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Are Constant Interest Rate Forecasts Modest Interventions? Evidence from an Estimated Open Economy DSGE Model of the Euro Area

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Abstract

This paper uses an estimated open economy DSGE model to examine if constant interest forecasts one and two years ahead can be regarded as modest policy interventions during the period $1993Q4 - 2002Q4$. An intervention is here defined to be modest if it does not trigger the agents to revise their expectations about the inflation targeting policy. Using univariate modesty statistics, we show that the modesty of the policy interventions depends on the assumptions about the uncertainty in the future shock realizations. In $1998Q4 - 2002Q4$, the two year constant interest rate projections turn out immodest when assuming uncertainty only about monetary policy shocks during the conditioning period. However, allowing non-policy shocks to influence the forecasts makes the interventions more modest, at least one year ahead. Using a multivariate statistic, however, which takes the joint effects of the policy interventions into consideration, we find that the conditional policy shifts all projections beyond what is plausible in the latter part of the sample ($1998Q4 - 2002Q4$), and thereby affects the expectations formation of the agents. Consequently, the constant interest rate assumption has arguably led to conditional forecasts at the two year horizon that cannot be considered economically meaningful during this period.

Keywords: Forecasting; Monetary policy; Open economy DSGE model; Policy interventions; Bayesian inference.

JEL Classification Numbers: E47; E52; C11; C53.

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1. Introduction

After the introduction of explicit inflation targets, many central banks produce two-year ahead forecasts of aggregate prices and quantities since monetary policy is thought to have delayed effects on economic activity. The rhetoric of some central banks, for example the Bank of England and Sveriges Riksbank, suggests that output and inflation forecasts are computed under the assumption that the interest rate is kept unchanged during the forecast horizon. If the constant interest rate (CIR) projection results in an inflation forecast above or below the target, the interest rate is adjusted accordingly - so called constant interest rate inflation forecast targeting. This suggests a *structural* monetary policy targeting rule of the form

$$\Delta R_t = \alpha \left[\mathrm{E}_t(\pi_{t+h} | R_{t+h} = ... = R_{t-1}, \widetilde{S}_t) - \pi^* \right] + \varepsilon_{R,t}, \tag{1}$$

where $\widetilde{S}_t$ is the state of the economy. If in period t the CIR inflation forecast h periods ahead, $\mathrm{E}_t(\pi_{t+h}|R_{t+h} = ... = R_{t-1}, \widetilde{S}_t)$, is one percent above(below) the inflation target π^*, the interest rate R_t is increased(decreased) by α percent. The inclusion of an error term $\varepsilon_{R,t}$ - a monetary policy shock - reflects the conventional assumption that the central bank does not exactly follow the targeting rule in each period.[1]

The inflation forecast from a forward-looking theoretical model economy depends in general on the state of the economy as well as the expected policy throughout the forecast horizon (see Appendix B). Inserting this into equation (1) implies that the *reduced form* representation of the monetary policy rule in an inflation targeting economy is given by

$$R_t = g'_{\widetilde{S}} \widetilde{S}_t + g_R R_{t-1} + \varepsilon_{R,t}, \tag{2}$$

where the response coefficients $g_{\widetilde{S}}$ and g_R depend on the structure of the theoretical model and the policy maker's preferences (i.e., how strongly the central bank reacts on CIR forecast deviations from the target, α), see also Honkapohja and Mitra (2004).[2] Thus, according to the reduced form version of the policy rule, the central bank should not keep the interest rate constant throughout the forecast horizon, even if the inflation forecast in the structural framework adopts such an assumption. Set aside the impact of the monetary policy shock, the central bank should respond to all state variables and shocks (collected in $\widetilde{S}_t$) that have an effect on the CIR forecast of inflation in period $t + h$. The interest rate will then change within the forecast horizon as the inflation target span in the structural rule moves forward in time, see Leitemo (2003). Keeping the reduced form interest rate constant throughout the forecast horizon when generating an inflation projection would therefore violate the assumptions made in the structural framework with the targeting rule (1).[3]

[1] The policy rule stated above could of course be extended with some measure of real economic activity, and also be generalized to convey the fact that central banks typically are concerned with the whole path of future inflation, and not just the inflation forecast of a given horizon. Honkapohja and Mitra (2004) study the properties of the rule in equation (1) within a learning framework, and find that the properties of the rule are satisfactory. Moreover, Honkapohja and Mitra show that if some measure of aggregate activity is introduced in the policy rule, the properties of the rule are even better. However, if the inflation forecast is contingent on R_t instead of R_{t-1}, Honkapohja and Mitra demonstrate that the rule can easily lead to indeterminacy of equilibria and instability under learning. Given that inflation forecasts are typically issued once every quarter, and that the interest rate decision is usually announced afterwards, we additionally think it is much more natural to condition on R_{t-1} rather than R_t.

[2] The truly reduced form of the model is obtained by, in turn, inserting the policy rule (2) into the model economy and solving the system using standard techniques.

[3] In Appendix B, we discuss the relation between the CIR inflation targeting rule (1) and the reduced form

Consequently, if central banks produce their forecasts in practice under the assumption of a constant interest rate using the reduced form solution of the model, this implies deviating from the structural targeting rule.[4] In this paper, we examine whether such constant interest rate forecasts are modest policy interventions using a forward-looking open economy DSGE model estimated for the Euro area. With modest, we here mean a policy intervention (e.g., the constant interest rate forecast) which does not significantly shift the agents' beliefs about the current policy regime. In particular we evaluate whether constant interest rate projections are perceived as being too far away from those of the estimated policy rule in the model using two variants of the modesty statistic developed by Leeper and Zha (2003).

The idea behind the statistic developed by Leeper and Zha (2003) is to compare the conditional forecast under a policy intervention with the unconditional forecast. If the interventions due to the alternative policy (i.e. the conditional forecast) cause the forecasted variables to deviate largely from their unconditional forecasts, the policy interventions are not perceived as being modest. This implies that the forecasts are not 'believable', and that the formation of expectations need to be incorporated when projecting under this alternative policy. In the framework of Leeper and Zha there is no uncertainty related to future shock realizations except for the policy shock during the conditioning period. We will, however, also consider a case where we allow for uncertainty induced by the other shocks in the model. Given that other shocks than the monetary policy shock account for a large part of the fluctuations in the system, the two cases are likely to yield different results. Once we allow for uncertainty regarding the future non-policy shocks, it is also possible to develop a multivariate generalization of the modesty statistic which accounts for the intervention effects on the joint movements of the variables.

The DSGE model used in the analysis is an open economy version of the closed economy DSGE model of Christiano, Eichenbaum and Evans (2005). As in Altig et al. (2003), we introduce a stochastic unit root technology shock, which enables us to work with trending data. Following Smets and Wouters (2003), the model is estimated with Bayesian techniques on data for the Euro area.[5] DSGE models estimated with Bayesian techniques are particularly interesting to use for this type of experiments for three reasons. First, they are based on optimizing firms and households so expectations about the monetary policy regime are important. Second, Smets and Wouters (2004) have shown that large scale DSGE models, such as the one used here, are quite good descriptions of the data. They have also shown that such models have good forecasting properties compared to Bayesian VARs, which are generally considered to be very good forecasting tools.[6] Third, and last, these models have a monetary transmission mechanism that are close in line with the conventional wisdom about the effects of monetary policy, and also

policy rule (2) in more detail. The reason why a CIR projection using the reduced form solution is not an appropriate forecast for the interest rate decision, is that the policy rule (1) will in general require a different interest rate in the subsequent periods due to the fact that the forecast targeting horizon moves forward. Forward-looking agents will realize that the policy rule (1) does not imply that the interest rate will remain constant during the period $t, t+1, ..., t+h$ and determine their expectations of policy accordingly. See the introduction in Leitemo (2003) for a more detailed discussion.

[4]This is only a correct procedure if the model of the economy is entirely backward-looking, so that no expectation formations effects are relevant for the private sector. In this case, the reduced form projections using equation (2) are the same as setting the interest rate constant during the forecast horizon and applying the structural policy rule on the resulting forecast. In a forward-looking model, the relevant forecasts for the central bank are generated by i) letting the interest rate follow the reduced form (2), ii) solve the model with (2) , and iii) use the reduced form solution of the complete model to calculate the policy consistent projections.

[5]Regarding the open economy assumption for the Euro area, see Adolfson et al. (2005), where the model is presented and discussed in detail.

[6]Preliminary results suggest that the open economy DSGE model used in this paper also compare favorably in out-of-sample forecasts relative to Bayesian VARs. This will be reported in detail elsewhere.

well in line with the results in the identified VAR literature, see, e.g., Rotemberg and Woodford (1997), and Christiano, Eichenbaum and Evans (2005).

Our main results can be summarized as follows. According to the Leeper and Zha statistic, which only allows for uncertainty about future monetary policy shocks, we find that constant interest rate forecasts are, in general, immodest at the 4 quarter horizon during $1998Q4-2001Q1$, and at the 8 quarter horizon they are all immodest from $1998Q4$ and onwards (the forecasts are evaluated during $1993Q4-2002Q4$). When we allow for uncertainty about all shocks, the univariate statistic indicates that all CIR forecasts are modest at the 4 quarter horizon. For the 8 quarter horizon, we find that the constant interest rate forecast is immodest during most quarters in 1999 for inflation, employment and output. So quite naturally, bringing in uncertainty about future non-policy shock realizations makes CIR forecasts more plausible in general, because it gets harder for the agents to figure out to what extent the differences between the conditional and unconditional forecast is due to policy or the other shocks hitting the economy.

The multivariate statistic, however, which takes all shock uncertainty into account and measures the effects on all variables jointly, suggests that the constant interest rate forecasts are immodest at the 4 quarter horizon during 1999, and from around 1999 and onwards at the 8 quarter horizon. The exact details are dependent on which set of variables that are considered.

The first important factor for why the constant interest rate interventions are often found to be immodest during the latter part of the sample but not in the first part, is that the policy rule has changed in such a way that the effects of monetary policy shocks are relatively stronger in the end of the sample compared to the first part. The second factor is differences related to the state of the economy over the sample period. According to our model, the economy appears to be relatively far from the steady state in the latter part of the sample (e.g., the nominal interest rate is very low compared to the steady state level), which makes the constant interest rate projections less plausible.

The paper is organized as follows. In Section 2, we briefly describe the theoretical open economy DSGE model, and report the whole sample estimation results. We present the testing framework and evaluate the conditional forecasts in Section 3. Finally, Section 4 provides some conclusions.

2. The estimated DSGE model

2.1. The theoretical model

This section gives an overview of the model and some key equations, and presents the log-linearized model. We refer to Adolfson, Laséen, Lindé and Villani (2005) for a more detailed description of the model.

Consumption preferences are subject to habit formation and the households attain utility from consuming a basket consisting of domestically produced goods and imported products. These products are supplied by domestic and importing firms, respectively. There is a continuum of households which attain utility from consumption, leisure and real cash balances. The preferences of household j are given by

$$\mathrm{E}_0^j \sum_{t=0}^{\infty} \beta^t \left[\zeta_t^c \ln \left(C_{j,t} - bC_{j,t-1}\right) - \zeta_t^h A_L \frac{(h_{j,t})^{1+\sigma_L}}{1+\sigma_L} + A_q \frac{\left(\frac{Q_{j,t}}{z_t P_t^d}\right)^{1-\sigma_q}}{1-\sigma_q} \right], \tag{3}$$

where $C_{j,t}$, $h_{j,t}$ and $Q_{j,t}/P_t^d$ denote the j^{th} household's levels of aggregate consumption, work effort and real cash holdings, respectively. To make the real balances stationary when the economy is growing they are scaled by z_t, the unit root technology shock. We allow for internal habit persistence by including $bC_{j,t-1}$. Aggregate consumption is assumed to be given by a basket of domestically produced and imported goods according to the following constant elasticity of substitution (CES) function:

$$C_t = \left[(1-\omega_c)^{1/\eta_c} \left(C_t^d \right)^{(\eta_c - 1)/\eta_c} + \omega_c^{1/\eta_c} \left(C_t^m \right)^{(\eta_c - 1)/\eta_c} \right]^{\eta_c/(\eta_c - 1)}, \tag{4}$$

where C_t^d and C_t^m are consumption of the domestic and imported good, respectively. ω_c is the share of imports in consumption, and η_c is the elasticity of substitution across consumption goods.

The households can save in domestic bonds and foreign bonds, and also hold cash. Following Benigno (2001), we assume that there is a premium on the foreign bond holdings which depends on the aggregate net foreign asset position of the domestic households. This ensures a well defined steady-state in the model.

The households invest in a basket of domestic and imported investment goods to form the physical capital stock, and decide how much capital services to rent to the domestic firms, given certain capital adjustment costs. These are costs to adjusting the investment rate as well as costs of varying the utilization rate of the physical capital stock. Total investment is assumed to be given by a CES aggregate of domestic and imported investment goods (I_t^d and I_t^m, respectively) according to

$$I_t = \left[(1-\omega_i)^{1/\eta_i} \left(I_t^d \right)^{(\eta_i - 1)/\eta_i} + \omega_i^{1/\eta_i} \left(I_t^m \right)^{(\eta_i - 1)/\eta_i} \right]^{\eta_i/(\eta_i - 1)}, \tag{5}$$

where ω_i is the share of imports in investment, and η_i is the elasticity of substitution across investment goods.

Further, along the lines of Erceg, Henderson and Levin (2000), each household is a monopoly supplier of a differentiated labour service which implies that they can set their own wage. This gives rise to a wage equation with Calvo (1983) stickiness.

