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Bank Mergers, Competition and Liquidity*

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Abstract

We model the impact of bank mergers on loan competition, reserve holdings and aggregate liquidity. A merger creates an internal money market that affects reserve holdings and induces financial cost advantages, but also withdraws liquidity from the interbank market. Loan market competition modifies the heterogeneity in the size of banks, thus affecting aggregate liquidity. Mergers among large banks tend to increase aggregate liquidity needs and thus the liquidity provision in monetary operations by the central bank.

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Keywords: Credit market competition, bank reserves, internal money market, banking system liquidity

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1 Introduction

The last decade has witnessed an intense process of consolidation in the financial sectors of many industrial countries. This 'merger movement', documented in a number of papers and official reports, was particularly concentrated among banking firms and occurred mostly within national borders.[1] As shown in Figure 1, in countries like Canada, Italy and Japan more than half of the banks combined forces over the 1990s.

[FIGURE 1 ABOUT HERE]

As a consequence, many countries (e.g., Belgium, Canada, France, the Netherlands, and Sweden) reached a situation of high banking concentration or faced a further deterioration of an already concentrated sector. As can be seen from Table 1, a small number of large banks often constitutes more than 70 per cent of the national banking sector.

[TABLE 1 ABOUT HERE]

This consolidation process raises a number of important questions. In this paper we concentrate on the potential consequences of this extensive consolidation process for the competitiveness of credit markets, reserve management and banking system liquidity. The conventional wisdom is that consolidation may lower liquidity needs and reduce activity in the interbank market. As stated in the G-10 'Report on Financial Sector Consolidation', '...by internalizing what had previously been interbank transactions, consolidation could reduce the liquidity of the market for central bank reserves, making it less efficient in reallocating balances across institutions and increasing market volatility' (Group of Ten, 2001, p. 20).[2] In this paper we show that this analysis misses a number of important effects.

[1] See, e.g., Boyd and Graham (1996), Berger et al. (1999), Hanweck and Shull (1999), Dermine (2000), ECB (2000), OECD (2000) and Group of Ten (2001).

[2] The effects of consolidation on interbank market liquidity are of course most pronounced in smaller countries with national money markets, such as Denmark, Sweden or Switzerland (private communication from central banks). For example, the Swiss banking system is now dominated by two main players. In order to moderate adverse effects on liquidity, the Swiss National Bank considerably facilitated foreign banks' access to the Swiss franc money market. "With this opening the influence of the main banks on the conditions in the money market was reduced. Their share of total outstanding liquidity transactions declined from more than 80% to now around 50%" (quote from the SNB Board Member Bruno Gehrig at the Jahresend-Mediengespräch of 8 December 2000, see http://www.snb.ch/d/aktuelles/referate/ref_001208_bge.html; translation by the authors).

First, in addition to the diversification effect there is an *internalization* effect. A merger creates an internal money market where liquidity can be reshuffled. On the one hand, the diversification effect related to the pooling of idiosyncratic liquidity shocks induces the merged banks to reduce reserves. On the other hand, the possibility to reshuffle reserves increases their marginal value, as one unit of reserve can be used to cover a liquidity demand at either bank. This leads the merged banks to increase reserves. We find that the internalization effect is stronger when the relative cost of refinancing is low, i.e., when the cost for banks of borrowing on the interbank market in case of liquidity shortage is low relative to the cost of raising deposits and keeping more reserves initially. In contrast, when the relative cost of refinancing is high, the diversification effect dominates and banks reduce reserve holdings. In both circumstances, the merged banks improve their liquidity situation, having lower liquidity risk and expected liquidity needs. Moreover, by lowering refinancing costs, the internal money market generates endogenous financial cost efficiencies, which reduce, ceteris paribus, the anti-competitive effects of mergers between banks. These results suggest that merged banks benefit from scope economies in their liquidity management by raising deposits in two imperfectly correlated deposit markets.

Second, the change in loan competition deriving from a merger also has important effects. It changes the size of banks, and this affects the aggregate demand for liquidity. When banks retain some market power through differentiation on the loan market, a merger modifies both loan rates and market shares. As known from the industrial organization literature, the overall effect of a merger on loan rates depends on how strong the increase in market power is relative to potential efficiency gains. What is most important for our analysis, though, is that the change of loan rates induces also a modification of the size of banks. The merged banks gain market share at the expense of competitors when loan rates fall, and lose market share otherwise. Thus, consolidation changes banks' balance sheets, creating (or reducing) heterogeneity. This has an important impact on the aggregate demand for liquidity.

The interbank market is affected by two channels working through changes in both banks' reserve holdings and loan competition. Higher reserve holdings increase aggregate liquidity supply, and vice versa. We call this mechanism the *reserve* channel. Concerning the change in the size of banks, we show that greater heterogeneity among banks increases the variance of the aggregate liquidity demand, thus leading, ceteris paribus, to higher aggregate liquidity needs. Whether this *asymmetry* channel works in the same or opposite direction from the reserve channel depends on the size of the relative cost of refinancing.

The model delivers several insights, which we interpret according to size of mergers and type of country or financial system. First, mergers between large banks leading to a 'polarization' of the banking system with large and small institutions are more likely to lead to higher aggregate liquidity needs than mergers involving small banks, since they increase the heterogeneity in banks' balance sheets. This result is particularly noteworthy in light of Table 1, which suggests that the banking sector consolidation of the 1990s led to greater asymmetry in the size of banks. Second, mergers are more likely, ceteris paribus, to increase aggregate liquidity needs in developing countries than in industrial ones, since they induce lower individual reserve holdings in less efficient markets, where banks face high refinancing costs. Third, the effects of consolidation on loan competition and aggregate liquidity tend to be complementary in industrial countries but not in developing ones. In fact, whereas mergers are likely to affect competition and liquidity in the same direction when the cost of refinancing is low (i.e., mergers between large banks are likely to increase both loan rates and expected aggregate liquidity needs, and vice versa for mergers involving small banks), they always push towards larger expected liquidity needs when the cost of refinancing is high, independently of the effect on loan competition.

To sum up, we show that the effect of consolidation on the interbank market crucially depends on how mergers affect both banks' reserve management and loan competition. The model stresses the functioning of an internal money market, which −contrary to conventional wisdom− may increase merged banks' optimal reserve holdings, while always reducing their financing costs and liquidity risk. This result finds empirical support in Hughes et al. (1996), who find that banks active in imperfectly correlated deposit markets have lower costs of controlling liquidity risk, especially after consolidation. Concerning the link between aggregate liquidity and loan competition, the model shows that imperfect loan market competition affects the size heterogeneity of the banking system and thus the volatility of the aggregate demand in the interbank market, since different sized banks have different liquidity needs. This result indicates that consolidation may affect negatively the interbank market, despite increasing individual reserve holdings and reducing individual liquidity risk; and that, be-sides reserve holdings and clearing needs, also the degree of loan competition may affect the impact of consolidation on the functioning of the interbank market and thus on the need and scope of central banks' interventions.

Relation with the literature

Our approach to study the joint implications of bank mergers for competition, individual and aggregate liquidity combines elements of the industrial organization literature on the implications of exogenous mergers under imperfect competition with the financial intermediation literature characterizing banks as liquidity providers. As in Deneckere and Davidson (1985) and Perry and Porter (1985), banks have incentives to merge to acquire market power. Differently from these papers, however, in our model banks' incentives to merge are also driven by financing cost advantages related to size, and in particular, by the gains from the optimal adjustment of reserve holdings due to the presence of an internal money market. In this sense, our paper also links the industrial organization literature on mergers with the contributions of Yanelle (1989, 1997) and Winton (1995, 1997) on the relation between competition and diversification in finite economies.

The field of research studying the role of banks as liquidity providers started with Diamond and Dybvig (1983). More recently Kashyap, Rajan and Stein (2002) describe the links between banks' liquidity provision to depositors and their liquidity provision to borrowers through credit lines; and Diamond (1997), discusses the relationship between the activities of Diamond-and-Dybvig-type banks and liquidity of financial markets. Concerning liquidity provision by public authorities, Holmstrom and Tirole (1998) analyze the role of government debt management in meeting the liquidity needs of the productive sector. This literature, however, has not yet considered the implications of imperfect competition and financial consolidation for private and public provision of liquidity, which . Our paper puts this at center stage, by studying how mergers change bank reserve holdings, aggregate liquidity fluctuations and the amount of liquidity that is provided by central banks in their monetary operations.

Several authors have studied the rationale for an interbank market and its effect on reserve holdings. For example, Bhattacharya and Gale (1987) show that banks can optimally cope with liquidity shocks by borrowing and lending reserves; but they also argue that moral hazard and adverse selection lead to under-investment in reserves. Bhattacharya and Fulghieri (1994) add that with some changed assumptions reserve holdings can also become excessive. These authors argue that the central bank has a role in healing these imperfections. Allen and Gale (2000) and Freixas et al. (2000) analyze how small unexpected liquidity shocks can lead to liquidity shortages in the banking system and thus, in the absence of a central bank, to contagious crises. We discuss how the likelihood and the extent of such shortages vary with changes in market structure when a central bank stands ready to offset private market liquidity fluctuations through monetary operations.

The paper is also related to the literature on internal capital markets. Gertner et al. (1994) and Stein (1997) discuss the efficiency-enhancing role of these internal markets. While Scharfstein and Stein (2000) and Rajan et al. (2000) warn that they might also become inefficient if internal incentive problems and power struggles lead to excessive cross-divisional subsidies, the empirical results of Graham et al. (2002) suggest that 'value destruction' in firms is not related to consolidation, supporting the idea of efficiently functioning internal capital markets. Concerning banks, Houston et al. (1997) provide evidence that loan growth at subsidiaries of US bank holding companies (BHCs) is more sensitive to the holding company's cash flow than to the subsidiaries' own cash flow; and Campello (2002) shows that the funding of loans by small affiliates of US BHCs is less sensitive to affiliate-level cash flows than independent banks of comparable size. Focusing on short-term assets, we show how the creation of an internal money market can cushion external liquidity shocks and how it affects banks' reserve choices and banking system liquidity. We also show that the financing cost advantages associated with the internal money market lead the merged banks, ceteris paribus, to be more aggressive on the loan market.

The remainder of the paper is structured as follows. Section 2 sets up the model. Section 3 derives the equilibrium before a merger ('status quo'). The subsequent section characterizes the effects of a merger on individual banks' behavior; and Section 5 looks at its implications for aggregate liquidity. Section 6 contains a discussion of the different scenarios for competition and liquidity effects of bank consolidation. Section 7 concludes. All proofs are in the Appendix.

2 The Model

Consider a three date ($T = 0, 1, 2$) economy with three classes of risk neutral agents: N banks ($N > 3$), numerous entrepreneurs, and numerous individuals. At date 0 banks raise funds from individuals in the form of retail deposits, and invest the proceeds in loans to entrepreneurs and in liquid short-term assets denoted as reserves. Thus, the balance sheet for each bank i is

$$L_i + R_i = D_i, \tag{1}$$

where L_i denotes loans, R_i reserves, and D_i deposits.

Competition in the loan market

Banks offer differentiated loans and compete in prices. The differentiation of loans may emerge from long-term lending relationships (see, e.g., Sharpe, 1990; Rajan, 1992), specialization in certain types of lending (e.g., to small/large firms or to different sectors) or in certain geographical areas. Following Shubik and Levitan (1980), we assume that each bank i faces a linear demand for loans given by

$$L_i = l - \gamma \left(r_i^L - \frac{1}{N} \sum_{j=1}^{N} r_j^L \right), \tag{2}$$

where r_i^L and r_j^L are the loan rates charged by banks i and j (with $j = 1, ..., i, ..., N$), and the parameter $\gamma \geq 0$ represents the degree of substitutability of loans. The larger γ the more substitutable are the loans. Note that expression (2) implies a constant aggregate demand for loans $\sum_{i=1}^{N} L_i = Nl$, as in Salop (1979).

Processing loans involves a per-unit provision cost c, which can be thought of as a set up cost or a monitoring cost. Loans mature at date 2 and yield nothing if liquidated before maturity.

