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# Derivation and Estimation of a New Keynesian Phillips Curve in a Small Open Economy*

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**Abstract**

In recent years, it has become increasingly common to estimate New Keynesian Phillips curves with a measure of firms' real marginal cost as the real driving variable. It has been argued that this measure is both theoretically and empirically superior to the traditional output gap. In this paper, a marginal-cost based New Keynesian Phillips curve is estimated on Swedish data by means of GMM and Full Information Maximum Likelihood. The results show that with real marginal cost in the structural equation the point estimates generally have the exptected positive sign, which is less frequently the case using the output gap in the Phillips curve equation. This suggests that real marginal cost might be a more adequate real explanatory variable for Swedish inflation than the output gap. However, standard errors in the estimations are large and it is in fact difficult to pin down a statistically significant relationship between either real marginal cost or the output gap and inflation.

*Keywords:* Inflation, New Keynesian Phillips curve, Real marginal cost, Small Open Economy, GMM, Full Information Maximum Likelihood.

*JEL Classification numbers:* E31, E32, C22.

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# 1 Introduction

Understanding inflation dynamics is of central concern for macroeconomists in general and for central banks in particular. In the New Keynesian literature, inflation is commonly explained by the level of economic activity and expectations of future inflation. Starting from an assumption of rigid nominal prices, inflation may be derived as a function of the expected future path of firms' real marginal cost. By making additional assumptions about technology and consumer preferences, which allows the derivation of a labor supply curve, one can pin down a proportionate relationship between real marginal cost and the level of output. Given this relationship, inflation may be expressed as a function of the output gap (deviation of actual output from potential output) and inflation expectations one period ahead, which is commonly referred to as the standard New Keynesian Phillips curve (NKPC).

However, reconciling the New Keynesian Phillips curve with empirical facts has not been entirely successful. For a start, the structural formulation of the NKPC implies that past inflation will have no impact on current inflation. This rhymes badly with the high degree of inflation persistence which is usually found in the data (see e.g. Christiano, Eichenbaum, Evans 2001, Mankiw 2001). One way to cope with this fact in the theoretical models has been to allow for a subset of firms that set prices according to a backward looking rule of thumb. As a result, also lagged inflation enters the Phillips curve.

Furthermore, it has proven difficult to establish an empirical link between estimates of the output gap and inflation; with quarterly data it has often not been possible to reject the hypothesis that the output gap has no importance for inflation (Chadha et al. 1992, Roberts 1997, 1998). This could of course be due to difficulties associated with measuring the output gap. Measuring the output gap by detrended GDP implicitly assumes that the natural rate of output is well approximated by a smooth trend. In reality, a wide variety of real shocks, such as productivity shocks, changes in attitudes towards labor supply etc., can produce fluctuations in the natural level of output. And these fluctuations may not be well approximated as smooth.

It could also be the case that the proportionate relationship between firms' real marginal cost and the level of aggregate activity is counterfactual. For this relationship to hold in practice there must not be any kind of labor market frictions. If for instance wages are rigid, this produces inertia in real marginal cost relative to the output gap (Galí, Gertler and Lopéz-Salido 2001). A high level of resource utilization would then lead to a delayed rise in firm's real marginal cost and in inflation.

To overcome the problem of identifying an empirical relationship between the output gap and inflation, a strand of papers have gone back one step and estimated a Phillips curve with real marginal cost as the driving force underlying changes in inflation. Real marginal cost has commonly been measured by the labor income share (or, equivalently, real unit labor costs). An advantage of this approach is that productivity shocks are automatically taken into account as they will be reflected in firms' marginal cost.

These papers have been more successful empirically. For instance, Galí and Gertler (1999) estimate a marginal cost based Phillips curve using Generalized Method of Moments (GMM) on US data and find that real marginal costs are indeed a statistically significant and quantitatively important determinant of inflation. They allow for both forward and backward looking price setting in the estimations. In subsequent work, Galí, Gertler and López-Salido (2001) provide evidence on the fit of this formulation of the Phillips curve also for the Euro area. Woodford (2001) produces series of predicted inflation based on expectations of detrended output and real unit labor costs using a reduced-form VAR to calculate expectations of the future. He finds that the set up with real unit labor cost gives a much better fit of actual inflation. Using similar estimation techniques, Sbordone (2002) draws the same conclusion.[1]

In this paper, a marginal cost based New Keynesian style of Phillips curve is estimated on quarterly Swedish data for the period 1986 - 2004. Since the Swedish economy is highly dependent on global developments, the standard NKPC model is adjusted for allowing international price developments to affect the domestic inflation rate. This is done by introducing imported inputs as an additional factor in the production function. As a result, firms' real marginal cost will be a function of both the labor income share and the share of imported inputs in production. The idea is to examine to what extent this Phillips curve can explain the development of Swedish inflation and, in particular, whether real marginal cost has a better explanatory power than the output gap also for Swedish inflation dynamics. The importance of lagged inflation in the Phillips curve is also studied.

The Phillips curve is estimated with GMM and Full Information Maximum Likelihood (FIML) techniques. In the FIML estimations, expectations of future inflation are solved for by setting up a complete model of the economy. However, in order to focus attention on the structural restrictions of the Phillips curve, the rest of the economy is represented by an unrestricted VAR system (as in Fuhrer and Moore 1995).

The estimation results show that it is difficult to pin down a statistically significant relationship between a real driving variable and inflation as suggested by the structural Phillips curve equation. As a robustness test I explore the effect of different choices of price index, of real variable in the Phillips curve and of the VAR set up and find that the result is fairly robust across various specifications. However, estimating a Phillips curve with real marginal cost as the real driving variable result in most cases in positive point estimates of the impact of labor share while using a measure of the output gap, in contrast, results in generally negative coefficients. This indicates that real marginal cost might be a more adequate real explanatory variable for Swedish inflation than the output gap. However, due to large standard errors in the estimations, this hypothesis can neither be rejected nor verified with standard statistical degree of certainty. Some possible reasons for the poor fit of this kind of New Keynesian Phillips curve on Swedish data are discussed in the conclusions.

The rest of the paper is organized as follows. The theoretical model - a marginal cost based Phillips curve in a small, open economy - is derived in Section 2. In Section 3, I motivate the choice of estimation methods (GMM and FIML) and present the results. Finally, in Section 4, some conclusions are drawn.

[1] Rudd and Whelan (2002), however, show that the results of Woodford and Sbordone are not robust to other specifications of the VAR.

## 2 The theoretical foundation

In this section, a structural relationship between current inflation and real marginal cost, expected and past inflation is derived. The model is in large parts a standard model used in the New Keynesian literature. It is adopted to a small open economy by allowing imported goods to be used in production. To keep the presentation simple, only the key equations are presented in this section. Detailed derivations are presented in Appendix A.

Nominal price rigidity is modeled as in Galí and Gertler (1999), who use a variant of the mechanism formulated by Calvo (1983). More specifically, in each period, a share $(1-\theta)$ of the firms is allowed to change prices while the remaining firms keep prices fixed. Of the firms who are allowed to change their prices, a fraction $(1-\omega)$ does so in an optimal, forward-looking manner while a fraction $\omega$ instead set the new price using a rule of thumb, which is based on past price developments. A motivation for this formulation is that the process of setting an optimal price is costly to firms (e.g. because of information gathering costs, decision making costs).

The model economy consists of a continuum of firms indexed by $i \in [0,1]$. Each firm is a monopolistic competitor and produces a differentiated good $Y_{it}$ that it sells at nominal price $P_{it}$. Each firm faces a constant elasticity demand function, i.e.

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t \tag{1}$$

where $Y_t$ and $P_t$ is aggregate demand and the aggregate price level respectively. All firms use the same production technology and need three inputs, labor $(L)$, capital $(K)$ and imported goods $(IM)$. The production function of firm i is given by

$$Y_{it} = A_t L_{it}^{\alpha} \left(IM_{it}^{\eta} K_{it}^{1-\eta}\right)^{1-\alpha} \tag{2}$$

where $0 < \alpha < 1$, $0 < \eta < 1$ and $A$ denotes technology. This function implies constant returns to scale with respect to all three production factors, but decreasing returns to increases in any combination of two production factors.

**The optimal flexible price**

Before deriving how a profit maximizing firm sets prices under nominal price rigidity, it is useful to derive the firm's optimal price under perfectly flexible prices.

Under flexible prices, the producer of good $Y_i$ chooses $P_i$ to maximize profits subject to the demand equation in (1). Formally,

$$\max_{P_{it}} \pi_{it} = P_{it} Y_{it} - MC_{it} Y_{it} \tag{3}$$

where $MC_{it}$ is the firm's marginal cost. The first order condition for the optimal choice of $P_{it}$ is

$$P_{it} = \frac{\varepsilon}{\varepsilon - 1} MC_{it} \tag{4}$$

Equation (4) states the standard result when assuming a constant elasticity demand function; in the absence of nominal rigidities, firms will set prices as a constant markup over current marginal cost. In other words, in case of flexible prices, real marginal cost, $mc_t^r$ would be constant and firms would always produce at the flexible price optimal level.