There is a continuum of intermediate domestic firms that each produce a differentiated good. These intermediate goods are sold to a retailer which transforms the intermediate products into a homogenous final good that in turn is sold to the households. The domestic firms determine the capital services and labour inputs used in their production which is exposed to unit root technology growth as in Altig et al. (2003). Production of the domestic intermediate good i follows

$$Y_{i,t} = z_t^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} H_{i,t}^{1-\alpha} - z_t \phi, \tag{6}$$

where z_t is a unit-root technology shock, ϵ_t is a covariance stationary technology shock, and $H_{i,t}$ denotes homogeneous labour hired by the i^{th} firm. Notice that $K_{i,t}$ is not the physical capital stock, but rather the capital services stock, since we allow for variable capital utilization in the model. Note also that a fixed cost is included in the production function to ensure that profits are zero in steady state.

The domestic firms, the importing and exporting firms all produce differentiated goods and set prices according to an indexation variant of the Calvo model. By including nominal rigidities in the importing and exporting sectors we allow for short-run incomplete exchange rate pass-through to both import and export prices, following for example Smets and Wouters (2002).

To simplify the analysis we adopt the assumption that the foreign prices, output (HP-detrended) and interest rate are exogenously given by an identified VAR(4) model. The fiscal policy variables - taxes on capital income, labour income, consumption, and the pay-roll, together with (HP-detrended) government expenditures - are assumed to follow an identified VAR(2) model.[7]

The first-order conditions of the households and the firms are log-linearized around the steady state, according to the following. The domestic (d), importing consumption (mc), importing investment (mi) and exporting (x) firms operating in this economy each have a particular Phillips curve:

$$\begin{aligned}\left(\widehat{\pi}_t^j - \widehat{\bar{\pi}}_t^c\right) &= \frac{\beta}{1+\kappa_j\beta}\left(\mathrm{E}_t\widehat{\pi}_{t+1}^j - \rho_\pi \widehat{\bar{\pi}}_t^c\right) + \frac{\kappa_j}{1+\kappa_j\beta}\left(\widehat{\pi}_{t-1}^j - \widehat{\bar{\pi}}_t^c\right) \\ &\quad -\frac{\kappa_j\beta\left(1-\rho_\pi\right)}{1+\kappa_j\beta}\widehat{\bar{\pi}}_t^c + \frac{(1-\xi_j)(1-\beta\xi_j)}{\xi_j\left(1+\kappa_j\beta\right)}\left(\widehat{mc}_t^j + \widehat{\lambda}_t^j\right),\end{aligned} \tag{7}$$

where $j = \{d, mc, mi, x\}$, $\widehat{\pi}_t^j = (\hat{P}_t^j - \hat{P}_{t-1}^j)$ denotes inflation in sector j, and $\widehat{\bar{\pi}}_t^c$ a time-varying inflation target of the central bank.[8] The ξ:s are the Calvo price stickiness parameters in each sector, and the κ:s are the indexation parameters.[9] $\widehat{\lambda}_t^d$, $\widehat{\lambda}_t^{mc}$, $\widehat{\lambda}_t^{mi}$, and $\widehat{\lambda}_t^x$ are stochastic AR(1) processes determining the time-varying markups in the four markets. The firms' marginal costs are defined as $\widehat{mc}_t^d = \alpha\left(\hat{\mu}_{z,t} + \hat{H}_t - \hat{k}_t\right) + \widehat{\bar{w}}_t + \hat{R}_t^f - \hat{\epsilon}_t$, $\widehat{mc}_t^{mc} = \widehat{P}_t^* + \widehat{S}_t - \widehat{P}_t^{mc}$, $\widehat{mc}_t^{mi} = \widehat{P}_t^* + \widehat{S}_t - \widehat{P}_t^{mi}$, and $\widehat{mc}_t^x = \widehat{P}_t^d - \widehat{S}_t - \widehat{P}_t^x$, respectively. $\hat{\mu}_{z,t}$ is the stochastic growth rate of the unit root technology shock, $\hat{H}_t$ hours worked, $\hat{k}_t$ the capital services stock, $\widehat{\bar{w}}_t$ the real wage, and $\hat{R}_t^f$ the effective nominal interest rate paid by the firms, reflecting the assumption that a fraction ν of the firms' wage bill has to be financed in advance (throughout the paper, we set $\nu = 1$). $\hat{\epsilon}_t$ is a stationary technology shock, $\widehat{P}_t^*$ the foreign price level and $\widehat{S}_t$ is the nominal exchange rate.

Under the assumption that those households that are not allowed to reoptimize their nominal wage in the current period instead update it according to the indexation scheme $W_{t+1} = \left(\pi_t^c\right)^{\kappa_w}\left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w)}\mu_{z,t+1}W_t$, the real wage equation can be written

$$\mathrm{E}_t\left[\begin{array}{c}\eta_0\widehat{\bar{w}}_{t-1} + \eta_1\widehat{\bar{w}}_t + \eta_2\widehat{\bar{w}}_{t+1} + \eta_3\left(\hat{\pi}_t^d - \widehat{\bar{\pi}}_t^c\right) + \eta_4\left(\hat{\pi}_{t+1}^d - \rho_{\widehat{\bar{\pi}}^c}\widehat{\bar{\pi}}_t^c\right) \\ +\eta_5\left(\hat{\pi}_{t-1}^c - \widehat{\bar{\pi}}_t^c\right) + \eta_6\left(\hat{\pi}_t^c - \rho_{\widehat{\bar{\pi}}^c}\widehat{\bar{\pi}}_t^c\right) \\ +\eta_7\hat{\psi}_{z,t} + \eta_8\hat{H}_t + \eta_9\hat{\tau}_t^y + \eta_{10}\hat{\tau}_t^w + \eta_{11}\hat{\zeta}_t^h\end{array}\right] = 0, \tag{8}$$

where $\hat{\pi}_t^c$ denotes CPI inflation, $\hat{\psi}_{z,t}$ the marginal utility of one additional income unit, $\hat{\tau}_t^y$ a labour income tax, $\hat{\tau}_t^w$ a pay-roll tax assumed to be paid by the households, and $\hat{\zeta}_t^h$ a labour supply shock. The η:s are composite parameters determined by the Calvo wage stickiness ξ_w, the pay-roll tax τ^w, the labour income tax τ^y, the labour supply elasticity σ_L, the wage markup λ_w, the wage indexation κ_w, and the discount factor β.

[7] It should be noted that Adolfson et al. (2005) report that the fiscal shocks have small dynamic effects in the model, presumably because these shocks are transitory and do not generate any wealth effects for the infinitively lived households.

[8] A hat denotes log-linearized variables throughout the paper (i.e, $\widehat{X}_t = dX_t/X$), and variables without time-subscript steady-state values. Variables denoted with small letters have been stationarized with the unit root technology shock.

[9] For the firms that are not allowed to reoptimize their price, we adopt the indexation scheme $P_{t+1}^j = \left(\pi_t^j\right)^{\kappa_j}\left(\bar{\pi}_{t+1}^c\right)^{1-\kappa_j}P_t^j$ where $j = \{d, mc, mi, x\}$.

The households' consumption preferences are subject to internal habit formation, which yields the following Euler equation for consumption expenditures:

$$\mathrm{E}_t\left[\begin{array}{c} -b\beta\mu_z\hat{c}_{t+1}+\left(\mu_z^2+b^2\beta\right)\hat{c}_t-b\mu_z\hat{c}_{t-1}+b\mu_z\left(\hat{\mu}_{z,t}-\beta\hat{\mu}_{z,t+1}\right)+ \\ +\left(\mu_z-b\beta\right)\left(\mu_z-b\right)\hat{\psi}_{z,t}+\frac{\tau^c}{1+\tau^c}\left(\mu_z-b\beta\right)\left(\mu_z-b\right)\hat{\tau}_t^c \\ +\left(\mu_z-b\beta\right)\left(\mu_z-b\right)\hat{\gamma}_t^{c,d}-\left(\mu_z-b\right)\left(\mu_z\hat{\zeta}_t^c-b\beta\hat{\zeta}_{t+1}^c\right) \end{array}\right]=0, \tag{9}$$

where $\hat{c}_t$ is consumption, $\hat{\tau}_t^c$ a consumption tax, $\hat{\gamma}_t^{c,d}$ the relative price between consumption and domestically produced goods, $\hat{\zeta}_t^c$ a consumption preference shock, b the habit persistence parameter, and μ_z is the steady-state growth rate.

By combining the first order conditions for the domestic and foreign bond holdings we obtain the following modified uncovered interest rate parity condition:

$$\widehat{R}_t-\widehat{R}_t^*=\mathrm{E}_t\Delta\widehat{S}_{t+1}-\widetilde{\phi}_a\widehat{a}_t+\widehat{\widetilde{\phi}}_t, \tag{10}$$

where $\widehat{R}_t$ is the domestic nominal interest rate, $\widehat{R}_t^*$ the foreign nominal interest rate, $\widehat{a}_t$ the net foreign asset position, and $\widehat{\widetilde{\phi}}_t$ a shock to the risk premium. Because of our assumption of imperfect integration in the international financial markets, the net foreign asset position enters.

The households' first order conditions for the physical capital stock ($\widehat{\bar{k}}_t$), investment ($\hat{\imath}_t$), and the utilization rate ($\widehat{u}_t=\widehat{k}_t-\widehat{\bar{k}}_t$, where $\widehat{k}_t$ denotes capital services) can be written:

$$\begin{array}{r} \hat{\psi}_{z,t}+\mathrm{E}_t\hat{\mu}_{z,t+1}-\mathrm{E}_t\hat{\psi}_{z,t+1}-\frac{\beta(1-\delta)}{\mu_z}\mathrm{E}_t\hat{P}_{k',t+1}+\hat{P}_{k',t} \\ -\frac{\mu_z-\beta(1-\delta)}{\mu_z}\mathrm{E}_t\hat{r}_{t+1}^k+\frac{\tau^k}{\left(1-\tau^k\right)}\frac{\mu_z-\beta(1-\delta)}{\mu_z}\mathrm{E}_t\hat{\tau}_{t+1}^k=0, \end{array} \tag{11}$$

$$\hat{P}_{k',t}+\hat{\Upsilon}_t-\hat{\gamma}_t^{i,d}-\mu_z^2\tilde{S}''\left[\left(\hat{\imath}_t-\hat{\imath}_{t-1}\right)-\beta\left(\hat{\imath}_{t+1}-\hat{\imath}_t\right)+\hat{\mu}_{z,t}-\beta\mathrm{E}_t\hat{\mu}_{z,t+1}\right]=0, \tag{12}$$

$$\widehat{u}_t=\frac{1}{\sigma_a}\widehat{r}_t^k-\frac{1}{\sigma_a}\frac{\tau^k}{\left(1-\tau^k\right)}\widehat{\tau}_t^k, \tag{13}$$

where $\hat{P}_{k',t}$ is the price of capital, $\widehat{r}_t^k$ the firms' real rental rate of capital services given by $\hat{r}_t^k=\hat{\mu}_{z,t}+\widehat{\bar{w}}_t+\hat{R}_t^f+\hat{H}_t-\hat{k}_t$, $\hat{\Upsilon}_t$ an investment specific technology shock, $\hat{\gamma}_t^{i,d}$ the relative price between investment and domestically produced goods, $\hat{\tau}_t^k$ a capital income tax, $\tilde{S}''$ the adjustment cost of changing investments, δ the depreciation rate, and σ_a the cost of varying the capital utilization rate.

The log-linearized law of motion for the physical capital stock is given by

$$\widehat{\bar{k}}_{t+1}=(1-\delta)\frac{1}{\mu_z}\widehat{\bar{k}}_t-(1-\delta)\frac{1}{\mu_z}\hat{\mu}_{z,t}+\left(1-(1-\delta)\frac{1}{\mu_z}\right)\hat{\Upsilon}_t+\left(1-(1-\delta)\frac{1}{\mu_z}\right)\hat{\imath}_t. \tag{14}$$

The evolution of net foreign assets at the aggregate level satisfies

$$\begin{aligned} \hat{a}_t \quad = \quad & -y^*\widehat{mc}_t^x-\eta_f y_t^*\widehat{\gamma}_t^{x,*}+y^*\hat{y}_t^*+y^*\widehat{\tilde{z}}_t^*+\left(c^m+i^m\right)\widehat{\gamma}_t^f \\ & -c^m\left[-\eta_c\left(1-\omega_c\right)\left(\gamma^{c,d}\right)^{-(1-\eta_c)}\widehat{\gamma}_t^{mc,d}+\widehat{c}_t\right] \\ & -i^m\left[-\eta_i\left(1-\omega_i\right)\left(\gamma^{i,d}\right)^{-(1-\eta_i)}\widehat{\gamma}_t^{mi,d}+\widehat{i}_t\right]+\frac{R}{\pi\mu_z}\hat{a}_{t-1}, \end{aligned} \tag{15}$$

where $\hat{y}_t^*$ denotes foreign output, $\widehat{\tilde{z}}_t^*$ is a stationary shock which measures the degree of asymmetry in the technological progress in the domestic economy versus the rest of the world, and

$\widehat{\gamma}_t^{x,*}$,$\hat{\gamma}_t^{f}$, $\widehat{\gamma}_t^{mc,d}$ and $\widehat{\gamma}_t^{mi,d}$ are relative prices defined as $\widehat{\gamma}_t^{x,*} = \hat{P}_t^x - \hat{P}_t^*$, $\hat{\gamma}_t^f = \hat{P}_t^d - \hat{S}_t - \hat{P}_t^*$, $\widehat{\gamma}_t^{mc,d} = \hat{P}_t^{mc} - \hat{P}_t^d$ and $\widehat{\gamma}_t^{mi,d} = \hat{P}_t^{mi} - \hat{P}_t^d$, respectively.

