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Modern Forecasting Models in Action: Improving Macroeconomic Analyses at Central Banks

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Abstract

There are many indications that formal methods are not used to their full potential by central banks today. In this paper we demonstrate how BVAR and DSGE models can be used to shed light on questions that policy makers deal with in practice using data from Sweden. We compare the forecast performance of BVAR and DSGE models with the Riksbank's official, more subjective forecasts, both in terms of the actual forecasts and root mean square errors. We also discuss how to combine model- and judgment based forecasts, and show that the combined forecast performs well out-of-sample. In addition, we show the advantages of structural analysis and use the models for interpreting the recent development of the inflation rate using historical decompositions. Lastly, we discuss the monetary transmission mechanism in the formal models, using impulse response functions and conditional forecasts.

Keywords: Bayesian inference; Combined forecasts; DSGE models; Forecasting; Monetary policy; Subjective forecasting; Vector autoregressions.
JEL classification: E52; E37; E47.

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1. Introduction

Since the early 1990s monetary policies in many countries have been focused on achieving a more narrow list of objectives than earlier. This has inspired researchers in macroeconomics to develop new tools for analyses of monetary policy, and the gap between academic research and policy implementation has decreased. Nevertheless, there are many indications that the use of formal methods based on academic research is far from its full potential at central banks. Sims' (2002) review of the analyses at four central banks suggests that there is still room for considerable improvements of the basis for policy decisions by exploiting the recent progress in economics and econometrics. There appears to be a gap between macroeconomics and reality, in the sense that researchers have developed, and used, methods to study macroeconomic data that have had rather little influence on how policy advisors at central banks view the world. If this is to change, the benefits of formal methods have to be clearly demonstrated in situations that policy advisors recognize.

Sims (2002) observes that the formal macro models apparently in use by central banks have up until now either been based on parameters that have been calibrated (rather than estimated), or on parameter estimates from a large number of different sector models, whose joint implications have not been thoroughly investigated. In addition, our experience is that judgments are typically added without exploring the consequences for other variables in the formal model, which creates large conceptual problems when aggregating sector-specific forecasts and analyzing their implications for future inflation prospects.

In this paper we examine how the new generation of formal models can be used in a policy institution like a central bank. The formal models that we discuss have not been "fit to data by ad hoc procedures that have no grounding in statistical theory" in the words of Sims. We compare official forecasts published by Sveriges Riksbank (the central bank of Sweden) with forecasts from two structural models, a dynamic stochastic general equilibrium (DSGE) model and an identified vector autoregression (VAR) estimated with Bayesian methods. We also discuss how the models can be used for story telling and for analysis of alternative policy scenarios, which is an important matter for central banks.

It is rather unusual that formal models are contrasted to the official forecasts of central banks. It is especially interesting to make this comparison given that the latter also include judgments and 'extra-model' information that is very hard to capture within a formal setting. Such an exercise can consequently give an assessment of how useful expertise knowledge is and shed light on the "role of subjective forecasting" which Sims (2002) raised. In his review only the official (Green Book) forecasts of the Federal Reserve were evaluated. We will supplement this analysis by evaluating the official and model forecasts for a small open economy (Sweden). Moreover, Smets and Wouters (2004) have shown that modern *closed economy* DSGEs (with various nominal and real frictions) have forecasting properties well in line with more empirically oriented models such as standard and Bayesian VARs (BVARs). However, this paper evaluates forecasts from DSGE and BVAR models that include *open economy* aspects and compares these with judgemental forecasts. Given the increased complexity of open economy models, and the different monetary policy transmission mechanism where exchange rate movements matter, our exercise adds an extra element to the analysis in Smets and Wouters (2004) and Sims (2002).

In the paper we also demonstrate how formal models can shed light on practical policy questions. This is done by letting the two models interpret the underlying reasons behind the recent economic development, using a historical decomposition of the forecast errors in each of the two models. We show that also a VAR model can be used to clarify what has happened in the economy as long as we are willing to impose some structure to identify the underlying shocks. However, a model with good statistical properties but with little economic theory (i.e., a VAR) may be a good forecasting tool but is probably too overparameterized to give precise answers about the monetary transmission mechanism. The large number of parameters results in imprecise parameter estimates which in turn generates large uncertainty bands for the impulse response functions. As a consequence, the true impact of monetary policy in the VAR is unclear. When we are interested in predictions conditioned upon alternative policy scenarios, such as optional future interest rate paths, an idea about how monetary policy is designed and how it affects the economy is required. By comparing impulse response functions we show that the DSGE model using structure from economic theory provides a much more reasonable transmission mechanism of monetary policy than the BVAR model, which is necessary for producing conditional forecasts that are meaningful from the perspective of a central banker.

The outline of the paper is as follows. Section 2 compares the official forecasts made by the Riksbank with forecasts from the two structural models, a Bayesian VAR model and a small open economy DSGE model estimated with Bayesian methods. We show the actual forecasts as well as the root mean square errors (RMSE) for inflation and the interest rate in the different setups. This section also looks further into the role of subjective forecasting by explicitly examining two episodes where the official and BVAR forecasts diverge. In Section 3 we show the limitations to a non-structural analysis and compare how the two models interpret the recent economic fluctuations by decomposing the forecast errors into different underlying sources. We then go on to contrasting the BVAR and DSGE models' different views about the effects of monetary policy using impulse response functions and conditional forecasts. Section 4 summarizes our views on the advantages of formal methods and on the reasons why such methods have not been more influential at central banks. This also involves some suggestions for future research and policy analysis.

2. Forecasting performance

This section provides an evaluation of the Riksbank's official inflation forecasts, which include subjective components based on judgments from both sector experts as well as the Executive Board, against the forecasts from two formal models, a Bayesian VAR model and a DSGE model for a small open economy. Both the actual forecasts and root mean square errors are examined for the period 1999Q1-2005Q4.[1] A similar evaluation of official forecasts and formal model forecasts has to our knowledge only been conducted for the Federal Reserve Board (see Sims, 2002; Altig et al., 1995).

Since the Riksbank's forecasts have until recently been intended to be conditioned on the assumption of a constant short term interest rate, we also look at the interest rate forecasts from the BVAR and DSGE models.[2] These are compared with implicit interest rate forecasts

[1]Official forecasts from the Riksbank can not be obtained on a quarterly basis before 1999Q1. Moreover, the DSGE model needs sufficiently many observations after the transition from a fixed exchange rate to the inflation targeting regime in 1993.

[2]Since October 2005 the Riksbank produces forecasts conditioned upon implicit forward rates, instead of the constant interest rate assumption.

calculated from (market-based) forward interest rates.[3] To characterize the forecasting advantage of the Riksbank's judgmental forecasts we also analyze two specific episodes where the different forecasting approaches diverge and where we a priori expect the sector experts have an informational advantage.

The Riksbank publishes official forecasts in quarterly Inflation Reports. The forecasts are not the outcome of a single formal model, but rather the result of a complex procedure with input both from many different kinds of models and judgments from sector experts and from the Riksbank's Executive Board. The forecast process during the evaluation period was of a recursive and iterative nature, where the foreign and financial variables entered first. Given these forecasts, the Swedish real variables were predicted. Typically the GDP forecast was an aggregation of the components of the GDP identity. The labor market variables entered in a third step, in which the productivity and unit labor cost variables were determined. Finally, predictions of CPI and core inflation, based mainly upon forecasts of import prices, unit labor cost and the output gap, ended the first forecast round. The Inflation Report only contains tables with yearly data, but monthly forecasts are presented in graphs on the Riksbank's web page. In this paper, we choose to look at quarterly averages of monthly observations.

The BVAR model contains quarterly data for the following seven variables: trade-weighted measures of foreign GDP growth (y_f), CPI inflation (π_f) and a short term interest rate (i_f), the corresponding domestic variables (y, π and i), and the level of the real exchange rate defined as $q = 100(s + p_f - p)$, where p_f and p are the foreign and domestic CPI levels (in logs) and s is the (log) trade-weighted nominal exchange rate. More details on the BVAR are provided in Appendix A.

The DSGE model is an extension of the closed economy models developed by Christiano, Eichenbaum and Evans (2005) and Altig, Christiano, Eichenbaum and Lindé (2003) to the small open economy setting in a way similar to Smets and Wouters (2002). The households consume and invest in baskets consisting of domestically produced goods and imported goods. We allow the imported goods to enter both aggregate consumption and aggregate investment. By including nominal rigidities in the importing and exporting sectors we allow for short-run incomplete exchange rate pass-through to both import and export prices. The foreign economy is exogenously given by a VAR for foreign inflation, output and the interest rate. The DSGE model is estimated with Bayesian methods using data on the following 15 variables: the GDP deflator, the real wage, consumption, investment, the real exchange rate, the short-run interest rate, hours, GDP, exports, imports, the consumption deflator, the investment deflator, foreign (i.e., trade-weighted) output, foreign inflation and the foreign interest rate. A more detailed description of the DSGE model can be found in Appendix B.

2.1. **Inflation forecasts.** The left column in Figure 1 presents the outcome of CPI inflation in Sweden 1998 – 2005 (bold line) together with the Riksbank's official forecasts (first row), and the forecasts from the BVAR model (second row) and the DSGE model (third row)

[3] We do not evaluate the GDP forecasts since no genuine real time data set has been put together. The absence of a real time data set implies that the formal models possibly have an information advantage relative to the official forecasts because the BVAR and DSGE forecasts are based on ex post data on GDP. (Preliminary GDP data are available with a delay of around one quarter, but they are subsequently revised.) This is not a problem with the other variables in the models, since data on prices, interest rates and exchange rates are available on a monthly basis and are not being revised. On the other hand the official forecasts have a small information advantage in some quarters, when the Inflation Reports have been published towards the end of the quarter and thus can be based on data on prices, interest rates and exchange rates from the early part of the same quarter.

for 1999Q1 – 2005Q4.[4] The visual impression is that the official and model forecasts have somewhat different properties. The Riksbank's forecasts appear to be rather "conservative"; most often they predict a very smooth development of inflation, and the changes in the forecast paths are relatively small from quarter to quarter. The BVAR model on the other hand seems to view the inflation process as more persistent, since forecast errors have a larger influence on subsequent forecasts. The Riksbank's forecast at the two years horizon are generally closer to the 2 per cent inflation target compared to the BVAR. The posterior median of the long-run inflation rate in the BVAR is only slightly below the target (1.95 per cent on average in the sequential forecasts), but the forecast horizon is in most periods not long enough for the BVAR forecasts to reach its long-run value.[5]

The top panel in Figure 2 shows the root mean squared errors (RMSE) for different forecast horizons ($1-8$ quarters ahead) of the yearly CPI inflation forecasts. The DSGE and the official forecasts have about the same precision for inflation forecasts made up to a year ahead, but on the somewhat longer horizons (5-8 quarters ahead) the forecasts from the DSGE model have better accuracy than the official inflation forecasts. The one-quarter-ahead BVAR forecasts on the other hand are slightly worse than both the official and the DSGE forecasts but perform very well 2-8 quarters ahead. It should also be noted that all three forecasting methods are better than a naive forecast of constant inflation (No Change). Figure 2 also displays the RMSE profile of a slightly more sophisticated benchmark forecasts: a forecast from a univariate AR(4) with the Riksbank's one-step-ahead forecast as initial value and the unconditional mean of the process restricted to the Riksbank's inflation target of two percent (AR, Infl. Target, RB Init).[6] We take the similar precision in the different inflation forecasts as evidence in favor of all three forecasting approaches. It is encouraging that the model forecasts also on shorter horizons are performing as good as the official forecasts. We will however return to the specific advantages and the role of subjective forecasting below.