Deposits, individual liquidity shocks and reserve holdings

Banks raise deposits in N distinct 'regions'. A region can be interpreted as a geographical area, a specific segment of the population, or an industry sector in which a bank specializes for its deposit business. There is a large number of potential depositors in every region, each endowed with one unit of funds at date 0. Depositors are offered demandable contracts, which pay just the initial investment in case of withdrawal at date 1 and a (net) rate r^D at date 2. The deposit rate r^D can be thought of as the reservation value of depositors (the return of another investment opportunity), or, alternatively, as the equilibrium rate in a competition game between banks and other deposit-taking financial institutions.

As in Diamond and Dybvig (1983), a fraction δ_i of depositors at each bank develops a preference for early consumption, and withdraws at date 1. The remaining $1-\delta_i$ depositors value consumption only at date 2, and leave their funds at the bank a period longer.[3] The fraction δ_i is assumed to be stochastic. Specifically, δ_i is uniformly distributed between 0 and 1, and it is i.i.d. across banks.[4] This introduces uncertainty at the level of each individual bank and in aggregate. All uncertainty is resolved at date 1, when liquidity shocks materialize.

Each bank keeps reserves R_i to face its date 1 *demand for liquidity* $x_i = \delta_i D_i$. Reserves represent a storage technology that transfers the value of investment from one period to the next. We may think of cash, reserve holdings at the central bank, or even short-term government securities and other safe and low yielding assets. (The interest rate on reserves needs not be zero.)

The stochastic nature of δ_i implies that the realized demand for liquidity x_i may exceed or fall short of R_i. Denoting as $f(x_i)$ the density function of x_i, from an ex ante perspective each bank faces a *liquidity risk* $-$ the probability to experience a liquidity shortage at date 1 $-$ given by

$$\phi_i = prob(x_i > R_i) = \int_{R_i}^{D_i} f(x_i)dx_i. \tag{3}$$

Banks have *expected liquidity needs* $-$ the expected sizes of liquidity shortages that need to be refinanced at date 1 $-$ equal to

$$\omega_i = \int_{R_i}^{D_i} (x_i - R_i)f(x_i)dx_i. \tag{4}$$

[3] The fraction δ_i can also be interpreted as a regional macro shock. For example, weather conditions may change the general consumption needs in a region, so that each depositor withdraws a fraction δ_i of his initial investment.

[4] We assume for simplicity that liquidity shocks are independent across banks, but all our results remain valid as long as liquidity shocks are not perfectly correlated.

Interbank refinancing and aggregate liquidity

At date 1 an interbank market opens where banks can either borrow or lend depending on whether they have shortages ($x_i < R_i$) or excesses ($x_i > R_i$) of reserves. We focus on the ultra-short interbank or money market, such as the unsecured market for wholesale deposits, where both banks and the central bank operate.[5] Since in this market rates are always in between the policy rates at which sound individual banks may receive(give) overnight deposits from(to) the central bank (e.g., the marginal lending and the deposit rates in the euro area, and the rate on primary credit in the US; see, e.g., Hartmann et al., 2001; and ECB, 2004), we assume that banks can borrow at a rate r^{IB} and lend at a rate r^{IL}, independently of the counterpart.

Given the presence of aggregate uncertainty, there may be an aggregate shortage or an aggregate excess of liquidity on the market. An aggregate shortage of private liquidity occurs whenever the aggregate demand for liquidity is higher than the aggregate supply of liquidity represented by the sum of individual banks' reserves, i.e., whenever

$$\sum_{i=1}^{N} x_i > \sum_{i=1}^{N} R_i. \tag{5}$$

Denoting as $X_i = \sum_{i=1}^{N} x_i$ the aggregate demand for liquidity with density function $f(X_i)$, we express the frequency with which aggregate private shortages occur through the *aggregate (or systemic) liquidity risk*

$$\Phi = prob\left(X_i > \sum_{i=1}^{N} R_i\right) = \int_{\sum R_i}^{\sum D_i} f(X_i)dX_i, \tag{6}$$

and the expected size through the *expected aggregate (or systemic) liquidity needs*

$$\Omega = \int_{\sum R_i}^{\sum D_i} \left(X_i - \sum_{i=1}^{N} R_i\right) f(X_i)dX_i. \tag{7}$$

[5] The most relevant and largest ultra-short market is the overnight market, in which banks exchange liquidity at the so-called 'overnight' or 'Fed funds' rates (e.g., bid and ask rates). Most central banks stabilize those market rates around an 'official rate' (e.g., the Fed Fund target rate in the US, and the minimum bid rate in the euro area) by adjusting the supply of liquidity to changes in the aggregate demand. Recent evidence indicates that central banks control overnight rates quite successfully (e.g., Carpenter and Demiralp, 2005; Pérez Quirós and Rodríguez Mendizábal, 2005).

The aggregate liquidity risk (6) and the expected aggregate liquidity needs (7) can then be interpreted as measures of the degree to which the banking system depends on the public supply of liquidity. Formulated differently, they represent indicators of the size of central bank operations in the implementation of monetary policy.

The timing of the model is summarized in Figure 2. At date 0 banks compete in prices in the loan market, choose reserve holdings, and raise deposits. After liquidity shocks materialize at date 1, the interbank market opens. At date 2 loans mature, and remaining claims from deposits and the interbank market are settled.

Figure 2: Timing of the model

T=0	T=1	T=2
price competition in the loan market, choice of reserves, deposits are raised	shocks δ_i materialize, interbank market opens	loans mature, claims are settled, and profits materialize

3 The Status Quo

In this section we characterize the equilibrium when all banks are identical. We start with noting two features of the model. First, bank runs never occur in this model. The illiquidity of loans together with $r^D > 0$ guarantees that depositors withdraw prematurely only if hit by liquidity shocks. Second, we assume that the loan market is sufficiently profitable (differentiated) for banks to borrow in the deposit and interbank markets. So, we can directly focus on the date 0 maximization problem.

With these considerations in mind, at date 0 each bank i chooses the loan rate r_i^L and the reserves R_i so as to maximize the following expected profit (for simplicity, the intertemporal discount factor is normalized to one):

$$\Pi_i = (r_i^L - c)L_i + \int_0^{R_i} r^{IL}(R_i - x_i)f(x_i)dx_i - \int_{R_i}^{D_i} r^{IB}(x_i - R_i)f(x_i)dx_i - r^D D_i(1 - E(\delta_i)). \tag{8}$$

The first term in (8) represents the profit from the loan market, the second term is the expected revenue from interbank lending at date 1 when the bank is in excess of reserves, the third term is the expected cost of refinancing at date 1 when the bank faces a shortage of reserves, and the fourth term is the expected repayment to depositors leaving their funds until date 2. Taken together, the last two terms represent bank i's *financing costs*.

For expositional convenience and without loss of generality, we set $r^{IL} = 0$ and denote r^{IB} simply as r^I. (No qualitative result depends on this simplification, which also captures the stylized fact that the interbank market is relatively 'passive' in that banks do not keep reserves to make profits, but only to protect themselves against liquidity shocks.)

The following proposition characterizes the symmetric equilibrium in the status quo.

Proposition 1 *The symmetric status quo equilibrium is characterized as follows:*

1. *Each bank sets a loan rate* $r^L_{sq} = \frac{l}{\gamma(\frac{N-1}{N})} + c_{sq}$, *where* $c_{sq} = c + \sqrt{r^I r^D}$*;*
2. *It has a loan market share* $L_{sq} = l$*;*
3. *If* $r^I > r^D$, *it keeps reserves* $R_{sq} = \left(\sqrt{\frac{r^I}{r^D}} - 1\right) L_{sq}$, *and raises deposits* $D_{sq} = L_{sq}\sqrt{\frac{r^I}{r^D}}$.

The equilibrium loan rate r^L_{sq} diverges from the total marginal cost c_{sq} via the mark up $\frac{l}{\gamma(\frac{N-1}{N})}$. This decreases with both the number of banks N and the loan substitutability parameter γ, while it increases with the level of loan demand l. The total marginal cost includes the loan provision cost c and the marginal financing cost $\sqrt{r^I r^D}$, i.e., the sum of the expected cost of refinancing and of raising deposits.

Equilibrium reserve holdings balance the marginal benefit of reducing the expected cost of refinancing with the marginal cost of increasing deposits, and they are positive as long as $r^I > r^D$. We restrict our attention to this plausible case. Both reserves and deposits increase with the interbank refinancing cost r^I and with the demand for loans L_{sq}, while they decrease with the deposit rate r^D. The ratio $\frac{r^I}{r^D}$ is *the relative cost of refinancing*, which will help us later on to distinguish various scenarios for liquidity effects. It is a measure of how costly refinancing at date 1 is relative to raising deposits and reserves at date 0.[6]

Two further implications of Proposition 1 are important for comparing this equilibrium with the post-merger equilibrium in the next section. First, using the balance sheet equality

[6]If $r^{IL} > 0$, the ratio would be $\frac{r^{IB} - r^{IL}}{r^D - r^{IL}}$.

(1), we can express equilibrium reserve holdings in terms of an *optimal reserve-deposit ratio* as

$$k_{sq} = \frac{R_{sq}}{D_{sq}} = \left(1 - \sqrt{\frac{r^D}{r^I}}\right). \tag{9}$$

Note that, whereas the equilibrium reserve holdings in Proposition 1 depend on the loan market outcome, the reserve-deposit ratio in (9) does not. To exploit this, in what follows we will mostly focus on this ratio. In practice, the ratios of liquid assets to customers' sight deposits or of liquid assets to total assets are among the most frequently used indicators by banks to assess their own liquidity situation (see, e.g., ECB, 2002, p. 22). Second, Proposition 1 implies the following corollary.

Corollary 1 *In the status quo equilibrium, each bank has liquidity risk* $\phi_{sq} = \sqrt{\frac{r^D}{r^I}}$ *and expected liquidity needs* $\omega_{sq} = \frac{r^D}{2r^I} D_{sq} = \frac{L_{sq}}{2}\sqrt{\frac{r^D}{r^I}}$.

The equilibrium liquidity risk ϕ_{sq} is increasing in the deposit rate r^D and decreasing in the refinancing cost r^I. An increase in r^D induces banks to reduce reserves and thus deposits. Lower reserves mean lower protection against early liquidity demand, while lower deposits reduce the size of such demand. As liquidity shocks hit only a fraction δ_i of deposits, the negative effect of lower reserves dominates, so that individual liquidity risk ϕ_{sq} increases. A similar mechanism explains the negative dependence of ϕ_{sq} on r^I, as well as the relationships between the expected liquidity needs ω_{sq}, the rates r^D and r^I, and the equilibrium demand for loans L_{sq}.

4 The Effects of a Merger on Individual Banks' Behavior

In this section we analyze what happens at the individual bank level when a merger takes place. The behavior of the merged banks changes in several ways. First, they can exchange reserves internally, which alters their way to insure against liquidity risk. Second, this 'internal money market' gives them a financing cost advantage, whose size is endogenously determined. Third, the merged banks may enjoy cost efficiencies in terms of lower loan provision costs. Fourth, they gain market power in setting loan rates. All these factors affect banks' equilibrium balance sheets and, in turn, the demand and supply of liquidity. We begin with discussing how the merger modifies banks' reserve holdings, and then we turn to its effects on loan market competition.

4.1 Internal Money Market and Choice of Reserves

We note first that the merger does not affect the optimal reserve-deposit ratio of the $N-2$ competitors. As they have the same cost structure as in the status quo, they still choose their reserve-deposit ratios according to (9), i.e., $k_c = k_{sq}$.

By contrast, the merged banks, say bank 1 and bank 2, choose a different reserve-deposit ratio. As their liquidity shocks are independently distributed, they can pool their reserves to meet the total demand for liquidity. Thus, as long as the two banks continue to raise deposits in two separate regions, the merger leaves room for an *internal money market* in which they can reshuffle reserves according to their respective needs. For simplicity, we assume a 'perfect' internal money market, so that exchanging reserves internally involves no cost, but all qualitative results go through as long as the internal money market is less costly than the interbank one. Proceeding in this way is motivated by recent empirical research suggesting that internal capital markets function relatively efficiently (see, e.g., Graham et al., 2002; Houston et al.,1997; and Campello, 2002).

