Indeed, it is straightforward to calculate the unconditional probability of obtaining a negative nominal funds rate for any given rule. For example, assuming an inflation target of 2 percent and an equilibrium real funds rate of 2.5 percent (which is obtained from the estimated constant term in the IS curve regression without de-meaned data), most of the optimized rules in table 3 give about a 20 percent probability of a negative interest rate. Clearly, these rules assume that nominal interest rates would be negative a non-negligible proportion of the time.

[34] In rational expectations models, difference rules appear to perform much better, e.g., Fuhrer and Moore [35] and Williams [81].

Still, for policy rule analysis, we view the simple imposition of an interest-rate non-negativity constraint as unsatisfactory in several respects. Technically, such a nonlinear constraint renders our analytical methods difficult if not infeasible, though simulation methods are available, see Fuhrer and Madigan [34] and Fair and Howrey [27]. More importantly, however, such a constraint, by limiting the degree to which the central bank can conduct expansionary monetary policy at low inflation rates, almost ensures dynamic instability in an otherwise linear model.[35] We do not view such instability as plausible. We think that there are always mechanisms by which the central bank can stimulate the economy even if short-term rates are near zero. Expansionary monetary policy could always be conducted by the injection of reserves through purchases of Treasury securities at all maturities (flattening the *entire* yield curve), or purchases of foreign exchange (unsterilized intervention), or even purchases (or financing) of corporate debentures and equity.[36] That is, our model, although not strictly true, may give a fairly accurate picture of the potential power of central banks. However, it must be admitted that there is little empirical basis for judging the performance of very low inflation economies in our sample.

5. Conclusions

An early working title of this paper was "Practical Inflation Targeting", by which we meant an exploration of plausible policy rules using a model of a form common at central banks. In this spirit, our examination of policy rules has been in part descriptive, and closely linked to what inflation-targeting central banks actually seem to be doing, as well as partly prescriptive, involving sifting and judging among various rules. From the latter perspective, our results suggest that certain simple forward-looking rules are able to perform quite well.

Of course, our prescriptive results about particular simple rules are conditional upon our particular model, and there is much room for extensions and improvements. Questions regarding parameter uncertainty and structural stability are crucial before the results can be taken too seriously; however, judging just from the results of this conference, questions about model uncertainty are likely an order of magnitude larger. Plausible model variation may strengthen our conclusions. For example, our model is backward-looking and has no explicit role for expectations and no "credibility effect" in the Phillips curve. An expectations channel for monetary

[35] Intuitively, with an estimated equilibrium real funds rate of 2.5 percent, if inflation ever falls to, say, -3 percent, then with a zero nominal funds rate, the real funds rate is still restrictive, so the output gap decreases and inflation falls even more.

[36] See the related discussion in Lebow [49].

through the Phillips curve would most likely make inflation easier to control and more self-stabilizing under inflation targeting.[37] In this sense, relative to some of the other papers at this conference, we are stacking the cards against inflation targeting. Nonetheless, there can be no substitute to actually investigating the robustness of our results across model specification.

However, we would like to emphasize that a forward-looking decision framework for inflation targeting can exhibit robustness to model variation. For example, as mentioned above, one implementation of inflation-forecast targeting is to choose from the set of conditional inflation forecasts (each based on a particular path for the instrument) the one that is most consistent with the inflation target—that is, approaches the inflation target at an appropriate rate, hits the inflation target at an appropriate horizon, and more generally, minimizes the loss function—and then follow the corresponding instrument path. The construction of conditional forecasts of course depends on the model used, but the procedure itself is robust to known model variation.[38] Put differently, targeting rules allow the coefficients of the implied instrument rules to change with structural shifts in the model. It is this decision framework that we have tried to capture in the optimal targeting rule in section 3.8.1, and in the simple inflation-forecast targeting rules in section 3.8.2. In contrast, any given optimal explicit instrument rule depends on the precise model assumed, and may be rather imperfect for a different model; any given reasonably robust explicit instrument rule may still be rather imperfect for a specific model.

[37] The analysis in Svensson [75] of inflation targeting in an open economy with forward-looking aggregate demand and supply confirms this.

[38] In a forward-looking model, constructing conditional inflation forecasts for arbitrary instrument paths imply some problems that are not present in a backward-looking model. Svensson [74] provides a solution.

A. Unconditional variances

The covariance matrix Σ_{YY} for the goal variables is given by

$$\Sigma_{YY} \equiv \mathrm{E}\left[Y_t Y_t'\right] = C \Sigma_{XX} C', \tag{A.1}$$

where Σ_{XX} is the unconditional covariance matrix of the state variables. The latter fulfills the matrix equation

$$\Sigma_{XX} \equiv \mathrm{E}\left[X_t X_t'\right] = M \Sigma_{XX} M' + \Sigma_{vv}. \tag{A.2}$$

We can use the relations $\mathrm{vec}(A + B) = \mathrm{vec}(A) + \mathrm{vec}(B)$ and $\mathrm{vec}(ABC) = (C' \otimes A)\ \mathrm{vec}(B)$ on (A.2) (where $\mathrm{vec}(A)$ denotes the vector of stacked column vectors of the matrix A, and $\otimes$ denotes the Kronecker product) which results in

$$\begin{aligned} \mathrm{vec}\left(\Sigma_{XX}\right) &= \mathrm{vec}\left(M \Sigma_{XX} M'\right) + \mathrm{vec}\left(\Sigma_{vv}\right) \\ &= \left(M \otimes M\right) \mathrm{vec}\left(\Sigma_{XX}\right) + \mathrm{vec}\left(\Sigma_{vv}\right). \end{aligned}$$