Although Smets and Wouters (2004) have shown that the forecasting performance of closed economy DSGE models compares quite favorably to more empirically oriented models such as vector autoregressive models, it is not evident that our DSGE model will have similar properties, given that the open economy dimensions add complexity to the model. In particular it is well known that uncovered interest rate parity (UIP) is rejected empirically, which may deteriorate the forecasting performance of an open economy DSGE model. The UIP condition in the DSGE model has therefore been modified to possibly allow for a negative correlation between the risk premium and expected exchange rate changes, see Adolfson et al. (2006) for details. Figure 2 reveals that our DSGE model has a forecasting performance for CPI inflation that is remarkably good. The DSGE model makes smaller forecast errors for CPI inflation 7 – 8 quarters ahead than both the BVAR forecasts and the official Riksbank forecasts. Part of this can be attributed to the modified UIP condition (see Adolfson et al., 2006), but we also believe that the fact that we are considering a stable regime makes the theoretical structure imposed by the DSGE model useful.

2.2. **Interest rate forecasts.** One difficulty when it comes to comparing official inflation forecasts from the Riksbank with model forecasts (or forecasts made by other institutions)

[4] The data in Figure 1 refer to annual inflation rates ($p_t - p_{t-4}$), just like the inflation published in the Inflation Reports, whereas the BVAR model is specified in terms of quarterly rates of change ($p_t - p_{t-1}$). The BVAR forecasts are summed up to fourth differences.

[5] It should be noted that the BVAR is parameterized such that it allows us to include a prior on the unconditional mean of the system, see Appendix A. For the inflation rate this prior agrees with the Riksbank's long-run inflation target of 2 percent.

[6] We are grateful to Jon Faust for suggesting the alternative benchmark forecasting procedure.

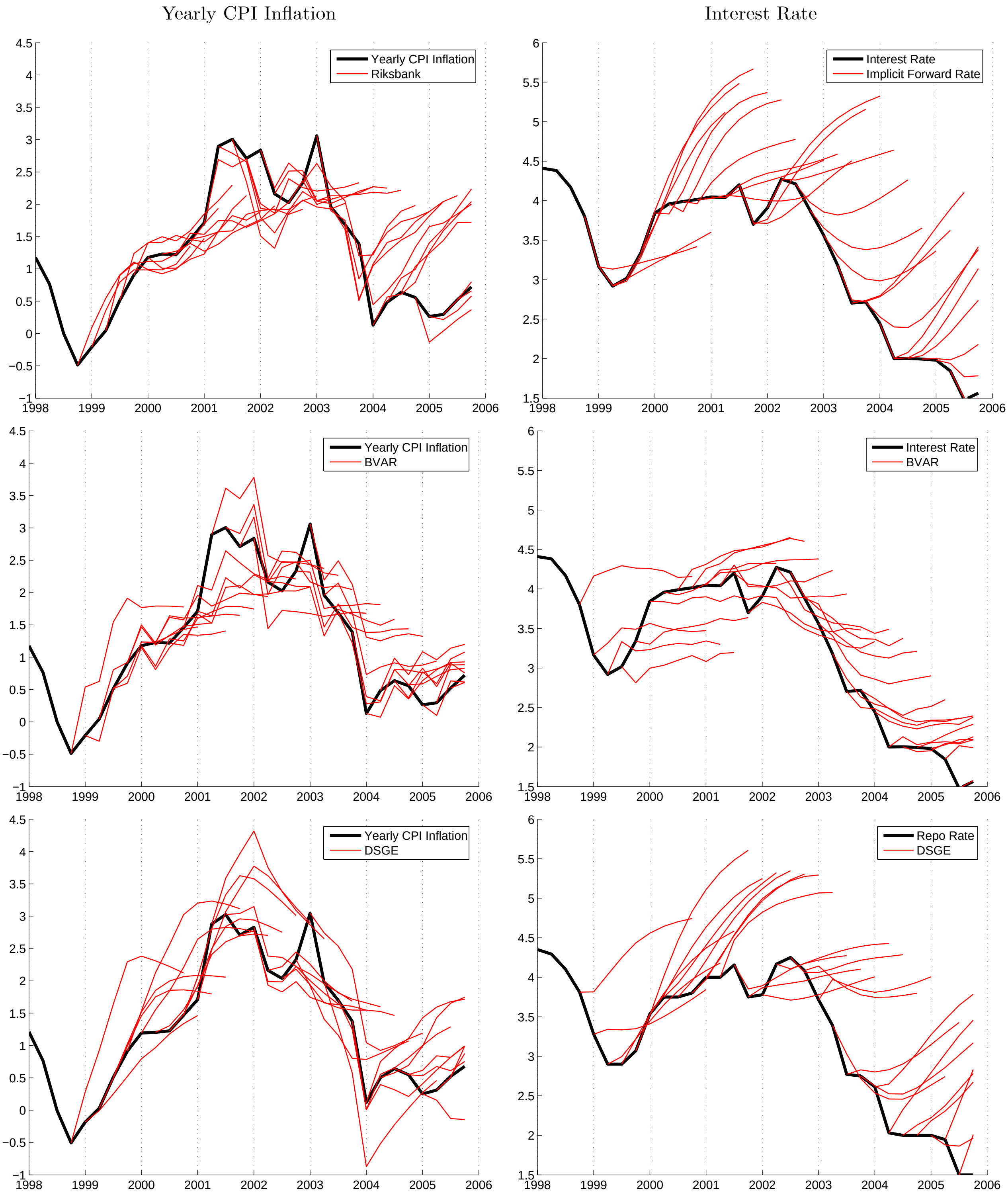

FIGURE 1. Sequential forecasts of yearly CPI inflation (left column) and of the interest rate (right column), 1999Q1-2005Q4, from the Riksbank (first row), the BVAR model (second row) and the DSGE model (third row).

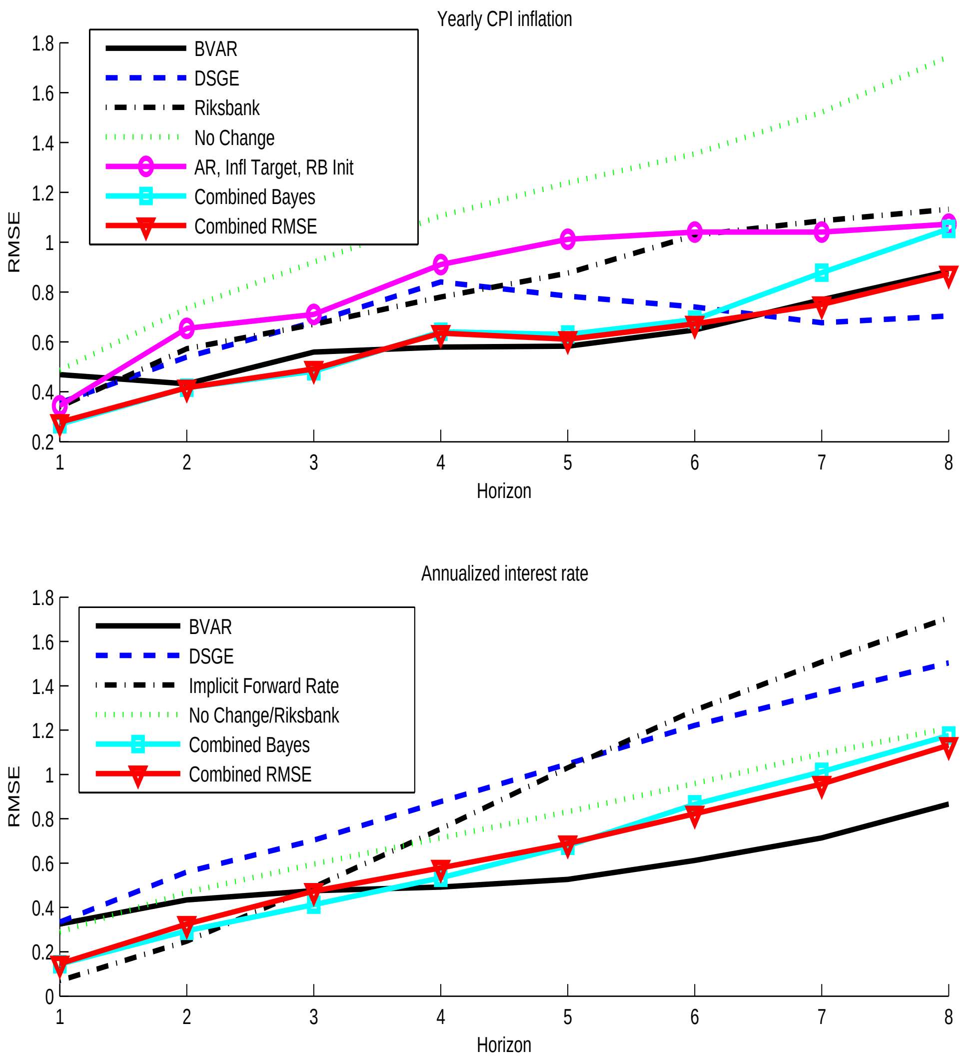

FIGURE 2. Root Mean Squared Error (RMSE) of yearly CPI inflation forecasts (top) and annualized interest rate forecasts (bottom) 1999Q2-2004Q3.

is that the Riksbank's forecasts have until recently been intended to be conditioned on the assumption that the short term interest rate (more specifically the bank's instrument, the repo rate) stays constant throughout the forecasting period.[7]

The right column in Figure 1 shows the three month interest rate (quarterly averages) along with expected interest rate paths derived from forward interest rates (first row), following the method described by Svensson (1995), and the interest rate forecasts from the BVAR model (second row). The DSGE model uses the repo rate (the Riksbank's instrument rate) and in

[7]It is less clear, given the subjective nature of the forecast, what assumptions are made about the interest rate beyond the forecast horizon. The implicit assumption most often seems to have been that the interest rate gradually returns to a level determined by some interest rate equation. Adolfson et al. (2005a) discuss difficulties with constant interest rate forecasts in a model-based environment. The Inflation Report from March 2005 also contains a discussion of the problems with constant interest rate forecasts, as well as forecasts of inflation and GDP growth that are not based on this assumption.

the right bottom panel we show the repo rate and the interest rate forecasts from the DSGE model.[8]

From the figure we see that the interest rate level has been rather stable during the sample period, why the constant interest rate assumption might not be that unfavorable. Moreover, it is extremely difficult to ensure that the official forecasts in practice have been conditioned on a constant interest rate path rather than some more likely path, given the subjective nature of these forecasts. Furthermore the impulse response functions from interest rate changes to inflation are typically fairly small with a substantial time-delay, at least in the BVAR model, which implies that the constant interest rate assumption at long horizons should have relatively small effects on the inflation forecast. Therefore, we believe our forecast comparisons are valid and that our conclusions will not be significantly affected by the constant interest rate assumption.