The log-linearized first order conditions for money balances and the households' cash holdings are, respectively:

$$\mathrm{E}_t\left[-\mu\hat{\psi}_{z,t} + \mu\hat{\psi}_{z,t+1} - \mu\hat{\mu}_{z,t+1} + \left(\mu - \beta\tau^k\right)\hat{R}_t - \mu\hat{\pi}_{t+1} + \frac{\tau^k}{1-\tau^k}\left(\beta - \mu\right)\hat{\tau}_{t+1}^k\right] = 0, \tag{16}$$

$$q_t = \frac{1}{\sigma_q}\left[\frac{\tau^k}{1-\tau^k}\widehat{\tau}_t^k - \widehat{\psi}_{z,t} - \frac{R}{R-1}\widehat{R}_{t-1}\right]. \tag{17}$$

The log-linearized aggregate resource constraint is

$$\begin{aligned} & (1-\omega_c)\left(\gamma^{c,d}\right)^{\eta_c}\frac{c}{y}\left(\hat{c}_t + \eta_c\widehat{\gamma}_t^{c,d}\right) + (1-\omega_i)\left(\gamma^{i,d}\right)^{\eta_i}\frac{i}{y}\left(\hat{\imath}_t + \eta_i\widehat{\gamma}_t^{i,d}\right) \\ & +\frac{g}{y}\hat{g}_t + \frac{y^*}{y}\left(\hat{y}_t^* - \eta_f\widehat{\gamma}_t^{x,*} + \widehat{\tilde{z}}_t^*\right) \\ = & \\ & \lambda_f\left(\hat{\epsilon}_t + \alpha\left(\hat{k}_t - \hat{\mu}_{z,t}\right) + (1-\alpha)\hat{H}_t\right) - \left(1-\tau^k\right)r^k\frac{\bar{k}}{y}\frac{1}{\mu_z}\left(\hat{k}_t - \widehat{\bar{k}}_t\right), \end{aligned} \tag{18}$$

where $\widehat{\gamma}_t^{i,d}$ is the relative price between investment and domestically produced goods.

To clear the loan market, the demand for liquidity from the firms (which are financing their wage bills) must equal the supplied deposits of the households plus the monetary injection by the central bank:

$$\nu\bar{w}H\left(\hat{\nu}_t + \widehat{\bar{w}}_t + \hat{H}_t\right) = \frac{\mu\bar{m}}{\pi\mu_z}\left(\hat{\mu}_t + \widehat{\bar{m}}_t - \hat{\pi}_t - \hat{\mu}_{z,t}\right) - q\hat{q}_t, \tag{19}$$

Following Smets and Wouters (2003a), monetary policy is approximated with the instrument rule

$$\begin{aligned} \widehat{R}_t \;=\; & \rho_R\widehat{R}_{t-1} + (1-\rho_R)\left[\widehat{\bar{\pi}}_t^c + r_\pi\left(\hat{\pi}_{t-1}^c - \widehat{\bar{\pi}}_t^c\right) + r_y\hat{y}_{t-1} + r_x\hat{x}_{t-1}\right] \\ & + r_{\Delta\pi}\left(\hat{\pi}_t^c - \hat{\pi}_{t-1}^c\right) + r_{\Delta y}\Delta\hat{y}_t + \varepsilon_{R,t}, \end{aligned} \tag{20}$$

where $\varepsilon_{R,t}$ is an uncorrelated monetary policy shock. Thus, the central bank is assumed to adjust the short term interest rate in response to deviations of CPI inflation from the time-varying inflation target $\left(\hat{\pi}_t^c - \widehat{\bar{\pi}}_t^c\right)$, the output gap ($\hat{y}_t$, measured as actual minus trend output), the real exchange rate ($\hat{x}_t$) and the interest rate set in the previous period. In addition, note that the nominal interest rate adjusts directly to the inflation target.

2.2. Estimation

To estimate the model we use quarterly Euro area data for the period $1970Q1$-$2002Q4$. The data set employed here was first constructed by Fagan et al. (2001).[10] We include a large set of variables when we estimate the model in order to facilitate identification of the parameters, and match the following 15 variables: the domestic inflation rate π_t; the growth rates in consumption Δc_t(Δ denotes the first difference operator), investment Δi_t, GDP Δy_t, exports $\Delta\widetilde{X}_t$, imports

[10] The Fagan data set includes foreign (i.e., rest of the world) output and inflation, but not a foreign interest rate. We therefore use the Fed funds rate as a proxy for R_t^*.

$\Delta\widetilde{M}_t$, the real wage $\Delta\overline{w}_t$, the consumption deflator $\pi_t^{def,c}$ and the investment deflator $\pi_t^{def,i}$; the real exchange rate x_t; the short-run interest rate R_t; employment E_t; foreign inflation π_t^*; the foreign interest rate R_t^*; and the growth rate in foreign output Δy_t^*.[11] The reason for modeling the real variables in growth rates is that the unit root technology shock induces a stochastic trend in the levels of these variables. To calculate the likelihood function of the observed variables we apply the Kalman filter.[12]

A number of parameters are kept fixed throughout the estimation procedure. Most of these parameters can be related to the steady-state values of the observed variables in the model, and are therefore calibrated so as to match the sample mean of these.[13]

Table 1 shows the assumptions for the prior distribution of the estimated parameters. The location of the prior distribution of the 51 estimated parameters corresponds to a large extent to those in Smets and Wouters (2003) and the findings in Altig et al. (2003) on U.S. data. For more details about our choice of prior distributions, see Adolfson et al. (2005).

The joint posterior distribution of all estimated parameters is obtained in two steps. First, the posterior mode and Hessian matrix evaluated at the mode is computed by standard numerical optimization routines. Second, draws from the joint posterior are generated using the Metropolis-Hastings algorithm (see Smets and Wouters (2003), and the references therein, for details). In Table 1 we report the posterior mode.

In Figures 1a and 1b, we report the posterior mode estimates of the impulse responses from a monetary policy shock. Figure 1a displays the responses based on estimates from five sample periods with different end-of-sample period (i.e., the sample is sequentially extended from $1980Q1 - 1993Q4$ two year at a time up to $1980Q1 - 2001Q4$). The responses appear to have changed considerably over the sample period. An almost equally sized unexpected increase in the nominal interest rate has more impact on both real and nominal variables in 1999 than in 1993. To understand why this occurs, we experiment with the parameters in the policy rule. Figure 1b displays the impulse responses using the $1993Q4$ estimate of the parameters, together with the responses when we use the structural parameters from the $1993Q4$ estimation but take the policy parameters from the $1999Q4$ estimate. Although the structural parameters are the same, the impulse responses change dramatically when the policy rule parameters are taken from the $1999Q4$ estimate. The impact of a monetary policy shock is greater in the latter case because the interest rate persistence is larger and the response to the output gap is weaker in the policy rule in 1999 than in 1993 (see Table 2). It is imperative to note that the changes in the impulse response functions are not due to changes in the structural or deep parameters. To see

[11] There is no (official) data on aggregate hours worked, $\hat{H}_t$, available for the euro area. Therefore, we use employment $\hat{E}_t$ in our estimations. Since employment is likely to respond more slowly to shocks than hours worked, we model employment using Calvo-rigidity (following Smets and Wouters, 2003a): $\Delta\hat{E}_t = \beta E_t \Delta\hat{E}_{t+1} + \frac{(1-\xi_e)(1-\beta\xi_e)}{\xi_e}\left(\hat{H}_t - \hat{E}_t\right)$. For reasons discussed in greater detail in Adolfson et al. (2005), we take out a linear trend in employment and the excess trend in imports and exports relative to the trend in GDP prior to estimation.

[12] We use the period $1970Q1$-$1979Q4$ to form a prior on the unobserved state variables in $1979Q4$, and then use the period $1980Q1$-$2002Q4$ for inference.

[13] The calibrated parameters are set to the following: the money growth $\mu = 1.01$; the discount factor $\beta = 0.999$; the depreciation rate $\delta = 0.013$; the capital share in production $\alpha = 0.29$; the share of imports in consumption and investment $\omega_c = 0.31$ and $\omega_i = 0.55$, respectively; the steady-state tax rates on labour income and consumption $\tau^y = 0.177$ and $\tau^c = 0.125$, respectively; government expenditure-output ratio 0.20. For reasons discussed in greater detail in Adolfson et al. (2005), we also set the substitution elasticity between domestic and imported goods $\eta_c = 5$ and the capital utilization parameter $\sigma_a = 10^6$. The calibrated parameters, together with some of the estimated parameters (e.g. μ_z, the steady-state growth rate) evaluated at the prior mode, imply a consumption-output ratio of 0.58, an investment-output ratio of 0.22, an import-output (and export-output) ratio of 0.25 in the steady state. Likewise, the quarterly gross interest rate (R) becomes 1.013, and the quarterly gross domestic inflation 1.005 in the steady state.

this clearly, we also plot the responses from the parametrization using the structural parameters from the 1999Q4 estimation and policy parameters from the 1993Q4 estimation. The change in the responses are very small compared to the case where all parameters come from the 1993Q4 estimation, as can be seen in Figure 1b. Differences in the responses to a monetary policy shock can thus be explained by changes in the policy parameters over the sample period we will be using in the subsequent evaluation.

3. Conditional forecasts

3.1. Framework

According to Leeper and Zha (2003), an intervention is modest if it "[...] does not significantly shift agents' beliefs about policy regime and does not induce the changes in behavior that Lucas (1976) emphasizes".[14] We evaluate whether constant interest rate forecasts are modest using three different statistics. First, we use the univariate modesty statistic developed by Leeper and Zha (2003) where it is assumed that no other shocks than the monetary policy shock hit the economy during the conditioning period. Second, we use their univariate statistic but allow for other shocks to hit the economy. We think that allowing for all types of shock uncertainty is more natural than assuming no uncertainty about the future economy during the conditional forecasting period. Third, by allowing other shocks to hit the economy, we can evaluate the conditional forecast against a multivariate modesty statistic that accounts for the effects of the conditional policy on the system as whole.[15]

The general idea behind the modesty statistic is the following. If the intervention is to be considered modest the conditional forecast (i.e., the constant interest rate forecast using the reduced form solution of the forward-looking model) should not deviate too much from that of the unconditional forecast (i.e., the forecast from the solution using the reduced form policy rule in (2) implied by the CIR inflation targeting rule). Thus, an implicit assumption behind the test is that the structural policy rule can change randomly, where a regime change cannot be directly observed by private agents but has to be inferred from the forecasting output of the central bank. Moreover, beliefs in the current policy regime must be firmly held by the private agents in order for the log-linear solution approximation to be accurate.

The univariate modesty statistic for variable y_i at horizon h is defined as

$$M_i^h(\bar{\varepsilon}_{T+1}^{T+h}) \equiv \frac{y_{i,T+h}(\bar{\varepsilon}_{T+1}^{T+h}) - \hat{y}_{i,T+h|T}}{Std[y_{i,T+h}(\varepsilon_{T+1}^{T+h})]}, \tag{21}$$

where $y_{i,T+h}(\bar{\varepsilon}_{T+1}^{T+h})$ is the realization of y_i at time $t = T + h$ if the structural shocks over the intervention period are $\bar{\varepsilon}_{T+1}^{T+h} = (\bar{\varepsilon}_{T+1}, ..., \bar{\varepsilon}_{T+h})$, and $\hat{y}_{i,T+h|T} = \mathrm{E}_T(y_{i,T+h})$ is the usual no-intervention forecast at time t. $M_i^h(\varepsilon_{T+1}^{T+h})$ follows a normal distribution with zero mean and unit variance if the ε_{T+1}^{T+h}-sequence is drawn from their no-intervention distribution. To

[14] Leeper and Zha (2003) use an identified VAR model whereas we use a DSGE model. However, since in both cases the reduced form is used to generate the forecasts, this is of no importance for the modesty statistic. If the policy intervention is immodest the agents are most likely going to reoptimize, and the reduced form solution of the model (VAR or DSGE) is no longer valid. That is, the Lucas critique applies even if the structural model is a DSGE model.

[15] One could also allow for parameter uncertainty, measurement errors and uncertainty about the current state, but we assume that agents operating in this economy, as well as the central bank, do not suffer from these sources of uncertainty. However, these sources of forecast uncertainty are small compared to uncertainty about future realizations of the shocks.

signal an intervention as immodest for variable y_i at horizon h, Leeper and Zha (2003) therefore advocate that the absolute value of the univariate statistic should be larger than two (i.e., $\left|M_i^h(\bar{\varepsilon}_{T+1}^{T+h})\right| > 2$). In the Leeper-Zha implementation of the statistic in (21), it is assumed that only monetary policy shocks are present in the conditioning periods, i.e. $\varepsilon_t = (\varepsilon_{R,t}, 0, ..., 0)'$, for $t = T+1, ...T+h$, where $\varepsilon_{R,t}$ is the policy shock. We will also consider the case where all shocks are allowed to be non-zero. Expressions of $M_i^h(\bar{\varepsilon}_{T+1}^{T+h})$ is given in Appendix A, both with and without non-policy shocks.

