Fiscal Policy and Interest Rates in a Small Open Economy

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Abstract

This paper contains an empirical investigation of the effects of fiscal policy on interest rates based on a conventional stochastic macro model designed for a small open economy. The empirical investigation undertaken utilizes data for Sweden, a country which has experienced very large fluctuations in the government budget deficits and in the short- and long-term nominal interest rates, thus providing a better empirical test than previous studies. According to the empirical results, larger budget deficits spell higher interest rates, as posited by conventional macroeconomic theory.

Keywords: Term structure of interest rates; Ricardian equivalence; budget deficits; small open economy; stationary and non-stationary time series.

JEL Classification Numbers: E12; E62; F41.

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1 Introduction

In this paper, I address the question of whether larger budget deficits produce higher interest rates. Theoretically, it is well known that the effects of changes in fiscal policy on the term structure of interest rates are ambiguous. The Ricardian equivalence theorem states that, for a given path of government consumption expenditures, individuals view budget deficits as postponed tax-liabilities. Therefore budget deficits do not alter wealth, desired consumption paths or interest rates. According to the more conventional view in macroeconomics, on the other hand, individuals do not fully internalize the future tax-liabilities, which implies that changes in government debt add to private wealth, influencing desired consumption paths and thus interest rates.[1] However, the empirical studies undertaken to date, mostly utilizing data for the United States, have not been able to supply either view with convincing evidence. Since the resolution of this issue is important for the design of macroeconomic policy, there is a need for more research in the field.

In this paper, I utilize data for Sweden to provide a good empirical answer to the question posed above. The reason why Sweden is an interesting case is that the country has experienced very large fluctuations in the government budget deficits and short- and long term nominal interest rates since the beginning of the 1980s. Consequently, this paper provides a high-powered empirical test compared to previous studies in the field.

The empirical approach in this investigation is close to that of Evans (1985, 1987a, 1987b, 1988). First, I make a survey of the results in the previous literature and try to draw some important lessons for the investigation in this paper. Second, I set up a conventional stochastic macro model, in which the term structure of nominal interest rates is determined in terms of different policy variables, and use this model to study the effects of fiscal policy. Since Sweden is best characterized as a small open economy, a conventional stochastic macro model for a small open economy is constructed. The reason for spending time on this is that there has been little attention paid to the effects of fiscal policy on the term structure in a small open economy.[2] In addition to providing a framework for the empirical research in this paper, this approach may also offer several important insights about this issue. For example, is it automatically the case that larger

[1] When the term "conventional" is used, reference is made to Keynesian or other non-Ricardian models with rational expectations.

[2] In a closed economy setting, Turnovsky (1989), develops and uses a stochastic macro model to study the effects of changes in macroeconomic policy on the term structure of real and nominal interest rates. A closely related paper is McCafferty (1986). Grinols and Turnovsky (1994) use stochastic calculus to study the interaction between exchange rates and interest rates in a small open economy, but without a term structure of interest rates explicitly incorporated. Finally, the seminal paper by Cox, Ingersoll and Ross (1985) contains the most general stochastic utility maximizing approach to the term structure of interest rates, but they do not explicitly consider changes in macroeconomic policy.

budget deficits produce higher interest rates even in a conventional model designed for a small open economy? Finally, I estimate the implied nominal interest rate regression equations on Swedish data, taking the lessons from the survey and the conventional model into account.

The empirical results suggest that larger budget deficits spell higher nominal interest rates. According to the empirical evidence, an increase in the budget deficit as a percentage of GDP by one percent leads to increases in the domestic short- and long term interest rates of approximately 0.20 percentage points after a period of two years.

The structure of the paper is as follows. The next section is a survey of different approaches used and empirical results obtained in the earlier literature. In Section 3, the model is developed and solved for the nominal interest rates. The quarterly and monthly data set are discussed in Section 4. In Section 5, some empirical issues are discussed and the empirical results for Sweden are presented. Some tentative conclusions are then finally drawn in Section 6.

2 Previous related studies

Previous results in the literature have been obtained within three types of approach. The first is termed "conventional", since it encompasses stochastic macro models, Keynesian or non-Ricardian, where agents are assumed to form their expectations rationally. The papers by Allen (1990, 1992) and Evans (1985, 1987a, 1987b, 1988) fall into this category.

Evans (1987a) develops a stochastic rational expectations model to study the effects of macroeconomic policy on real and nominal interest rates in a closed economy setting. In particular, he focuses on the validity of the proposition that larger budget deficits are associated with higher interest rates.[3] For a long sample period from the United States, he provides evidence inconsistent with this proposition. That is, larger present or expected government budget deficits do not significantly push up either nominal or real interest rates. The same conclusion is reached from a data set containing six countries (Evans, 1987b).[4] Finally, Evans (1988) investigates whether forward rates in the United States during the second world war were an increasing function of government debt. In the empirical tests, no evidence for such a positive relationship can be found; rather, there is a negative relationship.

Allen (1990) estimates a reduced form IS-LM-AS model using quarterly data on vari-

[3] Evans (1985) investigates the empirical relationship between nominal and real interest rates and current and past government budget deficits in the United States, and finds no positive association.

[4] The six countries are: Canada, France, West Germany, Japan, United Kingdom and the United States.

ous measures of the federal debt in the United States between 1961 and 1985, and finds that there is a positive and statistically significant linkage between government debt and a tax-adjusted short-term real interest rate. Allen (1992) models first differences in order to control for autocorrelation and intercept instability, and provides more empirical evidence of a positive and statistically significant relationship. There are several possible explanations for the different results obtained by Allen and Evans. Allen chooses not to model a reduced form for the inflation expectations. Instead he uses proxies in the estimated equation. Moreover, Allen primarily considers alternative measures of debt, while Evans focuses on different measures of deficits.

The second type of model attempts to test the Ricardian equivalence theorem more directly. The papers by Plosser (1982, 1987) are perhaps the most well known examples in a closed economy setting. In neither of his papers does Plosser find any statistically significant relation between deficits and interest rates in the United States. He interprets these findings as indirect evidence for the Ricardian view. Boothe and Reid (1989) extend the work of Plosser to the Canadian case, which they consider to be a small open economy. The empirical results of Boothe and Reid are also consistent with the previous studies undertaken by Evans and Plosser. In a political economy setting, Minford (1988) provides theoretical arguments against the Ricardian view. In brief, the argument is that different political parties tend to pursue policies designed to favor their own electorate. For instance, the "left" wing monetary policy will be more inflationary than the "right" wing, since the "right" electorate's nominal government bond holdings can be expropriated through unanticipated inflation. This will lead to a risk premium on nominal government bonds, which will be an increasing function of the size of the bond financed deficit. Minford then provides empirical evidence consistent with the predictions from the model, using annual data for the United Kingdom between 1920-1982.[5]

The third type of model considered in the literature is the so called "loanable funds" model. This type of model, which models interest rates as equilibrium responses to the demand and supply in the loan markets, is used, for example, by Cebula et al. (1988), de Haan and Zelhorst (1990), Cebula et al. (1990), Cebula and Rhodd (1993), Correia-Nunes and Stemitsiotis (1995) and Miller and Russek (1996).[6] The estimated equations in this literature are very similar; some nominal long-term interest rate is linearly related to a set of explanatory variables, including some measures of the expected inflation rate and

[5] It is notable that the parameterization in the model considered is not as parsimonious as in the other studies. He also includes a measure of inflation expectations, dummies for the second world war and the Korean war etc. in his regression model.

[6] Although not explicitly modeled, this literature generally acknowledge the effects of growing integration of world capital markets on the relationship between budget deficits and interest rates. Globalization of world financial markets in this context means that budget deficits may be financed by borrowing abroad, implying that the impact of deficits on national interest rates can be moderated.

government deficits and debts. Another characteristic of these studies is that they use annual data. The empirical evidence provided in this setting points in one direction: the level of nominal interest rates is positively related to government budget deficits.

So, which theoretical view is supported by the empirical evidence? Although the empirical results presented by Boothe and Reid, Evans, and Plosser are consistent with Ricardian equivalence, their investigations do not constitute a direct test. This stems from the fact that some of the assumptions underlying the theorem can be violated simultaneously, but work in different directions, so that even if Ricardian equivalence is not rejected by the data, one should only interpret the empirical results supporting Ricardian equivalence as a crude approximation of reality.[7] On the other hand, the papers which test loanable funds models, and the papers by Allen and Minford, seem to point in another direction, namely that government deficits and debts have a significant impact on short- and long-term nominal and real interest rates. Therefore, these papers provide some evidence for the conventional model, and against the Ricardian view.

What are the tentative conclusions as to why these discrepancies have occurred in the empirical evidence reported? The analyses summarized above suggest two important factors, which may account for the different empirical results. First of all, the data frequency seems to be important. In studies which have exploited lower frequency data, the evidence is more in favor of the conventional view and against the Ricardian view. Some economists have also argued that misleading estimates can result from fitting econometric models to data too finely disaggregated over time.[8] Secondly, the treatment of the expected inflation rate seems to be of considerable importance. In the studies surveyed, the results tend to be more supportive of the conventional model when a proxy has been used to account for the expected inflation rate, rather than a reduced form. By including a proxy for expected inflation, one does not capture the indirect effects of budget deficits via expected inflation on interest rates. In conventional models, a (temporary) increase in the budget deficit today, will are likely to raise the price level more today than expected future ones, leading to lower expected inflation rate today. As a result, the effects of budget deficit on interest rates are upward biased when expected inflation enter as a separate variable in the regression.

In this paper, we will account for these two important factors as follows. First, by using both monthly and quarterly data in the estimations, we will be able to investigate the potential sensitivity of the results with respect to the data frequency. Second, by using the theoretical model to solve for the expected inflation rate as a function of macroeconomic

[7] See Becker (1995) for a deeper discussion of this problem.

[8] For a discussion of the reasons, see Evans (1987a) and the references therein.

variables (e.g. the budget deficit) and use these macro variables in the regression rather than proxy for the expected inflation rate, we will be able to pin down the "true" effects of budget deficits on interest rates.

3 The yield curve in a conventional small open economy model

In this section, I construct and use a conventional stochastic macro model to illustrate the effects of fiscal policy on the term structure of nominal interest rates in a small open economy. The model is a straightforward small open economy extension of the model presented by Turnovsky (1989). For ease of exposition, no dynamics are explicitly considered, but of course, in an empirical analysis of real world data, dynamics are important. Therefore, one can view the parameters in the theoretical model below as stationary polynomials in the lag operator.

3.1 The model

The aggregate supply function, where output, measured as a deviation around its natural rate, depends upon the unanticipated change in the domestic price level is given by:

$$y_t = \beta \left(p_t - \mathrm{E}_{t-1} p_t\right) + \varepsilon_t^{AS}, \tag{1}$$

where y_t denotes real output gap in natural logs in time period t, p_t the price of y in natural logs and $\mathrm{E}_{t-1}p_t$ the conditional expectation of the price level in t conditional on all available information in $t-1$. In (1), ε^{AS} is interpreted as an exogenous white noise productivity shock.

Aggregate demand in the model is described by the IS-LM equations. The IS curve is given by

$$\begin{aligned} y_t &= -\lambda_1 r_t^l + \lambda_2 g_t + \lambda_3 D_t + \lambda_4 \left(s_t + p_t^* - p_t\right) + \varepsilon_t^{IS} \\ &\equiv -\lambda_1 r_t^l + X_t + \lambda_4 \left(s_t - p_t\right) + \varepsilon_t^{IS} \end{aligned} \tag{2}$$

where r^l denotes the domestic long-term real interest rate in natural units, g real government spending in natural logs, D real government budget deficit in natural units, s the nominal spot exchange rate in natural logs, p^* the foreign price level in natural logs and $X \equiv \lambda_2 g + \lambda_3 D + \lambda_4 p^*$ is just a convenient notation. As in Turnovsky (1989), the relevant interest rate in (2) is taken to be the domestic long-term real interest rate. The IS curve also captures the conventional mechanism that government budget deficits add to private wealth, influencing desired consumption paths and thus output and interest rates for a given exchange rate and a given domestic price level.

Money market equilibrium is described by the LM curve

$$m_t - p_t = \alpha y_t - \gamma i_t^s + \varepsilon_t^{LM}, \tag{3}$$

where m denotes the nominal money supply in natural logs and i^s the domestic nominal short-term interest rate. Thus, as in Turnovsky (1989), the demand for money is assumed to depend on the domestic short-term nominal interest rate. In (2) and (3), ε^{IS} and ε^{LM} are interpreted as real demand and money demand shocks, respectively. It is assumed that the parameters in (1), (2) and (3), denoted α, β, γ, λ_1, λ_2, λ_3 and λ_4, are all positive, which is standard in conventional macro models.

The financial part of the model involves the relationships between the domestic and foreign short- and long-term real and nominal interest rates.[9]

The Fisher equations which relate domestic nominal and real interest rates are

$$i_t^s = r_t^s + (\mathrm{E}_t p_{t+1} - p_t) \tag{4}$$

and

$$i_t^l = r_t^l + \frac{1}{2}(\mathrm{E}_t p_{t+2} - p_t) \tag{5}$$

where r^s = domestic short-term real interest rate in natural units and i^l = domestic long-term nominal interest rate in natural units.

The equations which describe the real and nominal term structures of interest rates are given by

$$r_t^l = \frac{1}{2}\left(r_t^s + \mathrm{E}_t r_{t+1}^s\right) \tag{6}$$

and

$$i_t^l = \frac{1}{2}\left(i_t^s + \mathrm{E}_t i_{t+1}^s\right). \tag{7}$$

The uncovered interest parity, UIP, condition, which relates the domestic short- and long-term nominal interest rates to their foreign counterparts, denoted i^{s^*} and i^{l^*}, and the expected one and two period changes in the nominal exchange rate are

$$i_t^s - i_t^{s^*} = \Delta \mathrm{E}_t s_{t+1} \tag{8}$$

and

$$i_t^l - i_t^{l^*} = \frac{1}{2}(\mathrm{E}_t s_{t+2} - s_t). \tag{9}$$

[9] In accordance with Turnovsky (1989), it is assumed that there exist two types of domestic and foreign (zero coupon) assets with one and two periods to maturity. It is then straightforward to derive (6), (7), (8) and (9) up to a constant risk-premium as simple asset pricing relationships.

In order to close the model, we need to make some additional assumptions. First, in conventional macro models, g and D are normally considered to be exogenous. We will adopt this approach throughout the theoretical analysis in this paper. Second, p^*, i^{s^*} and i^{l^*} will also be treated as exogenous to the domestic economy, which is quite natural in a small open economy framework. Finally, we need to specify a policy rule for m. In theoretical analysis, it is standard to assume that m is independent of the other exogenous variables. Since the main interest in this paper is the interaction between fiscal policy and the term structure, we adopt the conventional view in the theoretical part of the paper. However, in an analysis of real world data, this strategy may lead to problems, since, for instance, the monetary policy rule is unlikely to be independent of g and D. Therefore, the first and third assumptions are relaxed in the empirical analysis in this paper.

3.2 Determination of nominal interest rates

To derive analytical solutions for the endogenous variables i^s and i^l in terms of current and expected future values of the exogenous variables g, D, m, p^*, $i^{s^*}$and i^{l^*}, we proceed by first determining price level expectations, and then substitute the resulting expressions back into the system to solve for s_t. Finally, the solution for s_t can then be used in the UIP conditions to get the solutions for the short- and long-term domestic nominal interest rate differentials $i_t^s - i_t^{s^*}$ and $i_t^l - i_t^{l^*}$.[10]

By this procedure, the short- and long-term interest rate differential depends both on the as of period t and $t-1$ expected discounted sum of nominal money supplies, government expenditures and deficits, foreign price level and short-term nominal interest rates and the as of t expected discounted sum of foreign long-term nominal interest rates. More formally, let $\psi_j^{i^s,D}$ and $\psi_j^{i^l,D}$ measure the effects of (as of period t, unknown in period $t-1$) expected budget deficits $j = 0, 1, 2, ...$ periods ahead, $\mathrm{E}_t D_{t+j}$, on the short- and long-term interest rate differentials respectively (analogous notation for the other variables g, p^*, m, i^{s^*} and i^{l^*} as well). Since the $\psi_j^{i^s}$ and $\psi_j^{i^l}$ coefficients are quite messy to evaluate analytically for $j > 1$, I have made simulations conditional on some reasonable values for α, β, γ, λ_1 and λ_4 in order to get a feeling for the size and magnitude of the paths for them.[11] For simplicity, it is assumed that $\lambda_2 = \lambda_3 = 1$, so that $\psi_j^{i^s,g} = \psi_j^{i^s,D}$ and $\psi_j^{i^l,g} = \psi_j^{i^l,D}$ for all j. The resulting paths for $j = 0, 1, 2, ..., 40$ are depicted in Figures 1 and 2 for $i_t^s - i_t^{s^*}$ and $i_t^l - i_t^{l^*}$ respectively. In Figures 1 and 2, the dashed lines refer to

[10] All the derivations of the equations informally presented and analyzed in this section are provided in Appendix A.