Throughout our sample, forward interest rates have systematically overestimated the future interest rate level. In principle, this could be due to (possibly time-varying) term and risk premia, i.e., forward rates do not provide direct estimates of expectations. Turning to the interest rate forecasts from the DSGE model we see that future interest rates have been systematically overestimated also here. Comparing the predictions it appears as if the DSGE model's interest rate forecasts have been closer to the implicit forward interest rates than the BVAR model's forecasts, but the BVAR forecasts are much closer to the actual outcomes of the interest rate.

In the bottom panel of Figure 2 we compare the RMSEs for the interest rate forecasts. It can be seen that forward interest rates, a naive constant interest rate forecast and the BVAR model all have about the same precision for forecasts 3 – 4 quarters ahead whereas the DSGE has somewhat worse accuracy. During the last five years it has been easier to make good interest rate forecasts based on "reality" (the BVAR model) than on "macroeconomics" (the DSGE model). This of course raises some questions about how monetary policy is described in the DSGE model. Forward rates have better precision 1 – 2 quarters ahead, while the BVAR model makes much better forecasts for longer horizons than the other approaches. The existence of risk and term premia can probably explain part of the "forecast errors" from forward rates. That the BVAR model provides a more realistic picture at longer horizons than forward rates, which even seem to have lower predictive power than the constant interest rate assumption, may be interpreted as arguments against inflation forecasts conditioned on forward rates. This is nevertheless an assumption that the Bank of England, Norges Bank and Sveriges Riksbank have recently adopted. It should be emphasized, however, that our results have been obtained from a sample where the short term interest rate has been unusually low and stable in a historical context. It is thus possible that forward rates are more informative in periods when interest rates change more.

One hypothesis examined by Sims (2002) is that the central bank (i.e., in his case the Fed) can make good inflation forecasts because it has good information about its own future interest rate decisions. In this respect it is interesting to note that the Riksbank's official inflation forecasts up until a year ahead are about as good as the DSGE model, and in fact slightly worse than the BVAR 2-4 quarters ahead (cf. the top panel in Figure 2). It is perhaps not so surprising to find that the official forecasts 5-8 quarters ahead are worse than those from the BVAR or DSGE models, given that the bank's forecasts are intended to be conditioned on the assumption of a constant interest rate which is not a good forecast for longer

[8]The short term interest rate in the BVAR is thus the three month rate while the DSGE uses the repo rate, why a forecast comparison is somewhat difficult. The differences between the two rates are not large, however (cf. row 2 and 3 in Figure 1).

horizons. However, as noted above, whether the constant interest rate assumption has large effects on the forecasts of other variables in a model depends on the particular transmission mechanism of monetary policy.[9] From the DSGE model we see that it is apparently possible to make relatively good inflation forecasts without particularly good interest rate forecasts. One reason for this can be that the shocks that are important for the interest rate are not the same as those that determine inflation in the model. Another possibility is that the effects of monetary policy shocks on inflation are quite small, so that the interest rate is not crucial for the inflation forecast.

2.3. **The role of subjective forecasting.** To get a better understanding of the properties of the various approaches to forecasting it is interesting to take a closer look at some specific episodes. Such an exercise sheds light on "the role of subjective forecasting" raised by Sims (2002). A closer look at two Swedish episodes where the pure model forecast and the official Riksbank forecast (including judgments) differ provides useful information about whether sector experts can provide a better understanding of recent influences on inflation which perhaps are only temporary. In Figure 3 we therefore compare the Riksbank's official inflation forecasts and the corresponding BVAR forecasts on four different occasions pertaining to these two episodes (i.e., Figure 3 contains a subset of the information in Figure 1).[10]

The first episode concerns forecasts made immediately before and after the sudden increase in inflation in 2001Q2. One important factor behind this increase was an increase in food prices brought about by the mad cow and foot-and-mouth diseases. The inflation rate had however started to increase already during 1999, possibly because of a general improvement of the business cycle. In 2000Q4, the Riksbank's official forecasts implied a slowly increasing inflation rate over the next two years. The BVAR model suggested a somewhat stronger increase in inflation. This may reflect that the model forecasts attributed a larger weight to recent increases in inflation, while the subjective procedure, leading up to the Riksbank's official forecast, underestimated the persistence in changes in inflation. Because of the food price shock, both approaches exhibited large short-run prediction errors. The subjective approach, involving judgments from sector experts, proved to be very useful once this inflation shock had become apparent. At that time, 2001Q2, sector experts expected the food price shock to involve a persistent shock to the price level but with small further effects on annual inflation, and the Riksbank's forecasts from 2001Q2 were more in line with the actual outcome for the next few quarters. The BVAR model on the other hand treated the food price shock as any other inflation shock and overestimated its effects on inflation, mainly during 2001.

The second episode concerns inflation forecasts made in 2003Q1. Cold and dry weather had brought about extreme increases in electricity prices during the winter 2002/2003. This was a temporary shock to the price level, but it had persistent effects on annual inflation which became unusually low when electricity prices declined during spring and summer. The Riksbank's official forecasts described the decline in inflation extremely well for the first two quarters but underestimated the effects on the longer horizons, possibly because it was difficult to separate the effects of changes in energy prices (the oil price also fluctuated heavily) from the downward pressure on inflation from the relatively weak business cycle. In contrast, the BVAR model underestimated the drop in inflation at first but once the decline had started it more correctly predicted that inflation would be very low for the next $1-2$ years (cf. the forecasts from 2003:3 in Figure 3).

[9] For example, to the extent that there is a price puzzle in the impulse response functions, the unusually low outcomes of the interest rate during 2003-2004 may possibly explain why the Riksbank overpredicts inflation during these years.

[10] For ease of exposition we focus only on the Riksbank and BVAR forecasts here.

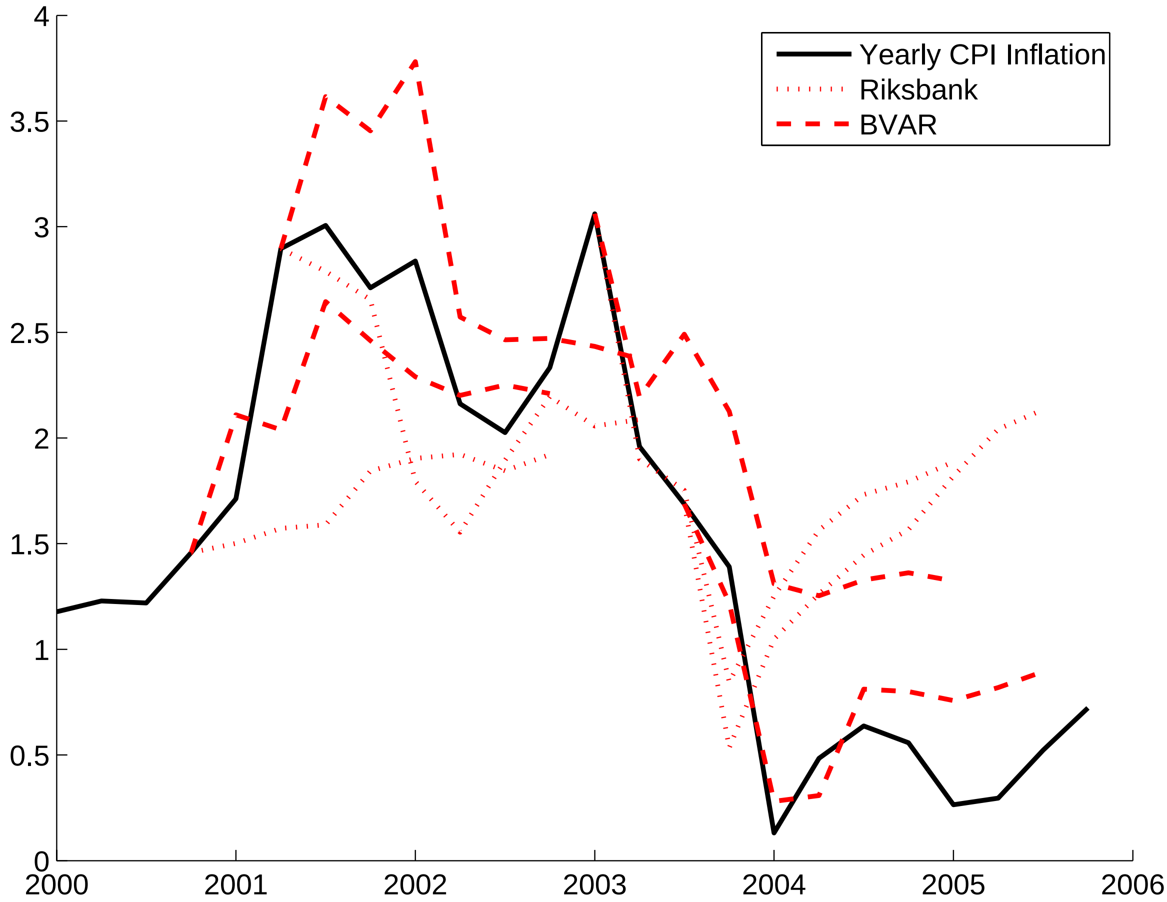

FIGURE 3. Riksbank and BVAR forecasts of yearly CPI inflation made on four specific occasions: 2000Q4, 2001Q2, 2003Q1 and 2003Q3.

These episodes nicely illustrate how formal statistical models and judgments by sector experts can complement each other. Subjective forecasts may sometimes be too myopic and pay too little attention to systematic inflation dynamics related to the business cycle or other historically important regularities. Model forecasts, on the other hand, cannot take enough account of specific unusual but observable events. BVAR models summarize regularities in the data that forecasters should pay attention to. At the same time, judgments from sector experts based on their detailed knowledge about the economy can be extremely useful, in particular when unusual shocks have hit the economy. It should therefore be useful both to develop sets of formal models that could regularly provide policy makers with updated forecasts, and also trying to include information from sector experts formally in these models. One possibility to implement the latter is to make use of the ideas in Waggoner and Zha (1999), who show how to construct conditional forecasts distributions where judgments from sector experts are included.

2.4. **Combined forecasts.** The previous subsections have contrasted model-based and judgment-based macroeconomic forecasts. Being equipped with several different forecasts, the natural question is of course: what can be gained from combining them into a single overall forecast? Combining a set of purely model-based forecasts is rather straightforward, especially within the Bayesian framework where the weights are given by posterior model probabilities (Draper, 1995). When at least one of the forecasting models cannot be represented by a probability model for the observed data, we need to resort to other solutions.

