Monetary Policy Analysis in Backward-Looking Models

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Abstract

In this paper, I investigate quantitatively how sensitive a typical backward-looking model used in monetary policy analysis is to the Lucas critique. To do this, I use an equilibrium business cycle model with a Taylor-type rule rule for nominal money growth. The backward-looking model displays considerable parameter instability, both from a statistical and economic point of view, when the parameters in the estimated monetary policy rule change. The findings suggest that the robustness of the conclusions in the literature on the relative merits of alternative monetary policy rules should be checked in an equilibrium framework.

Keywords: Lucas critique; real business cycle model; Taylor rules; aggregate supply; aggregate demand.

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1 Introduction

Recently, the empirical relevance of the Lucas (1976) critique has received increased attention. A possible explanation for this is the extensive use of backward-looking models in monetary policy analysis; cf. Ball (1997), Svensson (1997), Rudebusch and Svensson (1999) and Taylor (1999). In this class of models, where the structure of the model economy is assumed to be unaffected by changes in economic policy, a considerable amount of effort has been devoted to examining the relative merits of alternative proposed monetary policy rules. Now, if the Lucas critique is valid and quantitatively important, this type of policy experiments may produce misleading results.[1]

Estrella and Fuhrer (1999) argue that the Lucas critique is an empirically testable hypothesis. They provide evidence that when there is a change in monetary policy regime, some forward-looking models may be less stable than their better fitting backward-looking counterparts, which they argue is an observation inconsistent with the Lucas critique. Fuhrer (1997) also maintains that backward-looking behavior seems to be a better approximation of reality than forward-looking behavior. In addition, most - if not all - of the many papers which have used the concept of super exogeneity presented in Engle, Hendry and Richard (1983) to examine the Lucas critique empirically have found no evidence in favor of the proposition; see the survey by Ericsson and Irons (1995). But in a recent study, Lindé (1999) finds that the power of the super exogeneity test may be very low in small samples. Thus, it is still an open issue whether the Lucas critique is empirically important or not.[2]

Because the empirical relevance of the Lucas critique is still is an open question and hard to test on data, I think it is useful, as a first step, to investigate how important the

[1] Lucas's (1996) argument was that shifts in economic policy change how policy affects the economy since agents in the economy are forward-looking and adapt their expectations and behavior to the new policy stance. For this reason, Lucas concluded that reduced-form economic and econometric models cannot provide useful information about the actual consequences of alternative policies.

[2] Favero and Hendry (1992) have also studied the small samples properties of the super exogeneity test, using a non equilibrium model as data generating process. In contrast to the findings by Lindé (1999), their results suggest that the Lucas critique lacks force in practice and that the super exogeneity test has satisfactory power in small samples. The most plausible reasons why the results differ is that Favero and Hendry's analysis is restricted to a subset of the parameters in the monetary policy rule that is considered by Lindé, and the use of different data generating processes. More specifically, let the monetary policy Taylor-type interest rate rule be $R_t = R^* + \lambda_\pi (\pi_t - \pi^*) + \lambda_Y (\ln Y_t - \ln Y^*) + \rho_R R_{t-1} + \varepsilon_t$ where $\varepsilon \sim i.i.d.\ N\left(0, \sigma^2_{e^R}\right)$. Favero and Hendry consider shifts in R^* and σ^2_e one at a time, but not shifts in λ_π, λ_Y or ρ_R.

Lucas critique seems to be for these backward-looking models. More specifically, it is of interest to examine, first, if changes in the monetary policy rule lead to changes in the reduced form parameters that are economically important for policy analysis, and second, if the observed changes in the reduced form parameters are significant in a statistical sense.

In this paper, I examine these two questions in greater detail. My approach is to set up a modified version of Cooley and Hansen's (1995) real business cycle model with money.[3] The modification is that the model here includes government expenditures and a Taylor-type policy rule (see Taylor, 1993) for nominal money growth similar to the rule analyzed by McCallum (1984, 1988). It is shown in the paper that the rule for nominal money growth used in the model can be rewritten as a standard Taylor-type rule in the nominal interest rate. The policy rule for nominal money growth is then estimated using U.S. data for the recent periods in office of Federal Reserve's chairmen Arthur Burns, Paul Volcker, and Alan Greenspan following Judd and Rudebusch (1998).[4]

By calibrating the equilibrium model with the different estimated monetary policy regimes, I study the properties of the reduced-form parameters in the Rudebusch and Svensson (1999) model by means of simple Monte Carlo simulations. I also study the model implications for policy analysis of the changes in the model parameters. With policy analysis, I here mean the type of experiments that are often considered in the monetary policy literature, i.e. the long run effects (impulse responses and volatilities of inflation and output) of changes in monetary policy rules. Recent work by Leeper and Zha (1999) suggest that for more modest policy interventions - "within" a policy regime - the Lucas critique might be safely ignored.

The results in the paper are as follows. Firstly, it is shown that the reduced form parameters of the Rudebusch and Svensson model change in a statistically significant way

[3] The real business cycle (or equilibrium business cycle) literature initiated by Lucas (1975, 1977), Kydland and Prescott (1982) and others use models for policy analysis that are supposedly immune to the Lucas critique because they are equilibrium models with forward-looking behavior. However, later work has questioned the view that RBC-models with a representative agent are immune against the Lucas critique, see e.g. Geweke (1985) and Altissimo, Siviero and Terlizzese (2000).

[4] Judd and Rudebusch (1998) start out by noting that there is instability in the Fed reaction function. They then find support for the hypothesis that the monetary policy rule has varied systematically with the different periods in office of Fed chairmen Burns, Volcker, and Greenspan. As in their analysis, the period with chairman Miller is omitted here because of his very short tenure.

when the monetary policy rule changes. Secondly, the changes in the parameters are not only important according to a statistical criteria, they are also very important from an economic point of view. For instance, in the aggregate supply curve, the coefficient for output varies between -0.20 and 0.50 in the Rudebusch and Svensson model.[5] The changes in individual parameters are also such that the quantitative implications of the backward-looking model as a whole are largely affected when there is a regime shift. Thus, the Lucas critique is highly relevant when this type of model is used for policy analysis.

The structure of the paper is as follows. In the next section, I introduce the monetary equilibrium model, and indicate ways of computing the equilibrium. Estimation and calibration issues are addressed in Section 3. In Section 4, I present the Rudebusch and Svensson (1999) backward-looking model. Next, in Section 5, results of the Monte Carlo simulations regarding the sensitivity of the backward-looking models to the Lucas critique are reported. Section 6 concludes.

2 The equilibrium model

In this section, I describe and solve a slightly modified version of Cooley and Hansen's (1989, 1995) monetary equilibrium business cycle model. The model is a standard real business cycle model with some additional features. A stochastic nominal money supply interacts with a cash-in-advance technology and one-period nominal wage contracts, which creates short run real effects of nominal money supply shocks. As in Cooley and Hansen (1995), one period is one quarter.[6]

The difference between the model in this paper and the one in Cooley and Hansen (1995) is that the central bank is here assumed to use a policy rule when it decides on the nominal money supply growth in each period similar to that suggested by McCallum (1984, 1988). More specifically, the growth rate in nominal money supply in period

[5] The findings of this paper and Lindé (1999) also have some general implications for the empirical testing of the relevance of backward- versus forward-looking models. First, only the true forward-looking model will have parameters invariant to the monetary regime. With this in mind, the results in Estrella and Fuhrer (1999), suggesting that the Lucas critique is not important in practice (or more relevant for the forward than the backward-looking model) are likely due to model misspecification and/or that the stability tests have weak power in small samples.

[6] I would like to emphasize that the qualitative aspects of the results in the paper are not at all dependent on whether I calibrate the model to match quarterly or yearly data.

t is assumed to follow a Taylor-type policy rule and depends on the output gap, the difference between actual and targeted inflation rate (hereafter named inflation gap), an uncontrollable shock, and the growth rate in nominal money in period $t-1$. This specification is intended to capture the real world phenomenon that central banks use money supply to affect inflation and output gaps, although they act gradually and do not have perfect control of the process. It is shown that this monetary policy rule for nominal money growth can be rewritten as a Taylor rule for the nominal interest rate.

In the model I abstract from population and technological growth and represent all variables in per capita terms.

Finally, a notational comment; in the following, capital letters denote economy wide averages which the agent takes as given and small letters individual specific values which the agent internalizes.

2.1 An equilibrium monetary business cycle model

Infinitely many identical infinitely lived agents maximize expected utility with preferences summarized by

$$\begin{aligned} & \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{1t}, c_{2t}, h_t\right), \qquad (1) \\ u\left(c_{1t}, c_{2t}, h_t\right) \equiv & \ \alpha \ln(c_{1t}) + (1-\alpha) \ln\left(c_{2t}\right) - \gamma h_t \end{aligned}$$

where c_{1t} is consumption of the "cash good" in period t, c_{2t} is consumption of the "credit good," and h_t is the share of available time spent in employment which enters linearly in (1) because of the "indivisible labor" assumption (see Hansen, 1985). In (1), β is the subjective discount factor, γ the disutility the agent gets from working, while α reflects the trade-off between consumption of the cash and credit goods.