[11] The values for α and γ are taken from the empirical study by Goldfeld and Sichel (1990) and set to 0.6179 and 0.2170 respectively. λ_1 and β are taken from Söderlind (1997) and set to 5 and 500 respectively. λ_4 is taken from Hansson (1993) and set to 0.9644.

the $\psi_j^{i^s}$ and $\psi_j^{i^l}$ coefficients, while the solid lines refer to the accumulated effects in period $t+j$, measured as $\sum_{n=0}^{j}\psi_n^{i^s}$ and $\sum_{n=0}^{j}\psi_n^{i^l}$.

As can be seen from Figure 1, all the $\psi_j^{i^s}$ coefficients range from positive to negative values for $j=0,...,40$; for $j=0$, $\psi_0^{i^s,g}$, $\psi_0^{i^s,D}$ and $\psi_0^{i^s,p^*}$ are positive while $\psi_0^{i^s,m}$, $\psi_0^{i^s,s^*}$ and $\psi_0^{i^s,l^*}$ are negative, which can be demonstrated analytically. The coefficients for the foreign price level are very similar to those of government expenditures and budget deficit since the numerical value for λ_4 is close to 1. Turning to Figure 2, we find (as can also be shown analytically) that the coefficients now range from positive to negative and negative to positive values after the second period. It is interesting to note that the simulated paths for the budget deficit in Figures 1 and 2 compare well qualitatively with the closed economy results in Turnovsky (1989), in the sense that the $\psi_{t+j}^{i^s,D}$ and $\psi_{t+j}^{i^l,D}$ coefficients are both positive and negative, but poorly with Evans (1987a), where all the corresponding $\psi_j^{i^s,D}$ and $\psi_j^{i^l,D}$ coefficients are found to be greater than zero. It can be shown that this result is due to the introduction of a term structure within the model; see Turnovsky (1989) for a deeper discussion about the intuition.

From Figures 1 and 2, we also see that the accumulated effects of an as of t permanent change in g, D, p^* and i^{l^*} go towards zero when j increases. This result is due to the small open economy assumption; for example, after an as of t permanent change (unknown in period $t-1$) in the budget deficit, the nominal exchange rate today, s_t, and the as of t expected exchange rates in period $t+1$ and $t+2$, $\mathrm{E}_t s_{t+1}$ and $\mathrm{E}_t s_{t+2}$, change by the same amount. Via the UIP conditions (8) and (9), the effects on the short- and long-term interest rate differentials are then zero. This result is important since it suggests that the effects of budget deficits on interest rates in a small open economy framework are negligible if budget deficits can be characterized as (or close to) random walks.

Finally, the long-run accumulated effects of an as of t increase in money supply and the foreign short-term nominal interest rate are positive and exactly half as large on the long-term interest rate differential compared to the short-term interest differential. The intuition behind this result is that the as of $t-1$ expected price level in period t, $\mathrm{E}_{t-1}p_t$, is unaffected by an increase in as of t variables, so that s_t is unaffected in this respect by a permanent increase in m_t and $i_t^{s^*}$. But permanent increases in m_t and $i_t^{s^*}$ increase the as of t expected future price level in the periods $t+1$ and $t+2$, $\mathrm{E}_t p_{t+1}$ and $\mathrm{E}_t p_{t+2}$, in this respect and thereby also $\mathrm{E}_t s_{t+1}$ and $\mathrm{E}_t s_{t+2}$. Via the UIP conditions (8) and (9), we then get increases in $i_t^s - i_t^{s^*}$ and $i_t^l - i_t^{l^*}$.[12]

[12] However, this result is sensitive to the parameterization of the model. With the numerical assumptions about α, λ_1 and λ_4 here, it is the case that $\alpha(\lambda_1+2\lambda_4)-2>0$. But if $\alpha(\lambda_1+2\lambda_4)-2<0$, permanent increases in m_t and $i_t^{s^*}$ have negative accumulated effects on $i_t^s - i_t^{s^*}$ and $i_t^l - i_t^{l^*}$.

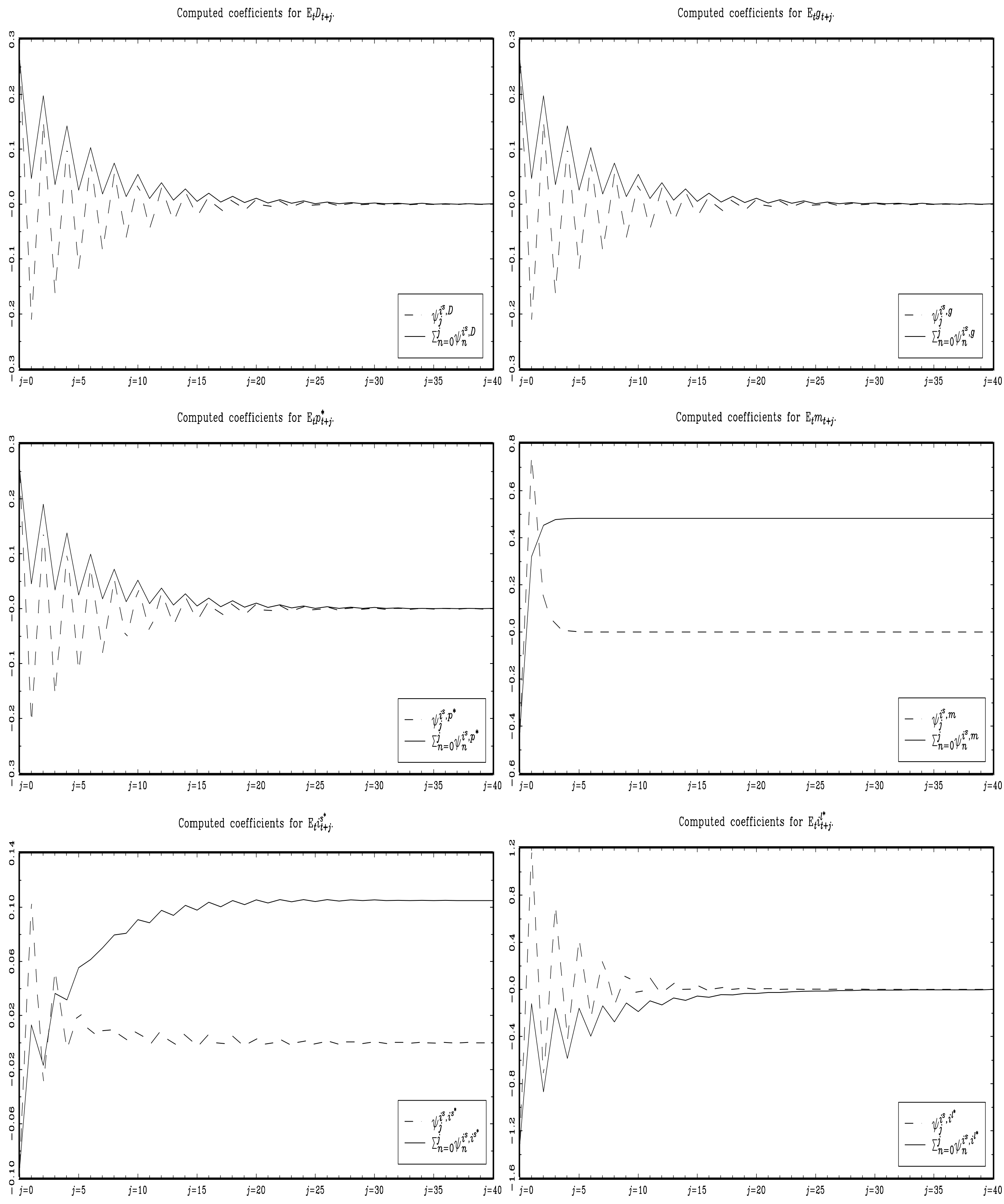

Figure 1: Effects on $i_t^s - i_t^{s^*}$ of exogenous variables j periods ahead.

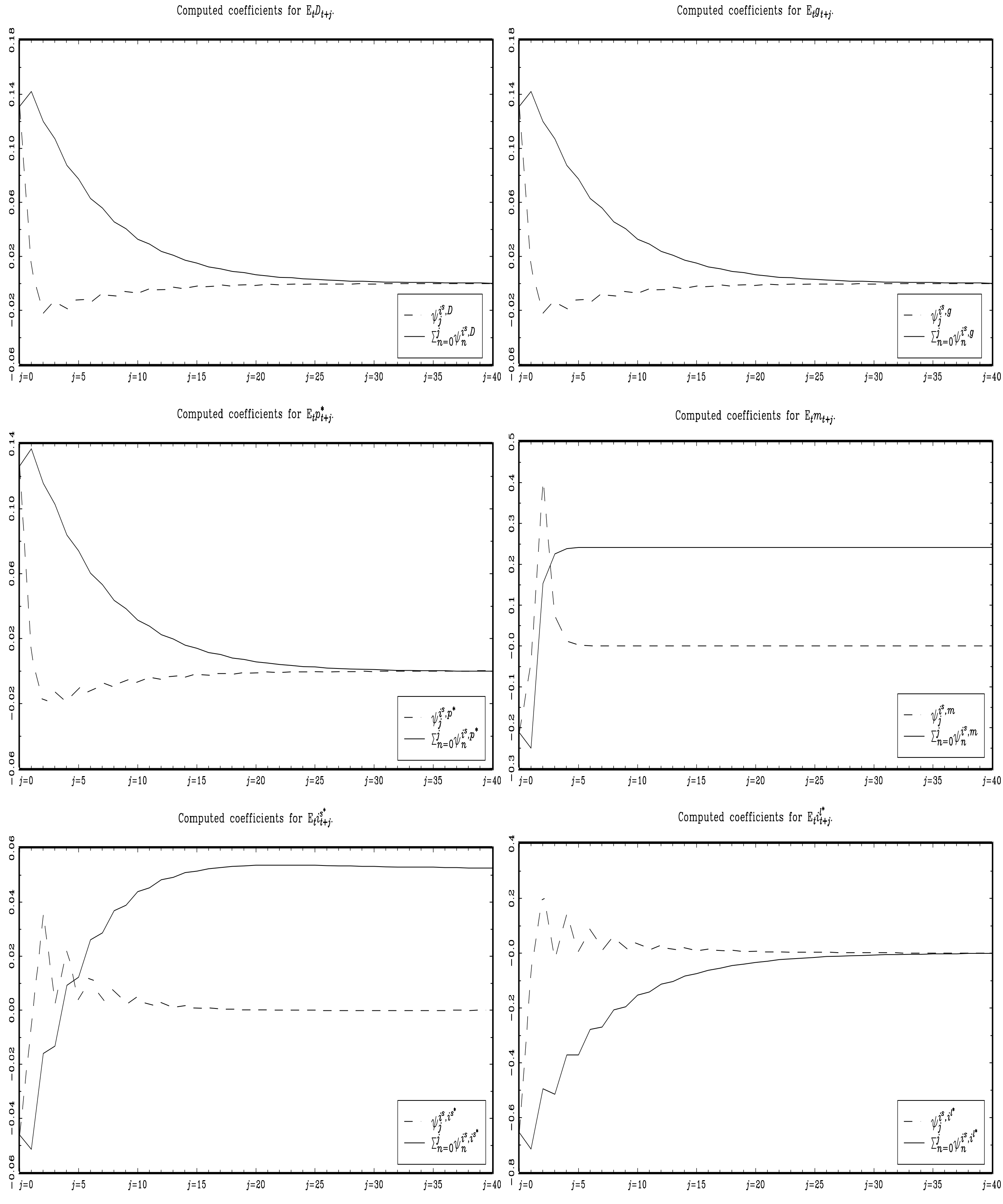

Figure 2: Effects on $i_t^l - i_t^{l^*}$ of exogenous variables j periods ahead.

3.3 Empirical implementation of the model

In order to generate empirical testable implications for the nominal short- and long-term interest rate differentials, we need to make some assumptions regarding the stochastic processes for the exogenous variables, and thus how about the expectations for these variables are formed.

Here, it is assumed that the exogenous vector of variables $\mathbf{z}_t^{\mathrm{T}} \equiv \left[p^* \; i^{s^*} \; i^{l^*} \; g \; D \; m\right]$ evolves according to a VAR(p) process

$$\mathbf{z}_{t+1} = \boldsymbol{\rho}^{\mathbf{z}}(L)\,\mathbf{z}_t + \boldsymbol{\varepsilon}^{\mathbf{z}}_{t+1}, \tag{10}$$

where $\boldsymbol{\rho}^{\mathbf{z}}(L) \equiv \sum_{i=0}^{p} \boldsymbol{\rho}^{\mathbf{z}}_i L^i$ and the errors in $\boldsymbol{\varepsilon}^{\mathbf{z}}$ are normally distributed and serially uncorrelated with $\mathrm{E}_t \boldsymbol{\varepsilon}^{\mathbf{z}}_{t+j} = \mathbf{0}$ for all $j > 0$ with a positive definite covariance matrix. This is a conventional assumption in empirical analysis, and for instance Evans (1987a) uses an assumption similar to (10). The specification in (10) relaxes the earlier assumption of independently distributed exogenous variables. For example, it allows for money supply to be determined by some policy function of the other exogenous variables.[13]

Using (10), it can be shown that the solutions for the interest rate differentials are of the following general form

$$i_t^r - i_t^{r^*} = \delta_0^r + \boldsymbol{\delta}^r(L)\mathbf{z}_t + \nu_t^r \tag{11}$$

where $\boldsymbol{\delta}^r(L) \equiv \left[\delta^{r,g}(L), \delta^{r,D}(L), \delta^{r,p^*}(L), \delta^{r,m}(L), \delta^{r,i^{s^*}}(L), \delta^{r,i^{l^*}}(L)\right]$ for $r = s,\ l$.[14] However, in this general case, nothing can be said about the sums of the individual parameters in the lag polynomials in $\boldsymbol{\delta}^r$; the sign and size of these sums will ultimately depend on the coefficients in $\boldsymbol{\rho}^{\mathbf{z}}(L)$, about which we know very little. In this sense, it is fair to say that it is essentially an empirical question whether larger government budget deficits lead to higher interest rates; that is, whether the sums of the coefficients in the lag polynomials $\delta^{s,D}(L)$ and $\delta^{l,D}(L)$, equal to $\sum \delta_i^{s,D} L^i$ and $\sum \delta_i^{s,D} L^i$ respectively, are positive or negative.

However, if we make the simplifying assumption that $\boldsymbol{\rho}^{\mathbf{z}}(L) \equiv \boldsymbol{\rho}^{\mathbf{z}}$, where $\boldsymbol{\rho}^{\mathbf{z}}$ is a diagonal matrix with the elements $\left[\rho^{p^*} \; \rho^{s^*} \; \rho^{l^*} \; \rho^g \; \rho^D \; \rho^m\right]$ in the diagonal, it is possible to draw further conclusions. In this case, the solution for the interest rate differentials is

$$\begin{aligned} i_t^r - i_t^{r^*} \;=\; & \delta_0^r + \delta_1^r(L)\,g_t + \delta_2^r(L)\,D_t + \delta_3^r(L)\,p_t^* - \delta_4^r(L)\,m_t - \delta_5^r(L) i_t^{s^*} - \\ & \delta_6^r(L) i_t^{l^*} + \delta_7^r(L)\,\Delta g_t + \delta_8^r(L)\,\Delta D_t + \delta_9^r(L)\,\Delta p_t^* + \delta_{10}^r(L)\,\Delta m_t + \\ & \delta_{11}^r(L)\Delta i_t^{s^*} + \nu_t^r. \end{aligned} \tag{12}$$

[13] Note also that from now on, dynamics are explicitly considered in the model. That is, we use the implicit assumption that all the parameters are stationary lag polynomials, i.e. $\alpha \equiv \alpha(L)$, $\beta \equiv \beta(L)$ and so forth.

[14] All the derivations of the equations presented in this section are provided in Appendix A.

for $r = s,\, l$. By introducing the notation

$$\begin{aligned}\boldsymbol{\delta}^r(L) &\equiv \left[\begin{array}{c}\delta_1^r(L)+\delta_7^r(L)(1-L),\, \delta_2^r(L)+\delta_8^r(L)(1-L),\, \delta_3^r(L)+\delta_9^r(L)(1-L),\\ -\delta_4^r(L)+\delta_{10}^r(L)(1-L),\; -\delta_5^r(L)+\delta_{11}^r(L)(1-L),\; -\delta_6^r(L)\end{array}\right] \\ &\equiv \left[\delta^{r,g}(L),\, \delta^{r,D}(L),\, \delta^{r,p^*}(L),\, \delta^{r,m}(L),\, \delta^{r,i^{s^*}}(L),\, \delta^{r,i^{l^*}}(L)\right]\end{aligned}$$

the solution can be written in the general form considered in (11). With these restrictive assumptions, the model has some nice implications. It is now the case that all the parameters in the lag polynomials $\delta_i^r(L)$ for $i = 1, ..., 6$ are positive provided that $\left\{\rho^g, \rho^D, \rho^{p^*}, \rho^m, \rho^{i^{s^*}}, \rho^{i^{l^*}}\right\} \in [0, 1)$. This implies that the sums of all the parameters in each of the polynomials $\delta^{r,g}(L)$, $\delta^{r,D}(L)$, $\delta^{r,p^*}(L)$, $\delta^{r,m}(L)$, $\delta^{r,i^{s^*}}(L)$ and $\delta^{r,i^{l^*}}(L)$ are also positive. However, except for $\delta^{r,i^{l^*}}(L)$, the same conclusion cannot be made for all the individual parameters in these polynomials; the sign of them can alternate over time. The reason is that changes in the exogenous variables have effects on the interest rate differentials via the lag polynomials $\delta_i^r(L)$ for $i = 7, ..., 11$, and that the signs of the parameters in these lag polynomials are ambiguous. Indeed, these theoretical predictions are different from those of Allen (1990, 1992) and Evans (1985, 1987a, 1987b), since their models did not imply these ambiguities for the individual parameters. The reason why these differences occur is that I have an aggregate supply function in the model, which makes it possible to explicitly solve for the price level expectations. The fact that the model considered includes a term structure of interest rates, and is designed for a small open economy, does not matter for this result.