Let $x_m = \delta_1 D_1 + \delta_2 D_2$ be the total demand for liquidity of the merged banks at date 1, $R_m = R_1 + R_2$ be their total reserves and $D_m = D_1 + D_2$ be their total deposits. The combined profits of the merged banks are then given by

$$\begin{aligned} \Pi_m &= (r_1^L - \beta c)L_1 + (r_2^L - \beta c)L_2 - \int_{R_m}^{D_m} r^I (x_m - R_m) f(x_m) dx_m \\ &\quad - r^D \left[D_1(1 - E(\delta_1)) + D_2(1 - E(\delta_2)) \right]. \end{aligned} \tag{10}$$

The first two terms in (10) represent the combined profits from the loan market, with β reflecting potential efficiency gains in the form of reduced loan provision costs, the third term is the total expected cost of refinancing, and the last one is the total expected repayment to depositors. The operation of the internal money market can be seen in the third term of (10), where demands for liquidity and reserves are pooled together.

A preliminary step before deriving their optimal reserve-deposit ratio is to understand the 'deposit market policy' of the merged banks. Whether they raise equal or different amounts in both regions affects the distribution of the demand for liquidity x_m, and thus the size of the expected cost of refinancing. We have the following lemma.

Lemma 1 *The merged banks raise an equal amount of deposits in each region, i.e.,* $D_1 = D_2 = \frac{D_m}{2}$.

Lemma 1 shows that the merged banks not only raise deposits in both regions, but they even do it symmetrically. Choosing equal amounts of deposits in both regions minimizes the variance of x_m and maximizes the benefits of diversification, thus reducing the expected refinancing cost. (We will come back to this point in Section 5 when studying the effect of the merger on aggregate liquidity demand.)

Given $D_1 = D_2$, the merged banks choose reserves R_m so as to maximize their combined profits in (10). Let $k_m = \frac{R_m}{D_m}$ be the reserve-deposit ratio for the merged banks and recall that k_{sq} is the one for banks in the status quo defined in (9). The following proposition compares these two ratios.

Proposition 2 *The merged banks choose a lower reserve-deposit ratio than in the status quo ($k_m < k_{sq}$) if the relative cost of refinancing is higher than a threshold ρ ($\frac{r^I}{r^D} > \rho$), and a higher one otherwise.*

The *diversification effect* of the internal money market leads the merged banks to reduce reserves, and this effect dominates as long as the relative refinancing cost is not too low. Contrary to conventional wisdom, however, Proposition 2 shows that the merged banks could also increase their optimal reserve-deposit ratio. The reason is that the typical diversification effect is offset by −what we call− an *internalization effect.* When choosing reserves, the merged banks take into account ('internalize') an externality, namely that each unit of reserves can now be used to cover a liquidity demand at either of them.

The merger modifies the demand for liquidity x_m of the merged banks relative to the demand for liquidity x_i of each individual bank in the status quo, and the relative cost of refinancing affects banks' reserve choices. As a sum of two independent liquidity shocks, x_m is more concentrated around the mean than x_i. Thus, the distribution of x_m gives a lower probability to events with very low and very high liquidity demand than that of x_i. If the ratio $\frac{r^I}{r^D}$ is very low, both the merged banks and each individual bank choose relatively small reserve-deposit ratios because refinancing is inexpensive. For any given small level of this ratio, however, the merged banks would be able to cover their demand for liquidity less frequently than the individual bank because of the thinner left tail of the distribution of x_m. The merged banks have therefore a higher marginal valuation of further reserve units and increase their reserve-deposit ratio k_m above k_{sq}.

The reverse happens if the relative cost of refinancing is high. In this case, all banks tend to have high reserve-deposit ratios. For any given large level of this ratio, the merged banks would experience liquidity shortages less often than an individual bank, because the right tail of the distribution of x_m is thinner than that of the distribution of x_i. This makes the merged banks have a lower marginal valuation of further reserve units, and it induces them to decrease their reserve-deposit ratio, i.e., the diversification effect dominates.

The change in the reserve-deposit ratio of the merged banks can also be interpreted in terms of better estimate of their liquidity needs. As the distribution of their demand for liquidity is more concentrated around the mean, the merged banks face less uncertainty about their liquidity shocks than before merging, and can therefore better assess the reserve-deposit ratio they should hold. This leads to an adjustment of reserves towards the improved estimate.[7]

To sum up, the internal money market implies that banks can benefit from scope economies in their liquidity management by raising deposits in two imperfectly correlated deposit markets. In this respect, our result is related to Kashyap et al. (2002), who show that combining the activities of lending and deposit taking produces synergies that allow banks to reduce the volume of liquid assets that banks need to hold to satisfy their customers' unexpected demands. However, whereas in their paper such an advantage in providing liquidity arises as a consequence of two imperfectly correlated markets on different sides of the balance sheet, in our model it emerges from two imperfectly correlated markets on the same side of the balance sheet.

4.2 Cost Structures, Choice of Loan Rates and Balance Sheets

We now examine how the merger modifies the equilibrium in the loan market and banks' balance sheets. Consider first banks' cost structures. As noted earlier, competitors have the same cost structure as in the status quo. Each of them pays a per-unit loan provision cost c and per-unit financing costs $\sqrt{r^I r^D}$ (from Proposition 1).

By contrast, the cost structures of the merged banks change in two ways. First, their loan provision costs reach βc, where the parameter $\beta \leq 1$ represents the potential non-financial efficiency gains that the merger induces for the processing of loans. The lower the

[7] We thank Loretta Mester for suggesting this interpretation.

parameter β the greater are the efficiency gains. The idea is to include, for example, the possibility for economies of scale, which are often put forward by bank managers in favor of mergers and have been questioned in the literature, as we discuss in Section 6.[8] Second, the emergence of the internal money market affects the merged banks' refinancing costs. We directly find the following result.

Lemma 2 *The merged banks have lower financing costs than the competitors.*

This advantage for the merged banks is endogenous to the model in that it is determined not only by diversification, but also by their optimal reserve readjustment. Notice that this result identifies a new motive to merge −in addition to the well-known market power and diversification−, that is the ability to save costs by optimally adjusting reserve holdings through an internal money market. This motive has empirical support in Hughes et al. (1996), who find that banks active in imperfectly correlated deposit markets −especially as a result of consolidation− have a lower cost of controlling liquidity risk. As in our model, the idea is that deposit volatility is endogenous, and banks can reduce their liquidity risk by appropriately adjusting deposit collection and reserve holdings.

The following proposition describes the post-merger equilibrium with symmetric behavior within the 'coalition' (merger) and among competitors.

Proposition 3 *The post-merger equilibrium with $r_1^L = r_2^L = r_m^L$ and $r_i^L = r_c^L$ for $i = 3, ..., N$ is characterized as follows:*

1. *Each merged bank sets a loan rate $r_m^L = \left(\frac{2N-1}{N-2}\right)\frac{l}{2\gamma} + \frac{(N-1)}{2N}c_c + \frac{(N+1)}{2N}c_m$, and each competitor sets $r_c^L = \left(\frac{N-1}{N-2}\right)\frac{l}{\gamma} + \frac{(N-1)}{N}c_c + \frac{1}{N}c_m$;*

2. *The merged banks have a total loan market share $L_m = \left(\frac{2N-1}{N}\right)l + \gamma\frac{(N-1)(N-2)}{N^2}(c_c - c_m)$, and each competitor has $L_c = \frac{(N-1)^2}{N(N-2)}l - \gamma\frac{(N-1)}{N^2}(c_c - c_m)$;*

3. *The merged banks raise total deposits $D_m = \frac{1}{1-k_m}L_m$, and each competitor raises $D_c = \frac{1}{1-k_c}L_c$;*

[8] We could also allow for $\beta > 1$, in which case the merger would even lead to diseconomies. Already the market power of merged banks tends to increase loan rates, and $\beta > 1$ would only strengthen this effect. So, none of our results would be qualitatively altered by further generalizing β.

where c_m, c_c are the total marginal costs of the merged banks and of the competitors, and k_m and k_c are their respective optimal reserve-deposit ratios.[9]

Since banks compete in strategic complements, in equilibrium the loan rates of competitors move in the same direction as the loan rates of the merged banks. Both r_m^L and r_c^L are a weighted average of the mark ups that banks can charge and of the total marginal costs c_m and c_c. All mark ups are higher than those in the status quo equilibrium (see r_{sq}^L in Proposition 1), but as the merged banks gain market power, they charge a higher mark up than competitors. By contrast, their total marginal cost c_m is lower than those of the competitors, as the merged banks benefit from lower financing costs (see Lemma 2) and potentially also from efficiency gains in the provision of loans. Thus, the effect of the merger on equilibrium loan rates depends on the relative importance of the increased market power of the merged banks as compared to their lower total marginal cost. Post-merger equilibrium loan rates increase when the merger induces small cost advantages relative to the increase in market power, whereas they decrease otherwise.

Loan market shares across banks change in line with loan rates. As the merged banks change their loan rates by more than competitors, their total loan market share shrinks when loan rates increase and it expands otherwise, i.e., $L_m < 2L_{sq} < 2L_c$ when $r_m^L > r_c^L$, and $L_m > 2L_{sq} > 2L_c$ otherwise.

The modification of loan market shares together with the change in the optimal reserve-deposit ratio described in Proposition 2 determines the effects on the size of banks' balance sheets (as measured by the amount of deposits). In the present set-up a merger breaks the symmetry in banks' balance sheets. Whereas in the status quo all banks have the same deposits D_{sq}, the merged banks have now in general different deposit sizes than competitors, i.e., $\frac{D_m}{D_c} \neq 2$.

4.3 Banks' Individual Liquidity Risk

An important implication of Propositions 2 and 3 is how the merger modifies banks' liquidity risks and expected liquidity needs. The results for competitor banks are quite straightforward. As they follow the same optimal reserve rule as in the status quo, they face the same

[9]The expressions for c_m, c_c are in the proof of this proposition; those for k_m and k_c are, respectively, in the proof of Proposition 2 and in equation (9).

liquidity risk $\phi_c = \phi_{sq} = \sqrt{\frac{r^D}{r^I}}$ (see Corollary 1). Their expected liquidity needs, however, change with their balance sheet, as $\omega_c = \frac{r^D}{2r^I} D_c$. The merged banks experience more far reaching changes in liquidity risks and needs.

Corollary 2 *The merged banks have lower liquidity risk than a single bank in the status quo.*

This result derives directly from Proposition 2. When the relative cost of refinancing is below the threshold ρ, the merged banks increase their reserve-deposit ratio and their liquidity risk goes down. In the other case, although they choose a lower reserve-deposit ratio than in the status quo, they still keep it sufficiently high to decrease the liquidity risk. This effect is so strong that the liquidity risk of the merged banks is not only lower than the risks of two banks in the status quo, but it is even lower than that of a single bank.

Corollary 3 *The merged banks have lower expected liquidity needs than in the status quo if $\frac{D_m}{D_{sq}} < h$, where $2 < h \leq 4$, and higher ones otherwise.*

The merger changes the merged banks' expected needs for three reasons. First, it creates the internal money market, which reduces ceteris paribus expected liquidity needs. Second, the merger modifies the merged banks' optimal reserve-deposit ratio, which reduces ceteris paribus expected liquidity needs when the relative cost of refinancing is low. Third, the merger changes the merged banks' deposits, and hence the size of their demand for liquidity. Corollary 3 shows that the first effect dominates unless cost advantages (efficiency gains and reduced financing costs) and competition in the loan market (degree of loan differentiation γ and number of banks N) are so strong that the merged banks increase their balance sheets substantially relative to two banks in the status quo. From an empirical perspective, such a strong balance-sheet expansion seems to be a less plausible scenario.

5 The Effects of a Merger on Aggregate Liquidity

Now that we have seen how a merger affects the behavior of individual banks, we can turn to its implications for the banking system as a whole. To see this, we analyze how changes in banks' reserve holdings and in loan market competition modify the aggregate supply and demand of liquidity.