The flow budget constraint facing the agent is

$$c_{1t} + c_{2t} + i_t + \frac{m_{t+1}}{P_t} + \frac{b_{t+1}}{P_t} = \left(\frac{W_t^c}{P_t}\right) h_t + R_t^K k_t + \frac{m_t}{P_t} + (1 + R_{t-1}) \frac{b_t}{P_t} + \frac{TR_t}{P_t} \qquad (2)$$

where i_t denotes the agent's investment, m_{t+1} and b_{t+1} the agent's holdings of nominal money and government bonds at the end of period t, P_t the aggregate price level, W_t^c the contracted nominal wage, R_t^K the gross real return on the capital stock k_t, R_{t-1} the nominal interest rate on government bonds between periods $t-1$ and t, and TR_t nominal lump-sum transfers (or taxes if negative) from the government.

The agent has the following cash-in-advance constraint for the cash-good c_{1t},

$$P_t c_{1t} = m_t + (1 + R_{t-1})\, b_t + TR_t - b_{t+1} \tag{3}$$

which always holds with equality since the nominal interest rate will always be positive in this model.

The government's budget constraint is

$$P_t G_t + TR_t = M_{t+1} - M_t + B_{t+1} - (1 + R_{t-1}) B_t \tag{4}$$

where G is exogenous public consumption expenditures, and M and B aggregate nominal money supply and government bonds. As in Cooley and Hansen (1995), I will assume that $B_t = 0$ for $t \geq 0$ and only use it to compute the nominal interest rate in the economy. It can be shown that the nominal interest rate in equilibrium is given by

$$R_t = \frac{\alpha}{1-\alpha} \frac{C_{2t}}{C_{1t}} - 1 \tag{5}$$

where C_{1t} and C_{2t} are aggregate consumption of the cash and credit goods, respectively.

Government consumption, G, in (4) is assumed to be generated by the following stationary AR(1)-process,

$$\ln G_{t+1} = \left(1 - \rho^{\ln G}\right) \ln \bar{G} + \rho^{\ln G} \ln G_t + \varepsilon_{t+1}^{\ln G},\ 0 < \rho^{\ln G} < 1,\ \varepsilon^{\ln G} \sim i.i.d.\ N\left(0, \sigma_{\ln G}^2\right). \tag{6}$$

Aggregate nominal money supply is assumed to evolve according to

$$M_{t+1} = e^{\mu_t} M_t \tag{7}$$

where the growth rate in nominal money supply in period t, defined as $\Delta \ln M_{t+1}$ and denoted μ_t, is assumed to be determined by

$$\begin{aligned} \mu_t &= \eta \mu_{t-1} - \lambda_\pi \left(\pi_t - \pi^*\right) - \lambda_Y \left(\ln Y_t - \ln Y^*\right) + \xi_t,\ 0 < \eta < 1, && (8) \\ \xi &\sim i.i.d.\ \textit{Log Normal},\ \mathrm{E}[\xi] = (1-\eta)\,\bar{\mu},\ \mathrm{Var}(\xi) = \sigma_\xi^2 \end{aligned}$$

where π_t is defined as $\ln P_t - \ln P_{t-1}$, and λ_π and λ_Y measure how the central bank reacts to deviations in the inflation $(\pi_t - \pi^*)$ and the output gap $(\ln Y_t - \ln Y^*)$, respectively.[7] The implicit assumption underlying the specification in (8) is that the central bank tries to stabilize inflation and/or output, and one might think of (8) as an implementable monetary policy rule for a central bank which has been attached a conventional quadratic loss function in the inflation and output gaps. For simplicity, we will also set π^* and $\ln Y^*$ in (8) equal to steady state nominal money supply growth $(\bar{\mu})$ and log of output $(\ln \bar{Y})$, respectively. The error term, ξ, as can be thought of as policy shocks from the perspective of the private sector. By introducing the persistence component $\eta\mu_{t-1}$, it is also assumed that the central bank reacts gradually to shocks which hit the economy.

The policy rule in (8) is not optimal. One important reason for choosing it nevertheless, is that is possible to derive a standard Taylor-type rule (see Taylor, 1993 and 1999) for the nominal interest rate within the equilibrium model given the functional form of (8). Log-linearizing (5), (3) and (19), and substituting these equations into (8), it is possible to derive

$$R_t = -\frac{\lambda_\pi \pi^* + \lambda_Y \ln Y^*}{\left(1 - \overline{\hat{P}}\bar{G}\right)\kappa_3} + \frac{1 + \lambda_\pi - \eta \mathrm{L}}{\left(1 - \overline{\hat{P}}\bar{G}\right)\kappa_3}\pi_t + \frac{\lambda_Y + \overline{\hat{P}}\bar{G}\left(1 - \eta\mathrm{L}\right)\left(1 - \mathrm{L}\right)\frac{\bar{Y}}{\bar{C}}}{\left(1 - \overline{\hat{P}}\bar{G}\right)\kappa_3}\ln Y_t + \left(1 + \eta - \eta\mathrm{L}\right)R_{t-1} + \varepsilon_t^R \tag{9}$$

where $\varepsilon_t^R \equiv \left[\frac{-\xi_t + \left(\frac{\bar{C}-\bar{G}}{\bar{C}}\right)\overline{\hat{P}}\bar{G}(1-\eta\mathrm{L})(1-\mathrm{L})\ln G_t - \delta\frac{\bar{K}}{\bar{C}}\overline{\hat{P}}\bar{G}(1-\eta\mathrm{L})(1-\mathrm{L})\ln I_t}{\left(1-\overline{\hat{P}}\bar{G}\right)\kappa_3}\right]$, $\kappa_3 = \frac{\bar{C}-\bar{C}_1}{\bar{C}} > 0$ (bar denotes steady state values) and L is the lag operator. Thus, it is possible to transform the Taylor inspired rule for nominal growth μ to a "standard" rule for the nominal interest rate R in the model. But here it only possible to use the rule for μ, because R is an endogenous equilibrium price.[8]

The production function is assumed to have constant returns to scale and be of Cobb-

[7] Although we assume that ξ is log normally distributed, we require that ξ has mean $(1-\eta)\bar{\mu}$, and variance σ_ξ^2 as seen in (8). By using that $\mathrm{E}[\xi] = e^{\mathrm{E}[\ln\xi] + \frac{1}{2}\mathrm{Var}(\ln\xi)}$ and that $\mathrm{Var}(\xi) = \mathrm{E}\left\{(\xi - \mathrm{E}[\xi])^2\right\} = \mathrm{E}\left[\xi^2\right] - [(1-\eta)\bar{\mu}]^2 = e^{2\mathrm{E}[\ln\xi] + \mathrm{Var}(\ln\xi)} - [(1-\eta)\bar{\mu}]^2$ since ξ is log-normally distributed, one can pin down the mean and the variance for $\ln\xi$ as $-\frac{1}{2}\ln\left(\sigma_\xi^2 + [(1-\eta)\bar{\mu}]^2\right) + 2\ln((1-\eta)\bar{\mu})$ and $\ln\left(\sigma_\xi^2 + [(1-\eta)\bar{\mu}]^2\right) - 2\ln((1-\eta)\bar{\mu})$ respectively.

[8] One potential problem with interpreting (9) as a standard Taylor-type rule is that the residual is presumably correlated with the arguments. However, this is not a specific issue for the model at hand, but rather a general problem that also has been acknowledged by many researchers, see e.g. Clarida, Gali and Gertler (1999) and McCallum and Nelson (1999).

Douglas type

$$Y_t = e^{\ln Z_t} K_t^{\theta} H_t^{1-\theta} \tag{10}$$

where K_t and H_t are aggregate (average) capital stock and hours worked, respectively, and Z_t the technology level which is assumed to follow a stationary AR(1)-process (in natural logs)

$$\ln Z_{t+1} = \rho^{\ln Z} \ln Z_t + \varepsilon_{t+1}^{\ln Z}, \, \varepsilon^{\ln Z} \sim i.i.d. \; N\left(0, \sigma_{\ln Z}^2\right). \tag{11}$$

Individual and aggregate investment in period t produces productive capital in period $t+1$ according to

$$k_{t+1} = (1-\delta)\, k_t + i_t \tag{12}$$

and

$$K_{t+1} = (1-\delta)\, K_t + I_t \tag{13}$$

where δ is the rate of capital depreciation.