**The optimal price with nominal price rigidity**

With restrictions on the possibility to adjust prices each period, optimal price setting must take expected future developments of demand and production conditions into account. The optimization problem of a firm which is drawn to reoptimize in a given period will be to set $P_{it}$ as to maximize expected profit over the horizon over which the price is expected to prevail. Formally, the optimization problem is

$$\max_{P_{it}} E_t \sum_{j=0}^{\infty} (\beta\theta)^j V_{t,t+j} \left[ \frac{P_{it}}{P_{t+j}} Y_{it+j} - \frac{MC_{it+j}}{P_{t+j}} Y_{it+j} \right] \tag{5}$$

subject to the demand equation in (1). $\theta$ is the probability of the firm not being allowed to change price in each period and $\beta^j V_{t,t+j}$ is the stochastic discount factor.

The solution to this optimization problem can, in log-linearized terms, be expressed as (see Appendix A for a detailed derivation)

$$p_{it} = (1 - \beta\theta) E_t \sum_{j=0}^{\infty} (\beta\theta)^j mc_{it+j} \tag{6}$$

Small letters denote log deviation from steady state. Equation (6) shows that when prices are rigid, rational firms will set prices as a markup over a weighted sum of current and expected future marginal cost.

By quasi-differencing (6), the optimal price in period t can instead be expressed as a function of current marginal cost and expectations of future prices.

$$p_{it} = (1 - \beta\theta) [mc_{it}] + \beta\theta E_t p_{it+1} \tag{7}$$

**The marginal cost function**

A common measure of real marginal cost in the New Keynesian literature is real unit labor cost. This results from assuming Cobb-Douglas technology with only labor and capital. With imported goods included in the production function along with labor and capital, as in (2), marginal cost will be a function of both wage costs and the cost of imported goods. In Appendix A, it is shown that a cost-minimizing firm will face the following marginal cost function (expressed as log deviation from steady state)

$$mc_{it} = \frac{1}{\alpha + \eta(1-\alpha)} [\alpha(w_t + l_{it} - y_{it}) + \eta(1-\alpha)(p_t^m + im_{it} - y_{it})] \tag{8}$$

where $w_t$ is the nominal wage level and $p_t^m$ is a price index of intermediate imported goods. Hence, (nominal) marginal cost is a function of both unit labor costs and the unit price of imports in production.

**The Phillips curve**

Turning to aggregate price dynamics, the fact that all firms who reoptimize in a given period will choose the same price justifies using the notation $p_t^f$ instead of $p_{it}$. The superscript $f$ emphasizes that the price setting is forward looking. Accordingly, from (7)

$$p_t^f = (1 - \beta\theta) [mc_t^r + p_t] + \beta\theta E_t p_{t+1}^f \tag{9}$$

where $mc_t^r$ is *real* marginal cost.

The general price level in period t, $p_t$, will be a weighted average of

- a share $(1-\theta)$ of firms which are allowed to change the price. Of these a share $(1-\omega)$ sets prices in an optimal, forward-looking manner $(p_t^f)$ and a share $\omega$ follows a rule of thumb and reset prices to adjust for last periods inflation.

- a share $(\theta)$ of firms who does not change the price.

  The assumed rule of thumb (again leaving out the subscript $i$ as the whole share $\omega$ of firms follow the same rule) is, in log-linearized form,

$$p_t^b = p_{t-1}^* + \pi_{t-1} \tag{10}$$

  where $p_{t-1}^*$ is an index of prices reset in period t-1 and $\pi_{t-1} = p_{t-1} - p_{t-2}$. Hence, firms which obey the rule of thumb set prices based on recent pricing behavior of its competitors, adjusted for recent inflation.[2]

The general price index evolves according to

$$p_t = (1-\theta)\, p_t^* + \theta p_{t-1} \tag{11}$$

and

$$p_t^* = (1-\omega)\, p_t^f + \omega p_t^b \tag{12}$$

Combining equations (10) - (12) with (9) yields the hybrid Phillips curve with current inflation as a function of both lagged inflation and expected inflation as well as real marginal costs (see Appendix A for a detailed derivation):

$$\pi_t = \xi mc_t^r + \lambda_f E_t \pi_{t+1} + \lambda_b \pi_{t-1} \tag{13}$$

where

$$\begin{aligned} \xi &= \tfrac{(1-\beta\theta)(1-\theta)(1-\omega)}{\phi} \\ \lambda_f &= \tfrac{\beta\theta}{\phi} \\ \lambda_b &= \tfrac{\omega}{\phi} \end{aligned} \tag{14}$$

with $\phi = \theta + \omega\left[1-\theta\left(1-\beta\right)\right]$ and

$$mc_t^r = \frac{1}{\alpha + \eta\,(1-\alpha)} \left[\alpha\,(w_t + l_t - p_t - y_t) + \eta\,(1-\alpha)\,(p_t^m + im_t - p_t - y_t)\right] \tag{15}$$

The two components of real marginal cost are $(w_t + l_t - p_t - y_t)$, which is real unit labor costs (or, equivalently, the labor income share) and $(p_t^m + im_t - p_t - y_t)$, the share of imported intermediate goods to production in current prices. When both these variables are at their flexible price levels, firms are producing at their desired production levels. Consequently, there will be no pressure from current production conditions for inflation to rise. When $mc_t^r$ is above (below) the flexible price level, the production level is higher (lower) than the flexible price level, and there will be a tendency for inflation to edge up (fall) as firms' who can will raise (lower) prices.

[2] This is the same rule of thumb as in Galí and Gertler (1999).

# 3 Estimating a Swedish Phillips Curve

In the following, the above derived Phillips curve will be estimated on Swedish data, that is

$$\pi_t = \kappa_1 ls_t + \kappa_2 ims_t + \lambda_f E_t \pi_{t+1} + \lambda_b \pi_{t-1} + \varepsilon_t \tag{16}$$

where $ls_t = \frac{w_t l_t}{p_t y_t}$, $ims_t = \frac{p_t^m i_t}{p_t y_t}$, $\kappa_1 = \xi \frac{\alpha}{\alpha + \eta(1-\alpha)}$ and $\kappa_2 = \xi \frac{\eta(1-\alpha)}{\alpha + \eta(1-\alpha)}$. With $\varepsilon_t$ I allow for the occurrence of supply shocks in period t.

The data cover the period 1986:1 to 2004:1 and is quarterly. Inflation is measured as the GDP deflator, which is the most theory-consistent price index (price increases on all produced goods and services). The inflation rate is expressed as the quarterly change in the price level.[3] As a robustness check, also CPI and a measure of underlying inflation, UND1X, are used in the estimations.[4] The model assumes that inflation in steady state is constant. To allow for the possibility of a time-varying steady state, estimations with detrended inflation (using a Hodrick-Prescott filter) are also performed. Trend inflation is then assumed to capture the (time-varying) steady state inflation rate.

Real unit labor cost, $ls_t$, is expressed as percentage deviation from the mean. The cost measure of imported goods, $ims_t$, should ideally capture the share of imported goods in *production*. However, total imports of goods is not easily divided into imports of intermediate goods and consumption goods respectively. Therefore, there are no time series of imported intermediate goods available. As a consequence, I choose to measure $ims_t$ as the share of all imports to production (also expressed as percentage deviation from the mean). This should be a reasonable proxy to the extent that the respective shares of imported intermediate goods and imported consumption goods have been fairly stable over time (in current prices).

As point of reference, the Phillips curve is also estimated with the output gap, $y_t$, as a measure of real activity (calculated as $\log(y_t)$ in deviation from a Hodrick-Prescott trend). A common approach when opening up the standard NKPC to foreign trade is to derive an expression under which the real exchange rate, $q_t$, enters the Phillips curve equation along with $y_t$ (see e.g. Svensson (1998)). As another robustness check, therefore, also $q_t$ (the log real exchange rate) is allowed to enter the Phillips curve along with the output gap. Finally, estimations are also performed with $mc_t^r$ defined as just the labor income share ($ls_t$), as in Galí et al. (2001). The main data used in the estimations are depicted in Figure 1.

As a first pass on the data, dynamic-cross correlations between inflation and the different cost measures are depicted in Figure 2. As can be seen, correlations are high both contemporaneously and with leads and lags between inflation and real unit labor costs. Dynamic correlations are overall lower between inflation and the output gap, which a priori suggests that a Phillips curve including real unit labor cost might do a better job in explaining Swedish inflation developments than a standard Phillips curve with the output gap. Dynamic correlations between inflation and the share of imports in production are overall *negative*. In fact, the import share has risen steadily since the beginning of the 1990's while inflation has first been falling and thereafter has remained low (see Figure 1). To a large

[3] In the estimations with GMM, I also experiment with inflation expressed in an annual rate.

