

Core inflation and monetary policy

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Abstract

What are the implications of targeting different measures of inflation? We extend a basic theoretical framework of optimal monetary policy under inflation targeting to include several components of CPI inflation, and analyze the implications of using different measures of inflation as target variable—core inflation, CPI excluding interest rates, and headline CPI inflation. Our main results are the following. *(i)* Barring the interest rate component, temporary shocks to inflation do not affect optimal monetary policy under any regime. *(ii)* Indirect (second-round) effects of disturbances on goal variables need to be accounted for properly. Simply excluding seemingly temporary disturbances from the reaction function risks leading to inappropriate policy responses. *(iii)* It may be optimal to respond to changes in one measure of inflation even if the target is defined in terms of another. *(iv)* The presence of the direct interest rate component in the CPI tends to push optimal monetary policy in an expansionary direction. The net effect, considering also the traditional channel, however, depends on the nature of the initial disturbance.

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1 Introduction

While the theoretical properties of inflation targeting are fairly well understood,¹ many unsolved issues remain as to its practical implementation. One important operational issue for inflation targeting central banks concerns what price index to use as a target variable. On the one hand, there is wide agreement that central banks should concentrate on stabilizing a broad index of prices, such as the consumer price index (CPI). On the other hand, the CPI typically includes a number of components that may make it undesirable as a goal for monetary policy (we discuss these perceived problems toward the end of this Introduction). An alternative strategy could therefore be for monetary policy to target a measure of the underlying trend rate of inflation, or “core inflation”.

Trying to strike a balance between these two considerations, inflation targeting central banks have reached different conclusions. For instance, Sveriges Riksbank (the central bank of Sweden) officially formulates its target in terms of CPI inflation, as does the Bank of Canada. The Reserve Bank of Australia, on the other hand, has until recently chosen to exclude certain volatile components of the CPI from its target, such as food and energy, indirect taxes, and home-mortgage costs, concentrating on the underlying rate of inflation. Other banks, for example the Bank of England and the Reserve Bank of New Zealand, have chosen an intermediate strategy, targeting CPI inflation exclusive of interest rates (referred to as RPIX in the UK and CPIX in New Zealand). Also, the European Central Bank (although not formally an inflation targeting central bank) defines its inflation target in terms of the Harmonised Index of Consumer Prices (HICP), which does not include interest rates.²

Despite the practical importance of the choice of price index, there have been few attempts to analyze the issue from a theoretical standpoint.³ The purpose of this paper, therefore, is to formulate a simple model of inflation targeting where the implications of having different measures of inflation in the central bank’s objective

¹See e.g. Svensson (1999a).

²Furthermore, some central banks have reached different conclusions at different points in time, an indication of the complex nature of the issue. Recently, the Riksbank has placed added emphasis on a measure of core inflation (see e.g. Sveriges Riksbank, March 2000). Due to recent changes in the official CPI, the Reserve Banks of Australia and New Zealand have now both switched to targeting CPI inflation (see Reserve Bank of Australia, 1998; Reserve Bank of New Zealand, 1999).

³A growing empirical literature on core inflation is concerned with issues of measurement, see e.g. Bryan and Cecchetti (1994), Quah and Vahey (1995), Bryan et al. (1997), Cecchetti (1997) and Apel and Jansson (1999).

function can be analyzed. The model we propose is an extension of a basic model of inflation targeting due to Svensson (1997, 1999b), in which there is only one measure of inflation. In our extension several different components of “CPI inflation” can be identified: (i) a core inflation rate, which is related to the level of real activity in the economy and which can be affected by monetary policy; (ii) purely exogenous inflation disturbances, e.g. changes in imported inflation, which cannot be affected by monetary policy; (iii) movements in inflation originating in fiscal policy actions, i.e. changes taxes and subsidies; and (iv) the direct effects of monetary policy on CPI via home-mortgage costs. With these four components, we define three different potential target measures: core inflation (i.e. only the first component), CPI inflation (all four components), and an intermediate measure, which we will call CPIX inflation, defined as CPI inflation excluding the interest rate component. Our purpose is to use this framework to analyze how optimal monetary policy is affected by the choice of inflation measure in the central bank’s objective function.

1.1 Perceived problems in the CPI

Since the consumer price index is a broadly based index, well-known and regularly published, it is a natural candidate as a target for monetary policy. At the same time, it includes a number of components that policymakers often perceive as problematic: temporary and exogenous disturbances (e.g. imported inflation), indirect taxes and the direct effects of monetary policy decisions. The volatility in these subcomponents of the CPI are frequently presented as undesirable, complicating the practical conduct of monetary policy.⁴

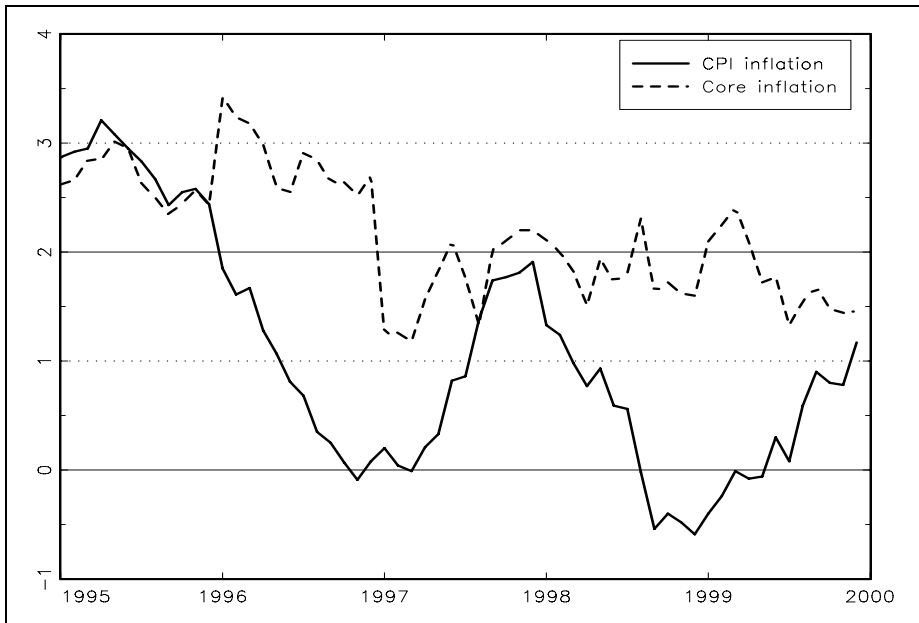
First, temporary disturbances and exogenous components make the CPI inflation rate more volatile than measures of core inflation. Therefore it is tempting to disregard these components when formulating monetary policy, in order to avoid large swings in the interest rate and excessive real volatility.⁵

Second, the presence of indirect taxes in the CPI means that changes in indirect taxes have a direct, possibly persistent, effect on CPI inflation. Monetary policymakers frequently worry about these effects, not wanting to respond to movements in inflation that do not affect the “underlying” inflation trend. At the same time, if these components are simply ignored, important indirect effects on aggregate de-

⁴See, e.g., Heikensten and Vredin (1998) or Heikensten (1999) for non-technical discussions of these practical issues.

⁵For example, Blinder (1997) argues that monetary policymakers should not respond to inflation components beyond their control, e.g. food and energy components, since attempting to offset the inflationary effects of these components leads to large variation in core inflation and real output.

Figure 1: Swedish CPI and core inflation, % per annum



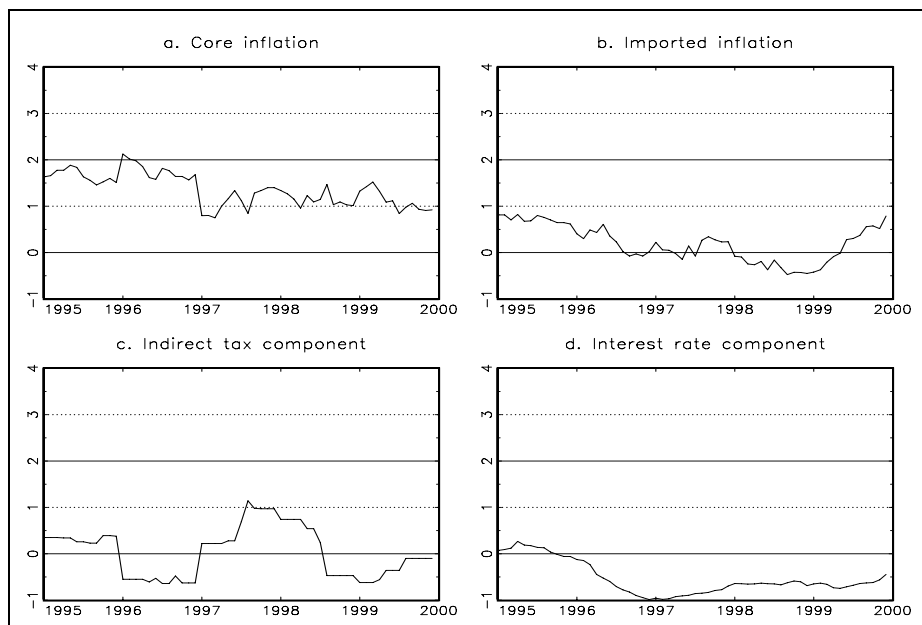
Yearly changes, monthly observations. Source: Statistics Sweden.

mand (which perhaps was the reason for the fiscal policy adjustment in the first place) and core inflation may be overlooked. Our model explicitly addresses this issue.

The third component of CPI inflation that complicates the formulation of monetary policy are the direct effects of policy itself on CPI inflation. In many countries the component of CPI that measures mortgage costs reflects the general level of interest rates. Therefore a policy move that raises interest rates will also raise inflation in the short run. If the central bank targets CPI inflation, should it respond to this increase by raising interest rates further, leading it to “chase its own tail” (Heikensten, 1999)? A second issue also arises: monetary policy now has immediate effects on the CPI inflation rate. Should the central bank use this channel to completely control inflation in the short term, by lowering the interest rate when inflation rises and vice versa?

To illustrate these issues, Figures 1 and 2 show the behavior of the Swedish CPI inflation rate and its main subcomponents from January 1995 to December 1999, along with Sveriges Riksbank’s inflation target of 2% and its tolerance interval of $\pm 1\%$. As seen in Figure 1, the CPI inflation rate has been very volatile over the period, whereas the core inflation rate (defined as domestic inflation excluding the direct effects of taxes and subsidies) is considerably smoother. The core inflation rate has been within the Riksbank’s tolerance interval during (almost) the entire

Figure 2: Contribution of subcomponents to Swedish CPI inflation, % per annum



Yearly changes, monthly observations. The components have been weighted according to their respective share of the CPI. Sources: Statistics Sweden, Sveriges Riksbank.

period, while the CPI inflation rate has strayed outside the interval for long periods.