The perfect competition zero profit maximizing conditions for the representative firm are

$$W_t^c = (1-\theta)\, e^{\ln Z_t} \left(\frac{K_t}{H_t}\right)^{\theta} P_t \tag{14}$$

and

$$R_t^K = \theta e^{\ln Z_t} \left(\frac{K_t}{H_t}\right)^{\theta - 1}. \tag{15}$$

The nominal wage W_t^c is assumed to be set at the end of period $t-1$ (see Cooley and Hansen (1995) for further details on the nominal wage arrangement) as

$$\ln W_t^c = \ln(1-\theta) + \mathrm{E}_{t-1} \ln Z_t + \theta \left(K_t - \mathrm{E}_{t-1} H_t\right) + \mathrm{E}_{t-1} P_t \tag{16}$$

where E_{t-1} denotes the conditional expectations operator on all relevant information in period $t-1$.[9] Moreover, households are assumed to transfer to the firms the right to choose aggregate hours worked in period t, H_t, to equate the marginal product of labor to the contracted wage rate. If we combine (14) and (16) in natural logarithms, using (11) below, we obtain

$$\ln H_t = \mathrm{E}_{t-1} \ln H_t + \frac{1}{\theta} \left(\ln P_t - \mathrm{E}_{t-1} \ln P_t\right) + \frac{1}{\theta} \varepsilon_t^{\ln Z}. \tag{17}$$

[9] Note that $\ln K_t$ is known at the end of period $t-1$ through the equilibrium decision rules (see Appendix A).

Similarly, one realizes that the natural logarithm of h_t for an agent in equilibrium is given by

$$\ln h_t = \mathrm{E}_{t-1} \ln H_t + \frac{1}{\theta} \left(\ln P_t - \mathrm{E}_{t-1} \ln P_t \right) + \frac{1}{\theta} \varepsilon_t^{\ln Z}. \tag{18}$$

The aggregate resource constraint

$$Y_t = C_{1t} + C_{2t} + I_t + G_t \equiv C_t + I_t + G_t \tag{19}$$

also holds in every period where C_t is total consumption.

2.2 Equilibrium in the model

The equilibrium in the model consists of a set of decision rules for the agents $\ln k_{t+1} = k \left(\mathbf{S}_t,\ \ln k_t,\ \ln \hat{m}_t \right)$, $\ln \hat{m}_{t+1} = \hat{m} \left(\mathbf{S}_t,\ \ln k_t,\ \ln \hat{m}_t \right)$ and $\ln h_t = h \left(\mathbf{S}_t,\ \ln k_t,\ \ln \hat{m}_t \right)$, and a set of aggregate decision rules $\ln K_{t+1} = K \left(\ \mathbf{S}_t \right)$, $\ln H_t = H \left(\mathbf{S}_t \right)$, $\ln \hat{P}_t = \hat{P} \left(\mathbf{S}_t \right)$ where $\mathbf{S}_t = \left[\ln Z_{t-1}, \varepsilon_t^{\ln Z}, \mu_{t-1}, \xi_t, \ln G_t, \ln K_t, \ln \hat{P}_{t-1} \right]'$ such that; (i) agents maximize utility, (ii) firms maximize profits, and (iii), individual decision rules are consistent with aggregate outcomes. Equilibrium condition (iii) implies that $k \left(\mathbf{S}_t,\ \ln K_t,\ 1 \right) = K \left(\ \mathbf{S}_t \right)$, $\hat{m} \left(\mathbf{S}_t,\ \ln K_t,\ 1 \right) = 1$, and $h \left(\mathbf{S}_t,\ \ln K_t,\ 1 \right) = H \left(\mathbf{S}_t \right)$ for all $\mathbf{S}_t$.

In Appendix A, I describe how to compute the equilibrium in this model.

3 Estimation and calibration

The parameters in the equilibrium model are determined in two ways. About half of the parameters (η, $\bar{\mu}$, σ_ξ^2, λ_π, λ_Y, $\rho^{\ln G}$, $\sigma_{\ln G}^2$ and $\bar{g} \equiv \frac{\bar{G}}{\bar{Y}}$) are estimated on U.S. data 1960-1997 with Instrumental Variables method (IV) and Ordinary Least Squares (OLS). The other half of the parameters (α, β, δ, γ, θ, $\rho^{\ln Z}$ and $\sigma_{\ln Z}^2$) are adapted from Cooley and Hansen (1995), and chosen so that the model's steady state properties are consistent with U.S. growth facts.

To estimate the parameters η, $\bar{\mu}$, σ_ξ^2, λ_π, and λ_Y in the monetary policy rule (8) for different Fed chairmen periods, I collected quarterly data on real gross national product per capita in natural logarithms ($\ln Y_t$), growth rate in nominal money supply (μ_t) and the inflation rate in the consumer price index (π_t). To compute measures of $\ln Y_t - \ln Y^*$ and

$\pi_t - \pi^*$, I simply filtered the series for output and inflation rate with the Hodrick-Prescott (H-P) filter (see Hodrick and Prescott, 1997).[10] It is standard to use H-P filtered output as measure of the output gap, but is less clear how to compute an appropriate measure of π^* from historical data as discussed by Judd and Rudebusch (1998).[11] Since the model does not distinguish between money controlled by the Fed (the monetary base, M0) and money used in private transactions (M2), I compromise between them and use M1 as a measure of money as in Cooley and Hansen (1989, 1995). The reason for estimating with IV rather than Ordinary Least Squares (OLS), is that OLS is likely to be a biased and inconsistent estimator due to the fact that we may have contemporaneous correlation between the error term and the regressors in (8). In terms of the theoretical model used in this paper, there will, via the equilibrium decision rules, be a positive correlation between the error term ξ_t and the regressors π_t and $\ln Y_t$ in (8). As instruments in the estimation, I therefore use $(\ln Y - \ln Y^*)_{t-1}$, μ_{t-1} and $(\pi - \pi^*)_{t-1}$ which are uncorrelated with the error term ξ_t in (8). In addition to that, the estimated λ_π and λ_Y will be correlated in general, why inference must be conducted with great care.

I estimate the monetary policy rule (8) with IV for the whole sample period ($1970Q1 - 1997Q4$), for chairman Burns' office period ($1970Q1 - 1978Q1$), chairman Volckers' office period ($1979Q3 - 1987Q2$), chairman Greenspans' office period ($1987Q3 - 1997Q4$), and omit chairman Miller as in Judd and Rudebusch (1998) because of his short tenure. The results of the estimations are reported in Table 1 (a constant is included in the regressions but is omitted from the table).

The D-W and Breusch-Godfrey statistics indicates presence of positive autocorrelation in the regressions, suggesting difficulties to interpret the significance levels of the estimates of η, λ_π and λ_Y. However, use of the asymptotic χ^2-distribution for the Breusch-Godfrey test is very likely to yield an oversized test (i.e. an exaggerated probability of rejecting a true null hypothesis of no autocorrelation) for sample sizes as small as the present ones. Simulated small sample adjusted p-values for the Breusch-Godfrey test confirm the

[10] I use the common value 1600 (quarterly data) for the smoothness coefficient λ in the H-P filter. See Appendix B for a detailed description of the raw data and data transformations.

[11] Although my approach regarding $\pi - \pi^*$ appears to be as good as any other considerable alternative (see Judd and Rudebusch), I have nevertheless experimented with other measures (such as the average inflation rate during a given chairmen's term), but it did not have any impact on the conclusions drawn in the paper.

Table 1: IV estimation results for the monetary policy rule (8).

Estimation period	$\hat{\eta}$	$\hat{\lambda}_\pi$	$\hat{\lambda}_Y$	$\hat{\sigma}_\xi$	$\bar{R}^2$	D-W	B-G χ^2 (4)	J-B	T
Whole	0.931 (0.033)	0.181 (0.093)	0.083 (0.092)	0.0138	0.89	1.39	33.33 (0.000)	0.400 (0.819)	112
Burns	0.515 (0.151)	0.182 (0.087)	−0.166 (0.148)	0.0073	0.73	1.96	14.17 (0.007)	0.715 (0.699)	33
Volcker	0.717 (0.116)	0.377 (0.203)	−0.137 (0.249)	0.0158	0.73	1.59	11.86 (0.019)	1.289 (0.525)	32
Greenspan	0.919 (0.054)	0.532 (0.540)	0.013 (0.262)	0.0153	0.91	0.98	17.37 (0.002)	1.249 (0.536)	42

Note: Standard errors in parenthesis for $\hat{\eta}$, $\hat{\lambda}_\pi$ and $\hat{\lambda}_Y$, and p-values in parenthesis for the Breusch-Godfrey autocorrelation test (null hypothesis no autocorrelation up to 4 lags) and the Jarque-Bera normality test (null hypothesis normally distributed residuals). A constant, $(\ln Y - \ln Y^*)_{t-1}$, μ_{t-1} and $(\pi - \pi^*)_{t-1}$ have been used as instruments. T denotes the number of observations in the regressions.

size problem, and result in a non-significant autocorrelation effect.[12] The p-value (i.e. the nominal significance level) for a joint F-test of the null hypothesis $H_0 : \eta = \lambda_\pi = \lambda_Y = 0$ is around 0 for all regimes, as indicated by the model's satisfactory fit during the subsamples (measured by the multiple correlation coefficient $\bar{R}^2$). Not surprisingly, we get the highest estimates of λ_π during chairmen Volcker and Greenspan periods of office, and the lowest for chairman Burns.