[4] UND1X is defined as CPI cleansed from certain components which are not directly determined by demand conditions (household mortgage interest expenditures and the direct effects of changes in indirect taxes and subsidies).

extent, the increase in the import share is owing to the successive depreciation of the Swedish exchange rate and thereby higher import prices. This development is not problematic from a theoretical point of view; theory predicts these price increases to spill over to domestic price inflation. However, the import share in production has also risen in *fixed prices*, probably to a large extent reflecting an increasing share of international trade. Such a structural change is not captured by the model (in terms of the model, it suggests that $\eta$ has increased over time). Estimations of the Phillips curve as in (16) may nevertheless still be justified as real marginal cost is expressed as *the sum of* real unit labor cost and the import share in production. To the extent that variations in the import share fail to explain much of the short term variations in inflation, the estimated elasticity of inflation with respect to this measure of imported inflation should be close to zero. However, to control for the possibility of a time-varying steady state level also in the import share and the labor share, estimations with detrended $ims_t$ and $ls_t$ are performed.

## 3.1 Estimations with GMM

In this section, the Phillips curve-relation in equation (16) is estimated with GMM.

The rationale for using GMM is the following. Using the fact that forward looking agents will form their expectations of future inflation in a rational fashion, it follows that $\pi_{t+1} = E_t \pi_{t+1} + \varepsilon_{t+1}$ where the expectational error, $\varepsilon_{t+1}$, will be uncorrelated with the set of information in period t used to form expectations about inflation one period ahead, $\Omega_t$. Accordingly, the following orthogonality condition must hold

$$cov\left(\Omega_t, \varepsilon_{t+1}\right) = E\left[\Omega_t \varepsilon_{t+1}\right] = 0 \tag{17}$$

By finding variables that are used when agents form their expectations about future inflation, i.e. are part of $\Omega_t$, the orthogonality condition can be written as

$$cov\left(z_t, \varepsilon_{t+1}\right) = E\left[z_t \varepsilon_{t+1}\right] = 0_{1*n(z)} \tag{18}$$

where $z_t$ is a vector of variables which form part of $\Omega_t$. The set of conditions can also be expressed as

$$E\left[f\left(x_t \beta\right) z_t\right] = 0_{1*n(z)} \tag{19}$$

where

$$f\left(x_t \beta\right) = \pi_t - \xi mc_t - \lambda_f \pi_{t+1} - \lambda_b \pi_{t-1} \tag{20}$$

This set of orthogonality conditions subsequently forms the basis for estimating the model by choosing parameters so as to minimize the corresponding sample moment. Valid instruments for $E_t \pi_{t+1}$ are variables dated $t$ or earlier, which on theoretical grounds can be judged to be part of the information set. In statistical terms, the instruments must be uncorrelated with the GMM residuals, which are essentially forecast errors.

However, in practice, the choice of instruments is often rather arbitrary as it amounts to using only a subset of the information variables.[5] In addition, there is a risk of misleading results in case of specification errors in the estimated equation, as pointed out by Rudd and Whelan (2001). Assume that

[5] Paul Söderlind, "Lecture Notes - Econometrics: GMM" (2001).

the *true model* for inflation includes only lags of inflation – as is often the case in empirical inflation equations – and no forward looking component. Yet, (16) is the equation which is being estimated and earlier lags of inflation are chosen as one of the instrument for $\pi_{t+1}$. Rudd and Whelan then shows that, as inflation is highly autocorrelated, this results in biased estimates with positive effects from $E_t\pi_{t+1}$ although the true model in this case is purely backward looking. Furthermore, Lindé (2002) shows by means of Monte Carlo simulations that GMM used on New-Keynesian sticky price models is likely to produce imprecise and biased estimates.

The GMM method will be used in this paper as an interesting comparison with the FIML estimations. The VAR used in the FIML estimations on the other hand implies more specific assumptions about inflation expectations than simple moment conditions, thus raising the risk of misspecification.

In the estimations, a restriction that $\lambda_f + \lambda_b = 1$ is imposed. Under the assumption that the discount factor, $\beta$, is close to one, this restriction implies that the share of backward and forward-looking firms sum to one.[6] The instrument set contains four lags of the variables in the Phillips curve and in some specifications also an exogenous variables with lags ( $y_t^*$, which denotes foreign trade-weighted GDP). The criterion for choosing instruments has been that they should pass the J-test of the overidentifying restrictions.

Results of the estimations using GMM are shown in Table 1. The table contains a number of model variations with regard to the specification of the real variable in the Phillips curve and choice of price index. The figures between parenthesis in the first three columns are the standard errors of the estimates. The number in parenthesis in the last colum is the p-value for the test of the overidentifying restrictions.

As can be seen from row 1 in Table 1 (which is the baseline case with real marginal cost measured as the sum of labor income share and the import share and with inflation measured with the GDP deflator) the estimated parameters of both $ls_t$ and $ims_t$ have the expected sign but are very small. The small point estimates of $\kappa_1$ and $\kappa_2$, are fairly robust across various specifications of the real driving variable and of the price index. The coefficients for the real variable are also in the lower range of estimated parameters compared to other studies.[7] One economic reading of the results would be that, with such low point estimates of the impact from the real variable, real economy developments are more or less unimportant for inflation dynamics. In a model with predetermined expectations this would be a possible conclusion. However, with forward looking expectations, such a strict reading of the impact on inflation is not impossible. What my results do indicate is a high degree of persistence in firms' price setting behaviour. In terms of the deep parameters of the model, the small estimates of $\kappa_1$ and $\kappa_2$ suggest a substantial degree of price stickiness (a high $\theta$). However, it is worth noting that this result is specific to the model set up in this paper. Altig, Christiano, Eichenbaum, Lindé (2004) show that in a model with firm-specific and predetermined capital, inflation may be persistent even though firms reoptimize frequently. The inertia in their model reflects that when firms do change prices they do so by a small amount.

[6] Mavroeidis (2005) shows that with certain properties of the non-modelled variables, the restriction $\lambda_f + \lambda_b = 1$ is necessary for the the model´s parameters to be identified

[7] Galí and Gertler (1999) estimate the parameter for the real marginal cost to be in the range 0.015 - 0.051 for U.S.data, while Galí, Gertler and López-Salido (2001) provide parameter estimates in the range 0.006 - 0.214 for the euro area.

**Table 1. GMM Estimates of Swedish Phillips Curve (quarterly rate of inflation).**

| Model specification | Instruments | Parameters | | | Test |
|---|---|---|---|---|---|
| | | $\kappa_1$ | $\kappa_2$ | $\lambda_f$ | J |
| *Open economy Phillips Curve* | | | | | |
| (1) $mc^r = f(ls, ims)$, | ls, ims, $\pi$ | 0.006 | 0.003 | 1.303 | 7.48 |
| $\pi$ = GDPdefl. | (-1 to -4) | (0.023) | (0.010) | (0.119) | (>0.75) |
| (2) $mc^r = f(ls, ims)$, | ls, ims, $\pi$ | -0.006 | -0.005 | 0.746 | 8.67 |
| $\pi$ = CPI | (-1 to -4) | (0.006) | (0.003) | (0.054) | (>0.75) |
| (3) $mc^r = f(ls, ims)$, | ls, ims, $\pi$ | 0.002 | -0.001 | 0.665 | 8.43 |
| $\pi$ = UND1X | (-1 to -4) | (0.005) | (0.002) | (0.091) | (>0.75) |
| (4) output gap (hp-filtered) | y, q, $\pi$, | -0.037 | 0.006 | 0.919 | 6.81 |
| and $q$, $\pi$ = GDP defl | (-1 to -4) | (0.023) | (0.004) | (0.067) | (>0.50) |
| (5) $mc^r = f(ls, ims)$, | ls, ims, $\pi$ | 0.009 | 0.001 | 1.294 | 7.36 |
| $\pi$ = GDPdefl. (dev. from trend) | (-1 to -4) | (0.023) | (0.010) | (0.113) | (>0.75) |
| (6) $mc^r = f(ls, ims)$ $ls$ and $ims$ | ls, ims, $\pi$ | 0.048 | -0.040 | 1.226 | 9.52 |
| as dev. from trend, $\pi$ = GDPdefl. | (-1 to -4) | (0.131) | (0.034) | (0.117) | (>0.75) |
| (7) $mc^r = f(ls, ims)$ $ls$ and $ims$ | ls, ims, $\pi$ | 0.010 | -0.029 | 1.238 | 7.32 |
| as dev. from trend, $\pi$ = GDPdefl. | (-1 to -4) | (0.127) | (0.034) | (0.115) | (>0.50) |
| (8) $mc^r = f(ls, ims)$, | ls, ims, $\pi$ | 0.026 | -0.005 | 1.770 | 6.52 |
| $\pi$ = GDPdefl., shortened sample | (-1 to -4) | (0.024) | (0.012) | (0.157) | (>0.50) |
| (9) $mc^r = f(ls, ims)$, | ls, ims, $\pi$ | 0.064 | 0.051 | 0.546 | 7.50 |
| $\pi$ = CPI, shortened sample | (-1 to -4) | (0.014) | (0.010) | (0.141) | (>0.50) |
| *Standard Phillips Curve* | | | | | |
| (10) $mc^r = f(ls)$, | ls, $\pi$, y, y* | -0.003 | – | 1.135 | 10.73 |
| $\pi$ = GDP defl. | (-1 to -4) | (0.018) | | (0.117) | (>0.50) |
| (11) output gap (hp-filtered), | ygap, $\pi$, y* | -0.074 | – | 1.405 | 5.92 |
| $\pi$ = GDP defl. | (-1 to -4) | (0.067) | | (0.198) | (>0.75) |
| (12) output gap (hp-filtered), | ygap, $\pi$ | -0.056 | – | 1.213 | 4.09 |
| $\pi$ = GDP defl., shortened sample | (-1 to -4) | (0.057) | | (0.153) | (>0.50) |
| (13) output gap (hp-filtered), | ygap, $\pi$ | -0.140 | | 1.301 | 8.32 |
| $\pi$ = CPI, shortened sample | (-1 to -4) | (0.048) | | (0.158) | (>0.50) |

Note: This table reports GMM estimates of equation (16). The data cover the sample period 1986:1-2004:4.