The multivariate modesty statistic, in turn, measures the intervention effects on all variables' forecasts taken together. A natural multivariate generalization of the modesty statistic may then be based on

$$M^h(\bar{\varepsilon}_{T+1}^{T+h}) \equiv [y_{T+h}(\bar{\varepsilon}_{T+1}^{T+h}) - \hat{y}_{T+h|T}]'\Omega_{T+h}^{-1}[y_{T+h}(\bar{\varepsilon}_{T+1}^{T+h}) - \hat{y}_{T+h|T}], \tag{22}$$

where $\Omega_{T+h} = \text{Cov}[y_{T+h}(\varepsilon_{T+1}^{T+h})]$. Since y_{T+h} is multivariate normally distributed it follows that the distribution of $M^h(\varepsilon_{T+1}^{T+h})$ under no intervention is a chi-squared distribution with p degrees of freedom, where p is the number of observed variables in the system. Thus, if $M^h(\bar{\varepsilon}_{T+1}^{T+h})$ lies far out in the right tail of the χ_p distribution we may say that the intervention $\bar{\varepsilon}_{T+1}^{T+h}$ is immodest. Rather than using a certain quantile of the χ_p distribution as a cut-off value (which varies with p), we will use the tail probability $\Pr[M^h(\varepsilon_{T+1}^{T+h}) \geq M^h(\bar{\varepsilon}_{T+1}^{T+h})]$ to determine whether or not an intervention is modest. A small probability, say below 0.05, signals an immodest intervention. It should be noted that the unexpected movements of the variables in the original Leeper-Zha framework are linearly dependent and Ω_{T+h} is singular. The multivariate statistic therefore only makes sense if non-policy shocks are allowed to be non-zero in the conditioning period. For more details, see Appendix A.

3.2. Results

Figure 2a and 2b show the univariate modesty statistic for the twelve domestic variables at 4 and 8 quarters horizon, when the forecast is conditioned on a constant interest rate. In Figure 2a we report the modesty statistic when no other shocks than the monetary policy shock hit the economy during the conditioning period (i.e., the original Leeper-Zha statistic). The Leeper-Zha statistic indicates that the constant interest rate interventions are immodest for almost all variables at 4 quarters horizon during 1998Q4 to 2001Q1. At the 8 quarter horizon the immodesty signal is even stronger. Here the statistic marks the constant interest rate intervention as immodest from 1998Q4 and onwards, for all variables except imports. Hence, constant interest forecasts are not meaningful to carry out in the latter part of the sample, according to the Leeper-Zha statistic.

Remember though that the monetary policy shocks are the only disturbances the agents perceive in the previous case. In Figure 2b we allow also for other shocks during the two year conditioning period. Allowing for uncertainty about the future economy naturally makes the agents more uncertain about the central bank's policy changes, which in turn increases the denominator in the modesty statistic (see equation 21). Consequently, the conditional interventions are in many more periods considered to be modest compared to when no other shocks are allowed. At four quarters horizon all of the variables indicate modest interventions (see Figure 2b). At eight quarters horizon, however, the inflation series (both domestic and the consumption and investment deflators), employment, interest rate, and output forecasts seem to display immodest interventions in 1999Q1-1999Q3. The inflation forecast also indicates an immodest intervention in 1999Q4. Figure 2c additionally shows the p-values of the multivariate modesty statistic. Taken the joint effects of the conditional forecasts into account, the interventions are more immodest (i.e., less plausible). The interventions turn out as immodest also at the four quarters horizon during 1999, at least when using the multivariate modesty statistic accounting for all variables. Using only the CPI inflation and output forecasts jointly, the modesty signal is a lot weaker at the four quarters horizon. However, at eight quarters horizon none of the interventions are modest during 1999 and some quarters during 2000 and 2001, according to this statistic. Accounting for all variables the multivariate modesty statistic at the two year horizon specifies all interventions from $1998Q4$ and onwards as immodest. This implies that the joint effects of the interventions are larger than unilaterally.

To understand the differences between the two univariate modesty statistics, Figures 3 and 4 depict the actual forecasts under a constant interest rate. Figures 3a and 3b show the unconditional and conditional forecasts when standing in $1999Q2$, without and with shock uncertainty, respectively. Figures 4a and 4b report the same forecasts when standing in $2001Q1$. The figures display the median and 95th percentile forecasts for the domestic variables. The unconditional forecast is depicted in grey and with a solid line, and the conditional forecast by dashed lines. In $1999Q2$, the two univariate statistics both indicate immodest interventions. However, as seen from Figure 3a the uncertainty bands of the forecasts are much smaller when no other shocks hit the economy during the conditioning period (i.e., the Leeper-Zha case). Consequently, the agents can make better inference about the policy maker's behavior and the Leeper-Zha statistic gives stronger indication of immodesty. The conditional forecasts are further away from the unconditional ones, compared to the case (in Figure 3b) where the agents are uncertain about several shocks that are hitting the economy.

In $2001Q1$ the indications of immodesty is less clear, at least in the case when other shocks are allowed to hit the economy during the conditioning period (see Figures 2b and 4b). Figure 4b shows that the median forecasts conditional on a constant interest rate lie well within the 95% uncertainty bands of the unconditional forecasts. Treating each variable independently would then suggest that the conditional forecasts are modest and thus economically meaningful. However, when taking the effects on all variables into account jointly, the interventions have in fact quite considerable impact on the model. That is, the multivariate statistic indicate immodesty also in $2001Q1$. To understand why the univariate and multivariate statistics yield slightly different results we look at plots of the bivariate conditional and unconditional forecasts.

Figure 5a and 5b show the univariate and bivariate forecasting distributions for inflation, output and the interest rate in $1999Q2$ and $2001Q1$, when allowing for uncertainty about the future shocks. The dashed lines denote the 4th quarter horizon in all panels, and the solid lines the 8th quarter horizon. The diagonal displays the unconditional univariate distributions (kernel density estimates) together with the median of the conditional forecast (vertical line). The off-diagonal shows the 68th and the 95th probability contours of the unconditional forecast distributions approximated by bivariate normal distributions, and the conditional forecasts represented by a cross, at four and eight quarters horizon. In $2001Q1$ the bivariate conditional forecast at eight quarters horizon of, for example, the interest rate and output lie far out of the 95th percentile of the unconditional bivariate forecast distribution (see Figure 5b). A similar pattern can be seen for the bivariate forecast distributions for inflation and the interest rate. At the same time the conditional forecasts of these variables are not nearly as extreme with respect to their respective marginal distributions (see the diagonal in Figure 5b). The effect on the joint movements of the variables is thus important for assessing the modesty of the intervention, and this aspect is captured in the multivariate statistic. It should also be noted that the multivariate statistic measures the degree of unexpected movements due to the intervention with respect to the 12-dimensional forecast distribution of all the domestic variables in the system, and that the bivariate distributions in Figure 5 therefore only provide a partial picture of the workings of this statistic.

4. Concluding remarks

Our starting point in this paper is that monetary policy in many countries can be described by a constant interest rate (CIR) inflation targeting rule of the type presented in equation (1). Given this, one important insight is that in forward-looking models, the CIR inflation targeting rule does not imply that the forecasts should be computed with a constant interest rate in practice. As noted by Leitemo (2003) and Honkapohja and Mitra (2004), a constant interest rate forecast is not compatible with a CIR inflation targeting rule in a forward-looking model because private agents update their expectations about policy during the forecast horizon.

Some central banks say, nonetheless, that they actually compute forecasts keeping the interest rate constant, which seems inconsistent with the CIR targeting rule they claim they have adopted. It is, however, unclear how the CIR forecasts are used in the interest rate decision process and to what extent central banks base their inflation reports on CIR forecasts or on letting the interest rate follow the reduced form policy rule (2) implied by CIR inflation targeting. This paper has therefore dealt with the following question: would private agents perceive CIR forecasts to be in line with the announced CIR targeting rule? The tool in our investigation was an estimated open economy DSGE model which was used to examine if CIR forecasts were modest policy interventions in the sense that they would not have given rise to private agents changing their perceptions about the structural monetary policy rule, following the framework in Leeper and Zha (2003). Our main findings are that CIR forecasts were not modest policy interventions during most of $1999-2002$ for the Euro area, looking at the forecasts of inflation and output at the eight quarter horizon. For a larger set of macroeconomic variables, the results speak against the use of CIR forecasts even more. Our interpretation of these results is that CIR forecasts are not a useful communication device of CIR targeting rules. Note also that some former CIR inflation forecast central banks (for example, the Bank of England) are today reconsidering the role of CIR forecasts and are moving towards forecasts that account for the private agents' perceptions of the future interest rate.

Appendix A. Statistics for modest interventions in state space models

This appendix details the implementation of the three modesty statistics discussed in the text in the state-space model

$$\begin{aligned} \xi_t &= F\xi_{t-1} + B\varepsilon_t \\ y_t &= A'x_t + H'\xi_t + w_t, \end{aligned} \tag{A.1}$$

where ξ_t is the partially unobserved vector of state variables and y_t is a p-dimensional vector of observed variables. The innovation sequence $\{\varepsilon\}_{t=1}^T$ and the measurement errors $\{w_t\}_{t=1}^T$ are assumed to be *iid* multivariate normal processes with zero mean and $\mathrm{Cov}(w_t) = R$ and $\mathrm{Cov}(\varepsilon_t) = \Sigma_\varepsilon$, respectively. See, e.g., Hamilton (1994, Ch. 13) for details.

We will first treat the univariate and multivariate modesty statistics in (21) and (22) in the case where all shocks are allowed during the conditioning period. We then move on to the univariate Leeper-Zha modesty statistic, with only monetary policy shocks. In order to compute the modesty statistics we need expressions for $\hat{y}_{T+h|T} = \mathrm{E}_T(y_{T+h})$, the baseline forecast, and $\Omega_{T+h} = \mathrm{Cov}[y_{T+h}(\varepsilon_{T+1}^{T+h})]$, the covariance matrix of y_{T+h} when the shocks are drawn from the model distributions. Note first that

$$\hat{y}_{T+h|T} = A'x_{T+h} + H'E_T(\xi_{T+h}),$$

where $\mathrm{E}_T(\xi_{T+h}) = F^h\hat{\xi}_{T|T}$ and $\hat{\xi}_{T|T} = \mathrm{E}_T(\hat{\xi}_T)$ is obtained from the Kalman filter (Hamilton, 1994, Ch. 13). Thus,

$$y_{T+h} - \hat{y}_{T+h|T} = H'(\xi_{T+h} - F^h\hat{\xi}_{T|T}) + w_{T+h},$$

and

$$\Omega_{T+h} = H'P_{T+h|T}H + R.$$

$P_{T+h|T}$ is obtained from the recursion

$$P_{T+i|T} = FP_{T+i-1|T}F' + B\Sigma_\varepsilon B',$$

where, under the assumption that the agents observe the past history of ξ_t, $P_{T|T}$ is the zero matrix.

If we assume that all shocks other than the monetary policy shock are zero as in Leeper and Zha (2003), the above formulae still applies if we replace the recursion for $P_{T+i|T}$ with

$$P_{T+i|T} = FP_{T+i-1|T}F' + \sigma_{\varepsilon_1}^2 B_1 B_1',$$

where $\sigma_{\varepsilon_1}^2$ is the variance of the monetary policy shocks and B_1 is the first column of B (assuming the monetary policy shock to be first shock in ε_t). Note that $\mathrm{rank}(P_{T+i|T}) = i$, for $i \geq 0$. This in turn implies that Ω_{T+h} has rank h and therefore that Ω_{T+h} is singular for all horizons h smaller than the number variables in y_t. Thus, with only a single shock to drive the system, the unexpected movements in the variables are linearly dependent and the multivariate modesty statistic makes no sense.

Appendix B. CIR interest rate policy rules in forward-looking models

To understand why CIR inflation targeting policy rules implies a systematic change of the interest rate during the forecast horizon in a forward-looking model, the reduced form policy rule is derived in this appendix. The DSGE model can be written on matrix form as

$$\begin{aligned} A_0\widetilde{Y}_t &= A_1\mathrm{E}_t\widetilde{Y}_{t+1} + B_0\theta_t + C\mathrm{E}_t\widetilde{R}_{t+1}, \\ \theta_t &= \rho\theta_{t-1} + \varepsilon_t, \end{aligned} \tag{B.2}$$

where $\widetilde{Y}_t$ is a $n_Y \times 1$ vector of the endogenous variables in the theoretical model (contemporaneous and lagged), θ_t is a vector of exogenous variables, and $\widetilde{R}_{t+1} = (R_{t+1},\ R_t,\ R_{t-1})'$. Finally, the model is closed by the inflation targeting rule in (1).

To derive the reduced form, we need to compute $\mathrm{E}_t(\pi_{t+h}|R_{t+h} = ... = R_{t-1}, \widetilde{S}_t) - \pi^*$, where $\widetilde{S}_t \equiv (\widetilde{Y}_t,\ \theta_t)'$. Without loss of generality, we here assume that $\pi^* = 0$. Assuming that A_0 is invertible, iterating forward h periods implies

$$\widetilde{Y}_t = \widetilde{A}_1^h\mathrm{E}_t\widetilde{Y}_{t+h} + \sum_{j=0}^{h-1}\widetilde{A}_1^j\widetilde{B}_0\rho^j\theta_t + \sum_{j=0}^{h-1}\widetilde{A}_1^j\widetilde{C}\mathrm{E}_t\widetilde{R}_{t+1+j}, \tag{B.3}$$

where $\widetilde{A}_1 = A_0^{-1}A_1$, $\widetilde{B}_0 = A_0^{-1}B_0$, and $\widetilde{C} = A_0^{-1}C$. Notice that $\widetilde{A}_1^0$ and ρ^0 are identity matrices. Equation (B.3) can be rearranged as

$$\mathrm{E}_t\widetilde{Y}_{t+h} = \widetilde{A}_1^{-h}\left(\widetilde{Y}_t - \sum_{j=0}^{h-1}\widetilde{A}_1^j\widetilde{B}_0\rho^j\theta_t - \sum_{j=0}^{h-1}\widetilde{A}_1^j\widetilde{C}\mathrm{E}_t\widetilde{R}_{t+1+j}\right), \tag{B.4}$$

provided that $\widetilde{A}_1^{-h}$ exists. Equation (B.4) shows that $\mathrm{E}_t\widetilde{Y}_{t+h}$ depends on the state of the economy, $\widetilde{S}_t$, as well as expectations about current and future monetary policy.