To examine if these parameter changes are in line with experiments conducted with interest rate rules, we insert the estimates of η, λ_π and λ_Y into the Taylor rule for the nominal interest rate in (9). It is then easy to verify that the resulting parameter changes in the Taylor rule for the nominal interest rate are well in line with typical parameter experiments considered in the interest rate rule literature.

To estimate $\rho^{\ln G}$ and $\sigma^2_{\ln G}$ in (6), I collected quarterly data series on real government expenditures on consumption and investment per capita in natural logarithms, and filtered the series with the Hodrick-Prescott (H-P) filter (see Hodrick and Prescott, 1997) to get a measure of $\ln G_t$. I then estimated (6) on the sample period $1960Q1$ to $1997Q4$ with

[12] The small sample adjusted B-G test statistics have been computed by: (i) estimating a VAR-model with 6 lags including the variables $\ln Y_t - \ln Y^*$, $\pi_t - \pi^*$ and μ_t (using likelihood ratio, autocorrelation and normality tests to determine the lag order) on data for the different periods; (ii) using the estimated VAR-model as a data generating process to simulate artificial samples of data; (iii) estimating the regression (8) on the simulated data with IV and then computing the associated B-G statistics. From the resulting distributions of B-G statistics, the small sample adjusted p-values are computed as the fraction of simulated B-G statistics that are larger than the estimated ones. The resulting p-values by this procedure are 0.683, 0.613, 0.663 and 0.458 for Whole sample, Burns, Volcker and Greenspan regimes respectively.

OLS with the result (standard error in parenthesis)

$$\ln G_t = \underset{(0.0485)}{0.8019} \ln G_{t-1} + \hat{\varepsilon}_t^{\ln G},\ \hat{\sigma}_{\ln G} = 0.009844,\ \text{D-W} = 1.93,\ \bar{R}^2 = 0.64,\ \text{B-G } \chi^2(4) = \underset{(0.0003)}{20.912}. \tag{20}$$

Although the D-W statistic is satisfactory, the Breusch-Godfrey test for autocorrelation in (20) is significant and shows tendencies of positive autocorrelation. But when I augmented the estimation with more lags on the dependent variable to remove this autocorrelation, I found that the estimated parameters were largely unaffected.

To compute values for $\bar{\mu}$ and $\bar{g}$, I took averages of quarterly nominal money growth and the ratio of government expenditures to gross national product to get 0.01310 and 0.21038 respectively.

γ is calibrated in the same way as in Cooley and Hansen (1995) and set so that hours worked as share of available time in steady state, $\bar{H}$, equals 0.30.[13]. The remaining parameters are directly taken from Cooley and Hansen; α is set to 0.84, β is set to 0.989, δ is set to 0.019, θ is set to 0.40 and $\rho^{\ln Z}$ and $\sigma_{\ln Z}$ are set to 0.95 and 0.00721 respectively.

4 The Backward-looking model

In this Section, I will briefly present the backward-looking model that I have chosen to study - the Rudebusch and Svensson (1999) model. The Rudebusch and Svensson model, which draws on the Svensson (1997) model, is intended to be a reasonable approximation of reality. It contains much richer dynamics than the simple Svensson model by allowing for four lags of inflation in the AS curve and two lags of output in the AD curve.

4.1 The Rudebusch and Svensson (1999) model

The Rudebusch and Svensson model is similar to many other models used for monetary policy analysis. It consists of aggregate supply (AS) and aggregate demand (AD) equations relating the output gap (the percentage deviation of output from its steady state

[13] Formally, we have that $\gamma = \frac{(1-\theta)\left(\alpha\beta\frac{1}{e^{\bar{\mu}}}+1-\alpha\right)\left(\frac{1-\beta(1-\delta)}{\beta\theta}\right)}{\bar{H}\left((1-\bar{g})\left(\frac{1-\beta(1-\delta)}{\beta\theta}\right)-\delta\right)}$, which can be used to compute $\gamma = 3.404$ given the values for the other parameters. This value is higher than Cooley and Hansen's value (2.53) since I have government expenditures in the model.

level) and the inflation rate to each other and a monetary policy instrument, the (short-run) interest rate. Formally, the model economy is described by the following equations

$$\begin{aligned} \pi_t &= \sum_{j=1}^{4} \alpha_{\pi,j}\pi_{t-j} + +\alpha_y y_{t-1} + \varepsilon_t^{\pi}, \\ y_t &= \beta_{y,1}y_{t-1} + \beta_{y,2}y_{t-2} + \beta_r \sum_{j=1}^{4} \tfrac{1}{4}(i-\pi)_{t-j} + \varepsilon_t^{y}. \end{aligned} \tag{21}$$

In (21), the first equation is the AS curve (or Phillips curve), where the (annualized) inflation rate π depends on past inflation rates, the output gap in the previous period and an exogenous supply shock ε^{π} (i.i.d. with zero mean and variance σ_{π}^2). The second equation in (21) is the AD curve, where the output gap y_t is related to past output gaps y_{t-1} and y_{t-2}, the average ex post real interest rate in the four previous periods, $\sum_{j=1}^{4} \frac{1}{4}(i-\pi)_{t-j}$, and an exogenous demand shock ε_t^y(i.i.d. with zero mean and constant variance). The central bank, which is assumed to control the nominal interest rate i_t, thus affects the inflation rate with a two period lag. The monetary transmission mechanism is via output to the inflation rate. In the Rudebusch and Svensson framework, the sum of the estimated $\alpha_{\pi,j}$'s is restricted to equal 1 to get an accelerationist Phillips curve where long-run monetary neutrality holds.

Rudebusch and Svensson estimate (21) on quarterly US data for the sample period $1961Q1$ to $1996Q2$. They cannot reject the hypothesis that $\sum_{j=1}^{4} \alpha_{\pi,j}$ equals 1 so they maintain that assumption throughout their analysis. But in the model framework here - when we have the equilibrium model as a data generating process - this restriction did not recieve econometric support in the estimations below, so I therefore allow for a α_{π} different from 1. Although Rudebusch and Svensson do not explicitly report any relevant statistics regarding the properties of ε^{π} and ε^{y}, they test for structural stability in the equations and cannot reject the null hypothesis of no instability.

Rudebusch and Svensson measure the inflation rate π_t, the output gap y_t and the ex post real interest rate $(i-\pi)_t$ in the following way. To get a measure of π, they compute $400(\ln p_t - \ln p_{t-1})$ where p is the quarterly chain-weighted GDP price index. y_t is measured as the percentage gap between real output and potential output $100((Y_t - Y_t^*)/Y_t^*)$. The ex post real interest rate (in period t) included in the AD curve is measured as

$\frac{1}{4}\sum_{j=1}^{4}(i-\pi)_{t-j}$ where i is the average quarterly federal funds rate and π is the inflation rate defined previously. All variables are then demeaned prior to estimation of the model economy; hence no constants are included in the regressions.

Assuming a quadratic objective function for the central bank over inflation and output (for example, $L_t \equiv \pi_t^2 + \lambda y_t^2 + \nu \Delta i_t^2$), it is possible solve the central bank's problem subject to the estimated model economy summarized by (21). The resulting decision rule for the nominal interest rate as a (linear) function of current inflations rate, output gap and lagged nominal interest rate are then used along with the estimated model economy to conduct policy analysis.

5 Parameter stability in the backward-looking model

In this section, I present the results of some simulation experiments designed to examine how robust the parameters in the backward-looking model are when there is a change in the monetary policy rule. I will discuss this issue from both a statistical as well as an economic point of view. I also present and motivate how the experiments have been carried out.

5.1 Testing strategy

To investigate whether the Lucas critique seems to be significant in a statistical sense for the backward-looking model, I have simulated the equilibrium model for the estimated monetary policy rules for nominal money growth and estimated the backward-looking model (21) on the simulated data.

The procedure in the simulations has been as follows:

1. Generate an artificial data set by simulating the equilibrium model for T periods under the assumption that the monetary policy rule changes completely unexpectedly after $T/2$ periods from one regime to another (for example, from Burns to Volcker and Burns to Greenspan).[14]

[14] The simulations are made in the GAUSS programming language, using the random number generator RDND with RDNDSEED set to 159425+$iter$ for $iter = 1, 2, ..., N$. To get a stochastic initial state in each simulation, the model is simulated for $T + 100$ periods, where the first 100 periods are then discarded.