Solving for $\mathrm{vec}\left(\Sigma_{XX}\right)$ we get

$$\mathrm{vec}\left(\Sigma_{XX}\right) = \left[I - \left(M \otimes M\right)\right]^{-1} \mathrm{vec}\left(\Sigma_{vv}\right). \tag{A.3}$$

B. The optimal instrument rule

The optimal instrument rule is the vector f in (3.7) that fulfills

$$f = -\left(R + \delta B' V B\right)^{-1} \left(U' + \beta B' V A\right),$$

where the 9×9 matrix V fulfills the Riccati equation

$$V = Q + Uf + f'U' + f'Rf + \delta M' V M,$$

where M is the transition matrix given by (3.8) and Q, U and R are given by

$$Q = C_X' K C_X, \ U = C_X' K C_i, \ R = C_i' K C_i.$$

Furthermore, the optimal value of (3.1) is

$$X_t' V X_t + \frac{\delta}{1-\delta} \mathrm{trace}\left(V \Sigma_{vv}\right), \tag{B.1}$$

where $\Sigma_{vv} = \mathrm{E}\left[v_t v_t'\right]$ is the covariance matrix of the disturbance vector.

For $\delta = 1$ the optimal value of (3.3) is

$$\mathrm{E}\left[L_t\right] = \mathrm{trace}\left(V \Sigma_{vv}\right). \tag{B.2}$$

C. An information lag

With our state-space setup, the information lag in (3.16) requires inserting π_{t-4} as a 10th state variable and forming the extended 1×10 state-variable vector

$$\tilde{X}_t = \begin{bmatrix} X_t \\ \pi_{t-4} \end{bmatrix}.$$

Then the restriction can be written

$$\begin{aligned} \tilde{g}\tilde{X}_t &= g_\pi \bar{\pi}_{t-1} + g_y y_{t-1} \\ \tilde{g} &= g_\pi(\tilde{e}_{2:4} + \frac{1}{4}\tilde{e}_{10}) + g_y \tilde{e}_6 \\ \tilde{f} &= h\tilde{e}_7 + \tilde{g} \\ i_t &= \tilde{f}\tilde{X}_t, \end{aligned}$$

where $\tilde{g}$ and $\tilde{f}$ are 1×10 row vectors, $\tilde{e}_j$ and $\tilde{e}_{j:k}$ are defined as e_j and $e_{j:k}$, except that they are 10×1 vectors.

D. An instrument rule that responds to a rule-consistent inflation forecast

Suppose $T \geqslant 3$ (we deal with $T = 2$ below.) Then we have to write the model in state-space form with forward-looking variables. We first note that, since in our model the first element in B is zero, the first equation in (3.4) is

$$\pi_{t+1} = A_{1\cdot}X_t + \nu_{1,t+1}, \tag{D.1}$$

where $A_{1\cdot}$ is the row vector $(a_{1k})_{k=1}^n$. Then π_{t+1} and $\pi_{t+1|t} = A_{1\cdot}X_t$ are predetermined. In order to write the system in state-space form, we now define the $(T-2)\times 1$ column vector of forward-looking variables, $x_t = (x_{lt})_{l=1}^{T-2}$, where

$$x_{lt} \equiv \pi_{t+l+1|t} \tag{D.2}$$

for $l = 1, .., T-2$. Observe that, for $l = 1, ..., T-3$, by the law of iterated expectations,

$$x_{l,t+1|t} = x_{l+1,t}, \tag{D.3}$$

whereas for $l = T-2$ we have

$$x_{T-2,t+1|t} = \pi_{t+T|t}. \tag{D.4}$$

Equation (D.3) gives us $T-3$ equations for the first $T-3$ forward-looking variables x_{lt}, $l = 1, ..., T-3$. We also need an equation for $x_{T-2,t}$. Lead equation (D.1) by one period, and take expectations in period t,

$$x_{1t} \equiv \pi_{t+2|t} = A_{1\cdot}X_{t+1|t} = A_{1\cdot}[AX_t + B(hi_{t-1} + \varphi\pi_{t+T|t})] = A_{1\cdot}(\tilde{A}X_t + \varphi Bx_{T-2,t+1|t}), \quad \text{(D.5)}$$

where

$$\tilde{A} = A + hBe_7, \quad \text{(D.6)}$$

where we have used (3.4), (3.17), and (D.4). Solve for $x_{T-2,t+1|t}$,

$$x_{T-2,t+1|t} = -\frac{1}{\varphi A_{1\cdot}B}A_{1\cdot}\tilde{A}X_t + \frac{1}{\varphi A_{1\cdot}B}x_{1t}, \quad \text{(D.7)}$$

which gives us the remaining equation (note that $A_{1\cdot}B$ is a scalar).

Thus, equations (D.3) and (D.7) give us $T-2$ equations for the $T-2$ forward-looking variables. With regard to the predetermined variables, we use (3.4), (3.17), (D.4), (D.6) and (D.7) to write,

$$\begin{aligned}
X_{t+1} &= \tilde{A}X_t + \varphi Bx_{T-2,t+1|t} \\
&= \tilde{A}X_t + \varphi B\left(-\frac{1}{\varphi A_{1\cdot}B}A_{1\cdot}\tilde{A}X_t + \frac{1}{\varphi A_{1\cdot}B}x_{1t}\right) \\
&= \left(I - \frac{1}{A_{1\cdot}B}BA_{1\cdot}\right)\tilde{A}X_t + \frac{1}{A_{1\cdot}B}Bx_{1t}. \quad \text{(D.8)}
\end{aligned}$$