Figure 2 shows the contribution of four subcomponents to CPI inflation, where the components have been weighted according to their share of the CPI.⁶ (The weighted components thus sum to CPI inflation in Figure 1.) The contribution of domestic core inflation has been between 1% and 2% throughout the period, while imported inflation has had a smaller impact. Changes in indirect taxes and interest rates have had a substantial effect on the variability of CPI inflation, the former with regular jumps and the latter with a persistent negative impact.⁷

1.2 Outline of the paper

The remainder of this paper is organized as follows. Section 2 presents the theoretical model, and demonstrates how numerical methods are used to calculate the optimal policy rule given a standard objective function for the central bank. Section 3 then analyzes the optimal response of monetary policy to different disturbances,

⁶The components' respective weights have varied somewhat over the sample period, but are approximately 63% for core inflation, 28% for imported inflation, and 9% for the interest rate component, while the tax component is added after weighting the other three components.

⁷This negative impact of interest rates on CPI inflation is primarily due to a persistent easing of monetary policy during 1996, which had a slow effect on mortgage costs.

comparing the responses across targeting regimes under different assumptions about some parameter values.

Section 3 presents the results in a rather detailed and technical manner. To provide some more intuition and to emphasize the policy implications of the model, Section 4 summarizes and discusses the results in less technical terms, while Section 5 contains some concluding remarks.

2 The model

The model used is based on the framework of Svensson (1997), and consists of dynamic aggregate supply and demand relationships, with lags in the transmission of monetary policy and persistent effects of supply and demand shocks.⁸ To this framework, we have added different components of the CPI inflation rate, as mentioned in the Introduction.

2.1 The economy

The CPI inflation rate (the percentage change in the CPI, measured as its deviation from the long-run average) is defined as the sum of four components: a core inflation rate, an exogenous inflationary process, the inflationary effects of fiscal policy actions, and the direct effects of monetary policy on the inflation rate (due to e.g. mortgage costs):⁹

$$\pi_t^{cpi} = \pi_t^c + \pi_t^x + \pi_t^g + \pi_t^i. \quad (1)$$

The core inflation rate π_t^c is determined by the level of real activity in the economy, as measured by the output gap y_t , and is positively related to last period's core inflation and output gap, plus an exogenous shock:

$$\pi_{t+1}^c = \pi_t^c + \alpha y_t + \varepsilon_{t+1}^c, \quad (2)$$

where $\alpha > 0$.

⁸Similar models or versions of this model have been used by e.g. Ball (1999), Rudebusch (1999) and Rudebusch and Svensson (1999a,b).

⁹This decomposition of the CPI is similar to that used by Sveriges Riksbank. In its quarterly *Inflation Reports*, movements in the CPI inflation rate are reported along with those in UND1X (CPI excluding interest rate expenditures and the direct effects of altered domestic indirect taxes and subsidies) and UNDINH (UND1X excluding prices on goods that are mainly imported). Our definition of core inflation corresponds most closely to changes in UNDINH, which is also the measure shown in Figures 1 and 2.

The exogenous inflationary process π_t^x represents the effects on inflation of price movements beyond the control of monetary policy, such as prices on commodities or imported goods. This inflationary process is assumed to follow a first-order autoregressive process:

$$\pi_{t+1}^x = \rho_x \pi_t^x + \varepsilon_{t+1}^x, \quad (3)$$

where $0 \leq \rho_x < 1$. When $\rho_x > 0$, a disturbance to the exogenous process has long-lived effects on the overall inflation rate, however without affecting core inflation.¹⁰

The fiscal policy component is also assumed to be exogenous to monetary policy and potentially persistent, following

$$\pi_{t+1}^g = \rho_g \pi_t^g + \varepsilon_{t+1}^g, \quad (4)$$

where $0 \leq \rho_g < 1$. As will be explained below, apart from adding to CPI inflation, fiscal policy also affects aggregate demand and thus future core inflation negatively. This component is intended to capture changes in indirect taxes and subsidies, decided upon by the government, and which have direct positive effects on overall inflation, but potentially also affects output. As shown shortly, we assume that the effect on output is negative, and lagged one period.

Finally, CPI inflation also includes the direct effects of monetary policy, π_t^i , representing home-mortgage costs, which are related to the general level of interest rates. Changes in the short interest rate set by the central bank then have a direct positive effect on CPI inflation:

$$\pi_{t+1}^i = \rho_i \pi_t^i + \varphi \Delta i_{t+1}, \quad (5)$$

where $0 \leq \rho_i < 1$ and $\varphi > 0$. The effect on overall CPI inflation may thus be persistent, depending on the value of ρ_i .

In the dynamic aggregate demand relationship, the output gap (the percentage deviation of output from its natural level) is positively related to past output, and negatively related to the previous period's monetary policy stance, as measured by the ex-ante real short interest rate. As mentioned above, it is also negatively related to the previous period's fiscal policy component:

$$y_{t+1} = \beta y_t - \gamma r_{t+1|t} - \kappa \pi_t^g + \varepsilon_{t+1}^y, \quad (6)$$

¹⁰One difference between the core inflation disturbance ε_t^c and the exogenous disturbance ε_t^x is that the latter shock introduces only exogenous persistence, while the former causes endogenous persistence (cf. Clarida et al., 1999).

where $0 < \beta < 1$; $\gamma, \kappa > 0$, and $r_{t+1|t}$ is the ex-ante real interest rate. The real interest rate is defined in terms of the core inflation rate, so $r_{t+1|t} \equiv i_t - \pi_{t+1|t}^c$, where i_t is the short nominal rate controlled by the central bank, and $\pi_{t+1|t}^c \equiv E_t \pi_{t+1}^c$, i.e. the expected core inflation rate in $t + 1$, given the information set in period t .¹¹ Note that a positive fiscal policy shock in period t has a direct positive effect on the CPI inflation rate (see equation(1)), but also a negative effect on the output gap in period $t + 1$, and on future core inflation. Finally, all shocks ε_{t+1}^j , $j = c, x, g, y$, are independent and identically distributed with zero means and constant variances σ_j^2 .

Monetary policy in this model is conducted by setting the short interest rate i_t , which by changing the short real rate will affect output in the next period. This in turn will affect the core inflation rate (and thus CPI inflation) in subsequent periods. Hence, this *traditional channel* from monetary policy to the inflation rate has a lag of two periods, via the output gap. However, since changes in the interest rate have a direct effect on CPI inflation, there is a second channel by which monetary policy affects inflation. We call this the *direct interest rate channel*.¹²

Note that this second channel suggests monetary policy responses opposite in sign to those implied by the traditional channel. For example, following a negative shock to inflation, the traditional channel suggests a lower interest rate (to boost demand) while the direct interest rate channel will suggest a higher interest rate (to raise the registered current inflation rate). The tension between these two channels is analyzed below.

2.2 Central bank objectives

The central bank is assigned a standard period loss function that is quadratic in deviations of a measure of inflation and the output gap from their constant targets (normalized to zero), and that also includes a preference for interest rate smoothing.¹³ The central bank may target one of three alternative measures of inflation: CPI inflation π_t^{cpi} , CPIX inflation $\pi_t^{cpi x}$ (i.e. CPI excluding the interest rate component), and core inflation π_t^c . The central bank's period loss function then is

$$L_t = \mu_c (\pi_t^c)^2 + \mu_{cpi x} (\pi_t^{cpi x})^2 + \mu_{cpi} (\pi_t^{cpi})^2 + \lambda y_t^2 + \nu (i_t - i_{t-1})^2, \quad (7)$$

¹¹Appendix C explores the implications of defining the real interest rate in terms of expected CPI inflation.

¹²This channel is similar to the exchange rate channel in open economy models, which also affects CPI inflation directly. See e.g. Batini and Haldane (1999) or Svensson (2000).

¹³As explained below, we include a weight on interest rate smoothing to avoid exploding solutions.

where $\pi_t^{cpi} \equiv \pi_t^{cpi} - \pi_t^i = \pi_t^c + \pi_t^g + \pi_t^x$, and where $(\mu_c, \mu_{cpi}, \mu_{cpi})$ are $\{0, 1\}$ variables, only one of them being non-zero at any time. The central bank thus either targets CPI inflation (with $\mu_{cpi} = 1$ and $\mu_c = \mu_{cpi} = 0$), CPIX inflation ($\mu_{cpi} = 1, \mu_c = \mu_{cpi} = 0$), or core inflation ($\mu_c = 1, \mu_{cpi} = \mu_{cpi} = 0$). The parameter $\lambda \geq 0$ measures the weight of output stabilization relative to inflation stabilization, and $\nu \geq 0$ measures the weight on interest rate smoothing.

Monetary policy in this model amounts to the central bank choosing a path for the short interest rate i_t , where the task is to minimize the expected discounted sum of future values of the loss function. Thus it minimizes the intertemporal loss function

$$\sum_{s=0}^{\infty} \delta^s E_t L_{t+s}, \quad (8)$$

where $0 \leq \delta \leq 1$ is a discount factor.

2.3 Optimal monetary policy

The central bank's optimization problem defined above cannot be solved analytically. Instead, numerical methods are used. For that purpose, the model is cast in terms of the standard stochastic linear-quadratic regulator problem (see e.g. Sargent, 1987). Thus, define a (6×1) vector x_t of state variables as

$$x_t \equiv \left[\pi_t^c \quad \pi_t^x \quad \pi_t^g \quad \pi_{t-1}^i \quad y_t \quad i_{t-1} \right]', \quad (9)$$

a (5×1) vector z_t of goal variables as

$$z_t \equiv \left[\pi_t^c \quad \pi_t^{cpi} \quad \pi_t^{cpi} \quad y_t \quad \Delta i_t \right]', \quad (10)$$

and a (6×1) vector ε_t of shocks as

$$\varepsilon_t \equiv \left[\varepsilon_t^c \quad \varepsilon_t^x \quad \varepsilon_t^g \quad 0 \quad \varepsilon_t^y \quad 0 \right]'. \quad (11)$$

Then the model (1)–(7) can be expressed in state-space form as

$$x_{t+1} = Ax_t + Bi_t + \varepsilon_{t+1}, \quad (12)$$

$$z_t = C_x x_t + C_i i_t, \quad (13)$$

$$L_t = z_t' K z_t, \quad (14)$$

where A is a (6×6) matrix, B is a (6×1) vector, C_x is a (5×6) matrix, C_i a (5×1) vector, and K is a (5×5) diagonal matrix with diagonal elements $(\mu_c, \mu_{cpi}, \mu_{cpi}, \lambda, \nu)$ (see Appendix A.1 for details).