Assuming that the interest rate is kept equal to R_{t-1} during the forecasting horizon $t + 1, ...t + h$, we have that $\widetilde{R}_{t+h} = ... = \widetilde{R}_t = (R_{t-1},\ R_{t-1},\ R_{t-1})'$, which implies that (B.4) can be rewritten as

$$\mathrm{E}_t(\widetilde{Y}_{t+h}|R_{t+h} = ... = R_{t-1}, \widetilde{S}_t) = S_Y(h)\widetilde{Y}_t - S_\theta(h)\theta_t - S_R(h)R_{t-1}, \tag{B.5}$$

where $S_Y(h) \equiv \widetilde{A}_1^{-h}$, $S_\theta(h) \equiv S_Y(h)\sum_{j=0}^{h-1}\widetilde{A}_1^j\widetilde{B}_0\rho^j$, and $S_R(h) \equiv S_Y(h)\sum_{j=0}^{h-1}\widetilde{A}_1^j\widetilde{C}\ \iota$, $\iota = (1, 1, 1)'$.

To obtain the reduced form policy rule in equation (2) of the main text, we insert (B.5) into the structural policy rule, and by defining

$$\begin{aligned} g_{\widetilde{S}} &\equiv (S_Y(h), -S_\theta(h))'\tilde{\alpha}, \\ g_R &\equiv 1 - \tilde{\alpha}' S_R(h), \end{aligned}$$

where $\tilde{\alpha} \equiv (0, ..., \alpha, ..., 0)'$ is a $n_Y \times 1$ vector with zeros except in the location of π_t in $\widetilde{Y}_t$, we have equation (2).

Finally, the reduced-form solution of the model that are used for forecasting purposes, are obtained by solving (B.2) and (2) using standard methods (e.g., the Anderson and Moore (1985) algorithm).

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Table 1: Prior and posterior distributions

Parameter		Prior distribution			Sample period 1980Q1-2002Q4 Posterior distribution				
		type	mean*	std.dev. /df	mode	std. dev. (Hessian)	mean	5%	95%
Calvo wages	ξ_w	beta	0.675	0.050	0.697	0.047	0.690	0.607	0.766
Calvo domestic prices	ξ_d	beta	0.675	0.050	0.883	0.015	0.891	0.862	0.921
Calvo import cons. prices	ξ_{mc}	beta	0.500	0.100	0.463	0.059	0.444	0.345	0.540
Calvo import inv. prices	ξ_{mi}	beta	0.500	0.100	0.740	0.040	0.721	0.641	0.792
Calvo export prices	ξ_x	beta	0.500	0.100	0.639	0.059	0.612	0.506	0.717
Calvo employment	ξ_e	beta	0.675	0.100	0.792	0.022	0.787	0.741	0.827
Indexation wages	κ_w	beta	0.500	0.150	0.516	0.160	0.497	0.258	0.739
Indexation domestic prices	κ_d	beta	0.500	0.150	0.212	0.066	0.217	0.095	0.362
Index..import cons. prices	κ_{mc}	beta	0.500	0.150	0.161	0.074	0.220	0.084	0.418
Index..import inv. prices	κ_{mi}	beta	0.500	0.150	0.187	0.079	0.231	0.098	0.405
Indexation export prices	κ_x	beta	0.500	0.150	0.139	0.072	0.185	0.069	0.347
Markup domestic	λ^d	inv. gamma	1.200	2	1.168	0.053	1.222	1.122	1.383
Markup imported cons.	λ^{mc}	inv. gamma	1.200	2	1.619	0.063	1.633	1.526	1.751
Markupimported invest.	λ^{mi}	inv. gamma	1.200	2	1.226	0.088	1.275	1.146	1.467
Investment adj. cost	$\tilde{S}''$	normal	7.694	1.500	8.732	1.370	8.670	6.368	10.958
Habit formation	b	beta	0.650	0.100	0.690	0.048	0.708	0.608	0.842
Subst. elasticity invest.	η_i	inv. gamma	1.500	4	1.669	0.273	1.696	1.393	2.142
Subst. elasticity foreign	η_f	inv. gamma	1.500	4	1.460	0.098	1.486	1.340	1.674
Technology growth	μ_z	trunc. normal	1.006	0.0005	1.005	0.000	1.005	1.004	1.006
Capital income tax	τ_k	beta	0.120	0.050	0.137	0.042	0.135	0.072	0.200
Labour pay-roll tax	τ_w	beta	0.200	0.050	0.186	0.050	0.197	0.118	0.286
Risk premium	$\tilde{\phi}$	inv. gamma	0.010	2	0.145	0.047	0.252	0.139	0.407
Unit root tech. shock	ρ_{μ_z}	beta	0.850	0.100	0.723	0.106	0.698	0.526	0.852
Stationary tech. shock	ρ_{ε}	beta	0.850	0.100	0.909	0.030	0.886	0.810	0.939
Invest. spec. tech shock	ρ_{Υ}	beta	0.850	0.100	0.750	0.041	0.720	0.638	0.796
Asymmetric tech. shock	$\rho_{\tilde{z}*}$	beta	0.850	0.100	0.993	0.002	0.992	0.986	0.995
Consumption pref. shock	ρ_{ζ^c}	beta	0.850	0.100	0.935	0.029	0.892	0.722	0.964
Labour supply shock	ρ_{ζ^h}	beta	0.850	0.100	0.675	0.062	0.676	0.565	0.774
Risk premium shock	$\rho_{\tilde{\phi}}$	beta	0.850	0.100	0.991	0.008	0.955	0.922	0.991
Imp. cons. markup shock	$\rho_{\lambda^{mc}}$	beta	0.850	0.100	0.978	0.016	0.970	0.943	0.991
Imp. invest. markup shock	$\rho_{\lambda^{mi}}$	beta	0.850	0.100	0.974	0.015	0.963	0.931	0.989
Export markup shock	ρ_{λ^x}	beta	0.850	0.100	0.894	0.045	0.886	0.789	0.961
Unit root tech. shock	σ_z	inv. gamma	0.200	2	0.130	0.025	0.137	0.099	0.185
Stationary tech. shock	σ_{ε}	inv. gamma	0.700	2	0.452	0.082	0.519	0.361	0.756
Invest. spec. tech. shock	σ_{Υ}	inv. gamma	0.200	2	0.424	0.046	0.469	0.389	0.561
Asymmetric tech. shock	$\sigma_{\tilde{z}*}$	inv. gamma	0.200	2	0.203	0.031	0.217	0.166	0.276
Consumption pref. shock	σ_{ζ^c}	inv. gamma	0.200	2	0.151	0.031	0.157	0.108	0.224
Labour supply shock	σ_{ζ^h}	inv. gamma	0.050	2	0.095	0.015	0.098	0.075	0.128
Risk premium shock	$\sigma_{\tilde{\phi}}$	inv. gamma	0.400	2	0.130	0.023	0.183	0.128	0.246
Domestic markup shock	σ_{λ^d}	inv. gamma	0.300	2	0.130	0.012	0.132	0.111	0.157
Imp. cons. markup shock	$\sigma_{\lambda^{mc}}$	inv. gamma	0.300	2	2.548	0.710	2.882	1.737	4.463
Imp. invest. markup shock	$\sigma_{\lambda^{mi}}$	inv. gamma	0.300	2	0.292	0.079	0.354	0.218	0.550
Export markup shock	σ_{λ^x}	inv. gamma	0.300	2	0.977	0.214	1.124	0.772	1.604
Monetary policy shock	σ_R	inv. gamma	0.150	2	0.133	0.013	0.135	0.113	0.160
Inflation target shock	$\sigma_{\bar{\pi}^c}$	inv. gamma	0.050	2	0.044	0.012	0.053	0.032	0.081
Interest rate smoothing	ρ_R	beta	0.800	0.050	0.874	0.021	0.881	0.844	0.915
Inflation response	r_{π}	normal	1.700	0.100	1.710	0.067	1.730	1.577	1.876
Diff. infl response	$r_{\Delta\pi}$	normal	0.300	0.100	0.317	0.059	0.310	0.212	0.411
Real exch. rate response	r_x	normal	0.000	0.050	-0.009	0.008	-0.009	-0.024	0.006
Output response	r_y	normal	0.125	0.050	0.078	0.028	0.104	0.051	0.168
Diff. output response	$r_{\Delta\pi}$	normal	0.0625	0.050	0.116	0.028	0.128	0.081	0.177
Log marginal likelihood					-1909.34				

*Note: For the inverse gamma distribution, the mode and the degrees of freedom are reported. Also, for the parameters $\lambda^d, \lambda^{mc}, \lambda^{mi}, \eta_i, \eta_f$ and μ_z the prior distributions are truncated at 1. A posterior sample of 550,000 draws was generated from the posterior of which the first 50,000 draws were discarded as burn-in. Convergence was checked using standard diagnostics such as CUSUM plots and ANOVA on parallel simulation sequences.

Table 2: Policy parameters from sequential estimations (posterior mode estimates)

Parameter		End-of-sample period								
		1993Q4	1994Q4	1995Q4	1996Q4	1997Q4	1998Q4	1999Q4	2000Q4	2001Q4
Interest rate smoothing	ρ_R	0.790	0.799	0.826	0.835	0.863	0.891	0.888	0.896	0.881
		(0.031)	(0.032)	(0.029)	(0.028)	(0.025)	(0.017)	(0.017)	(0.016)	(0.019)
Inflation response	r_π	1.750	1.735	1.721	1.706	1.700	1.716	1.713	1.713	1.726
		(0.082)	(0.074)	(0.088)	(0.072)	(0.091)	(0.060)	(0.091)	(0.091)	(0.076)
Diff. infl response	$r_{\Delta\pi}$	0.314	0.306	0.289	0.268	0.264	0.245	0.253	0.256	0.305
		(0.068	(0.069)	(0.068)	(0.065)	(0.064)	(0.060)	(0.059)	(0.057)	(0.058)
Real exch. rate response	r_x	-0.027	-0.030	-0.034	-0.033	-0.027	-0.012	-0.016	-0.018	-0.023
		(0.008)	(0.009)	(0.009)	(0.008)	(0.009)	(0.006)	(0.006)	(0.007)	(0.008)
Output response	r_y	0.092	0.116	0.114	0.123	0.121	0.042	0.042	0.046	0.069
		(0.021)	(0.024)	(0.026)	(0.025)	(0.035)	(0.015)	(0.014)	(0.015)	(0.024)
Diff. output response	$r_{\Delta\pi}$	0.131	0.146	0.149	0.145	0.143	0.164	0.159	0.164	0.134
		(0.032)	(0.031)	(0.030)	(0.028)	(0.028)	(0.025)	(0.024)	(0.023)	(0.028)

Note: The estimation starts out in 1980Q1 in all cases. The approximative posterior standard deviation from the Hessian at the posterior mode is reported in parenthesis.