Winkler (1981) proposes an alternative Bayesian approach which may be used to combine model-based and judgment-based forecasts. Winkler's procedure is described in detail in Appendix C. The method assumes that the forecast errors from the different forecasts at a specific time period follow a multivariate normal distribution with zero mean and covariance matrix Σ. It is further assumed that the forecast errors are independent over time. The optimal weights on the individual forecasts can then be shown to be a simple function of Σ^{-1}. This means that the weights depend not only on the relative precision of the forecasts, but also on the correlation between forecast errors. It should be noted that while the weights sum to unity, some weights may be negative. Negative weights arise quite naturally, especially when the forecast errors are highly correlated (Winkler, 1981), but we shall for convenience in interpretation restrict all weights to be non-negative. The results do not change substantially if we allow for negative weights.

The forecast error covariance matrix Σ needs to be estimated from the realized forecast errors available at the time of the formation of the combined forecast. This means that Σ needs to be estimated from a small number of observations. We use a prior distribution to stabilize the estimate of Σ (see Appendix C for details). The assumption of unbiased forecast errors does not seem to hold for the DSGE and implicit forward rate forecasts of the interest rate (see Figure 4), which may have consequences for the combined forecasts. We therefore also consider a combined forecast with weights inversely related to the univariate mean squared forecast errors (MSE), which includes any potential bias. We will form weights separately for each variable and forecast horizon.

Figures 5 shows the sequential weighting schemes for CPI inflation. Note that the weights are equal on all three forecasts in the beginning of the evaluation period where there are not enough realized forecast errors to estimate Σ. From Figure 2 we see that the two combined CPI inflation forecasts perform very well at all forecast horizons. The excellent performance of the combined forecasts at the first quarter horizon is particularly noteworthy. The weights for CPI inflation in the two different weighting schemes are similar on the first and second quarter horizons, at least in the latter part of the evaluation period. At the longer horizons there are fewer forecast errors for constructing the weights and larger biases in the forecasts. This causes the two weighting schemes to differ much more than at the shorter horizons. We want to stress that we only use those forecast errors which were actually available at the time of the forecast. This means that the RMSE evaluation at for example the eight quarter horizon only uses weights up to $2003Q4$.

Turning to the short interest rate we see again that combining forecasts is a good idea. The RMSE of the two combined forecasts are low for all forecast horizons, beaten only slightly by the implicit forward rate at the first two horizons and the BVAR forecast on the longer horizons (see the lower panel of Figure 2). The sequential forecast weights are again more stable at the two shortest horizons compared to the longer ones, see Figure 6. From Figure 6 one can also see that the superiority of the implicit forward rate at the first and second quarter horizons is immediately picked up by both weighting schemes.

What can then be learnt from all this? We have shown that the precision in the Riksbank's inflation forecasts within a year ahead is about the same as in the DSGE model, but that the accuracy in the forecasts one to two years ahead is better in both the DSGE and BVAR models. However, the episode comparison in Section 2.3 made clear that judgments from sector experts can be useful in the short-run, especially when unusual disturbances to the economy occur. Given the low RMSEs on the combined inflation forecast we therefore think

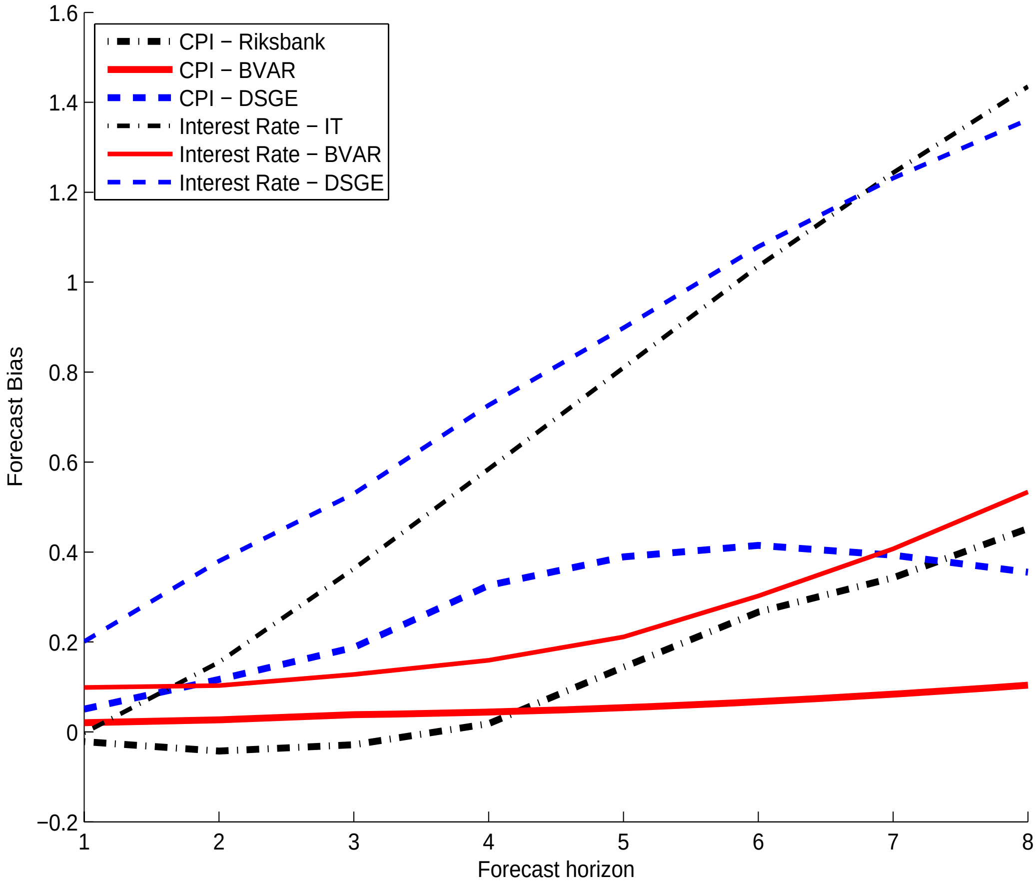

Figure 4. Forecast bias.

it is useful for the central bank to merge these different features into a single combined forecast using the Bayesian weighting schemes.

3. Advantages of structural analysis

3.1. **Historical decompositions.** In Section 2 we have looked at the usefulness of BVAR and DSGE models as basis for discussions about inflation forecasts and interest rate forecasts. But policy makers are also very interested in understanding the factors that have brought the economy to where it is right now. To be able to explain this we need a structural model that can disentangle the underlying causes for the recent economic development. In the DSGE model all shocks are given an economic interpretation, while the BVAR model requires additional identifying restrictions. To exemplify on the use of models for structural analysis, we let the two models interpret the low inflation rate in Sweden during 2003 and 2004, by decomposing the model projections into the various shocks that have driven the output and inflation development.

In Figure 7 we report the actual outcome of GDP growth, inflation and the short term interest rate together with projections from the BVAR model. The first row of Figure 7 reports forecasts made in 2003Q1 under the assumption that no shocks would hit the Swedish economy during 2003 to 2005. It can be seen that parts of the increase in GDP growth and the decreases in inflation and the interest rate were expected but that actual inflation turned

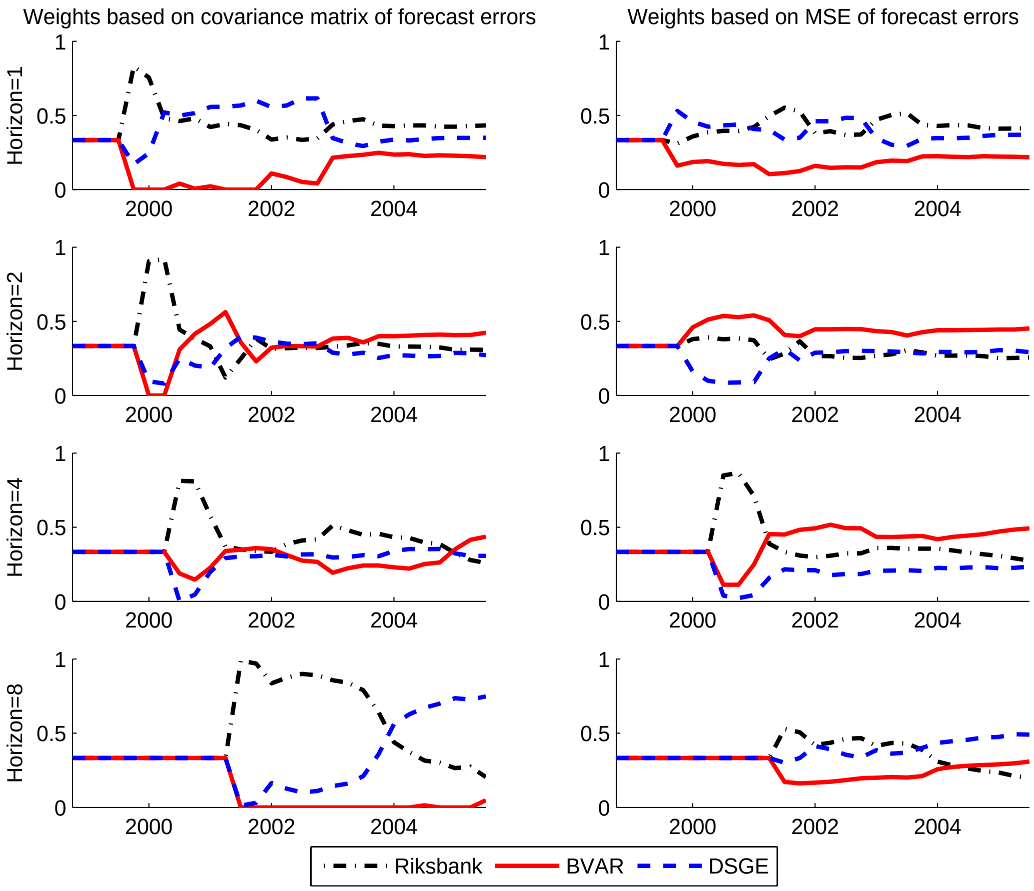

FIGURE 5. Sequential weighting schemes for yearly CPI inflation.

out a lot lower than anticipated by the BVAR model. From the second row of Figure 7, where we have added the "foreign" shocks (identified by the BVAR model ex post)[11] to the BVAR model's no-shock (expected) scenario, we can see that the sudden drop in inflation during 2003 and the lower GDP growth during the first quarters of 2003 were mainly due to "foreign" shocks hitting the economy. The last row of Figure 7 shows the effects of "domestic" shocks, which are simply identified residually as the parts of the forecast errors that are not accounted for by "foreign" shocks. From the last row we see that with only "domestic" shocks the BVAR model overestimates inflation during 2003, although the overall macroeconomic growth seems to be well captured. At longer horizons the picture is more complicated, however. Seemingly the "domestic" shocks are an important source behind the forecast errors in 2004 and 2005 (higher GDP growth and lower inflation) but it should be kept in mind that these paths are contingent upon having only "domestic" shocks also during 2003. By comparing rows two and

[11]The "foreign" shocks are identified through the assumption that foreign GDP growth, foreign inflation and the foreign interest rate are strictly exogenous. Formally, this implies, among other things, that the foreign variables are ordered before all domestic variables in the Choleski decomposition. In addition, the forecast error decompositions in Figure 7 are based on the assumption that the real exchange rate is ordered last in the Choleski decomposition. The results are however not much affected if we instead assume that real exchange rate shocks are treated as entirely "foreign". The results are available upon request. No attempt is made to identify the individual "foreign" shocks.