As shown in Appendix A.2, the optimal monetary policy rule is to set the short interest rate as a linear function of the current state variables,¹⁴

$$i_t = f x_t. \tag{15}$$

The resulting dynamics of the system then are

$$x_{t+1} = M x_t + \varepsilon_{t+1}, \tag{16}$$

$$z_t = C x_t, \tag{17}$$

where

$$M \equiv (A + Bf) \tag{18}$$

$$C \equiv (C_x + C_i f). \tag{19}$$

3 Analysis

This section contains a detailed and rather technical analysis of the numerical solutions to our model—the reader who is interested mainly in the policy implications will find a summary in Section 4. Rather than presenting the solution in one sweep, we will proceed in a sequential manner. To understand the logic of this, the framework we have proposed in Section 2 can be seen as consisting of a workhorse model (Svensson, 1997) with three additional components of CPI inflation, namely:

1. An exogenous component, π_t^x , with only direct effects on CPIX and CPI inflation.
2. An exogenous component, π_t^g , with both direct and indirect effects on CPIX and CPI inflation, the latter effect via the output gap and core inflation.
3. An interest rate component, π_t^i , with direct effects only on CPI inflation.

(Furthermore, all of these components are either temporary or persistent, depending on the ρ -parameters.) Therefore, when examining the effects of these additional features, we focus on them one at a time. Thus we begin the analysis in this section with the effects of the purely exogenous process in isolation, setting $\kappa =$

¹⁴Since our model (with expected core inflation in the real interest rate) contains no forward-looking variables, the solutions under commitment and discretion coincide. As shown in Appendix C, when the expected CPI inflation rate enters the real interest rate, the model contains some forward-looking elements, so we confine the analysis to the discretionary case.

Table 1: Parameter values

Fixed structural parameters		Varying structural parameters		Preference parameters	
α	0.34	ρ_x	{ 0, 0.3 }	δ	0.95
β	0.77	κ	{ 0, 0.15 }	λ	{ 0, 0.5 }
γ	0.40	ρ_g	{ 0, 0.3 }	ν	0.001
		φ	{ 0, 0.1 }		
		ρ_i	{ 0, 0.3 }		

Values on α , β , and γ are from Orphanides and Wieland (1999).

$\rho_g = 0$ (excluding any effects from π_t^g) and $\varphi = \rho_i = 0$ (excluding the interest rate component π_t^i), and varying ρ_x . Next we study the effects of the fiscal policy component π_t^g in isolation, letting $\kappa > 0$, but $\rho_x = \varphi = \rho_i = 0$, and varying ρ_g . Finally, when analyzing the interest rate component π_t^i , we will set $\varphi > 0$, but $\rho_x = \kappa = \rho_g = 0$ and vary ρ_i . Only in the end do we consider all components at once, to see to what extent they reinforce or offset each other.

Table 1 shows the parameter values that are used in the numerical solutions: the first pair of columns contains the structural parameters that remain fixed throughout the analysis, the second pair contains the structural parameters that are allowed to vary, and the third pair of columns contains preference parameters. The values for the fixed structural parameters α , β , and γ are taken from Orphanides and Wieland (1999), who estimate a similar model (although without the different definitions of inflation) on data from the Euro area. The parameters ρ_x , κ , ρ_g , φ , and ρ_i are assigned values on a more *ad hoc* basis. The central bank is assumed to discount the future at a rate of 5% per period, and its weight on output stabilization is set in accordance with both strict inflation targeting ($\lambda = 0$) and flexible inflation targeting ($\lambda = 0.5$). Finally, to avoid explosive solutions under some targeting regimes, the central bank is assigned a small preference for interest rate smoothing, so $\nu = 0.001$ throughout.¹⁵

3.1 Purely exogenous disturbances

We thus begin by analyzing the effects of adding an exogenous inflation process to the baseline model in isolation, setting $\kappa = \rho_g = \varphi = \rho_i = 0$. Table 2 reports

¹⁵If $\nu = 0$, optimal policy under strict CPI inflation targeting can completely stabilize CPI inflation by fully exploiting the direct interest rate channel. Since such policy entails reducing the interest rate when inflation is high, it leads to exploding paths for the interest rate and the output gap, as well as for core and CPIX inflation, and can be avoided by assigning a small preference for interest rate smoothing.

Table 2: Reaction functions, exogenous disturbances only

λ	Regime	π_t^c	π_t^x	π_t^g	π_{t-1}^i	y_t	i_{t-1}
(a) <i>Temporary disturbance: $\rho_x = 0$</i>							
0	All	6.582	0.000	–	–	3.931	0.035
0.5	All	3.558	0.000	–	–	3.087	0.008
(b) <i>Persistent disturbance: $\rho_x = 0.3$</i>							
0	Core	6.582	0.000	–	–	3.931	0.035
	CPIX/CPI	6.582	0.429	–	–	3.931	0.035
0.5	Core	3.558	0.000	–	–	3.087	0.008
	CPIX/CPI	3.558	0.109	–	–	3.087	0.008

Parameters are set to $\rho_x \in \{0, 0.3\}$, $\kappa = \rho_g = \varphi = \rho_i = 0$. A dash (–) means that the coefficient is zero by construction.

the optimal reaction functions (i.e. the elements of the vector f in equation (15)) for the three targeting regimes under two scenarios: in panel (a) disturbances to the exogenous inflation process have only temporary effects (so $\rho_x = 0$), while in panel (b) the effects are persistent ($\rho_x = 0.3$). Since we suppress the interest rate component, CPIX targeting and CPI targeting will coincide, and we will refer to this as the CPIX/CPI regime.

How should the central bank respond to disturbances to the exogenous inflation process? Consider first purely temporary exogenous disturbances in panel (a) of Table 2. The optimal response to such shocks in all regimes is to do nothing (i.e. the coefficients in the column for π_t^x are all zero), both under strict and flexible inflation targeting. This result is a natural consequence of the time lag of monetary policy in this particular model. Interest rate changes affect the inflation rate with a two-period lag; therefore policy must be guided by the inflation (and output) forecasts. Thus there is no reason to respond to purely temporary disturbances to inflation, since these affect neither future inflation nor the forecast. Furthermore, note that optimal monetary policy responds to movements in core inflation also under CPIX/CPI targeting, and not to the targeted variables themselves, i.e. not to the exogenous process which is a component of both CPIX and CPI inflation. This is because current core inflation is a better predictor of future CPI inflation than is current CPI, since core inflation excludes temporary disturbances.

Consider next the case of persistent exogenous disturbances in panel (b). Now the regimes differ. When core inflation is the goal variable, policy needs not respond to the exogenous inflation disturbance since the core inflation rate is unaffected.

Table 3: Reaction functions, fiscal policy disturbances only

λ	Regime	π_t^c	π_t^x	π_t^g	π_{t-1}^i	y_t	i_{t-1}
(a) <i>Temporary disturbance: $\rho_g = 0$</i>							
0	All	6.582	–	–0.330	–	3.931	0.035
0.5	All	3.558	–	–0.366	–	3.087	0.008
(b) <i>Persistent disturbance: $\rho_g = 0.3$</i>							
0	Core	6.582	–	–0.344	–	3.931	0.035
	CPIX/CPI	6.582	–	0.086	–	3.931	0.035
0.5	Core	3.558	–	–0.368	–	3.087	0.008
	CPIX/CPI	3.558	–	–0.260	–	3.087	0.008

Parameters are set to $\kappa = 0.15$, $\rho_g \in \{0, 0.3\}$, $\rho_x = \varphi = \rho_i = 0$. A dash (–) means that the coefficient is zero by construction.

Under CPIX/CPI targeting, on the other hand, a positive exogenous disturbance has persistent effects on the goal variables, so the central bank must act to offset these by depressing future output and core inflation.¹⁶ Thus, when there are persistent exogenous disturbances the regimes will differ, since the exogenous process affects the respective goal variables differently.

3.2 Fiscal policy

We next consider the effects of adding so-called fiscal policy disturbances, i.e. exogenous disturbances with both direct and indirect effects on inflation, setting $\rho_x = \varphi = \rho_i = 0$. Again CPIX and CPI targeting will coincide. Table 3 presents the optimal reaction functions in this scenario. When the fiscal policy shock is temporary—panel (a)—the optimal response in all regimes is to lower the interest rate after a positive disturbance, to offset the effects on output and thus on future inflation.¹⁷

Persistence in the fiscal policy shock—panel (b)—hardly changes this conclusion under core inflation targeting.¹⁸ Under CPIX/CPI targeting, on the other

¹⁶This is the mechanism discussed by Blinder (1997), and mentioned in footnote 5 above. However, in our framework, it only applies when the exogenous shocks to CPI inflation are persistent.

¹⁷Without interest rate smoothing (i.e. when $\nu = 0$, the effects on output would be completely neutralized by setting the coefficient to $-\kappa/\gamma = -0.375$; here the response is slightly smaller, so some effects on output remain.

¹⁸Again, without interest rate smoothing, the coefficient on π_t^g would remain at -0.375 , as the negative effects on future levels of output are offset by continued expansionary policy. Now, with a weight on interest rate smoothing, that path is slightly altered, since changes in the interest rate lead to disutility for the central bank.

Table 4: Reaction functions, direct interest rate effect only

λ	Regime	π_t^c	π_t^x	π_t^g	π_{t-1}^i	y_t	\dot{i}_{t-1}
<i>(a) Temporary interest rate effect: $\rho_i = 0$</i>							
0	Core/CPIX	6.582	–	–	0.000	3.931	0.035
	CPI	4.509	–1.532	–1.532	0.000	3.025	0.169
0.5	Core/CPIX	3.558	–	–	0.000	3.087	0.008
	CPI	3.199	–0.622	–0.622	0.000	2.834	0.068
<i>(b) Persistent interest rate effect: $\rho_i = 0.3$</i>							
0	Core/CPIX	6.582	–	–	0.000	3.931	0.035
	CPI	4.671	–1.594	–1.594	–0.172	3.194	0.073
0.5	Core/CPIX	3.558	–	–	0.000	3.087	0.008
	CPI	3.279	–0.646	–0.646	–0.073	2.916	0.031

Parameters are set to $\varphi = 0.1, \rho_i \in \{0, 0.3\}, \rho_x = \kappa = \rho_g = 0$. A dash (–) means that the coefficient is zero by construction.

hand, policy must also counteract the effects on future inflation, pushing the interest rate in the opposite, contractionary, direction. When $\lambda = 0$ the contractionary effect dominates, causing the net effect to be a rise in the interest rate. Under flexible inflation targeting ($\lambda = 0.5$), however, the negative effects on output are more undesirable, so the net effect is a fall in the interest rate.

