Price-level targeting versus inflation targeting in a forward-looking model

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Abstract

This paper examines a price-level target in a model with a forward-looking Calvo-Taylor Phillips curve. Contrary to conventional wisdom, it is found that price-level targeting leads to a better trade-off between inflation and output-gap variability than inflation targeting, when the central bank acts under discretion. In some cases, price-level targeting under discretion results in the same equilibrium as inflation targeting under commitment.

Keywords: Monteary policy, price-level targeting, inflation targeting

JEL Classification: E52, E58

1 Introduction

Several central banks use inflation targeting as their monetary policy strategy.[1] Since inflation is equal to the change in the price-level, it is natural to consider an explicit price-level target as an alternative. Although similar in spirit, this alternative has only been pursued in Sweden in the thirties.[2] Common arguments for inflation targeting apply

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[1]Examples are Bank of England, Bank of Canada and Sveriges Riksbank.

[2]For a description of this episode, see Berg and Jonung [1].

equally well, or better, to price-level targeting. For example, it is argued that inflation targeting reduces uncertainty about future price-developments, which is beneficial to consumers and firms when planning purchases and investments. This uncertainty, however, is reduced even further with a price-level target.[3]

This paper will take a slightly different, complementary, approach, in that it will argue that even if society is concerned about inflation (and output), it might still beneficial to delegate a price-level target to the central bank.

The conventional wisdom emerging from the discussion about price-level targeting seems to be that price-level targeting should be avoided because it generates unnecessary variability in the output gap. In some recent papers, the relative merits of inflation targeting and price-level targeting have been debated, and some results cast doubts on this conventional wisdom. Svensson [17] found that price-level targeting delivers a better outcome (lower variability of inflation) than inflation targeting, when the central bank acts under discretion. This result is shown for a Lucas-type Phillips curve and requires some (realistic) output gap persistence. Woodford [18] found that for an inflation targeting central bank, the optimal policy under commitment is characterized by a significant degree of interest rate inertia. His results suggest that given a central bank with no commitment, assigning a loss-function with the interest rate as an explicit argument induces the central bank to partly mimic the commitment solution, since there is an explicit reason for smoothing interest rates under the new loss-function. Jensen [8] finds that in some instances, nominal income growth targeting can dominate inflation targeting for the same reason as pointed out by Woodford; that is by creating inertial behavior of interest rates, which is a feature of the commitment solution.

The fact that both Clarida, Gali and Gertler [3] and Woodford [18] find the price level to be stationary under inflation targeting with commitment is of great importance. This directs the attention to the possibility of an explicit price-level target being preferable when the central bank acts under discretion (since the price level is stationary when there is a price-level target).

This paper compares price-level targeting and inflation targeting under discretion,

[3] For a thorough discussion about pros and cons of price-level targeting, see Duguay [5].

and finds that the outcome of the discretionary inflation targeting case can be improved by assigning a price-level target to the central bank. Thus, the question is the same as in Svensson [17], but posed in a model where forward-looking behavior is emphasized. The emphasis on forward-looking elements will turn out to play an important role when considering price-level targeting.

Recently, Clarida, Gali and Gertler [3] have stressed that in forward-looking models, gains from commitment are possible also when the central bank aims at the natural rate of unemployment. This paper makes an attempt to see whether these benefits can be reached when no such commitment device exists.

The main result in this paper is that price-level targeting delivers a more favorable trade-off between inflation and output-gap variability than does inflation targeting. With no (exogenous) persistence in the inflation process the commitment solution can always be fully implemented through an appropriate price-level targeting regime.

The mechanism behind these results is the restraining effect of expectations. The private sector realizes that the central bank's incentive to offset shocks increases with a price-level target, since the price level is persistent. Therefore, reduced expectations about future inflation are beneficial for the central bank when the economy is hit by a cost-push shock.