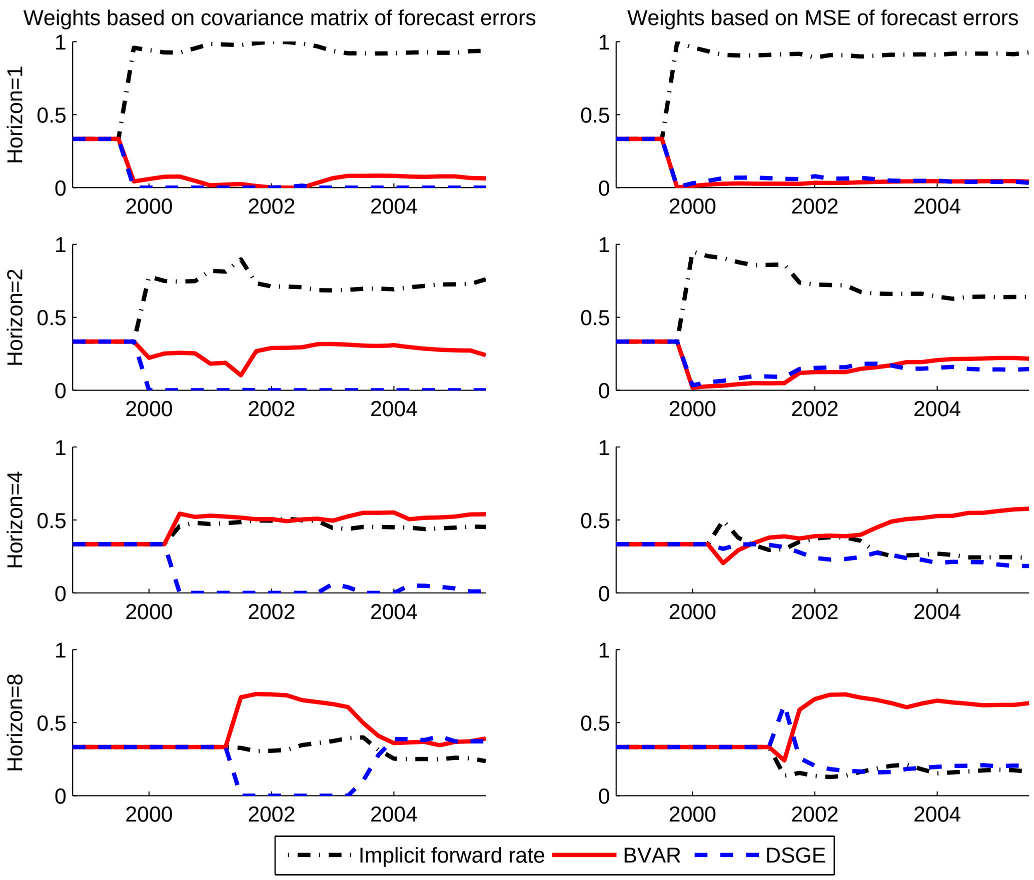

FIGURE 6. Sequential weighting schemes for the interest rate.

three we see that this scenario is less plausible for 2003. In principle, it is nonetheless possible to get much more information about the shocks from the BVAR model than this, if we are willing to make other identifying assumptions.[12]

In Figure 8 we instead decompose the forecast errors from the DSGE model. The first row shows the outcome of GDP growth, inflation and the interest rate along with the predictions from the DSGE model under the assumption that no shocks are hitting the economy (i.e., the corresponding information to the first row of Figure 7 which was based on the BVAR model). Both models overestimated inflation and the interest rate, but the DSGE model also overrated GDP growth to a somewhat larger extent. The other rows in Figure 8 show the "ex post forecasts" from the DSGE model when we add the model's estimates of different kinds of shocks during 2003 – 2005 to the original forecasts from 2003Q1: monetary policy shocks (second row), technology shocks (third row), mark-up shocks (fourth row), foreign shocks (fifth row), preference shocks (sixth row), and fiscal policy shocks (last row). It can be seen that when the estimated technology shocks and foreign shocks are individually taken into account, the "ex post forecasts" of inflation from the DSGE model are rather close to

[12]Other identifying restrictions may involve, e.g., restrictions on long run impulses as in King, Plosser, Stock and Watson (1991). Jacobson, Jansson, Vredin and Warne (2001) present some results based on such restrictions from a VAR model using similar data as this paper. Alternatively, restrictions may be imposed directly on the impulse response functions as suggested by Canova and de Nicoló (2002) and Uhlig (2004).

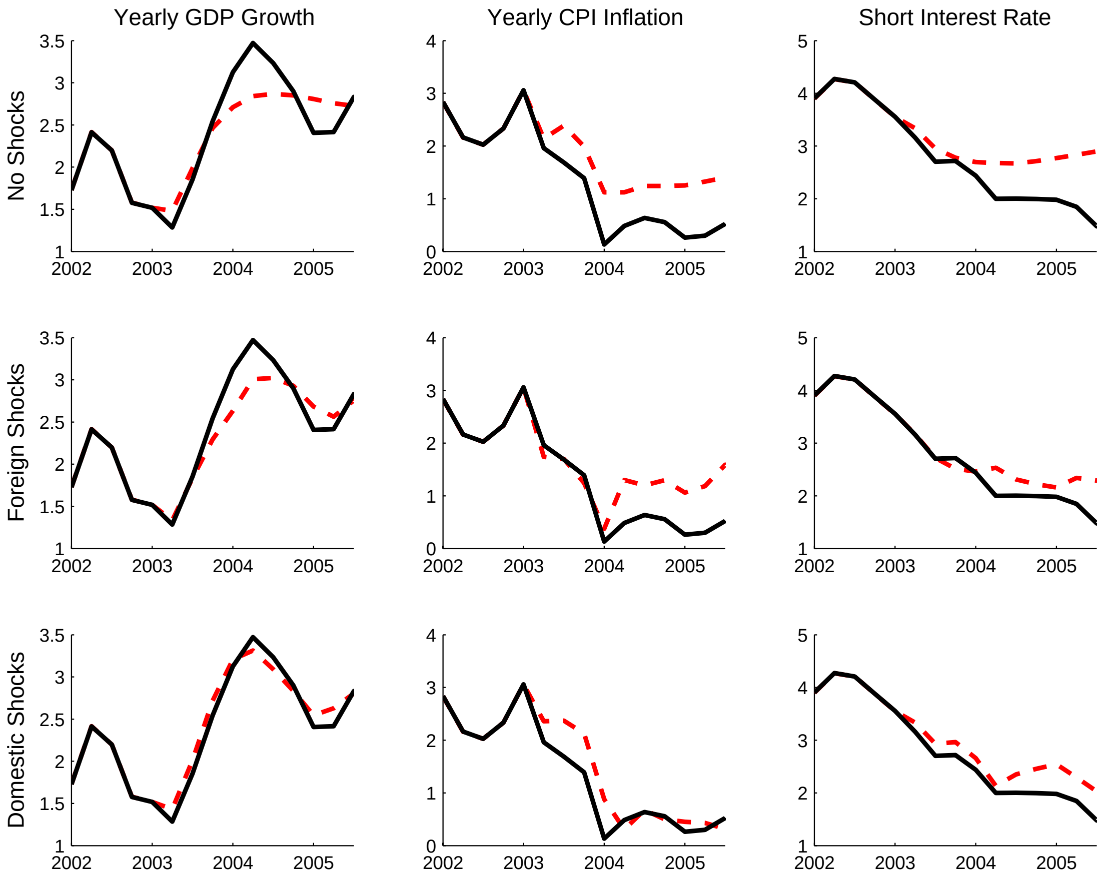

FIGURE 7. Predictions for 2003Q2-2004Q3 from the BVAR model with only subsets of the shocks active during the forecasting period.

the outcome. The DSGE model thus supports the finding from the BVAR model, that a large part of the forecast errors during 2003 were due to foreign shocks. In 2004 and 2005, when the BVAR model suggested that foreign shocks were less important, the DSGE model attributes a large part of the low inflation to both foreign shocks and domestic technology shocks. Interestingly the model suggests that increased competition (i.e., lower markups) is not an important factor for understanding the low inflation outcome.

We find the results from these exercises very promising. Not only can the BVAR and DSGE models make forecasts that have, on average, equally or better precision as the Riksbank's official more subjective forecasts. They can also, ex post, decompose the forecast errors in ways that are informative for policy makers and advisers.

3.2. **The effects of monetary policy shocks and conditional forecasts.** In the previous sections we have shown that the BVAR model, which has good statistical properties but uses very little economic theory, is a fine forecasting tool but is also to some extent able to explain what has happened in the economy as long as we are willing to place some identifying assumptions on the shocks. However, in order to explain what is likely to happen in the future in a policy context, an idea of how monetary policy is designed and how it influences the economy is required. To answer questions about the monetary transmission mechanism we need to add further structure. To work with an identified model is especially important in a