3.3 Direct interest rate effects

Finally, consider Table 4, which shows the optimal reaction functions when only the interest rate component π_t^i is added (i.e. when $\rho_x = \kappa = \rho_g = 0$). Optimal policy under core and CPIX targeting—which now will coincide by construction—is not affected by the presence of the interest rate component, regardless of its degree of persistence. Thus the relevant rows in panels (a) and (b) are identical to the corresponding rows in Table 2.¹⁹

In contrast, if CPI is the goal variable, optimal monetary policy is very much affected by the presence of an interest rate component in the CPI: the response to all shocks is now different. Now interest rate changes have a contemporaneous effect on the CPI inflation rate (increases in the interest rate lead to higher current CPI inflation, and vice versa), meaning that the direct interest rate channel in isolation

¹⁹If the ex ante real interest rate is defined in terms of the expected CPI inflation rate, this is no longer true, since inflation expectations are directly affected by changes in the interest rate. See Appendix C.

gives the central bank an incentive to *lower* the interest rate after an inflationary shock. The net effect on the policy response, however, depends on the nature of the initial shock. After positive shocks with persistent effects (such as those to core inflation and output), motivating a higher interest rate, this contractionary move is moderated by the presence of the direct interest rate channel.²⁰ Thus the coefficients on core inflation and output in panel (a) of Table 4 are smaller under CPI targeting than under core/CPIX targeting. On the other hand, if the shock has only temporary effects (such as those to the exogenous process or the fiscal policy component, which with only the traditional channel would not prompt any policy reaction at all) the presence of the direct interest rate channel leads to a more activist, expansionary, monetary policy. Policy should now respond also to purely temporary shocks, to offset some of the effects on current CPI inflation. Thus the coefficients on π_t^x and π_t^g in Table 4 are no longer zero, even though these components have only temporary effects on CPI inflation.

The effect of persistence in the interest rate component—panel (b)—is to magnify the policy response: if the optimal response when $\rho_i = 0$ is to increase (decrease) the interest rate, a positive value of ρ_i will lead to further increases (decreases) in the current interest rate. The intuition behind this is straightforward. Persistence in the interest rate component means that interest rate changes cause movements in future CPI inflation of the same sign, which must be offset by movements in core inflation of the opposite direction. In a sense, monetary policy is less potent, since the effects of the direct interest rate channel partly offsets the traditional negative effect of an interest rate increase on future CPI inflation.

In sum, the presence of the direct interest rate channel pushes the optimal policy response following positive shocks in an expansionary direction (and vice versa). This has a moderating effect on the response to persistent disturbances²¹ (but de-

²⁰If the contractionary response is very mild, the net effect may be an expansionary move, as would be the case with shocks to a persistent exogenous component. This is not shown in Table 4, since neither the exogenous component nor the fiscal policy component is persistent.

²¹When φ is very small, this is no longer true. Instead the existence of the interest rate component makes policy *more aggressive*. This can be understood in the following way. Let $\rho_i = 0$. Then CPI inflation is given by $\pi_t^{cpi} = \pi_t^{cpi x} + \varphi \Delta i_t$, and its variance is

$$\text{Var}(\pi_t^{cpi}) = \text{Var}(\pi_t^{cpi x}) + \varphi^2 \text{Var}(\Delta i_t) + 2\varphi \text{Cov}(\pi_t^{cpi x}, \Delta i_t).$$

For the parameter values used in this paper, $\text{Cov}(\pi_t^{cpi x}, \Delta i_t)$ is negative (since CPIX inflation falls when the interest rate is raised), but $\varphi^2 \text{Var}(\Delta i_t)$ is larger than $2\varphi \text{Cov}(\pi_t^{cpi x}, \Delta i_t)$ in absolute value. Thus less interest rate variability will lead to lower CPI variability and thus to higher utility under CPI targeting. This holds true for all φ larger than 0.05. However, when φ is very small, the covariance term may well dominate the variance term, so less interest rate variability reduces

Table 5: Reaction functions, full model

λ	Regime	π_t^c	π_t^x	π_t^g	π_{t-1}^i	y_t	i_{t-1}
0	Core	6.582	0.000	-0.344	0.000	3.931	0.035
	CPIX	6.582	0.429	0.086	0.000	3.931	0.035
	CPI	4.671	-0.573	-0.809	-0.172	3.194	0.073
0.5	Core	3.558	0.000	-0.368	0.000	3.087	0.008
	CPIX	3.558	0.109	-0.260	0.000	3.087	0.008
	CPI	3.279	-0.242	-0.560	-0.073	2.916	0.031

Parameters are set to $\kappa = 0.15$, $\varphi = 0.1$, $\rho_x = \rho_g = \rho_i = 0.3$.

creasingly so with the degree of persistence in the interest rate component), but leads to more activism following temporary disturbances. In the latter case, policy induces variability in the real sector.

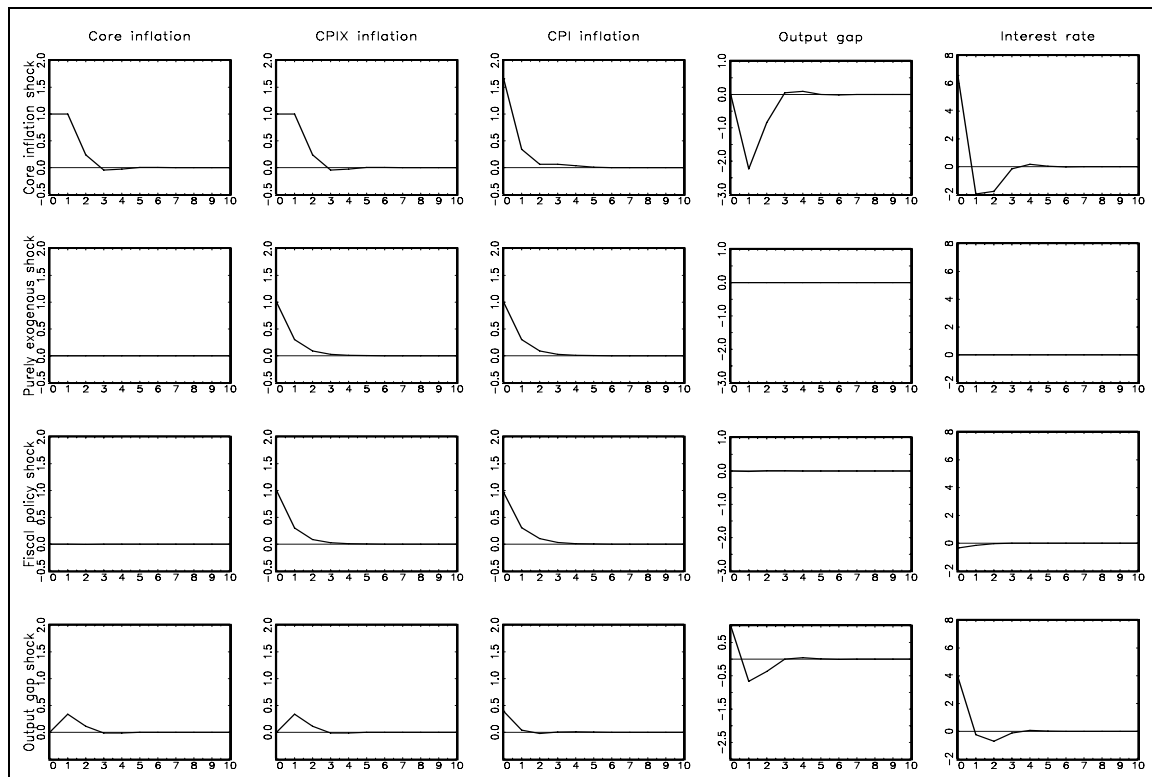
Finally, note that although the central bank does take the direct effects of interest rates on CPI inflation into account when formulating policy, it does not exploit the direct channel fully. As long as the central bank puts some weight on output or interest rate stabilization, it is never optimal to completely control inflation by exploiting the interest rate channel, since such a policy would lead to instability in both output and the interest rate (see also footnote 15).

3.4 The full model

Table 5 contains the optimal reaction functions when all three additional features of our model are present, and with some persistence (i.e. $\rho_x = \rho_g = \rho_i = 0.3$). In the case of core and CPIX inflation targeting, all coefficients can be found in Tables 2–3, and there is therefore no further need to comment on them. Only in the case of CPI inflation targeting do the exogenous disturbances and the interest rate component simultaneously affect individual coefficients in the reaction function. For example, the coefficient on the purely exogenous process π_t^x under CPI targeting is a combination of the contractionary influence of the persistent disturbance (raise the interest rate to offset positive future values of π_t^x ; see Table 2) and the expansionary influence from the direct interest rate channel (lower the interest rate to offset the increase in current and future CPI inflation; see Table 4), the net effect here being

utility for the central bank. Thus the response to core inflation and output is more aggressive under CPI targeting. (We are grateful to Lars Svensson for pointing out this potential property of the model.)

Figure 3: Response to shocks, strict core inflation targeting



expansionary. The coefficient on the fiscal policy component π_t^g is similarly the product of opposing forces.

3.4.1 Impulse responses

For the full model, impulse responses for all target variables—the three inflation measures, the output gap, and the interest rate—are presented in Figures 3–5. These figures represent the three targeting regimes under strict inflation targeting. The impulse responses under flexible inflation targeting are very similar, and are discussed briefly in Appendix B.

When core inflation is the goal variable in Figure 3, the policy responses to core inflation and output gap disturbances (in the first and last rows) are the same as in Svensson (1997): the interest rate is raised to push core inflation back to target quickly. Here, when there is a small preference for interest rate stabilization, core inflation is almost back on target after three periods.²² The interest rate response is characterized by large swings, since the strong initial policy response creates a deep recession which is later offset by expansionary policy. There is no response to purely

²²With no weight on interest rate smoothing, core inflation would be back on target after two periods, i.e. as soon as possible.