By combining (D.8), (D.3) and (D.7), we can now write the system in state-space form,

$$\begin{bmatrix} X_{t+1} \\ x_{t+1|t} \end{bmatrix} = D\begin{bmatrix} X_t \\ x_t \end{bmatrix} + \begin{bmatrix} v_{t+1} \\ 0 \end{bmatrix}, \quad \text{(D.9)}$$

where the $(n+T-2)\times(n+T-2)$ matrix D is given by

$$D = \begin{bmatrix} \left(I - \frac{1}{A_{1\cdot}B}BA_{1\cdot}\right)\tilde{A} & \frac{1}{A_{1\cdot}B}Bu_{n+1} \\ D_{21} & D_{22} \end{bmatrix},$$

where u_k, $k = 1, ..., n+T-2$ is an $1\times(n+T-2)$ row vector with element k equal to unity and all other elements equal to zero, and where the $(T-2)\times n$ matrix D_{21} and the $(T-2)\times(T-2)$ matrix D_{22} are given by

$$D_{21} = \begin{bmatrix} 0_{(T-3)\times n} \\ -\frac{1}{\varphi A_{1\cdot}B}A_{1\cdot}\tilde{A} \end{bmatrix}, \quad D_{22} = \begin{bmatrix} 0_{(T-3)\times 1} \quad I_{T-3} \\ \frac{1}{\varphi A_{1\cdot}B}u_{n+1} \end{bmatrix},$$

where $0_{k\times m}$ is a $k\times m$ matrix of zeros and I_m is an $m\times m$ identity matrix.

The system (D.9) can then be solved with the help of known algorithms, for instance the one in Klein [47]. The solution results in a $(T-2)\times n$ matrix H, expressing the forward-looking variables as a linear function of the state-variables,

$$x_t = HX_t. \tag{D.10}$$

The dynamics of the predetermined variable are then given by

$$X_{t+1} = (D_{11} + D_{12}H)X_t + v_{t+1}, \tag{D.11}$$

where D_{11} and D_{12} are the obvious submatrices of D. It furthermore follows that

$$x_{t+1|t} = D_{21}X_t + D_{22}x_t = (D_{21} + D_{22}H)X_t.$$

From (3.17) and (D.4) follows that the equilibrium instrument rule can be written

$$\begin{aligned} i_t &= fX_t \\ f &= he_7 + \varphi u_{n+T-2}(D_{21} + D_{22}H). \end{aligned}$$

Then we can use f in (3.8) and (3.9) and proceed as in the other cases. The matrix M in (3.8) will of course equal the matrix $(D_{11} + D_{12}H)$ in (D.11).

For $T = 2$, by (3.17) and D.5, we directly get

$$\begin{aligned} \pi_{t+2|t} &= A_{1}.(\tilde{A}X_t + \varphi B\pi_{t+2|t}) \\ &= \frac{1}{1-\varphi A_{1}.B}A_{1}.\tilde{A}X_t, \end{aligned}$$

hence,

$$\begin{aligned} i_t &= hi_{t-1} + \frac{\varphi}{1-\varphi A_{1}.B}A_{1}.\tilde{A}X_t, \\ f &= he_7 + \frac{\varphi}{1-\varphi A_{1}.B}A_{1}.\tilde{A}. \end{aligned}$$

Table 1. Model Responses to a Funds Rate Increase
(Annual average difference from baseline in percentage points)

	Years after funds rate increase		
	1	2	3
Output Gap			
MPS	−.07	−.45	−.99
Our Model	−.07	−.41	−.66
Inflation			
MPS	−.00	−.03	−.26
Our Model	−.00	−.08	−.25

Note: The MPS results are from table II.1 in Mauskopf [54].

Table 2. Model Selection Criteria

	SIC	AIC
Inflation Equation		
VAR	736.8	698.8
Our Model	705.0	690.3
Output Equation		
VAR	652.2	617.1
Our Model	639.7	630.9

Table 3. Results on Volatility and Loss with Various Rules
($\lambda = 1,\ \nu = 0.5$)

Rule	Std$[\bar{\pi}_t]$	Std$[y_t]$	Std$[i_t - i_{t-1}]$	Loss	Rank
Optimal	2.15	2.24	1.68	11.08	1
L$(\bar{\pi}_t, y_t)$ $g_\pi = 2.72,\ g_y = 1.57$	2.18	2.24	1.74	11.27	5
L$(\bar{\pi}_{t+T\|t}(i_{t-1}))$ $T = 8;\ g_\pi = 2.55$	2.42	2.27	2.07	13.15	18
L$(\bar{\pi}_{t+T\|t}(i_{t-1}), y_t)$ $T = 8;\ g_\pi = 2.53,\ g_y = 0.29$	2.44	2.15	2.20	13.01	17
S$(\bar{\pi}_t, y_t)$ $g_\pi = 2.37,\ g_y = 1.44,\ h = 0.14$	2.18	2.25	1.68	11.23	4
S$(\bar{\pi}_{t+T\|t}(i_{t-1}))$ $T = 8;\ g_\pi = 1.89,\ h = 0.46$	2.15	2.47	1.53	11.89	12
S$(\bar{\pi}_{t+T\|t}(i_{t-1}), y_t)$ $T = 8;\ g_\pi = 1.54,\ g_y = 0.45,\ h = 0.60$	2.15	2.25	1.68	11.09	2
S$(\pi_{t+T\|t})$					
$T = 8;\ \varphi = 2.62,\ h = 0.32$	2.15	2.45	1.53	11.77	11
$T = 12;\ \varphi = 3.65,\ h = 0.38$	2.13	2.41	1.55	11.58	10
$T = 16;\ \varphi = 5.52,\ h = 0.41$	2.13	2.40	1.57	11.51	7
SIFT(T)					
$T = 8$	1.40	2.84	7.44	37.65	22
$T = 12$	1.81	2.44	3.15	14.17	19
$T = 16$	2.21	2.27	2.03	12.05	13
FIFT(T)					
$T = 8;\ c = 0.72$	2.24	1.82	5.31	22.41	21
$T = 12;\ c = 0.39$	2.17	2.11	2.72	12.86	16
$T = 16;\ c = 0.01$	2.22	2.26	2.02	12.05	13
SIFTS(T)					
$T = 8;\ h = 0.59$	1.51	3.39	3.88	21.29	20
$T = 12;\ h = 0.45$	1.87	2.60	1.94	12.16	15
$T = 16;\ h = 0.31$	2.24	2.34	1.47	11.57	8
FIFTS(T)					
$T = 8;\ c = 0.66,\ h = 0.71$	2.15	2.26	1.86	11.42	6
$T = 12;\ c = 0.35,\ h = 0.47$	2.18	2.28	1.59	11.17	3
$T = 16;\ c = 0.00,\ h = 0.31$	2.24	2.34	1.47	11.57	8