In addition, if $\boldsymbol{\rho}^{\mathbf{z}} = \mathbf{I}_6$, then the variables g, D, p^* and i^{l^*} do not have any effects either on the short- or long-term interest rate differentials, and in this case, only changes in money supply and the foreign nominal short-term interest rate influence $i_t^s - i_t^{s^*}$ and $i_t^l - i_t^{l^*}$ via changes in the expected price level. The intuition behind this result is straightforward. Consider, for example, an increase in the budget deficit in period t. If ρ^D is equal to one, then the nominal exchange rate today, s_t, and the expected exchange rates in the periods $t+1$ and $t+2$, $\mathrm{E}_t s_{t+1}$ and $\mathrm{E}_t s_{t+2}$, will be fully adjusted downwards by the same amount (s_t appreciates), thus leaving $i_t^s - i_t^{s^*}$ and $i_t^l - i_t^{l^*}$ unaffected via the UIP conditions. But if ρ^D is less than one, then the nominal exchange rate s_t is still fully adjusted, while $\mathrm{E}_t s_{t+1}$ and $\mathrm{E}_t s_{t+2}$ are only partially adjusted downwards, thus increasing $i_t^s - i_t^{s^*}$ and $i_t^l - i_t^{l^*}$ via the UIP conditions since $\Delta \mathrm{E}_t s_{t+1}$ and $\mathrm{E}_t s_{t+2} - s_t$ become positive. Moreover, since it is plausible to assume that all the parameters ρ^g, ρ^{p^*}, ρ^m, $\rho^{i^{s^*}}$ and $\rho^{i^{l^*}}$ are equal or very close to one, we do not expect any of these variables to have any large level effects on the short- and long-term interest rate differentials. It also seems reasonable to assume that ρ^D is high, but slightly less than one, which implies that an increase in the budget deficit will increase $i_t^s - i_t^{s^*}$ and $i_t^l - i_t^{l^*}$ by a relatively small amount. Thus, if ρ^D is sufficiently close to one, the effect of changes in the budget deficit on $i_t^s - i_t^{s^*}$ and $i_t^l - i_t^{l^*}$ will be almost zero, independently of the monetary policy rule. However, it should be emphasized that these last results are due to the small open economy feature of the model.

The most striking implication of the derivations above is that a simple conventional macroeconomic model may offer a possible explanation for the lack of empirical relationship between government budget deficits and interest rates. That is, when empirical analyses based on (11) are carried out, one might readily obtain "wrong" results, because, as argued above, even in the simplest case when the exogenous variables are assumed to follow univariate autoregressive processes, it may very well be the case that the sum of the elements in the lag polynomials for D_t is very close to zero.

(11) provides the framework for the empirical investigation that follows below, and it should therefore be noted that the exogenous shocks ν_t^r are very likely to be serially correlated over time.[15]

4 Data

Since the Swedish financial markets were heavily regulated until the beginning of the 1980s, it is hard to acquire good interest rate data for long samples for Sweden. In this paper, a three-month government Treasury bill and a five- to ten-year government Treasury bond are used as measures of i^s and i^l (both expressed as effective yields), and data of good quality on these two series are only available from January 1982 and the middle of February 1984 respectively.[16] Accordingly, the data frequencies which can be exploited in the analysis must be rather high, in order to get a sufficient number of observations. Monthly frequency until June 1996 then gives 174 observations, while quarterly frequency gives at the most only 58. This means that the monthly frequency is desirable, and almost every data series that is needed is also available on monthly frequency. Unfortunately the highest frequencies for g and y are quarterly.[17] Hence, in order to be able to use monthly data in the regressions, some kind of interpolation for these two variables is necessary. In this paper, it is assumed that: (i) the quarterly values of g are uniformly distributed over the months within each quarter; (ii) the monthly distribution of y within the quarters follows the private industrial production, denoted x,

[15] Since I consider the parameters to be stationary lag polynomials, i.e. $\alpha \equiv \alpha(L)$, $\beta \equiv \beta(L)$ and so forth, the model implies that the error terms in (11), ν_t^s and ν_t^l, are moving average (MA) terms.

[16] To get an indication of the robustness with respect to the choice of maturity for the short-term Treasury bill for the empirical investigation, other Treasury bills with one, six and twelve months to maturity have been examined, and since the results were unaffected they are not reported.

[17] From now on, y denotes the gross domestic product, GDP, and not the log of the output gap.

for which data are available on a monthly basis, according to the scheme $y_{m,t} = \kappa_t x_{m,t}$ where $\kappa_t \equiv \frac{y_{q,j(t)}}{x_{q,j(t)}}$ and $j(t) = 1$ for all $t = 1, 2, 3$, $j(t) = 2$ for all $t = 4, 5, 6$ etc.[18] Therefore, in order to get a feeling for the validity of the interpolation, both monthly and quarterly data are used in this paper. Another justification for using both quarterly and monthly data is that the survey of earlier empirical literature suggested that a different choice of data frequency has been important for different empirical evidence. By using both frequencies here, we take this aspect into account. There are two principal reasons for the need to use y. First, we want to detrend the data series and only consider the business cycle component of the variables g, D and m. A very natural way to accomplish this is to divide the relevant variables by y. Second, the presumption is made that the Swedish economy has the property of homogeneity; that is, doubling government consumption and deficits and nominal money supply and the size of the economy leaves the interest rate differentials unaffected. Evans (1987a) uses the same approach.

Since Sweden had a fixed exchange rate regime between 1982 and November 1992, and thus for the greater part of the sample period, "currency-basket" weighted foreign short- and long-term interest rates (both expressed as effective yields) and price levels have been constructed to obtain measures of i^{s^*}, i^{l^*} and p^* during the whole sample period. When there has been no possibility of acquiring interest rate data for certain countries during limited periods in the sample, the "currency-basket" weights have been normalized to one.[19] Moreover, since the foreign long-term interest rates were only available as monthly averages, averages have been utilized for the other interest rates as well.[20] The calculated series for i^{s^*}, i^{l^*}, $i^s - i^{s^*}$ and $i^l - i^{l^*}$, and g, D, m and p^* are depicted in Figures 3 and 4 respectively.

Summary statistics for quarterly and monthly data are given in Tables 1 and 2 respectively.[21] In general, Tables 1 and 2 show that the sample autocorrelations are very high and taper off very slowly over time, with the possible exception of i^s, i^{s^*}, i^l, i^{l^*}, $i^s - i^{s^*}$, $i^l - i^{l^*}$ and D. This pattern is normally an indication that the variables may

[18] To test the sensitivity of these assumptions for the analysis, an alternative method suggested by Litterman (1983) with better properties from a statistical viewpoint has been tested to generate g and y on monthly frequencies, but since the qualitative conclusions were unaffected, they are not reported.

[19] This has not been a significant problem though; countries which together make up at least 67.10 percent in the beginning and up to 100 percent at the end of the sample period are included in the calculation of the foreign long-term interest rate. The corresponding figures for the short-term interest rate are 100 percent until November 1992, and thereafter between 79.9 to 97.54 percent.

[20] Note that the macroeconomic variables g, D, m and p^* have been subjected to seasonal adjustment. Since there seemed to be tendencies of changing seasonal pattern in most of the data series, the X11-method was used to deseasonalize the data. However, since the large and changing monthly seasonal variation in the private industrial production (x_m), used to generate a measure of monthly GDP (y_m), could not be sufficiently well deseasonalized with the X11-method, I used deseasonalized raw data on y_q and x_m to calculate y_m.

[21] Exact definitions and sources of all the variables used are given in Appendix B.

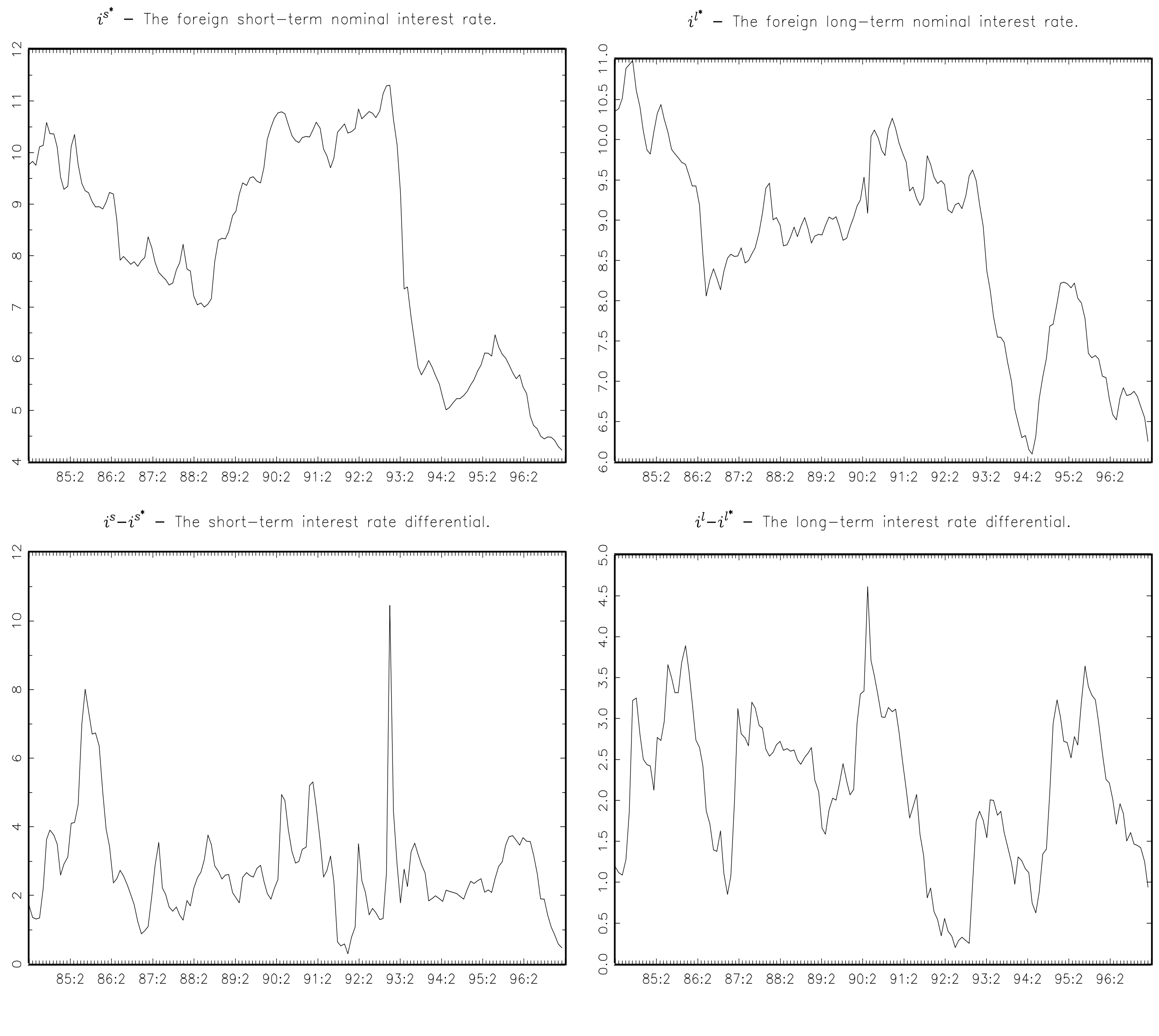

Figure 3: Monthly data on nominal interest rates.

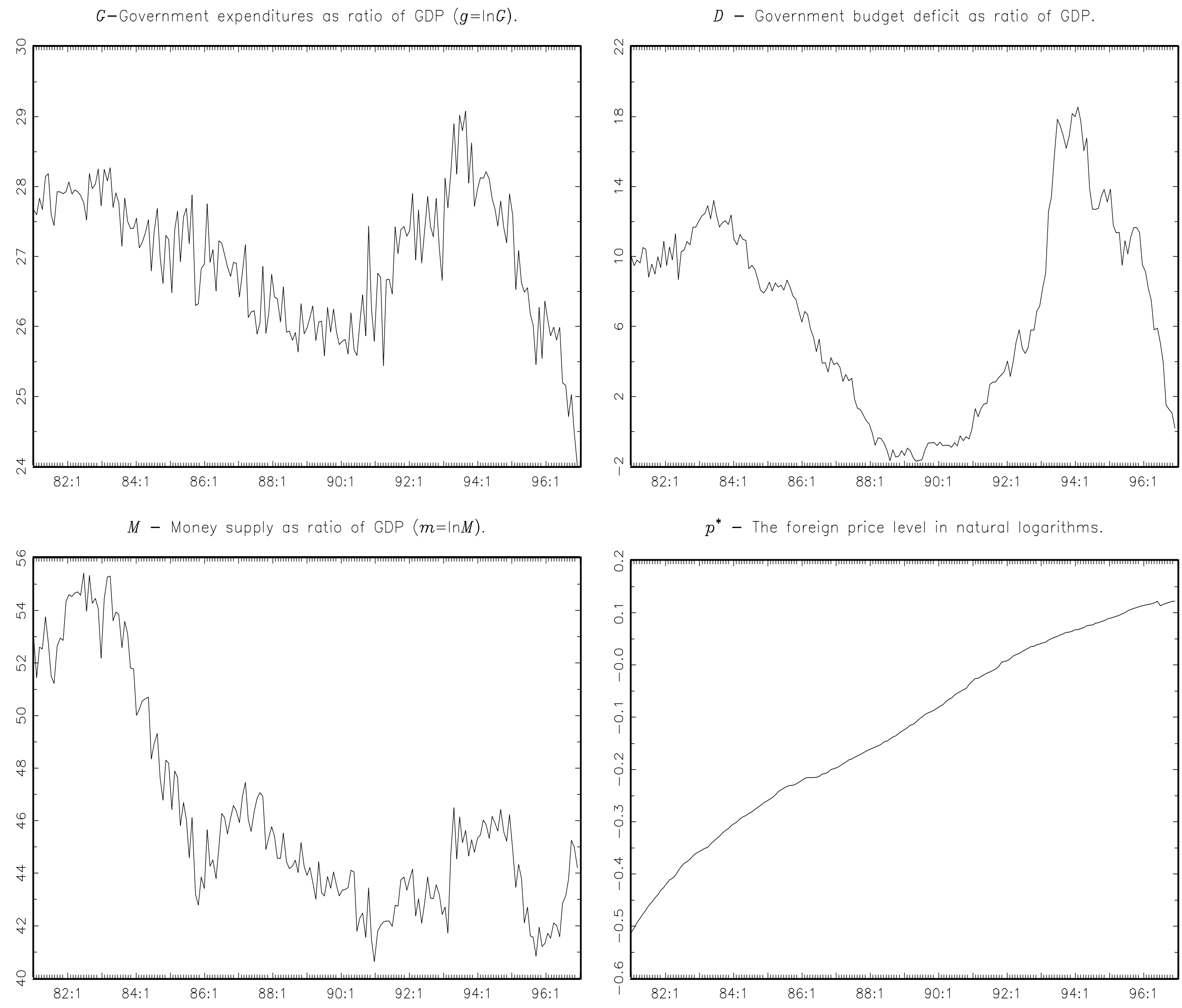

Figure 4: Monthly data on seasonally adjusted macrovariables.

Table 1: Summary statistics for quarterly data.

			Sample autocorrelations					
Variable	Mean	Std.dev.	$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$	$\hat{\rho}_4$	$\hat{\rho}_8$	$\hat{\rho}_{12}$
i^s	10.92	2.68	0.82	0.64	0.50	0.33	0.12	-0.12
i^{s^*}	8.60	2.36	0.89	0.73	0.61	0.49	0.22	-0.06
$i^s - i^{s^*}$	2.65	1.29	0.56	0.23	0.03	-0.23	-0.15	0.03
i^l	10.80	1.71	0.84	0.62	0.39	0.21	0.20	0.14
i^{l^*}	9.13	1.69	0.94	0.85	0.76	0.65	0.40	0.28
$i^l - i^{l^*}$	2.13	0.91	0.75	0.43	0.18	-0.07	0.03	-0.09
g	3.26	0.05	0.94	0.89	0.85	0.80	0.58	0.37
D	5.53	4.55	0.95	0.88	0.81	0.71	0.35	-0.05
m	3.85	0.12	0.97	0.94	0.91	0.88	0.75	0.65
p^*	-0.15	0.20	0.95	0.90	0.85	0.80	0.61	0.45

Note: g, D, m and p^* have been subjected to seasonal adjustment. g, m and p^* are in natural logs.

be non-stationary. They reveal that the premium has on average been higher on $i^s - i^{s^*}$ compared to $i^l - i^{l^*}$. The variance is also higher for the bills, both Swedish and foreign, than the bonds. Among the macroeconomic variables, the variance in D is much higher than the variability in g and m. This is a remarkable fact, since Plosser (1987) reports variabilities in g and m which exceed the variability in D by large amounts for the United States. For instance, Plosser (1987) reports that the ratios between the standard deviation in g to D and in m to D are 7.3 and 2.1 on the monthly frequency. In this data set, the corresponding figures are 0.35 and 0.81. Of course, the high volatility in D reflects the dramatic swings in the Swedish budget deficits, which can be seen in Figure 4.