Figure 4: Response to shocks, strict CPIX inflation targeting

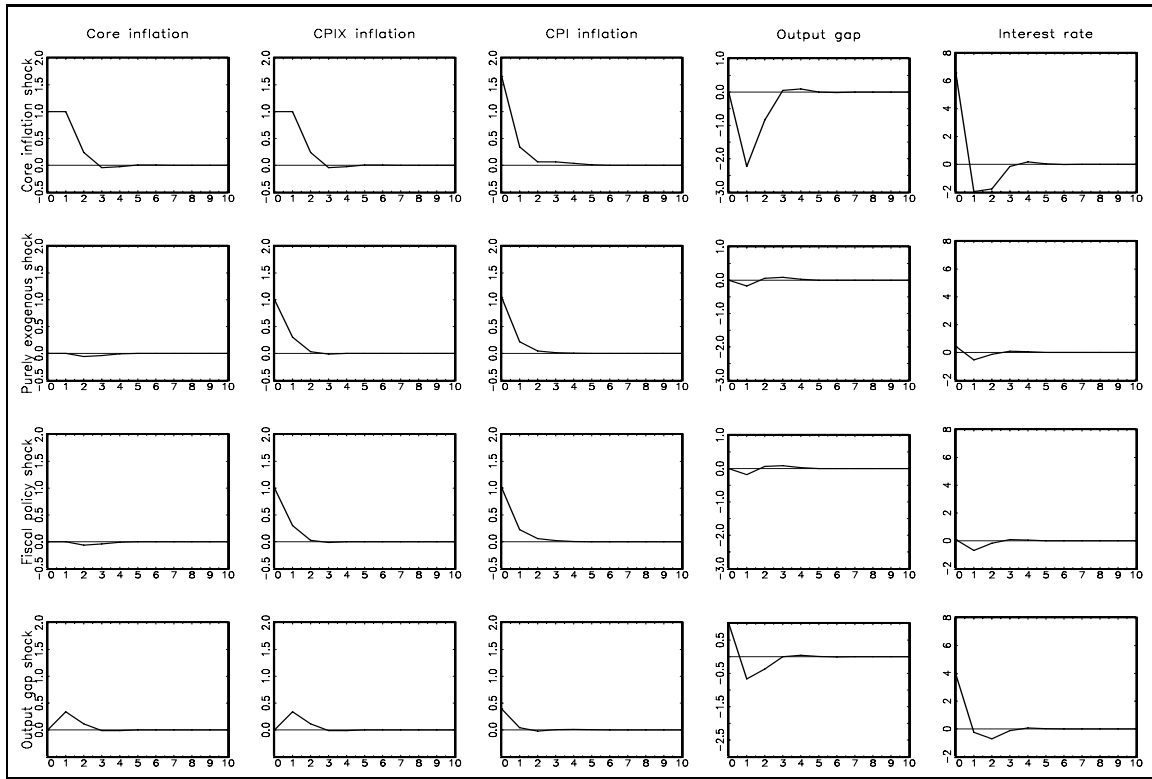


Figure 5: Response to shocks, strict CPI inflation targeting

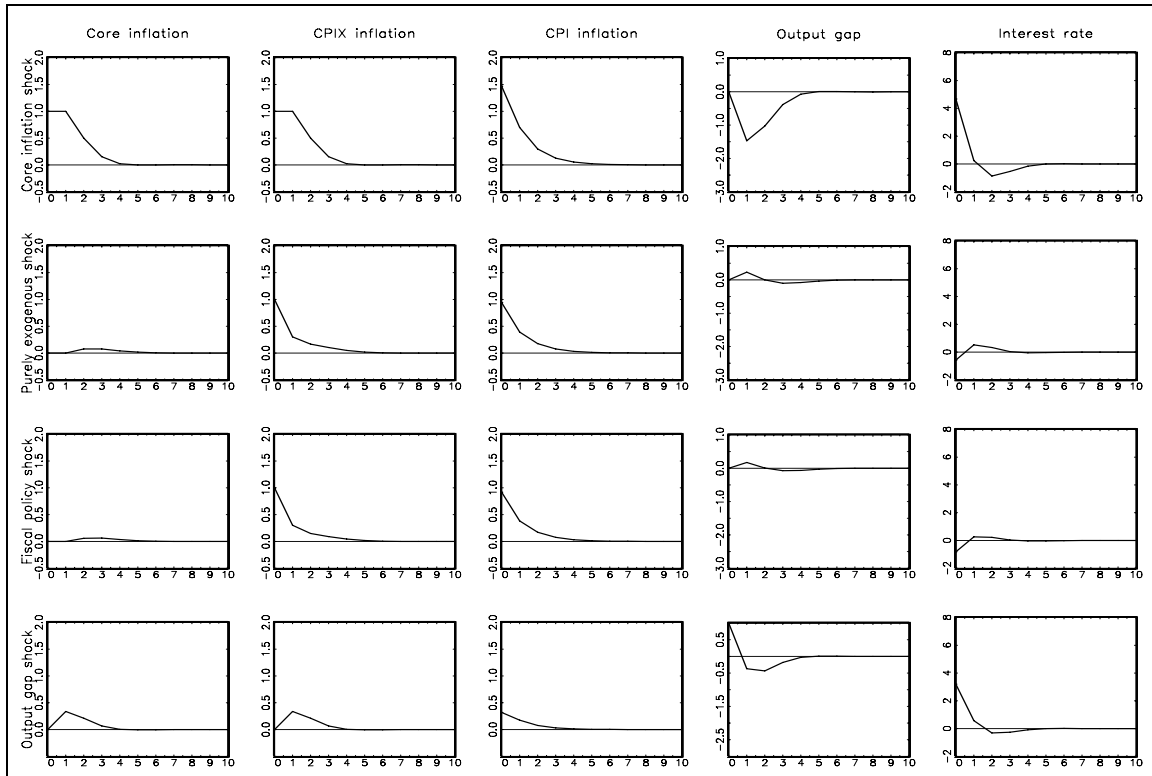


Table 6: Unconditional variances of goal variables, benchmark case

λ	Regime	π_t^c	$\pi_t^{cpi,x}$	π_t^{cpi}	y_t	i_t	Δi_t
0	Core	2.19	4.39	5.17	7.28	66.59	152.80
	CPIX	2.20	4.37	5.24	7.37	67.47	154.91
	CPI	2.46	4.69	4.99	4.83	35.17	64.35
0.5	Core	2.92	5.12	5.35	3.07	22.63	40.53
	CPIX	2.92	5.11	5.36	3.08	22.63	40.48
	CPI	3.06	5.27	5.43	2.90	20.46	33.42

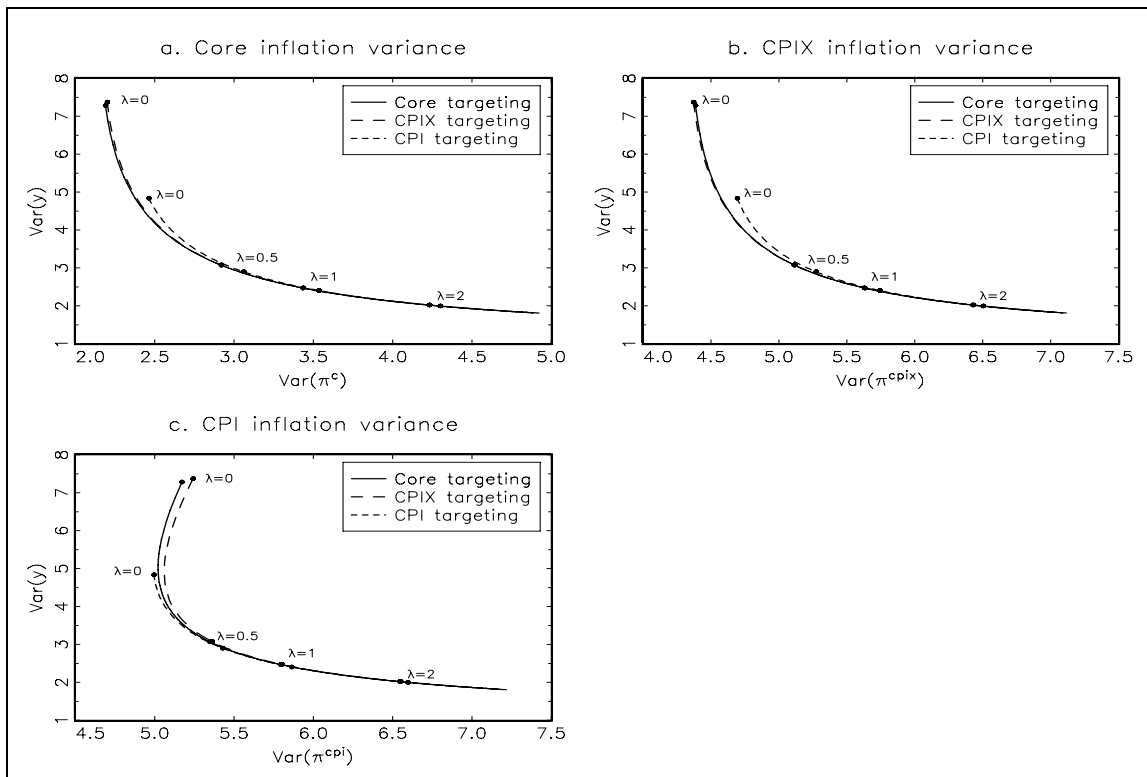
Parameters are set to $\kappa = 0.15$, $\varphi = 0.1$, $\rho_x = \rho_g = \rho_i = 0.3$. The variances of all shocks are set to unity.

exogenous inflation disturbances (in the second row), although these are persistent, since they have no effects on core inflation. After a fiscal policy disturbance (in the third row), there is a small expansionary response to neutralize the effects on output (and thus on future core inflation). As noted above, due to the weight on interest rate changes, the output effects are not entirely offset, so there is some remaining movement (although barely visible) in the output gap after a fiscal policy shock.

When CPIX inflation is the goal variable in Figure 4, the responses to core inflation and output gap disturbances are the same as under core inflation targeting. The response to an exogenous inflation disturbance is different, however: future rates of core inflation must be depressed to offset the persistent effects of the disturbance on CPIX inflation. After a fiscal policy disturbance, policy in addition needs to respond to the negative effects on the output gap, so the initial tightening is smaller. As a consequence, CPIX inflation is back on target faster than under core targeting after both exogenous and fiscal policy disturbances.

Figure 5 shows the impulse responses under strict CPI inflation targeting. There is now a milder response to both core inflation and output disturbances, due to the direct effects of the interest rate change on CPI inflation, as discussed earlier. As a consequence, CPI inflation is slightly lower in the first period than under the alternative regimes, but moves back to target more slowly, and the interest rate path is smoother than in the previous figures. Relative to CPIX targeting, the response to exogenous disturbances and fiscal policy is more expansive in the first period, since the central bank wants to minimize the increase in current CPI inflation, leading to stronger movements in the output gap, or policy-induced real variability.

Figure 6: Variance frontiers



3.4.2 Variances of goal variables

The properties of the model may also be summarized by examining the unconditional variances of the goal variables presented in Table 6 (see Appendix A.3 for details on how these are calculated). Moving from core to CPIX inflation targeting naturally increases the variance of core inflation and decreases that of CPIX inflation. However, the variances of CPI inflation, the output gap and the interest rate also increase, since policy responds to more disturbances under CPIX targeting. Under CPI targeting, the variances of CPI inflation, output, and the interest rate are substantially lower than in the other two regimes, while both core and CPIX inflation naturally become more volatile. Moving from strict to flexible inflation targeting, the variances of all inflation measures naturally increase, whereas those of output and the interest rate fall in all regimes. Note also that the variances under core and CPIX targeting are now virtually identical.