2. Estimate (21) with OLS on the first $1, ..., T/2$ observations in the simulated sample. Denote the estimated parameter vectors $\hat{\boldsymbol{\beta}}_{AS}$ (for aggregate supply) and $\hat{\boldsymbol{\beta}}_{AD}$ (aggregate demand) respectively.

3. Estimate (21) with OLS on the last $T/2 + 1, ..., T$ observations in the simulated sample. Denote the estimated parameter vectors $\hat{\boldsymbol{\alpha}}_{AS}$ and $\hat{\boldsymbol{\alpha}}_{AD}$ respectively.

4. Use a version of the F-test, often called the Chow (1960) breakpoint test, to examine if the null hypotheses

$$\begin{aligned} H_0 &: \boldsymbol{\alpha}_{AS} = \boldsymbol{\beta}_{AS}, \\ H_0 &: \boldsymbol{\alpha}_{AD} = \boldsymbol{\beta}_{AD}, \end{aligned}$$

and

$$H_0 : (\boldsymbol{\alpha}_{AS} = \boldsymbol{\beta}_{AS}) \text{ and } (\boldsymbol{\alpha}_{AD} = \boldsymbol{\beta}_{AD})$$

are rejected on appropriate significance levels.

5. Repeat Step 1 to Step 4 many (N) times to compute probabilities for how often the null hypotheses are maintained for the given significance level.

6. To get correct significance levels, Step 1 to Step 5 above are carried out twice. In the first round, small sample critical values are computed under the (true) null hypotheses $H_0 : \boldsymbol{\alpha}_{AS} = \boldsymbol{\beta}_{AS}$ and $H_0 : \boldsymbol{\alpha}_{AD} = \boldsymbol{\beta}_{AD}$ (that is, compute the distribution of F-statistics although there has been no regime shift). In the second round, these adjusted critical values are used in the F-test.

7. Now, if the computed probabilities in Step 5 (in the second round) of rejecting parameter stability are higher/lower than the given significance levels, the Lucas critique is/is not relevant in this model in a statistical sense.

The critical assumptions in Step 1 to Step 6 are clearly made in Step 1 to 3, and I would like to briefly comment on them. First, I have chosen to change monetary policy regime in the middle of the sample. The motivation behind this choice is that it gives the highest possible power in the testing. Secondly, I have chosen to model the once and for all change in monetary policy regime as a completely unexpected shift in the estimated monetary policy rule where I let the economy bring the state vector from the last period in the previous regime (period $T/2$) to the first period in the new regime (period $T/2+1$). Third, it is assumed that the monetary regime is perfectly credible and expected to last forever. By this procedure, I implicitly assume a first order Markov chain for the different monetary policy regimes where I let the diagonal elements in the transition matrix approach unity. The second and third assumptions are very convenient since they allows me to use the same decision rules for the first $T/2$ periods and then change to new decision rules in the beginning of period $T/2+1$ for the remaining $T/2$ periods. Finally, I have chosen to use OLS as the estimation method - although it can be argued that it is an inconsistent estimator here - since it is a simple, fast and widely used method. I have made some experiments with a consistent estimator (the IV method), but the results were largely unaffected.

5.2 Results

The results of this exercise for the different estimated monetary policy rules for sample size $T = 200$ (corresponding to 50 years of quarterly data), are provided in Table 2. I have some experiments with other sample sizes ($T = 100$ and $T = 400$), but the results were not much affected. In the estimations, all the variables involved in the regressions have been measured in precisely the same way as by Rudebusch and Svensson, whose measurement procedure was presented in Section 4.1.[15]

As seen in Table 2, the probabilities of rejecting the null hypothesis of parameter stability between regimes are clearly higher than the given significance levels in most cases. For the AS curve, we see that the probabilities of rejecting parameter stability are found to be low between the Burns and Volcker regimes and vice versa (0.088 and 0.153 respectively at the 10 percent level), indicating that the Lucas critique is not quantitatively important in for the AS curve in these cases in a statistically significant way. This is quite

[15] To be able to generate reliable small sample critical values under the null (when there is no regime shift) in the first round, the model has been simulated $N = 100,000$ times. Note that the probabilities in the diagonal (when there is no regime shift) for the AS- and AD-curves equal 0.10 (0.05) at the 10 (5) percent significance level since the same shock realizations have been used in the second round.

Table 2: *F*-test probabilities for rejecting the null hypothesis of parameter stability in the Rudebusch and Svensson (1999) model in (21) at various significance levels.

	Significance level 10 percent				Significance level 5 percent			
	Comparison regime							
	WS	B	V	G	WS	B	V	G
Benchmark regime	*The aggregate supply function*; $H_0 : \boldsymbol{\alpha}_{AS} = \boldsymbol{\beta}_{AS}$							
Whole sample (WS)	0.100	0.850	0.748	0.689	0.050	0.765	0.652	0.573
Burns (B)	0.725	0.100	0.088	0.539	0.557	0.050	0.054	0.335
Volcker (V)	0.730	0.154	0.100	0.520	0.487	0.055	0.050	0.276
Greenspan (G)	0.732	0.736	0.561	0.100	0.590	0.574	0.426	0.050
Benchmark regime	*The aggregate demand function*; $H_0 : \boldsymbol{\alpha}_{AD} = \boldsymbol{\beta}_{AD}$							
Whole sample	0.100	0.424	0.359	0.139	0.050	0.262	0.216	0.064
Burns	0.295	0.100	0.106	0.146	0.167	0.050	0.060	0.074
Volcker	0.238	0.102	0.100	0.117	0.109	0.043	0.050	0.049
Greenspan	0.156	0.318	0.256	0.100	0.084	0.199	0.160	0.050
Benchmark regime	*Either AS- or AD-curve;* $H_0 : (\boldsymbol{\alpha}_{AS} = \boldsymbol{\beta}_{AS})$ and $(\boldsymbol{\alpha}_{AD} = \boldsymbol{\beta}_{AD})$							
Whole sample	N.C.	0.904	0.824	0.728	N.C.	0.819	0.714	0.597
Burns	0.826	N.C.	0.150	0.619	0.649	N.C.	0.083	0.388
Volcker	0.808	0.211	N.C.	0.591	0.556	0.077	N.C.	0.314
Greenspan	0.776	0.805	0.660	N.C.	0.623	0.640	0.498	N.C.

Note: NC is shorthand notation for not computed. The Chow (1960) statistic underlying the computation of the probabilities is defined as $\frac{\left(\hat{\sigma}_T^2 - \frac{T_1}{T}\hat{\sigma}_{T_1}^2 - \frac{T_2}{T}\hat{\sigma}_{T_2}^2\right)/k}{\left(\frac{T_1}{T}\hat{\sigma}_{T_1}^2 + \frac{T_2}{T}\hat{\sigma}_{T_2}^2\right)/(T-2k)}$ and it follows the F-distribution with $k, T - 2k$ degrees of freedom where k is the number of parameter restrictions that are being tested, T the total number of observations ($T \equiv T_1 + T_2$) and $\hat{\sigma}_T^2$, $\hat{\sigma}_{T_1}^2$, and $\hat{\sigma}_{T_2}^2$denote the estimated standard error of the regression during both monetary regimes, the first monetary regime, and the second monetary regime respectively. The small sample critical values are $\{3.02, 3.22, 2.75, 2.85\} \,/\, \{5.07, 4.85, 5.20, 4.46\}$ and $\{3.86, 4.45, 4.64, 3.86\} \,/\, \{6.49, 6.15, 7.03, 5.56\}$ for the Whole sample, Burns, Volcker and Greenspan regimes at the 10 and 5 percent significance levels for the AS/AD equations respectively, whereas the asymptotic critical values are $\{2.11, 1.88\}$ and $\{2.65, 2.26\}$ at the 10 and 5 percent significance levels for $T = 200$ and $k = \{3, 5\}$. The small sample critical values are generated under the null hypothesis in a first round of $N = 100,000$ simulations, while the probabilities reported in the table are computed from a second round of simulations (again, $N = 100,000$) where the small sample critical values are used in the testing.

natural since we can see in Table 1 that the estimated monetary policy rules for Burns and Volcker are similar (low η, negative λ_Y). Turning to the AD curve, we find that the probabilities for rejecting parameter stability are in general lower than for the AS counterparts, implying that the AD curve is less sensitive to the Lucas critique than the AS curve. In particular, the statistical significance of the Lucas critique between the Burns and Volcker regimes is again ambiguous. Looking at both the parameter estimates in Table 1 and the probabilities in Table 2, one conclusion seems to be that in particular the parameters η and λ_Y are most important for the AS curve while η seems to be most important for the AD curve.

However, the most interesting hypothesis to test - because both the AS and AD curve are used in policy analysis - is the null hypothesis of instability in either the AS or the AD curve. In Table 2, the results for this hypothesis clearly indicate that the parameters in the Rudebusch and Svensson model as a whole are not exogenous to the parameters in the monetary policy rule (and thus the central banks optimization problem).