Table 4. Results on Volatility and Loss with Various Rules
($\lambda = 0.2,\ \nu = 0.5$)

Rule	Std[$\bar{\pi}_t$]	Std[y_t]	Std[$i_t - i_{t-1}$]	Loss	Rank
Optimal	1.97	2.64	1.55	6.47	1
L($\bar{\pi}_t, y_t$) $g_\pi = 3.17,\ g_y = 1.22$	2.00	2.61	1.65	6.71	10
L($\bar{\pi}_{t+T\|t}(i_{t-1})$) $T = 8;\ g_\pi = 2.65$	2.37	2.28	2.17	9.00	17
L($\bar{\pi}_{t+T\|t}(i_{t-1}), y_t$) $T = 8;\ g_\pi = 2.69,\ g_y = -0.25$	2.36	2.41	2.10	8.92	16
S($\bar{\pi}_t, y_t$) $g_\pi = 2.34,\ g_y = 1.03,\ h = 0.30$	2.00	2.64	1.56	6.60	9
S($\bar{\pi}_{t+T\|t}(i_{t-1})$) $T = 8;\ g_\pi = 1.63,\ h = 0.69$	1.97	2.75	1.53	6.58	8
S($\bar{\pi}_{t+T\|t}(i_{t-1}), y_t$) $T = 8;\ g_\pi = 1.42,\ g_y = 0.16,\ h = 0.74$	1.97	2.64	1.55	6.48	2
S($\pi_{t+T\|t}$)					
$T = 8;\ \varphi = 2.35,\ h = 0.62$	1.97	2.73	1.53	6.55	7
$T = 12;\ \varphi = 3.86,\ h = 0.71$	1.97	2.69	1.54	6.50	5
$T = 16;\ \varphi = 8.33,\ h = 0.47$	1.97	2.68	1.54	6.49	4
SIFT(T)					
$T = 8$	1.40	2.84	7.44	31.21	22
$T = 12$	1.81	2.44	3.15	9.42	19
$T = 16$	2.21	2.27	2.03	7.95	14
FIFT(T)					
$T = 8;\ c = 0.69$	2.13	1.87	5.38	19.68	21
$T = 12;\ c = 0.24$	1.99	2.24	2.88	9.10	18
$T = 16;\ c = 0.00$	2.21	2.27	2.03	7.95	14
SIFTS(T)					
$T = 8;\ h = 0.71$	1.62	3.84	3.34	11.16	20
$T = 12;\ h = 0.60$	1.93	2.74	1.60	6.51	6
$T = 16;\ h = 0.45$	2.28	2.39	1.25	7.11	12
FIFTS(T)					
$T = 8;\ c = 0.53,\ h = 0.79$	1.98	2.67	1.67	6.74	11
$T = 12;\ c = 0.08,\ h = 0.60$	1.98	2.65	1.52	6.48	2
$T = 16;\ c = 0.00,\ h = 0.45$	2.28	2.39	1.25	7.11	12

Table 5. Results on Volatility and Loss with Various Rules
($\lambda = 5,\ \nu = 0.5$)

Rule	Std[$\bar{\pi}_t$]	Std[y_t]	Std[$i_t - i_{t-1}$]	Loss	Rank
Optimal	2.65	1.86	2.36	26.99	1
L($\bar{\pi}_t, y_t$) $g_\pi = 2.15,\ g_y = 2.17$	2.69	1.89	2.16	27.46	4
L($\bar{\pi}_{t+T\|t}(i_{t-1})$) $T = 8;\ g_\pi = 2.13$	2.69	2.22	1.70	33.32	17
L($\bar{\pi}_{t+T\|t}(i_{t-1}), y_t$) $T = 8;\ g_\pi = 2.24,\ g_y = 1.40$	2.80	1.83	2.69	28.17	7
S($\bar{\pi}_t, y_t$) $g_\pi = 1.26,\ g_y = 2.35,\ h = -0.11$	2.68	1.88	2.27	27.39	3
S($\bar{\pi}_{t+T\|t}(i_{t-1})$) $T = 8;\ g_\pi = 2.03,\ h = 0.06$	2.67	2.23	1.59	33.29	16
S($\bar{\pi}_{t+T\|t}(i_{t-1}), y_t$) $T = 8;\ g_\pi = 1.78,\ g_y = 1.27,\ h = 0.31$	2.65	1.87	2.31	27.15	2
S($\pi_{t+T\|t}$)					
$T = 8;\ \varphi = 2.62,\ h = -0.15$	2.65	2.21	1.62	32.81	15
$T = 12;\ \varphi = 3.16,\ h = -0.11$	2.61	2.19	1.65	32.06	12
$T = 16;\ \varphi = 3.91,\ h = -0.09$	2.59	2.18	1.67	31.78	11
SIFT(T)					
$T = 8$	1.40	2.84	7.44	69.88	22
$T = 12$	1.81	2.44	3.15	37.89	20
$T = 16$	2.21	2.27	2.03	32.59	13
FIFT(T)					
$T = 8;\ c = 0.81$	2.64	1.70	5.15	34.71	18
$T = 12;\ c = 0.64$	2.70	1.91	2.49	28.61	8
$T = 16;\ c = 0.44$	2.79	2.02	1.78	29.86	10
SIFTS(T)					
$T = 8;\ h = 0.35$	1.44	3.04	5.07	61.07	21
$T = 12;\ h = 0.15$	1.82	2.47	2.69	37.50	19
$T = 16;\ h = 0.02$	2.21	2.27	1.99	32.59	13
FIFTS(T)					
$T = 8;\ c = 0.80,\ h = 0.52$	2.63	1.87	2.52	27.48	5
$T = 12;\ c = 0.64,\ h = 0.22$	2.71	1.95	1.94	28.15	6
$T = 16;\ c = 0.44,\ h = 0.03$	2.79	2.03	1.71	29.85	9