Figure 1a: Impulse responses from different estimation periods

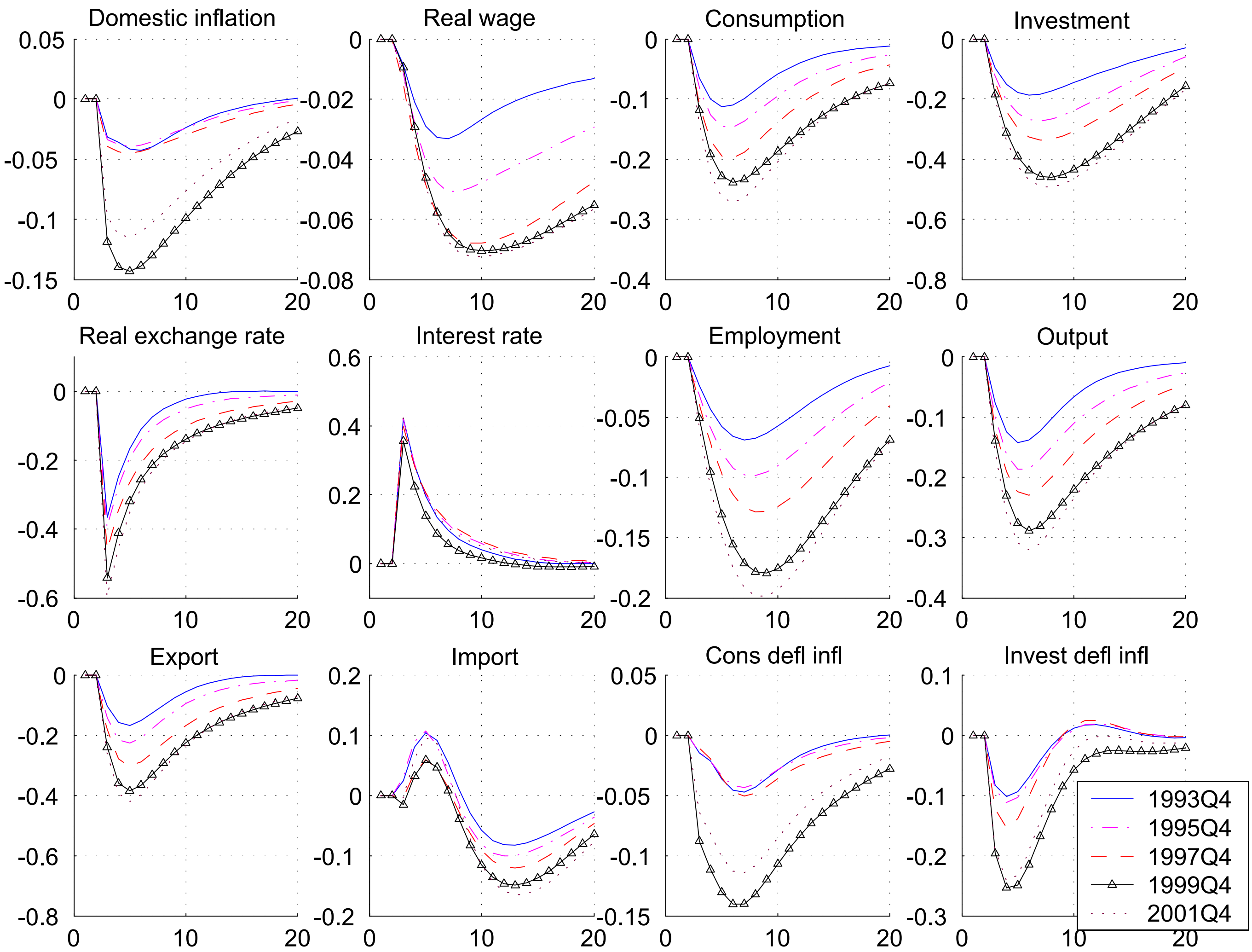


Figure1b: Impulse responses from different policy parameters

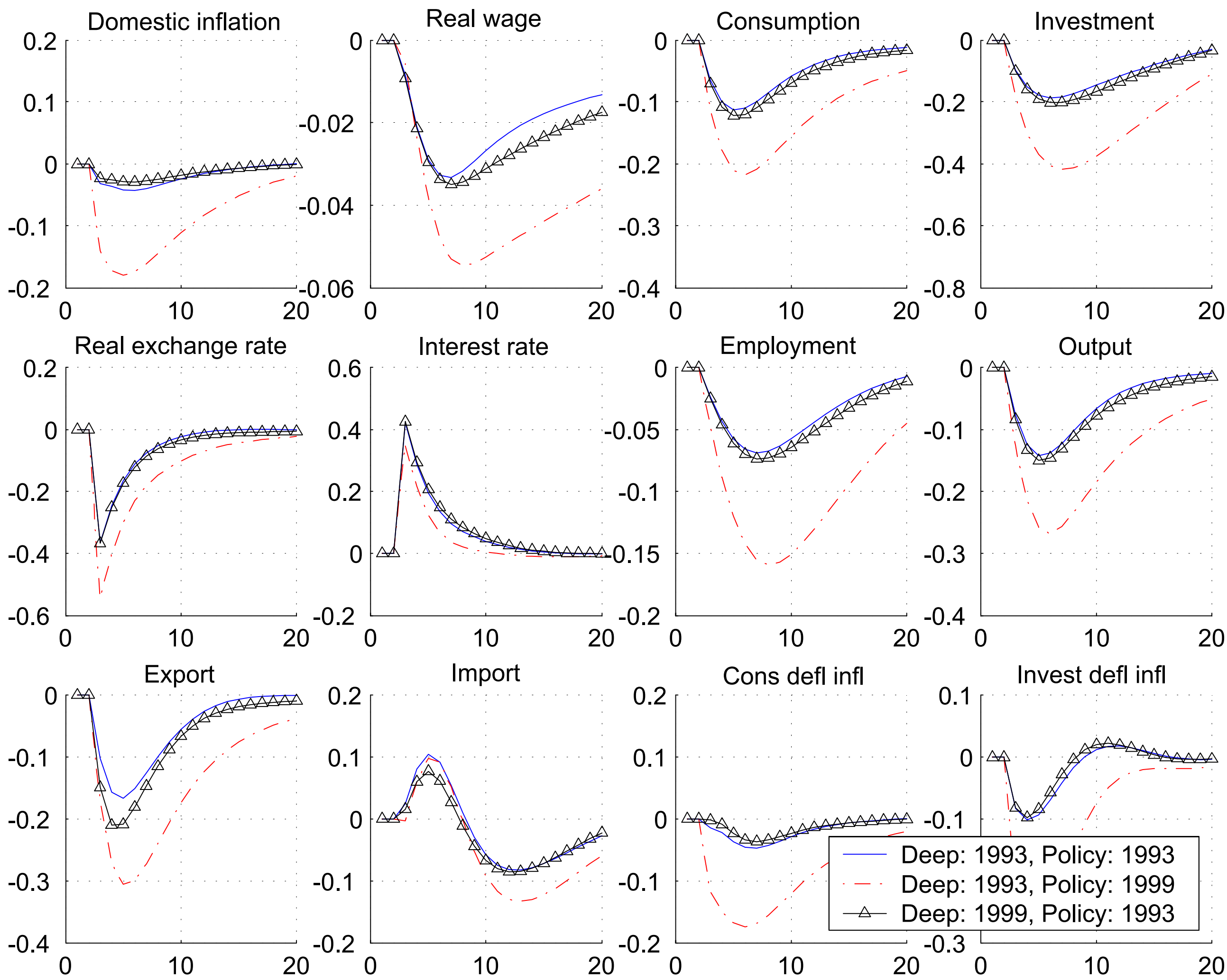


Figure 2a: Univariate modesty statistics without shock uncertainty (Leeper-Zha)

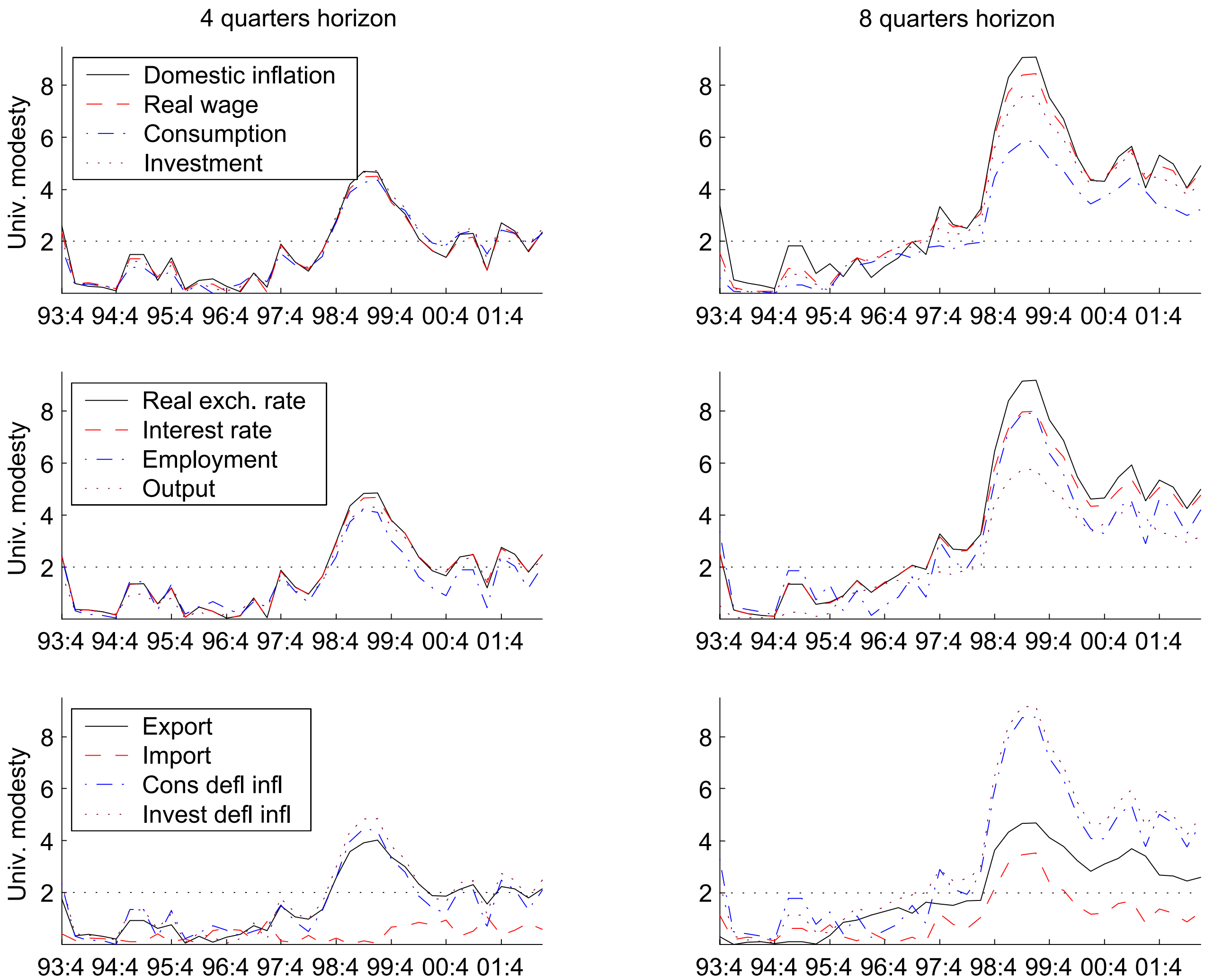


Figure 2b: Univariate modesty statistic with shock uncertainty

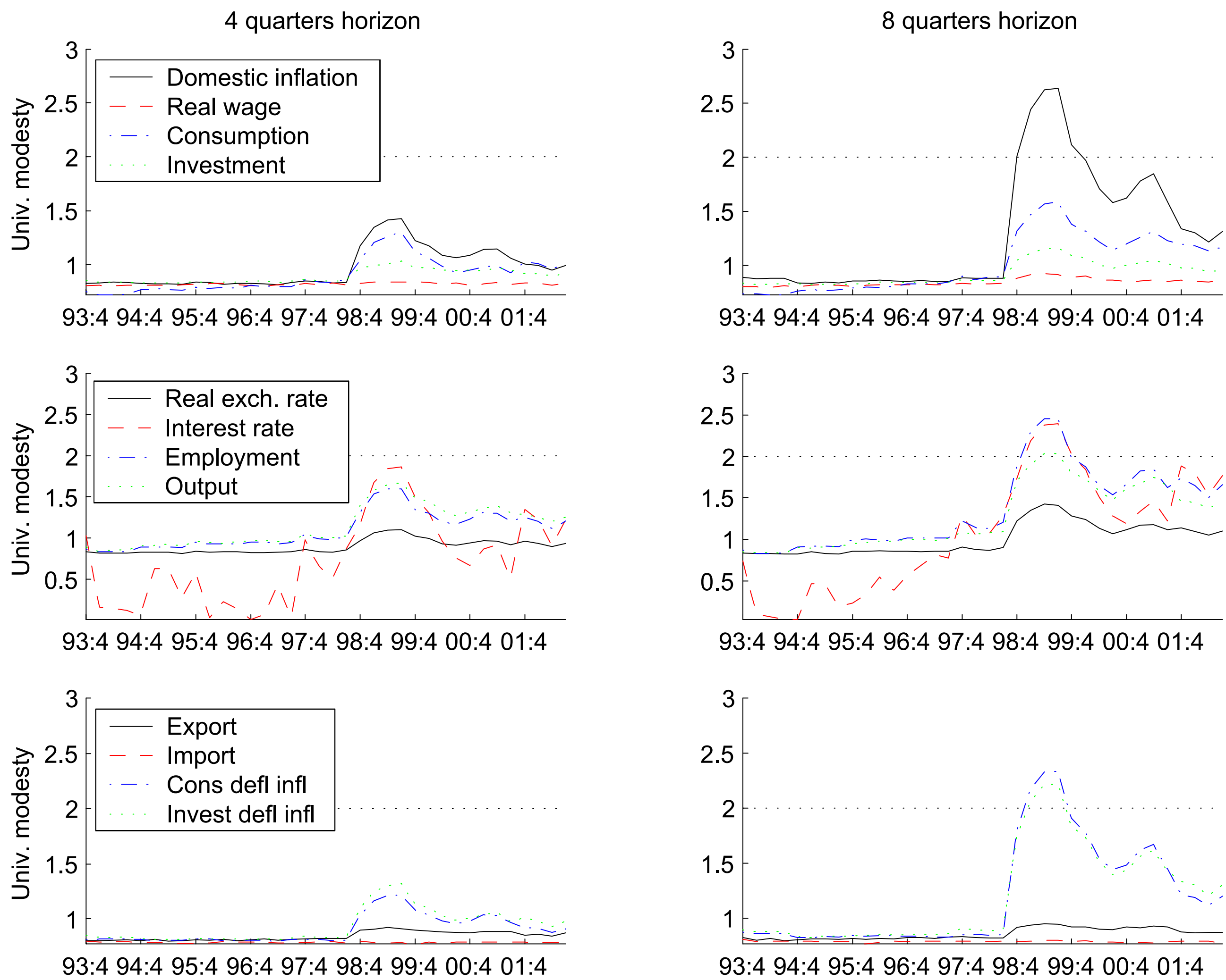


Figure 2c: Multivariate modesty statistic with shock uncertainty
(left column: four quarters, right column: eight quarters)