Finally, Figure 6 shows the variance frontiers in terms of the variance of output against the variances of the three inflation measures under the three regimes. These frontiers are constructed by tracing out the variance in output and each inflation measure as λ increases from 0 to 3. Figure 6a shows the variance frontiers for the

three regimes in $\text{Var}(y_t)\text{--}\text{Var}(\pi_t^c)$ space, Figure 6b in $\text{Var}(y_t)\text{--}\text{Var}(\pi_t^{cpi})$ space, and Figure 6c in $\text{Var}(y_t)\text{--}\text{Var}(\pi_t^{cpi})$ space. By construction, the variance frontier of regime X will dominate (lie closer to the origin) in $\text{Var}(y)\text{--}\text{Var}(X)$ space. Nonetheless, the figures provide interesting information as to how far apart the frontiers are, and what happens to the “other” inflation measures under a particular regime.

As we have mentioned before, the differences between the regimes more or less disappear as the bank cares more about output stabilization (i.e. as λ increases). Hence, in Figure 6, the frontiers almost coincide when λ is above 0.5. For smaller values of λ , however, differences arise. Looking at panels (a) and (b), the variance frontiers for core and CPIX targeting typically lie inside that for CPI targeting, but reach further up toward the north-west. Thus, the variances of core and CPIX inflation are lower under core and CPIX targeting, whereas the variance of output (for a given λ) is considerably larger, especially when λ is small.

Panel (c) shows that the variance of CPI inflation naturally is smallest under CPI targeting, but as inflation targeting becomes more strict, the frontiers under core and CPIX targeting bend up toward north-east. Thus, the variance of CPI inflation increases under more strict core/CPIX targeting, and the variance of output becomes substantially larger. This is because core and CPIX targeting entails more interest rate volatility. Thus, relatively strict targeting of core or CPIX inflation leads to considerably larger variance in both CPI inflation and output than CPI inflation targeting.

4 Policy implications

Having gone through the optimal response of monetary policy to various disturbances under three different regimes and a wide range of parameter configurations, we in this section try to draw some policy implications.²³

Result 1 *Barring the interest rate component, purely temporary shocks to inflation do not affect optimal monetary policy under any regime. Removing such shocks (if such can be found) from the targeted inflation rate therefore has no effect on the conduct of monetary policy.*

The existence of purely temporary disturbances to the CPI is thus not a valid reason in itself to use core inflation as a goal variable if the intent is for monetary policy to avoid responding to temporary shocks. Due to the control lag of monetary policy,

²³See also Apel et al. (1999) for a non-technical discussion of some of the main arguments.

the central bank should not respond to these shocks under any regime, since their effects disappear before policy can respond (see Table 2). However, while monetary policy will be identical under the three regimes in the face of temporary shocks, the degree of target fulfillment will of course differ. The regime with the most narrow measure may appear to be more successful than a regime with a target measure that includes the temporary shock.²⁴

Result 2 *Indirect (second-round) effects of disturbances on goal variables need to be accounted for properly. Simply excluding seemingly temporary disturbances from the reaction function risks leading to inappropriate policy responses.*²⁵

In our model, although the direct effects of an indirect tax increase is to raise inflation, the indirect effects on inflation via depressed output dominate the direct effect in the longer run. Thus, the central bank should lower the interest rate following a fiscal policy disturbance, even though the initial effect on inflation is positive (see Table 3). Another example is when a temporary shock affects inflation expectations, and thus the real interest rate. Such shocks must be offset under all regimes.

Result 3 *It may be optimal to respond to changes in one measure of inflation (e.g. core inflation) even if the target is defined in terms of another (e.g. CPI inflation).*

Put differently, the presence of a particular measure of inflation in the optimal policy rule does not necessarily imply that this measure enters the central bank's objective function. For example, optimal policy responds to changes in core inflation also under CPIX and CPI targeting, but not to all changes in CPIX/CPI inflation (see Table 2). This is a consequence of the control lag of monetary policy: when policy is guided by the forecasts of goal variables, it should respond to the determinants of these forecasts, and not to the goal variables themselves.²⁶

Result 4 *The presence of a direct interest rate component in the CPI presents the central bank with a second channel for affecting inflation. The effect of this direct interest rate channel on optimal monetary policy in isolation is expansionary, i.e.*

²⁴If credibility is perfect this should not matter; it should be easily explained to the public that the difference lies outside the control of the central bank. However, with imperfect credibility one cannot rule out that target fulfillment is of importance. To analyze issues such as these, however, a different model framework is needed.

²⁵See also Eitrheim and Wulfsberg (1999) for a related discussion.

²⁶See e.g. Svensson (1997, 1999b) for a discussion.

following a positive disturbance to inflation, the central bank will want to lower the interest rate to limit the movement in current CPI inflation. The net effect, considering also the traditional channel, depends on the nature of the initial disturbance.

After shocks with persistent effects, when the traditional channel suggests a contractionary response, the net effect typically is contractionary, although more moderate (see Table 4, the columns for π_t^c and y_t). In this case, policy with a CPI inflation target is less variable than with a CPIX or core inflation target, and real variability is correspondingly reduced (see Table 6). After temporary shocks, where the traditional channel suggests an unchanged interest rate, the net effect is expansionary (Table 4, the columns for π_t^x and π_t^g). In this case, policy with a CPI inflation target is more variable than with a CPIX or core inflation target, inducing greater real variability. Furthermore, as soon as the central bank is only slightly concerned with output or interest rate stability, it does not exploit the direct interest rate channel fully. As discussed in Result 4, however, it does take this channel into account when formulating policy.

5 Concluding remarks

In this paper we formulate a framework for analyzing an important operational issue in inflation targeting, namely the choice of price index to target. Our model may be seen as an extension of Svensson (1997) where we account for several different components of inflation. We can thus distinguish between not only “core” and overall CPI inflation, but also an intermediate definition (CPIX inflation) where interest rate components due to home-mortgage costs are excluded from the CPI. Using this model we examine optimal monetary policy under three different regimes: core inflation targeting, CPIX inflation targeting and CPI inflation targeting.

The results are summarized in a non-technical manner in Section 4. One set of results concerns the optimal response to disturbances to what we call “exogenous” inflationary processes (such as imported inflation or changes in indirect taxes). If these disturbances have only temporary effects, the optimal response is an unchanged interest rate, regardless of inflation measure in the objective function (barring the interest rate component). If the disturbances have persistent effects, the regime matters: under core inflation targeting monetary policy should not respond, while under CPIX or CPI targeting policy must offset the long-run effects of the exogenous disturbance. If, in turn, the exogenous disturbances also have indirect effects on

inflation (an example being an increase in indirect taxes which not only adds to inflation, but also depresses future output), monetary policy under all regimes must act to boost aggregate demand, so a central bank targeting core inflation will also respond to such disturbances.

Another set of results concerns the effects of an interest rate component in the CPI. This opens up a new channel whereby monetary policy can affect CPI inflation: the direct interest rate channel. However, to make use of this channel, the interest rate should move in the opposite direction to what the traditional channel dictates (i.e. the interest rate is lowered when inflation is high). Our analysis suggests that the presence of the interest rate component in isolation has an expansionary effect on policy. The net effect, however, considering also the traditional channel, depends on the nature of the initial disturbance.

A Solving the benchmark model

A.1 State-space representation

In the model of Section 2, the real interest rate is defined in terms of the core inflation rate:

$$r_{t+1|t} = i_t - \pi_{t+1|t}^c. \quad (20)$$

Using this and the definition of core inflation from (2) in the output gap (6) yields

$$\begin{aligned} y_{t+1} &= \beta y_t - \gamma (i_t - \pi_t^c - \alpha y_t) - \kappa \pi_t^g + \varepsilon_{t+1}^y \\ &= (\beta + \alpha \gamma) y_t - \gamma (i_t - \pi_t^c) - \kappa \pi_t^g + \varepsilon_{t+1}^y. \end{aligned} \quad (21)$$

Then the state-space representation is

$$x_{t+1} = Ax_t + Bi_t + \varepsilon_{t+1}, \quad (22)$$

where x_t is a state vector:

$$x_t \equiv \left[\pi_t^c \quad \pi_t^x \quad \pi_t^g \quad \pi_{t-1}^i \quad y_t \quad i_{t-1} \right]', \quad (23)$$

ε_{t+1} is a vector containing the shocks:

$$\varepsilon_{t+1} \equiv \left[\varepsilon_{t+1}^c \quad \varepsilon_{t+1}^x \quad \varepsilon_{t+1}^g \quad 0 \quad \varepsilon_{t+1}^y \quad 0 \right]', \quad (24)$$

and the matrix A and the vector B are given by

$$A \equiv \begin{bmatrix} 1 & 0 & 0 & 0 & \alpha & 0 \\ 0 & \rho_x & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_g & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_i & 0 & -\varphi \\ \gamma & 0 & -\kappa & 0 & \beta + \alpha \gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (25)$$

$$B \equiv \left[0 \quad 0 \quad 0 \quad \varphi \quad -\gamma \quad 1 \right]'. \quad (26)$$

Defining a vector of goal variables as

$$z_t \equiv \left[\pi_t^c \quad \pi_t^{cpi x} \quad \pi_t^{cpi} \quad y_t \quad \Delta i_t \right]', \quad (27)$$

these can be expressed in terms of the state variables and the instrument as

$$\begin{aligned} z_t &= C_x x_t + C_i i_t \\ &= \begin{bmatrix} C_x & C_i \end{bmatrix} \begin{bmatrix} x_t \\ i_t \end{bmatrix}, \end{aligned} \quad (28)$$

where

$$C_x \equiv \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & \rho_i & 0 & -\varphi \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}, \quad (29)$$

and

$$C_i \equiv \begin{bmatrix} 0 & 0 & \varphi & 0 & 1 \end{bmatrix}'. \quad (30)$$

Then the loss function (7) is

$$L_t = z_t' K z_t, \quad (31)$$

where K is a diagonal preference matrix with diagonal elements $(\mu_c, \mu_{cpi}, \mu_{cpi}, \lambda, \nu)$.