Table 6. Results on Volatility and Loss with Various Rules
($\lambda = 1,\ \nu = 0.1$)

Rule	Std[$\bar{\pi}_t$]	Std[y_t]	Std[$i_t - i_{t-1}$]	Loss	Rank
Optimal	1.96	2.12	3.02	9.25	1
L($\bar{\pi}_t, y_t$) $g_\pi = 3.43,\ g_y = 2.50$	2.01	2.18	2.71	9.51	5
L($\bar{\pi}_{t+T\|t}(i_{t-1})$) $T = 8;\ g_\pi = 3.46$	2.11	2.35	2.99	10.86	20
L($\bar{\pi}_{t+T\|t}(i_{t-1}), y_t$) $T = 8;\ g_\pi = 3.41,\ g_y = 1.00$	2.18	2.05	3.53	10.18	11
S($\bar{\pi}_t, y_t$) $g_\pi = 2.80,\ g_y = 2.80,\ h = -0.16$	2.00	2.15	2.90	9.46	4
S($\bar{\pi}_{t+T\|t}(i_{t-1})$) $T = 8;\ g_\pi = 3.15,\ h = 0.31$	1.94	2.47	2.47	10.51	18
S($\bar{\pi}_{t+T\|t}(i_{t-1}), y_t$) $T = 8;\ g_\pi = 2.79,\ g_y = 1.06,\ h = 0.47$	1.96	2.14	2.98	9.29	2
S($\pi_{t+T\|t}$)					
$T = 8;\ \varphi = 5.01,\ h = -0.01$	1.94	2.45	2.49	10.37	13
$T = 12;\ \varphi = 7.99,\ h = 0.06$	1.92	2.41	2.55	10.13	9
$T = 16;\ \varphi = 13.66,\ h = 0.09$	1.91	2.39	2.58	10.04	8
SIFT(T)					
$T = 8$	1.40	2.84	7.44	15.54	22
$T = 12$	1.81	2.44	3.15	10.19	12
$T = 16$	2.21	2.27	2.03	10.41	14
FIFT(T)					
$T = 8;\ c = 0.61$	1.95	1.97	5.54	10.75	19
$T = 12;\ c = 0.27$	2.02	2.21	2.84	9.78	7
$T = 16;\ c = 0.00$	2.21	2.27	2.03	10.41	14
SIFTS(T)					
$T = 8;\ h = 0.34$	1.43	3.03	5.14	13.86	21
$T = 12;\ h = 0.11$	1.82	2.46	2.80	10.15	10
$T = 16;\ h = 0.06$	2.20	2.26	2.15	10.41	14
FIFTS(T)					
$T = 8;\ c = 0.60,\ h = 0.45$	1.95	2.13	3.08	9.30	3
$T = 12;\ c = 0.27,\ h = 0.13$	2.03	2.23	2.46	9.73	6
$T = 16;\ c = 0.00,\ h = -0.06$	2.20	2.26	2.15	10.41	14

Table 7. Results on Volatility and Loss with Various Rules
$(\lambda = 1,\ \nu = 1)$

