Modelling systemic financial sector and sovereign risk

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This article introduces a new framework for macroprudential analysis using a risk-adjusted balance sheet approach that supports policy efforts aimed mitigating systemic risk from linkages between institutions and the extent to which they precipitate or amplify general market distress. In this regard, the systemic contingent claims analysis ('Systemic CCA') framework helps quantify the magnitude of general solvency risk and government contingent liabilities by combining the individual risk-adjusted balance sheets of financial institutions and the dependence between them. An example of Systemic CCA applied to the US financial sector delivers useful insights about the magnitude of systemic losses and potential public sector costs from market-implied contingent liabilities. Stress tests using this framework are presented. Applications to European banks and the stress testing of systemic risk are also described. Finally, the banking and sovereign risk analysis is applied to Sweden, and joint banking sector and sovereign stress testing applications are shown. The paper concludes with new directions for a framework of integrated stress testing of banking and sovereign risk, with macrofinancial feedbacks, and monetary and fiscal policy analysis. Future research would ideally explore directions in using CCA-based economic output value and Systemic CCA to promote economic growth and financial stability, as well as the relationship to fiscal and debt management dynamics.

I. Introduction

The complex interactions, spillovers and feedbacks of the global crisis that began in 2007 remind us how important it is to improve our analysis and modelling of financial crises and sovereign risk. This article provides a broad framework to examine how vulnerabilities can build up and suddenly erupt in a financial crisis, with potentially disastrous feedback effects for sovereign debt and economic growth. The article discusses lessons from the crisis and new directions for research on modelling financial crises and sovereign risk. It shows how risk management tools and contingent claims analysis (CCA) can be applied in new ways to measure and analyse financial system and sovereign risk. A new framework ("Systemic CCA") is presented, which can help the measurement, analysis and management of financial sector systemic risk, tail-risk, and associated government implicit and explicit guarantees (contingent liabilities).

This article begins with a brief overview of the crisis of 2007-2011 which describes key features and market events, the actions of the authorities, and feedbacks from the markets to the real economy. This is followed by a section on what has been missing in the measurement and analysis of financial crises and sovereign risk. This includes a discussion of the need for better measurement and analysis of risk exposures, balance sheet risk, interconnectedness and contagion. Conceptual frameworks that can better analyse risk exposures and risk-adjusted balance sheets are presented. The article shows how risk management tools and contingent claims analysis (CCA) can be applied in new ways to the financial system, to economic sectors and to the national economy. CCA is a valuable tool to improve systemic financial sector and sovereign risk management. Next, a new framework ("Systemic CCA") is presented, which can help the measurement, analysis and management of financial sector systemic risk, tail-risk, and associated government implicit and explicit guarantees (contingent liabilities). An example of the Systemic CCA for the US financial sector, as well as similar applications of the model in the context of the European and Swedish banking sector, are provided. The next section shows how this can be used to analyse potential (non-linear) destabilising feedback processes between the financial sector and the sovereign balance sheet. Finally, the systemic risk dynamics are interlinked with important new measures of risk-adjusted economic output value via the CCA balance sheets and put-call parity relationships.

II. Key features of the global financial crisis and shortcomings of traditional analysis

A. KEY FEATURES AND STAGES OF THE CRISIS, 2007-2011

The crisis can be divided into four stages: Stage 1 – Buildup of vulnerabilities; Stage 2 – Run on shadow banking system; Stage 3 – Lehman bankruptcy and global financial crisis/great recession; and Stage 4 – Sovereign debt crisis.

In the first stage of the crisis, the surge in new credit created from securitising subprime mortgages in the US contributed to the upward spiral of higher house prices, and eventually to speculation and a bubble in the housing market. Poor regulation meant discipline in mortgage lending eroded from a loosening of lending standards. As initial low "teaser" rates expired and adjustable rate mortgage interest payments increased, many households could not afford to pay their mortgages. Eventually, the surge of house prices slowed and many borrowers defaulted.

Structured finance and regulatory rules created incentives for regulatory arbitrage which allowed for a reduction in the capital cushion across the financial system. This strategy of creating such off-balance sheet vehicles was part of the "originate and distribute" model that allowed banks to hold less capital than if the assets were held on-balance sheet. The structured assets placed in these off-balance sheet vehicles were financed by very short-term funding, in large part by commercial paper.

While the crisis started with a credit shock from defaults by subprime borrowers in the United States in mid-2007, there are additional features which amplified the subprime credit shock and turned it into such a serious crisis. The second stage of the crisis in 2007 can be thought of as a *run on the shadow or parallel banking system*. The conditions needed for a run are: (i) a negative credit shock from subprime borrowers; (ii) illiquid structured credit without transparent values, (iii) very short-term funding of longer maturity assets (maturity transformation); and, (iv) the lack of a lender of last resort to key institutions in what had grown into a very sizable "parallel banking system" (outside the US banking sector) (Loeys and Cennella, 2008).

The build-up in leverage, financed by wholesale short-term funding, was a key contributing factor to the severity of the crisis. The leverage in securitised products does not come from the products themselves but from how they are funded (collateralised debt obligations, CDOs, themselves merely redistribute risk). By 2007, short-dated funding of longer maturity assets outside of the regulated banking system in the US economy were about USD 5.9 trillion (Loeys and Cennella, 2008).[1] Overall, this maturity transformation outside of the banking world amounted to 40% of total maturity transformation in the US financial system in 2007. Yet there was no official lender of last resort to this "parallel banking system." The vulnerabilities were building from 2003 to 2007, but didn't erupt into a full-blown crisis until mid-2007, when lenders stopped providing short-dated funding to SIVs, conduits, and ABCPs.[2] This was similar to a run.

The third stage of the crisis began in September 2008, when financial markets and the rating agencies decided Lehman Brothers was near bankruptcy. The US Treasury tried to arrange financial support but decided not to participate in a bailout. Lehman declared bankruptcy on 14 September 2008, which was the largest bankruptcy in the history of the world.[3] Prime money market funds (MMFs) that held the USD 4 billion Lehman commercial paper and USD 20 billion short-term debt had to write down these assets when Lehman went bankrupt. This led one money market fund to "break the buck"[4]– shaking confidence in the supposedly safe prime MMFs and prompting intense redemption pressures from institutional investors. Falling confidence induced a precipitous pull-back from MMFs, engendering a downward spiral in confidence in the financial system. World stock markets plunged, wiping out USD 1 trillion in market value. The crisis rapidly spilled over internationally. Several banks in the UK, Belgium and other countries were taken over by their governments. Depositors started a run on an Icelandic bank, the Icelandic króna fell by over 60%, and the three largest Icelandic banks had to be nationalised, triggering a

1 This USD 5.9 trillion was composed of: (1) broker-dealers funding through repos and customer deposits (USD 2.2 trillion); commercial paper issued by ABS issuers and finance companies (USD 1.4 trillion); (3) auction rate securities (USD 900 billion); and repo funding by hedge funds (USD 1.3 trillion). Overall, this maturity transformation outside of the banking world amounted to 40% of total maturity transformation in the US financial system in 2007.

2 SIV is special investment vehicle and ABCP is asset-backed commercial paper.

3 The Finance Crisis and Rescue, page 7, Rotman School of Management, October 2008.

4 'Breaking the buck' refers to closing with a net asset value of less than one U.S. dollar.

sovereign debt crisis. Bank lending to Eastern Europe and the Baltics led to distress in some EU and Nordic banks in 2009.

Extensive government support via liability guarantees, capital injections and economic stimulus packages was initiated to counteract the sharp recession caused by the spillovers from the crisis globally. Many governments significantly increased their borrowing, raising sovereign debt levels simultaneously with declines in tax revenues, higher expenditures and increasing fiscal deficits.

The fourth stage of the crisis, which emerged in 2010, is the sovereign debt crisis. This appeared first in the euro area (Greece, Portugal, Ireland), before morphing into wider concerns about UK and US debt sustainability. Sovereign debt and fiscal issues and banking sector risks are intertwined. Banking risks spilled over, increasing sovereign risk via increased contingent liabilities to banks (this was particularly extreme in Ireland). As sovereign credit risks rise, the value of government support to banks becomes more uncertain, and sovereign spreads can spill over, increasing bank borrowing costs. Large scale banking rollover and refinancing needs and high sovereign borrowing needs occurred simultaneously in 2010 in many countries. By August 2011, there was serious concern about sovereign risk in Italy, Spain and also France, triggering concern about the viability of the euro area single currency.

B. SHORTCOMINGS OF TRADITIONAL ANALYSIS

Traditional macroeconomic and banking models do not adequately measure risk exposures of financial institutions and sovereigns and cannot be used to understand the transmission and amplification of risk within and between balance sheets in the economy. Traditional macroeconomic analysis of the government and central bank is almost entirely flow or accounting balance-sheet based. Sovereign debt analyses focus on debt sustainability (stocks, flows and debt to GDP). A fundamental point is that accounting balance sheets or a flow-of-funds do not indicate risk exposures, which are forward-looking.[5] A risk exposure measures how much can be lost over a forward-looking time horizon with an estimated probability. There has been extensive work on linking the default risk of corporations with macroeconomic models (for example, Schuermann et al., 2006). However, a key risk exposure that macroeconomists have frequently left out of their models is default risk in the financial sector. As pointed out by Charles Goodhart, "the study of financial fragility has not been well served by economic theory. Financial fragility is intimately related to probability of default. Default is hard to handle analytically being a discontinuous, nonlinear event so most macro models [and their underlying] transversality assumptions exclude the possibility of default."[6] Default risk models and risk-adjusted balance sheets of financial institutions are needed to analyse financial fragility and contingent liabilities.

5 Robert C. Merton (2002) pointed out that "Country risk exposures give us important information about the dynamics of future changes that cannot be inferred from the standard 'country accounting statements,' either the country balance sheet or the country income flow-of-funds statements".