Table 2: Summary statistics for monthly data.

			Sample autocorrelations						
Variable	Mean	Std.dev.	$\hat{\rho}_1$	$\hat{\rho}_6$	$\hat{\rho}_{12}$	$\hat{\rho}_{18}$	$\hat{\rho}_{24}$	$\hat{\rho}_{30}$	$\hat{\rho}_{36}$
i^s	10.92	2.76	0.91	0.59	0.30	0.18	0.12	0.04	-0.11
i^{s^*}	8.60	2.36	0.97	0.73	0.48	0.32	0.22	0.11	-0.06
$i^s - i^{s^*}$	2.65	1.46	0.76	0.16	-0.20	-0.24	-0.10	0.05	0.02
i^l	10.81	1.71	0.96	0.59	0.18	0.08	0.20	0.30	0.11
i^{l^*}	9.35	1.67	0.98	0.84	0.64	0.48	0.40	0.31	0.18
$i^l - i^{l^*}$	2.15	0.93	0.92	0.41	-0.08	-0.15	-0.01	0.14	-0.10
g	3.27	0.06	0.80	0.77	0.71	0.59	0.47	0.42	0.32
D	5.63	4.74	0.98	0.89	0.72	0.56	0.37	0.17	-0.02
m	3.91	0.12	0.95	0.91	0.85	0.78	0.74	0.71	0.67
p^*	-0.15	0.20	0.98	0.90	0.80	0.70	0.61	0.53	0.45

Note: g, D, m and p^* have been subjected to seasonal adjustment. g, m and p^* are in natural logs.

To summarize, the data set utilized in this paper implies that the regressions for the short- and long-term interest rate differentials on the government budget deficit will have high power compared to previous studies.

5 Estimation and empirical results

This section deals with the problem of how to estimate (11) in an appropriate way and then reports the results of the regressions.

5.1 Non-stationarity

As already noted in section 4, one striking feature of the sample autocorrelations is that they start at very high values and then taper off very gradually, possible exceptions being $i^s - i^{s^*}$, $i^l - i^{l^*}$ and D. This pattern is generally an indication that the time series are non-stationary. Banerjee et al. (1993) discuss the properties of the regression estimates obtained when some of the variables are integrated, and find that what is often called balance in the regression is an important property. This means that when the dependent variable is stationary, the explanatory variables should also be integrated of order zero or cointegrated.[22] Consequently, there is a need to test the integration order of the variables involved in the regressions.

To test the integration order of the individual series, I have used the augmented Dickey-Fuller (ADF) procedure and applied the practical guidelines proposed in Hamilton (1994), which means that a constant and/or linear trend is included in the regression if the variable displays a non-zero mean and/or sign of linear trend in the observed sample (see Figures 3 and 4). For which variables a constant and/or trend is included in the regressions are reported in Tables 3 and 4 below. In the ADF procedure, H_0 is the hypothesis that the series under consideration is non-stationary, which in practice implies an estimated $\hat{\phi}$ in Tables 3 and 4 not significantly lower than zero. The ADF test results for the variables in level form are reported in Table 3.

As seen from Table 3, the null hypothesis that the variables are non-stationary can only be rejected for $i^s - i^{s^*}$, $i^l - i^{l^*}$ and D on reasonable significance levels. Although we can reject the hypothesis that D is non-stationary, the estimated autoregressive coefficients are close to one, implying that our empirical estimate of ρ^D is also close to one. These findings are consistent with what we expected a priori, and discussed in Section 3.2. Thus, according to the theoretical model (11), the effects of government budget deficits on the nominal interest rate differentials are likely to be relatively small.

In order to determine the integration order for the other variables, we proceed to the $I(1)$ tests. With quarterly and monthly frequencies, both first differences on seasonally

[22] Note: If a variable needs to be differentiated exactly k times to achieve stationarity, then the variable is $I(k)$, integrated of order k. It follows that a stationary variable is $I(0)$. If for a particular variable $k > 0$, where k is a positive integer, then it is said to be non-stationary.

Table 3: Augmented Dickey-Fuller tests of integration order on levels.

	Quarterly frequency					Monthly frequency				
Variable	const/ trend	T	p	$\hat{\phi}$	t-value	const/ trend	T	p	$\hat{\phi}$	t-value
i^s	yes/yes	51	4	-0.105	-1.465	yes/yes	158	9	-0.060	-1.581
i^{s^*}	yes/yes	56	3	-0.056	-1.116	yes/yes	166	13	-0.012	-1.063
$i^s - i^{s^*}$	yes/no	48	7	-0.794	-3.218**	yes/no	157	10	-0.306	-3.588***
i^l	yes/yes	49	1	-0.175	-2.508	yes/yes	147	6	-0.039	-1.965
i^{l^*}	yes/yes	60	5	-0.050	-1.574	yes/yes	185	13	-0.016	-1.554
$i^l - i^{l^*}$	yes/no	47	3	-0.375	-3.475**	yes/no	143	10	-0.120	-3.173**
g	yes/yes	98	7	-0.067	-2.176	yes/yes	293	24	-0.065	-2.210
D	yes/no	93	8	-0.097	-3.208**	yes/no	297	20	-0.032	-3.142***
p^*	yes/yes	62	4	-0.087	-1.149	yes/yes	196	5	-0.010	-2.342
m	yes/yes	92	13	-0.127	-1.827	yes/yes	294	23	-0.110	-2.539

Note: g, D, p^* and m have been subjected to seasonal adjustment as described in Appendix B. T is the number of observations included in the test. $*\,(**)\,[***]$ indicates that H_0: $Z \sim I(k)$ where $k > 0$ is rejected at the 10 (5) [1] percent significance level. McKinnon (1991) critical values are used.

adjusted data or annual changes on seasonal unadjusted data can be utilized in the tests. One of the aims of using annual changes is to eliminate most of the seasonal variability prior to estimation. In addition, the series obtained are often easier to interpret than first difference series, where the seasonal variability often completely swamps the remaining variability.[23] Thus, for i^{s^*} and i^{l^*}, the tests are based on first differences, and for g, p^* and m on annual changes of seasonally unadjusted data.[24]

The overall impression from Table 4 is that the null hypothesis is firmly rejected, and together with the test results in Table 3, it is concluded that i^{s^*}, i^{l^*}, g, p^* and m are non-stationary and integrated of order one. It is a relief to note the resemblance of the results for the different frequencies.

Table 4: Augmented Dickey-Fuller tests of integration order on differences.

	Quarterly frequency					Monthly frequency				
Variable	const/ trend	T	p	$\hat{\phi}$	t-value	const/ trend	T	p	$\hat{\phi}$	t-value
Δi^{s^*}	yes/no	58	0	-0.60	-4.938***	yes/no	166	12	-0.55	-3.649***
Δi^{l^*}	yes/no	60	4	-1.19	-6.138***	yes/no	184	13	-0.81	-4.770***
Δg	yes/no	100	1	-0.26	-3.099***	yes/no	283	22	-0.24	-1.952**
Δp^*	yes/no	59	4	-0.07	-2.964**	yes/no	177	12	-0.03	-3.360***
Δm	yes/no	90	11	-0.43	-3.445**	yes/no	283	22	-0.41	-3.865***

Note: The tests are performed on first differences for i^{s^*} and i^{l^*}, and on annual changes on seasonally unadjusted data for g, p^* and m. T is the number of observations included in the test. $*\,(**)\,[***]$ indicates that H_0: $Z \sim I(k)$ where $k > 1$ is rejected at the 10 (5) [1] percent significance level. McKinnon (1991) critical values are used.

[23] Furthermore, note that $1 - L^4 = (1 - L)\left(1 + L + L^2 + L^3\right)$, which shows that an analysis based on annual changes can be regarded as an analysis based on first differences on seasonally adjusted data.

[24] The $I(1)$ tests are not executed for i^s and i^l, since these variables are not individually involved in the regressions.

The ADF tests above have shown that the dependent variables involved in the regression (11) are stationary, but that every explanatory variable except D is non-stationary. Consequently, we have the undesirable unbalanced regression case, where some variables involved are stationary and some non-stationary. Therefore, all the non-stationary variables in (11) are rewritten in difference form (i^{s^*} and i^{l^*} in first differences; annual changes for g, p^* and m), whereas the stationary variables ($i^s - i^{s^*}$, $i^l - i^{l^*}$ and D) are in levels in the regression analysis to get balanced regressions.[25]

5.2 Econometric Issues

The estimated regression equations for the short- and long-term interest rate differentials include a lag polynomial in the stationary variable D, and lag polynomials in the stationary differences for the $I(1)$ variables. Since the coefficient sums on the regressors in (11) have appropriate probability limits only if enough lagged values are included in the regressions, one should not be too parsimonious. On the other hand, the more extraneous regressors included, the less power there is to test hypotheses. These two competing considerations have been balanced by including lagged values up to 3 years. The insignificant lagged values of each variable were then removed so that the most important dynamics were captured in the final estimated equations.[26] To give an indication of the estimated model's goodness of fit, the adjusted sample coefficients of determination, $\bar{R}^2$, are provided.

Before turning to the estimation results presented in Tables 6 and 7, a comment on the method used in the estimations is in order. First, estimation with OLS is based on the assumptions that the error term is uncorrelated with the regressors and that the regressors are weakly exogenous with respect to the dependent variables. However, it is easy to argue that aggregate money, demand and supply shocks, contained in the residual ν_t^r, also have contemporaneous effects on the regressors. For example, consider a positive aggregate demand shock. Nominal interest rates and output rise simultaneously. The increased

[25] An alternative approach would be to estimate a vector error correction model (VECM) for the whole system $\left[p^* \; i^{s^*} \; i^{l^*} \; g \; D \; m \; i^s \; i^l\right]$ with the Johansen method; see Johansen (1988) and Johansen and Juselius (1990). In this study, however, I have chosen to use the restrictions from the conventional model directly to enable comparability with previous studies (e.g. Correia-Nunes and Stemitsiotis, 1995 and Evans, 1985, 1987a and 1987b). Moreover, as was noted in Section 3.3, if the variables p^*, i^{s^*}, i^{l^*}, g and m follow unit root processes (i.e., the null that $\rho^{p^*}(L)$, $\rho^{i^{s^*}}(L)$, $\rho^{i^{l^*}}(L)$, $\rho^{g}(L)$ and $\rho^{m}(L)$ equal 1 cannot be rejected) , the conventional model suggested that these variables should not have any long-run influence on $i^s - i^{s^*}$ and $i^l - i^{l^*}$. This suggests that the information loss of not using a VECM may not be severe. Finally, we have only about 50 quarterly observations, which is not too much data when estimating a VECM (see Gredenhoff and Jacobson, 1999).

[26] In the estimations, I use Almon lags with no end point restrictions and allow for a third degree polynomial. The lag in effect may, by this procedure, be distributed as a straight line, a parabola or an "s-curve".

output reduces some components of government spending, increases tax revenue and thus lessens the budget deficit. Furthermore, the monetary authorities may accommodate some of the increased money demand that the higher spending induces. As a result, $i^s - i^{s^*}$ and $i^l - i^{l^*}$ rise while g and D are falling and m is rising endogenously. As a consequence of this, the OLS estimates of the coefficient sums in (11) are very likely to be inconsistent. The inconsistencies can be serious and of either sign, depending on how important each source of endogeneity is. In this paper, I have overcome this problem in three ways. First, for the sample period considered, I think it is reasonably fair to say that exogenous influences have been important for g and D. Second, relatively high frequency data have been used so that the endogeneity effects from the shocks in the residual on the regressors are likely to be relatively small. Moreover, the regressions have been estimated with the Two-Stage Least Squares (2SLS) method with correction for serial correlation suggested by Fair (1970). Fair shows that consistent estimates can be obtained when the residuals are serially correlated, if lagged values of the regressors and the dependent variable are used as instruments and the estimated residual is explicitly modelled as an ARMA(p, q) process. The reason not to use 2SLS without serial correction is, as discussed in Section 3.3, that we expect the residuals to be serially correlated. Therefore, an augmented ARMA(p, q)-process $\nu_t^r = \rho_1^{\nu^r} \nu_{t-1}^r + ... + \rho_p^{\nu^r} \nu_{t-p}^r + \varepsilon_t^{\nu^r} + \theta_1^{\nu^r} \varepsilon_{t-1}^{\nu^r} + ... + \theta_q^{\nu^r} \varepsilon_{t-q}^{\nu^r}$ was included in the 2SLS estimations of (11), until the Ljung-Box (LB) statistic indicated absence of serial correlation in the residuals.[27]

Finally, since I have a limited number of observations in the regressions, I have simulated the critical values reported in Tables 6 and 7 below to get the correct small sample significance levels. In the simulations, I first estimated and then simulated (10) on quarterly and monthly data to get a sample of the same size as used in the regressions reported in Tables 5 and 6 for the independent variables, then which were used to generate $i^s - i^{s^*}$ and $i^l - i^{l^*}$. I then used the simulated dependent and independent variables to estimate the regressions in the Tables 5 and 6. To get small sample distributions for the coefficient sums, I repeated this procedure until the simulated distributions converged in mean and variance.[28] To get a feeling for the importance of the small sample significance levels, the asymptotic t-statistics are also provided in parentheses.

[27] Another estimation method which produces consistent estimates of the coefficient sums in (11) is the so called Two-Step Two-Stage Least Squares (2S2SLS) method proposed by Cumby et al. (1983). However, when I tested 2S2SLS and the 2SLS method with correction for serial correction, I found that the results were very similar. But since the latter method was much simpler to implement, due to the fact that coefficient sums were estimated with Almon lags, it was used in the final regressions.

[28] In practice, it took approximately 1000 repetitions on both the monthly and quarterly frequency for the simulated distributions to converge according to the mean-variance criteria.

5.3 Results

Table 5 reports that the coefficient sums for D are indeed positive and strongly statistically significant on the quarterly frequency. The estimated coefficient sums are 0.20 and 0.25, suggesting that a one percentage unit increase in the government budget deficit as a ratio of GDP leads to an increase in the short- and long-term nominal interest rate differentials by 0.20 and 0.25 percentage points respectively after two years' time. These figures are close to point estimates reported by Correia-Nunes and Stemitsiotis (1995) for Japan (0.21), Germany (0.22) and Ireland (0.22), but are lower than their estimate for the United States (0.79) using yearly data.

Table 5: Quarterly 2SLS with correction for serial correlation regressions.

	$i^s - i^{s^*}$		$i^l - i^{l^*}$	
	Coefficient sum	Lag length	Coefficient sum	Lag length
Δg	$-\ 0.270^{***}$ (-2.59)	0	$-\ 0.103^{**}$ (-1.76)	0
Δp^*	1.973^{***} (4.53)	0	0.908^{***} (5.44)	10
Δm	$-\ 0.076$ (-0.76)	12	0.048^{**} (1.63)	0
Δi^{s^*}	$-\ 1.594$ (-1.15)	12	2.456^{***} (3.20)	9
Δi^{l^*}	$-\ 0.515^{***}$ (-1.95)	0	$-\ 10.945^{***}$ (-6.12)	10
D	0.200^{**} (3.07)	8	0.249^{***} (7.09)	8
c	$-\ 5.813^{***}$ (-3.39)		$-\ 2.253^{**}$ (-3.27)	
Dummy	2.679^{***} (4.58)			
p, q	1,4		0,0	
$\bar{R}^2$	0.77		0.88	

Note: Simulated critical limits. c denotes the constant term and *Dummy* is a dummy variable equal to 1 1992:3 - 1992:4 and 0 otherwise. $*$ $(**)$ $[***]$ indicates that the coefficient is statistically significant at the 10 (5) [1] percent level according to the simulated distribution. Asymptotic t-statistics within parentheses. The samples consist of 44 and 47 observations, respectively. Lagged dependent and explanatory variables have been used as instruments. p and q denote the order of the ARMA(p, q) process for the residual in the estimations.

Among the other regressors, the short-run dynamics for g and p^* are most important, although their estimated parameters have opposite signs. A dummy variable has also been included in the regression for the short-term interest rate differential to capture the effects of the interventions of Sveriges Riksbank (Bank of Sweden) on the market for short-term bills in September to November 1992. This "intervention effect" is easily seen in Figure 3.

On the monthly frequency, as seen from Table 6, the estimated coefficient sums for D are still positive and highly significant, although they are lower than in the quarterly regressions. This can be taken as an indication of that lower (quarterly or yearly) data frequencies are more supportive for the conventional view than higher (monthly), as noted in the survey of previous studies (see Section 2). But here, due to the large sample variability for the budget deficit, we were able to identify positive effects of the budget deficit on interest rates also on monthly data.

Table 6: Monthly 2SLS with correction for serial correlation regressions.