From an economic point of view, it is of particular interest to examine whether this instability is economically meaningful. To shed light on this issue, Table 3 reports the OLS estimation results of the model (21) on simulated data.[16]

We see from Table 3 that the results in Table 2 are confirmed. From an economic point of view, the estimated parameters in the model, in particular for the AS curve, are heavily affected by changes in the monetary policy rule. The output parameter in the AS equation varies from about -0.20 to 0.50 and the real interest rate coefficient in the AD equation, although low in general, also alters in sign. From an econometric point of view, the estimated equations often pass (in about $80-90$ percent on average) statistical tests for autocorrelation, as indicated by the Breusch-Godfrey statistics for autocorrelation. The adjusted r-squares are also satisfactory in most cases, the exception being the low adjusted r-square for the AS curve for the Greenspan regime. The reason for this is the estimated high value for λ_π during the Greenspan regime, which drives

[16] I have not been able to solve analytically for the reduced-form parameters as functions of the monetary policy rule parameters η, λ_π and λ_Y (and the other parameters in the equilibrium model). Therefore, I have been forced to estimate them on simulated data. I expanded the number of simulations until the (estimated) parameters coefficients converged in mean down to five digits, which required slightly less than 100000 simulations for $T = 200$.

Table 3: OLS estimation of the Rudebusch and Svensson model (21) for different regimes on simulated data.

Regime	Estimation output for the AS curve								
	$\alpha_{\pi,1}$	$\alpha_{\pi,2}$	$\alpha_{\pi,3}$	$\alpha_{\pi,4}$	α_y	$\bar{R}^2$	D-W	$\hat{\sigma}$ (%)	B-G $\chi^2(4)$
Whole sample	0.438	0.203	0.109	0.037	-0.199	0.78	2.02	0.11	2.87 (0.924)
Burns	0.062	0.133	0.062	0.041	0.496	0.36	2.03	0.20	1.68 (0.922)
Volcker	0.136	0.140	0.051	0.022	0.411	0.35	2.01	0.29	1.36 (0.956)
Greenspan	0.173	0.076	0.043	0.022	-0.003	0.10	2.00	0.07	5.14 (0.865)

Regime	Estimation output for the AD curve						
	$\beta_{y,1}$	$\beta_{y,2}$	β_r	$\bar{R}^2$	D-W	$\hat{\sigma}$ (%)	B-G $\chi^2(4)$
Whole sample	0.8240	0.0990	-0.0146	0.81	2.01	0.05	5.10 (0.855)
Burns	0.4743	0.3319	0.0172	0.51	2.12	0.08	10.37 (0.488)
Volcker	0.4764	0.3267	-0.0405	0.50	2.11	0.11	10.44 (0.503)
Greenspan	0.6935	0.2141	-0.0136	0.76	2.03	0.04	4.83 (0.869)

Note: $\hat{\sigma}$ denotes standard error of regression. The values within parentheses measure the likelihood that the computed test statistics are insignificant (significance level 5 percent) for the Breusch-Godfrey's χ^2-test (null hypotheses no 4th order autocorrelation). All the statistics reported are averages of $N = 100,000$ simulations of sample size $T = 200$.

down the autocorrelation (and the volatility) of the inflation rate. Consequently, all in all, an econometrician who estimates this model from the data would not immediately reject it for statistical reasons in most cases.

To sum up, the simulation results for the null hypothesis of instability in either the AS or the AD curve clearly indicate that the parameters in the Rudebusch and Svensson model as a whole are not exogenous to the central banks optimization problem (and thus the parameters in the monetary policy rule) using an equilibrium model with forward-looking agents as a data generating process. Thus, the Lucas critique applies strongly to this model according to the equilibrium model.

6 The quantitative importance of the Lucas critique in policy analysis

The results of the stability tests in the previous subsection do not necessarily imply that the Lucas critique is severe for the model considered as a system, they only imply the Lucas critique is highly relevant for individual parameters. When there is a monetary regime shift, there is a possibility that the resulting parameter changes in the AS- and AD-curves along with the interest rate Taylor-type rule (9) are such that the Lucas critique is not important for the model implications when it is considered as a system. This section examines this issue by computing impulse response functions using the estimated AS- and AD-curves from Table 3 along with estimates of the interest rate rule (9).

Figure 1 shows the impulse response functions for the nominal interest rate R, the output gap y and the inflation gap π for the different shocks in the model. The first column depicts the effects of a monetary policy shock in (9) defined as $\varepsilon_t^R = 1$ and $\varepsilon_{t+j}^R = 0$ for $j > 0$, the second the effects of a demand shock to the AD-curve ($\varepsilon_t^y = 1$ and $\varepsilon_{t+j}^y = 0$ for $j > 0$), and the third the effects of a supply shock to the AS-curve ($\varepsilon_t^\pi = 1$ and $\varepsilon_{t+j}^\pi = 0$ for $j > 0$). From Figure 1, we see that the impulse response for the interest rate for a monetary policy shock are most persistent for the "whole sample" and Greenspan regimes. Looking at the results in Table 1, this is also what one would expect since the estimated persistence coefficient η is highest for these regimes. We also see that the effects on the output and inflation gap of a monetary policy shock are not too large. At least not in comparison with demand shocks, which have much larger (and persistent) effects on inflation and output gaps. Interest rates are most sensitive to supply shocks, although the effects evaporate faster than demand shocks. A general impression from Figure 1 is that the impulse responses in Figure 1 differ a lot, which is an indication that the Lucas critique is also very relevant for the model considered as a system.

In Figure 2, I have quantified the importance of the Lucas critique for the output gap and inflation gaps of a monetary policy shock as follows:

1. Let $\mathbf{IRF}(\{AS_{\mathrm{WS}}, AD_{\mathrm{WS}}\} \,|TR_{\mathrm{WS}})$ denote the (true) impulse responses for output and inflation gaps of a monetary policy shock when estimates of the AS- and AD-

Figure 1: Impulse responses in the backward-looking model for different monetary regimes.