6 Charles Goodhart on the occasion of a presentation at the IMF (2005).

Sovereign default risk models are needed and should be used together with the financial sector risk models. Models that *integrate* credit, market and liquidity risks into financial and sovereign crisis models in one framework were not used in the run-up to the global crisis. Also, risk appetite changes in markets, at the global or regional level, affect spreads across corporations, banks and sovereigns. Risk appetite changes are a key crisis component that is not built into traditional approaches (but which is an integral part of the risk-adjusted balance sheet/CCA models).

What are needed are better frameworks to model macrofinancial risk transmission, macroeconomic flows, and financial and sovereign risks together in an integrated way. To mitigate and manage financial sector risk and sovereign risk, new risk analytic tools and broader regulatory frameworks are needed.[7] Recent work has shown that financial sector risk indicators, such as the systemic expected losses or system default risk from CCA, have significant predictive power for GDP and the output gap (see Garcia et al., 2008 and forthcoming). This is most likely due to a credit channel process and a risk appetite channel. When CCA risk indicators for banks are low, i.e. low probability of default, then credit growth is higher, which boosts economic growth, and risk appetite is high. When banks are distressed and expected losses are high (default probabilities are high), then credit growth, GDP growth and risk appetite are likely to decrease as a result.[8]

Policymakers did look at certain aspects of interconnections in the financial sector, but, in light of the financial crisis, it is clear they lacked the correct data, analytical tools or authority to take appropriate action. Going forward, more attention needs to be paid to the linkages between financial sector risk exposures and sovereign risk exposures and their potential interactions and spillovers to other sectors in the economy or internationally. There should be more emphasis on the use of system-wide stress-testing approaches to evaluate vulnerabilities and the potential impact of self-fulfilling negative market dynamics. Improvements are needed in modelling destabilisation processes and what Robert Merton calls "destructive feedback loops" caused by situations where a guarantor provides a guarantee, the obligations of which the guarantor may not be able to meet precisely in those states of the world in which it is called on to pay.[9]

In summary, the financial crisis that began in 2007 has its roots in excessive leverage and maturity transformation in the shadow banking system, which led to large scale risk transmission and spillovers and, ultimately, large scale risk transfer to the sovereign. What is needed, going forward, is much better macrofinancial risk analysis, more use of risk-adjusted balance sheets (for financial institutions and sovereigns), improved systemic risk monitoring (which necessitates broader and more detailed data collection), and policy tools

7 This is similar to what some central bankers call a "macroprudential approach" to financial stability.

8 Garcia et al. (2008) find that including CCA financial sector risk indicators in monetary policy models (in the Taylor rule), interest rate reactions to the financial risk indicator leads to lower inflation volatility and lower output volatility in an application to Chile.

9 Robert C. Merton, Keynote speech, ECB, December 1, 2008. "Observations on Risk Propagation and the Dynamics of Macro Financial Crises: A Derivatives Perspective."

to help mitigate systemic risk. While there are many new ways to integrate risk-adjusted balance sheets with macroeconomic and financial stability models, this article will focus on their use in financial sector and sovereign risk analysis.

III. Contingent Claims Analysis (CCA)

Contingent Claims Analysis (CCA) represents a generalisation of the option pricing theory (OPT) pioneered by Black and Scholes (1973), as well as Merton (1973), and, thus, is forward-looking by construction, providing a consistent framework based on current market conditions rather than on historical experience.[10] When applied to the analysis and measurement of credit risk, it is commonly called the Merton Model.

CCA determines the risk-adjusted balance sheet of firms, based on three principles. They are: (i) the values of liabilities (equity and debt) are derived from assets; (ii) liabilities have different priority (i.e. senior and junior claims); and, (iii) assets (such as the present value of income flows and proceeds from asset sales) follow a stochastic process. Assets may be above or below promised payments on debt which constitute a default barrier. When there is a chance of default, the repayment of debt is considered "risky," to the extent that it is not guaranteed in the event of default. Risky debt is composed of two parts, the default-free value of debt, and deposits minus the "expected loss to bank creditors" from default over a specific time horizon, which can be expressed as the value of a put option.

The value of assets may be above or below promised payments on debt which constitute a default barrier at a given point in time. A CCA framework is a risk-adjusted balance sheet concept. It is an integrated framework relating bank asset values to equity value, default risk and bank funding costs. This concept of measuring credit risk has a wide spectrum of applications. CCA can help central banks analyse and manage the financial risks of the economy. The basic analytical tool is the risk-adjusted (CCA) balance sheet, which shows the sensitivity of the enterprise's assets and liabilities to external "shocks." At the national level, the sectors of an economy can be viewed as interconnected risk-adjusted balance sheets with portfolios of assets, liabilities and guarantees—some explicit and others implicit. Traditional approaches have difficulty analysing how risks can accumulate gradually and then suddenly erupt in a full-blown crisis. The CCA approach is well-suited to capturing such "non-linearities" and to quantifying the effects of asset-liability mismatches within and across institutions. Risk-adjusted CCA balance sheets facilitate simulations and stress testing to evaluate the potential impact of policies to manage systemic risk.

The following sections provide a description of CCA for individual banks, measures of market-implied contingent liabilities, systemic CCA, and CCA applied to the measurement of spillover effects between banks and sovereign default risk.

10 Although market prices are subject to market conditions not formally captured in this approach, they endogenise the capital structure impact of government interventions.

A. CCA FOR INDIVIDUAL BANKS

In order to understand individual risk exposures (and associated public sector contingent liabilities) in times of stress, CCA is first applied to construct risk-adjusted (economic) balance sheets for financial institutions.

In its basic concept, CCA quantifies default risk on the assumption that owners of equity in leveraged firms hold a call option on the firm's value after outstanding liabilities have been paid off. The concept of a risk-adjusted balance sheet is instrumental in understanding default risk. More specifically, the total market value of firm assets, A, at any time, t, is equal to the sum of its equity market value, E, and its risky debt, D, maturing at time T.[11] The asset value follows a random, continuous process and may fall below the value of outstanding liabilities, which constitutes the bankruptcy level ('default threshold' or 'distress barrier') B.[12] B is defined as the present value of promised payments on debt discounted at the risk-free rate. The value of risky debt is equal to default-free debt minus the present value of expected loss due to default. These uncertain changes in future asset value, relative to promised payments on debt, are the driver of credit and default risk. Indeed, default happens when assets are insufficient to meet the amount of debt owed to creditors at maturity.

In this framework, market-implied expected losses associated with outstanding liabilities can be valued as an implicit put option, with its cost reflected in a credit spread above the risk-free rate that compensates investors for holding risky debt. The put option value is determined by the duration of the total debt claim, the leverage of the firm, and the volatility of its asset value (see Appendix 1).

In the traditional way of analysing bank balance sheets, a change in accounting assets results in a one-for-one change in book equity. The traditional bank accounting balance sheet has accounting assets on the left and liabilities consisting of book equity and the book value of debt and deposits on the right. When assets change, the full change affects book equity.

Traditional bank accounting balance sheet

Assets	**Liabilities**
Accounting assets	Debt and deposits
(cash, reserves, loans, credits, other exposures)	Book equity

In conventional definition of credit risk, the concept of "expected losses" refers to exposures on the asset side of the bank's balance sheet, such as loans, mortgages, and non-cash claims (derivatives and contingent assets). This traditional expected loss is frequently calculated as a probability of default (PD) times a loss given default (LGD) times the exposure at default (EAD). The expected losses of different exposures are aggregated (using certain assumptions regarding correlation, etc.) and used as an input into loss distribution calculations which are, in turn, used for the estimation of regulatory capital.

11 We identify contingent liabilities based on the standard Black-Scholes-Merton (BSM) framework of capital structure-based option pricing theory (OPT). See Merton, (1974).

12 Moody's KMV CreditEdge defines this barrier equal to total short-term debt plus one-half of long-term debt.

In the risk-adjusted (CCA) balance sheet context, however, changes in assets are directly linked to changes in the market value of equity *and* the expected losses in an *integrated framework*. A decline in the value of assets increases expected losses to creditors and leads to a less than one-to-one decline in the market value of equity; the amount of change in equity depends on the severity of financial distress, the degree of leverage, and the volatility of assets. The amount of increase can be very high when banks are in severe financial distress. While expected loss in this case also relates to the total debt and deposits on the full bank balance sheet, the underlying "exposure" is represented by the default-free value of the bank's total debt and deposits. The expected loss to creditors is a "risk exposure" in the risk-adjusted balance sheet.

Risk-adjusted (CCA) balance sheet

Assets	**Liabilities**
Market value of assets (A)	Risky debt (D)
(cash, reserves, value of "risky" assets)	(= default-free value of debt and deposits minus expected losses to bank creditors)
	Market value of equity (E)

The risk-adjusted bank balance sheet and the traditional accounting bank balance sheet can be reconciled if uncertainty about the default risk is ignored. The accounting balance sheet can be "derived" from the special case of the risk-adjusted balance sheet—the case in which uncertainty is set to zero (i.e. the bank's assets have no volatility). With zero volatility on the balance sheet, the expected loss to bank creditors goes to zero and equity becomes book equity. The "risk exposure" becomes zero (Gray et al, 2007 and 2008; Gray and Malone, 2008).

The risk-adjusted balance sheet of the banks can quantify the impact on the bank borrowing cost of higher (or lower) levels of equity, the impact of changes in global risk appetite, and of government guarantees:

- Lower levels of the market value of equity are directly related to higher bank funding costs. There is increasing interest in indicators that use the market value of equity as a measure of financial fragility.[13]
- The impact of changes in global or regional risk appetite on the values of bank expected losses to creditors, bank funding costs, and bank equity can be measured. Lower risk appetite causes investors to flee from "risky" investments to safer forms of investment – this raises borrowing costs around the world for corporate, sovereign and household borrowers etc. As the CCA framework quantifies the impact of changes in risk appetite, stress test scenarios can include stressing changes in global or regional risk appetite (see Appendix 2).

13 For example, Haldane (2011) states that "market-based metrics of bank solvency could be based around the market rather than book value of capital…..e.g., [the] ratio of a bank's market capitalisation to its total assets. …Market-based measures of capital offered clear advance signals of impending distress beginning April 2007…..replacing the book value of capital by the market value lowers errors by half. Market measures provide both fewer false positives and more reliable advance warnings of future banking distress."