On statistical grounds, inference about the impact of $\kappa_1$ and $\kappa_2$ is even more uncertain. Standard errors of the estimates are generally large and the estimates are in no case significantly different from zero when I estimate over the full sample period. In other words, statistically it is difficult to pin down a significant relationship between any of the real variables and inflation as suggested by the structural Phillips curve equation. Nevertheless, it is worth noting that using the output gap in the Phillips curve yields point estimates of $\kappa_1$ which are negative while the estimates with real marginal cost in the equation in all cases but one give a positive estimate of $\kappa_1$. This is a similar picture as in Galí and Gertler (1999), where the authors argue that a measure of real marginal cost outperforms the output gap in the estimation of the Phillips curve in the sense that it enters with the expected, positive sign.[8]

In specifications (8) and (9) as well as (12) and (13) in the standard Phillips curve I use a shorter sample, from 1995 and onwards. Following the shift to an inflation targeting regime in 1993, the new inflation target of two per cent inflation became fully effective in 1995. However, it may be noted that theory in itself does not suggest that such regime shifts should introduce a break in the structural relationship between inflation and the real driving variable. With a sample starting in 1995, the estimated impact of the real variables are on average larger than when the sample starts in 1986. And the point estimate of $\kappa_1$ is positive when labor share enters the equation and negative when the output gap does, also over this shorter period. However, the shorter sample only covers 37 observations and standard errors remain large.

In line with Galí and Gertler (1999), I find that expectations about future inflation are more important for explaining inflation than past inflation developments. In fact, a predominant role for forward looking expectations is a robust result across all specifications. However, in some setups the estimate of $\lambda_f$ is even larger than 1. Given the restrictions I impose on the parameters, this implies an implausible negative estimate of $\lambda_b$, i.e. that an increase in inflation one period would act to *lower* inflation the next period. This casts a general doubt on whether the New Keynesian Phillips curve with staggered price setting yields a correct specification of Swedish inflation dynamics. A badly specified model could be an explanation for the obtained estimates. However, this is a problem particularly when inflation is defined using the GDP deflator. Using CPI inflation or UND1X inflation yield more realistic results.

When allowing for time-varying steady state values of the labor share and the import share, the results indicate a substantially larger impact of the labor share (specification (6) in Table 1). However, neither in this case is it possible to reject a hypothesis of parameter values of zero for the real variables. In addition, allowing also for a time-varying steady state value of inflation (specification (7) in Table 1), again reduces the importance for labor share.

A closed economy set up for the Phillips curve (specification (10) to (14) in Table 1) also yield negative estimates of $\kappa_1$ as well as $\lambda_f$ in excess of 1. This indicates, as expected, that external influences on Swedish inflation need to be taken into account to improve the validity of the results.

In Table 2 below, I have used inflation expressed in yearly rates instead. This is incoherent with the theoretical set up (see below), but can often be seen in empirical studies on inflation dynamics. To render comparisons with the results in Table 1 more straightforward, the inflation rate has been scaled down to a quarterly equivalent rate. It can be noted that the estimated parameters of the real variable (real marginal cost or the output gap) are now positive across all specifications. However, the coefficients remain small and standard errors of the estimates large.

[8] I have also experimented with estimating the deep parameters (with $\beta$ calibrated to be close to 1). However, standard errors of the estimates remained large and the implied estimates of $\kappa_1$, $\kappa_2$ and $\lambda_f$ were in the same range as in Table 1.

**Table 2. GMM Estimates of Swedish Phillips Curve (annual rate of inflation).**

| Model specification | Instruments | Parameters | | | Test |
|---|---|---|---|---|---|
| | | $\kappa_1$ | $\kappa_2$ | $\lambda_f$ | J |
| *Open economy Phillips Curve* | | | | | |
| (1) $mc^r = f(ls, ims)$ | ls, ims, $\pi$, y* | 0.004 | 0.001 | 0.354 | 9.94 |
| $\pi$ = GDPdefl. | (-1 to -4) | (0.004) | (0.002) | (0.052) | (>0.50) |
| (2) $mc^r = f(ls, ims)$ | ls, ims, $\pi$, y*, | 0.004 | 0.002 | 0.351 | 8.73 |
| $\pi$ = CPI | (-1 to -4) | (0.003) | (0.001) | (0.036) | (>0.50) |
| (3) $mc^r = f(ls, ims)$ | ls, ims, $\pi$, y* | 0.000 | 0.001 | 0.493 | 11.85 |
| $\pi$ = UND1X | (-1 to -4) | (0.002) | (0.001) | (0.046) | (>0.75) |
| (4) output gap (hp-filtered) | ygap, q, $\pi$, | 0.004 | 0.000 | 0.668 | 9.85 |
| and $q$, $\pi$ = GDP defl. | y*(-1 to -4) | (0.010) | (0.002) | (0.100) | (>0.50) |
| (5) $mc^r = f(ls, ims)$ | ls, ims, $\pi$, y* | 0.002 | 0.001 | 0.350 | 10.04 |
| $\pi$ = GDPdefl. (dev. from trend) | (-1 to -4) | (0.004) | (0.002) | (0.053) | (>0.50) |
| (6) $mc^r = f\left(\sum_{t=0}^{3} ls_{t-i}, \sum_{t=0}^{3} ims_{t-i}\right)$ | ls, ims (-4 to -7) | 0.003 | 0.001 | 0.722 | 10.36 |
| $\pi$ = GDPdefl. | $\pi$, y* (-1 to -4) | (0.005) | (0.001) | (0.096) | (>0.50) |
| *Standard Phillips Curve* | | | | | |
| (7) $mc^r = f(ls)$ | ls, $\pi$, y* | 0.001 | – | 0.399 | 9.63 |
| $\pi$ = GDPdefl. | (-1 to -4) | (0.003) | | (0.044) | (>0.75) |
| (8) output gap (hp-filtered) | ygap, $\pi$,y*, | 0.003 | – | 0.781 | 8.10 |
| $\pi$ = GDPdefl. | (-1 to -4) | (0.011) | | (0.134) | (>0.75) |

Note: This table reports GMM estimates of equation (16). The data cover the sample period 1986:1-2004:4.

As regards the relative importance of lagged and expected inflation, Table 2 shows that specfications with inflation at an annual rate overall point to a higher degree of inflation persistence (i.e. higher value of $\lambda_b$). However, this is likely due to a misspecification of the structural model. As shown in Section 2, the theoretically correct measure of inflation in the New Keynesian Phillips curve is price changes between two quarters (given that quarterly data are used). Adhering to the structural model, while at the same time expressing inflation in annual changes, requires adjusting (16) to allow for four lags of the real driving variable (see Appendix B). When I allow for these adjustments in specification (6), the estimated size of $\lambda_b$ becomes much smaller and in better accordance with the results in Table 1. The conclusion drawn above, that GMM estimations show that expected inflation is more important for inflation dynamics in Sweden than past inflation, thus remains valid.

### 3.2 Estimation with FIML

The VAR(2) model used in the FIML estimations is specified as

$$Y_t = C + \delta_1 D_{923} + \delta_2 D_{931} + \sum_{i=1}^{2} \Upsilon_i Y_{t-i} + \sum_{i=1}^{2} \Psi_i \pi_{t-i} + \sum_{i=1}^{2} \Gamma_i Y_{t-i}^* + \eta_t \tag{21}$$

where $Y_t = \left[ \left( y_t - \bar{y}_t \right) \; ls_t \; ims_t \;\; r_t \; q_t \right]'$. $\left( y_t - \bar{y}_t \right)$ is the (Hodrick-Prescott filtered) output gap of domestic real GDP, $r_t$ is the three month nominal interest rate and $q_t$ is the (log of) real trade-weighted exchange rate. $D_{923}$ is a dummy variable equal to 1 in 1992:3 and 0 otherwise (to control for the extremely high interest rates level during the currency crisis in the autumn of 1992). $D_{931}$ is a dummy variable equal to 1 in 1993:1 and thereafter (intended to capture possible structural shifts in connection with the shift to a new exchange rate regime).[9] $\pi_t$ is the inflation rate measured as the GDP deflator, expressed as quarterly rate of change. $Y_t^*$ is a vector of exogenous variables, namely $Y_t^* = [y_t^* \; \pi_t^* \; r_t^*]'$. $y_t^*$ denotes (the log of) the foreign trade-weighted (TCW) real GDP, $\pi_t^*$ is foreign trade-weighted CPI inflation and $r_t^*$ is the foreign trade-weighted 3-month nominal interest rate.[10]

Two close the system, also a foreign VAR(2) with the variables in the $Y_t^*$ vector is included. I put no restrictions on the VAR equations as the intention is to focus on estimating the structural Phillips curve equation. (In Appendix C, the model is presented in state-space form.) The system of equations comprised of the unrestricted VAR and the Phillips curve relation are solved by using Paul Söderlind´s algorithm for models with rational expectations (see Söderlind 1999).