The paper proceeds as follows. Section 2 presents the model. Section 3 contains a summary of the optimal policy for the different regimes. Comparisons are made in section 4 and section 5 presents some conclusions.

2 The forward looking model

The model has the following standard Phillips curve relating inflation, π, to the output gap, x, and expected future inflation, $\pi_{t+1|t}$[4]

$$\pi_t = \beta\pi_{t+1|t} + \kappa x_t + u_t, \tag{1}$$

where u_t is an exogenous shock. Equation (1) is the central equation in what has become a work-horse model, dating back to Calvo [2], recently derived and extended by Rotemberg

[4]The notation $t+1|t$ means conditional expectation of $t+1$, given information at t.

and Woodford [13] and thoroughly examined in Clarida, Gali and Gertler [3]. Today's inflation is affected by two components. First, expected future inflation enters due to price rigidity. Second, prices reflect the marginal cost conditions, due to monopolistic competition. This is captured by the inclusion of the output gap, acting as a proxy for labor-market conditions affecting wages and thus, marginal cost. Finally, the "cost-push" shock, u_t, can be viewed as any effect on real marginal cost through channels other than the output gap.

This model can be compared with the model in Svensson [17], which is of Lucas-type. In that model, it is the current inflation surprise that affects the output gap. In the present model, instead, expected *future* inflation and hence the forward-looking elements of monetary policy drive the results.

At this stage, I choose to abstract from transmission lags and what is labeled as endogenous persistence in the literature. Although these issues are an important part of practical monetary policy making, the main focus of the paper is to examine the effects of forward-looking behavior. Gali and Gertler [7] finds empirical support for the relevance of forward-looking behavior, whereas its importance is questioned in Fuhrer [6].

Many empirical papers find that in order to fit the data, some persistence in the inflation process must be introduced. To avoid ignoring this issue completely, I introduce exogenous persistence in the cost push shock captured by an AR(1) process (as in Clarida, Gali and Gertler [3]). Thus,

$$u_t = \rho u_{t-1} + \varepsilon_t, \tag{2}$$

where ε_t is iid with zero mean and variance σ_ε^2.

The Phillips curve is often coupled with an equation relating the interest rate to the output gap. Unless interest rate smoothing is considered, however, this equation is redundant for solving the model in the sense that the problem is separable. First, a solution to the model treating the output gap (instead of the interest rate) as the control variable is found. Then, the "redundant" equation is used to find the path of the nominal interest rate that is consistent with the optimal output gap path. If interest rate smoothing (captured by adding an interest rate term to the loss function) is considered, the problem is no longer separable. Then, the variability of the interest rate must be explicitly weighted against the variability of the output gap and inflation, and the variability of the interest rate depends on the elasticity of the output gap with respect to the interest rate. Since interest rate smoothing is not considered in this paper, the Phillips curve gives a complete description of the dynamics of interest.

The central bank behavior is assumed to be minimizing the expected loss, i.e.

$$\min_{x_t} \mathrm{E}_t \left(1-\beta\right) \sum_{i=0}^{\infty} \beta^i L_{t+i}, \tag{3}$$

where L_t will take on different forms, depending on whether inflation targeting or price-level targeting is pursued. Society's loss-function take has form

$$L_t = \frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right).$$

To evaluate different policies, my focus is on the average performance, measured by the unconditional expected value of the loss function. To simplify the exposition of several of the results in this paper, it is convenient to express the loss function in terms of variances of inflation and the output gap. When $\beta \to 1$, we get the following interpretation of the expected value of (3)

$$\mathrm{E}\left(L_t\right) = \mathrm{Var}\left(\pi_t\right) + \lambda \mathrm{Var}\left(x_t\right). \tag{4}$$

Two different regimes for conducting monetary policy will be considered. One way of thinking about these regimes is in terms of delegation in the sense of Rogoff [12]. Society delegates a regime (defined in terms of a loss-function) to an independent central bank. Assuming this delegation to be enforceable, for example by finding a central banker with appropriate preferences or by conditioning the re-election of the governor on performance evaluated against the assigned objectives, the implications of the different regimes are explored. Strategic delegation should thus be thought of as "targeting" in the sense of Persson and Tabellini [10].