- During the crisis, implicit and explicit government guarantees had an important impact on reducing bank borrowing costs (and shifting risk to the sovereign balance sheet) which can be measured in the CCA framework.

It is important to measure expected losses to bank creditors in order to understand the drivers of changes in bank funding costs and in financial stability. Higher bank borrowing costs lead to higher lending rates for corporates and households, to credit rationing, and lower credit growth. This can have a negative impact on economic output, which can, in turn, feed back, causing further distress in the banking system. Higher expected losses to creditors raise bank borrowing costs. Lenders may cut off credit and induce severe liquidity problems that can spread through the whole financial system. Bank creditors can incur losses which might contribute to financial instability. Higher expected systemic losses can transfer risk to the government via guarantees and the costs of resolving failed banks.

STRESS TESTING USING CCA

For stress testing, three different methods can be used to model the macrofinancial linkages affecting individual expected losses. Macro variables and changes in risk appetite can be linked to CCA balance sheets and used for stress testing in several ways. In the first model ("satellite model"), the historical sensitivity of expected losses to creditors (or other CCA risk indicators) is estimated from several macroeconomic variables (such as short-term and long-term interest rates, real GDP and unemployment) and bank-specific variables (net interest income, operating profit before taxes, credit losses, leverage and funding gap) using some econometric approach, such as a dynamic panel regression specification (IMF, 2010b, 2010c, 2011b, 2011f, and 2011g). In the second model ("structural model"), the value of implied assets of each bank is adjusted by forecasts of operating profit and credit losses as updated inputs into the calibrated bank CCA model in order to determine changes in expected losses, funding costs, the CCA capital ratio (i.e. market value of equity to market value of assets) and other useful outputs (IMF, 2011c, 2011d, 2011f and 2011g). The third way to link macro variables is to estimate the historical relationships of the macrofactors to changes in the bank market value of assets (which is done in Moody's KMV Global Correlation and Portfolio Manager models).

B. MEASURING MARKET-IMPLIED CONTINGENT LIABILITIES FROM THE FINANCIAL SECTOR

The implicit put option calculated for each financial institution from equity market and balance sheet information using CCA can be combined with information from credit default swap (CDS) markets to estimate the government's contingent liabilities. If guarantees do not affect equity prices in a major way (especially when firms are in distress), implicit guarantees reduce default risk, so that the price of insuring against default, which is expressed as CDS spreads for contracts at different maturity tenors, captures only the expected loss *retained* by the financial institution – and borne by unsecured senior

creditors. Thus, the implied CDS spread is generally higher than the actual CDS spreads due to the impact of explicit and implicit guarantees.

Hence, the scope of market-implied guarantees affecting firm valuation can be defined as the difference between the total expected loss (i.e. the value of a put option derived from the firm's equity price) and the value of an implicit put option derived from the firm's CDS spread, which reflects expected losses associated with the default net of any financial guarantees. This allows one to measure the time pattern of the government's market-implied contingent liabilities and the retained risk in the banking sector (see Appendix 1).[14]

C. MEASURING SYSTEM-WIDE CCA ('SYSTEMIC CCA')

In order to assess systemic risk (and the underlying joint default risk), however, a simple summation of implicit put options would presuppose perfect correlation, i.e. a coincidence of defaults. While it is necessary to move beyond "singular CCA" by accounting for the dependence structure of individual balance sheets and associated contingent claims, the estimation of systemic risk through correlation becomes exceedingly unreliable in the presence of "fat tails".[15]

The Systemic CCA framework (Gray and Jobst, 2010 and forthcoming; Gray and others, 2010; IMF, 2011g) extends the risk-adjusted balance sheet approach in order to quantify the systemic financial sector risk jointly posed by the interlinkages between institutions, including the time-varying dependence of default risk. Under this approach, the magnitude of systemic risk depends on the firms' size and interconnectedness in a multivariate framework. This methodology models the joint market-implied expected losses of multiple institutions with "too-big-to-fail" properties as a portfolio of individual contingent claims (with individual risk parameters).[16] By accounting for the dependence structure of individual bank balance sheets and associated contingent claims, this approach can be used to quantify the contribution of specific institutions to the dynamics of the components of

14 For a more detailed exposition, see Gray and Jobst (2010a and forthcoming) and IMF (2010a). While this definition of market-implied contingent liabilities provides a useful indication of possible sovereign risk transfer, the estimation of the alpha-value depends on a variety of assumptions that influence the assessment of the likelihood of government support, especially at times of extreme stress during the credit crisis. The extent to which the put option values differ from the ones implied by CDS spreads might reflect distortions stemming from the modelling choice (and the breakdown of efficient asset pricing in situations of illiquidity), changes in market conditions, and the capital structure impact of crisis interventions, such as equity dilution in the wake of capital injections by the government, beyond the influence of explicit or implicit guarantees.

15 Correlation describes the complete dependence structure between two variables correctly only if the joint (bivariate) probability distribution is elliptical—an ideal assumption rarely encountered in practice. This is especially true in times of stress, when default risk is highly skewed, and higher volatility inflates conventional correlation measures automatically (as covariance increases disproportionately to the standard deviation), so that large extremes may even cause the mean to become undefined. In these instances, default risk becomes more frequent and severe than suggested by the standard assumption of normality—i.e., there is a higher probability of large losses and more extreme outcomes.

16 The Systemic CCA framework can be decomposed into two sequential estimation steps. First, the market-implied potential losses (and associated change in existing capital levels) are estimated for each sample bank using an advanced form of contingent claims analysis (CCA). Then, these individual estimates are aggregated in a multivariate set-up in order to derivate estimates of joint expected losses and changes in capital levels.

systemic risk (at different levels of statistical confidence),[17] how this systemic risk affects the systemic expected losses and government's contingent liabilities, and how policy measures may influence the size and allocation of this systemic risk over time.

Systemic CCA generates estimates of expected and unexpected losses from systemic financial sector risk, as well as measures of extreme risk. These estimates are based on the multivariate density of each bank's individual marginal distribution of market-implied expected losses and their dependence structure within a system of financial institutions. Accounting for both linear and non-linear dependence and its effect on joint expected losses can deliver important insights about the joint tail risk of multiple entities. Large shocks are transmitted across entities differently than small shocks. As opposed to the traditional (pairwise) correlation-based approach, this method of measuring "tail dependence" is better suited to analysing extreme linkages of multiple (rather than only two) entities, because it links the univariate marginal distributions of expected losses (and associated liabilities) in a way that formally captures both linear and non-linear dependence in joint tail risk behaviour over time.[18]

In addition, the Systemic CCA framework can be used for stress testing. By modelling how macroeconomic conditions and bank-specific income and loss elements (net interest income, fee income, trading income, operating expenses and credit losses) have influenced the changes in the financial institution's market-implied expected losses (as measured by implicit put option values), it is possible to link a particular macroeconomic path to financial sector performance in the future.

D. ADAPTING CCA TO THE SOVEREIGN AND FRAMEWORK FOR INTERACTIONS AND FEEDBACK BETWEEN THE FINANCIAL SECTOR AND THE SOVEREIGN

The CCA approach can be adapted to the sovereign, but the procedure for doing so generally depends on whether one is dealing with an emerging market sovereign, which may possess significant foreign debt, usually denominated in hard currency, or a developed country sovereign, in which most or all debt is issued in local currency (see Gray et al., 2007, Gapen et al., 2008, Gray and Malone, 2008, Gray and Jobst, 2010a, and IMF, 2010a). Our application of the sovereign CCA focuses on developed country sovereigns, especially European sovereigns, such as Greece. The value of sovereign debt can be seen as having two components, the *default-free value* (promised payment value) and the *expected loss* associated with default in the event the assets are insufficient to meet the promised payments. The value of sovereign assets at time horizon *T*, relative to the promised payments on sovereign debt (the sovereign debt or distress barrier), is

17 The contribution to systemic (joint tail risk) is derived as the partial derivative of the multivariate density relative to changes in the relative weight of the univariate marginal distribution of each bank at the specified percentile.

18 As an integral part of this approach, the marginal distributions fall within the domain of Generalized Extreme Value Distribution, GEV (Coles et al., 1999; Poon et al., 2003; Stephenson, 2003; Jobst, 2007). Sample banks in each jurisdiction based on the multivariate distribution (MGEV) of joint CDS spread movements defined by a non-parametric dependence function (Gray and Jobst, 2009 and 2010; Jobst and Kamil, 2008). As opposed to a simple copula approach, this method does not generate a single, time-invariant dependence parameter.

the driver of these expected losses. There is a random element to the way the sovereign asset value evolves over time. The application of the sovereign CCA model to developed country sovereigns requires us to infer the value of sovereign assets—because the value of sovereign assets is not directly observable—based upon measures of expected losses on sovereign debt derived from the *full term structure of sovereign spreads.* See Appendix 3 for details.

The previous discussion and illustration of Systemic CCA points out the importance of measuring the government's contingent liabilities to banks and accounting for the dependence structure of the portfolio of such contingent liabilities using a framework that can capture time variation. The full set of interlinked risk exposures between the government and financial sector should be analysed in a comprehensive framework.

A stylised framework starts with the economic (i.e. risk-adjusted) balance sheets of the financial sector (portfolio of financial institutions) and is then linked to, and interacts with, the government's economic balance sheet.[19] For example, distressed financial institutions can lead to large government contingent liabilities, which, in turn, reduce government assets and lead to a higher risk of default on sovereign debt. Table 1 below shows the key linkages between the financial sector and the government. The economic balance sheet items in italics reflect the risk exposures of the government to the financial sector. The government has provided financial guarantees associated with expected losses due to default, it may have provided asset guarantees, it may have injected capital and have an equity stake in the banks. All of these form the government's risk exposures to the financial sector. Note that these risk exposures consist of portfolio financial institutions. These, in turn, affect the economic value of the government's assets and may affect the government's own default risk and borrowing spreads. Risk interactions and feedbacks can be analysed with this type of framework.