Rule	Std$[\bar{\pi}_t]$	Std$[y_t]$	Std$[i_t - i_{t-1}]$	Loss	Rank
Optimal	2.27	2.29	1.33	12.17	1
L$(\bar{\pi}_t, y_t)$ $g_\pi = 2.44,\ g_y = 1.23$	2.29	2.28	1.42	12.49	8
L$(\bar{\pi}_{t+T\mid t}(i_{t-1}))$ $T = 8;\ g_\pi = 2.24$	2.60	2.24	1.79	14.99	17
L$(\bar{\pi}_{t+T\mid t}(i_{t-1}), y_t)$ $T = 8;\ g_\pi = 2.23,\ g_y = 0.07$	2.61	2.20	1.82	14.97	16
S$(\bar{\pi}_t, y_t)$ $g_\pi = 1.12,\ g_y = 1.04,\ h = 0.27$	2.29	2.30	1.34	12.33	4
S$(\bar{\pi}_{t+T\mid t}(i_{t-1}))$ $T = 8;\ g_\pi = 1.47,\ h = 0.54$	2.27	2.47	1.24	12.82	12
S$(\bar{\pi}_{t+T\mid t}(i_{t-1}), y_t)$ $T = 8;\ g_\pi = 1.18,\ g_y = 0.30,\ h = 0.65$	2.27	2.30	1.34	12.18	2
S$(\pi_{t+T\mid t})$					
$T = 8;\ \varphi = 1.92,\ h = 0.45$	2.26	2.45	1.25	12.71	10
$T = 12;\ \varphi = 2.52,\ h = 0.50$	2.25	2.42	1.26	12.54	9
$T = 16;\ \varphi = 3.63,\ h = 0.53$	2.25	2.41	1.27	12.48	7
SIFT(T)					
$T = 8$	1.40	2.84	7.44	65.29	22
$T = 12$	1.81	2.44	3.15	19.13	19
$T = 16$	2.21	2.27	2.03	14.11	15
FIFT(T)					
$T = 8;\ c = 0.77$	2.45	1.75	5.21	36.19	21
$T = 12;\ c = 0.47$	2.30	2.04	2.64	16.43	18
$T = 16;\ c = 0.12$	2.32	2.20	1.95	14.02	14
SIFTS(T)					
$T = 8;\ h = 0.66$	1.56	3.62	3.54	28.08	20
$T = 12;\ h = 0.56$	1.91	2.70	1.68	13.77	13
$T = 16;\ h = 0.45$	2.28	2.39	1.25	12.47	6
FIFTS(T)					
$T = 8;\ c = 0.69,\ h = 0.79$	2.28	2.33	1.47	12.77	11
$T = 12;\ c = 0.40,\ h = 0.59$	2.27	2.31	1.31	12.19	3
$T = 16;\ c = 0.05,\ h = 0.45$	2.32	2.36	1.23	12.46	5

Table 8. Results for Conference Rules
($\lambda = 1,\ \nu = 0.5$)

Rule	Std[$\bar{\pi}_t$]	Std[y_t]	Std[$i_t - i_{t-1}$]	Loss
a. With Contemporaneous Information; $i_t = hi_{t-1} + g_\pi \bar{\pi}_t + g_y y_t$				
Rule I(1); D($\bar{\pi}_t, y_t$)		Dynamically unstable		
$g_\pi = 3.00,\ g_y = 0.80,\ h = 1.00$				
Rule II(1); D($\bar{\pi}_t, y_t$)		Dynamically unstable		
$g_\pi = 1.20,\ g_y = 1.00,\ h = 1.00$				
Rule III(1); L($\bar{\pi}_t, y_t$)	3.46	2.25	0.71	17.25
$g_\pi = 1.50,\ g_y = 0.50,\ h = 0.00$				
Rule IV(1); L($\bar{\pi}_t, y_t$)	3.52	1.98	1.03	16.86
$g_\pi = 1.50,\ g_y = 1.00,\ h = 0.00$				
Rule V(1); S($\bar{\pi}_t, y_t$)		Dynamically unstable		
$g_\pi = 1.20,\ g_y = 0.06,\ h = 1.30$				
Optimal L($\bar{\pi}_t, y_t$)	2.18	2.24	1.74	11.27
$g_\pi = 2.72,\ g_y = 1.57,\ h = 0.00$				
Optimal D($\bar{\pi}_t, y_t$)	3.85	3.80	1.07	30.42
$g_\pi = 0.07,\ g_y = 0.27,\ h = 1.00$				
b. With Lagged Information; $i_t = hi_{t-1} + g_\pi \bar{\pi}_{t-1} + g_y y_{t-1}$				
Rule I(2); D($\bar{\pi}_{t-1}, y_{t-1}$)		Dynamically unstable		
$g_\pi = 3.00,\ g_y = 0.80,\ h = 1.00$				
Rule II(2); D($\bar{\pi}_{t-1}, y_{t-1}$)		Dynamically unstable		
$g_\pi = 1.20,\ g_y = 1.00,\ h = 1.00$				
Rule III(2); L($\bar{\pi}_{t-1}, y_{t-1}$)	3.62	2.40	0.72	19.07
$g_\pi = 1.50,\ g_y = 0.50,\ h = 0.00$				
Rule IV(2); L($\bar{\pi}_{t-1}, y_{t-1}$)	3.63	2.14	1.04	18.29
$g_\pi = 1.50,\ g_y = 1.00,\ h = 0.00$				
Rule V(2); S($\bar{\pi}_{t-1}, y_{t-1}$)		Dynamically unstable		
$g_\pi = 1.20,\ g_y = 0.06,\ h = 1.30$				
Optimal L($\bar{\pi}_{t-1}, y_{t-1}$)	2.38	2.44	1.69	13.03
$g_\pi = 2.50,\ g_y = 1.50,\ h = 0.00$				
Optimal D($\bar{\pi}_{t-1}, y_{t-1}$)				
$g_\pi = 0.04,\ g_y = 0.21,\ h = 1.00$	4.96	4.21	0.87	42.75