A.2 Optimal policy

The central bank's loss function can be expressed as

$$\begin{aligned} L_t &= z_t' K z_t \\ &= \begin{bmatrix} x_t' & i_t' \end{bmatrix} \begin{bmatrix} C_x' \\ C_i' \end{bmatrix} K \begin{bmatrix} C_x & C_i \end{bmatrix} \begin{bmatrix} x_t \\ i_t \end{bmatrix} \\ &= x_t' C_x' K C_x x_t + x_t' C_x' K C_i i_t + i_t' C_i' K C_x x_t + i_t' C_i' K C_i i_t \\ &\equiv x_t' Q x_t + x_t' U i_t + i_t' U' x_t + i_t' R i_t, \end{aligned} \quad (32)$$

where

$$Q \equiv C_x' K C_x \quad (33)$$

$$U \equiv C_x' K C_i \quad (34)$$

$$R \equiv C_i' K C_i. \quad (35)$$

Now the problem is re-formulated into the usual form, and we can go on solving it using standard methods (cf. Chow, 1975; Ljungqvist and Sargent, 1997).

The control problem is

$$J(x_t) = \min_{i_t} \{x_t' Q x_t + x_t' U i_t + i_t' U' x_t + i_t' R i_t + \delta E_t J(x_{t+1})\}, \quad (36)$$

subject to the transition equation

$$x_{t+1} = A x_t + B i_t + \varepsilon_{t+1}. \quad (37)$$

Since the objective function is quadratic and the constraint linear, the value function will be of the form

$$J(x_t) = x_t'Vx_t + w, \quad (38)$$

where V is a positive semi-definite matrix and w is a scalar. The Bellman equation is then given by

$$x_t'Vx_t + w = \min_{i_t} \left\{ x_t'Qx_t + x_t'Ui_t + i_t'U'x_t + i_t'Ri_t + \delta \left[x_{t+1|t}'Vx_{t+1|t} + w \right] \right\}. \quad (39)$$

Using $x_{t+1|t} = Ax_t + Bi_t$, the first-order condition for the minimization problem is²⁷

$$2U'x_t + 2Ri_t + 2\delta B'V(Ax_t + Bi_t) = 0, \quad (40)$$

and rearranging, we get

$$(R + \delta B'VB)i_t = -(U' + \delta B'VA)x_t, \quad (41)$$

leading to the decision rule

$$i_t = fx_t, \quad (42)$$

where

$$f \equiv -(R + \delta B'VB)^{-1}(U' + \delta B'VA). \quad (43)$$

Substituting the transition rule and the decision rule into the Bellman equation (39), we get

$$\begin{aligned} x_t'Vx_t + w &= x_t'Qx_t + x_t'Ui_t + i_t'U'x_t + i_t'Ri_t \\ &\quad + \delta [(Ax_t + Bi_t)'V(Ax_t + Bi_t) + w] \\ &= x_t'Qx_t + x_t'Ufx_t + x_t'f'U'x_t + x_t'f'Rfx_t \\ &\quad + \delta [(Ax_t + Bfx_t)'V(Ax_t + Bfx_t) + w] \\ &= x_t'[Q + Uf + f'U' + f'Rf + \delta M'VM]x_t + \delta w, \end{aligned} \quad (44)$$

where $M = A + Bf$. Thus the solution is characterized by the Ricatti equation

$$V = Q + Uf + f'U' + f'Rf + \delta M'VM, \quad (45)$$

²⁷Use the rules $\partial x'Ax/\partial x = (A + A')x$, $\partial y'Bz/\partial y = Bz$, and $\partial y'Bz/\partial z = B'y$, and the fact that R and V are symmetric. See, e.g. Ljungqvist and Sargent (1997).

where

$$M \equiv A + Bf, \quad (46)$$

$$f \equiv -(R + \delta B'VB)^{-1}(U' + \delta B'VA). \quad (47)$$

Finally, the goal variables can be written as

$$z_t = Cx_t, \quad (48)$$

where $C \equiv C_x + C_i f$.

A.3 Unconditional variances

Following Rudebusch and Svensson (1999b, Appendix A), since the goal variables are related to the state variables by equation (48), the covariance matrix of the goal variables is given by

$$\Sigma_{zz} = C\Sigma_{xx}C', \quad (49)$$

where Σ_{xx} is the unconditional covariance matrix of the state variables. Since the dynamics of the state variables is given by

$$x_{t+1} = Mx_t + \varepsilon_{t+1}, \quad (50)$$

Σ_{xx} fulfills

$$\Sigma_{xx} = M\Sigma_{xx}M' + \Sigma_{\varepsilon\varepsilon}, \quad (51)$$

where $\Sigma_{\varepsilon\varepsilon}$ is the unconditional covariance matrix of the shock vector ε_{t+1} , and so is a diagonal matrix with diagonal elements $(\sigma_c^2, \sigma_g^2, \sigma_x^2, 0, \sigma_y^2, 0)$.

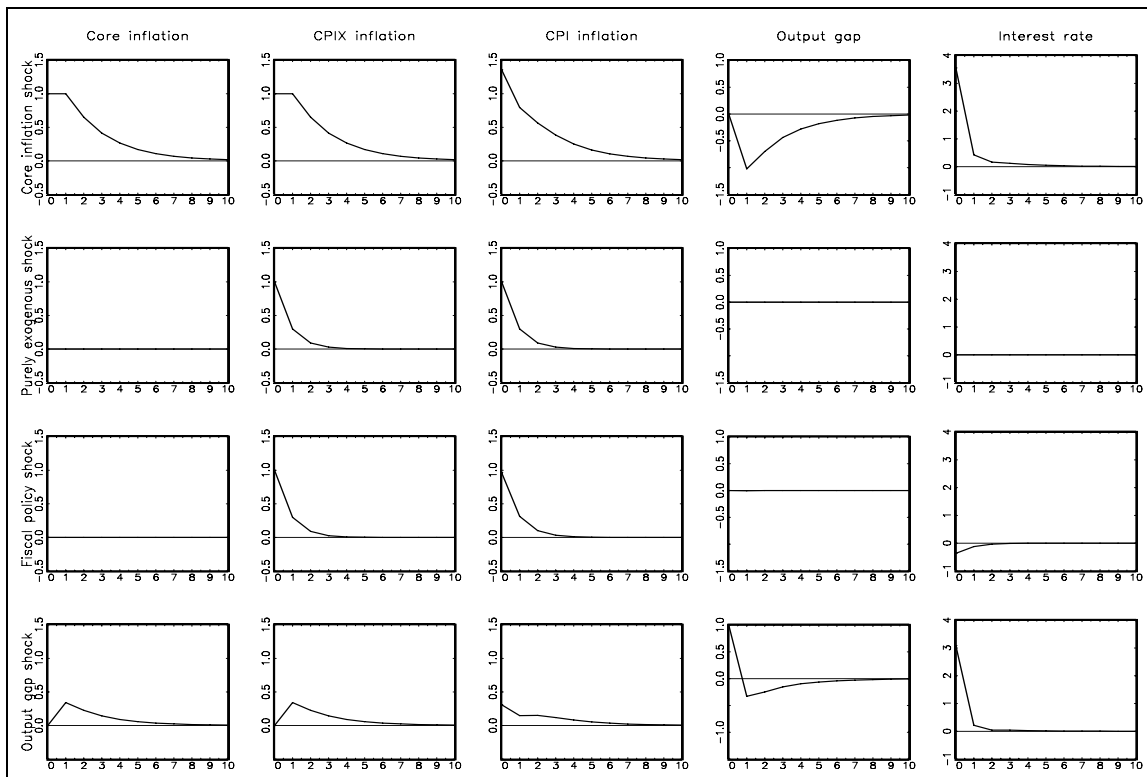
To solve equation (51), we rewrite the matrices in terms of stacked column vectors and solve for $\text{vec}(\Sigma_{xx})$:²⁸

$$\begin{aligned} \text{vec}(\Sigma_{xx}) &= \text{vec}(M\Sigma_{xx}M') + \text{vec}(\Sigma_{\varepsilon\varepsilon}) \\ &= (M \otimes M) \text{vec}(\Sigma_{xx}) + \text{vec}(\Sigma_{\varepsilon\varepsilon}) \\ &= (I - M \otimes M)^{-1} \text{vec}(\Sigma_{\varepsilon\varepsilon}), \end{aligned} \quad (52)$$

where $\text{vec}(A)$ denotes the vector of stacked columns of the matrix A and \otimes denotes the Kronecker product.

²⁸Note that $\text{vec}(A + B) = \text{vec}(A) + \text{vec}(B)$, and $\text{vec}(ABC) = (C' \otimes A) \text{vec}(B)$.

Figure 7: Response to shocks, flexible core inflation targeting



B Impulse responses under flexible inflation targeting

Figures 7–9 show the impulse responses of the full model under flexible inflation targeting, i.e. when the central bank puts a positive weight ($\lambda = 0.5$) on stabilizing output. As is well known, the added emphasis on real stability will yield responses to core inflation and output disturbances that are more moderate than under strict inflation targeting, whereby inflation and output move back to target more slowly, and the interest rate is less volatile. The responses following shocks to the exogenous process under CPIX and CPI targeting are also somewhat milder, again a natural consequence of the added concern for output stabilization.

The responses following fiscal policy shocks also change when the weight on output stabilization increases. Under CPIX targeting, when the optimal response is the result of two opposing forces—an expansionary force to offset any effects on output, and a contractionary force to depress future core inflation—the contractionary force becomes more costly than under strict inflation targeting. Hence the response is more expansionary under flexible CPIX inflation targeting (comparing Figures 4 and 8). The response under CPI targeting is less expansionary, however: an increased weight on output stabilization implies that the central bank cares less