19 There are three types of accounts for any entity, including a financial institution or a government: flow/income accounts; accounting balance sheets; and economic risk-adjusted balance sheets. All three need to be analysed. In the economic risk-adjusted balance sheets of financial institutions or governments, assets always equal liabilities (which include equity). In simple terms, Assets + Guarantees – Equity – (Default-free Debt – Expected Loss due to Default) = 0.

Table 1. Linkages between the financial sector and sovereign balance sheets

FINANCIAL SECTOR	GOVERNMENT
ASSETS	
Assets/loans + Liquid assets/reserves + *Asset guarantees*	Present value of fiscal surplus and guarantee fees + *Equity (government-owned)* + Other assets
LIABILITIES	
– *Equity (non-government)* – *Equity (government-owned)*	– Credit owed to central bank – *Asset guarantees*
– Default-free debt and deposits + (1-α) **Expected Losses (EL) due to default*	– α* *Expected Losses (EL) due to default*
– Present value of guarantee fees	– Default-free sovereign debt + *Expected Losses (EL) due to sovereign default*
ASSETS MINUS LIABILITIES	
0	0

Source: Gray et al. (2010).

Negative feedback effects could arise in a situation in which the financial system is outsized compared to the government, and distress in the financial system triggers a large increase in government financial guarantees. These contingent liabilities to the government due to guarantees, can lead to a rise in sovereign spreads. Banks' spreads depend on retained risk, which is lower given the application of government guarantees, and also on the creditworthiness of the sovereign (as a result of fiscal sustainability and debt service burden), as investors view the bank's risk and sovereign risk as intertwined. Concern that the government balance sheet will not be strong enough for it to make good on guarantees could lead to deposit withdrawals or a cutoff of credit to the financial sector, triggering a destructive feedback in which both bank and sovereign spreads increase.[20] In some situations, this vicious circle can spiral out of control, resulting in the inability of the government to provide sufficient guarantees to banks and leading to a systemic financial crisis and a sovereign debt crisis.

Fiscal, banking and other problems can cause distress for the government, which can transmit risk to holders of government debt. Holders of sovereign debt have a claim on the value of the debt minus the potential credit loss, the value of which is dependent on the level of assets of the sovereign.[21] A sudden stop in access to foreign funding (inability to rollover short-term debt and to borrow) can dramatically increase credit spreads for the sovereign and for banks. A vicious spiral of increasing bailout costs for banks, possible currency devaluation, and the inability of the sovereign to borrow can lead to the default of both banks and the sovereign.

20 The Iceland crisis of 2008 is a case in point.

21 See Gapen et al. (2005), Gray et al. (2007), Gray and Malone (2008), and Gray and Jobst (2010b) for more detail on sovereign CCA models.

IV. Applications

This section describes three applications of CCA and Systemic CCA with examples of stress testing. The first example summarises the findings from the recent US FSAP (IMF, 2010b), where the Systemic CCA approach premiered as an IMF stress-testing approach. The subsequent cases illustrated the application of CCA to the banking systems in Europe and the Swedish banking sector, with a particular focus on spillover effects between banks and the fiscal conditions.

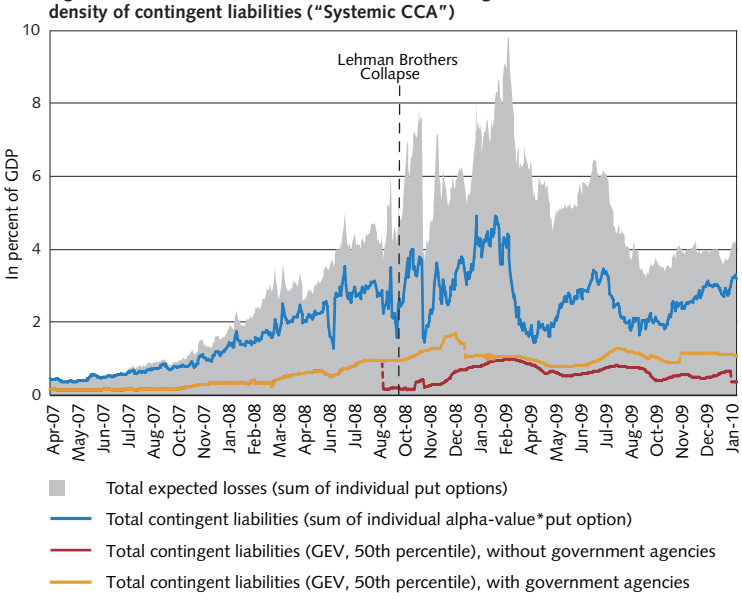
A. APPLICATION TO THE US FINANCIAL SECTOR

This section describes the results from applying the Systemic CCA framework to the financial sector of the United States (IMF, 2010b and 2010c). It uses market and balance sheet information about commercial banks, investment banks, insurance companies and special purposes financial institutions (the 36 largest institutions), using daily data from 1 January 2007 to late January 2010.[22] We apply the enhanced version of the Merton model (see above) with implied asset volatility derived from equity options to determine the CCA-based risk-adjusted balance sheets and one-year CDS spreads as the basis for calculating associated market-implied contingent liabilities.

Figure 1 shows total expected losses (area) and government contingent liabilities (line) for all 36 institutions; both are highest between the periods just after Lehman's collapse in September 2008 and the end of July 2009. The analysis suggests that markets expected that, on average, more than 50 per cent of total expected losses could have been transferred to the government in the event of default. A simple summation of expected losses and contingent liabilities, however, ignores the fact that the realisation of defaults does not happen concurrently, i.e. it does not capture intertemporal changes in the dependence structure between this 'portfolio' of financial institutions.

22 Key inputs used were the daily market capitalisation of each firm (from Bloomberg), the default barrier estimated for each firm based on quarterly financial accounts (from Moody's *KMV CreditEdge*), the risk-free rate of interest (at 3 per cent), a one-year time horizon, and one-year credit default swap (CDS) spreads (from *Markit)*. Outputs were the expected losses (i.e. the implicit put option value over a one-year horizon) and the contingent liabilities (i.e. alpha*implicit put option).

Figure 1. United States: financial sector – total contingent liabilities and multivariate density of contingent liabilities ("Systemic CCA")

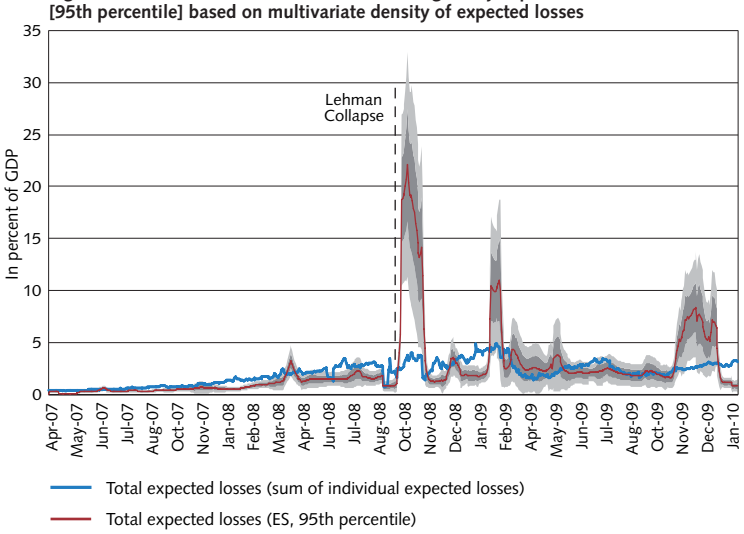


Note. Sample period: 3 January 2007-29 January 2010 (743 obs.) of individual put option values (i.e. expected losses) of 36 sample banks, insurance companies, and other financial institutions.

Source: IMF (2010c).

The median of the joint distribution is much lower than the simple summation of individual contingent liabilities, which underscores the importance of accounting for the dependence structure when measuring systemic risk. With the dependence structure included, the median value of joint contingent liabilities is much lower than the total contingent liabilities obtained from summation. There are two 50th percentile lines in Figure 1. The solid line shows results for the case where government-sponsored financing agencies were de facto nationalised (which warranted their exclusion from the sample on 8 September 2008, which is marked by the sharp drop in the line before Lehman Brothers declared bankruptcy a little more than a week later). Controlling for the time-varying dependence structure between sample firms, the expected joint contingent liabilities peaked at about 1 per cent of GDP at the end of March 2009, averaging 0.5 per cent of GDP over the sample period. The second, dashed, 50th percentile line shows the case where these government-sponsored financing agencies are left in the sample (note that daily equity prices were still available but it can be argued that information may be much less informative).

Figure 2. United States: financial sector – average daily Expected Shortfall (ES) [95th percentile] based on multivariate density of expected losses



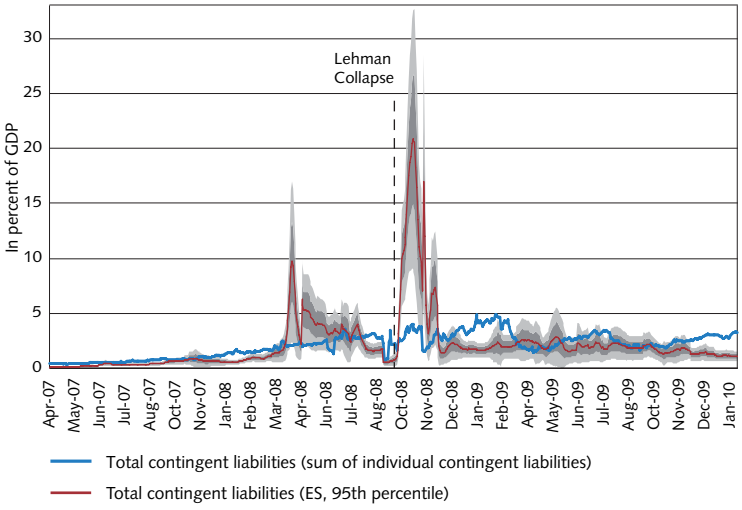
Note. Sample period: 3 January 2007-29 January 2010 (743 obs.) of individual put option values (i.e. expected losses) of 36 institutions. The red line shows the expected shortfall (ES) for the entire sample at a 95th percentile threshold within a confidence band of one and two standard deviations (grey areas).