Each regime will imply a different response to shocks which, in turn, will imply different time series properties for inflation and the output gap. Given the interpretation of the loss-function discussed above, my focus will be on the variance of inflation and the output gap. The relative performance of the two regimes will be evaluated against the true social loss function. In particular, we will assume the existence of a "true" λ, that is, the relative weight on output-stabilization.

The central bank is assumed to lack commitment, in the strict sense of not being able to credibly announce future actions inconsistent with the assigned loss function. Nevertheless, the commitment solution for the social loss-function is calculated as a benchmark, in order to see how close the different discretionary policies come this solution. In order to evaluate the social loss-function, the next section calculates the implied variances of inflation and the output gap for each case.

3 Solving the model

It is assumed that the objectives of monetary policy are delegated by government to an otherwise independent central bank. In the case of inflation targeting, the loss function will take the form

$$L_t = \pi_t^2 + \lambda x_t^2. \tag{5}$$

The loss function corresponding to a price-level target is

$$L_t = p_t^2 + \tilde{\lambda} x_t^2. \tag{6}$$

This is what Svensson [16] labels "flexible" inflation (price-level) targeting. It is important to note the distinction between these two loss functions; the first corresponds to "true" preferences, the second does not. Rotemberg and Woodford [14] shows that (5) is an approximation of the true social welfare function.[5] Later in the paper, the different strategies for monetary policy will be evaluated against each other.

The optimal choice of the output gap (the control variable) and the evolution of the price level (the state variable) can be expressed on a similar form in all three cases

$$x_t = -cp_{t-1} - du_t$$

$$p_t = ap_{t-1} + bu_t,$$

[5] See also the discussion in Woodford [18].

where a, b, c and d will be determined by minimizing the respective loss-function defined by (5) and (6). Furthermore, appendix B shows that given $a < 1$, the variance of inflation and the output gap will take the form[6]

$$\text{Var}(\pi_t) = e^2 \sigma_u^2 \tag{7}$$

$$\text{Var}(x_t) = f^2 \sigma_u^2 \tag{8}$$

$$e = \frac{2b^2(1-\rho)}{(1-a\rho)(1+a)}$$

$$f = \frac{b^2c^2(1+a\rho) + d^2(1-a^2)(1-a\rho) + 2\rho bcd(1-a^2)}{(1-a^2)(1-a\rho)}$$

The rest of this section examines the different cases. First, we consider the social benchmark, that is, inflation targeting under commitment. It is assumed that the central bank cannot commit, and thus the subsequent paragraph's examine inflation targeting and price-level targeting under discretion.

3.1 Social benchmark: Inflation targeting under commitment

In the first best case, the central bank has complete credibility and is able to commit. Thus, it can credibly announce any future path for the output gap, and thus affect the public sector's expectations about future inflation with these statements. In this paper, this is not the environment the central bank is assumed to face, which is the reason why the next section deals with discretion. It is still interesting to calculate the commitment case as a benchmark for evaluating the regimes under discretion.

Following Currie and Levine [4], Woodford [18] and the appendix in Clarida, Gali and Gertler [3], define the following Lagrangian.

$$\min_{\{x_i\}_{i=t}^{\infty}} \text{E}_t \left\{ \sum_{i=0}^{\infty} \frac{\beta^i}{2} \left(\pi_{t+i}^2 + \lambda x_{t+i}^2\right) + \phi_{t+i}\left(\pi_{t+i} - \kappa x_{t+i} - \beta \pi_{t+i+1} - u_{t+i}\right) \right\}$$

[6] This is true for inflation targeting under commitment and for price-level targeting. For inflation targeting under discretion, $a = 1$ and the variance calculation is then trivial, as will become clear in the next section.