We identify two channels. The first one we call *reserve channel*, as it works through changes in reserve holdings. When looking at the system as a whole, the distinction between the internal money market of the merged banks and the interbank market is blurred, and the total supply of liquidity is composed of the sum of all banks' reserve holdings. Nevertheless, the existence of the internal money market affects the total supply of liquidity through the change in the reserve holdings of the merged banks. The second channel is an *asymmetry channel*, which affects the distribution of the aggregate liquidity demand. This channel originates in the heterogeneity of balance sheets across banks, which $-$as shown above$-$ depends on both the different amounts of reserves and the different loan market shares that banks have after the merger.

We start with analyzing each of the two channels in isolation. Then we examine how they interact in determining aggregate liquidity risk and expected aggregate liquidity needs.

5.1 Asymmetry Channel without Internal Money Market

To isolate the working of the asymmetry channel, we assume for a moment that the merged banks cannot make use of the internal money market. In this case, they do not have any financing cost advantages, and they choose the same optimal reserve rule as their competitors. As a consequence, the asymmetry in banks' balance sheets originates only from the different distribution of market shares resulting from loan competition.

As all banks continue to choose reserves according to (9) and as the aggregate demand for loans is inelastic, the merger does not affect the total amounts of reserves and deposits, thus leaving the aggregate supply of liquidity unchanged. The heterogeneity of banks' balance sheets, however, modifies the aggregate liquidity demand, which changes from $X_{sq} = \sum_{i=1}^{N} \delta_i D_{sq}$ in the status quo to $X_m = \delta_1 \frac{D_m}{2} + \delta_2 \frac{D_m}{2} + \sum_{i=3}^{N} \delta_i D_c$ after the merger. Both X_{sq} and X_m are weighted sums of N uniform random variables, but in the first case weights are equal and in the second case they differ (according to deposit sizes). This brings us to the main result about the asymmetry channel.

Proposition 4 *Suppose the merged banks do not exchange reserves internally. Then, the aggregate liquidity effects of the merger are as follows:*

1. *The merger decreases aggregate liquidity risk if the relative cost of refinancing is below a threshold σ ($\frac{r^I}{r^D} < \sigma < \rho$), and increases it otherwise;*

2. The merger always increases expected aggregate liquidity needs.

The intuition behind Proposition 4 is as follows. As already mentioned for Lemma 1, moving from a uniformly weighted sum of random variables (in the status quo) to a heterogeneously weighted sum of random variables (after merger) increases the variance of the total sum. Thus, as Figure 3 illustrates, the distribution of X_{sq} gives lower probability to extreme events – very low and very high realizations of the aggregate liquidity demand – than that of X_m.

This change in the distribution of X_m reduces the aggregate liquidity risk if the relative cost of refinancing is low (below the threshold σ), because it increases the probability that the aggregate liquidity demand is below the total supply. This is illustrated in Figure 3, where total reserves –indicated by the vertical line $\sum_{i=1}^{N} R_i$– are low and the area $1 - \Phi_m$ is larger than the diagonally striped area $1 - \Phi_{sq}$. The opposite happens when the relative cost of refinancing is high.

[FIGURE 3 ABOUT HERE]

Proposition 4 also states that the merger always increases the expected amount of public liquidity needed. The reason is that the expected aggregate liquidity needs depend not only on the frequency with which aggregate liquidity demand exceeds aggregate supply, but also on the magnitude of each excess. As noted earlier, the merger increases the variance of the distribution of X_m and thus the probability of events with very low and very high demands. If banks do not hold reserves, these increases offset each other and the expected aggregate liquidity needs are the same before and after the merger. By contrast, when banks hold positive reserves, they can cover the events with low aggregate liquidity demand. Hence, the higher probability of extreme events with high aggregate liquidity demand is not outweighed any more by the higher frequency of low demand events, and the expected aggregate liquidity needs grow.

5.2 Interaction with the Reserve Channel

In this section we reintroduce the possibility for the merged banks to use the internal money market. We first analyze how this affects aggregate liquidity through the reserve channel.

Denote as

$$K_m = \frac{R_m + \sum_{i=3}^{N} R_c}{D_m + (N-2)D_c} = \frac{k_m D_m + \sum_{i=3}^{N} k_c D_c}{D_m + (N-2)D_c} \tag{11}$$

the aggregate reserve-deposit ratio after the merger. Since competitors choose the same ratio as in the status quo ($k_c = k_{sq}$), the change in K_m is solely determined by the change in the merged banks' reserve-deposit ratio. Hence, it follows from Proposition 2 that K_m increases when the relative cost of refinancing is relatively low (because then $k_m > k_{sq}$), whereas it decreases otherwise. The following lemma describes how the change in the aggregate reserve-deposit ratio alone affects aggregate liquidity.

Lemma 3 *Suppose the merger does not cause any asymmetry in banks' balance sheets ($D_m = 2D_c$). Then, it decreases aggregate liquidity risk and expected aggregate needs if the relative cost of refinancing is below ρ, and it increases them otherwise.*

When the merger does not generate asymmetry across banks' balance sheets, it affects aggregate liquidity only through the reserve channel. The aggregate liquidity supply changes, whereas the aggregate liquidity demand remains the same. Thus, the merger reduces both aggregate liquidity risk and expected aggregate liquidity needs when the aggregate liquidity supply increases through a higher reserve-deposit ratio of the merged banks. The opposite happens when the aggregate liquidity supply falls.

When the merger generates the internal money market and asymmetry across banks, both the asymmetry and the reserve channel are at work. Depending on the size of the relative cost of refinancing, the two channels can reinforce or offset each other. Therefore, we consider the cases of high and low relative cost of refinancing separately.

Proposition 5 *If the relative cost of refinancing is above ρ, the merger increases both aggregate liquidity risk and expected aggregate liquidity needs.*

When the relative cost of refinancing is rather high, the asymmetry channel and the reserve channel work in the same direction. The asymmetry channel increases the variance of the aggregate liquidity demand, and the reserve channel reduces the aggregate liquidity supply through the lower reserve holdings of the merged banks. Both these effects make the system more vulnerable to liquidity shortages and more dependent on public liquidity provision.

Proposition 6 *If the relative cost of refinancing is below ρ, then the following holds:*

1. *There exists a critical level of the relative cost of refinancing $g \in (\sigma, \rho)$ such that the merger reduces aggregate liquidity risk if the cost of refinancing is below such critical level, and increases it otherwise;*

2. *For any small level of asymmetry induced by the merger, there exists a set G of values of the relative cost of refinancing, with $G \subset (1, \rho)$, for which the merger reduces expected aggregate liquidity needs.*

When the cost of refinancing is relatively low, the reserve and the asymmetry channels drive aggregate liquidity in opposite directions, and the net effect depends on their relative strength. As shown in Lemma 3, the reserve channel reduces both aggregate liquidity risk and expected liquidity needs. As stated in Proposition 4, however, the asymmetry channel always increases expected aggregate liquidity needs, whereas it reduces aggregate liquidity risk only if the relative cost of refinancing is sufficiently low.

Thus, when the two channels interact, the merger reduces aggregate liquidity risk for a larger range of parameter values than in Proposition 4, where only the asymmetry channel is active. Similarly, it increases aggregate liquidity risk in a larger range of parameter values than in Lemma 3, where only the reserve channel is present.

As for the expected aggregate liquidity needs, the reserve channel dominates when the asymmetry induced by the merger is sufficiently small. Thus, there is a range of values of the relative cost of refinancing for which the merger reduces expected aggregate liquidity needs. The larger the asymmetry in banks' balance sheets, the larger is this range of parameters in which the merger increases expected aggregate liquidity needs.

How relevant are these different scenarios for changes in aggregate liquidity? One way to proceed is to associate the level of the relative cost of refinancing with different countries or financial systems. For example, in industrial countries with relatively sizable and developed financial systems one would expect this cost to be rather low. In contrast, in developing or emerging countries with less developed financial systems this cost may be quite high. Then, Proposition 5 suggests that in the latter group of countries bank consolidation may lead to a deterioration of aggregate liquidity. Proposition 6 indicates instead that both a deterioration and an improvement of aggregate liquidity are possible in industrial countries, depending on whether the reserve or the asymmetry channel dominates. The asymmetry channel may dominate when consolidation takes the form of mergers between large banks leading to a 'polarization' of the banking system. As Table 1 shows, this seems to have occurred in several industrial countries during the 1990s, as consolidation enlarged the share of the largest players, thus increasing the asymmetry among banks. Whether this development was strong enough to actually worsen aggregate liquidity significantly, and in particular how this 'polarization' related to reserve changes, is an empirical issue which would be interesting to address in future research.

6 The Relationship between Competition and Aggregate Liquidity

We now discuss more in detail how loan market competition and reserve choices interact in determining loan rates and aggregate liquidity (for simplicity, here interpreted only as expected aggregate liquidity needs), and how liquidity effects relate to competition effects.

At the individual bank level, the loan market equilibrium affects banks' reserve holdings (in absolute terms) by determining the amount of deposits required to finance loans, and hence the size of liquidity demands at any given level of reserves. Equilibrium reserve holdings determine banks' financing costs –the sum of the expected cost of refinancing and of the expected repayment to depositors–, and thereby influence the loan market equilibrium. At the aggregate level, loan market competition affects the degree of asymmetry in banks' balance sheets through the distribution of equilibrium loan market shares.

Table 2 summarizes the possible effects of mergers on both loan rates r^L and expected aggregate liquidity needs Ω, as described in Propositions 3, 5 and 6. The rows of the table indicate whether a merger is characterized by low or high efficiency gains in terms of both reduced loan provision costs and lower financing costs ($\frac{c_m}{c_c}$ high or low); the two columns show the cases of high and low relative cost of refinancing $\frac{r^I}{r^D}$.

[TABLE 2 ABOUT HERE]

As Table 2 shows, the model predicts several scenarios, depending on the value of the parameters. The effect of mergers on expected aggregate liquidity needs is ambiguous when the relative cost of refinancing is low, whereas it is always negative when the relative cost of refinancing is high. Concerning competition, mergers increase loan rates when efficiency gains are small relative to the increase in market power, and, vice versa, decrease loan rates when efficiency gains are large.

What can we say about the plausibility of the different scenarios displayed in Table 2? As already indicated above, one may associate low refinancing costs with industrial countries and high refinancing costs with developing or emerging countries. Moreover, one may relate the magnitude of efficiency gains to the size of mergers. Even if there is an ongoing debate in the literature on whether efficiency gains (and, in particular, scale economies) exhaust at large or small sizes of output, the empirical consensus seems still to be that mergers between small banks produce larger efficiency gains than mergers between large banks.[10]

One plausible scenario in industrial countries (low $\frac{r^I}{r^D}$) is therefore the occurrence of mergers between large banks leading to higher loan rates and expected aggregate liquidity needs, as they do not realize sufficient efficiency gains and induce greater asymmetry in the banking system (one case in cell I). Differently, the occurrence of mergers between small banks in industrial countries is likely to reduce both loan rates and expected aggregate liquidity needs, as smaller mergers may realize more efficiency gains relative to the increase in market power and make the banking system more homogenous (one case in cell II). For developing countries (high $\frac{r^I}{r^D}$), cells III and IV suggest that mergers would always increase expected aggregate needs, whereas the effect on loan rates may still depend on their sizes.

The interesting features of these results are that mergers are likely, ceteris paribus, to increase expected aggregate liquidity needs more in developing countries than in industrial ones, as they lead to lower reserve holdings for higher cost of refinancing; and that there is more complementarity between competition and liquidity in industrial countries than in developing ones. In terms of policy implications, these results suggest that policies aiming at protecting loan market competition may also prevent the adverse effects of consolidation for interbank liquidity in industrial countries, but not necessarily in developing countries.

[10] A substantial amount of empirical research has been spent on measuring the efficiency gains generated by bank mergers, but results are not unanimous (see, e.g., the surveys of Carletti et al., 2002; and Rhoades, 1994 and 1998). Whereas the mainstream literature shows that banks exhaust potential scale economies at modest levels of size (see, e.g., Berger et al., 1987; Berger and Humphrey, 1991; and Wheelock and Wilson, 2001), other studies (e.g., Berger and Mester, 1997; and Hughes et al., 2001) find that there are scale economies also at large asset sizes if one takes changes in risk into account.

7 Conclusions

This paper analyzes the impact of bank mergers on credit market competition, reserves and banking system liquidity. A merger creates an internal money market, which modifies merged banks' optimal reserves holdings, either decreasing them through a diversification effect or −more surprisingly− increasing them through an internalization effect. In both situations, merged banks benefit from scope economies in their liquidity management, and they lower their financing costs and liquidity risk.