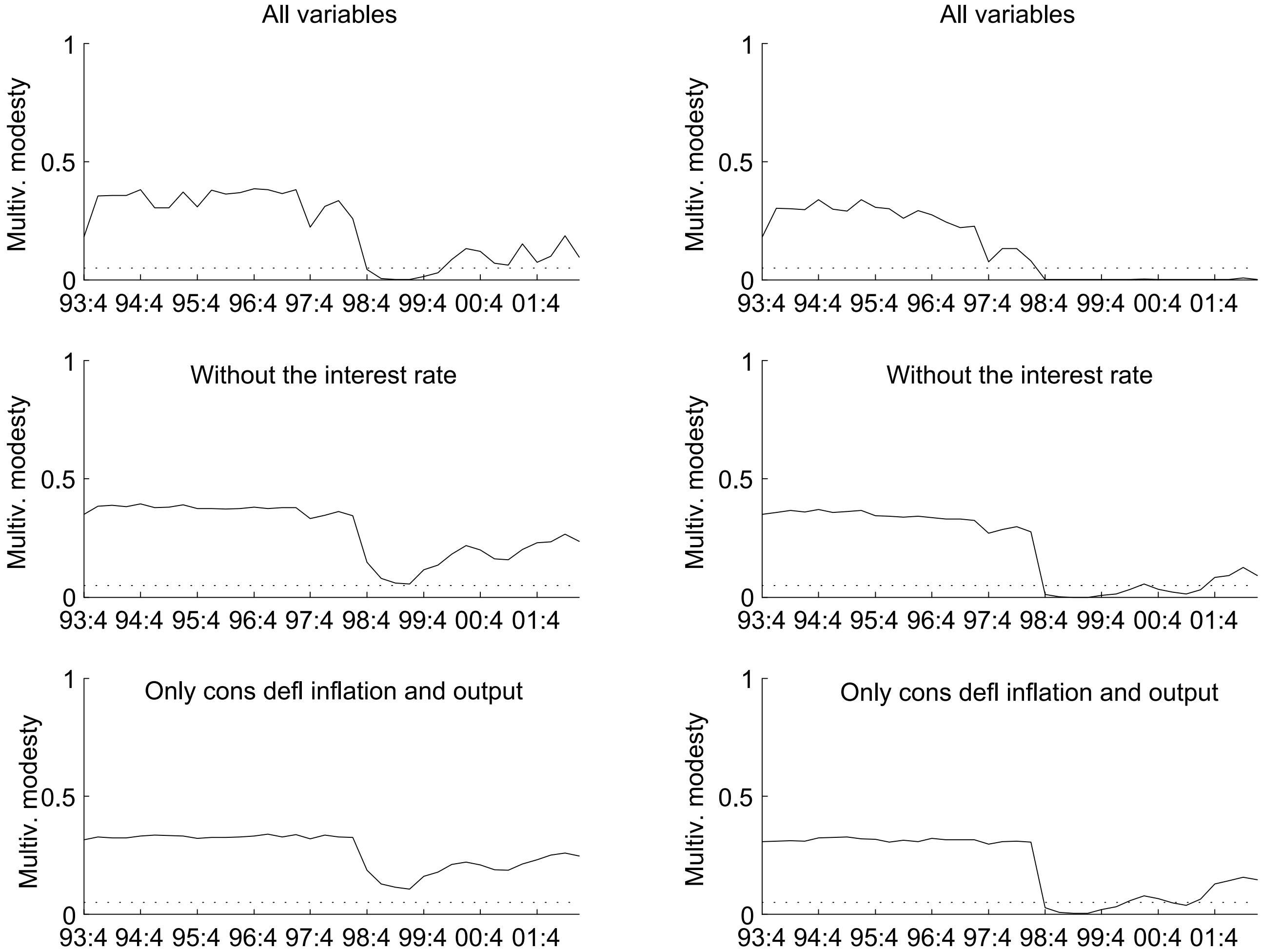


Figure 3a: Unconditional and conditional forecasts in 1999Q2, without shock uncertainty (Leeper-Zha)

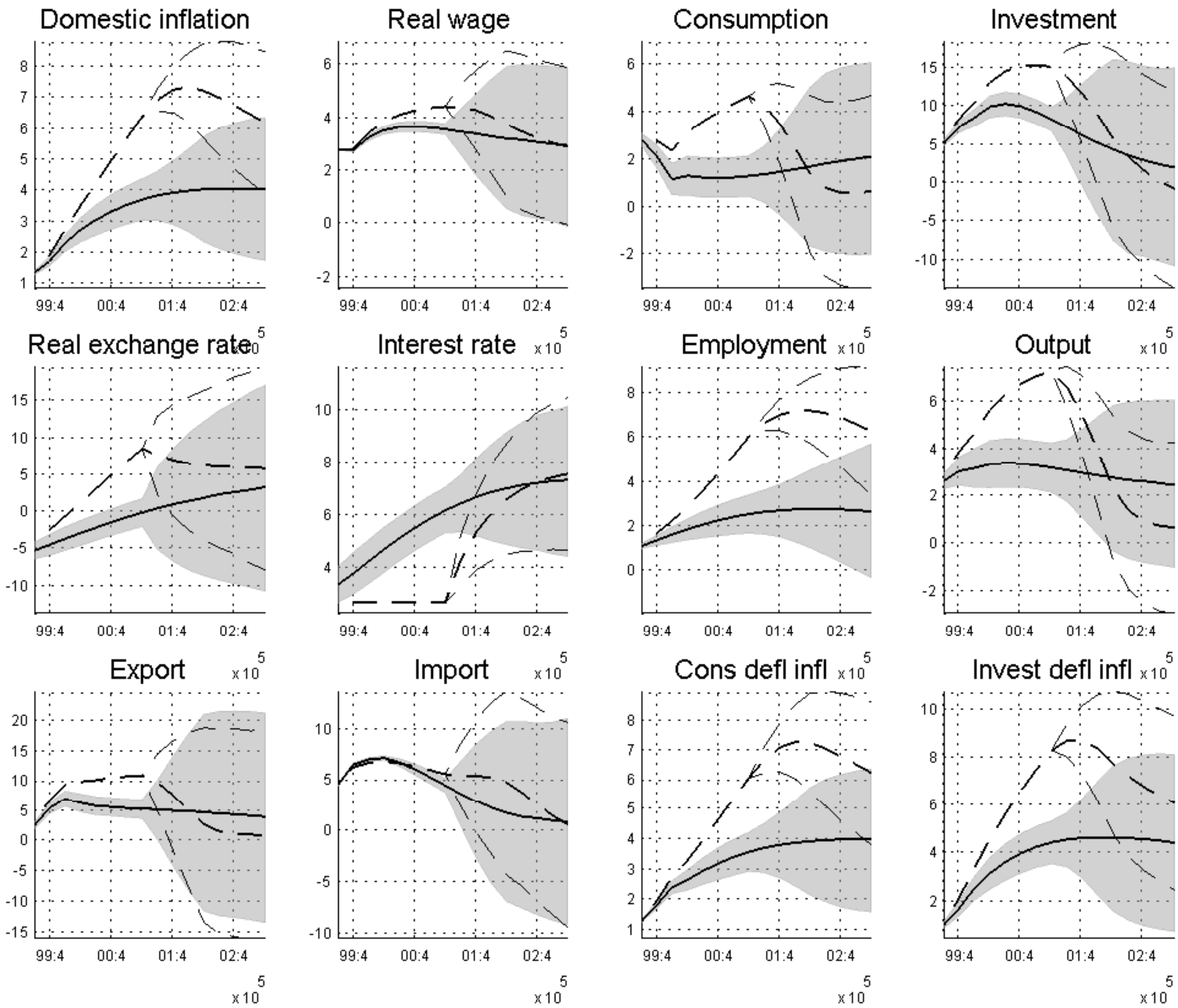


Figure 3b: Unconditional and conditional forecasts in 1999$Q2$, with shock uncertainty

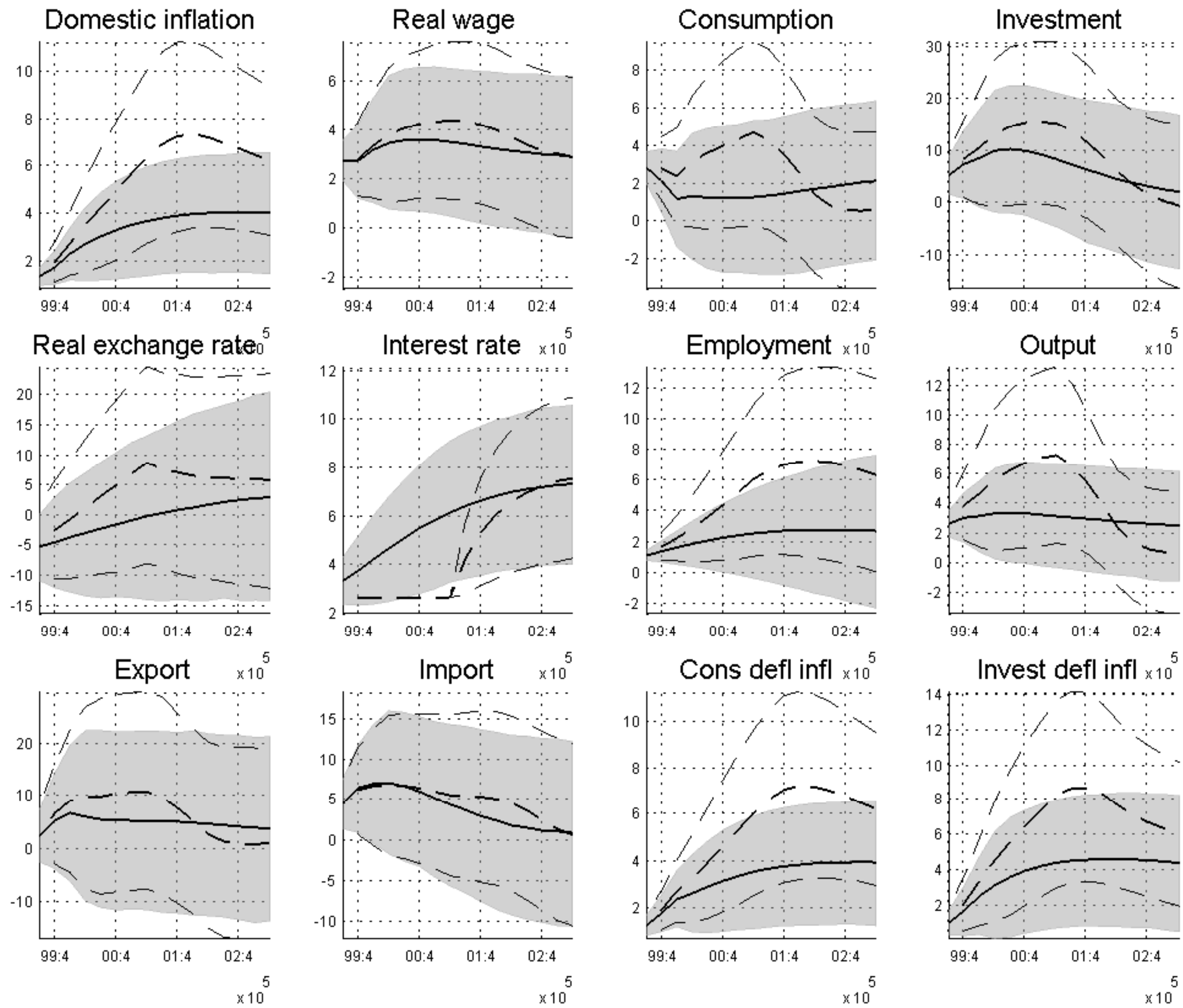


Figure 4a: Unconditional and conditional forecasts in 2001Q1, without shock uncertainty (Leeper-Zha)

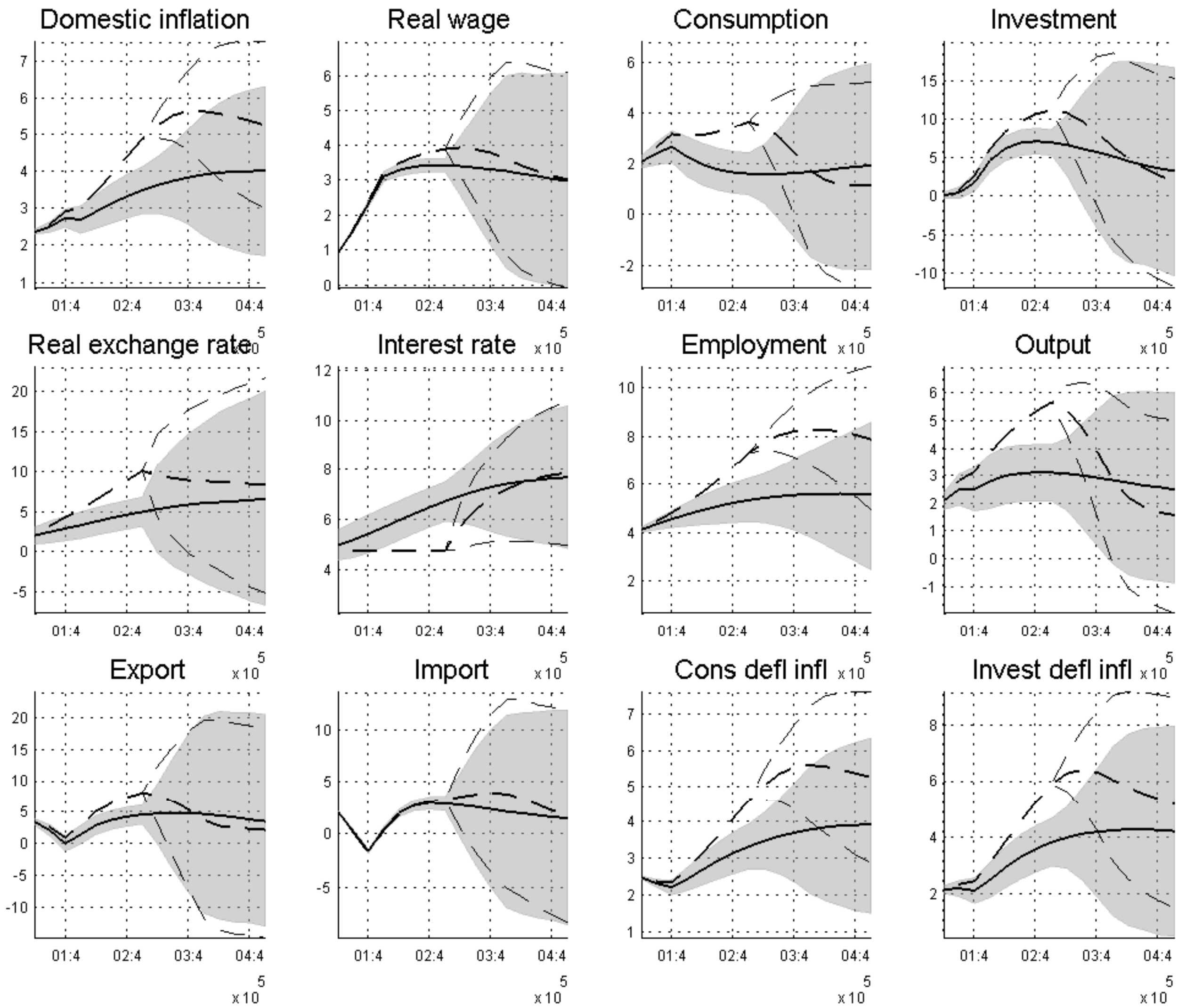


Figure 4b: Unconditional and conditional forecasts in 2001$Q1$, with shock uncertainty