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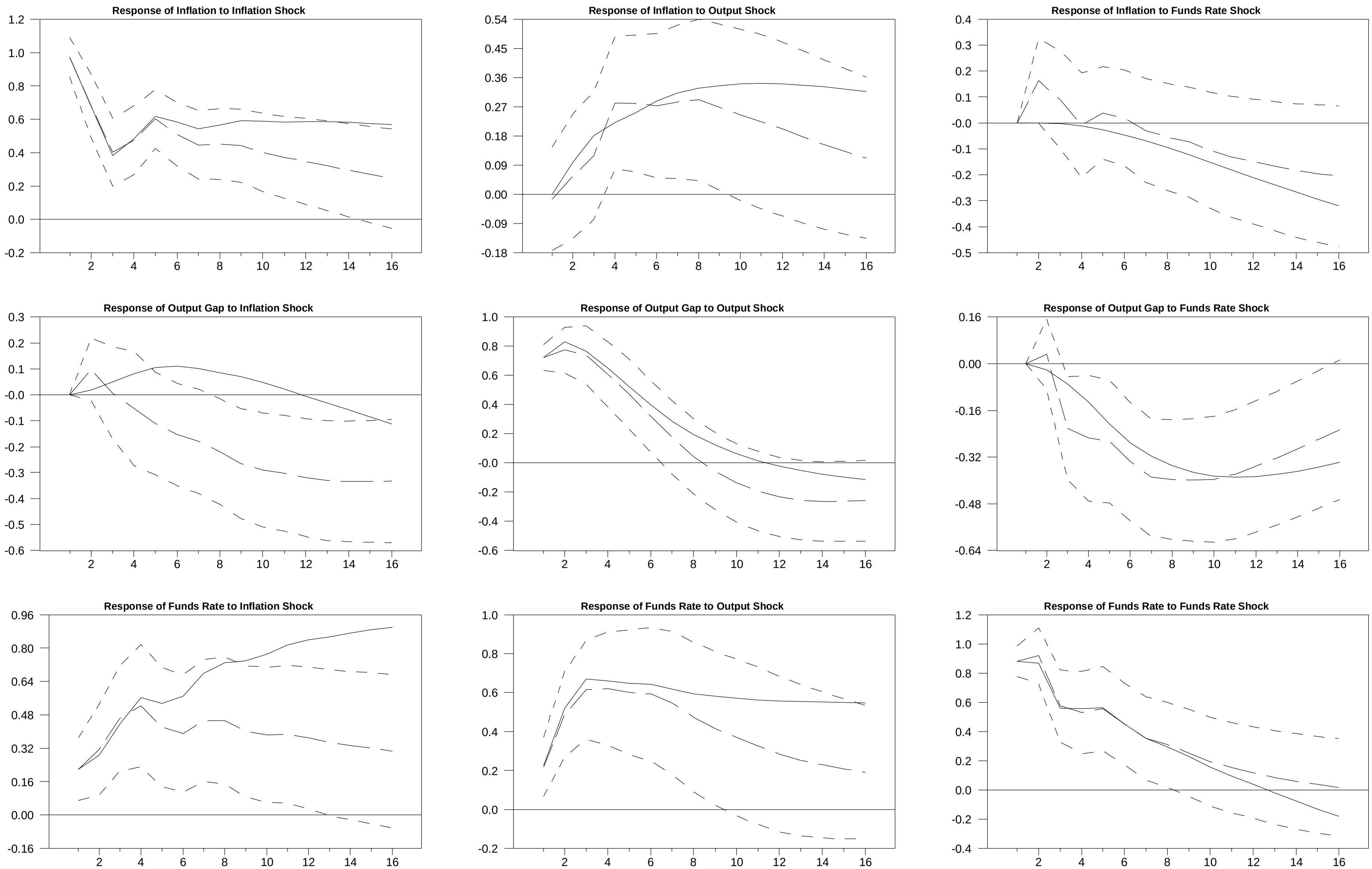
Figure 1

VAR and Structural Model Impulse Responses



Notes: The impulse responses of the structural model (ammended with the VAR interest rate equation) are shown as the solid lines. The impulse responses of the VAR are shown as long-dashed lines and their 95 percent confidence intervals are shown as short-dashed lines.