As shown in appendix C, the optimal policy, represented by the optimal choice for the output gap (the control variable), is given by

$$x_t = -c^* p_{t-1} - d^* u_t$$

with

$$\begin{aligned} c^* &= \frac{(1 - a^*\beta)(1 - a^*)}{\kappa} \\ d^* &= \frac{1 - b^*[1 + \beta(1 - \rho - a^*)]}{\kappa}. \end{aligned} \tag{9}$$

This choice of the output gap gives an evolution of the price level given by

$$p_t = a^* p_{t-1} + b^* u_t \tag{10}$$

$$a^*(\lambda) = \frac{(\lambda(1+\beta)+\kappa^2)\left(1 - \sqrt{1 - 4\beta\left(\frac{\lambda}{\lambda(1+\beta)+\kappa^2}\right)^2}\right)}{2\lambda\beta} \tag{11}$$

$$b^* = \frac{a^*}{1 - \beta a^* \rho}.$$

Appendix F shows that

$$\begin{aligned} \lim_{\lambda \to 0} a^*(\lambda) &= 0 \\ \lim_{\lambda \to \infty} a^*(\lambda) &= 1. \end{aligned}$$

Thus,

$$0 \leq a^*(\lambda) \leq 1.$$

The case when $\lambda \to 0$ corresponds to a strict inflation target, whereas $\lambda \to \infty$ corresponds to a strict output-gap target. Thus, the price level is stationary except in the special case when the central bank only cares about stabilizing the output gap ($\lambda \to \infty$).

Consider a one-time positive shock to the inflation rate. Since $c^* > 0$, the optimal policy requires the central bank to maintain the control variable (the output gap) below the steady state value (of zero) as long as the (log of the) price level remains above the steady state value (zero), even when no further shocks hit the economy. This is the gradual response found by Woodford and Clarida et.al. As we shall see later, this gradual response will not be a feature under discretionary inflation targeting, but it will turn out to be the case with a price-level target. In the case of inflation targeting with commitment, the intuition is that with a gradual (credible) response, expectations of future inflation can be affected through the forward-looking component of the Phillips curve, thereby reducing the amount of activism needed to stabilize inflation.

Using a^*, b^*, c^* and d^* in (7) and (8) gives

$$\mathrm{Var}\left(\pi_t\right) = \left(e^*\right)^2 \sigma_u^2 \tag{12}$$

$$\mathrm{Var}\left(x_t\right) = \left(f^*\right)^2 \sigma_u^2. \tag{13}$$

3.2 Inflation targeting, discretion

Next, let us turn to the discretionary setting. Since no credible promise can be made, and there is no endogenous state variable under inflation targeting discretion, the value function can be written as

$$\begin{aligned} V\left(u_t\right) &= \mathrm{E}_t\left[\min_{x_t} \frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right) + \beta V\left(u_{t+1}\right)\right] \\ &= \gamma_0 + \gamma_1 u_t + \frac{\gamma_2}{2} u_t^2, \end{aligned}$$

where the minimization is subject to (1) and γ_0, γ_1 and γ_2 remains to be determined. The control variable x_t will be a linear function of the exogenous variables. Furthermore, the forward-looking variable (being a linear function of the exogenous variable and the control variable) will have the form ($\hat{a} = 1$, $\widehat{c} = 0$).

$$\pi_t = \hat{b} u_t \tag{14}$$

$$x_t = -\hat{d} u_t \tag{15}$$

Appendix D shows that

$$\begin{aligned}\hat{b} &= \frac{\lambda}{\kappa^2+\lambda(1-\beta\rho)} \\ \hat{d} &= \frac{\kappa}{\lambda}\hat{b}=\frac{\kappa}{\kappa^2+\lambda(1-\beta\rho)}.\end{aligned} \tag{16}$$