Source: IMF (2010c).

After the collapse of Lehman Brothers, the extreme tail risk in the system increased sharply. The point estimates of the 95th percentile expected shortfall of extreme risk jumped to more than 20 percent of GDP in the months after the Lehman collapse (see Figure 2). The shaded bands show the one and two standard deviation bands around the estimate. In other words, during this period of exceptional systemic distress, market prices implied a minimum loss of 20 per cent of GDP with a probability of 5 per cent over a one-year time horizon. The magnitude of such tail risk dropped to under 2 per cent of GDP during the course of 2009.

The joint tail risk measure of contingent liabilities shows spikes in April 2008 and October 2008, indicating a high government exposure to financial sector distress. After controlling for the market perception (via CDS prices) of the residual risk retained in the financial sector, we find that the potential tail risk transferred to the government exceeded 9 per cent of GDP in April 2008 (in the wake of the Bear Stearns rescue) and almost reached 20 per cent of GDP in October 2008 (see Figure 3). The red line shows the 95th percentile expected shortfall within a confidence band of one and two standard deviations (grey areas). This spike in April 2008 is absent in the earlier chart showing expected losses (Figure 3), illustrating the distinction of expected losses and contingent liabilities for the purpose of systemic risk measurement. The bailout of Bear Stearns led to expectations of public support and induced highly correlated expectations of government support across numerous institutions, while residual risk outside anticipated public sector support was considered less susceptible to co-movements in asset prices.

Figure 3. United States: financial sector – average daily Expected Shortfall (ES) [95th percentile] based on multivariate density of contingent liabilities ("Systemic CCA")



Note. Sample period: 3 January 2007-29 January 2010 (743 obs.) Contingent liabilities (alpha*expected losses) of 36 sample institutions. The red line shows the expected shortfall (ES) for the entire sample at a 95th percentile threshold within a confidence band of one and two standard deviations (grey areas).

Source: IMF (2010c).

The systemic risk from contingent liabilities was considerable during the credit crisis. For the whole period from 1 April 2007 to 29 January 2010, the average contingent liabilities at the 50th and the 95th percentile levels amounted to 0.5 per cent and 1 per cent of GDP respectively.

This model is also used for forward-looking stress testing. The historical sensitivity of the bank-expected losses to macro variables is estimated (nominal and real GDP growth, real consumption, output gap, unemployment rate, housing prices, 3 month LIBOR-treasury rate spread). Secondly, for each bank, the baseline/adverse scenarios of implicit expected losses are extrapolated based on their joint historical sensitivity derived from a dynamic factor model. The baseline scenario used the IMF World Economic Outlook for 2010, and the adverse scenario assumed slower GDP growth, unemployment at 10 per cent and a further fall in house prices. The multivariate density of both expected losses and government contingent liabilities is then estimated using the marginal distributions of forecasted implicit put option values and their dependence structure for each quarter until the end of 2014 according to the Systemic CCA model. Results are shown in Table 2 below (IMF, 2010b and 2010c).

Table 2. United States – FSAP stress test results: systemic expected losses and market-implied contingent liabilities

Systemic CCA of financial sector – average systemic risk from expected losses and contingent liabilities, forecasting period, 2010 Q1-2014 Q4
(in USD billions unless indicated otherwise)

	50TH PERCENTILE	VAR (95%)	ES (95%)
Baseline scenario			
Market-implied contingent liabilities	31	92	180
Market-implied expected losses	75	219	429
Adverse scenario			
Market-implied contingent liabilities	41	130	382
Market-implied expected losses	97	308	910

Source: IMF (2010b and 2010c).

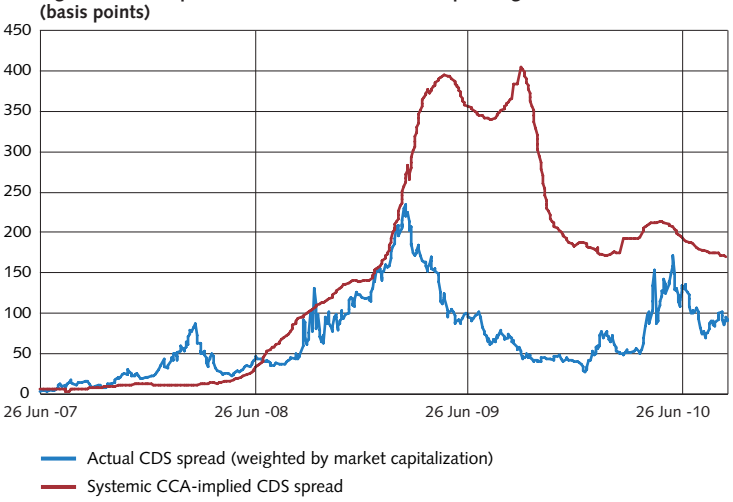
Stress test results for expected losses show the median of projected expected losses under the baseline scenario of USD 75 billion, USD 219 billion at the VaR 95 per cent level, and Expected Shortfall (ES) is higher.[23] For the adverse scenario, losses are USD 97 billion and VaR 95 per cent level USD 308 billion. Stress test results for market-implied contingent liabilities under the baseline are USD 31 billion, and USD 92 billion at the VaR 95 per cent level. For the adverse scenario, contingent liabilities are USD 41 billion, and USD 130 billion at the VaR 95 per cent level.

B. APPLICATION TO EUROPEAN BANKING SECTORS

The CCA model was applied to banks in the euro area. The CCA-implied CDS spread is generally higher than actual CDS spreads due to the impact of explicit and implicit guarantees. This is illustrated in Figure 5 for the top six banks in Europe. The gap between the CCA-implied spread and the actual CDS was largest in 2009 following the actions of authorities to guarantee bank senior debt.

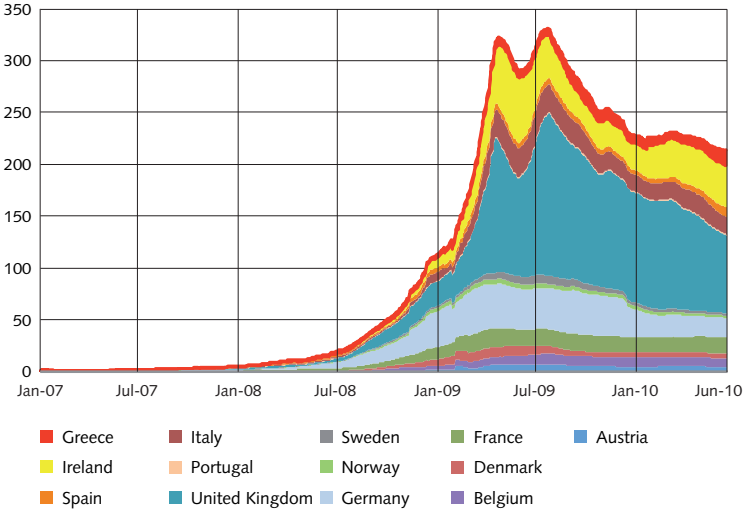
23 VaR (Value at Risk) is a widely-used risk measure. VaR is defined as a threshold value such that the probability that the loss over the given time horizon exceeds this value. ES (Expected Shortfall) is the expected value of the tail loss beyond the specific VaR level.

Figure 4. CCA implied CDS vs. actual CDS for Europe's largest six banks (basis points)



Source: IMF staff estimates.

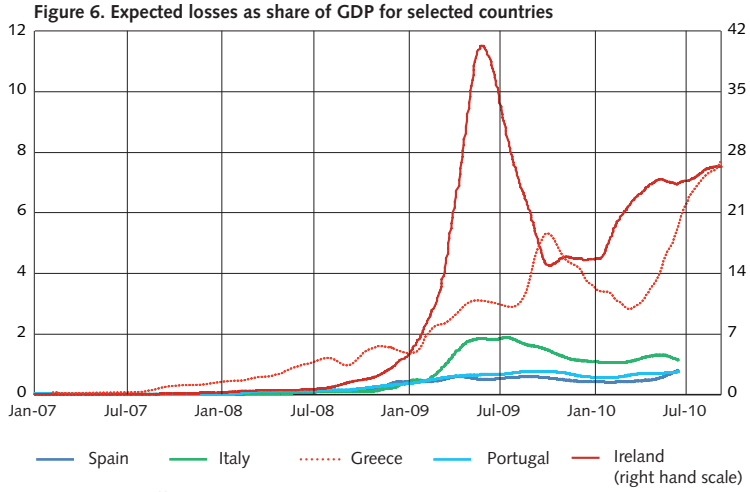
Figure 5. European banking system expected losses EUR billions, Jan. 2007-June 2010



Note. Sample period: 1 March 2005-18 June 2010 (1,075 obs.) of individual put option values. Sample institutions are 37 large commercial banks from the euro area plus Denmark, Sweden, Norway and the United Kingdom as shown above. The time series shows the 50th percentile of the multivariate density generated from extreme value univariate marginals (Generalized Extreme Value Distribution (GEV)) and a non-parametrically identified time-varying dependence structure of sample banks within each country.

Sources: IMF staff estimates, Gray and Jobst (2010).

Figure 6. Expected losses as share of GDP for selected countries



Source: IMF staff estimates.

The CCA model was applied to the banking systems in 13 euro area countries. The CCA model for the largest banks in each country was calibrated, and the Systemic CCA model estimates for each national banking sector were subsequently aggregated by applying the aggregation mechanism of Systemic CCA once again. The time pattern of the expected losses (50th percentile) is shown in Figure 5. While the UK is largest contributor in absolute terms, given the size of the system, this amount, if scaled by GDP, becomes much smaller when compared to Ireland, for instance. Figure 6 shows that the expected losses (as a percentage of GDP) are less than 2 per cent of GDP in Italy, Spain and Portugal, while in Greece they are 6 per cent of GDP. In Ireland, the range is from 20 to 40 per cent of GDP (right hand scale, RHS, is expected losses as a share of GDP for Ireland only).