The change in merged banks' reserve holdings, together with the change in the size of banks' balance sheets due to loan competition, affect the functioning of the interbank market. Changes in reserve holdings modify aggregate liquidity supply, while increased balance-sheet asymmetry raises aggregate liquidity needs by altering the distribution of the aggregate liquidity demand. These reserve and asymmetry channels can work in the same or in the opposite directions, depending on the cost of refinancing in the money market as compared to the cost of financing through retail deposits. We conclude that mergers between large banks tend to increase aggregate liquidity needs, although the risk of adverse liquidity effects of bank consolidation is likely to be more relevant in developing countries than in industrial countries; and that there is more complementarity between competition and liquidity in industrial countries than in developing ones.

The model implies some empirical hypotheses, which would be interesting to test in future research. While the competition effects of bank mergers are already quite well covered in the empirical literature, the same does not apply to the liquidity effects. On the individual level it would be interesting to estimate the effects of mergers on reserve holdings, and in particular the role of refinancing costs for the sign of the reserve changes. On the aggregate level, it would be important to examine how heterogeneity in bank sizes relates to liquidity fluctuations.

Several features of the model deserve further discussion. We introduce a merger in a situation where all banks are identical ex ante. This means that the merger leads to some degree of heterogeneity in banks' sizes. Even though this direction appears consistent with the bank merger movement of the 1990s, as shown in Table 1, not every merger leads to a more asymmetric banking system. For example, in a situation where the system is composed of a group of small banks and another group of large banks, mergers among the small banks would have the opposite effect. This configuration reverses the functioning of the asymmetry channel. A merger that makes the banking system more symmetric is, ceteris paribus, more likely to moderate expected aggregate liquidity needs. Even in this situation, however, financial consolidation may still cause greater liquidity risk and larger expected aggregate liquidity needs, if it induces a reduction of banks' reserve holdings.

The interbank market works in a very simple way. In the ultra-short interbank market, the central bank adjusts the liquidity supply to accommodate changes in the aggregate demand, and banks can always meet the repayment to depositors without suffering any liquidity crisis. In a similar spirit, long-term loans are totally illiquid, or, equivalently, the costs of liquidation are higher than the relative cost of refinancing. This framework allows us to focus on pure liquidity issues, and isolate reserve management from other considerations. An interesting extension of the model would be to analyze the functioning of other, long term, interbank markets, where the central bank would not be active and banks could modify the liquidity supply only by selling their long-term assets. We leave this for future research.

Appendix

Proof of Proposition 1

Using Leibniz's rule and (1), from (8) we obtain the first order conditions with respect to the choice variables r_i^L and R_i:

$$\frac{\partial \Pi_i}{\partial r_i^L} = L_i + (r_i^L - c)\frac{\partial L_i}{\partial r_i^L} - \left[\frac{r^I}{2}\frac{L_i^2 + 2L_iR_i}{(L_i+R_i)^2} + \frac{r^D}{2}\right]\frac{\partial L_i}{\partial r_i^L} = 0, \text{ for } i = 1...N, \tag{12}$$

$$\frac{\partial \Pi_i}{\partial R_i} = r^D(L_i+R_i)^2 - r^I L_i^2 = 0, \text{ for } i = 1...N. \tag{13}$$

Solving (13) for R_i gives

$$R_i = \left(\sqrt{\frac{r^I}{r^D}} - 1\right)L_i. \tag{14}$$

Solving (12) for r_i^L in a symmetric equilibrium where $r_i^L = r_{sq}^L$ for $i = 1...N$ after substituting (2) and (14) gives

$$l + (r_{sq}^L - c - \sqrt{r^I r^D})(-\gamma\frac{N-1}{N}) = 0,$$

from which r_{sq}^L and c_{sq} follow. Substituting then r_{sq}^L in (2) gives L_{sq}, and through (14) R_{sq}. Substituting R_{sq} and L_{sq} in (1), we obtain D_{sq}. Q.E.D.

Proof of Corollary 1

Solving (3) and (4) gives $\phi_i = 1 - \frac{R_i}{D_i}$ and $\omega_i = \frac{(R_i)^2}{2D_i} - R_i + \frac{D_i}{2}$. Substituting the expressions for R_{sq} and D_{sq}, we obtain ϕ_{sq} and ω_{sq} as in the corollary. Q.E.D.

Proof of Lemma 1

We proceed in two steps. First, we show that the variance of the liquidity demand x_m of the merged banks is minimized when deposits are raised symmetrically in the two regions. Second, we show that the expected liquidity needs of the merged banks (and therefore their refinancing costs) are lower when deposits are symmetric.

Step 1. Define the liquidity demand of the merged banks as

$$x_m = \delta_1 \alpha D_m + \delta_2(1-\alpha)D_m,$$

where $\alpha \in [0,1]$ indicates the fraction of deposits that the merged banks raise in one region and $(1-\alpha)$ the fraction they raise in the other region. Since δ_1 and δ_2 are independent and $Var(\delta_1) = Var(\delta_2)$, the variance of x_m is simply

$$\begin{aligned} Var\,(x_m) &= \alpha^2 D_m^2 Var(\delta_1) + (1-\alpha)^2 D_m^2 Var(\delta_2) \\ &= Var(\delta_1)[\alpha^2 D_m^2 + (1-\alpha)^2 D_m^2]. \end{aligned}$$

Differentiating it with respect to α, we obtain

$$\frac{\partial Var\left(x_m\right)}{\partial \alpha} = 2D^2 Var(\delta_1)(2\alpha - 1) = 0,$$

which has a minimum at $\alpha = \frac{1}{2}$.

Step 2. Define now the liquidity demand of the merged banks as

$$x_{ma} = \delta_1 \alpha D_m + \delta_2 (1-\alpha) D_m,$$

when $\alpha \neq \frac{1}{2}$, and as

$$x_{ms} = \delta_1 \frac{D_m}{2} + \delta_2 \frac{D_m}{2}$$

when $\alpha = \frac{1}{2}$. Applying the general formula in Bradley and Gupta (2002) to our case, the density functions of x_{ma} and x_{ms} can be written as (assume $\alpha < \frac{1}{2}$ without loss of generality):

$$f_{ma}(x_{ma}) = \begin{cases} \frac{x_{ma}}{\alpha(1-\alpha)D_m^2} & \text{for } x_{ma} \leq \alpha D_m \\ \frac{1}{(1-\alpha)D_m} & \text{for } \alpha D_m < x_{ma} \leq (1-\alpha) D_m \\ \frac{D_m - x_{ma}}{\alpha(1-\alpha)D_m^2} & \text{for } x_{ma} > (1-\alpha) D_m, \end{cases}$$

$$f_{ms}(x_{ms}) = \begin{cases} \frac{4x_m}{D_m^2} & \text{for } x_{ms} \leq D_m/2 \\ \\ \frac{4(D_m - x_{ms})}{D_m^2} & \text{for } x_{ms} > D_m/2. \end{cases} \tag{15}$$

Since $\alpha < \frac{1}{2}$, $f_{ma}(x_{ma})$ is steeper than $f_{ms}(x_{ms})$ both for $x_{ma} \leq \alpha D_m$ and for $x_{ma} > (1-\alpha)D_m$. This implies that the two density functions do not cross in these intervals, whereas they do it in two points in the interval $\alpha D_m < x_{ma} \leq (1-\alpha)D_m$. Given that they are symmetric around the same mean $D_m/2$ with $Var\left(x_{ma}\right) > Var\left(x_{ms}\right)$, it is:

$$\begin{aligned} F_{ma} &> F_{ms} \text{ for } R_m < \frac{D_m}{2}, \\ F_{ma} &< F_{ms} \text{ for } R_m > \frac{D_m}{2}, \end{aligned} \tag{16}$$

where $F_{ma} = \Pr(x_{ma} < R_m)$ and $F_{ms} = \Pr(x_{ms} < R_m)$.

Denote now as ω_{ma} and ω_{ms} the expected liquidity needs of the merged banks with asymmetric deposits and symmetric deposits respectively. We have

$$\begin{aligned} \omega_{ma} - \omega_{ms} &= \int_{R_m}^{D_m} (x_{ma} - R_m) f_{ma}(x_{ma}) d(x_{ma}) - \int_{R_m}^{D_m} (x_{ms} - R_m) f_{ms}(x_{ms}) d(x_{ms}) \\ &= \int_{R_m}^{D_m} x_{ma} f_{ma}(x_{ma}) d(x_{ma}) - \int_{R_m}^{D_m} x_{ms} f_{ms}(x_{ms}) d(x_{ms}) \\ &\quad - R_m(1 - F_{ma}(R_m)) + R_m(1 - F_{ms}(R_m)). \end{aligned} \tag{17}$$

Differentiating (17) with respect to R_m gives

$$\begin{aligned}\frac{d(\omega_{ma}-\omega_{ms})}{dR_m} &= -R_m f_{ma}(R_m)+R_m f_{ms}(R_m)-(1-F_{ma}(R_m))\\ &\quad +R_m f_{ma}(R_m)+(1-F_{ms}(R_m))-R_m f_{ms}(R_m)\\ &= F_{ma}(R_m)-F_{ms}(R_m).\end{aligned}$$

From (16) it follows $\frac{d(\omega_{ma}-\omega_{ms})}{dR_m}>0$ for $R_m<\frac{D_m}{2}$ and $\frac{d(\omega_{ma}-\omega_{ms})}{dR_m}<0$ otherwise. This, along with $\omega_{ma}-\omega_{ms}=0$ both for $R_m=0$ and for $R_m=D_m$ implies $\omega_{ma}-\omega_{ms}>0$ for all $R_m\in[0,D_m]$. Q.E.D.

Proof of Proposition 2

The demand for liquidity of the merged banks, $x_m=\delta_1\frac{D_m}{2}+\delta_2\frac{Dm}{2}$, has density function as in (15). Using Leibniz's rule, the equality $D_m=R_m+L_1+L_2$, and the ratio $k_m=\frac{R_m}{D_m}$, from (10) we can express the first order condition $\frac{\partial\Pi_m}{\partial R_m}=0$ as

$$\begin{cases}\frac{8}{3}k_m^3-4k_m^2+1=\frac{r^D}{r^I} & \text{for } k_m\leq 1/2\\ \\ \frac{8}{3}(1-k_m)^3=\frac{r^D}{r^I} & \text{for } k_m>1/2.\end{cases}\tag{18}$$

The term on the LHS of the equalities is the marginal benefit of increasing the reserve-deposit ratio, that is the reduction in the expected need of refinancing induced by a marginal increase of the reserve ratio. The term on the RHS of the equalities is the ratio between the marginal cost of raising reserves r^D and the marginal cost of refinancing r^I. From (18), we obtain:

$$k_m=\begin{cases}z(r^I,r^D) & \text{for } r^I\leq 3r^D\\ \\ 1-\sqrt[3]{\frac{3}{8}\frac{r^D}{r^I}} & \text{for } r^I>3r^D,\end{cases}\tag{19}$$

where $z(r^I,r^D)$ is the solution of the equation $z^3-\frac{3}{2}z^2+\frac{3}{8}(1-\frac{r^D}{r^I})=0$ in the interval $(0,\frac{1}{2}]$ increasing in the ratio $\frac{r^I}{r^D}$. Since $f(0)>0$, $f(1/2)<0$ and $f'(z)<0$, $z(r^I,r^D)$ is the unique real solution.

To compare k_m with k_{sq}, we rearrange k_{sq} given in (9) as

$$(1-k_{sq})^2=\frac{r^D}{r^I},\tag{20}$$

where, as before, the LHS is the marginal benefit of increasing the reserve-deposit ratio and the RHS is the ratio between the marginal cost of raising deposits and holding reserves r^D and the marginal cost of refinancing r^I.

Denote as $f(k_m)$ the LHS of (18) and as $f(k_{sq})$ the LHS of (20). Plotting $f(k_m)$ and $f(k_{sq})$ for k_{sq} and k_m between 0 and 1, we get Figure 4.