Thus, a positive shock to inflation will, to some extent, be offset by a negative output gap. For later comparison, it is convenient to rewrite (14) in terms of the price level.

$$p_t = p_{t-1}+\hat{b}u_t. \tag{17}$$

From (14) and (15), it is evident that

$$\text{Var}(\pi_t)=\hat{b}^2\sigma_u^2 \tag{18}$$

$$\text{Var}(x_t)=\hat{d}^2\sigma_u^2 \tag{19}$$

with

$$\sigma_u^2=\frac{1}{1-\rho^2}\sigma_\varepsilon^2.$$

It is easy to see that the variance of inflation is increasing with λ, while the variance of the output-gap is decreasing with λ. This is intuitive, it means that if more weight is put on output-gap stabilization, the output-gap will vary less and inflation will vary more.

By comparing (16) and (9), it is possible to evaluate the response of the output-gap to the cost-push shock. It turns out that $\hat{d}>d^*$ (non trivial) which means that there is a *stabilization bias*, though no inflation bias. The reason is that when no credible promise can be made, the central bank will have to do all the stabilization in the current period, thus ends up "over-reacting" compared to the case of commitment.

Is there a role for a conservative central banker with discretion and an inflation target, although we do not have an overambitious output target? It turns out that the answer is yes, if there is persistence ($\rho>0$). This claim will be proved below.

3.3 Price-level targeting

In this section, a price-level target is considered.[7,8] The loss-function takes the form

$$L_t = \frac{1}{2}\left(p_t^2 + \tilde{\lambda} x_t^2\right)$$

where $\tilde{\lambda}$ is the weight delegated to the central bank together with the price-level target. When society delegates the loss function, there is no reason why the relative weight on output stabilization must equal the true weight. Rogoff [12] showed that assigning a lower λ than society's true value (that is, a more conservative central banker) reduced the inflation bias. Here, there is no bias, since the output-gap target was assumed to be consistent with the natural rate of unemployment. However, as will be clear from the results below, different values of $\tilde{\lambda}$ will affect the trade-off between inflation and output-gap *variability* (whereas Rogoff's result was in terms of the *level* of inflation).

Rewriting the Phillips curve (1) in terms of the price level yields

$$p_t - p_{t-1} = \kappa x_t + \beta\left(p_{t+1|t} - p_t\right) + u_t.$$

To solve the model, note that there are now two state variables. In this case, the price level from the previous period also enters as a state variable. Intuitively, this is due to the fact that actions affecting the price level will persist. In the case of inflation targeting, an increase in inflation today will not affect inflation tomorrow, whereas an increase in the price level today will affect the price level tomorrow. This helps clarifying the difference between a price-level target and targeting inflation at zero. In the latter case, a temporary deviation from the target will not affect future losses, while in the price-level targeting case, a temporary deviation from target will have to be countered with an offsetting deviation in the future.

With two state variables, the loss-function will take the following form:

$$V\left(p_{t-1}, u_t\right) = \mathrm{E}_t\left\{\min_{x_t}\left[\frac{1}{2}\left(p_t^2 + \tilde{\lambda} x_t^2\right) + \beta V\left(p_t, u_{t+1}\right)\right]\right\}$$

[7] It is possible to consider a trend in the price level by defining the loss function as the deviation of the price level from trend. This will not affect the variances of inflation and the output gap.