Once the model is solved, the likelihood function is computed for any set of parameters in the Phillips curve (the coefficients in the VAR are estimated separately and held fixed in the FIML estimations). FIML estimation is valid under the assumption that the innovations in the model are joint normally distributed with mean zero. Finally, a sequential quadratic programming algorithm is used to find the set of parameters that maximize the value of the likelihood-function. The same restriction as in the GMM estimation, i.e. $\lambda_f + \lambda_b = 1$, is imposed.

The results of the FIML estimations are shown in Table 3. The numbers in the first three columns are the point estimates of the parameters with standard errors in paranthesis. The last column shows the result of a likelihood ratio test of the hypothesis of a completely forward-looking model.

The overall picture of the results is the same as in the GMM estimations. In other words, the real driving variable is estimated to be very small, suggesting a large amount of price-stickiness. The estimates of $\kappa_1$ and $\kappa_2$ are broadly within the same range as in Table 1. However, standard errors are generally somewhat smaller, although still large. Measuring inflation as CPI (specification (2)), the impact of labor share on inflation is positive and significantly different from zero. The estimated parameter on the import share is negative in this setup, contrary to what one may expect on theoretical grounds but in line with the factual negative correlation between inflation and the import share over the sample period. The estimates of $\lambda_f$ indicate a predominant role for forward-looking expectations which is in line with the GMM results.

[9] In the autumn of 1992, the fixed exchange rate policy was abandoned and the Swedish krona was allowed to float.

[10] I have also experimented with including (the log of) domestic real GDP in level instead of as deviation from potential, as this is a more common set up in VAR esimations. However, this did not change the results in any significant way.

**Table 3. FIML Estimates of Swedish Phillips Curve (quarterly rate of inflation).**

| Model specification | Parameters | | | Hypothesis test |
|---|---|---|---|---|
| | $\kappa_1$ | $\kappa_2$ | $\lambda_f$ | of $\lambda_f = 1$, $\chi^2(1)$ |
| *Open economy Phillips curve* | | | | |
| (1) $mc^r = f(ls, ims)$, | -0.001 | 0.008 | 1.431 | 0.865 |
| $\pi =$ GDP defl. | (0.001) | (0.001) | (0.230) | |
| (2) $mc^r = f(ls, ims)$, | 0.014 | -0.004 | 0.890 | 0.012 |
| $\pi =$ CPI | (0.006) | (0.002) | (0.091) | |
| (3) $mc^r = f(ls, ims)$, | 0.007 | 0.000 | 0.987 | 0.001 |
| $\pi =$ UND1X | (0.005) | (0.002) | (0.120) | |
| (4) $mc^r = f(ls, ims)$, $\pi =$ GDP defl. | 0.015 | 0.001 | 0.877 | 0.009 |
| two dummies in Phillips curve | (0.008) | (0.003) | (0.092) | |
| (5) $(y_t - \bar{y})$ (hp-filtered) | 0.007 | 0.006 | 0.510 | 0.923 |
| and q, $\pi =$ GDP defl. | (0.008) | (0.004) | (0.084) | |
| (6) $mc^r = f(ls, ims)$, $\pi =$ GDP defl. | 0.018 | -0.003 | 1.531 | 0.109 |
| (deviation from hp-trend) | (0.016) | (0.005) | (0.284) | |
| (7) $mc^r = f(ls, ims)$, $ls$ and $ims$ as | -0.008 | 0.007 | 1.475 | 0.542 |
| dev. from trend, $\pi =$ GDP defl | (0.026) | (0.008) | (0.266) | |
| *Standard Phillips curve* | | | | |
| (8) $mc^r = f(ls)$ | -0.004 | – | 1.450 | 0.097 |
| $\pi =$ GDP defl. | (0.009) | | (0.245) | |
| (9) $(y_t - \bar{y})$ (hp-filtered), | 0.012 | – | 1.233 | 0.458 |
| $\pi =$ GDP defl. | (0.003) | | (0.437) | |

Note: This table reports FIML estimates of the system of equations equation constituted of the Phillips curve equation (equation (16)), the domestic VAR (equation (21)) and the foreign VAR (equation (A 40)). The data cover the sample period 1986:1-2004:4.

In fact, the hypothesis of a completely forward-looking model cannot be rejected under any of the setups in Table 3. This is most likely a consequence of the fact that there are large residuals. As a result, the log likelihood function becomes flat and inference uncertain. With regard to the definition of real variable, FIML estimations can neither verify, with any statistical degree of certainty, that real marginal cost has better explanatory power for Swedish inflation than the output gap.

In specification (4), I allow for a structural shift in the Phillips curve at the time of change of exchange rate regime. I use the same dummies as for the variables in the VAR, i.e. a spike dummy in 1992:3 and a regime shift dummy from 1993:1 and onwards. The estimated equation when introducing these dummies gives point estimates of both $\kappa_1$and $\kappa_2$ with the expected positive signs and in the case of $\kappa_1$ significance at the 10 per cent level. However, the results are not distinctly different from the the result of the estimations without dummies in the Phillips curve. This is not entirely surprising. As can be observed in Figure 3, the sharp reduction in the inflation rate in the beginning of the 1990´s actually coincides with a marked fall in real unit labor costs. This suggests that the fall in inflation at this time may be explained within the New Keynesian framework.

As seen in specification (7), the FIML estimations support the GMM results that detrending $ls_t$ and $ims_t$ to control for time-varying steady state values of these variables does not particularly improve the fit of the estimations.

Finally, as regards the robustness of the VAR, Rudd and Whelan (2002) demonstrated that in the studies of Woodford (2001) and Sbordone (2002) who use small, unrestricted VAR models to solve for inflation expectations when estimating Phillips curves on US data, the results were sensitive to different VAR specifications. That does not appear to be the case in the Swedish data. I have experimented with different lag lengths in the VAR and with some changes as to the included variables, but the results were found to be qualitatively similar in the sense that the impact from the real variable in the Phillips curve is small and not statistically significant.

# 4 Conclusions

The estimations in this study suggest that a New Keynesian Phillips curve with staggered price setting - augmented to a small open economy by allowing for the use of imported goods in production - offers insufficient explanations for the development of Swedish inflation over the period studied. In fact, it has not been possible to pin down a statistically significant relationship between a real variable and inflation during the period studied (1986-2004).

The claim by inter alia Galí and Gertler (1999) that a marginal cost based Phillips curve has a better potential for explaining short term inflation than an output gap based Phillips curve was tested on Swedish data. It was noted that the contemporaneous correlation is fairly strong and positive between real unit labor cost and inflation, and stronger than between the output gap and inflation. This suggested, a priori, that a measure of firms' real marginal cost including real unit labor cost might capture firms' resource utilization and, hence, their inclination to raise prices, better than an output gap based Phillips curves. However, this stronger relationship could not be statistically verified in either GMM estimations or FIML estimations. It was, nevertheless, noted that in the GMM estimations the point estimate of the labor share parameter in most cases had the exptected positive sign while using the output gap resulted in point estimates which where in most cases negative. Even though the standard errors were generally too large for robust inference to be drawn, the sign of the point estimates on the real variables suggests that labor share might be a better representation of the real variable driving inflation than the output gap also in a Swedish Phillips curve.

The lack of a statistically significant impact of real activity on inflation is shared with other studies of the Swedish Phillips curve relationship (e.g. Hallsten 2000). One likely reason for this result is that the time span used in studies of the Swedish economy, commonly from the beginning or middle of the 1980's, is too short. Empirical studies of the Phillips curve in the US economy are commonly based on time series from the 1960 and onwards (in the study by Galí, Gertler and López-Salido on the Euro area, from 1970). This evidently reduces the standard errors and increases the possibility to draw solid conclusions from the data.

Another possibility is that the link between real activity and inflation has indeed been less stable in Sweden. Structural changes in the economy over the last twenty years may have led to breaks in the relationship between real activity and inflation (even though, as noted above, on strictly theoretical grounds it is not evident that structural changes would not be reflected in firms price setting behaviour). As regards the shift to a floating exchange rate in 1992, I tested for a possible structural break in the inflation process by introducing dummies in the regression equation in the FIML estimation and by estimating over a shorter sample with GMM. This improved the fit of the regressions but standard errors in general remained large.