An example of stress testing using systemic CCA for banking systems in the 13 European countries is shown in Table 3. First, historical sensitivity of the bank median expected losses to macro variables was estimated (real GDP growth and unemployment rate). Second, for each country banking sector, the baseline/stress scenario of median expected losses was projected, based on its historical sensitivity derived from a dynamic factor model. Stress scenario projections were based on an annual decrease of 1.5 percentage points in GDP growth and an increase in unemployment of 1.5 and 1.0 percentage points respectively. The results show that, under the baseline, losses fall to EUR 114 billion for the first year and then to EUR 89 billion in the second year. However, under the stress scenario, the expected losses are EUR 165 billion for the first year and EUR 219 billion in the second year.

Table 3. Stress testing systemic risk of European banking systems

EURO AREA BANKING SECTOR STRESS SCENARIOS	MEDIAN EXPECTED LOSSES, EURO BILLION
Estimation Period (Historical)	
Pre-crisis December 2005 to September 2008	6
Sept 15 to December 30 2008	47
January 1 to March 2010	135
Sovereign Crisis: March 1 to July 2010	123
Projection Period 1st year (2010 Q3-2011 Q2)	
Baseline Scenario	114
Stress Scenario	165
Projection Period 2nd year (2011 Q3-2012 Q2)	
Baseline Scenario	89
Stress Scenario	219

Source: IMF staff estimates.

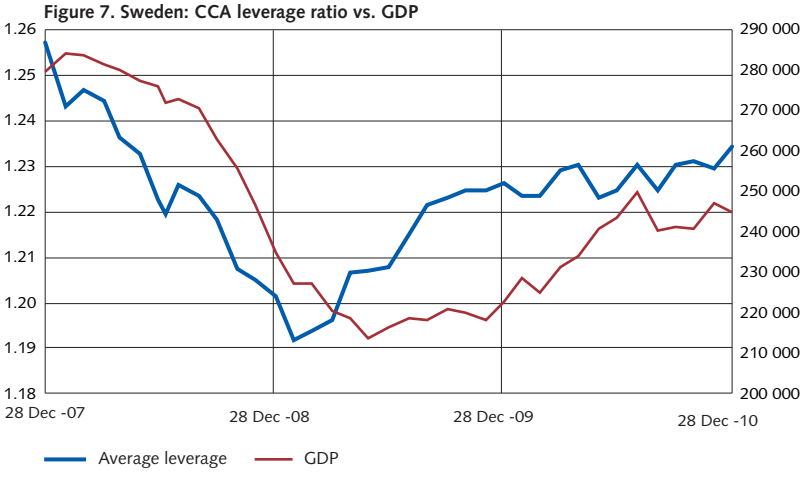
C. APPLICATION TO SWEDEN

CCA models for each of the four banks were first calibrated, and then expected losses for each were estimated. The CCA model for each bank used equity market and balance sheet information (including some inputs from Moody's KMV Credit Edge for each bank) to calibrate the key parameters of the CCA model (bank asset level, asset volatility, bank debt distress barrier, skew, kurtosis, and a volatility adjustment parameter).[24]

One key CCA risk indicator is the ratio of market capitalisation to the market value of assets. All banks show the same pattern, with a low point reached in early 2009. What is very interesting is how this indicator leads GDP. This is common – financial sector indicators of this type contain forward-looking information and relate to credit and risk appetite channels that affect GDP. See Figure 7, showing how the CCA leverage ratio (equity/assets) for the four banks leads GDP.

24 The four banks are Swedbank, Svenska Handelsbanken, Nordea, and Skandinaviska Enskilda Banken (SEB).

Figure 7. Sweden: CCA leverage ratio vs. GDP

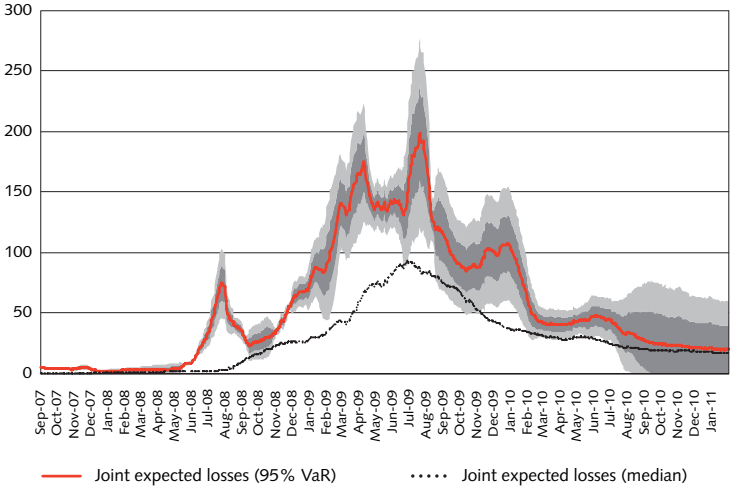


Source: IMF (2011d).

THE SYSTEMIC CCA METHODOLOGY FOR THE FOUR LARGEST COMMERCIAL BANKS IN SWEDEN

Over a sample period from September 2007 to January 2011, we estimate the magnitude of expected losses for all banks, and quantify the individual banks' contributions to systemic bank distress. Figure 8 shows the estimation results of the Systemic CCA-derived multivariate density of expected losses (i.e. the full value of the implicit put option). This is the median of the multivariate distribution of losses and the 95 per cent VaR (tail risk). The risk horizon is one year. July and August 2009 defined peak events (5 per cent chance of losses of SEK 200 billion over the coming year).

Figure 8. Sweden: banking sector – total sum and multivariate distribution function of expected losses (50th percentile and 95th percentile)
SEK, billions

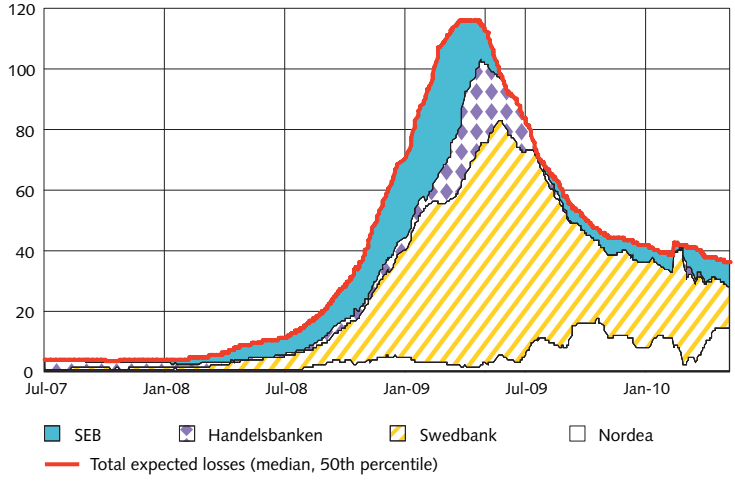


Note. Sample period: 1 January 2007-11 February 2010 of individual of individual put option values. The red line shows the daily Value-at-Risk (VaR) estimate for the entire sample at the 95th percentile within a confidence band of one and two standard errors (grey areas). The multivariate density is generated from univariate marginals, which conform to the Generalized Extreme Value Distribution (GEV) and a non-parametrically identified time-varying dependence structure.

Source: IMF (2011d).

The contribution of each bank to the median expected losses, 50th percentile, is shown in Figure 9. It clearly shows that Swedbank was the largest contributor, suggesting that the dynamics of market prices have anticipated the rising risk profile of Swedbank.

Figure 9. Sweden: individual contribution to daily Value-at-Risk (VaR) point estimates of expected losses ("Systemic CCA") at the 50th percentile
SEK, billions



Source: IMF (2011d).

The results of the balance sheet stress tests were used to estimate changes in bank assets. Bank-by-bank profits before loan losses, and bank-by-bank loan losses, adjusted for taxes and dividends, give the changes in bank assets for the stress test scenarios for each year from 2011 to 2014. In addition, the global market price of risk (a measure of global risk appetite) was projected for baseline and adverse (based on historical relationships to GDP, see Appendix 2 for details). Thus the changes in bank assets (and associated change in bank asset volatility) and the scenarios for the market price of risk form inputs to the CCA bank models, while the outputs are the expected losses to creditors and the market value of equity for each bank annually over the 2011 to 2014 period, from the base date of end 2010 (IMF, 2011c).

The simple sum of expected losses to bank creditors increases in the adverse scenario. They increase from SEK 89 billion at the end of 2010 to nearly 180 billion under the adverse scenario. This is significantly lower than the sum of expected losses, which peaked at SEK 375 billion in 2009.

APPLICATION OF CCA TO SWEDEN SOVEREIGN

In order to calibrate the sovereign risk-adjusted balance sheet, the implied value of sovereign assets and sovereign asset volatility needs to be calculated from observable information (the procedure is described in Appendix 3). The inputs are the sovereign debt default barrier and the term structure of the sovereign CDS spreads on 30 December 2010. The sovereign default barrier is the present value of the promised principle and interest payments on Swedish sovereign debt discounted at the risk free rate (3 per cent was used).

It is informative to see the evolution of the term structure of sovereign CDS spreads over the crisis. Before the crisis, one-year spreads were 8 basis points (bps) and 10-year spreads were 11 bps. During the crisis, on 9 December 2008, the one-year spreads were 120 bps, while 10-year spreads were 158 bps. The spreads have dropped down and, as of 30 December 2010, the five-year spreads were 30 bps.

The time patterns of principal and interest payments on Swedish sovereign debt from Bloomberg were used to estimate the sovereign debt default barrier, which was SEK 629 billion at the end of 2010. Using the CDS spreads and the debt default barrier, the procedure described above yields an implied sovereign asset equal to SEK 1 006 billion. Using end-2010 FX reserves of USD 37.9 billion (equal to SEK 256 billion), the PV of the primary fiscal surplus 2011 to 2016 (using data from the IMF) is estimated at SEK 457 billion, and implicit contingent liabilities to the financial sector are SEK 75 billion. The remainder (other assets) is estimated to be SEK 351 billion. Now we have all the components to estimate the impact on sovereign spreads from changes in financial sector contingent liabilities and changes in risk appetite in the stress test scenarios. Table 4 shows the results of a joint banking system and sovereign stress test with a baseline scenario (WEO 2010 forecast) and adverse (lower growth) scenario. The higher banking-system expected losses translate into higher contingent liabilities and higher sovereign spreads. The higher market price of risk increases both bank expected losses and sovereign spreads.