	$i^s - i^{s^*}$		$i^l - i^{l^*}$	
	Coefficient sum	Lag length	Coefficient sum	Lag length
Δg	0.114*** (3.20)	0	0.007* (0.80)	0
Δp^*	0.160 (0.85)	11	0.090 (0.57)	24
Δm	− 0.195*** (−4.18)	4	− 0.064 (−1.25)	12
Δi^{s^*}	8.410*** (3.84)	24	5.491*** (3.17)	24
Δi^{l^*}	− 10.127*** (−3.23)	16	− 12.611*** (−3.55)	24
D	0.142*** (3.39)	24	0.113*** (3.13)	24
c	1.347 (1.74)		1.280 (1.97)	
Dummy	3.667*** (7.27)			
p, q	0,1		0,3	
$\bar{R}^2$	0.76		0.88	

Note: Simulated critical limits. c denotes the constant term and *Dummy* is a dummy variable equal to 1 1992:09 and 1992:11, 0 otherwise. $*$ ($**$) [$* * *$] indicates that the coefficient is statistically significant at the 10 (5) [1] percent level according to the simulated distribution. Asymptotic t-statistics within parentheses. The samples consist of 148 in both regressions. Lagged dependent and explanatory variables have been used as instruments. p and q denote the order of the ARMA(p, q) process for the residual in the estimations.

Comparison of the Tables 5 and 6 also reveals that the lag length effect of D is the same in both the quarterly and monthly regressions. For the other variables, the most pronounced difference is that the estimated coefficient for g is positive, in contrast to the quarterly regressions. This may be an indication that our interpolation measure of G is flawed on the monthly frequency. Unlike the quarterly regressions, the estimated coefficient sums for m are now negative/positive and statistically significant/insignificant for $i^s - i^{s^*}/i^l - i^{l^*}$. We also see that i^{s^*} and i^{l^*} now become highly significant in the regression for $i^s - i^{s^*}$, while the short-run dynamics for p^* still are positive but not statistically significant. Finally, the estimated parameter for the dummy variable is higher, since the variable can be defined in a more appropriate way with monthly data.

What is then the general impression of the estimation results of the conventional macro model reported in Tables 5 and 6? First, we notice that the model in (11) did not suggest sign uniqueness of the coefficient sums estimated above. Consequently, one cannot reject the conventional macro model either on the basis that the estimated coefficient sums for the variables were not statistically significant or because they have the "wrong" sign. Rather, I would like to argue that the goodness-of-fit criterion for the model should be used to evaluate the model as a whole. The $\bar{R}^2$ values reported in Tables 5 and 6 are high, but we need a comparison with an alternative model, in order to get a measure of the model's within-sample forecasting accuracy. Here, I followed the approach in Meese and Rogoff (1983), and used the $\bar{R}^2$ values generated by a random walk with a drift to form a basis for a comparison. The corresponding $\bar{R}^2$ values for the short- and long-term interest differentials on quarterly and monthly frequency were {0.17, 0.58} and {0.50, 0.86} respectively. In all cases they are lower than the ones in Tables 5 and 6. Thus, it is tempting to argue that the empirical evidence presented here also supports the conventional model in general; at least, the empirical results are not obviously inconsistent with the predictions of the conventional model.

In accordance with many other countries, Sweden went from a fixed to a managed floating exchange rate regime in November 1992, and in January 1993, the Swedish central bank announced the new inflation targeting/floating exchange rate regime. In this paper, I have used data from both the fixed and floating regimes to get sufficiently many observations in the regressions. Therefore, it is desirable to test whether the structures of the regressions reported in Tables 5 and 6 are the same after the regime shift. The standard test available for this purpose is the Chow test, the basic idea of which can be described as comparing the results of separate estimation in the two subperiods, fixed and floating regime periods, and on the basis of the complete period; in the latter case assuming that the structure of the model is unchanged.[29]

Table 7: Chow test for structural stability.

	Quarterly regressions		Monthly regressions	
Test statistic	$i^s - i^{s^*}$	$i^l - i^{l^*}$	$i^s - i^{s^*}$	$i^l - i^{l^*}$
F^{obs}	1.511	1.380	0.892	1.196
p-value	0.241	0.273	0.652	0.244

Note: n_{fl} is equal to 13 ($1993Q2 - 1996Q2$) on quarterly data and 41 (1993:2 – 1996:6) on monthly data, while n_{fi} is equal to 31 ($1985Q3 - 1993Q1$) and 34 ($1984Q4 - 1993Q1$) on quarterly data and 107 (1984:03 – 1993:1) on monthly data for $i^s - i^{s^*}$ and $i^l - i^{l^*}$ respectively.

As can be seen from Table 7, we can in no case reject the null hypothesis of an unchanged structure at reasonable significance levels. It therefore seems as if the results

[29] Since the original Chow-test is impossible to use in our case due to the short floating exchange rate regime period, I have used a modification of the test, sometimes called the Chow forecast test. If the model structure is unchanged, the statistic $F^{obs} = \frac{\hat{\sigma}_T^2 + (\hat{\sigma}_T^2 - \hat{\sigma}_{fi}^2)\frac{(n_{fi}-k)}{n_{fl}}}{\hat{\sigma}_{fi}^2}$ follows the F-distribution with n_{fl} and $n_{fi} - k$ degrees of freedom, where n_{fi} = number of observations in the fixed exchange rate regime period, n_{fl} = number of observations in the managed floating exchange rate regime period, k = number of estimated parameters, $\hat{\sigma}_T^2$ = estimated residual variance in the complete period and $\hat{\sigma}_{fi}^2$ = estimated residual variance in the fixed exchange rate regime period. If the residual variance is unchanged, the value of F^{obs} is 1. A change in structure should lead to a large residual variance for the complete period, with a consequent F^{obs} that is larger than 1.

reported in Tables 5 and 6 are robust with respect to the exchange rate/monetary regime shift in Sweden.

6 Concluding remarks

In this paper, I have tried to shed light upon the empirical relation between nominal interest rates and government budget deficits. The strategy employed is similar to that of Evans (1985, 1987a, 1987b, 1988) in the sense that I have used a conventional macro model as my point of departure for the empirical investigation. But on the basis of a survey, I have also taken into account what seem to be the most important lessons from the previous empirical literature.

The survey suggests two factors that may account for the different empirical results in the previous literature. First of all, the treatment of the expected inflation rate seems to be of considerable importance; the results tend to be more supportive for the conventional view when a proxy is used to account for the expected inflation rate, rather than a reduced form. Second, the data frequency seems to be important. In studies which have exploited lower frequency data, the evidence is more in favor of the conventional than the Ricardian view. In order to control for the first factor, I have constructed and used a conventional model in which it is possible to solve for the rational inflation expectations analytically. To take the latter factor into proper account, I have used both quarterly and monthly data in the estimations.

The theoretical analysis shows that the conventional macro model developed here, in contrast to the findings in the previous literature, does not imply sign uniqueness for the sum of the elements in the parameter polynomials for the budget deficit in the regression equations for the short- and long-term nominal interest rates. However, in the special case when the budget deficit is assumed to follow an AR(p) process, the model implies that the coefficient sum of the elements in the parameter polynomials for the budget deficit should be positive, although individual elements may be negative, given that the budget deficit is a stationary process. Thus, the model offers three important insights for empirical investigations of this issue. First, it stresses the importance of a careful determination of the number of lags for the regressors; if an insufficient number of lags for the government budget deficit are included in the regressions, the estimated coefficient sums may be close to zero. Second, if the persistence in the budget deficit is sufficiently high, the estimated coefficient sums for the budget deficit will be close to zero regardless of how many lags one includes in the estimations; however, it should be emphasized that this is a (small) open economy result. Third, it is essentially an empirical question as to whether larger budget deficits are associated with higher interest rates or not. Consequently, the lack of a robust finding between budget deficits and interest rates should not necessarily be interpreted as evidence against the conventional view and indirect support for the Ricardian equivalence theorem, as claimed, for instance, by Evans (1987a) and Plosser (1987).

The empirical study utilizes data for Sweden, a small open economy with extremely high sample variability for the government budget deficit compared to previous studies. Thus, the empirical results here ought to be more reliable than those of previous studies. The results presented in the paper, which seems robust over time, provide evidence for the conventional view in macroeconomics; larger government budget deficits produce higher nominal interest rates.

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Appendix A Theoretical Derivations

A.1 Derivation of the expected price level

Leading (2) one and two periods, taking E_t and differences, one derives

$$\lambda_1 \Delta \mathrm{E}_t r^l_{t+2} = \lambda_4 \Delta \mathrm{E}_t s_{t+2} + \Delta \mathrm{E}_t X_{t+2} - \Delta \mathrm{E}_t p_{t+2}$$

since $\mathrm{E}_t y_{t+j} = 0$ for all $j = 1, 2, \ldots$. Substituting (6), (4), (3) and (8) into the expression above, yields the following difference equation in the expected price level

$$\begin{aligned} \mathrm{E}_t p_{t+1} &= -d_1 \Delta \mathrm{E}_t X_{t+2} + d_1 \lambda_4 \mathrm{E}_t i^{s^*}_{t+1} + d_2 \mathrm{E}_t m_{t+1} - \\ & \frac{d_2 \lambda_1}{\lambda_1 + 2\lambda_4} \mathrm{E}_t m_{t+3} + d_2 \gamma \mathrm{E}_t p_{t+2} + \frac{d_1 \lambda_1}{2 d_2 \gamma} \mathrm{E}_t p_{t+3} - \frac{d_1 \lambda_1}{2} \mathrm{E}_t p_{t+4} \end{aligned} \tag{A.1}$$

where by definition $d_1 \equiv \frac{2\gamma}{(1+\gamma)(\lambda_1 + 2\lambda_4)} > 0$ and $d_2 \equiv \frac{1}{1+\gamma} > 0$. Since γ, λ_1 and λ_4 are assumed to be positive, (A.1) converges forward if speculative bubbles are ruled out. It is then straightforward to show that the stable solution for the expected price level is given by

$$\mathrm{E}_t p_{t+1} = \sum_{j=0}^{\infty} \psi^{p,X}_j \mathrm{E}_t X_{t+1+j} + \tfrac{1}{1+\gamma} \sum_{j=0}^{\infty} \left(\tfrac{\gamma}{1+\gamma}\right)^j \mathrm{E}_t m_{t+1+j} + \sum_{j=0}^{\infty} \psi^{p,s^*}_j \mathrm{E}_t i^{s^*}_{t+1+j} \tag{A.2}$$

where

$$\begin{aligned} \psi^{p,X}_0 &\equiv d_1 > 0,\ \psi^{p,X}_1 \equiv d_2 \gamma \psi^{p,X}_0 - \psi^{p,X}_0 < 0,\ \psi^{p,X}_2 \equiv d_2 \gamma \psi^{p,X}_1 + \frac{d_1 \lambda_1}{2 d_2 \gamma} \psi^{p,X}_0 \gtrless 0, \\ \psi^{p,X}_3 &\equiv d_2 \gamma \psi^{p,X}_2 + \frac{d_1 \lambda_1}{2 d_2 \gamma} \psi^{p,X}_1 - \frac{d_1 \lambda_1}{2} \psi^{p,X}_0, \ldots,\ \psi^{p,X}_j = d_2 \gamma \psi^{p,X}_{j-1} + \frac{d_1 \lambda_1}{2 d_2 \gamma} \psi^{p,X}_{j-2} - \frac{d_1 \lambda_1}{2} \psi^{p,X}_{j-3}, \end{aligned}$$

$$\begin{aligned} \psi^{p,s^*}_0 &\equiv d_1 \lambda_4 > 0,\ \psi^{p,s^*}_1 \equiv d_2 \gamma \psi^{p,s^*}_0 > 0,\ \psi^{p,s^*}_2 \equiv d_2 \gamma \psi^{p,s^*}_1 + \frac{d_1 \lambda_1}{2 d_2 \gamma} \psi^{p,s^*}_0 > 0, \\ \psi^{p,s^*}_3 &\equiv d_2 \gamma \psi^{p,s^*}_2 + \frac{d_1 \lambda_1}{2 d_2 \gamma} \psi^{p,s^*}_1 - \frac{d_1 \lambda_1}{2} \psi^{p,s^*}_0, \ldots,\ \psi^{p,s^*}_j = d_2 \gamma \psi^{p,s^*}_{j-1} + \frac{d_1 \lambda_1}{2 d_2 \gamma} \psi^{p,s^*}_{j-2} - \frac{d_1 \lambda_1}{2} \psi^{p,s^*}_{j-3}. \end{aligned}$$

A.2 Solution for the nominal exchange rate

Insert (5), (9) and (1) into (2) for r^l_t and p_t to get

$$\begin{aligned} y_t &= \frac{\beta(\lambda_1 + 2\lambda_4)}{\lambda_1 + 2(\beta + \lambda_4)} s_t + \frac{2\beta}{\lambda_1 + 2(\beta + \lambda_4)} \left(X_t - \lambda_1 i^{l^*}_t\right) - \\ & \frac{\beta(\lambda_1 + 2\lambda_4)}{\lambda_1 + 2(\beta + \lambda_4)} \mathrm{E}_{t-1} p_t + \frac{\beta \lambda_1}{\lambda_1 + 2(\beta + \lambda_4)} \mathrm{E}_t p_{t+2} + \\ & \frac{\lambda_1 + 2\lambda_4}{\lambda_1 + 2(\beta + \lambda_4)} \varepsilon^{AS}_t + \frac{2\beta}{\lambda_1 + 2(\beta + \lambda_4)} \varepsilon^{IS}_t - \frac{\beta \lambda_1}{\lambda_1 + 2(\beta + \lambda_4)} \mathrm{E}_t s_{t+2}. \end{aligned}$$

By substituting (8) and (1) for i_t^s and p_t into (3) and rearranging, we have

$$y_t = -\frac{\beta\gamma}{1+\alpha\beta}s_t + \frac{\beta}{1+\alpha\beta}\left(m_t - \mathrm{E}_{t-1}p_t\right) + \frac{\beta\gamma}{1+\alpha\beta}\left(i_t^{s^*} + \mathrm{E}_t s_{t+1}\right) + \frac{\beta}{1+\alpha\beta}\left(\frac{\varepsilon_t^{AS}}{\beta} - \varepsilon_t^{LM}\right).$$

Combining the two expressions above gives the solution for s_t

$$\begin{aligned} s_t \;=\; & -\tfrac{2(1+\alpha\beta)}{d_3}X_t + \tfrac{\lambda_1+2(\beta+\lambda_4)}{d_3}m_t + \tfrac{\gamma(\lambda_1+2(\beta+\lambda_4))}{d_3}i_t^{s^*} + \\ & \tfrac{2(1+\alpha\beta)\lambda_1}{d_3}i_t^{l^*} + \tfrac{(\alpha(\lambda_1+2\lambda_4)-2)\beta}{d_3}\mathrm{E}_{t-1}p_t - \tfrac{(1+\alpha\beta)\lambda_1}{d_3}\mathrm{E}_t p_{t+2} + \tfrac{2-\alpha(\lambda_1+2\lambda_4)}{d_3}\varepsilon_t^{AS} - \tfrac{2(1+\alpha\beta)}{d_3}\varepsilon_t^{IS} - \\ & \tfrac{\lambda_1+2(\beta+\lambda_4)}{d_3}\varepsilon_t^{LM} + \underbrace{\tfrac{\gamma(\lambda_1+2(\beta+\lambda_4))}{d_3}}_{\equiv\phi_1^s>0}\mathrm{E}_t s_{t+1} + \underbrace{\tfrac{(1+\alpha\beta)\lambda_1}{d_3}}_{\equiv\phi_2^s>0}\mathrm{E}_t s_{t+2} \end{aligned} \tag{A.3}$$

as a second order difference equation where $d_3 \equiv (1+\alpha\beta+\gamma)(\lambda_1+2\lambda_4)+2\gamma\beta$. By ruling out speculative bubbles, it can be verified that (A.3) always converges forward provided that $\beta \geq 1$ and $0 < \alpha \leq 5.5$ for all $\{\gamma\ \lambda_1\ \lambda_4\} \in R_{++}^3$. Straightforward recursions on (A.3), using $\mathrm{E}_t\varepsilon_{t+j}^{AS} = \mathrm{E}_t\varepsilon_{t+j}^{IS} = \mathrm{E}_t\varepsilon_{t+j}^{LM} = 0$ for all $j > 0$, then gives the stable solution for s_t as

$$\begin{aligned} s_t \;=\; & k - \sum_{j=0}^{\infty}\psi_j^{s,X}\mathrm{E}_t X_{t+j} + \sum_{j=0}^{\infty}\psi_j^{s,m}\mathrm{E}_t m_{t+j} + \gamma\sum_{j=0}^{\infty}\psi_j^{s,m}\mathrm{E}_t i_{t+j}^{s^*} + \\ & \lambda_1\sum_{j=0}^{\infty}\psi_j^{s,X}\mathrm{E}_t i_{t+j}^{l^*} + \psi_0^{s,p}\mathrm{E}_{t-1}p_t + \sum_{j=1}^{\infty}\psi_j^{s,p}\mathrm{E}_t p_{t+j} - \tfrac{\psi_0^{s,p}}{\beta}\varepsilon_t^{AS} - \psi_0^{s,X}\varepsilon_t^{IS} - \psi_0^{s,m}\varepsilon_t^{LM} \end{aligned} \tag{A.4}$$

where by construction

$$\begin{aligned} \psi_0^{s,X} &\equiv \tfrac{2(1+\alpha\beta)}{d_3} > 0,\ \psi_1^{s,X} \equiv \phi_1^s\psi_0^{s,X} > 0,\ \psi_2^{s,X} \equiv \phi_1^s\psi_1^{s,X} + \phi_2^s\psi_0^{s,X} > 0,\ ..., \\ \psi_j^{s,X} &= \phi_1^s\psi_{j-1}^{s,X} + \phi_2^s\psi_{j-2}^{s,X} > 0\ \forall j \geq 2, \end{aligned}$$

$$\begin{aligned} \psi_0^{s,m} &\equiv \tfrac{\lambda_1+2(\beta+\lambda_4)}{d_3} > 0,\ \psi_1^{s,m} \equiv \phi_1^s\psi_0^{s,m} > 0,\ \psi_2^{s,m} \equiv \phi_1^s\psi_1^{s,m} + \phi_2^s\psi_0^{s,m} > 0,\ ..., \\ \psi_j^{s,m} &= \phi_1^s\psi_{j-1}^{s,m} + \phi_2^s\psi_{j-2}^{s,m} > 0\ \forall j \geq 2, \end{aligned}$$

$$\begin{aligned} \psi_0^{s,p} &\equiv \tfrac{(\alpha(\lambda_1+2\lambda_4)-2)\beta}{d_3} \gtreqless 0,\ \psi_1^{s,p} \equiv \phi_1^s\psi_0^{s,p} \gtreqless 0,\ \psi_2^{s,p} \equiv \phi_1^s\psi_1^{s,p} - \tfrac{(1+\alpha\beta)\lambda_1}{d_3} + \phi_2^s\psi_0^{s,p} \gtreqless 0, \\ \psi_3^{s,p} &= \phi_1^s\psi_2^{s,p} + \phi_2^s\psi_1^{s,p} \gtreqless 0,\ ...,\ \psi_j^{s,p} = \phi_1^s\psi_{j-1}^{s,p} + \phi_2^s\psi_{j-2}^{s,p} \gtreqless 0\ \forall j \geq 3. \end{aligned}$$