Finally, it is of course also possible that the measure of marginal cost used in this paper, real unit labor cost and the cost of imported inputs, is a poor proxy for firms' true real marginal cost. For instance, Rotemberg and Woodford (1999) discuss a number of reasons why firm's real marginal cost may vary more with resource utilization in the economy than real unit labor cost (such as adjustment cost of labor, the existence of overhead labor and other fixed costs in production). This opens up the possibility that firms' 'true' real marginal cost in practice covaries with inflation to a higher degree than does real unit labor costs. To the extent that such costs are more important in the Swedish economy than e.g. the American, it would offer an additional explanation for the poorer fit in the estimations on Swedish data than on US data. In order to improve the empirical fit of a Swedish Phillips curve, it would then seem important to develop alternative measures of real marginal cost in order to increase its realism.

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# Appendix

## A. The New Keynesian Phillips curve with staggered price setting adopted to a small open economy

### The optimal price with nominal price rigidity

With restrictions on the possibility of firms to change prices, the optimization problem can be written

$$\max_{P_{it}} E_t \sum_{j=0}^{\infty} (\beta\theta)^j V_{t,t+j} \left[ \frac{P_{it}}{P_{t+j}} Y_{it+j} - \frac{MC_{it+j}}{P_{t+j}} Y_{it+j} \right] \tag{A 1}$$

subject to demand

$$Y_{it+j} = \left( \frac{P_{it}}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j} \tag{A 2}$$

The first order condition to the maximization problem is

$$E_t \sum_{j=0}^{\infty} (\beta\theta)^j V_{t,t+j} Y_{t+j} \left[ \frac{1-\varepsilon}{P_{t+j}} \left( \frac{P_{it}}{P_{t+j}} \right)^{-\varepsilon} + \frac{\varepsilon}{P_{t+j}} \frac{MC_{it+j}}{P_{t+j}} \left( \frac{P_{it}}{P_{t+j}} \right)^{-(1+\varepsilon)} \right] = 0 \tag{A 3}$$

which after some algebra may be written as

$$P_{it} E_t \sum_{j=0}^{\infty} (\beta\theta)^j V_{t,t+j} Y_{t+j} P_{t+j}^{\varepsilon-1} = \frac{\varepsilon}{\varepsilon-1} E_t \sum_{j=0}^{\infty} (\beta\theta)^{jj} V_{t,t+j} Y_{t+j} MC_{t+j} P_{t+j}^{\varepsilon-1} \tag{A 4}$$

Log-linearizing equation (A 4) yields

$$p_{it} = (1-\beta\theta)\, E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, mc_{it+j} \tag{A 5}$$

where small letters denote log-deviation from steady state. Quasi-differencing (A 5) gives

$$\begin{aligned} p_{it} &= (1-\beta\theta)\,(mc_{it}) + (1-\beta\theta)\, E_t \sum_{j=0}^{\infty} (\beta\theta)^j \, mc_{it+j} \\ &= (1-\beta\theta)\, mc_{it} + \beta\theta E_t p_{it+1} \end{aligned} \tag{A 6}$$

### The marginal cost function

The cost minimization problem of the firm is (with capital held fixed)

$$\min W_t L_{it} + P_t^m IM_{it} \tag{A 7}$$

subject to

$$L_{it}^{\alpha} IM_{it}^{\eta(1-\alpha)} = Y_{it} \tag{A 8}$$

(For simplicity, $A$ and $K$ in the production function are set equal to 1.) First order conditions of the Lagrangian are

$$W_t = \lambda\alpha \frac{Y_{it}}{L_{it}} \tag{A 9}$$

$$P_t^m = \lambda\eta\,(1-\alpha)\, \frac{Y_{it}}{IM_{it}} \tag{A 10}$$

$$L_{it}^{\alpha} IM_{it}^{\eta(1-\alpha)} = Y_{it} \tag{A 11}$$

Solving this for the factor demand equations gives

$$L_{it} = \left[\frac{\alpha}{\eta(1-\alpha)}\right]^{\frac{\eta(1-\alpha)}{\alpha+\eta(1-\alpha)}} W_t^{\frac{-\eta(1-\alpha)}{\alpha+\eta(1-\alpha)}} (P_t^m)^{\frac{\eta(1-\alpha)}{\alpha+\eta(1-\alpha)}} Y_{it}^{\frac{1}{\alpha+\eta(1-\alpha)}} \tag{A 12}$$

$$IM_{it} = \left[\frac{\alpha}{\eta(1-\alpha)}\right]^{\frac{-\alpha}{\alpha+\eta(1-\alpha)}} W_t^{\frac{\alpha}{\alpha+\eta(1-\alpha)}} (P_t^m)^{\frac{-\alpha}{\alpha+\eta(1-\alpha)}} Y_{it}^{\frac{1}{\alpha+\eta(1-\alpha)}} \tag{A 13}$$

The cost function, given the above cost minimizing choices of production factors, is

$$C_{it} = \left[\left(\frac{\alpha}{\eta(1-\alpha)}\right)^{\frac{\eta(1-\alpha)}{\alpha+\eta(1-\alpha)}} + \left(\frac{\alpha}{\eta(1-\alpha)}\right)^{\frac{-\alpha}{\alpha+\eta(1-\alpha)}}\right] W_t^{\frac{\alpha}{\alpha+\eta(1-\alpha)}} (P_t^m)^{\frac{\eta(1-\alpha)}{\alpha+\eta(1-\alpha)}} Y_{it}^{\frac{1}{\alpha+\eta(1-\alpha)}} \tag{A 14}$$

and the marginal cost function with respect to $Y_{it}$ is given by

$$MC_{it} = \gamma W_{it}^{\frac{\alpha}{\alpha+\eta(1-\alpha)}} (P_t^m)^{\frac{\eta(1-\alpha)}{\alpha+\eta(1-\alpha)}} Y_{it}^{\frac{1}{\alpha+\eta(1-\alpha)}-1} \tag{A 15}$$

where $\gamma = \left[\left(\frac{\alpha}{\eta(1-\alpha)}\right)^{\frac{\eta(1-\alpha)}{\alpha+\eta(1-\alpha)}} + \left(\frac{\alpha}{\eta(1-\alpha)}\right)^{\frac{-\alpha}{\alpha+\eta(1-\alpha)}}\right] \frac{1}{\alpha+\eta(1-\alpha)}$.

Finally, substituting for $Y_{it} = L_{it}^{\alpha} IM_{it}^{\eta(1-\alpha)}$ gives

$$MC_{it} = \gamma \left(\frac{W_t L_{it}}{Y_{it}}\right)^{\frac{\alpha}{\alpha+\eta(1-\alpha)}} \left(\frac{P^m IM_{it}}{Y_{it}}\right)^{\frac{\eta(1-\alpha)}{\alpha+\eta(1-\alpha)}} \tag{A 16}$$

In log-linearized terms, the marginal cost function in (A 16) is approximately equal to

$$mc_{it} = \frac{1}{\alpha+\eta(1-\alpha)} \left[\alpha(w_t + l_{it} - y_{it}) + \eta(1-\alpha)(p_t^m + im_{it} - y_{it})\right] \tag{A 17}$$

where small letters denote log-deviation from steady state. The corresponding expression in real terms is

$$MC_{it}^r = \gamma \left(\frac{W_t L_{it}}{P_t Y_{it}}\right)^{\frac{\alpha}{\alpha+\eta(1-\alpha)}} \left(\frac{P^m IM_{it}}{P_t Y_{it}}\right)^{\frac{\eta(1-\alpha)}{\alpha+\eta(1-\alpha)}} \tag{A 18}$$

And in log-linearized terms expressed as deviation from steady state (A 18) will be approximately equal to

$$mc_t^r = \frac{1}{\alpha+\eta(1-\alpha)} \left[\alpha(w_t + l_t - p_t - y_t) + \eta(1-\alpha)(p_t^m + im_t - p_t - y_t)\right] \tag{A 19}$$

**Aggregate price developments**

All firms which set an optimal price do so in a manner consistent with (22), i.e.

$$p_t^f = (1-\beta\theta)\left[mc_t^r + p_t\right] + \beta\theta E_t p_{t+1}^f \tag{A 20}$$

The rule of thumb for firms which are not allowed to reoptimize is (in log-linearized form)

$$p_t^b = p_{t-1} + \pi_{t-1} \tag{A 21}$$

The aggregate price level evolves according to (in log-linearized form)

$$p_t = (1-\theta) p_t^* + \theta p_{t-1} \tag{A 22}$$

where $p_t^*$ is an index of newly set prices according to

$$p_t^* = (1-\omega)\, p_t^f + \omega p_t^b \tag{A 23}$$

**The hybrid Phillips curve**

By combining (A 21) and (A 22), the difference between the rule of thumb price, $p_t^b$, and the current price level, $p_t$, can be expressed as

$$p_t^b - p_t = -\pi_t + \frac{1}{(1-\theta)}\pi_{t-1} \tag{A 24}$$

Solving (A 22) for $p_t^*$, inserting this expression in (A 23) and subtracting $p_t$ from both sides yields

$$\frac{\theta}{(1-\theta)}\pi_t = (1-\omega)\left(p_t^f - p_t\right) + \omega\left(p_t^b - p_t\right) \tag{A 25}$$