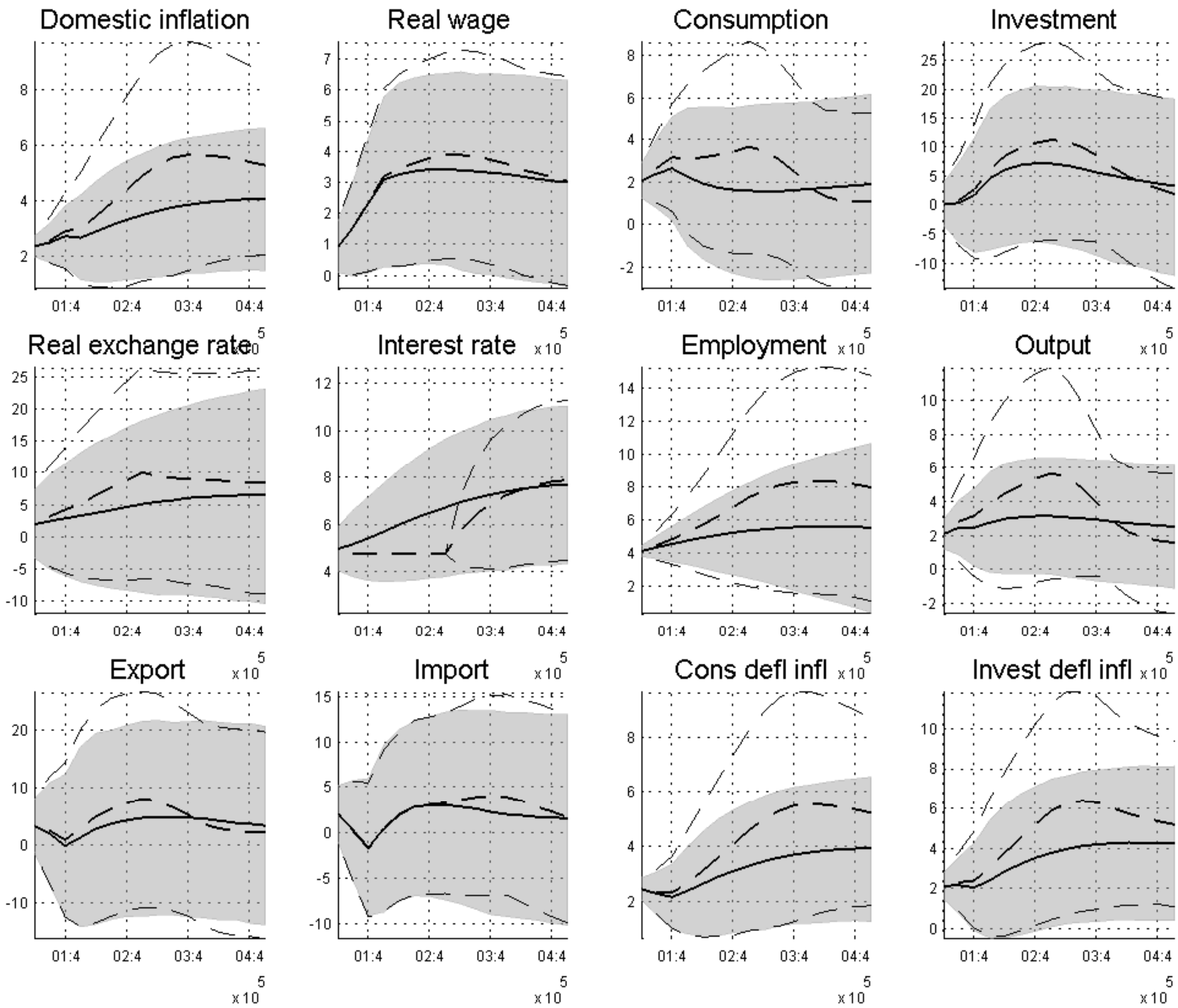
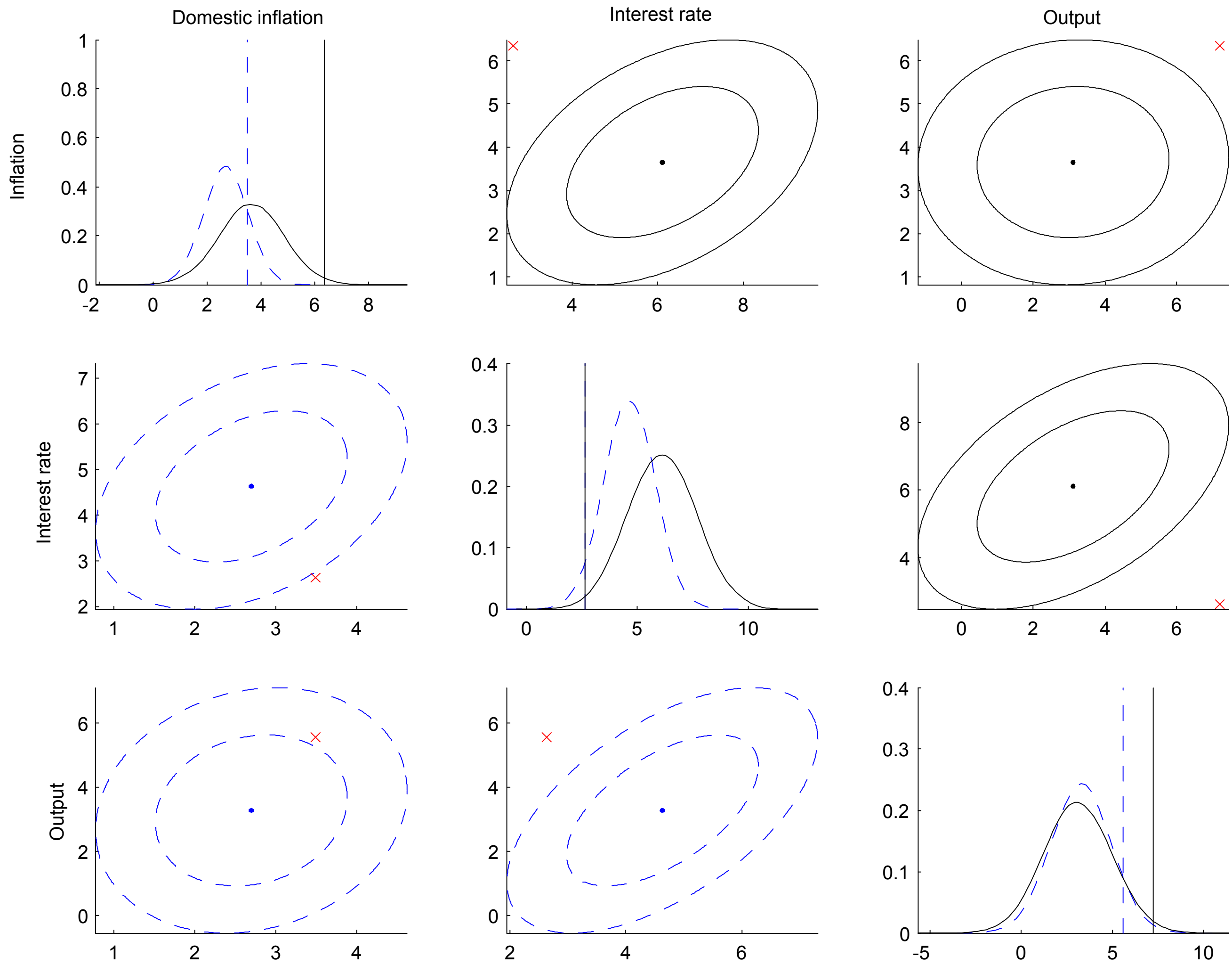
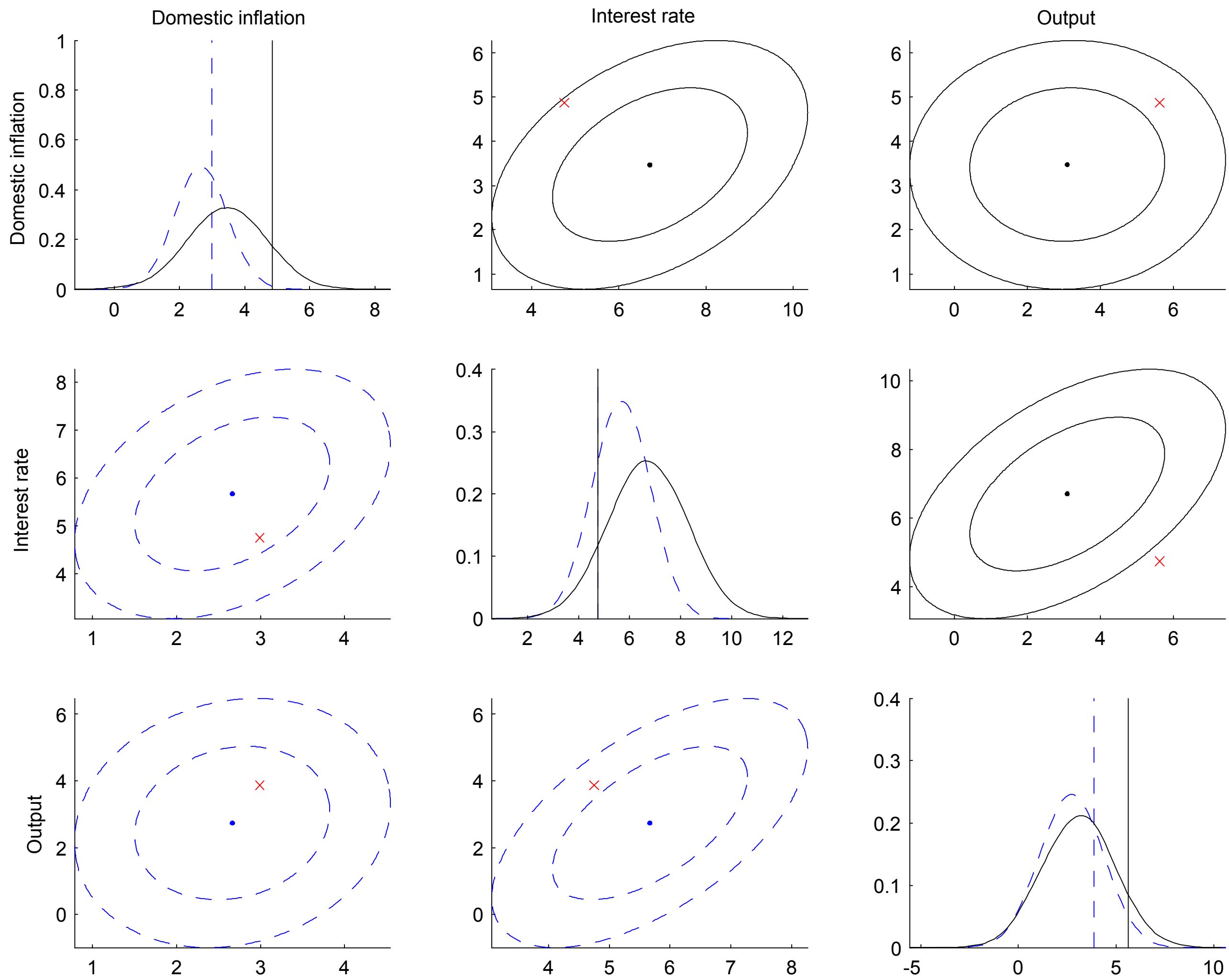


Figure 5a: Univariate and bivariate forecast distributions in 1999Q2



Note: The diagonal graphs display the univariate unconditional marginal forecast distributions and the off-diagonal graphs the bivariate unconditional forecast distributions at the four (dashed) and eight (solid) quarter horizon. The median conditional forecasts are displayed as lines (diagonal graphs) and crosses (off-diagonal graphs).

Figure 5b: Univariate and bivariate forecast distributions in 2001Q1



Note: The diagonal graphs display the univariate unconditional marginal forecast distributions and the off-diagonal graphs the bivariate unconditional forecast distributions at the four (dashed) and eight (solid) quarter horizon. The median conditional forecasts are displayed as lines (diagonal graphs) and crosses (off-diagonal graphs).

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