Table 4. Joint banking system and sovereign stress testing

	BANKING SYSTEM EXPECTED LOSSES (SUM) SEK, BILLIONS		SOVEREIGN SPREAD, FIVE YEAR, IN BASIS POINTS	
Historical				
Pre-crisis	8		10	
2008	60		145	
2009	190		130	
End 2010	89		30	
Projections	BASELINE SCENARIO	ADVERSE SCENARIO	BASELINE SCENARIO	ADVERSE SCENARIO
End 2011	85	180	29	45
End 2012	83	150	28	85
End 2013	80	120	27	77
End 2014	77	98	26	70

Sources: IMF staff estimates and IMF (2011c).

V. Further extensions going forward: integrating macrofinancial stress testing and policy analysis

Going forward, the type of analysis described above could be extended to integrate financial sector and sovereign risk analysis with macrofinancial feedbacks to perform stress testing and policy analysis, as well as monetary and fiscal policy analysis. This framework links some of the important components of financial sector systemic risk analysis to sovereign risk analysis to help evaluate fiscal policies and link the financial sector risk

indicators to GDP and output gap and thus link into the monetary policy models. The fact that CCA financial risk indicators have predictive power for GDP and output gap means that this framework is useful for macrofinancial linkages and feedback as well as monetary policy models. Such integrated risk models could stress test shocks to banking and sovereign balance sheets and evaluate the policy responses on capital requirements of banks, guarantees, fiscal policy and macroprudential regulation, all within one framework. Using economy-wide CCA can also provide new measures of economic output – the present value of risk-adjusted GDP (see Gray et al. (2010) for details).

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Appendix 1. The Contingent Claims Analysis (CCA) approach—standard definition

In the first structural specification of CCA, commonly referred to as the Black-Scholes-Merton (BSM) framework (or in short, the "Merton model") of capital structure-based option pricing theory (OPT) (Black and Scholes, 1973; Merton, 1973 and 1974), total value of firm assets follows a stochastic process and may fall below the value of outstanding liabilities. Thus, the asset value $A(t)$ at time t describes a continuous asset process so that the physical probability distribution of the end-of-period value is

$$A(T-t) \sim A(t)\exp\left\{\left(r_A + \sigma_A^2/2\right)(T-t) + \sigma_A\sqrt{T-t}\tilde{z}\right\},$$

for time to maturity T-t. More specifically, $A(t)$ is equal to the sum of its equity market value, $E(t)$, and its risky debt, $D(t)$, so that $A(t) = E(t) + D(t)$. The term r_A is the risk free rate of interest, σ_A is the volatility of the sovereign asset, z is the stochastic term equal to standard normal distribution mean zero standard deviation of one. Default occurs if $A(t)$ is insufficient to meet the amount of debt owed to creditors at maturity, which constitutes the bankruptcy level ("default threshold" or "distress barrier").

The equity value $E(t)$ is the value of an implicit call option on the assets, with an exercise price equal to default barrier. It can be computed as the value of a call option $E(t) = A(t)\Phi(d_1) - Be^{-r(T-t)}\Phi(d_2)$, with $d_1 = \left[\ln\left(A(t)/B\right) + \left(r + \sigma_A^2/2\right)(T-t)\right]\left(\sigma_A\sqrt{T-t}\right)^{-1}$, $d_2 = d_1 - \sigma_A\sqrt{T-t}$, asset return volatility σ_A, and the cumulative probability $\Phi(.)$ of the standard normal density function. Both the asset, $A(t)$, and asset volatility, σ_A, are valued *after* the dividend payouts. The value of risky debt is equal to default-free debt minus the present value of expected loss due to default,

$$D(t) = Be^{-r(T-t)} - P_E(t).$$

Thus, the present value of market-implied expected losses associated with outstanding liabilities can be valued as an implicit put option, which is calculated with the default threshold B as strike price on the asset value $A(t)$ of each institution. Thus, the present value of market-implied expected loss can be computed as

$$P_E(t) = Be^{-r(T-t)}\Phi(-d_2) - A(t)\Phi(-d_1),$$

over time horizon $T–t$ at risk-free discount rate r, subject to the duration of debt claims, the leverage of the firm, and asset volatility.[25] Since the implicit put option $P_E(t)$ can be decomposed into the risk-neutral probability of default *(PD)* and the loss given default *(LGD)*,

$$P_E = \Phi(-d_2)\left(1 - \frac{\Phi(-d_1)}{\Phi(-d_2)}\frac{A(t)}{Be^{-rT}}\right)Be^{-r(T-t)} = PD \times LGD,$$

There is no need to introduce the potential inaccuracy of assuming a certain loss given default (LGD). The risk-neutral default probability is RNDP. We can use the equations above to see that the spread can also be written as

$$s = -\frac{1}{T}\ln(1 - RNDP \times LGD).$$

Another important factor that drives spreads of banks (as well as corporates and sovereigns) and affects bank funding costs is the change in global risk appetite. The *market price of risk* (MPR, see Box 1) is an important parameter in *CCA* formulas, which changes when global risk appetite changes. It is a barometer of the level of risk appetite and is used to translate from the real to risk-neutral default probability. In the *CCA* model developed by Moody's KMV, the market price of risk is empirically calculated. It uses the capital asset pricing model, together with the *CCA* model, to estimate the market price of risk (MPR) as, $\lambda = \rho_{A,M} SR$,
where λ is the market price of risk, $\rho_{A,M}$ is the correlation of the bank's asset return with the global market and is the global market Sharpe ratio. Appendix 2 provides the derivation and the details.

The market-implied expected losses calculated for each financial institution from equity market and balance sheet information using the *CCA* can be combined with information from credit default swap *(CDS)* markets to estimate the government's contingent liabilities. The put option value $P_{CDS}(t)$ using credit default swap *(CDS)* spreads reflects the expected losses associated with default net of any financial guarantees, i.e., residual default risk on unsecured senior debt and can be written as

$$P_{CDS}(t) = \left(1 - \exp\left(-\left(s_{CDS}(t)/10{,}000\right)\left(B/D(t) - 1\right)\left(T - t\right)\right)\right)Be^{-r(T-t)}.$$

25 Note that the above option pricing method for $P_E(t)$ does not incorporate skewness, kurtosis, and stochastic volatility, which can account for implied volatility smiles of equity prices. More advanced option pricing techniques have been incorporated in the CCA (Gray and Jobst, forthcoming; IMF, 2011g).

The linear adjustment $(B/D(t) - 1)$ is needed if outstanding debt B trades either above (below) par value D, which decreases (increases) the CDS spread $s_{CDS}(t)$ (in basis points) due to an implicit recovery rate of the CDS contract at notional value and below (above) the recovery rate implied by the market price $D(t)$. This negative (positive) difference ("basis") between the CDS spread and the corresponding bond spread represents the ratio between recovery *at face value (RFV)*, which underpins the CDS spread calculation, and recovery *at market value (RMV)*, which applies to the commensurate bond spread.[26] $P_{CDS}(t)$ above is derived by rearranging the specification of the CDS spread

$$s_{CDS}(t) = -(T-t)^{-1} \ln\left(1 - P_{CDS}(t) / Be^{-r(T-t)}\right) \times \left(B/D(t) - 1\right) \times 10,000$$

under the risk-neutral measure, assuming a survival probability

$$1 - \bar{p} = \exp\left(-\int_0^t h(u)\,du\right) = \exp(-ht)$$

at time t with cumulative default rate p, and a constant hazard rate $s(t)_{CDS} \approx h$. Then $P_{CDS}(t)$ can be used to determine the fraction

$$\alpha(t) = 1 - P_{CDS}(t) / P_E(t)$$

of total potential loss due to default, $P_E(t)$, covered by implicit guarantees that depress the CDS spread below the level that would otherwise be warranted for the option-implied default risk.[27] In other words, $\alpha(t) P_E(t)$ is the fraction of default risk covered by the government (i.e. its contingent liability) and $(1 - \alpha(t)) P_E(t)$ is the risk retained by an institution and reflected in the CDS spreads. Thus, the time pattern of the government's contingent liability and the retained risk in the financial sector can be measured.

26 We approximate the change in recovery value based on the stochastic difference between the standardised values of the fair value CDS (FVCDS) spread and the fair value option-adjusted spread (FVOAS) reported by *Moody's KMV* (MKMV). Both FVOAS (FVCDS) are credit spreads (in bps) over the London Interbank Offered Rate for the bond (CDS) of a particular company, calculated by MKMV's valuation model based on duration (term) of t years (where t=1 to 10 in one-year increments). Both spreads imply an LGD determined by the industry category. In practice, this adjustment factor is very close to unity for most of the cases, with a few cases where the factor is within a 10 per cent range (0.9 to 1.1).

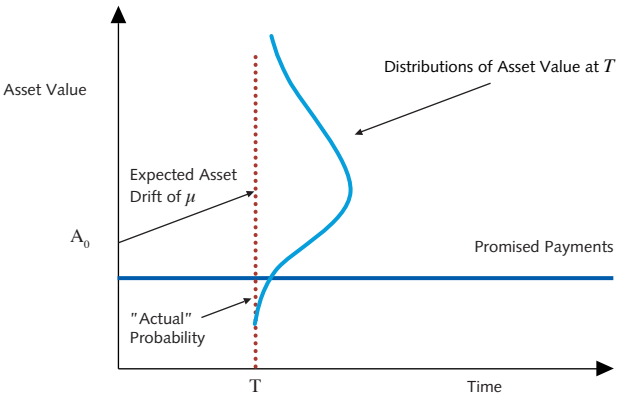
27 Note that the estimation assumes a European put option, which does not recognise the possibility of premature execution. This might overstate the actual expected losses inferred from put option values in comparison with the put option derived from CDS spreads.