From (A 20) it follows that

$$p_t^f - p_t = (1-\beta\theta)\, mc_t^r - \beta\theta p_t + \beta\theta E_t p_{t+1}^f \tag{A 26}$$

Solving (A 25) for $p_t^f$, leading the equation one period, expressing it in expectational terms and using the expression for $p_t^b - p_t$ in (A 24) yields

$$E_t p_{t+1}^f = \frac{\theta + \omega(1-\theta)}{(1-\theta)(1-\omega)} E_t\pi_{t+1} - \frac{\omega}{(1-\theta)(1-\omega)}\pi_t + E_t p_{t+1} \tag{A 27}$$

Substituting for the expression for $E_t p_{t+1}^f$ in (A 27) in (A 26) yields

$$p_t^f - p_t = (1-\beta\theta)\, mc_t^r + \left[\beta\theta + \beta\theta\left(\frac{\theta+\omega(1-\theta)}{(1-\theta)(1-\omega)}\right)\right] E_t\pi_{t+1} - \frac{\beta\theta\omega}{(1-\theta)(1-\omega)}\pi_t \tag{A 28}$$

Substituting for (A 28) and (A 24) in (A 25) finally yields, after some algebra, the hybrid Phillips curve

$$\pi_t = \frac{(1-\beta\theta)(1-\theta)(1-\omega)}{\theta+\omega[1-\theta(1-\beta)]} mc_t^r + \frac{\beta\theta}{\theta+\omega[1-\theta(1-\beta)]} E_t\pi_{t+1} + \frac{\omega}{\theta+\omega[1-\theta(1-\beta)]}\pi_{t-1} \tag{A 29}$$

## B. The New Keynesian Phillips Curve with inflation expressed as annual rate

In the New Keynesian Phillips curve, $\pi_t$ measures price changes between period t and t-1. Accordingly, if a period t is taken to respresent one quarter, $\pi_t$ corresponds to the quarterly rate of inflation. If, instead, inflation is measured as the yearly change of a price index (observed at a quarterly frequency), the model can be adopted as shown below. The NKPC Phillips curve will then include lags of the variable measuring real activity.

Let

$$\bar{\pi}_t = \pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3} \tag{A 30}$$

be the annual inflation rate in period t. For any period, it holds that (using equation (16))

$$\pi_{t-j} = \lambda_f E_{t-j}\pi_{t+1-j} + \lambda_b\pi_{t-1-j} + \kappa_1 ls_{t-j} + k_2 ims_{t-j} + \varepsilon_{t-j} \tag{A 31}$$

Substituting for (A 31) in (A 30) gives

$$\begin{aligned}\bar{\pi}_t &= \pi_t + \pi_{t-1} + \pi_{t+2} + \pi_{t-3} \\ &= \lambda_f E_t \pi_{t+1} + \lambda_b \pi_{t-1} + \kappa_1 ls_t + k_2 ims_t + \varepsilon_t \\ &\quad + \lambda_f E_{t-1} \pi_t + \lambda_b \pi_{t-2} + \kappa_1 ls_{t-1} + k_2 ims_{t-1} + \varepsilon_{t-1} \\ &\quad + \lambda_f E_{t-2} \pi_{t-1} + \lambda_b \pi_{t-3} + \kappa_1 ls_{t-2} + k_2 ims_{t-2} + \varepsilon_{t-2} \\ &\quad + \lambda_f E_{t-3} \pi_{t-2} + \lambda_b \pi_{t-3} + \kappa_1 ls_{t-3} + k_2 ims_{t-3} + \varepsilon_{t-3}\end{aligned} \tag{A 32}$$

Rearranging (22) yields

$$\begin{aligned}\bar{\pi}_t &= \lambda_f \left(E_t \pi_{t+1} + E_{t-1} \pi_t + E_{t-2} \pi_{t-1} + E_{t-3} \pi_{t-2}\right) + \lambda_b \bar{\pi}_{t-1} \\ &\quad + \kappa_1 \left(ls_t + ls_{t-1} + ls_{t-2} + ls_{t-3}\right) \\ &\quad + \kappa_2 \left(ims_t + ims_{t-1} + ims_{t-2} + ims_{t-3}\right) + \left(\varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2} + \varepsilon_{t-3}\right)\end{aligned} \tag{A 33}$$

For $j > 0$, next periods inflation can be decomposed as a sum of expected inflation and an error term.

$$\pi_{t-j} = E_{t-1-j} \pi_{t-j} + \epsilon_{t-j} \tag{A 34}$$

When solving (A 34) for $E_{t-1-j}\pi_{t-j}$ and using this in (22) one gets

$$\begin{aligned}\bar{\pi}_t &= \lambda_f \left(E_t \pi_{t+1} + \pi_t - \epsilon_t + \pi_{t-1} - \epsilon_{t-1} + \pi_{t-2} - \epsilon_{t-2}\right) + \lambda_b \bar{\pi}_{t-1} \\ &\quad + \kappa_1 \left(ls_t + ls_{t-1} + ls_{t-2} + ls_{t-3}\right) \\ &\quad + \kappa_2 \left(ims_t + ims_{t-1} + ims_{t-2} + ims_{t-3}\right) + \left(\varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2} + \varepsilon_{t-3}\right)\end{aligned} \tag{A 35}$$

Leading (A 30) one period and expressing it in expectational form yields

$$E_t \bar{\pi}_{t+1} = E_t \pi_{t+1} + \pi_t + \pi_{t-1} + \pi_{t-2} \tag{A 36}$$

Finally, collecting the error terms in (22) and using (A 36) leads to the following expression

$$\begin{aligned}\bar{\pi}_t &= \lambda_f E_t \bar{\pi}_{t+1} + \lambda_b \bar{\pi}_{t-1} + \kappa_1 \left(ls_t + ls_{t-1} + ls_{t-2} + ls_{t-3}\right) \\ &\quad + \kappa_2 \left(ims_t + ims_{t-1} + ims_{t-2} + ims_{t-3}\right) \\ &\quad - \lambda_f \left(\epsilon_t + \epsilon_{t-1} + \epsilon_{t-2}\right) + \left(\varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2} + \varepsilon_{t-3}\right) \\ &= \lambda_f E_t \bar{\pi}_{t+1} + \lambda_b \bar{\pi}_{t-1} + \kappa_1 \left(ls_t + ls_{t-1} + ls_{t-2} + ls_{t-3}\right) \\ &\quad + \kappa_2 \left(ims_t + ims_{t-1} + ims_{t-2} + ims_{t-3}\right) + \left(\eta_t + \eta_{t-1} + \eta_{t-2} + \eta_{t-3}\right)\end{aligned} \tag{A 37}$$

where the error term, $\eta$, is a sum of the disturbance term in the Phillips curve equation and the expectational error of inflation (in period t and in previous periods).

## C. The full model in state-space form

The system of equations can be expressed in the following state-space form.

(1) The Phillips curve equation

$$\pi_t = \kappa y_t + +\lambda_f E_t \pi_{t+1} + \lambda_b \pi_{t-1} + \varepsilon_t \tag{A 38}$$

(2) An unrestricted VAR for $Y_t = \left[ \left( y_t - \bar{y}_t \right) \;\; ls_t \;\; ims_t \;\; r_t \;\; q_t \right]'$

$$Y_t = C + \delta_1 D_{923} + \delta_2 D_{931} + \sum_{i=1}^{2} \Upsilon_i Y_{t-i} + \sum_{i=1}^{2} \Psi_i \pi_{t-i} + \sum_{i=1}^{2} \Gamma_i Y^*_{t-i} + \eta_t \tag{A 39}$$

(3) A foreign VAR where $Y^*_t = [y^*_t \;\; \pi^*_t \;\; r^*_t]'$

$$Y^*_t = C + \delta_1 \Gamma_t + \sum_{i=1}^{2} \Lambda_i Y^*_{t-i} + \eta^*_t \tag{A 40}$$

and $\Gamma_t$ is a linear time trend.

Let $X_{t10*1} = [Y_t \; Y_{t-1}]'$ and $X^*_{t6*1} = \left[ Y^*_t \; Y^*_{t-1} \right]'$ and let $\widetilde{\eta}_{t9*1}$ be a vector collecting all disturbances i.e $\widetilde{\eta}_t = [\eta_{t5*1} \;\; \eta^*_{t3*1} \;\; \varepsilon_t]'$, then the model can be written in state-space form as

$$A^0_{28*28} \begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix}_{28*1} = \widetilde{A}_{28*28} \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix}_{28*1} + \begin{bmatrix} \widetilde{\epsilon}_{t+1} \\ 0 \end{bmatrix}_{28*1} \tag{A 41}$$

where $\widetilde{\epsilon}_{t+1} = \begin{bmatrix} \widetilde{\eta}_{t+19*1} & 0_{18*1} \end{bmatrix}'$.

The vector of predetermined variables is defined as $x_{1t} = \begin{bmatrix} \widetilde{\eta}_{t9*1} & X_{t-110*1} & X^*_{t-16*1} & \pi_{t-1} & \pi_{t-2} \end{bmatrix}'_{27*1}$ and the vector of forward looking variables as $x_{2t} = [\pi_t]$.