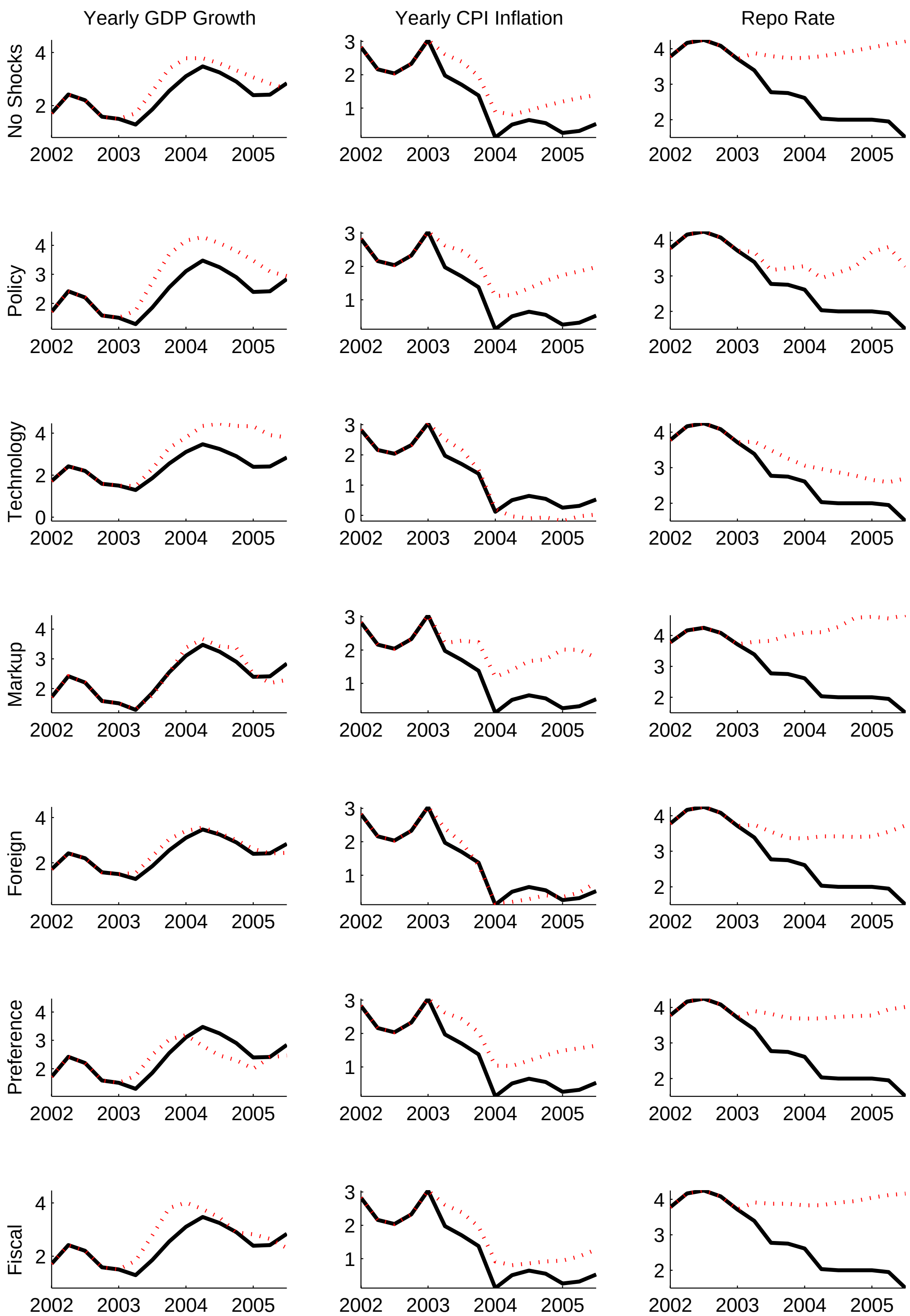

FIGURE 8. Actual values (——) and predictions (· · ·) for 2003Q2-2004Q3 from the DSGE model with only subsets of the shocks active during the forecasting period.

central bank environment where experiments such as predictions conditioned upon alternative interest rate paths are carried out. To see how the links between the interest rate setting and inflation outcomes differ between the BVAR and DSGE models we study impulse response functions but also conditional forecasts in this section.

We start by examining the responses to a one standard deviation interest rate shock in the two models.[13] Figure 9 shows the effects on the interest rate, inflation, output growth and the real exchange rate in the BVAR model[14], and Figure 10 shows the corresponding effects in the DSGE model.

Qualitatively there are some similarities between the effects of an interest rate shock in the BVAR and DSGE models. For example, an increase in the interest rate appreciates the real exchange rate in both models. However, there are also discrepancies between the models. Although an increase of the interest rate gives rise to a decline in output growth and inflation, consistent with typical prejudices, the responses in the BVAR are not significant in contrast to the DSGE model. Moreover, the DSGE model fulfills long-run nominal neutrality whereas there is no such restriction in the BVAR. Further, the quantitative differences are very large. The DSGE model supports the conventional wisdom: if the interest rate is unexpectedly raised by 0.35 percentage points (and then gradually lowered), the maximum effect on inflation is around 0.15 percentage points and is recorded after about $1 - 1\frac{1}{2}$ years. The effects in the BVAR model are much smaller and typically insignificant.

Although there are reasons to expect that monetary policy shocks are more credibly identified in the DSGE model, the differences in impulse response functions and forecasting properties between the BVAR and the DSGE model may create difficulties when using the models in the policy process, even if each individual model compares well to the subjective forecasts and rests on solid methodological grounds.

To illustrate this point, we show the differences between the unconditional and conditional inflation forecasts produced in 2004Q4 from the BVAR and DSGE models in the top panel of Figure 11. The conditional forecasts are generated by injecting monetary policy shocks to keep the interest rate path equal to the implicit forward rate. If we compare the unconditional inflation forecasts, the differences may not seem very large. Both models predict that inflation will increase over the next two years, from around 0.6 per cent to around 1.1 – 1.4 per cent. The DSGE and BVAR models however make quite different predictions about the development of the short term interest rate (see the lower panel of Figure 11).[15] The DSGE model expects the interest rate to increase from 2 per cent to around 3.5 per cent, while the BVAR model expects the interest rate to stay below 2.5 per cent. In order to make the inflation forecasts more comparable, and useful for policy analysis and advice, it makes sense to compute forecasts that are conditioned on the same development of the interest rate. We choose to condition on the interest rate path implied by the forward rate (which has recently

[13]In the BVAR model, this is implemented through exogenous shocks to the interest rate in a Choleski decomposition where the interest rate is ordered after all other variables except the real exchange rate (see Appendix A). Other non-recursive identifying restrictions (e.g. allowing the central bank to react to changes in the real exchange rate within the period, but not to the two GDP variables) gave similar results. In the DSGE model, exogenous shocks are added to the central bank's reaction function (i.e., a monetary policy shock).

[14]We report results for two different BVAR models, one where inflation is measured in terms of CPI and another where inflation is instead measured in terms of "core" (or "underlying") inflation, which excludes the effects on CPI from changes in indirect taxes, subsidies and interest rate costs on mortgage loans. Results from the latter are shown in the right hand graphs of Figure 9. The CPI effects show a temporary "price puzzle" which may depend on the response of mortgage costs, although no such puzzle arises for CPI in the DSGE model.

[15]The solid line is the three month interest rate which is the short term interest rate in the BVAR model, while the DSGE model uses the Riksbank's repo rate, given by the dashed line.

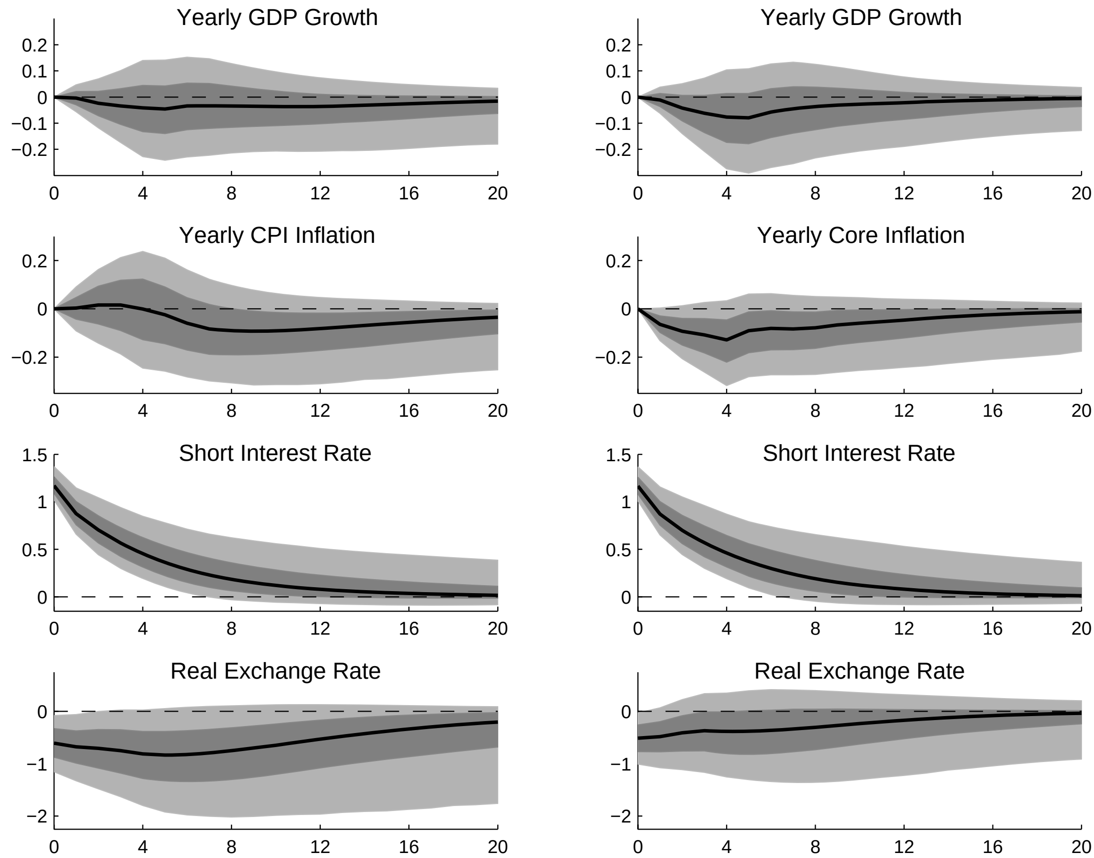

Figure 9. Posterior median impulse response functions for the domestic variables in the BVAR model with 68% and 95% probability bands. The graphs to the left is for a system with CPI inflation. The graphs to the right is for a system with UND1X inflation (a measure of core inflation).

been used by both the Bank of England and Sveriges Riksbank to compute inflation and output forecasts in their Inflation Reports). The forward rate is very close to the interest rate forecast from the BVAR model during the first year, but during the second year the DSGE model's interest rate path rapidly approaches that of the forward rate (see the lower panel of Figure 11). Relative to the unconditional forecasts of inflation these conditional forecasts thus imply a more expansionary monetary policy (lower interest rate) in the DSGE model, and a less expansionary policy (higher interest rate) in the BVAR model. This implies, in turn, that there is a larger difference between the conditional inflation forecasts than between the unconditional forecasts. The DSGE model now predicts that inflation will be close to 2 per cent two years ahead, while the BVAR model still predicts an inflation rate of around 1.1 per cent. Given the large uncertainty which these forecasts are associated with, this may not seem like a very large difference, but for policy purposes, when the inflation target is 2.0 percent, the difference is large enough to question both models in the policy process. The reason why these differences can occur is not surprising, given our earlier findings. From the impulse responses we saw that the effects of monetary policy shocks were larger in the DSGE model than in the BVAR.

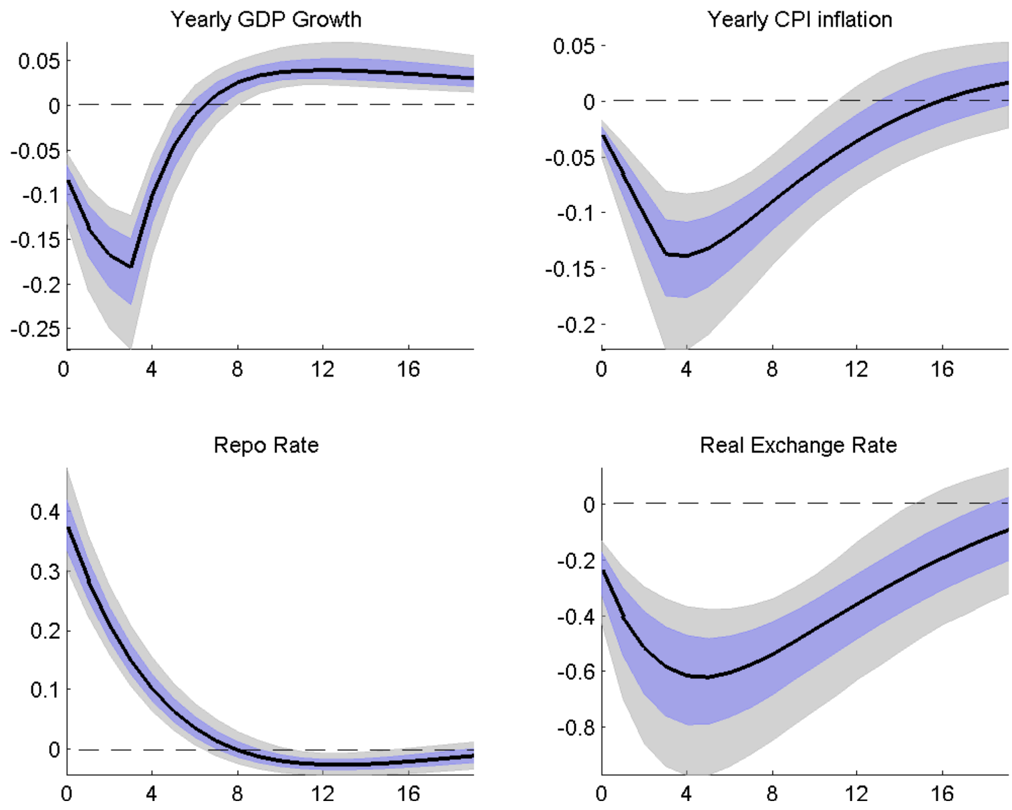

FIGURE 10. Posterior median impulse response functions in the DSGE model with 68% and 95% probability bands.