Combining (A.4) and (A.2) together with the definitions above gives

$$\begin{aligned} s_t \;=\; & -\sum_{j=0}^{\infty}\psi_j^{s,X}\mathrm{E}_t X_{t+j} + \sum_{n=1}^{\infty}\psi_n^{s,p}\sum_{j=0}^{\infty}\psi_j^X\mathrm{E}_t X_{t+n+j} + \psi_0^{s,p}\sum_{j=0}^{\infty}\psi_j^X\mathrm{E}_{t-1}X_{t+j} + \\ & \sum_{j=0}^{\infty}\psi_j^{s,m}\mathrm{E}_t m_{t+j} + d_2\sum_{n=1}^{\infty}\psi_n^{s,p}\sum_{j=0}^{\infty}\left(\tfrac{\gamma}{1+\gamma}\right)^j\mathrm{E}_t m_{t+n+j} + d_2\psi_0^{s,p}\sum_{j=0}^{\infty}\left(\tfrac{\gamma}{1+\gamma}\right)^j\mathrm{E}_{t-1}m_{t+j} + \\ & \gamma\sum_{j=0}^{\infty}\psi_j^{s,m}\mathrm{E}_t i_{t+j}^{s^*} + \sum_{n=1}^{\infty}\psi_n^{s,p}\sum_{j=0}^{\infty}\psi_j^{s^*}\mathrm{E}_t i_{t+n+j}^{s^*} + \psi_0^{s,p}\sum_{j=0}^{\infty}\psi_j^{s^*}\mathrm{E}_{t-1}i_{t+j}^{s^*} + \\ & \lambda_1\sum_{j=0}^{\infty}\psi_j^{s,X}\mathrm{E}_t i_{t+j}^{l^*} - \left(\tfrac{\psi_0^{s,p}}{\beta}\varepsilon_t^{AS} + \psi_0^{s,X}\varepsilon_t^{IS} + \psi_0^{s,m}\varepsilon_t^{LM}\right). \end{aligned} \tag{A.5}$$

the solution for the nominal exchange rate in (A.5). Thus, similar to (A.2), the nominal exchange rate in period t is the discounted sum of all as of t and $t-1$ expected future nominal money supplies, foreign price levels and short and long-term nominal interest rates, government expenditures and budget deficits plus some current disturbances.

A.3 Derivation of the short- and long-term interest rate differentials

From (A.4), the one period expected change in the nominal exchange rate is

$$\begin{aligned}\Delta \mathrm{E}_t s_{t+1} &= -\sum_{j=0}^{\infty}\psi_j^{s,X}\Delta \mathrm{E}_t X_{t+1+j} + \sum_{j=0}^{\infty}\psi_j^{s,m}\Delta \mathrm{E}_t m_{t+1+j} + \\ &\gamma\sum_{j=0}^{\infty}\psi_j^{s,m}\Delta \mathrm{E}_t i_{t+1+j}^{s^*} + \lambda_1\sum_{j=0}^{\infty}\psi_j^{s,X}\Delta \mathrm{E}_t i_{t+1+j}^{l^*} + \sum_{j=1}^{\infty}\psi_j^{s,p}\Delta \mathrm{E}_t p_{t+1+j} + \\ &\psi_0^{s,p}\left(\mathrm{E}_t p_{t+1} - \mathrm{E}_{t-1} p_t\right) + \tfrac{\psi_0^{s,p}}{\beta}\varepsilon_t^{AS} + \psi_0^{s,X}\varepsilon_t^{IS} + \psi_0^{s,m}\varepsilon_t^{LM}.\end{aligned} \tag{A.6}$$

Using (A.2) to substitute for $\Delta \mathrm{E}_t p_{t+1+j}$, $\mathrm{E}_t p_{t+1}$ and $\mathrm{E}_{t-1} p_t$ gives

$$\begin{aligned}\Delta \mathrm{E}_t s_{t+1} &= -\sum_{j=0}^{\infty}\psi_j^{s,X}\Delta \mathrm{E}_t X_{t+1+j} + \sum_{j=1}^{\infty}\psi_j^{s,p}\sum_{n=0}^{\infty}\psi_n^{p,X}\Delta \mathrm{E}_t X_{t+1+j+n} + \\ &\sum_{j=0}^{\infty}\psi_j^{s,m}\Delta \mathrm{E}_t m_{t+1+j} + \sum_{j=1}^{\infty}\psi_j^{s,p} d_2\sum_{n=0}^{\infty}\left(\tfrac{\gamma}{1+\gamma}\right)^n \Delta \mathrm{E}_t m_{t+1+j+n} + \\ &\gamma\sum_{j=0}^{\infty}\psi_j^{s,m}\Delta \mathrm{E}_t i_{t+1+j}^{s^*} + \sum_{j=1}^{\infty}\psi_j^{s,p}\sum_{n=0}^{\infty}\psi_n^{p,s^*}\Delta \mathrm{E}_t i_{t+1+j+n}^{s^*} + \\ &\lambda_1\sum_{j=0}^{\infty}\psi_j^{s,X}\Delta \mathrm{E}_t i_{t+1+j}^{l^*} + \\ &\psi_0^{s,p}\left(\sum_{j=0}^{\infty}\psi_j^{p,X}\mathrm{E}_t X_{t+1+j} + \sum_{j=0}^{\infty}\psi_j^{p,s^*}\mathrm{E}_t i_{t+1+j}^{s^*} + d_2\sum_{j=0}^{\infty}\left(\tfrac{\gamma}{1+\gamma}\right)^j \mathrm{E}_t m_{t+1+j}\right) - \\ &\psi_0^{s,p}\left(\sum_{j=0}^{\infty}\psi_j^{p,X}\mathrm{E}_{t-1} X_{t+j} + \sum_{j=0}^{\infty}\psi_j^{p,s^*}\mathrm{E}_{t-1} i_{t+j}^{s^*} + d_2\sum_{j=0}^{\infty}\left(\tfrac{\gamma}{1+\gamma}\right)^j \mathrm{E}_{t-1} m_{t+j}\right) + \\ &\tfrac{\psi_0^{s,p}}{\beta}\varepsilon_t^{AS} + \psi_0^{s,X}\varepsilon_t^{IS} + \psi_0^{s,m}\varepsilon_t^{LM},\end{aligned}$$

which after some algebraic manipulations can be rewritten as

$$\Delta \mathrm{E}_t s_{t+1} = \lambda_2\sum_{j=0}^{\infty}\psi_j^{is,X}\mathrm{E}_t g_{t+j} - \lambda_2\psi_0^{s,p}\sum_{j=0}^{\infty}\psi_j^{p,X}\mathrm{E}_{t-1} g_{t+j} + \tag{A.7}$$

$$\lambda_3 \sum_{j=0}^{\infty} \psi_j^{i^s,X} \mathrm{E}_t D_{t+j} - \lambda_3 \psi_0^{s,p} \sum_{j=0}^{\infty} \psi_j^{p,X} \mathrm{E}_{t-1} D_{t+j} +$$
$$\lambda_4 \sum_{j=0}^{\infty} \psi_j^{i^s,X} \mathrm{E}_t p^*_{t+j} - \lambda_4 \psi_0^{s,p} \sum_{j=0}^{\infty} \psi_j^{p,X} \mathrm{E}_{t-1} p^*_{t+j} +$$
$$\sum_{j=0}^{\infty} \psi_j^{i^s,m} \mathrm{E}_t m_{t+j} - d_2 \psi_0^{s,p} \sum_{j=0}^{\infty} \left(\tfrac{\gamma}{1+\gamma}\right)^j \mathrm{E}_{t-1} m_{t+j} +$$
$$\sum_{j=0}^{\infty} \psi_j^{i^s,s^*} \mathrm{E}_t i^{s^*}_{t+j} - \psi_0^{s,p} \sum_{j=0}^{\infty} \psi_j^{p,s^*} \mathrm{E}_{t-1} i^{s^*}_{t+j} +$$
$$\sum_{j=0}^{\infty} \psi_j^{i^s,l^*} \mathrm{E}_t i^{l^*}_{t+j} + \tfrac{\psi_0^{s,p}}{\beta} \varepsilon_t^{AS} + \psi_0^{s,X} \varepsilon_t^{IS} + \psi_0^{s,m} \varepsilon_t^{LM}$$

where by definition the ψ^{i^s}-coefficients are

$$\begin{aligned}
\psi_0^{i^s,X} &\equiv \psi_0^{s,X} > 0,\ \psi_1^{i^s,X} \equiv \left(\psi_1^{s,X} - \psi_0^{s,X}\right) - \psi_1^{s,p}\psi_0^{p,X} + \psi_0^{s,p}\psi_0^{p,X}, \\
\psi_2^{i^s,X} &\equiv \left(\psi_2^{s,X} - \psi_1^{s,X}\right) + \left(\psi_1^{s,p}\psi_0^{p,X} - \left(\psi_1^{s,p}\psi_1^{p,X} + \psi_2^{s,p}\psi_0^{p,X}\right)\right) + \psi_0^{s,p}\psi_1^{p,X}, \\
\psi_3^{i^s,X} &\equiv \left(\psi_3^{s,X} - \psi_2^{s,X}\right) + \begin{pmatrix} \left(\psi_1^{s,p}\psi_1^{p,X} + \psi_2^{s,p}\psi_0^{p,X}\right) - \\ \left(\psi_1^{s,p}\psi_2^{p,X} + \psi_2^{s,p}\psi_1^{p,X} + \psi_3^{s,p}\psi_0^{p,X}\right) \end{pmatrix} + \psi_0^{s,p}\psi_2^{p,X},\ \ldots, \\
\psi_j^{i^s,X} &= \left(\psi_j^{s,X} - \psi_{j-1}^{s,X}\right) + \left(\psi_1^{s,p}\psi_{j-2}^{p,X} + \psi_2^{s,p}\psi_{j-3}^{p,X} + \ldots + \psi_{j-2}^{s,p}\psi_1^{p,X} + \psi_{j-1}^{s,p}\psi_0^{p,X}\right) - \\
&\quad \left(\psi_1^{s,p}\psi_{j-1}^{p,X} + \psi_2^{s,p}\psi_{j-2}^{p,X} + \ldots + \psi_{j-1}^{s,p}\psi_1^{p,X} + \psi_j^{s,p}\psi_0^{p,X}\right) + \psi_0^{s,p}\psi_{j-1}^{p,X}\ ,
\end{aligned}$$

and

$$\begin{aligned}
\psi_0^{i^s,m} &\equiv -\psi_0^{s,m} < 0,\ \psi_1^{i^s,m} \equiv -\left[\left(\psi_1^{s,m} - \psi_0^{s,m}\right) + d_2\psi_1^{s,p} - d_2\psi_0^{s,p}\right], \\
\psi_2^{i^s,m} &\equiv -\left[\left(\psi_2^{s,m} - \psi_1^{s,m}\right) + d_2\left(\left(\psi_1^{s,p}\tfrac{\gamma}{1+\gamma} + \psi_2^{s,p}\right) - \psi_1^{s,p}\right) - d_2\psi_0^{s,p}\tfrac{\gamma}{1+\gamma}\right], \\
\psi_3^{i^s,m} &\equiv -\left[\left(\psi_3^{s,m} - \psi_2^{s,m}\right) + d_2\begin{pmatrix} \left(\psi_1^{s,p}\left(\tfrac{\gamma}{1+\gamma}\right)^2 + \psi_2^{s,p}\tfrac{\gamma}{1+\gamma} + \psi_3^{s,p}\right) - \\ \left(\psi_1^{s,p}\tfrac{\gamma}{1+\gamma} + \psi_2^{s,p}\right) \end{pmatrix} - d_2\psi_0^{s,p}\left(\tfrac{\gamma}{1+\gamma}\right)^2\right],\ \ldots, \\
\psi_j^{i^s,m} &= -\left[\begin{array}{c} \left(\psi_j^{s,m} - \psi_{j-1}^{s,m}\right) + d_2\left(\psi_1^{s,p}\left(\tfrac{\gamma}{1+\gamma}\right)^{j-1} + \psi_2^{s,p}\left(\tfrac{\gamma}{1+\gamma}\right)^{j-2} + \ldots + \psi_{j-1}^{s,p}\left(\tfrac{\gamma}{1+\gamma}\right) + \psi_j^{s,p}\right) - \\ d_2\left(\psi_1^{s,p}\left(\tfrac{\gamma}{1+\gamma}\right)^{j-2} + \psi_2^{s,p}\left(\tfrac{\gamma}{1+\gamma}\right)^{j-3} + \ldots + \psi_{j-2}^{s,p}\left(\tfrac{\gamma}{1+\gamma}\right) + \psi_{j-1}^{s,p}\right) - d_2\psi_0^{s,p}\left(\tfrac{\gamma}{1+\gamma}\right)^{j-1} \end{array}\right],
\end{aligned}$$

and

$$\begin{aligned}
\psi_0^{i^s,s^*} &\equiv -\gamma\psi_0^{s,m} < 0,\ \psi_1^{i^s,s^*} \equiv -\left[\gamma\left(\psi_1^{s,m} - \psi_0^{s,m}\right) + \psi_1^{s,p}\psi_0^{p,s^*} - \psi_0^{s,p}\psi_0^{p,s^*}\right], \\
\psi_2^{i^s,s^*} &\equiv -\left[\gamma\left(\psi_2^{s,m} - \psi_1^{s,m}\right) + \left(\psi_1^{s,p}\psi_1^{p,s^*} + \psi_2^{s,p}\psi_0^{p,s^*}\right) - \psi_1^{s,p}\psi_0^{p,s^*} - \psi_0^{s,p}\psi_1^{p,s^*}\right],
\end{aligned}$$

$$\psi_3^{i^s,s^*} \equiv - \left[\begin{array}{c} \gamma\left(\psi_3^{s,m} - \psi_2^{s,m}\right) + \left(\psi_1^{s,p}\psi_2^{p,s^*} + \psi_2^{s,p}\psi_1^{p,s^*} + \psi_3^{s,p}\psi_0^{p,s^*}\right) - \\ \left(\psi_1^{s,p}\psi_1^{p,s^*} + \psi_2^{s,p}\psi_0^{p,s^*}\right) - \psi_0^{s,p}\psi_2^{p,s^*} \end{array} \right], ...,$$

$$\psi_j^{i^s,s^*} = - \left[\begin{array}{c} \gamma\left(\psi_j^{s,m} - \psi_{j-1}^{s,m}\right) + \left(\psi_1^{s,p}\psi_{j-1}^{p,s^*} + \psi_2^{s,p}\psi_{j-2}^{p,s^*} + ... + \psi_{j-1}^{s,p}\psi_1^{p,s^*} + \psi_j^{s,p}\psi_0^{p,s^*}\right) \\ -\left(\psi_1^{s,p}\psi_{j-2}^{p,s^*} + \psi_2^{s,p}\psi_{j-3}^{p,s^*} + ... + \psi_{j-2}^{s,p}\psi_1^{p,s^*} + \psi_{j-1}^{s,p}\psi_0^{p,s^*}\right) + \psi_0^{s,p}\psi_{j-1}^{p,s^*} \end{array} \right],$$