Appendix 2. CCA with the market price of risk

MODELLING DEFAULT RISK

Let us start with the evolution of bank assets over time horizon t relative to the promised payments on the debt (default free value of the debt and deposits). The value of assets at time t is $A(t)$. The asset return process is $dA/A = \mu_A dt + \sigma_A \varepsilon \sqrt{t}$, where μ_A is the drift rate or asset return, σ_A is equal to the standard deviation of the asset return, and ε is normally distributed, with zero mean and unit variance. The probability distribution at time T is shown in Figure A1(a) below.

Figure A1 (a). Modelling default risk



Default occurs when assets fall to or below the promised payments, B_t. The probability of default is $A_t \leq B_t$ so that

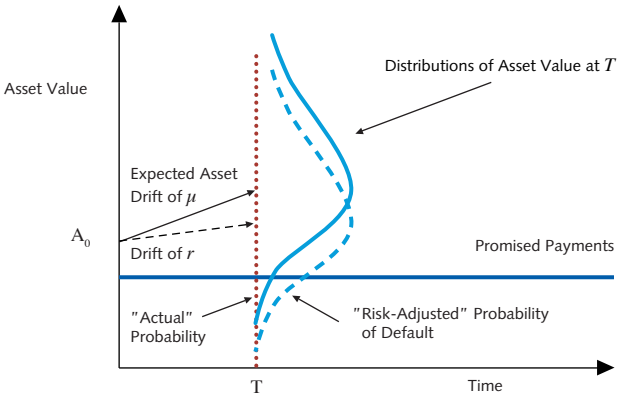
$$\Pr(A_t \leq B_t) = \Pr\left(A_0 \exp\left[\left(\mu_A - \sigma_A^2/2\right)t + \sigma_A \varepsilon \sqrt{t}\right] \leq B_t\right) = \Pr\left(\varepsilon \leq -d_{2,\mu}\right).$$

Since, $\varepsilon \sim \Phi(0,1)$ the "actual" probability of default is, $N(-d_{2,\mu})$ where $d_{2,\mu} = \left[\ln\left(A_0/B_t\right) + \left(\mu_A - \sigma_A^2/2\right)t\right]\left(\sigma_A\sqrt{t}\right)^{-1}$. The "actual" probability of default is the area below the line (promised payment, i.e. the default barrier).

Shown in Figure A1 (b) below is the probability distribution (dashed line) with drift of the risk-free interest rate, *r*. The risk adjusted probability of default is $N(-d_2)$. The area below the distribution in Figure A1 (a) is the "actual" probability of default. The asset-return probability distribution used to value contingent claims is not the "actual" one but the "risk-neutral" probability distribution, which is the dashed line in Figure A1(b) with expected rate of return *r*, the risk-free rate. Thus, the "risk-neutral" probability of default is larger than the actual probability of default for all assets which have an actual expected return (μ) greater than the risk-free rate *r* (that is, a positive risk premium).[28]

28 See Merton (1992, pp.334-343; 448-450).

Figure A1 (b). Actual vs. risk-neutral default probabilities



These two risk indicators are related by the *market price of risk*, λ:

$$\Phi\left(-d_{2,\mu}\right)=\Phi\left(-d_2-\lambda\sqrt{t}\right)$$

The market price of risk reflects investors' risk appetite. It is the "wedge" between the real and risk neutral default probability. It can be estimated in several ways. One way is the use of the capital asset pricing model (CAPM) model to estimate the market price or risk is shown in Box 1 so that:

$$\left(\mu_A-r\right)/\sigma_A=\lambda$$

where $\rho_{A,M}$ is the correlation of the asset return with the market and SR is the market Sharpe Ratio.

Box 1. Market price of risk (MPR)

A two moment CAPM is used to derive the market price of risk (developed and used in Moody's KMV model). This CAPM states that the excess return of a security is equal to the beta β of the security times the market risk premium $\mu - r$, so that

$$\mu_A - r = \beta(\mu_M - r)$$

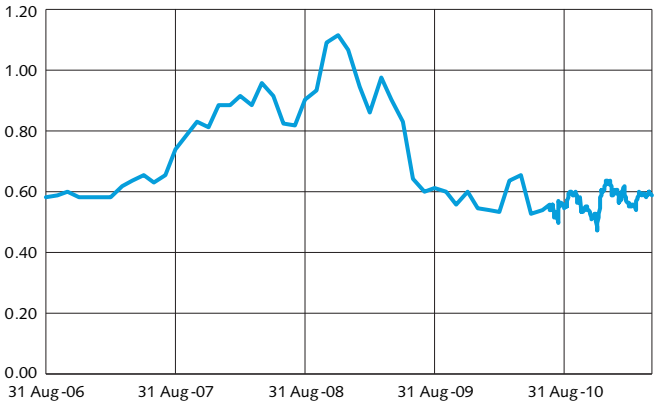
Beta is equal to the correlation of the asset with the market times the volatility of the asset divided by the volatility of the market.

$$\beta = \frac{\text{cov}(r_A, r_M)}{\text{var}(r_M)} = \rho_{A,M}\frac{\sigma_A}{\sigma_M} \quad \text{So, } \mu_A - r = \rho_{A,M}\sigma_A(\mu_M - r)/\sigma_M = \rho_{A,M}\sigma_A SR$$

Here SR is the Market Sharpe ratio, the market risk premium per standard deviation of market risk, and, thus, $(\mu_A - r)/\sigma_A = \rho_{A,M} SR$.

According to MKMV data, $\rho_{A,M}$ is usually around 0.5 to 0.7 (calculated bank by bank in the *MKMV* Credit Edge model) and the around 0.55 to 1.2 during the last few years.[29] The main driver of the market price of risk in this model is the global Sharpe ratio. The correlation does not change much over time, but the SR changed considerably, see Figure A2 below.

Figure A2. Global Sharpe ratio



A higher global Sharpe ratio is associated with higher average volatility for Swedish banks. There is systemic impact on volatility in addition to the idiosyncratic change in volatility described in Appendix 1. For the Swedish banks, average volatility is around 16 per cent

29 See MKMV (2003), Crouhy et al., Galai and Mark (2000).

(annualised) when the Sharpe ratio is 0.6, but increases to 23 per cent when the Sharpe ratio reaches 1.1. This systemic increase in volatility is included in the scenarios, empirically the change in Sharpe ratio times 0.09 gives the incremental change in volatility (measured as a fraction).[30]

30 Changes in risk appetite affect risk perceptions going forward affecting the dynamics of the market price of risk. The market price of risk, over a one-year horizon is $\lambda = \rho_{A,M} SR$ and it provides a way to translate between the actual default probability (EDF) and the risk-neutral default probability.

Appendix 3. Interaction and feedback between sovereign CCA balance sheet and the financial sector: potential destabilisation processes

The CCA framework can be used to calibrate sovereign balance sheets and be integrated with banking sector balance sheets in a simple but illustrative framework to show the interaction and potential destabilisation of values of spreads and risks in both the sovereign and banking sectors. In the absence of measureable equity and equity volatility, such as in the case of a developed country sovereign, including where there are assets and debt all in the same currency, the term structure of sovereign spreads can be used to estimate implied sovereign assets and asset volatility and calibrate market-implied sovereign risk adjusted balance sheets.

Sovereign spreads are related to the sovereign implicit put option, P_{Sov}, and sovereign default barrier, B_{Sov} (or threshold that debt restructuring is triggered) in the following way. Rearranging the formula for the sovereign implicit put option gives:

$$\frac{P_{Sov}}{B_{Sov}e^{-rT}} = N(-d_2) - \frac{A_{Sov}}{B_{sov}}\frac{1}{e^{-rT}}N(-d_1)$$

Inserting this equation into the equation for sovereign spreads and using (i) an estimate of the sovereign default barrier from debt data, and (ii) the full term structure of the sovereign CDS, (CDS for years 1, 3, 5, 7 and 10) one can estimate the implied sovereign assets, A_{Sov}, and implied sovereign asset volatility, σ_A that most closely matches the sovereign spread term structure. The sovereign asset value can be broken down into its key components: Reserves (R); net fiscal asset or present value of the primary fiscal surplus (PVPS); implicit and explicit contingent liability (αPut_{Bank}); and "Other" remainder items, i.e. $A_{Sov,t=0} = R + PVPS - \alpha Put_{Bank} + Other$. The value of the foreign currency reserves can be observed and the contingent liabilities can be estimated from the banking sector CCA models (i.e. systemic CCA). Subtracting these from the implied sovereign asset and subtracting an estimate of the present value of the expected primary surplus allows us to calculate the residual (Other). There are a number of government assets and various unrealised liabilities, pension and healthcare obligations, which are not known but are aggregated in "Other", which may include contingent financial support from other governments or multilaterals or other backstop assets (e.g. land or other public sector assets of value). We can use this valuation formula to evaluate the effects of changes in reserves, the primary fiscal balance, and the implicit banking sector guarantee on the sovereign asset value. This can be used with changes in the composition of short-term and long-term debt for stress tests to evaluate changes in sovereign credit spreads and other values and risk indicators.

The spreads for the banks can be seen as a function of the implicit put option, P_{Bank} (derived from equity information) times the fraction of risk retained by the banks (as

described in the systemic CCA section above) plus a premium (δ) if high sovereign spreads spill over to increase bank spreads.

$$s_{Bank} = -\frac{1}{T}\ln(1-\frac{(1-\alpha)P_{Bank}}{B_{Bank}e^{-rT}})+\delta$$

This simple model shows the ways in which sovereign and bank spreads can interact and potentially lead to a destabilisation process. If sovereign spreads increase, this can lead to an increase in bank spreads for several reasons: (i) the credibility of sovereign guarantees decreases (alpha goes down); (ii) the implicit bank put option could increase as the value of the bank's holdings of government debt decrease; (iii) the bank default barrier may increase due to higher borrowing costs as the premium (δ) increases (and if banks can't roll over debt). Prospects of a much more fragile banking system can feed back on sovereign spreads via several possible channels, e.g. large and increasing bank guarantee/bailout costs that may overwhelm the budget, reduced ability of sovereigns to borrow from banks and potential crowding-out effects.