[FIGURE 4 ABOUT HERE]

The curves $f(k_m)$ and $f(k_{sq})$ cross only once at $k_{sq} = k_m = \frac{5}{8}$. Substituting this value in (18) or (20) gives $k_{sq} = k_m$ when $\frac{r^I}{r^D} = \frac{64}{9} \equiv \rho$. Thus, $k_m > k_{sq}$ if $\frac{r^I}{r^D} < \rho$, and $k_m < k_{sq}$ otherwise. Q.E.D.

Proof of Lemma 2

From the last two terms in (8), we can express the financing costs of competitors as

$$\frac{r^I}{2} \frac{L_c^2}{(R_c + L_c)} + \frac{r^D}{2} (R_c + L_c). \tag{21}$$

Using $\frac{R_c}{D_c} = k_c$ and $\frac{L_c}{D_c} = 1 - k_c$ in (21) and rearranging terms, we obtain

$$\frac{r^I (1 - k_c)^2 + r^D}{2(1 - k_c)}. \tag{22}$$

Analogously, from the last two terms in (10), using $\frac{R_m}{D_m} = k_m$ and $\frac{L_m}{D_m} = 1 - k_m$, we obtain the financing costs of the merged banks as

$$\begin{cases} \frac{r^I(3-6k_m+4k_m^3)+3r^D}{6(1-k_m)} & \text{for } r^I \leq 3r^D \\ \\ \frac{4r^I(1-k_m)^3+3r^D}{6(1-k_m)} & \text{for } r^I > 3r^D. \end{cases} \tag{23}$$

It is easy to check that when the merged banks set k_m at the level which is optimal for competitors, the financing costs of the merged banks are always lower than the ones of the competitors. A fortiori this must be true when they set k_m to minimize their financial costs. Q.E.D.

Proof of Proposition 3

The merged banks choose r_1^L and r_2^L to maximize (10) while competitors choose r_i^L to maximize (8) where the subscript i is now c. Define from the financing costs in Lemma 2 ((22) and (23)) the total marginal costs of the competitors and the merged banks as

$$c_c = c + \frac{r^I (1 - k_c)^2 + r^D}{2(1 - k_c)} \tag{24}$$

and

$$c_m = \begin{cases} \beta c + \frac{r^I(3-6k_m+4k_m^3)+3r^D}{6(1-k_m)} & \text{for } r^I \leq 3r^D \\ \\ \beta c + \frac{4r^I(1-k_m)^3+3r^D}{6(1-k_m)} & \text{for } r^I > 3r^D, \end{cases} \tag{25}$$

respectively. Using the expressions for k_m and k_c in (19) and (20), those for c_c and c_m in (24) and (25), $D_m = R_m + L_1 + L_2$ and $D_c = R_c + L_c$, we can write the expected profits for the merged banks and competitors when reserves are chosen optimally as

$$\Pi_m = r_1^L L_1 + r_2^L L_2 - c_m(L_1 + L_2)$$

$$\Pi_c = (r_1^L - c_c)L_c,$$

where

$$L_m = L_1 + L_2 = \left[l - \gamma \left(r_1^L - \frac{1}{N}\sum_{j=1}^{N} r_j^L \right) \right] + \left[L - \gamma \left(r_2^L - \frac{1}{N}\sum_{j=1}^{N} r_j^L \right) \right], \tag{26}$$

and L_c is given by (2). The first order conditions are then given by

$$\frac{\partial \Pi_m}{\partial r_h^L} = L_h + (r_1^L - c_m)\frac{\partial L_1}{\partial r_h^L} + (r_2^L - c_m)\frac{\partial L_2}{\partial r_h^L} = 0 \text{ for } h = 1, 2 \tag{27}$$

$$\frac{\partial \Pi_c}{\partial r_i^L} = L_c + (r_i^L - c_c)\frac{\partial L_c}{\partial r_i^L} = 0 \text{ for } i = 3...N. \tag{28}$$

We look at the post-merger equilibrium where $r_1^L = r_2^L = r_m^L$ and $r_i^L = r_c^L$. Substituting (26) in (27) and (2) in (28), we obtain the best response functions as

$$r_m^L = \frac{l}{2\gamma(\frac{N-2}{N})} + \frac{c_m}{2} + \frac{r_c^L}{2}. \tag{29}$$

$$r_c^L = \frac{l}{\gamma(\frac{N+1}{N})} + (\frac{N-1}{N+1})c_c + \frac{2}{N+1}r_m^L. \tag{30}$$

Solving (29) and (30) gives the post-merger equilibrium loan rates r_m^L and r_c^L. Substituting r_m^L and r_c^L respectively in (26) and in (2) gives the equilibrium L_m and L_c. Analogously, we derive D_m and D_c. Q.E.D.

Proof of Corollary 2

Using (15), we can express the liquidity risk for the merged banks as

$$\phi_m = \Pr(x_m > R_m) = \begin{cases} 1 - \int_0^{R_m} \frac{4x_m}{D_m^2} dx_m & \text{for } r^I \leq 3r^D \\ \\ \int_{R_m}^{D_m} \frac{4(D_m - x_m)}{D_m^2} dx_m & \text{for } r^I > 3r^D. \end{cases}$$

Solving the integrals, we obtain $\phi_m = 1 - 2\frac{R_m^2}{D_m^2}$ for $r^I \leq 3r^D$ and $2 - 4\frac{R_m}{D_m} + 2\frac{R_m^2}{D_m^2}$ for $r^I > 3r^D$. Substituting $k_m = \frac{R_m}{D_m}$ implies

$$\phi_m = \begin{cases} 1 - 2k_m^2 & \text{for } r^I \leq 3r^D \\ \\ 2(1-k_m)^2 & \text{for } r^I > 3r^D. \end{cases}$$

Substituting k_m as in (19), we can express the merged banks' resiliency as

$$1 - \phi_m = \begin{cases} 2[z(r^I, r^D)]^2 & \text{for } r^I \leq 3r^D \\ \\ 1 - 2(\sqrt[3]{\frac{3}{8}\frac{r^D}{r^I}})^2 & \text{for } r^I > 3r^D. \end{cases}$$

Similarly, from Corollary 1 we can write a bank's individual resiliency in the status quo as $1 - \phi_{sq} = k_{sq} = 1 - \sqrt{\frac{r^D}{r^I}}$. Plotting these expressions as a function of the ratio $\frac{r^I}{r^D}$, one immediately sees that $1 - \phi_m > 1 - \phi_{sq}$ always holds, so that $\phi_m < \phi_{sq}$. The plot is available from the authors upon request. Q.E.D.

Proof of Corollary 3

Using (15), we can express the expected liquidity needs for the merged banks as

$$\omega_m = \begin{cases} \int_{R_m}^{\frac{D_m}{2}} (x_m - R_m) \frac{4x_m}{D_m^2} dx_m + \int_{\frac{D_m}{2}}^{D_m} (x_m - R_m) \frac{4(D_m - x_m)}{D_m^2} dx_m & \text{for } r^I \leq 3r^D \\ \\ \int_{\frac{D_m}{2}}^{D_m} (x_m - R_m) \frac{4(D_m - x_m)}{D_m^2} dx_m & \text{for } r^I > 3r^D. \end{cases}$$

Solving the integrals, we obtain $\omega_m = \frac{D_m}{2} - R_m + \frac{2}{3}\frac{R_m^3}{D_m^2}$ for $r^I \leq 3r^D$ and $\frac{2}{3}\frac{(D_m - R_m)^3}{D_m^2}$ for $r^I > 3r^D$. Substituting $k_m = \frac{R_m}{D_m}$, we obtain

$$\omega_m = \begin{cases} \left(\frac{1}{2} - k_m + \frac{2}{3}k_m^3\right) D_m & \text{for } r^I \leq 3r^D \\ \\ \frac{2}{3}(1-k_m)^3 D_m & \text{for } r^I > 3r^D. \end{cases}$$

To compare ω_m with $2\omega_{sq}$, we substitute (19) in the above expression for ω_m and (20) in the expression for ω_{sq} as in Corollary 1. We obtain:

$$\omega_m - 2\omega_{sq} = \begin{cases} \left(\frac{1}{2} - k_m + \frac{2}{3}k_m^3\right) D_m - (1-k_{sq})^2 D_{sq} & \text{for } r^I \leq 3r^D \\ \\ \frac{r^D}{r^I}\left(\frac{D_m}{4} - D_{sq}\right) & \text{for } r^I > 3r^D. \end{cases}$$

For $r^I > 3r^D$ it is immediate to see that $\omega_m - 2\omega_{sq} < 0$ if $\frac{D_m}{D_{sq}} < 4$. For $r^I \leq 3r^D$, $\omega_m - 2\omega_{sq}$ can be rearranged as

$$\omega_m - 2\omega_{sq} = (1 - k_{sq})^2 D_{sq} \left[\frac{\left(\frac{1}{2} - k_m + \frac{2}{3}k_m^3\right)}{(1 - k_{sq})^2} \frac{D_m}{D_{sq}} - 1 \right].$$

Suppose for a moment $k_m = k_{sq}$ and $D_m = 2D_{sq}$. Then, the expression simplifies to $k_{sq}^2 D_{sq} \left(\frac{4}{3}k_{sq} - 1\right)$, which is negative because $k_{sq} < 1/2$. To see that this holds also for $k_m > k_{sq}$, we use (20) and rewrite $\omega_m - 2\omega_{sq}$ as

$$\omega_m - 2\omega_{sq} = \frac{r^D}{r^I} D_{sq} \left[\frac{r^I}{r^D} \left(\frac{1}{2} - k_m + \frac{2}{3}k_m^3 \right) \frac{D_m}{D_{sq}} - 1 \right].$$

Denote now $A = \left(\frac{1}{2} - k_m + \frac{2}{3}k_m^3\right)$. Since A is decreasing in k_m and $k_m > k_{sq}$ for $r^I \leq 3r^D$, it follows $\omega_m - 2\omega_{sq} < 0$ when $D_m = 2D_{sq}$. The same holds for $\frac{D_m}{D_{sq}} < 2$. By plotting the expression $(\frac{r^I}{r^D} A \frac{D_m}{D_{sq}} - 1)$ for $\frac{D_m}{D_{sq}} > 2$ and $\frac{r^I}{r^D} \in (1, 3]$, one sees that there is a level $h \in (2, 4)$ of the ratio $\frac{D_m}{D_{sq}}$ such that $\omega_m \leq 2\omega_{sq}$ if $\frac{D_m}{D_{sq}} \leq h$, and $\omega_m > 2\omega_{sq}$ otherwise. The plot is available from the authors upon request. Q.E.D.