and finally

$$\psi_0^{i^s,l^*} \equiv -\lambda_1\psi_0^{s,X} < 0,\ \psi_1^{i^s,l^*} \equiv -\lambda_1\left(\psi_1^{s,X} - \psi_0^{s,X}\right),\ ...,\ \psi_j^{i^s,l^*} \equiv -\lambda_1\left(\psi_j^{s,X} - \psi_{j-1}^{s,X}\right)$$

for all $j \geq 3$. If (A.7) is substituted into (8), utilizing the definitions

$$\psi_j^{i^s,g} \equiv \lambda_2\psi_j^{i^s,X},\ \psi_j^{i^s,D} \equiv \lambda_3\psi_j^{i^s,X},\ \psi_j^{i^s,p^*} \equiv \lambda_4\psi_j^{i^s,X},$$

we have the solution for the short-term interest rate differential as

$$\begin{aligned} i_t^s - i_t^{s^*} = & \sum_{j=0}^{\infty}\psi_j^{i^s,g}\mathrm{E}_t g_{t+j} - \lambda_2\psi_0^{s,p}\sum_{j=0}^{\infty}\psi_j^{p,X}\mathrm{E}_{t-1}g_{t+j} + \\ & \sum_{j=0}^{\infty}\psi_j^{i^s,D}\mathrm{E}_t D_{t+j} - \lambda_3\psi_0^{s,p}\sum_{j=0}^{\infty}\psi_j^{p,X}\mathrm{E}_{t-1}D_{t+j} + \\ & \sum_{j=0}^{\infty}\psi_j^{i^s,p^*}\mathrm{E}_t p_{t+j}^* - \lambda_4\psi_0^{s,p}\sum_{j=0}^{\infty}\psi_j^{p,X}\mathrm{E}_{t-1}p_{t+j}^* + \\ & \sum_{j=0}^{\infty}\psi_j^{i^s,m}\mathrm{E}_t m_{t+j} - d_2\psi_0^{s,p}\sum_{j=0}^{\infty}\left(\tfrac{\gamma}{1+\gamma}\right)^j\mathrm{E}_{t-1}m_{t+j} + \\ & \sum_{j=0}^{\infty}\psi_j^{i^s,s^*}\mathrm{E}_t i_{t+j}^{s^*} - \psi_0^{s,p}\sum_{j=0}^{\infty}\psi_j^{p,s^*}\mathrm{E}_{t-1}i_{t+j}^{s^*} + \\ & \sum_{j=0}^{\infty}\psi_j^{i^s,l^*}\mathrm{E}_t i_{t+j}^{l^*} + \tfrac{\psi_0^{s,p}}{\beta}\varepsilon_t^{AS} + \psi_0^{s,X}\varepsilon_t^{IS} + \psi_0^{s,m}\varepsilon_t^{LM}. \end{aligned} \quad \text{(A.8)}$$

Repeating the procedure above for $\mathrm{E}_t s_{t+2} - s_t$ gives

$$\begin{aligned} \mathrm{E}_t s_{t+2} - s_t = & \lambda_2\sum_{j=0}^{\infty}\psi_j^{i^l,X}\mathrm{E}_t g_{t+j} - \lambda_2\psi_0^{s,p}\sum_{j=0}^{\infty}\psi_j^{p,X}\mathrm{E}_{t-1}g_{t+j} + \\ & \lambda_3\sum_{j=0}^{\infty}\psi_j^{i^l,X}\mathrm{E}_t D_{t+j} - \lambda_3\psi_0^{s,p}\sum_{j=0}^{\infty}\psi_j^{p,X}\mathrm{E}_{t-1}D_{t+j} + \\ & \lambda_4\sum_{j=0}^{\infty}\psi_j^{i^l,X}\mathrm{E}_t p_{t+j}^* - \lambda_4\psi_0^{s,p}\sum_{j=0}^{\infty}\psi_j^{p,X}\mathrm{E}_{t-1}p_{t+j}^* + \\ & \sum_{j=0}^{\infty}\psi_j^{i^l,m}\mathrm{E}_t m_{t+j} - d_2\psi_0^{s,p}\sum_{j=0}^{\infty}\left(\tfrac{\gamma}{1+\gamma}\right)^j\mathrm{E}_{t-1}m_{t+j} + \end{aligned} \quad \text{(A.9)}$$

$$\sum_{j=0}^{\infty}\psi_j^{il,s^*}\mathrm{E}_t i_{t+j}^{s^*} - d_2\psi_0^{s,p}\sum_{j=0}^{\infty}\psi_j^{p,s^*}\mathrm{E}_{t-1} i_{t+j}^{s^*} +$$
$$\sum_{j=0}^{\infty}\psi_j^{il,l^*}\mathrm{E}_t i_{t+j}^{l^*} + \left(\tfrac{\psi_0^{s,p}}{\beta}\varepsilon_t^{AS} + \psi_0^{s,X}\varepsilon_t^{IS} + \psi_0^{s,m}\varepsilon_t^{LM}\right),$$

where

$$\begin{aligned}
\psi_0^{il,X} &\equiv \psi_0^{s,X} = \psi_0^{is,X} > 0,\ \psi_1^{il,X} \equiv \psi_1^{s,X} - \psi_1^{s,p}\psi_0^{p,X},\\
\psi_2^{il,X} &\equiv \left(\psi_2^{s,X} - \psi_0^{s,X}\right) - \left(\psi_1^{s,p}\psi_1^{p,X} + \psi_2^{s,p}\psi_0^{p,X}\right) + \psi_0^{s,p}\psi_0^{p,X},\\
\psi_3^{il,X} &\equiv \left(\psi_3^{s,X} - \psi_1^{s,X}\right) + \left(\psi_1^{s,p}\psi_0^{p,X} - \left(\psi_1^{s,p}\psi_2^{p,X} + \psi_2^{s,p}\psi_1^{p,X} + \psi_3^{s,p}\psi_0^{p,X}\right)\right) + \psi_0^{s,p}\psi_1^{p,X},\ ...,\\
\psi_j^{il,X} &= \left(\psi_j^{s,X} - \psi_{j-2}^{s,X}\right) + \left(\psi_1^{s,p}\psi_{j-3}^{p,X} + \psi_2^{s,p}\psi_{j-4}^{p,X} + ... + \psi_{j-3}^{s,p}\psi_1^{p,X} + \psi_{j-2}^{s,p}\psi_0^{p,X}\right) -\\
&\quad \left(\psi_1^{s,p}\psi_{j-1}^{p,X} + \psi_2^{s,p}\psi_{j-2}^{p,X} + ... + \psi_{j-1}^{s,p}\psi_1^{p,X} + \psi_j^{s,p}\psi_0^{p,X}\right) + \psi_0^{s,p}\psi_{j-2}^{p,X},
\end{aligned}$$

and

$$\begin{aligned}
\psi_0^{il,m} &\equiv -\psi_0^{s,m} = \psi_0^{is,m} < 0,\ \psi_1^{il,m} \equiv -\left[\psi_1^{s,m} + d_2\psi_1^{s,p}\right],\\
\psi_2^{il,m} &\equiv -\left[\left(\psi_2^{s,m} - \psi_0^{s,m}\right) + d_2\left(\psi_1^{s,p}\tfrac{\gamma}{1+\gamma} + \psi_2^{s,p}\right) - d_2\psi_0^{s,p}\right],\\
\psi_3^{il,m} &\equiv -\left[\left(\psi_3^{s,m} - \psi_1^{s,m}\right) + d_2\left(\left(\psi_1^{s,p}\left(\tfrac{\gamma}{1+\gamma}\right)^2 + \psi_2^{s,p}\tfrac{\gamma}{1+\gamma} + \psi_3^{s,p}\right) - \psi_1^{s,p}\right) - d_2\psi_0^{s,p}\tfrac{\gamma}{1+\gamma}\right],\ ...,\\
\psi_j^{il,m} &= -\left[\begin{array}{c}\left(\psi_j^{s,m} - \psi_{j-2}^{s,m}\right) + d_2\left(\psi_1^{s,p}\left(\tfrac{\gamma}{1+\gamma}\right)^{j-1} + \psi_2^{s,p}\left(\tfrac{\gamma}{1+\gamma}\right)^{j-2} + ... + \psi_{j-1}^{s,p}\left(\tfrac{\gamma}{1+\gamma}\right) + \psi_j^{s,p}\right) -\\ d_2\left(\psi_1^{s,p}\left(\tfrac{\gamma}{1+\gamma}\right)^{j-3} + \psi_2^{s,p}\left(\tfrac{\gamma}{1+\gamma}\right)^{j-4} + ... + \psi_{j-3}^{s,p}\left(\tfrac{\gamma}{1+\gamma}\right) + \psi_{j-2}^{s,p}\right) - d_2\psi_0^{s,p}\left(\tfrac{\gamma}{1+\gamma}\right)^{j-2}\end{array}\right],
\end{aligned}$$

and

$$\begin{aligned}
\psi_0^{il,s^*} &\equiv -\gamma\psi_0^{s,m} = \psi_0^{is,s^*} < 0,\ \psi_1^{is,s^*} \equiv -\left[\gamma\psi_1^{s,m} + \psi_1^{s,p}\psi_0^{p,s^*}\right],\\
\psi_2^{il,s^*} &\equiv -\left[\gamma\left(\psi_2^{s,m} - \psi_0^{s,m}\right) + \left(\psi_1^{s,p}\psi_1^{p,s^*} + \psi_2^{s,p}\psi_0^{p,s^*}\right) - \psi_0^{s,p}\psi_0^{p,s^*}\right],\\
\psi_3^{il,s^*} &\equiv -\left[\gamma\left(\psi_3^{s,m} - \psi_1^{s,m}\right) + \left(\psi_1^{s,p}\psi_2^{p,s^*} + \psi_2^{s,p}\psi_1^{p,s^*} + \psi_3^{s,p}\psi_0^{p,s^*}\right) - \psi_1^{s,p}\psi_0^{p,s^*} - \psi_0^{s,p}\psi_1^{p,s^*}\right],\ ...,\\
\psi_j^{il,s^*} &= -\left[\begin{array}{c}\gamma\left(\psi_j^{s,m} - \psi_{j-2}^{s,m}\right) + \left(\psi_1^{s,p}\psi_{j-1}^{p,s^*} + \psi_2^{s,p}\psi_{j-2}^{p,s^*} + ... + \psi_{j-1}^{s,p}\psi_1^{p,s^*} + \psi_j^{s,p}\psi_0^{p,s^*}\right) -\\ \left(\psi_1^{s,p}\psi_{j-3}^{p,s^*} + \psi_2^{s,p}\psi_{j-4}^{p,s^*} + ... + \psi_{j-3}^{s,p}\psi_1^{p,s^*} + \psi_{j-2}^{s,p}\psi_0^{p,s^*}\right) - \psi_0^{s,p}\psi_{j-2}^{p,s^*}\end{array}\right],
\end{aligned}$$

and finally

$$\begin{aligned}
\psi_0^{il,l^*} &\equiv -\lambda_1\psi_0^{s,X} = \psi_0^{is,l^*} < 0,\ \psi_1^{il,l^*} \equiv -\lambda_1\psi_1^{s,X},\ \psi_2^{il,l^*} \equiv -\lambda_1\left(\psi_2^{s,X} - \psi_0^{s,X}\right),\ ...,\\
\psi_j^{il,l^*} &\equiv -\lambda_1\left(\psi_j^{s,X} - \psi_{j-2}^{s,X}\right),
\end{aligned}$$

for all $j \geq 4$. If (A.9) is substituted into (9), utilizing the definitions

$$\psi_j^{il,g} \equiv \frac{\lambda_2}{2}\psi_j^{il,X},\ \psi_j^{il,D} \equiv \frac{\lambda_3}{2}\psi_j^{il,X},\ \psi_j^{il,p^*} \equiv \frac{\lambda_4}{2}\psi_j^{il,X},$$

we have the solution for the long-term interest rate differential as

$$\begin{aligned} i_t^l - i_t^{l^*} = & \sum_{j=0}^{\infty} \psi_j^{il,g} \mathrm{E}_t g_{t+j} - \tfrac{\lambda_2 \psi_0^{s,p}}{2} \sum_{j=0}^{\infty} \psi_j^{p,X} \mathrm{E}_{t-1} g_{t+j} + \\ & \sum_{j=0}^{\infty} \psi_j^{il,D} \mathrm{E}_t D_{t+j} - \tfrac{\lambda_3 \psi_0^{s,p}}{2} \sum_{j=0}^{\infty} \psi_j^{p,X} \mathrm{E}_{t-1} D_{t+j} + \\ & \sum_{j=0}^{\infty} \psi_j^{il,p^*} \mathrm{E}_t p^*_{t+j} - \tfrac{\lambda_4 \psi_0^{s,p}}{2} \sum_{j=0}^{\infty} \psi_j^{p,X} \mathrm{E}_{t-1} p^*_{t+j} + \\ & \sum_{j=0}^{\infty} \psi_j^{il,m} \mathrm{E}_t m_{t+j} - \tfrac{d_2 \psi_0^{s,p}}{2} \sum_{j=0}^{\infty} \left(\tfrac{\gamma}{1+\gamma}\right)^j \mathrm{E}_{t-1} m_{t+j} + \\ & \sum_{j=0}^{\infty} \psi_j^{il,s^*} \mathrm{E}_t i^{s^*}_{t+j} - \tfrac{d_2 \psi_0^{s,p}}{2} \sum_{j=0}^{\infty} \psi_j^{p,s^*} \mathrm{E}_{t-1} i^{s^*}_{t+j} + \\ & \sum_{j=0}^{\infty} \psi_j^{il,l^*} \mathrm{E}_t i^{l^*}_{t+j} + \tfrac{1}{2}\left(\tfrac{\psi_0^{s,p}}{\beta}\varepsilon_t^{AS} + \psi_0^{s,X}\varepsilon_t^{IS} + \psi_0^{s,m}\varepsilon_t^{LM}\right). \end{aligned} \tag{A.10}$$

A.4 Proof of (11)

Follows by induction from the derivations in the next section. ■

A.5 Derivation of (12)

Inserting (10) (under the simplifying assumption that $\boldsymbol{\rho}^{\mathbf{z}}(L) \equiv \boldsymbol{\rho}^{\mathbf{z}}$, where $\boldsymbol{\rho}^{\mathbf{z}}$ is a diagonal matrix with the elements $\left[\rho^{p^*}\ \rho^{s^*}\ \rho^{l^*}\ \rho^{g}\ \rho^{D}\ \rho^{m}\right]$ in the diagonal) in (A.2), recognizing that $\mathrm{E}_t \boldsymbol{\varepsilon}^{\mathbf{z}}_{t+s} = 0\ \forall s > 0$, gives the solution for the expected price level j periods in the future as

$$\begin{aligned} \mathrm{E}_t p_{t+j} = & \tfrac{2\gamma\left(1-\rho^X\right)}{\left(\lambda_1\left(1-(\rho^X)^2\right)+2\lambda_4\right)(1+\gamma(1-\rho^X))} \left(\rho^X\right)^j X_t + \\ & \tfrac{1}{1+\gamma(1-\rho^m)} \left(\rho^m\right)^j m_t + \tfrac{2\gamma\lambda_4}{\left(\lambda_1\left(1-\left(\rho^{s^*}\right)^2\right)+2\lambda_4\right)\left(1+\gamma\left(1-\rho^{s^*}\right)\right)} \left(\rho^{s^*}\right)^j i_t^{s^*}. \end{aligned} \tag{A.11}$$