Figure 8: Response to shocks, flexible CPIX inflation targeting

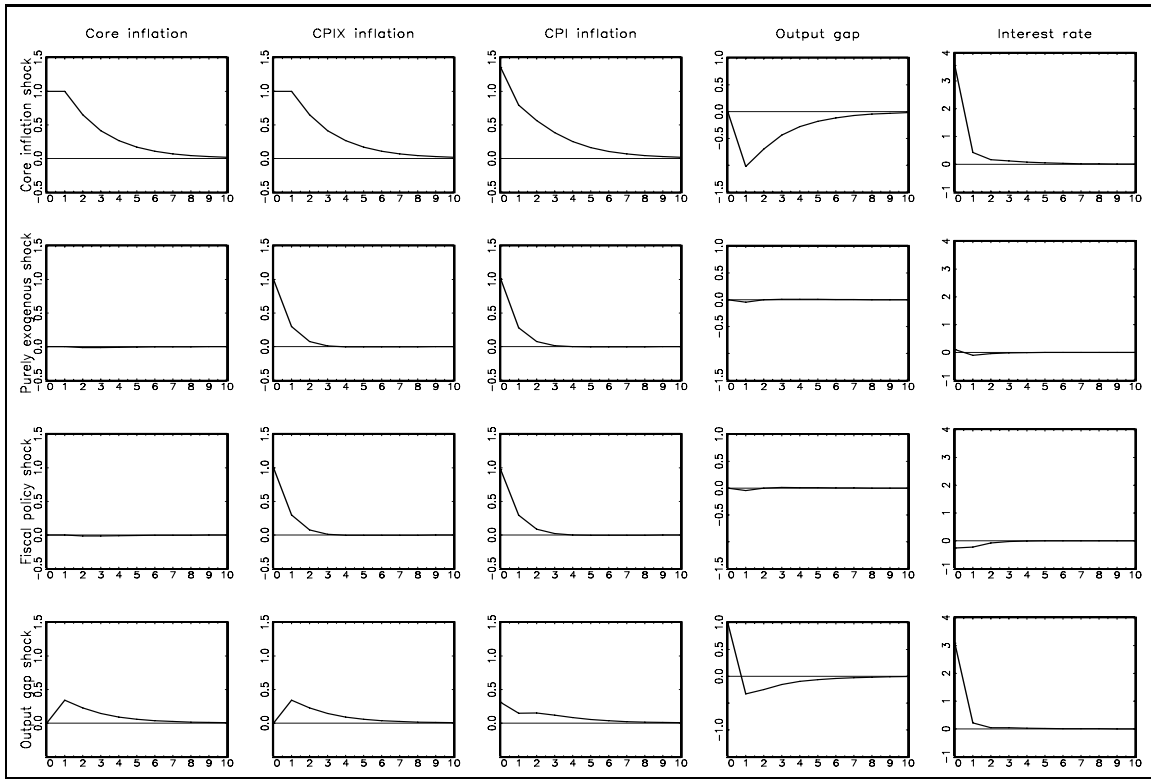
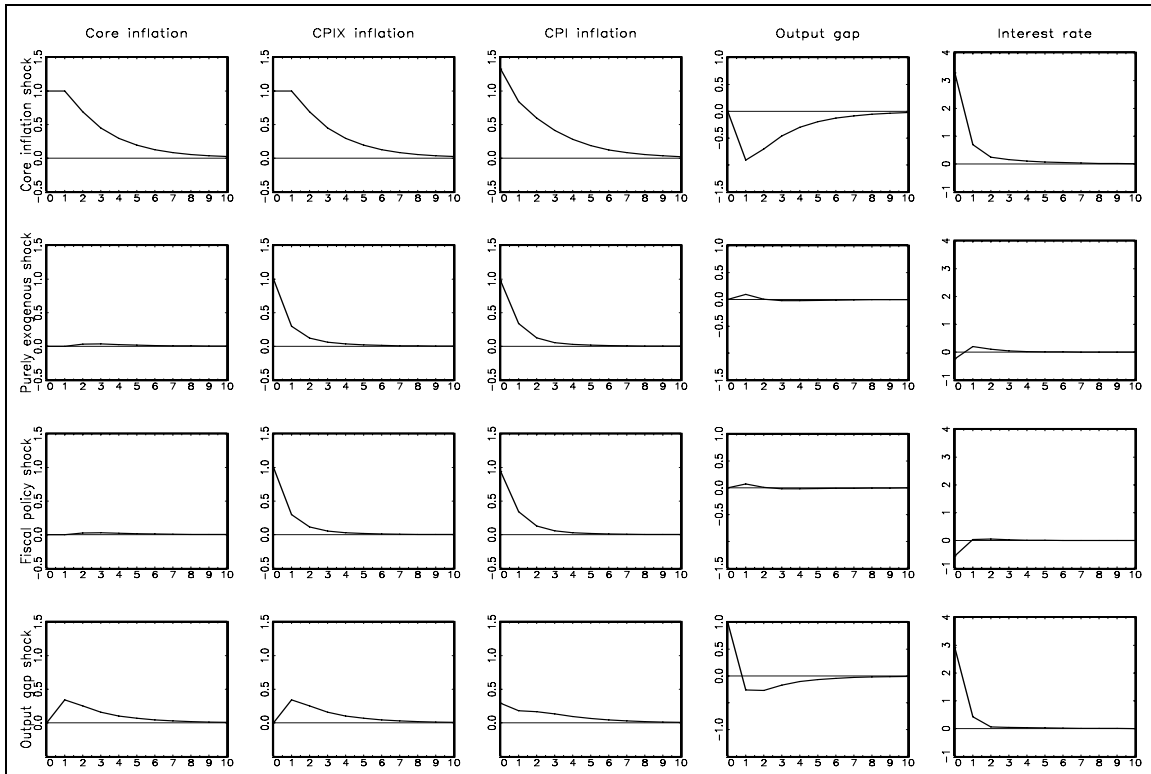


Figure 9: Response to shocks, flexible CPI inflation targeting



about the effects on current CPI inflation, hence the expansionary effect from the direct interest rate channel is smaller.

C Expected CPI inflation in the real interest rate

In the model of Section 2, the ex ante real interest rate is defined in terms of the expected core inflation rate, i.e. $r_{t+1|t} \equiv i_t - \pi_{t+1|t}^c$. Disturbances then affect policy only to the extent that they affect the target inflation rates directly. An equally plausible assumption is that private agents care about the expected CPI inflation rate when making their consumption and production choices. Then the relevant real interest rate is defined in terms of the expected CPI inflation rate:

$$r_{t+1|t} \equiv i_t - \pi_{t+1|t}^{cpi}, \quad (53)$$

where, from equations (1)–(5), the expected CPI inflation rate is

$$\pi_{t+1|t}^{cpi} = \pi_t^c + \alpha y_t + \rho_x \pi_t^x + \rho_g \pi_t^g + \rho_i \pi_t^i + \varphi \Delta i_{t+1|t}, \quad (54)$$

since $\pi_{t+1|t}^c = \pi_t^c + \alpha y_t$. Thus, three new components of the real interest rate can be identified: $\rho_x \pi_t^x$, $\rho_g \pi_t^g$ and $\rho_i \pi_t^i + \varphi \Delta i_{t+1|t}$, implying that purely exogenous and fiscal policy disturbances as well as the current interest rate change have an immediate effect on the real interest rate via inflation expectations. These added mechanisms will lead to a different conduct of monetary policy under all three regimes.

The output gap is now given by

$$\begin{aligned} y_{t+1} = & (\beta + \alpha\gamma) y_t - \gamma (i_t - \pi_t^c) + \gamma \rho_x \pi_t^x + \gamma \rho_g \pi_t^g \\ & + \gamma \rho_i \pi_t^i + \varphi \gamma (i_{t+1|t} - i_t) - \kappa \pi_t^g + \varepsilon_{t+1}^y. \end{aligned} \quad (55)$$

Using

$$\pi_t^i = \rho_i \pi_{t-1}^i + \varphi \Delta i_t, \quad (56)$$

the state-space representation is

$$x_{t+1} = A_0 x_t + B_0 i_t + B_1 i_{t+1|t} + \varepsilon_{t+1}, \quad (57)$$

where

$$A_0 \equiv \begin{bmatrix} 1 & 0 & 0 & 0 & \alpha & 0 \\ 0 & \rho_x & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_g & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_i & 0 & -\varphi \\ \gamma & \gamma\rho_x & \gamma\rho_g - \kappa & \gamma\rho_i^2 & \beta + \alpha\gamma & -\varphi\gamma\rho_i \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (58)$$

$$B_0 \equiv \begin{bmatrix} 0 & 0 & 0 & \varphi & -\gamma(1 + \varphi - \varphi\rho_i) & 1 \end{bmatrix}', \quad (59)$$

$$B_1 \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & \varphi\gamma & 0 \end{bmatrix}'. \quad (60)$$

Comparing with the standard form, we have the expression $B_1 i_{t+1|t}$ on the right-hand side, including the expected future interest rate. Following Svensson (2000), we handle it in the following way. Assuming that monetary policy is made under discretion, the optimal policy rule is

$$i_t = f x_t, \quad (61)$$

where the decision vector f remains to be determined. Then the expected future interest rate is

$$\begin{aligned} i_{t+1|t} &= f x_{t+1|t} \\ &= f (x_{t+1} - \varepsilon_{t+1}), \end{aligned} \quad (62)$$

so we can rewrite (57) as

$$(I - B_1 f) x_{t+1} = A_0 x_t + B_0 i_t + (I - B_1 f) \varepsilon_{t+1}, \quad (63)$$

or, solving for x_{t+1} ,

$$x_{t+1} = A x_t + B i_t + \varepsilon_{t+1}, \quad (64)$$

where

$$A \equiv (I - B_1 f)^{-1} A_0 \quad (65)$$

$$B \equiv (I - B_1 f)^{-1} B_0. \quad (66)$$

We can then go on solving the model using the same methods as above, but the solution is found by iterating also on (65)–(66) in addition to equations (45)–(47).

Table 7: Reaction functions, CPI in real interest rate

λ	Regime	π_t^c	π_t^x	π_t^g	π_{t-1}^i	y_t	\dot{i}_{t-1}
0	Core	6.235	0.259	-0.065	0.078	3.716	0.006
	CPIX	6.235	0.670	0.347	0.078	3.716	0.006
	CPI	4.521	-0.333	-0.557	-0.100	3.060	0.048
0.5	Core	3.334	0.276	-0.069	0.083	2.892	-0.021
	CPIX	3.334	0.378	0.033	0.083	2.892	-0.021
	CPI	3.108	0.034	-0.267	0.010	2.756	0.002

Real interest rate defined as $r_{t+1|t} \equiv i_t - \pi_{t+1|t}^{cpi}$. Parameters are set to $\kappa = 0.15, \varphi = 0.1, \rho_x = \rho_g = \rho_i = 0.3$.

The optimal reaction functions in this alternative model have been computed for the full model, i.e. with all parameters positive as in Table 5. These reaction functions are shown in Table 7.²⁹ The main effect of this alternative specification of the ex ante real interest rate is to make monetary policy more potent: a given change in the nominal interest rate translates into a larger effect on the real interest rate.³⁰ This stronger effect of policy moderates the optimal response to all disturbances under all regimes.

Furthermore, monetary policy will now respond to exogenous disturbances and fiscal policy shocks for yet another reason, namely that these shocks lead to changes in inflation expectations (see equation (54)). Therefore a positive exogenous or fiscal policy shock must be countered by an increase in the interest rate, to keep the real rate unaffected. A critical difference in comparison with the model in Section 2 is consequently that policy responds to the exogenous inflation component π_t^x also under core inflation targeting, to offset the effects on the real interest rate.

²⁹The variances of goal variables are largely unaffected, and are therefore not reported.

³⁰After a policy tightening, the central bank is expected to lower the interest rate back towards the initial level. Therefore, the expected change in the interest rate is negative, so a policy tightening leads to lower expected inflation. Tightening policy thus raises the nominal interest rate and lowers expected inflation, leading to a stronger effect on the real interest rate.

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