Proof of Proposition 4

This proof is a generalization of that of Lemma 1. Let D_{tot} denote the total deposits $ND_{sq} = D_m + (N - 2)D_c$, and let R_{tot} denote the total reserves $NR_{sq} = R_m + (N - 2)R_c$. Applying the general formula for the distribution of a weighted sum of uniformly distributed random variables in Bradley and Gupta (2002) to our model, we obtain the density functions of the aggregate liquidity demands in the status quo $f_{sq}(X_{sq})$ and after the merger $f_m(X_m)$ as

$$f_{sq}(X_{sq}) = \frac{1}{(N-1)!(D_{sq})^N} \sum_{i=0}^{N} \left[(-1)^i \binom{N}{i} (X_{sq} - iD_{sq})_+^{N-1} \right],$$

$$f_m(X_m) = \frac{\sum_{i=1}^{N-2} \left[(-1)^i \binom{N-2}{i-1} (X_m - D_m - (i-1)D_c)_+^{N-2} + \binom{N-2}{i} (X_m - iD_c)_+^{N-2} \right]}{(N-2)! D_m (D_c)^{N-2}}.$$

The two density functions are plotted in Figure 3. The density $f_{sq}(X_{sq})$ is more concentrated around the mean than $f_m(X_m)$. To verify that this is always the case, we compare the variances of X_{sq} and X_m, which are given by

$$Var\,(X_{sq}) = \sum_{i=1}^{N} D_{sq}^2 Var(\delta_i),$$

$$\begin{aligned} Var\left(X_m\right) &= \frac{D_m^2}{4}Var(\delta_1) + \frac{D_m^2}{4}Var(\delta_m) + \sum_{i=3}^{N} D_c^2 Var(\delta_i) \\ &= Var(\delta_i)\left[\frac{D_m^2}{2} + \sum_{i=3}^{N} D_c^2\right] \end{aligned}$$

because $Var(\delta_1) = Var(\delta_2) = Var(\delta_i)$. Since $D_m + \sum_{i=3}^{N} D_c = \sum_{i=1}^{N} D_{sq}$, one obtains $\left[\sum_{i=1}^{2} \frac{D_m^2}{4} + \sum_{i=3}^{N} D_c^2\right] > \sum_{i=1}^{N} D_{sq}^2$ by Lagrangian maximization. Hence, it is always $Var\left(X_m\right) > Var\left(X_{sq}\right)$. Since $f(X_{sq})$ and $f(X_m)$ are well behaved (they approach a normal distribution), they intersect only in two points.[11] This, along with the symmetry of the two density functions around the same mean $E[X_m] = E[X_{sq}] = \frac{D_{tot}}{2}$ and $Var\left(X_m\right) > Var\left(X_{sq}\right)$, implies

$$\Phi_{sq} = \Pr(X_{sq} > R_{tot}) > \Phi_m = \Pr(X_m > R_{tot}) \text{ for any } R_{tot} < \frac{D_{tot}}{2},$$

and vice versa for $R_{tot} > \frac{D_{tot}}{2}$. Using Proposition 1, $R_{tot} = NR_{sq}$, and (1), we obtain that $R_{tot} < \frac{D_{tot}}{2}$ if $\frac{r^I}{r^D} < 4 \equiv \sigma$. The first statement follows.
Using the definition in (7), we have

$$\begin{aligned} \Omega_m - \Omega_{sq} &= \int_{R_{tot}}^{D_{tot}} (X_m - R_{tot}) f_m(X_m) d(X_m) - \int_{R_{tot}}^{D_{tot}} (X_{sq} - R_{tot}) f_{sq}(X_{sq}) d(X_{sq}) \\ &= \int_{R_{tot}}^{D_{tot}} X_m f_m(X_m) d(X_m) - \int_{R_{tot}}^{D_{tot}} X_{sq} f_{sq}(X_{sq}) d(X_{sq}) \\ &\quad - R_{tot}(1 - F_m(R_{tot})) + R_{tot}(1 - F_{sq}(R_{tot})). \end{aligned}$$

Deriving it with respect to R_{tot} gives

$$\begin{aligned} \frac{d(\Omega_m - \Omega_{sq})}{dR_{tot}} &= -R_{tot} f_m(R_{tot}) + R_{tot} f_{sq}(R_{tot}) - (1 - F_m(R_{tot})) \\ &\quad + R_{tot} f_m(R_{tot}) + (1 - F_{sq}(R_{tot})) - R_{tot} f_{sq}(R_{tot}) \\ &= F_m(R_{tot}) - F_{sq}(R_{tot}). \end{aligned}$$

As showed earlier, $F_m(R_{tot}) - F_{sq}(R_{tot}) > 0$ for $R_{tot} < \frac{D_{tot}}{2}$ and $F_m(R_{tot}) - F_{sq}(R_{tot}) < 0$ for $R_{tot} > \frac{D_{tot}}{2}$. Also, $F_m(0) = F_{sq}(0) = 0$ and $F_m(R_{tot}) = F_{sq}(R_{tot}) = 0$. This implies $\Omega_m - \Omega_{sq} > 0$ for all $R_{tot} \in [0, D_{tot}]$. The second statement follows. Q.E.D.

Proof of Lemma 3

Suppose first $\frac{r^I}{r^D} < \rho$. In this range, the aggregate reserve/deposit ratio in the status quo (which coincides with the individual banks' deposit ratio) is smaller than the one after merger; i.e.,

$$k_{sq} = \frac{R_{sq}}{D_{sq}} = \frac{\sum_{i=1}^{N} R_{sq}}{ND_{sq}} < K_m$$

[11] A formal proof that this is the case is in Manzanares (2002).

because $k_m > k_c = k_{sq}$. Consider now the aggregate liquidity risk. When $D_m = 2D_c$, this is given by

$$\Phi_{sq} = prob\left(\sum_{i=1}^{N} \delta_i D_{sq} > \sum_{i=1}^{N} R_{sq}\right) = prob(X' < k_{sq})$$

in the status quo, and by

$$\Phi_m = prob\left(\sum_{i=1}^{N} \delta_i D_c > R_m + \sum_{i=3}^{N} R_c\right) = prob(X' < K_m),$$

after the merger, where $X' = \sum_{i=1}^{N} \frac{\delta_i}{N}$. Since $K_m > k_{sq}$, it follows $\Phi_m < \Phi_{sq}$.

We can then express the expected aggregate liquidity needs in the status quo as

$$\Omega_{sq} = \int_{k_{sq}ND_{sq}}^{ND_{sq}} (X_{sq} - k_{sq}ND_{sq}) f(X_{sq}) d(X_{sq}) = ND_{sq} \int_{k_{sq}}^{1} (X^{'} - k_{sq}) f(X') d(X').$$

Applying the same logic, the post-merger expected aggregate liquidity needs are

$$\begin{aligned} \Omega_m &= ND_c \int_{K_m}^{1} (X^{'} - K_m) f(X') d(X') \\ &= ND_{sq} \left(1 + (K_m - k_{sq})\right) \int_{K_m}^{1} (X^{'} - K_m) f(X') d(X'), \end{aligned}$$

where we have used $D_m = 2D_c$ and $D_m + (N-2)D_c = ND_c = ND_{sq} + (K_m - k_{sq})ND_{sq}$. Given $K_m > k_{sq}$, we can write the expected aggregate liquidity needs as

$$\begin{aligned} \Omega_{sq} &= ND_{sq} \left[\int_{K_m}^{1} (X^{'} - k_{sq}) f(X') d(X') + \int_{k_{sq}}^{K_m} (X^{'} - k_{sq}) f(X') d(X') \right] \\ &= ND_{sq} \left[\begin{array}{c} \int_{K_m}^{1} (X^{'} - K_m) f(X') d(X') + (K_m - k_{sq}) \int_{K_m}^{1} f(X') d(X') + \\ \int_{k_{sq}}^{K_m} (X^{'} - K_m) f(X') d(X'), \end{array} \right] \end{aligned}$$

and, after rearranging and simplifying, we have

$$\Omega_m - \Omega_{sq} = ND_{sq} \left[\begin{array}{c} (K_m - k_{sq}) \int_{K_m}^{1} (X^{'} - K_m - 1) f(X') d(X') \\ - \int_{k_{sq}}^{K_m} (X^{'}_{sq} - K_m) f(X') d(X') \end{array} \right] < 0$$

because $(X^{'} - K_m - 1) < 0$. Analogous steps can be followed for the case $\frac{r^I}{r^D} > \rho$. Q.E.D.

Proof of Proposition 5

Proposition 4 implies that if $k_m = k_{sq}$, then $\Phi_m > \Phi_{sq}$ and $\Omega_m > \Omega_{sq}$ for any $\frac{r^I}{r^D} > \rho$. A fortiori this must be true in equilibrium where $k_m < k_{sq}$ (Φ_m and Ω_m are decreasing in K_m, which falls with k_m). Q.E.D.

Proof of Proposition 6

Statement 1. From the proof of Proposition 4, $K_m = k_{sq}$ implies $\Phi_m = \Phi_{sq}$ when $\frac{r^I}{r^D} = \sigma$, and $\Phi_m < \Phi_{sq}$ when $\frac{r^I}{r^D} < \sigma$. Since $K_m > k_{sq}$ in the range $\frac{r^I}{r^D} < \rho$, it is $\Phi_m < \Phi_{sq}$ when $\frac{r^I}{r^D} = \sigma$. The strict inequality and continuity imply that there must exist a neighborhood where $\frac{r^I}{r^D} > \sigma$ and $\Phi_m < \Phi_{sq}$. For $\frac{r^I}{r^D} > \rho$, $\Phi_m > \Phi_{sq}$ (from Proposition 5); hence, there must exist a critical level $g \in (\sigma, \rho)$ (with $\sigma < \rho$ from the proofs of Propositions 2 and 4) such that as $\Phi_m < \Phi_{sq}$ if $\frac{r^I}{r^D} < g$, and $\Phi_m > \Phi_{sq}$ otherwise. The first statement follows.

Statement 2. From Proposition 2, $k_m = k_{sq}$ for $\frac{r^I}{r^D} = 1$ and $\frac{r^I}{r^D} = \rho$, and $k_m > k_{sq}$ for $1 < \frac{r^I}{r^D} < \rho$. This induces the same relation between K_m and k_{sq}, so that $K_m - k_{sq}$ is first increasing and then decreasing in the interval $\frac{r^I}{r^D} \in (1, \rho)$. By Proposition 4, when $D_m \neq 2D_c$ there is a neighborhood of $\frac{r^I}{r^D} = 1$ where $\Omega_m - \Omega_{sq} > 0$. Also, when $\frac{r^I}{r^D} = \rho$ and $D_m \neq 2D_c$, $\Omega_m > \Omega_{sq}$. When $\frac{r^I}{r^D} = 1$, it is always $\Omega_m = \Omega_{sq} = \frac{D_{tot}}{2}$. From Lemma 3, when $D_m = 2D_c$ it is $\Omega_m - \Omega_{sq} < 0$ for all $\frac{r^I}{r^D} \in (1, \rho)$ and $\Omega_m = \Omega_{sq}$ when $\frac{r^I}{r^D} = \rho$. By continuity, if one fixes a sufficiently small level of asymmetry in the deposit bases across banks ($D_m - 2D_c$ sufficiently small), then $\Omega_m - \Omega_{sq} > 0$ in an immediate neighborhood of $\frac{r^I}{r^D} = 1$. Given that $K_m - k_{sq}$ is increasing around $\frac{r^I}{r^D} = 1$, there will be a higher ratio $\frac{r^I}{r^D}$, named $\underline{g}$, such that if the merger generates that asymmetry when $\frac{r^I}{r^D} = \underline{g}$, then $\Omega_m - \Omega_{sq} = 0$ and $\Omega_m - \Omega_{sq} < 0$ in the immediate right neighborhood. Again by continuity, $\Omega_m - \Omega_{sq} > 0$ in an immediate neighborhood of $\frac{r^I}{r^D} = \rho$. Given that $K_m - k_{sq}$ is decreasing around $\frac{r^I}{r^D} = \rho$, there will be a smaller ratio $\frac{r^I}{r^D}$, named $\overline{g}$, such that, when $\frac{r^I}{r^D} = \overline{g}$, then $\Omega_m - \Omega_{sq} = 0$ and $\Omega_m - \Omega_{sq} < 0$ in the immediate left neighborhood. The second statement follows. Q.E.D.