## D. Data appendix

### Data description

Real unit labor cost is the (seasonally adjusted) wage sum of the total economy, including social charges, divided by GDP in current prices. The time series were obtained from Statistics Sweden.

The import share is the (seasonally adjusted) total import of goods in current prices divided by GDP in current prices. Data were obtained from Statistics Sweden.

The interest rate is the three Treasury bill rate (source: the Riksbank).

The exchange is the nominal exchange rate (TCW-weighted) deflated by weighted relative consumer price indexes (source: the Riksbank).

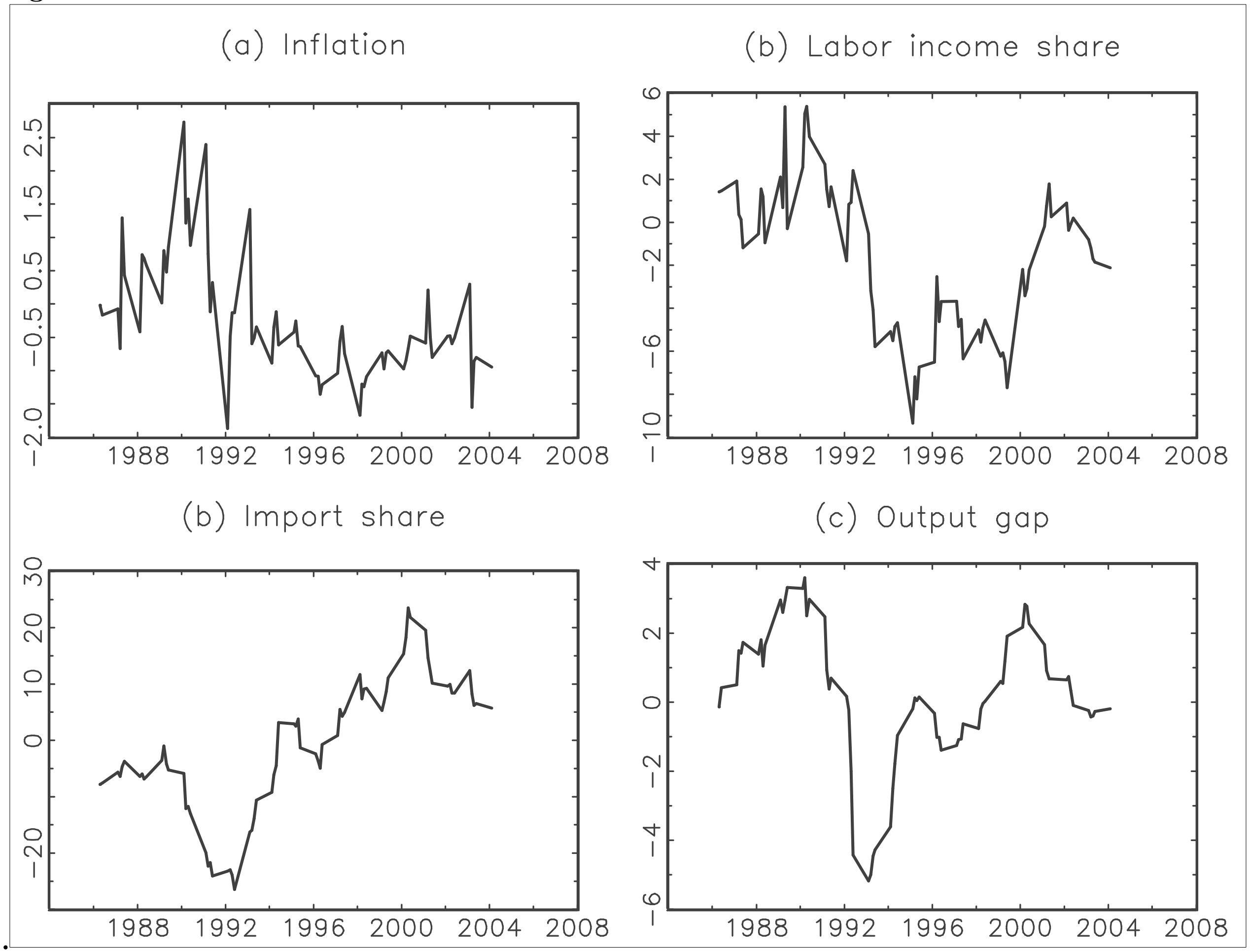
The output gap is calculated as the detrended GDP. The trend is calculated with a Hodrick-Prescott filter. Source: the Riksbank.

Inflation is the CPI price index and UND1X (CPI excluding net of household mortgage interest expenditure and the direct effect of changes in indirect taxes and subsidies). Source: Statistics Sweden.

Foreign output is foreign real GDP weighted with TCW weights (seasonally adjusted). Source: the Riksbank.

The foreign interest rate is the 3-month nominal interest rate and foreign inflation is CPI inflation, both weighted with TCW weights. Source: the Riksbank.

**Figure 1. Plot of data.**

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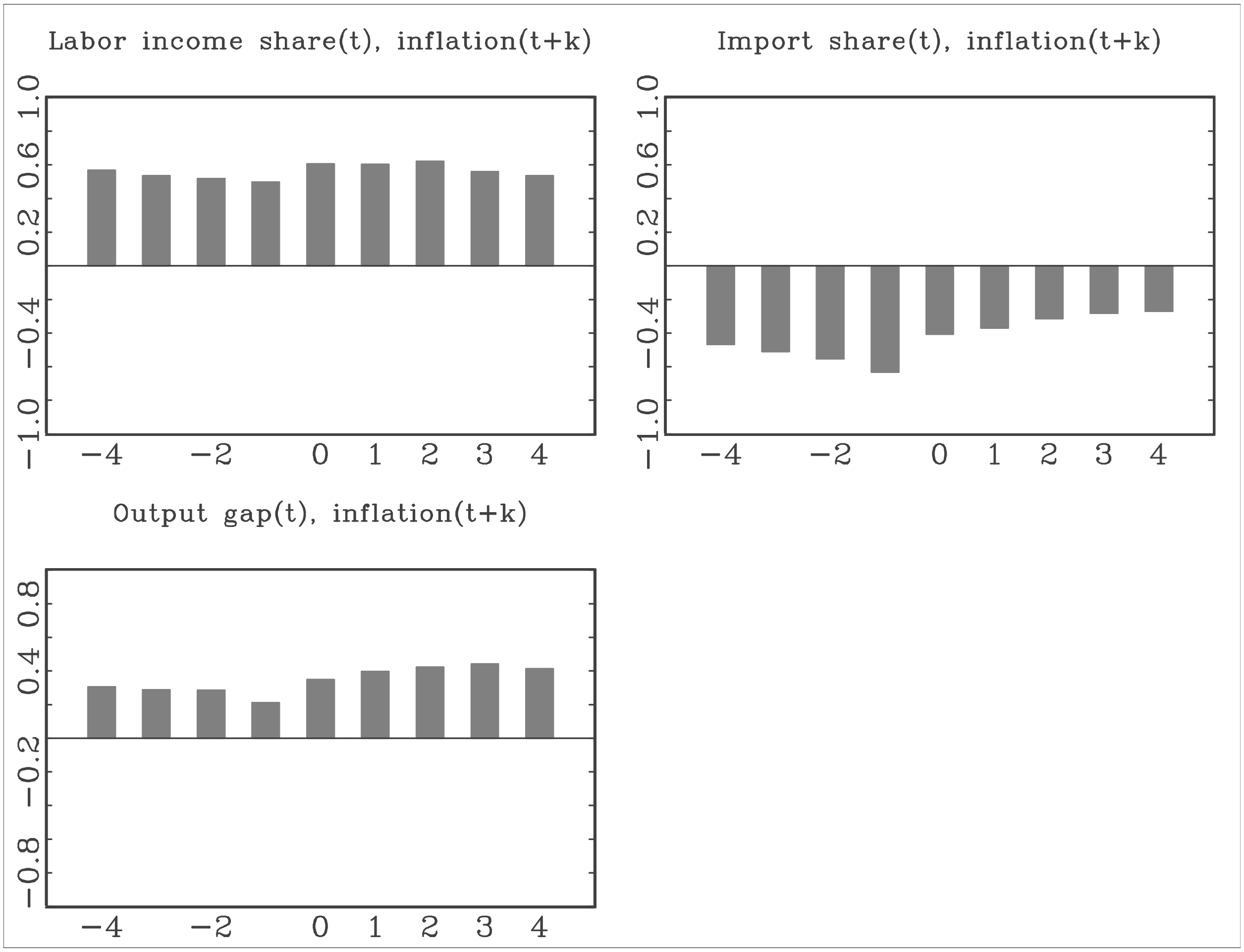
Note: Inflation, the labor income share and the import share are plotted as deviations from mean.The output gap is calculated using a Hodrick-Prescott filter.

**Figure 2. Dynamic cross correlations.**




Note: Correlations are calculated on the full sample. i.e. 1986-2004.

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