Now, after some considerable algebra, it can be shown that (A.11), (10) and (A.5) imply that

$$\begin{aligned} s_t = & -d_4 X_t + d_5 m_t + d_6 i_t^{s^*} + d_7 i_t^{l^*} + \tfrac{(\alpha(\lambda_1+2\lambda_4)-2)\beta}{d_3} \mathrm{E}_{t-1} p_t \\ & -\psi_0^{s,p}\varepsilon_t^{AS} - \psi_0^{s,X}\varepsilon_t^{IS} - \psi_0^{s,m}\varepsilon_t^{LM} \end{aligned} \tag{A.12}$$

$$= -d_4 X_t + d_5 m_t + d_6 i_t^{s^*} + d_7 i_t^{l^*} + \underbrace{\frac{2(\alpha(\lambda_1+2\lambda_4)-2)\beta\gamma\rho^X\left(1-\rho^X\right)}{d_3\left(\lambda_1\left(1-(\rho^X)^2\right)+2\lambda_4\right)(1+\gamma(1-\rho^X))}}_{\equiv d_8} X_{t-1} +$$

$$\underbrace{\frac{(\alpha(\lambda_1+2\lambda_4)-2)\beta\rho^m}{d_3(1+\gamma(1-\rho^m))}}_{\equiv d_9} m_{t-1} + \underbrace{\frac{2(\alpha(\lambda_1+2\lambda_4)-2)\beta\gamma\lambda_4\rho^{s^*}}{d_3\left(\lambda_1\left(1-\left(\rho^{s^*}\right)^2\right)+2\lambda_4\right)\left(1+\gamma\left(1-\rho^{s^*}\right)\right)}}_{\equiv d_{10}} i_{t-1}^{s^*} -$$

$$\frac{\psi_0^{s,p}}{\beta}\varepsilon_t^{AS} - \psi_0^{s,X}\varepsilon_t^{IS} - \psi_0^{s,m}\varepsilon_t^{LM}$$

where by definition

$$d_4 \equiv \frac{2\gamma^2(\lambda_1+2(\beta+\lambda_4))\left(1-\rho^X\right)\left[\left(1+\alpha\beta\left(1-\rho^X\right)\right)(\lambda_1+2\lambda_4)+2\beta\rho^X\right]}{d_3\left(\lambda_1\left(1-(\rho^X)^2\right)+2\lambda_4\right)\left(\gamma(\lambda_1+2(\beta+\lambda_4))(1-\rho^X)+(1+\alpha\beta)\left(\lambda_1\left(1-(\rho^X)^2\right)+2\lambda_4\right)\right)(1+\gamma(1-\rho^X))} + \frac{2(1+\alpha\beta)\gamma\left[\left(\left(1-\rho^X\right)(\lambda_1+2\lambda_4)+\left(1+\alpha\beta\gamma\left(1-\rho^X\right)\right)\left(\lambda_1\left(1-\left(\rho^X\right)^2\right)+2\lambda_4\right)\right)(\lambda_1+2\lambda_4)+2\beta\left(\lambda_1\left(1-\left(\rho^X\right)^3\right)+2\lambda_4\right)\right]}{d_3\left(\lambda_1\left(1-(\rho^X)^2\right)+2\lambda_4\right)\left(\gamma(\lambda_1+2(\beta+\lambda_4))(1-\rho^X)+(1+\alpha\beta)\left(\lambda_1\left(1-(\rho^X)^2\right)+2\lambda_4\right)\right)(1+\gamma(1-\rho^X))},$$

$$d_5 \equiv \frac{(\lambda_1+2(\beta+\lambda_4))\left[\gamma(1-\rho^m)(d_3+2\beta)+\alpha\beta\gamma\rho^m(\lambda_1(1-\rho^m)+2\lambda_4)+(1+\alpha\beta+\gamma)\left(\lambda_1\left(1-(\rho^m)^2\right)+2\lambda_4\right)\right]}{d_3\left(\gamma(\lambda_1+2(\beta+\lambda_4))(1-\rho^m)+(1+\alpha\beta)\left(\lambda_1\left(1-(\rho^m)^2\right)+2\lambda_4\right)\right)(1+\gamma(1-\rho^m))},$$

$$d_6 \equiv \frac{\gamma^2(\lambda_1+2(\beta+\lambda_4))\left[d_3\left(1-\rho^{s^*}\right)\left(\lambda_1\left(1-\left(\rho^{s^*}\right)^2\right)+2\lambda_4\right)+2\beta\lambda_4\left(2\left(1-\rho^{s^*}\right)+\alpha\rho^{s^*}\left(\lambda_1\left(1-\rho^{s^*}\right)+2\lambda_4\right)\right)\right]}{d_3\left(\lambda_1\left(1-\left(\rho^{s^*}\right)^2\right)+2\lambda_4\right)\left(\gamma(\lambda_1+2(\beta+\lambda_4))\left(1-\rho^{s^*}\right)+(1+\alpha\beta)\left(\lambda_1\left(1-\left(\rho^{s^*}\right)^2\right)+2\lambda_4\right)\right)\left(1+\gamma\left(1-\rho^{s^*}\right)\right)} + \frac{2\gamma\lambda_4\left[2d_3\lambda_4+(1+\alpha\beta)\beta\lambda_4\left(\lambda_1\left(1-\left(\rho^{s^*}\right)^2\right)+2\lambda_4\right)\right]}{d_3\left(\lambda_1\left(1-\left(\rho^{s^*}\right)^2\right)+2\lambda_4\right)\left(\gamma(\lambda_1+2(\beta+\lambda_4))\left(1-\rho^{s^*}\right)+(1+\alpha\beta)\left(\lambda_1\left(1-\left(\rho^{s^*}\right)^2\right)+2\lambda_4\right)\right)\left(1+\gamma\left(1-\rho^{s^*}\right)\right)}$$

and

$$d_7 \equiv \frac{2(1+\alpha\beta)\lambda_1}{\left(\gamma(\lambda_1+2(\beta+\lambda_4))\left(1-\rho^{s^*}\right)+(1+\alpha\beta)\left(\lambda_1\left(1-\left(\rho^{s^*}\right)^2\right)+2\lambda_4\right)\right)}.$$

From the definitions of d_4, d_5, d_6 and d_7, it follows that they are all positive as long as the elements in $\boldsymbol{\rho}^{\mathbf{z}}$ lie between 0 and 1. However, the signs of d_8, d_9 and d_{10} are ambiguous, and depend on whether $\alpha(\lambda_1+2\lambda_4)-2$ are $\lesseqgtr 0$. If $\alpha(\lambda_1+2\lambda_4)-2>0$, then they are all positive and vice versa. (A.12) implies that

$$\begin{aligned}\Delta \mathrm{E}_t s_{t+1} &= d_4\left(1-\rho^X\right)X_t - d_5\left(1-\rho^m\right)m_t - d_6\left(1-\rho^{s^*}\right)i_t^{s^*} - \\ &\quad d_7\left(1-\rho^{l^*}\right)i_t^{l^*} + d_8\Delta X_t + d_9\Delta m_t + d_{10}\Delta i_t^{s^*} + \\ &\quad \frac{\psi_0^{s,p}}{\beta}\varepsilon_t^{AS} + \psi_0^{s,X}\varepsilon_t^{IS} + \psi_0^{s,m}\varepsilon_t^{LM}\end{aligned} \tag{A.13}$$

and

$$\begin{aligned}\mathrm{E}_t s_{t+2} - s_t &= d_4\left(1-\left(\rho^X\right)^2\right)X_t - d_5\left(1-(\rho^m)^2\right)m_t - \\ &\quad d_6\left(1-\left(\rho^{s^*}\right)^2\right)i_t^{s^*} - d_7\left(1-\left(\rho^{l^*}\right)^2\right)i_t^{l^*} + d_8\rho^X\Delta X_t - d_8\left(1-\rho^X\right)X_{t-1} + \\ &\quad d_9\rho^m\Delta m_t - d_9\left(1-\rho^m\right)m_{t-1} + d_{10}\rho^{s^*}\Delta i_t^{s^*} - d_{10}\left(1-\rho^{s^*}\right)i_{t-1}^{s^*} + \\ &\quad \frac{\psi_0^{s,p}}{\beta}\varepsilon_t^{AS} + \psi_0^{s,X}\varepsilon_t^{IS} + \psi_0^{s,m}\varepsilon_t^{LM}.\end{aligned} \tag{A.14}$$

If we use the implicit assumption that all the parameters are stationary lag polynomials, i.e. $\alpha \equiv \alpha(L)$, $\beta \equiv \beta(L)$ and so forth, together with the definition of X, (A.13) and (A.14) can be rewritten in the following form

$$\begin{aligned}\Delta \mathrm{E}_t s_{t+1} &= \delta_1^s(L)\, g_t + \delta_2^s(L)\, D_t + \delta_3^s(L)\, p_t^* - \delta_4^s(L)\, m_t - \delta_5^s(L) i_t^{s^*} - \\ &\quad \delta_6^s(L) i_t^{l^*} + \delta_7^s(L)\, \Delta g_t + \delta_8^s(L)\, \Delta D_t + \delta_9^s(L)\, \Delta p_t^* + \delta_{10}^s(L)\, \Delta m_t + \delta_{11}^s(L) \Delta i_t^{s^*} + \\ &\quad \psi_0^{s,p}(L)\, (\beta(L))^{-1} \varepsilon_t^{AS} + \psi_0^{s,X}(L) \varepsilon_t^{IS} + \psi_0^{s,m}(L) \varepsilon_t^{LM}\end{aligned} \tag{A.15}$$

and

$$\begin{aligned}\mathrm{E}_t s_{t+2} - s_t &= \delta_1^l(L)\, g_t + \delta_2^l(L)\, D_t + \delta_3^l(L)\, p_t^* - \delta_4^l(L)\, m_t - \delta_5^l(L) i_t^{s^*} - \\ &\quad \delta_6^l(L) i_t^{l^*} + \delta_7^l(L)\, \Delta g_t + \delta_8^l(L)\, \Delta D_t + \delta_9^l(L)\, \Delta p_t^* + \delta_{10}^l(L)\, \Delta m_t + \delta_{11}^l(L) \Delta i_t^{s^*} + \\ &\quad \psi_0^{s,p}(L)\, (\beta(L))^{-1} \varepsilon_t^{AS} + \psi_0^{s,X}(L) \varepsilon_t^{IS} + \psi_0^{s,m}(L) \varepsilon_t^{LM}\end{aligned} \tag{A.16}$$

where

$$\begin{aligned}\delta_1^s(L) &\equiv d_4(L)\left(1-\rho^X\right) = d_{4,0}\left(1-\rho^X\right) + d_{4,1}\left(1-\rho^X\right)L + \ldots + d_{4,p}\left(1-\rho^X\right)L^p, \;\ldots\;, \\ \delta_{11}^s(L) &\equiv d_{10}(L) = d_{10,0} + d_{10,1}L + \ldots + d_{10,p}L^p\end{aligned}$$

and

$$\begin{aligned}\delta_1^l(L) &\equiv \underbrace{d_{4,0}\left(1-\left(\rho^X\right)^2\right)}_{\equiv \delta_{1,0}^l} + \left[d_{4,1}\left(1-\left(\rho^X\right)^2\right) - d_{8,0}\left(1-\rho^X\right)\right] L + \\ &\quad \left[d_{4,2}\left(1-\left(\rho^X\right)^2\right) - d_{8,1}\left(1-\rho^X\right)\right] L^2 + \ldots + \\ &\quad \underbrace{\left[d_{4,p}\left(1-\left(\rho^X\right)^2\right) - d_{8,p-1}\left(1-\rho^X\right)\right] L^p}_{\equiv \delta_{1,p}^l} - \underbrace{d_{8,p}\left(1-\rho^X\right) L^{p+1}}_{\equiv \delta_{1,p+1}^l}, \;\ldots\;, \\ \delta_{11}^l &\equiv d_{10}(L)\rho^{s^*} = d_{10,0}\rho^{s^*} + d_{10,1}\rho^{s^*}L + \ldots + d_{10,p}\rho^{s^*}L^p.\end{aligned}$$

Clearly, all the elements in the $\delta_1^s(L)$, ... , $\delta_6^s(L)$ polynomials in (A.15) are positive if $\left\{\rho^g, \rho^D, \rho^{p^*}, \rho^m, \rho^{i^{s^*}}, \rho^{i^{l^*}}\right\} \in [0,1)$ since the $d_{i,j}$ for all $i = 1, ..., 6$ and $j = 0, ..., p$ are positive. But, the signs of $\delta_7^s(L)$, ... , $\delta_{11}^s(L)$ are not uniquely determined and the $\delta_{i,j}^s$:s for all $i = 7, ..., 11$ and $j = 0, ..., p$ can be both positive or negative even if $\left\{\rho^g, \rho^D, \rho^{p^*}, \rho^m, \rho^{i^{s^*}}\right\} \in [0,1)$. The story about the lag polynomials in (A.16) is somewhat different, although it still holds that $sign\left(\delta_i^l(L)\right) = sign\left(\delta_i^s(L)\right)$ for all $i = 1, ..., 11$. The difference stems from the fact that it is no longer certain that all the elements in the $\delta_1^l(L)$, $\delta_2^l(L)$, $\delta_3^l(L)$, $\delta_4^l(L)$ and $\delta_5^l(L)$ polynomials for $L \geq 1$ are positive, which can be directly seen from the definitions of $\delta_1^l(L)$, $\delta_2^l(L)$, $\delta_3^l(L)$, $\delta_4^l(L)$ and $\delta_5^l(L)$ above.

Finally, combining (8) with (A.15) and (9) with (A.16), introducing the definitions $\delta_0^s \equiv \frac{\sigma q^2}{2}$, $\delta_0^l \equiv k_l$, $\nu_t^s \equiv \psi_0^{s,p}(L)\left(\beta\left(L\right)\right)^{-1}\varepsilon_t^{AS} + \psi_0^{s,X}(L)\varepsilon_t^{IS} + \psi_0^{s,m}(L)\varepsilon_t^{LM}$ and $\nu_t^l \equiv \frac{1}{2}(\psi_0^{s,p}(L)\left(\beta\left(L\right)\right)^{-1}\varepsilon_t^{AS} + \psi_0^{s,X}(L)\varepsilon_t^{IS} + \psi_0^{s,m}(L)\varepsilon_t^{LM})$, establishes (12) for $r = s, l$.

Appendix B The data set

This appendix contains a comprehensive description of the data set utilized in the paper. Below, Table B.1 describes the raw data series and Tables B.2 and B.3 the generation of composite variables on a monthly and quarterly basis.

Table B.1: The raw data set.

Variable	Sample period	Frequency	Source
1,3,6,12 month STB	1982:01-1996:12	daily	Sveriges Riksbank
1,3,6,12 month FEB	1982:01-1992:11	daily	Lindberg and Söderlind (1994)
1,3,6,12 month FEB	1992:12-1996:12	daily	O.c., Sveriges Riksbank
SGB5Y	1984:02-1986:12	monthly	Sveriges Riksbank
SGB10Y	1987:01-1996:12	monthly	OECD MEI
FGB520Y	1980:01-1996:10	monthly	O.c., OECD MEI
CPI	1970:01-1996:10	monthly	IFS
FCPI	1980:01-1996:10	monthly	O.c., Findata
IIP	1960:01-1996:10	monthly	OECD MEI
IIPSA	1960:01-1996:10	monthly	OECD MEI
GDP	1970:1-1979:4	quarterly	SNEPQ-database
GDP	1980:1-1996:2	quarterly	OECD MEI
GDPSA	1980:1-1996:2	quarterly	OECD MEI
PGDP	1970:1-1979:4	quarterly	SNEPQ-database
PGDP	1980:1-1996:2	quarterly	OECD MEI
GC	1970:1-1979:4	quarterly	SNEPQ-database
GC	1980:1-1996:2	quarterly	OECD MEI
GDEBT	1950:01-1996:09	monthly	Swedish National Debt Office
M3	1960:01-1996:10	monthly	OECD MEI

Note: All real macroeconomic variables are measured in 1991 prices. O.c. stands for own calculations. Abbreviations: STB=Yield on Swedish Treasury bills, FEB=Yield on SEK "basket" weighted foreign ECU bills, SGB5Y=Average yields on 5-year Swedish government bonds, SGB10Y=Average yields on 10-year Swedish government bonds, FGB520Y=Average yields on SEK "basket" weighted 5- to 20-year foreign government bonds, CPI=Swedish consumer price index, FCPI=SEK "basket" weighted foreign consumer price index, IIP=Swedish private industrial production index, IIPSA=Seasonally adjusted IIP, GDP=Swedish gross domestic product, GDPSA=Seasonally adjusted GDP, PGDP= Swedish Implicit GDP deflator, GC=Swedish government consumption (investments not included), GDEBT=Nominal value of the Swedish government debt in SEK, M3=Nominal Swedish M3.

Table B.2: Generation of composite data series on quarterly frequency.

Variable	Sample period	Calculation formula
$i^s - i^{s^*}$	1982:1 - 1996:4	Average 3 months STB-FEB
$i^l - i^{l^*}$	1984:1 - 1986:4	SGB5Y-FGB520Y
$i^l - i^{l^*}$	1987:1 - 1996:3	SGB10Y-FGB520Y
g	1970:1 - 1996:2	ln((CG/GDP)*100)
D	1970:1 - 1996:2	((GDEBT-GDEBT(-4))/(GDP*PGDP))*100
p^*	1980:1 - 1996:3	ln(FCPI)
m	1970:1 - 1996:2	ln((M3/(GDP*PGDP))*100)

Note: g, d and m are then subject to seasonal adjustment with the X11-method, as described in Section 4.

Table B.3: Generation of composite data series on monthly frequency.

Variable	Sample period	Calculation formula
$i^s - i^{s^*}$	1982:01 - 1996:12	Average 3 months STB-FEB
$i^l - i^{l^*}$	1984:02 - 1986:12	SGB5Y-FGB520Y
$i^l - i^{l^*}$	1987:01 - 1996:10	SGB10Y-FGB520Y
g	1970:01 - 1996:06	ln(((CGMSA)/GDPMSA)*100)
D	1970:01 - 1996:06	(((GDEBT-GDEBT(-12))/CPI)/(GDPMSA*3))*100
p^*	1980:01 - 1996:10	ln(FPCI)
m	1970:01 - 1996:06	ln(((M3SA/CPI)/(GDPMSA*3))*100)

Note: CGSA and M3SA denote CG and M3 seasonally adjusted with the X11-method respectively. For the period 1970:1 - 1979:4, GDPSA are GDP seasonally adjusted with the X11-method and GDPMSA then denotes the monthly GDPSA figures, generated as described in Section 4. Since the X11-method can only adjust monthly data up to the length of 20 years, the seasonal adjustment of CG and M3 has been divided into the subperiods 1970:01 - 1979:12 and 1980:01 - 1996:06 respectively.