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Table 1: Bank concentration ratios in industrial countries, 1980-1998

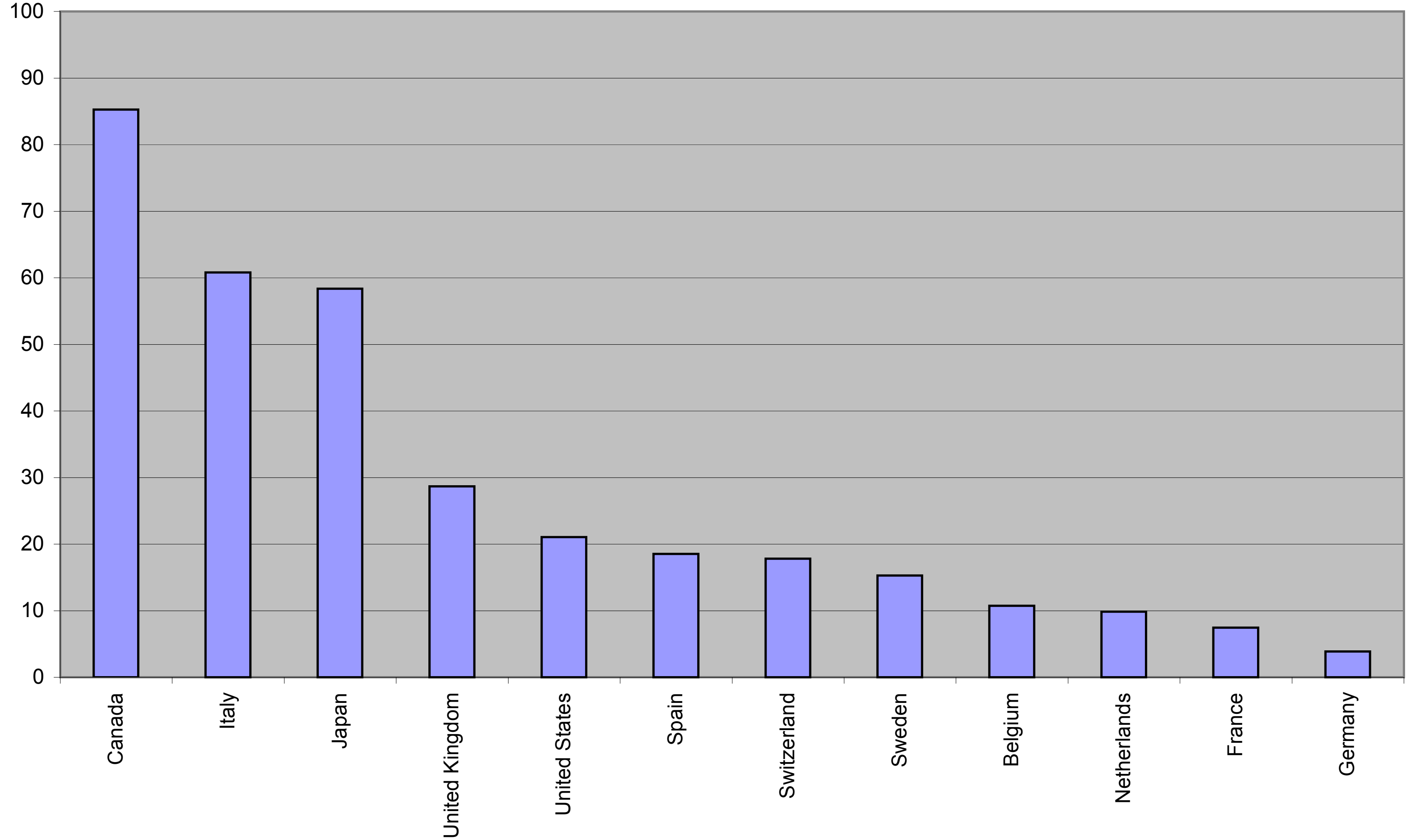
	1980	1990	1998	Change 1980-1990	Change 1990-1998
Europe					
Belgium	53.4	48.0	66.7	-5.4	18.7
France	n.a.	51.9	70.2	n.a.	18.3
Germany	n.a.	17.1	18.8	n.a.	1.7
Italy	n.a.	(25.9)	38.3	n.a.	12.4
Netherlands	n.a.	73.7	81.7	n.a.	8.0
Spain	38.1	38.3	(47.2)	0.2	8.9
Sweden	n.a.	62.0	84.0	n.a.	22.0
Switzerland	n.a.	53.2	(57.8)	n.a.	4.6
United Kingdom	n.a.	43.5	35.2	n.a.	-8.3
North America					
Canada	n.a.	60.2	77.7	n.a.	17.5
United States	14.2	11.3	26.2	-2.9	14.0
Pacific Rim					
Australia	76.5	72.1	73.9	-4.4	1.8
Japan	28.5	31.8	30.9	3.3	-0.9

Notes: Concentration ratios are defined as the share of the five largest banks in total bank deposits (in %). Values in parentheses are for 1992 (Italy) or 1997 (Spain, Switzerland). Changes are in percentage points (Spain and Switzerland 1990-1997, Italy 1992-1998). n.a.=not available. *Source*: Group of Ten, 2001

Table 2: Effects of a merger on loan rates and expected aggregate liquidity needs

	$\frac{r^I}{r^D}$ low	$\frac{r^I}{r^D}$ high
$\frac{c_m}{c_c}$ high	I $\Omega \uparrow\downarrow$ $r^L \uparrow$	III $\Omega \uparrow$ $r^L \uparrow$
$\frac{c_m}{c_c}$ low	II $\Omega \uparrow\downarrow$ $r^L \downarrow$	IV $\Omega \uparrow$ $r^L \downarrow$

Figure 1: Banking consolidation in industrial countries, 1990-99



Note *Number of domestic M&As between banks (1990-99) divided by the average number of banks (1990-99) times 100. Australia has been excluded for data consistency.*

Source *Group of Ten, 2001.*

Figure 3: Aggregate liquidity risk before merger, Φ_{sq}, and after merger, Φ_m

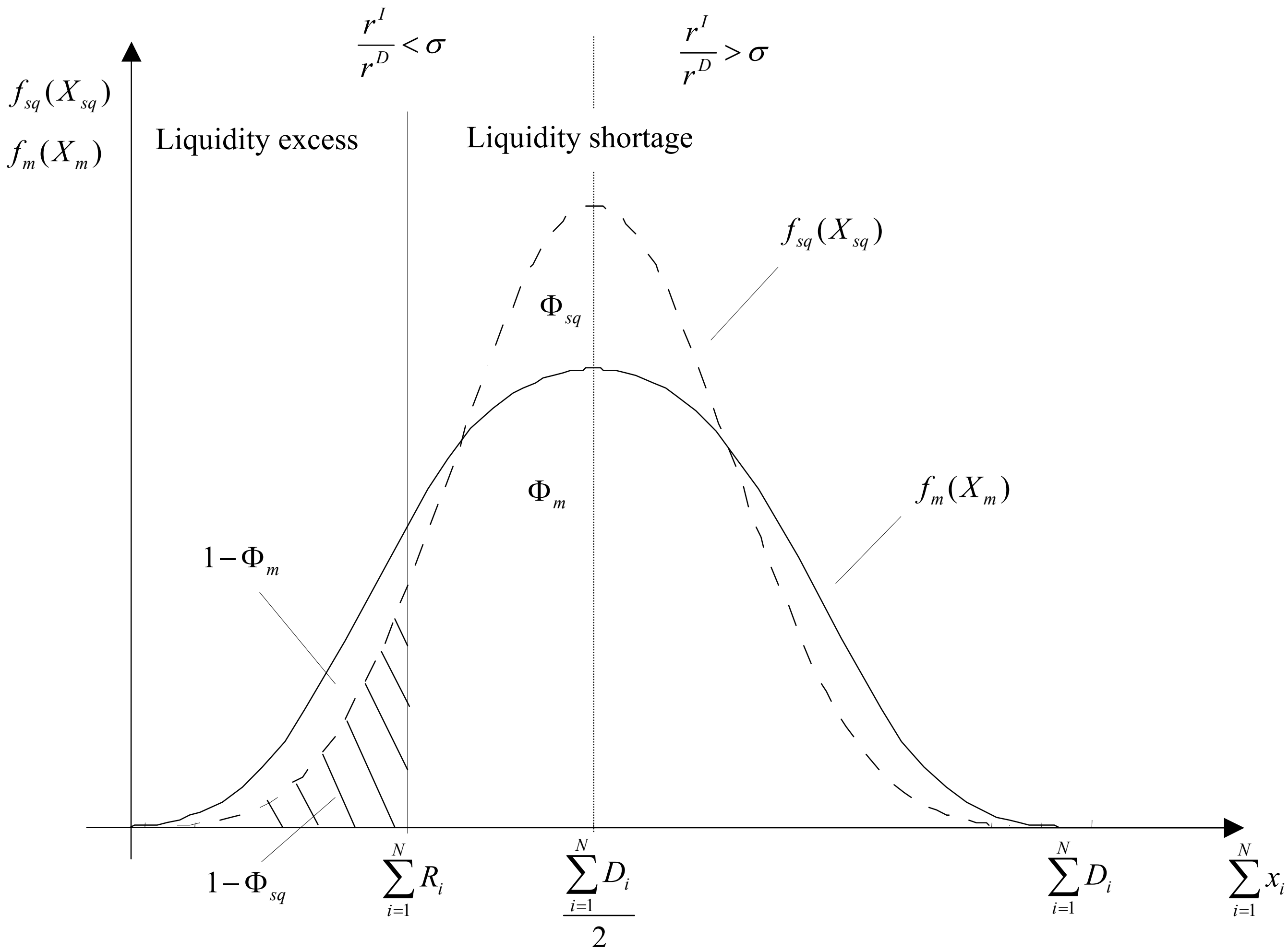
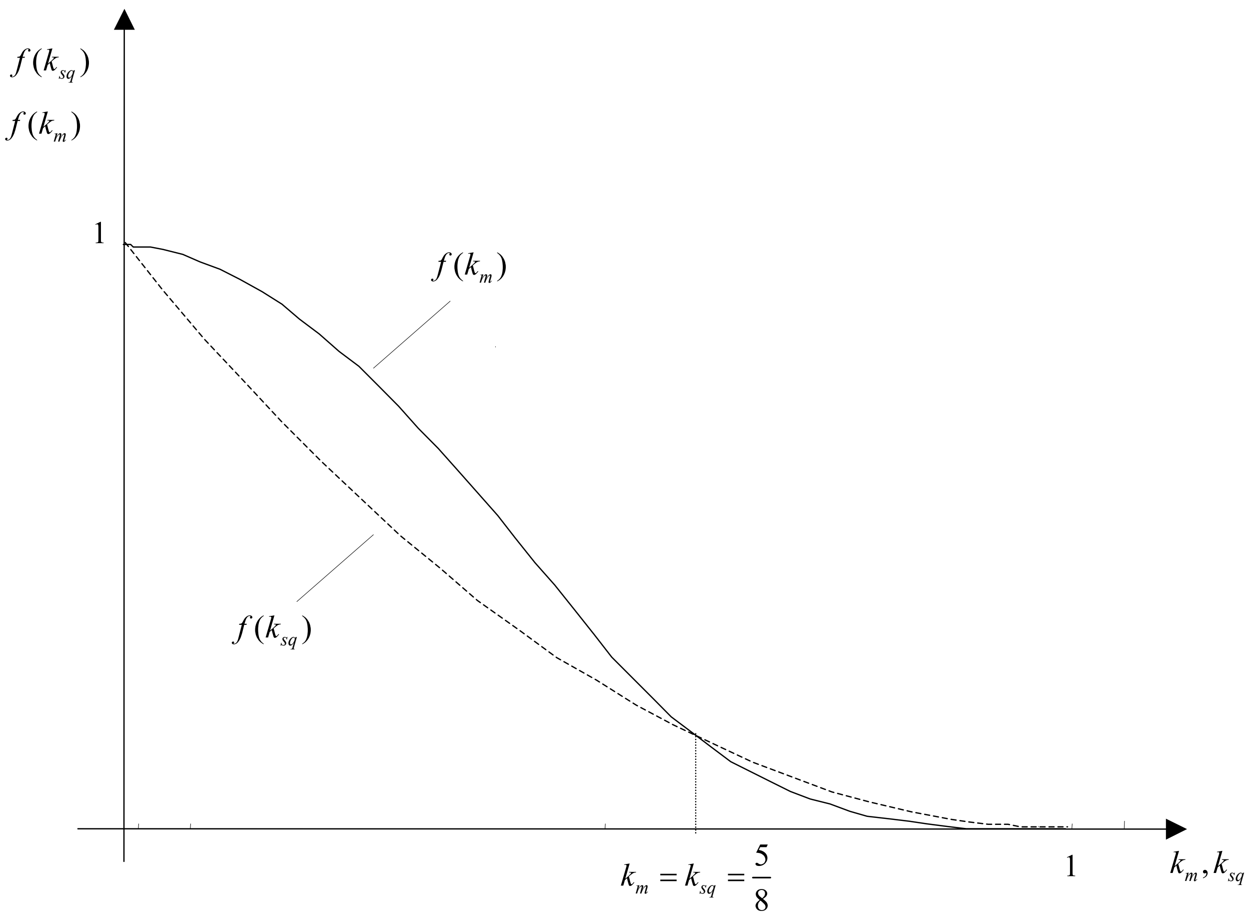


Figure 4: Marginal benefits of higher reserve-deposit ratios for the merged banks, $f(k_m)$, and for banks in the status quo, $f(k_{sq})$.



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