4. CONCLUDING REMARKS

The theme of this paper is that modern macroeconomic tools like BVAR and DSGE models deserve to be used more in real-time forecasting and for policy advice at central banks, a message which has earlier been emphasized by Altig et al. (1995). We have shown that it is possible to construct and use BVAR and DSGE models that make about as good inflation forecasts as the much more complicated judgmental procedure typically used at central banks. In our view, central banks should use formal models - VARs and DSGEs - as benchmarks for forecasts and policy advice, and to summarize the implications of the continuous flow of new information about the state of the economy that central bank economists are exposed to.

We want to emphasize that we do not view our results as arguments against the use of judgments in monetary policy analysis. The key question here is not to dispute judgments versus formal models but rather how to coherently combine the BVAR and DSGE models with beliefs about the current conditions. Our results suggest that it would be beneficial to incorporate judgments into the formal models, so that the forecasts reflect both judgments and historical regularities in the data.[16] We have analyzed a Bayesian weighting scheme based on the different methods' forecast errors to combine the judgmental and model forecasts. The

[16]Svensson (2005) offers a theoretical analysis of the links between judgments and monetary policy. One way to include judgments in the formal models is to approach this in a Bayesian manner. However, it is less clear how to translate the provided form of judgment into a usable prior distribution.

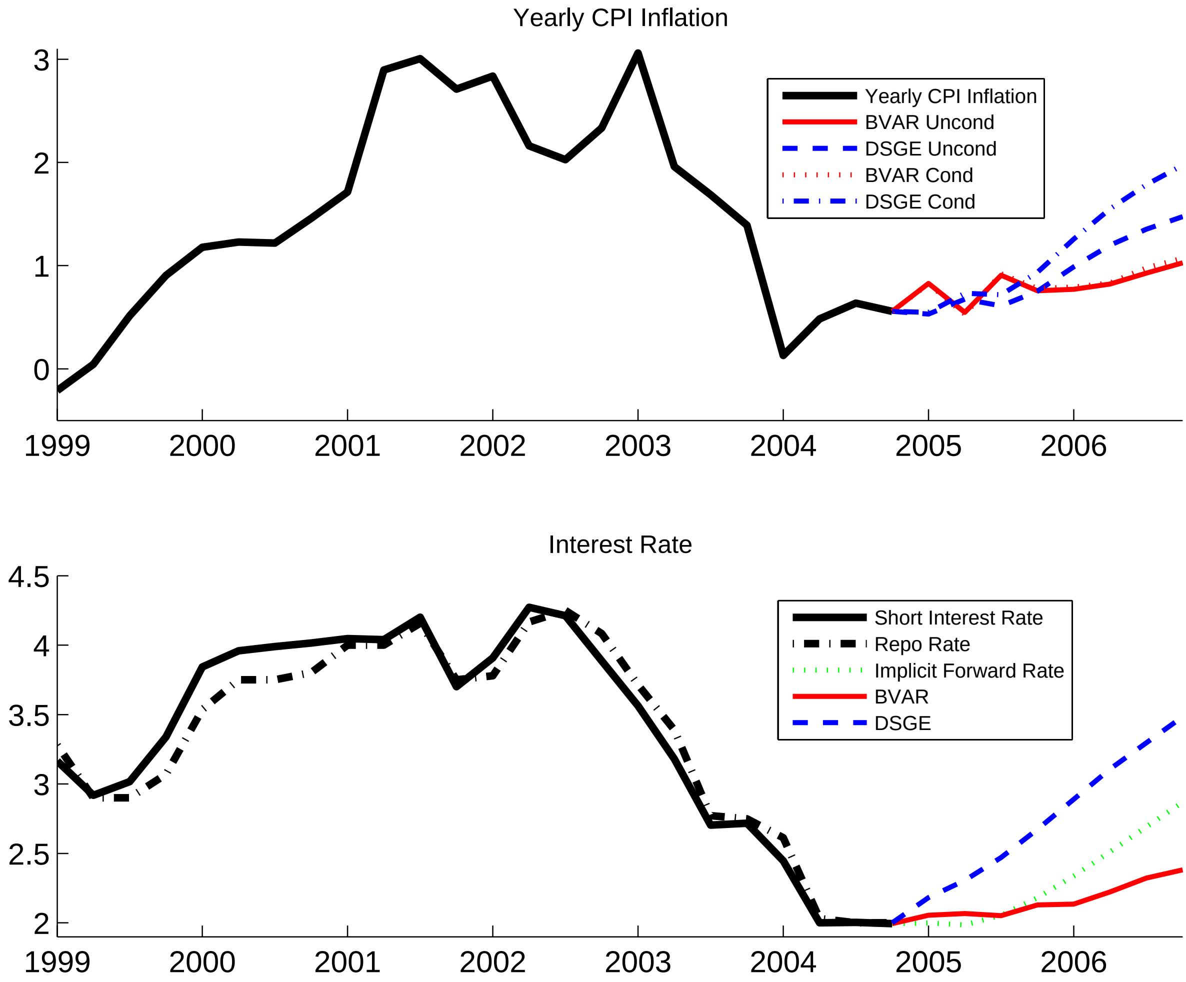

FIGURE 11. Forecasts of yearly CPI inflation (upper panel) and the interest rate (lower panel) produced in 2004Q4 from the BVAR and DSGE models.

results are promising: the root mean squared errors of the combined forecasts of the CPI and the interest rate are consistently low on all evaluated forecast horizons. As an alternative one could incorporate short-run judgments by sector experts directly into the models by exploiting the methodology suggested in Waggoner and Zha (1999).

There are a number of gains of using formal models in the policy analysis; they make it possible to *decompose forecast errors* and provide a tool to characterize the *uncertainty* involved in statements about the future development in the economy. Formal models make it possible to quantify the imprecision and uncertainties involved in the forecasting process. Formal models also serve as a *learning mechanism*, where lessons about the complex interdependencies in the economy can be accumulated. Our results suggest, for example, that subjective forecasts may be too myopic and not take enough account of important historical regularities in the data. Our present version of the DSGE model, on the other hand, may reflect problems with interest rate determination in financial markets, i.e., the empirical failure of the expectation hypothesis and the UIP condition, see the discussion in e.g. Faust (2005). When properties of various forecast approaches are discussed in this way, the models used also become devices for communication, perhaps primarily within central banks. When the economists work with some common models they believe in, it is easier to avoid being trapped in inefficient "battles of anecdotes".[17]

There are of course also limitations to the use of the current generation of modern macroeconomic models for policy purposes. Policy makers are often interested in *details* about the state of the current economy. Formal models cannot possibly cover all details within a tractable consistent framework. Neither can of course sector experts, but their insights about details often lead policy makers to rely on advice and forecasts from the experts rather than from the models. Another problem is that there are *gaps between different models*. Different models give quite different forecasts and imply different policy recommendations. Researchers are not typically bothered by that, as long as the models are considered good. Policy makers are of course bothered. For some policy purposes, we therefore think it makes sense to weight various models according to their empirical performance. We have discussed a procedure to combine forecasts which may also be used when some of the forecasts do not come from a well-specified formal model, which is typically the case in policy work. Another way to bridge the gap between formal models is to use Bayesian prior distributions which incorporate identical prior information on features that are common to the models, such as the steady state of the system (Villani, 2005) or impulse response functions (Del Negro and Schorfheide, 2004). The problems with *model misspecification* also suggest it is worthwhile to think about policies that are robust between different types of models, as suggested by Levin, Wieland and Williams (2003), and not place all emphasis on the outcome of one single model. Implementing the approach of Del Negro and Schorfheide (2004) is one way to account for model misspecification in DSGE models.

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Appendix A. The BVAR model

The BVAR model contains quarterly data on the following seven variables: trade-weighted measures of foreign GDP growth (y_f), CPI inflation (π_f) and a short term interest rate (i_f), the corresponding domestic variables (y, π and i), and the level of the real exchange rate defined as $q = 100(s + p_f - p)$, where p_f and p are the foreign and domestic CPI levels (in logs) and s is the (log of the) trade-weighted nominal exchange rate.

The BVAR model used in this paper is of the form

$$\Pi(L)(x_t - \Psi d_t) = A\varepsilon_t, \tag{A.1}$$

where $x = (y_f, \pi_f, r_f, y, \pi, r, q)'$ is a n-dimensional vector of time series, $\Pi(L) = I_n - \Pi_1 L - ... - \Pi_k L^k$, L is the usual back-shift operator with the property $Lx_t = x_{t-1}$. The structural disturbances $\varepsilon_t \sim N_n(0, I_n)$, $t = 1, ..., T$, are assumed to be independent across time. We impose restrictions on $\Pi(L)$ such that the foreign economy is exogenous. A is the lower-triangular (Choleski) contemporaneous impact matrix, such that the covariance matrix Σ of the reduced form disturbances decomposes as $\Sigma = AA'$. We have also experimented with non-recursive identifying restrictions, in which case the equations are normalized with the Waggoner-Zha rule (Waggoner and Zha, 2003b) and the Gibbs sampling algorithm in Waggoner and Zha (2003a) is used to sample from the posterior distribution. The deterministic component is $d_t = (1, d_{MP,t})'$, where

$$d_{MP,t} = \begin{cases} 1 & \text{if } t < 1993Q1 \\ 0 & \text{if } t \geq 1993Q1 \end{cases},$$

is a shift dummy to model the abandonment of the fixed exchange rate and the introduction of an explicit inflation target in $1993Q1$. Since the data are modelled on a quarterly frequency we use $k = 4$ lags in the analysis. Larger lag lengths gave essentially the same results, with a slight increase in parameter uncertainty.