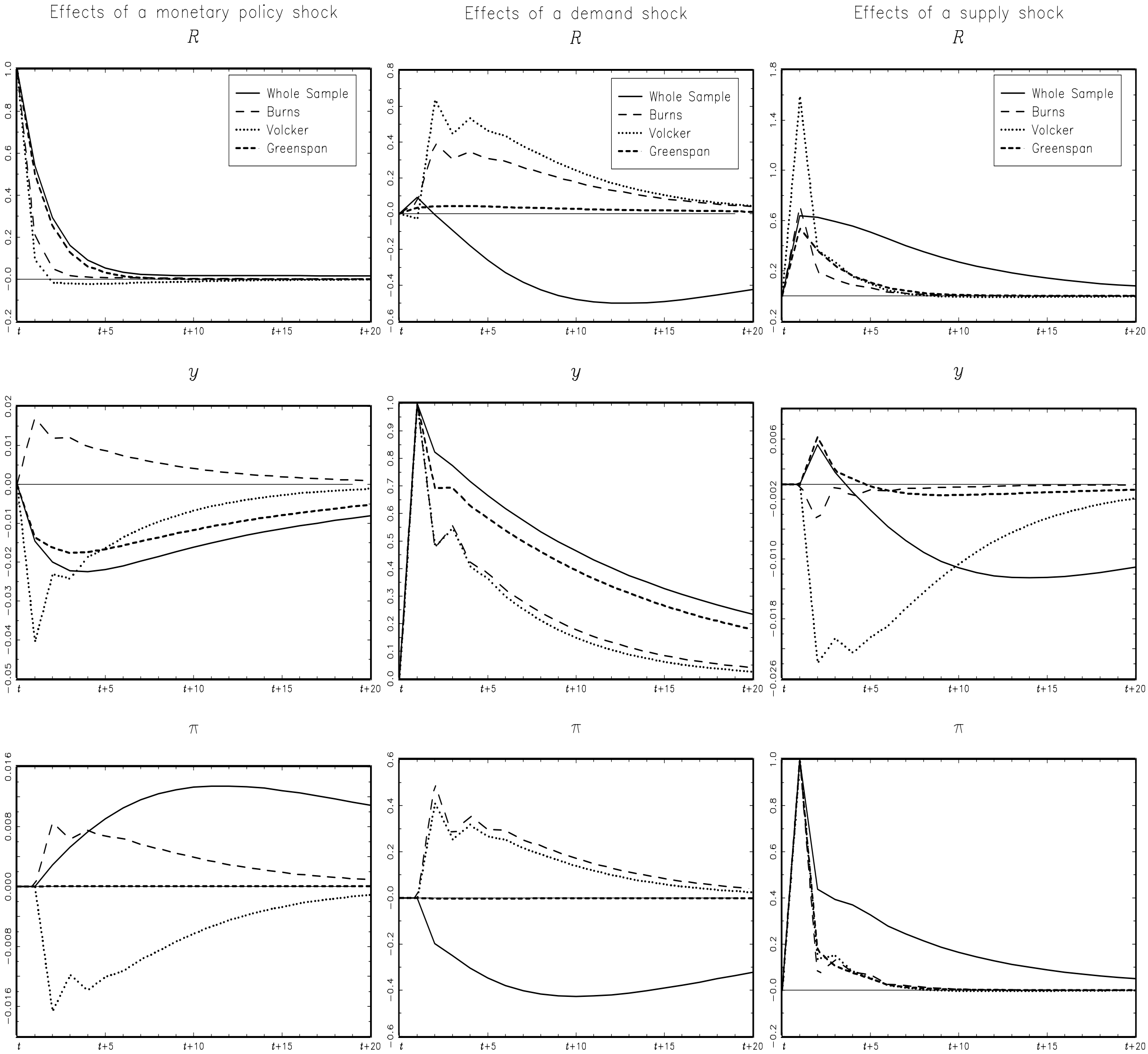
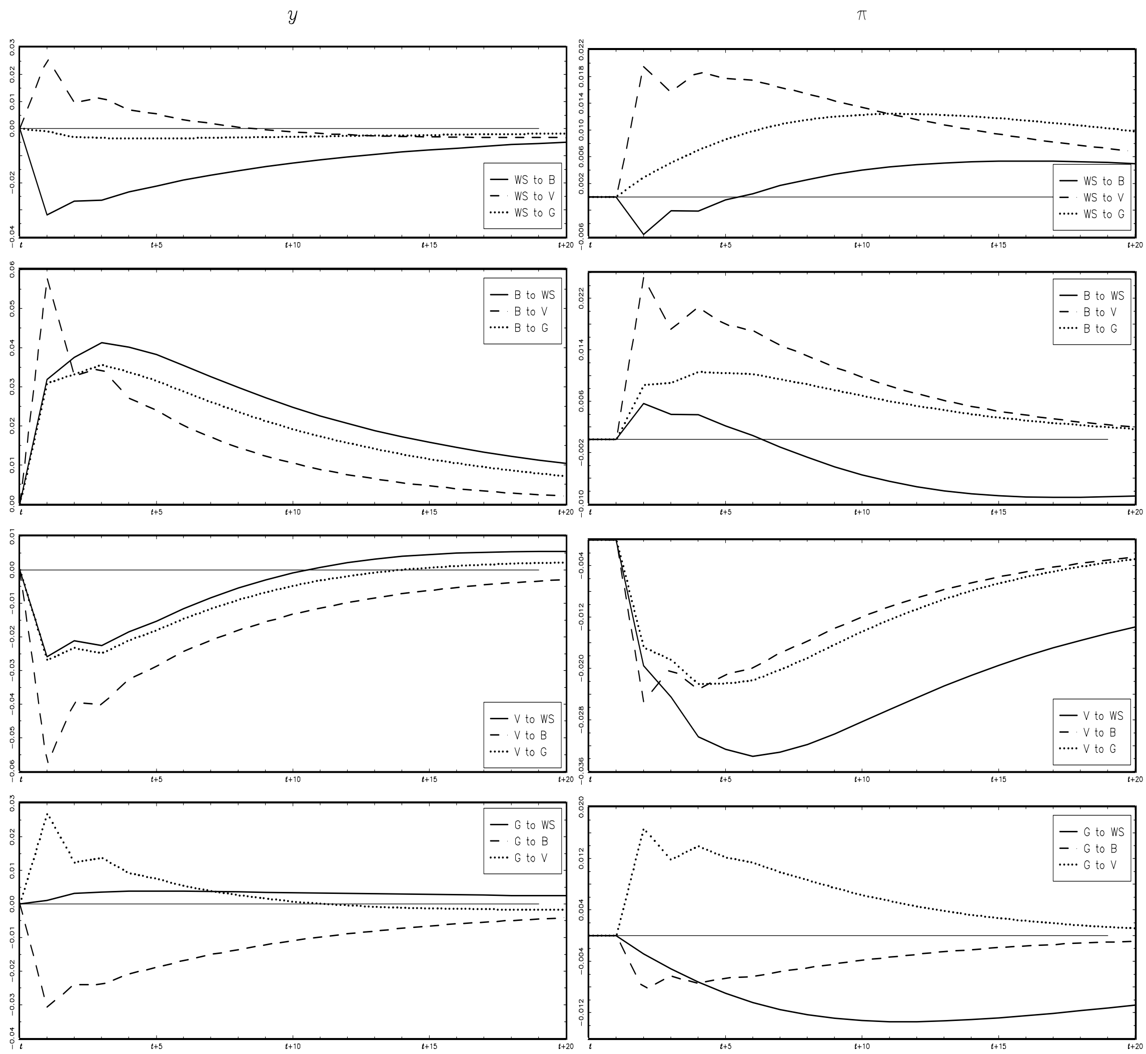


Figure 2: Deviations from true impulse responses for different regime shifts.



curves for the whole sample regime are being used along with estimates of the monetary policy rule (9) for the whole sample regime.

2. Then $\mathbf{IRF}(\{AS_{\mathrm{WS}},\, AD_{\mathrm{WS}}\}\,|TR_{\mathrm{B}}) - \mathbf{IRF}(\{AS_{\mathrm{B}},\, AD_{\mathrm{B}}\}\,|TR_{\mathrm{B}})$ is the relevant quantitative measure of the Lucas critique when a policy regime shift from "whole sample" to Burns is considered.

The assumptions behind this measure are that the policymaker knows the slopes of the AS- and AD-curves in the existing regime, and that the policymaker uses this information to compute the effects on the economy of applying alternative Taylor-type interest rate rules.

Figure 2 depicts this measure for the output and inflation gaps for all various combinations of regime shifts for a monetary policy shock. The differences are not large in absolute values, a result which we expected since the impulse responses for π and y for temporary monetary policy shock were found to be relatively small as can be seen in Figure 1. However, we clearly see that the differences in the impulse responses as share of the true impulse responses, e.g. $(\mathbf{IRF}\,(\{AS_{\mathrm{WS}},\, AD_{\mathrm{WS}}\}\,|TR_{\mathrm{B}}) - \mathbf{IRF}\,(\{AS_{\mathrm{B}},\, AD_{\mathrm{B}}\}\,|TR_{\mathrm{B}}))\ /\ \mathbf{IRF}(\{AS_{\mathrm{B}},\, AD_{\mathrm{B}}\}\,|TR_{\mathrm{B}})$, are very large (over 1 in most cases, indicating a 100 percent deviation in impulse response functions or more) in almost every case. This verifies the quantitative importance of the Lucas critique in the backward-looking model, at least on longer horizons when the agents have sufficient information to understand that a regime shift has occurred.[17]

7 Concluding remarks

It seems like if the Lucas critique is potentially very important quantitatively, at least on longer horizons, for the type of backward-looking models that has been extensively used in the recent literature on monetary policy rules and, not least, for the type of questions that this literature addresses. More research, using other equilibrium models, e.g. the

[17] The quantitative long-run importance of the Lucas critique can be verified by computing the impulse response functions for a permanent monetary policy shock, defined as $\varepsilon^R_{t+j} = 1$ for $j \geq 0$. Although not reported, it can be understood from the results in Figure 2 that the differences becomes very large over time (after 1 year, say).

limited participation model advocated by Christiano, Eichenbaum and Evans (1999), are warranted to examine the robustness of the results here which are based on the Cooley and Hansen (1995) monetary equilibrium model. If the results here are valid, then the results from the large literature regarding the relative merits of alternative monetary policy rules using backward-looking models might be misleading. Thus, it seems to be a good research idea to check the robustness of the conclusions from this literature in a dynamic general equilibrium framework.

One may argue that the results are driven by incorrectly specified monetary policy regimes (and thus rules). I have done some experiments with attaching the central bank a standard quadratic loss function in inflation and output volatility and solved numerically for the implied coefficients in the monetary policy rule that are consistent with minimization for different relative weights on inflation and output volatility.[18] This approach, which is more standard in the literature, do not affect the qualitative conclusions in the paper.

Moreover, the paper may be criticized for using a monetary rule for nominal money growth rather than a rule for the nominal interest rate. But I think that the results in this paper are robust against this critique for two reasons. First, it is possible to map analytically the rule for the nominal money growth to a "standard" Taylor-type rule for the interest rate. Second, the parameter changes in the monetary policy rule for nominal money growth are not large in comparison with typical parameter changes in interest rate rules.