Figure 2
Policy Rule Frontiers

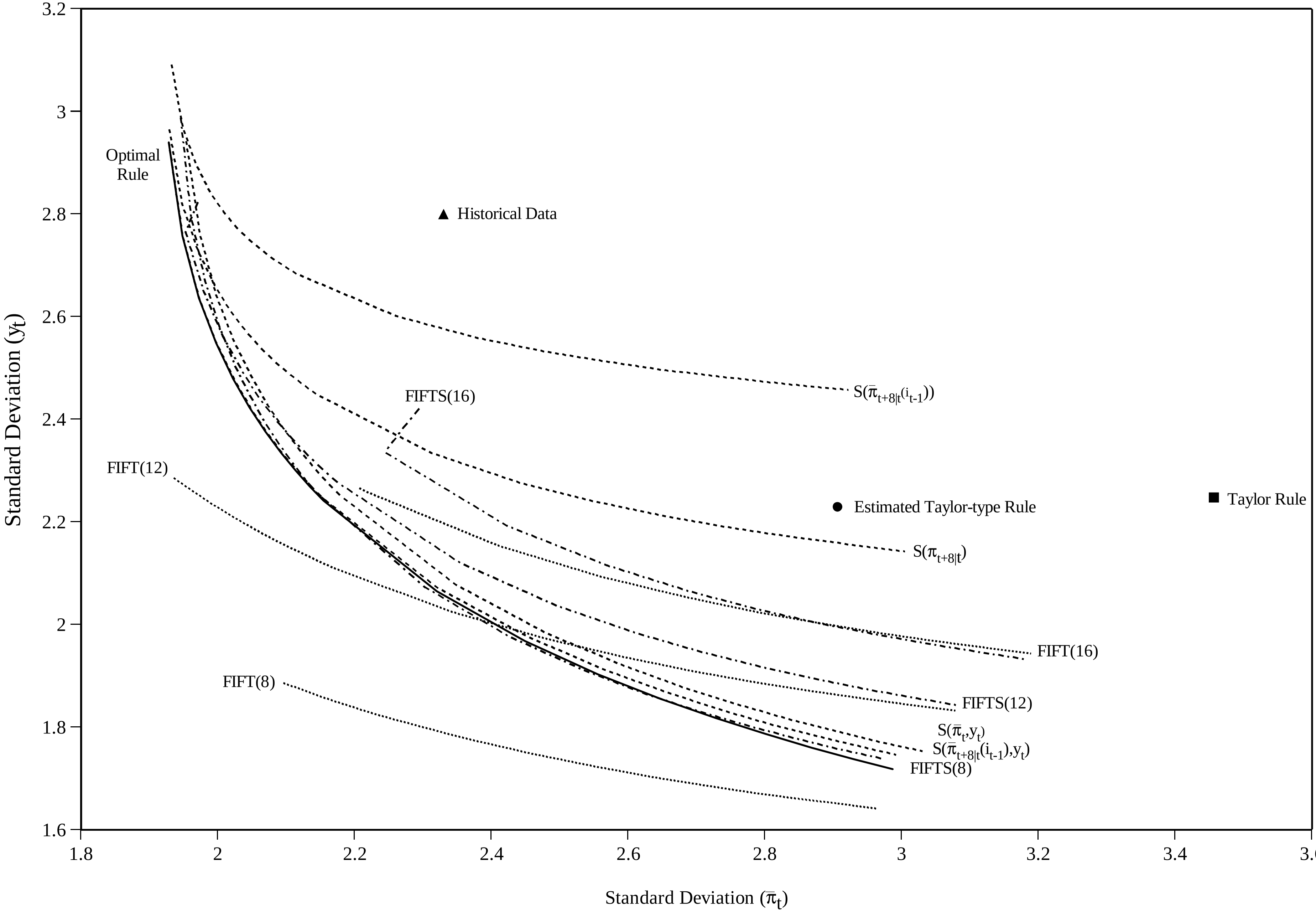


Figure 3
Impulse Responses for Smoothing Rules
($\lambda = 1$, $\nu = 0.5$)

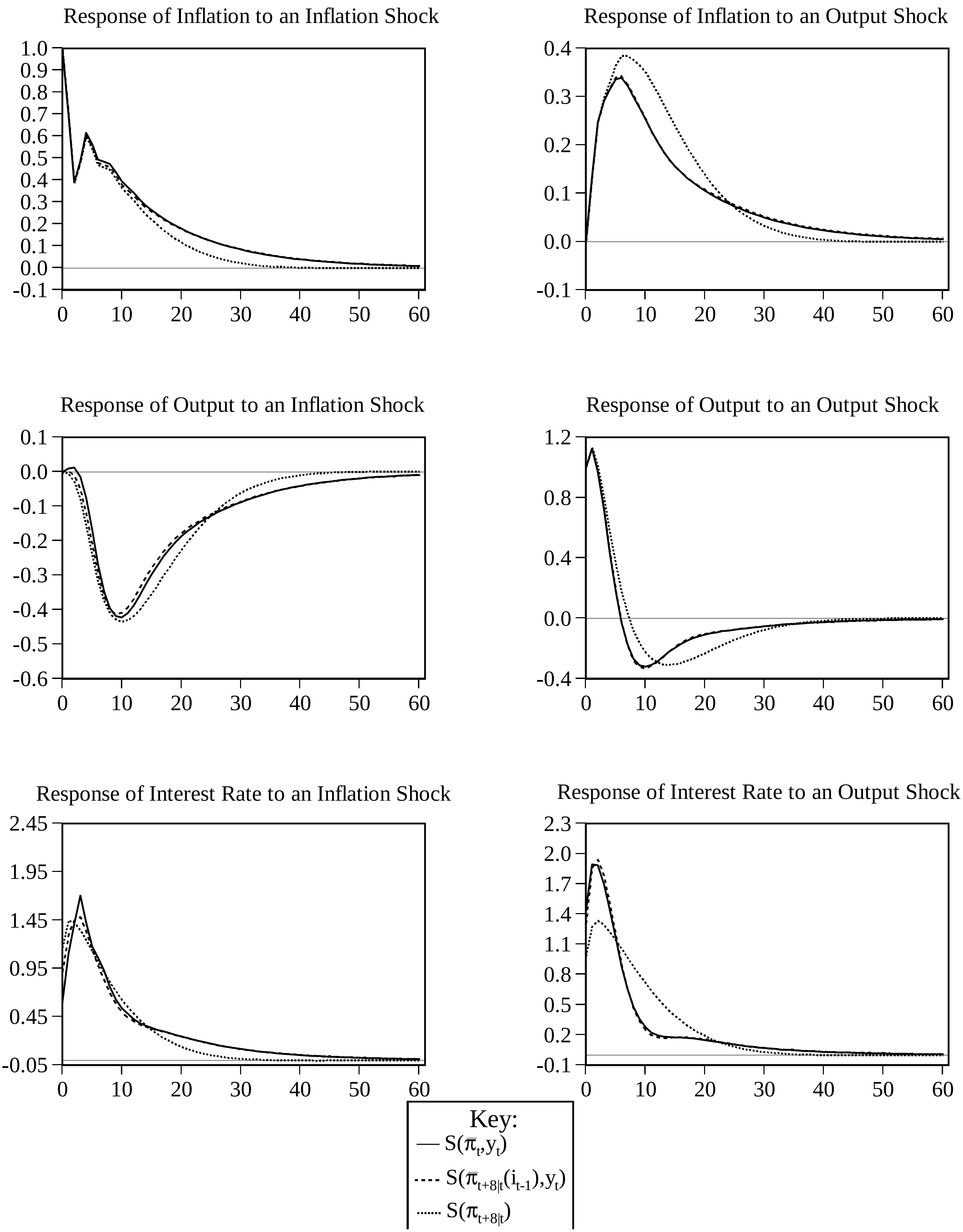


Figure 4
Impulse Responses for Inflation Targeting Rules
($\lambda = 1$, $\nu = 0.5$)