The somewhat non-standard parametrization of the VAR model in (A.1) is non-linear in its parameters, but has the advantage that the unconditional mean, or steady state, of the process is directly specified by Ψ as $E_0(x_t) = \Psi d_t$. This allows us to incorporate prior beliefs directly on the steady state of the system, *e.g.* the information that the steady state inflation is likely to be close to the Riksbank's inflation target. To formulate a prior on Ψ, note that the specification of d_t implies the following parametrization of the steady state

$$E_0(x_t) = \begin{cases} \psi_1 + \psi_2 & \text{if } t < 1993Q1 \\ \psi_1 & \text{if } t \geq 1993Q1 \end{cases},$$

where ψ_i is the ith column of Ψ. The elements in Ψ are assumed to be independent and normally distributed *a priori*. The 95% prior probability intervals are given in Table A.1.

Table A.1. 95% prior probability intervals of Ψ

	y_f	π_f	r_f	y	π	r	q
ψ_1	$(2,3)$	$(1.5,2.5)$	$(4.5,5.5)$	$(2,2.5)$	$(1.7,2.3)$	$(4,4.5)$	$(385,400)$
ψ_2	$(-1,1)$	$(1.5,2.5)$	$(1.5,2.5)$	$(-0.5,0)$	$(4.3,5.7)$	$(3,5.5)$	$(-50,0)$

The prior proposed by Litterman (1986) will be used on the dynamic coefficients in Π, with the default values on the hyperparameters in the priors suggested by Doan (1992): overall tightness is set to 0.2, cross-equation tightness to 0.5 and a harmonic lag decay with a hyperparameter equal to one. See Litterman (1986) and Doan (1992) for details. Litterman's prior was designed for data in levels and has the effect of shrinking the process toward the univariate random walk model. We therefore set the prior mean on the first own lag to zero for all the variables in growth rates. The two interest rates and the real exchange rate are assigned a prior which centers on the $AR(1)$ process with a dynamic coefficient equal to 0.9. The usual random walk prior is not used here as it is inconsistent with having a prior on the steady state. Finally, the usual non-informative prior $|\Sigma|^{-(n+1)/2}$ is used for Σ.

The posterior distribution of the model's parameters and the forecast distribution of the seven endogenous variables were computed numerically by sampling from the posterior distribution with the Gibbs sampling algorithm in Villani (2005).

Appendix B. The estimated DSGE model

In this appendix, we give a very brief presentation of the DSGE model. For a more detailed description of the model, we refer the reader to Adolfson et al. (2005b).

The model extends the closed economy DSGE model of Christiano, Eichenbaum and Evans (2005) and Altig et al. (2003) by incorporating open economy aspects into it in a way similar to Smets and Wouters (2002). There is a continuum of households which attain utility from consumption, leisure and real cash balances. Consumption preferences are subject to habit formation and households consume a basket consisting of domestically produced goods and imported products. These products are supplied by domestic and importing firms, respectively. The preferences of household j are given by

$$\mathrm{E}_0^j \sum_{t=0}^{\infty} \beta^t \left[\zeta_t^c \ln\left(C_{j,t} - bC_{j,t-1}\right) - \zeta_t^h A_L \frac{(h_{j,t})^{1+\sigma_L}}{1+\sigma_L} + A_q \frac{\left(\frac{Q_{j,t}}{z_t P_t^d}\right)^{1-\sigma_q}}{1-\sigma_q} \right], \tag{B.1}$$

where $C_{j,t}$, $h_{j,t}$ and $Q_{j,t}/P_t^d$ denote the j^{th} household's levels of aggregate consumption, work effort and real cash holdings, respectively. To make the real balances stationary when the economy is growing they are scaled by z_t, the unit root technology shock. We allow for internal habit persistence by including $bC_{j,t-1}$. Aggregate consumption is assumed to be given by a basket of domestically produced and imported goods according to the following constant elasticity of substitution (CES) function:

$$C_t = \left[(1-\omega_c)^{1/\eta_c} \left(C_t^d\right)^{(\eta_c-1)/\eta_c} + \omega_c^{1/\eta_c} \left(C_t^m\right)^{(\eta_c-1)/\eta_c} \right]^{\eta_c/(\eta_c-1)}, \tag{B.2}$$

where C_t^d and C_t^m are consumption of the domestic and imported good, respectively. ω_c is the share of imports in consumption, and η_c is the elasticity of substitution across consumption goods.

The households can save in domestic bonds and foreign bonds, and also hold cash. Following Benigno (2001), we assume that there is a premium on the foreign bond holdings which depends on the aggregate net foreign asset position of the domestic households. This ensures a well defined steady-state in the model.

The households invest in a basket of domestic and imported investment goods to form the physical capital stock, and decide how much capital services to rent to the domestic firms, given capital adjustment costs. These are costs to adjusting the investment rate. Total investment is assumed to be given by a CES aggregate of domestic and imported investment goods (I_t^d and I_t^m, respectively) according to

$$I_t = \left[(1-\omega_i)^{1/\eta_i} \left(I_t^d\right)^{(\eta_i-1)/\eta_i} + \omega_i^{1/\eta_i} \left(I_t^m\right)^{(\eta_i-1)/\eta_i} \right]^{\eta_i/(\eta_i-1)}, \tag{B.3}$$

where ω_i is the share of imports in investment, and η_i is the elasticity of substitution across investment goods.

Further, along the lines of Erceg, Henderson and Levin (2000), each household is a monopoly supplier of a differentiated labour service which implies that they can set their own wage. This gives rise to a wage equation with Calvo (1983) stickiness.

There is a continuum of intermediate domestic firms that each produce a differentiated good. These intermediate goods are sold to a retailer which transforms the intermediate products into a homogenous final good that in turn is sold to the households. The domestic firms determine the capital services and labour inputs used in their production which is exposed to unit root technology growth as in Altig et al. (2003). Production of the domestic intermediate good i follows

$$Y_{i,t} = z_t^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} H_{i,t}^{1-\alpha} - z_t \phi, \tag{B.4}$$

where z_t is a unit-root technology shock, ϵ_t is a covariance stationary technology shock, $K_{i,t}$ the capital stock„ and $H_{i,t}$ denotes homogeneous labour hired by the i^{th} firm. Note that a fixed cost is included in the production function to ensure that profits are zero in steady state.

The domestic firms, the importing and exporting firms all produce differentiated goods and set prices according to an indexation variant of the Calvo model. By including nominal rigidities in the importing and exporting sectors we allow for short-run incomplete exchange rate pass-through to both import and export prices, following for example Smets and Wouters (2002).

To simplify the analysis we adopt the assumption that the foreign prices, output (HP-detrended) and interest rate are exogenously given by an identified VAR(4) model. The fiscal policy variables - taxes on capital income, labour income, consumption, and the pay-roll, together with (HP-detrended) government expenditures - are assumed to follow an identified VAR(2) model.

Appendix C. Combining judgemental and model forecasts

Suppose that we have available forecasts, at a given forecast horizon, from k different forecasting methods over T different time periods. Let $\hat{x}_{jt}$ denote the jth method's forecast of a variable x_t, and $e_{jt} = \hat{x}_{jt} - x_t$ the corresponding forecast error, where $j = 1, ..., k$ and $t = 1, ..., T$. The question here is how to merge these k forecasts into a single combined forecast. Following Winkler (1981), we shall assume that the vector of forecast errors from the k methods, $e_t = (e_{1t}, ..., e_{kt})'$, can be modelled as independent draws from a multivariate normal distribution with zero mean and covariance matrix Σ. This implies that $\hat{x}_t \sim N_k(x_t u, \Sigma)$, where $\hat{x}_t = (\hat{x}_{1t}, ..., \hat{x}_{kt})'$ is the vector of forecasts of x_t from the k forecasting methods, and $u = (1, ..., 1)'$. We use an uninformative (uniform) prior on x_t and an inverted Wishart density for Σ a priori: $\Sigma \sim IW(\Sigma_0, v)$, where $\Sigma_0 = E(\Sigma)$ and $v \geq k$ is the degrees of freedom parameter. The prior on Σ is important as historical forecast errors are limited, and an estimate of Σ is typically unreliable. This is particularly important when the correlations in Σ are large, which is often the case with forecast errors from competing methods. As v increases, the prior becomes more and more concentrated around Σ_0. The specification of Σ_0 and v is discussed below.

The posterior mean of the true value x_t, which is the natural combined forecast for a Bayesian, can now be shown to be a linear combination of the individual forecasts (Winkler, 1981)

$$E(x_t|\hat{x}_t) = \sum_{j=1}^{k} w_{jt} \hat{x}_{jt}, \tag{C.1}$$

where the weights of the forecasting methods are given by

$$w_t' = (w_{1t}, ..., w_{kt}) = \frac{u'\tilde{\Sigma}_t^{-1}}{u'\tilde{\Sigma}_t^{-1}u}, \tag{C.2}$$

and the posterior estimate of Σ is

$$\tilde{\Sigma}_t = E(\Sigma|e_1, ..., e_t) = \frac{v}{t+v}\Sigma_0 + \frac{t}{t+v}\hat{\Sigma}_t,$$

where $\hat{\Sigma}_t$ is the usual unbiased estimator of a covariance matrix.

The weights w_t sum to unity at all dates t, but they need not be positive. Negative weights may result quite naturally, as explained in Winkler (1981), especially when the forecasts are positively correlated across methods.

Strictly speaking, the weighting scheme in (C.2) is only known to be the Bayesian solution under the assumption that forecast errors are independent and unbiased. The independence assumption is likely to be violated for forecast errors beyond the first horizon. We will continue to assume independent forecast errors at all forecast horizons for simplicity. An alternative approach would be to stick to the weighing scheme in (C.2), but with a more sophisticated $\hat{\Sigma}_t$ estimate which accounts for autocorrelation, e.g. the Newey-West estimator. This procedure is unlikely to be a Bayesian solution, however, and suffers also from the drawback that the Newey-West estimator is likely to be unstable when the history of available forecast errors is short. The second assumption behind (C.2) is that forecasts are unbiased. This does not seem to be supported for the implicit forward rate or the DSGE's interest rate forecast at longer horizons, and can potentially have a large effect on the combined forecast. We therefore also look at an ad hoc method for combining forecasts with weights inversely proportional to the mean squared errors (MSE) from past forecasts. Note that this method ignores that forecasts errors of different methods are typically correlated.

We need to determine Σ_0 and v in the Inverted Wishart prior for Σ. We will use the following parametrization of the prior mean of Σ

$$\Sigma_0 = \sigma_0^2 \begin{pmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{pmatrix},$$

which leaves σ_0, ρ and v to be specified. It seems fair to expect all forecasting methods to produce fairly correlated forecasts so that ρ is comparatively large, and also that it increases with the forecast horizon (most methods will produce long run forecasts which are quite close to their steady state value, and the steady states in the different methods should not be too different). Also σ_0 should increase with the forecast horizon. We will assume that both ρ and σ_0 increase linearly with the forecast horizon (σ_0 ranges from 0.5 to 1, and ρ equals 0.5 at the first forecast horizon and 0.92 at the 8th horizon). Finally, we need to pin down the overall precision in the prior, the degrees of freedom parameter, v. We set $v = 50$, which gives us a 95% prior probability interval for ρ at the first horizon equal to $(0.4, 0.75)$. The results are robust to non-drastic variations in the prior.

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