Another limitation of the paper is that I have implicitly assumed that the institutional design of the economy (that is, the one period nominal wage contacting assumption in the model) is unaffected by the monetary policy regime shifts. This assumption can be motivated by the real world observation that institutions, for example labor market arrangements, change very slowly over time.[19] Consequently, the effects of institutional

[18] The loss function (LF) is $L_t = \frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda(\ln Y_t - \ln Y^*)^2 + \nu(\Delta i_t)^2\right]$ and the coefficients η, λ_π and λ_Y in the monetary policy rule (8) are chosen so that they are consistent with minimization of LF for values of λ equal to 0, 0.2, 1 and 5 given $\nu = 1$ as in Rudebusch and Svensson (1999). This policy rule is not optimal (see Flodén, 1999) in this setting, but rather a "second-best" rule given a restriction of the central banks information set of the exogenous driving forces in the economy.

[19] If a sufficiently long period is covered (50 years or so), the effects of institutional changes may be larger; see for instance Fregert and Jonung (1998). When considering the effects of the Lucas critique

changes when testing for the Lucas critique in practice should be of “second order”, while the effects of monetary policy regime shift examined in this paper should be of “first order”.

here, however, I have in mind a shorter period (1 to 10 years or so).

Appendix A Computation of equilibrium

In order to make all variables in the deterministic version of the model above converge to a steady state, I transform the nominal variables by dividing m_{t+1} and P_t with M_{t+1}, and m_t with M_t. If we introduce the notation

$$\hat{m}_{t+s} \equiv \frac{m_{t+s}}{M_{t+s}} \text{ and } \hat{P}_{t+s} \equiv \frac{P_{t+s}}{M_{t+s+1}}$$

and use the transformations to rewrite the equations (2), (3), (4), (8), (17) and (18), the representative agent's optimization problem can, following Hansen and Prescott (1995), be expressed as the recursive dynamic programming problem:

$$V\left(\mathbf{S}_t, \hat{m}_t, k_t\right) \equiv \max_{\{\hat{m}_{t+1}, h_t, k_{t+1}\}} \left[\alpha \ln(c_{1t}) + (1-\alpha)\ln\left(c_{2t}\right) - \gamma h_t + \beta \mathrm{E}_t V\left(\mathbf{S}_{t+1}, \hat{m}_{t+1}, k_{t+1}\right)\right]$$

$$s.t. \;\; (10)\,, \;\; (6)\,, \;\; (11)\,, \tag{A.1}$$

$$\begin{aligned}
c_{1t} &= \frac{\hat{m}_t + e^{\mu_t} - 1}{e^{\mu_t}\hat{P}_t} - G_t, \\
c_{2t} &= (1-\theta)\, e^{\ln Z_t}\left(\frac{K_t}{H_t}\right)^{\theta} h_t + \left(1 + R_t^K - \delta\right) k_t - k_{t+1} - \frac{\hat{m}_{t+1}}{\hat{P}_t}, \\
\mu_t &= \tfrac{\eta}{1+\lambda_\pi}\mu_{t-1} - \tfrac{\lambda_\pi}{1+\lambda_\pi}\left(\ln \hat{P}_t - \ln \hat{P}_{t-1} - \pi^*\right) - \tfrac{\lambda_Y}{1+\lambda_\pi}\left(\ln Y_t - \ln Y^*\right) + \tfrac{1}{1+\lambda_\pi}\xi_t, \\
H_t - \mathrm{E}_{t-1}\ln H_t &= \tfrac{1}{\theta(1+\lambda_\pi)+(1-\theta)\lambda_Y}\left(\ln \hat{P}_t - \mathrm{E}_{t-1}\ln \hat{P}_t\right) + \tfrac{1+\lambda_\pi-\lambda_Y}{\theta(1+\lambda_\pi)+(1-\theta)\lambda_Y}\varepsilon_t^{\ln Z} + \tfrac{\xi_t-(1-\eta)\bar{\mu}}{\theta(1+\lambda_\pi)+(1-\theta)\lambda_Y}, \\
h_t - \mathrm{E}_{t-1}\ln H_t &= \tfrac{1}{\theta(1+\lambda_\pi)+(1-\theta)\lambda_Y}\left(\ln \hat{P}_t - \mathrm{E}_{t-1}\ln \hat{P}_t\right) + \tfrac{1+\lambda_\pi-\lambda_Y}{\theta(1+\lambda_\pi)+(1-\theta)\lambda_Y}\varepsilon_t^{\ln Z} + \tfrac{\xi_t-(1-\eta)\bar{\mu}}{\theta(1+\lambda_\pi)+(1-\theta)\lambda_Y}, \\
\ln K_{t+1} &= K\left(\mathbf{S}_t\right),\; \ln H_t = H\left(\mathbf{S}_t\right),\; \ln \hat{P}_t = \hat{P}\left(\mathbf{S}_t\right).
\end{aligned}$$

In (A.1), $\mathbf{S}_t$ is a 1×8 row vector which contains all the aggregate state variables $\ln Z_{t-1}$, $\varepsilon_t^{\ln Z}$, $\ln G_t$, μ_{t-1}, ξ_t, $\ln K_t$, $\ln \hat{P}_{t-1}$ and a constant term. If $\lambda_\pi = 0$, then $\ln \hat{P}_{t-1}$ vanishes in $\mathbf{S}_t$.[20] In maximization of (A.1), the agent takes the economy-wide aggregate (average) variables as given. The functions K, $\hat{P}$ and H describe the relationship perceived by agents between the aggregate decision variables and the state of the economy. As the solution to the problem in (A.1), we have the agent's decision rules $\ln k_{t+1} = k\left(\mathbf{S}_t,\ \ln k_t,\ \ln \hat{m}_t\right)$,

[20] Note that the household budget constraint on line 4 in (A.1) incorporates the fact that the contracted nominal wage divided by the price level equals the equilibrium marginal product of labor since firms unilaterally determine hours worked in period t.

$\ln \hat{m}_{t+1} = \hat{m}(\mathbf{S}_t,\ \ln k_t,\ \ln \hat{m}_t)$ and $\ln h_t = h(\mathbf{S}_t,\ \ln k_t,\ \ln \hat{m}_t)$. The competitive equilibrium is obtained when the individual and average decision rules coincide for $\ln k_t = \ln K_t$ and $\ln \hat{m}_{t+1} = \ln \hat{m}_t = 0$.

Since it is impossible to derive the decision rules analytically, I have used the same method as Cooley and Hansen (1995) and computed the decision rules numerically by approximating the original problem with a second order Taylor expansion around the constant steady state values in the nominal-growth adjusted economy. As a consequence of this approximation, the method produces linear decision rules (in natural logarithms for K_{t+1}, H_t and $\hat{P}_t$). The algorithm utilized is described in detail in Hansen and Prescott (1995).

Appendix B Data sources and definitions

In this appendix, I provide the sources of the data collected in Table B.1 below.

Table B.1: The data set.

Variables	Sample period	Source
GNP	1960Q1-1997Q4	FRED database, Federal Reserve Bank of St. Louis
GEC	1960Q1-1997Q4	FRED database, Federal Reserve Bank of St. Louis
M1	1959Q1-1997Q4	FRED database, Federal Reserve Bank of St. Louis
POP	1960-1996	OECD Main Economic Indicators
CPI	1959Q1-1997Q4	FRED database, Federal Reserve Bank of St. Louis

Note: All real macroeconomic variables are measured in 1992 billion U.S. dollars. Abbreviations; GNP denotes real (fixed, seasonally adjusted) gross national product; GEC real (chained, seasonally adjusted) government consumption and investment; M1 (not seasonally adjusted) nominal money supply 1; CPI (not seasonally adjusted) consumer price index; POP average U.S. population (for 1997, POP is set equal to average gross growth rate times the value for 1996).

The transformations made to generate the variables used in Table 1 are displayed in Table B.2.

Table B.2: Generation of composite quarterly data series.

Variable	Sample period	Calculation formula
$\ln Y$	1960Q1-1997Q1	$\ln(\text{GNP}/\text{POP})$
μ	1960Q1-1997Q4	$\ln(\text{M1}_t/\text{M1}_{t-4})$
π	1960Q1-1997Q4	$\ln(\text{CPI}_t/\text{CPI}_{t-4})$
$\ln G$	1960Q1-1997Q4	$\ln(\text{GEC}/\text{POP})$

Note: To get measures of $\ln Y - \ln Y^*$, $\ln G$ and $\pi - \pi^*$, $\ln Y$, $\ln G$ and π are then subject to Hodrick-Prescott filtering with the smoothness coefficient λ set to 1600.

To compute measures of the ratio of government expenditures to output and the growth rate in nominal money supply in steady state, $\bar{g}$ and $\bar{\mu}$ respectively, I computed the sums $\frac{1}{152}\sum_{t=1960Q1}^{1997Q4}(\mathrm{GEC}_t/\mathrm{GNP}_t)$ and $\frac{1}{152}\sum_{t=1960Q1}^{1997Q4}(1+\mu_t)^{\frac{1}{4}}-1$.

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