[8] It can be shown that the inflation bias generated from an overambitious output-gap target (not present in this paper) in the inflation targeting case does not appear in the price-level targeting case, as shown by Kiley [9] and Svensson [17].

$$u_t = \rho u_{t-1} + \varepsilon_t.$$

Appendix E shows that the state variable we are interested in will also be a linear function of the state variables

$$p_t = \tilde{a} p_{t-1} + \tilde{b} u_t, \tag{20}$$

where the coefficients are defined by the following equations:

$$\tilde{a} = \frac{\omega \tilde{\lambda}}{\kappa^2 + \omega^2 \tilde{\lambda} + \beta \tilde{\lambda} (1 - \omega \tilde{a})} \tag{21}$$

$$\tilde{b} = \frac{\omega \tilde{\lambda} + \beta \rho \tilde{\lambda} \left[\tilde{b} - \left(1 + \beta \rho \tilde{b} \right) + \omega \tilde{b} \right]}{\kappa^2 + \omega^2 \tilde{\lambda} + \beta \tilde{\lambda} (1 - \omega \tilde{a})} \tag{22}$$

$$\omega = 1 + \beta (1 - \tilde{a}).$$

Note that $\tilde{a}$ is independent of ρ, that is, the degree of persistence in the shock process.[9]

Precisely in the same way as for the inflation targeting case under commitment, it can be shown that

$$\begin{aligned} \lim_{\tilde{\lambda} \to 0} \tilde{a} \left(\tilde{\lambda} \right) &= 0 \\ \lim_{\tilde{\lambda} \to \infty} \tilde{a} \left(\tilde{\lambda} \right) &= 1, \end{aligned}$$

which, again, means that

$$0 \leq \tilde{a} \left(\tilde{\lambda} \right) < 1.$$

[9] To show that a solution exists, we can restrict attention to the interval $x \in [0, 1]$, define

$$f(x) = \frac{\omega \tilde{\lambda}}{\kappa^2 + \omega^2 \tilde{\lambda} + \beta \tilde{\lambda} (1 - \omega x)}$$

with $\omega(x) = 1 + \beta(1 - x)$. Note that $1 \leq \omega(x) < 2$, and also that $0 \leq \omega x < 1$ when $x \in [0, 1]$. This, together with the observation that $\omega \tilde{\lambda} \leq \omega^2 \tilde{\lambda}$ ensures that $f(x)$ maps x back into $[0, 1]$. Since $[0, 1]$ is a compact closed set and $f(x)$ is continuous in x we can thus by Brouwer's fix point theorem conclude that there is a solution.

This exercise is done in appendix E. Notice that this means that the price level follows an AR(1) process and is stationary, except in the case when $\lambda \to \infty$ (a strict output gap target). Also, note that if there is no persistence in the residual process (i.e. $\rho = 0$), then $\tilde{a} = \tilde{b}$. The solution for the control variable x_t is given by

$$x_t = -\tilde{c}p_{t-1} - \tilde{d}u_t$$

where

$$\begin{aligned} \tilde{c} &= \frac{(1-\tilde{a}\beta)(1-\tilde{a})}{\kappa} \\ \tilde{d} &= \frac{1-\tilde{b}\left[1+\beta(1-\rho-\tilde{a})\right]}{\kappa}. \end{aligned}$$

As in the previous section, to find the variances of inflation and the output gap, use $\tilde{a}$, $\tilde{b}$, $\tilde{c}$ and $\tilde{d}$ in (33) and (34).

$$Var(\pi_t) = \tilde{e}^2\sigma_u^2 \tag{23}$$

$$Var(x_t) = \tilde{f}^2\sigma_u^2. \tag{24}$$

4 Comparing results

The main purpose of this paper is to examine the relative performance of an inflation target to a price-level target. The essential insight is gained from comparing (20) with (10) and (17).

		Defined in
Inflation targeting, commitment	$p_t = a^* p_{t-1} + b^* u_t$	(10)
Inflation targeting, discretion	$p_t = p_{t-1} + \hat{b}u_t$	(17)
Price-level targeting, discretion	$p_t = \tilde{a}p_{t-1} + \tilde{b}u_t$	(20)

These equations define the optimal solution in terms of the price level for inflation targeting under commitment and discretion, and price-level targeting under discretion. All other results such as variances of inflation and the output gap will be based on these equations. An implication of this is that if equations (10) and (20) are shown to be the same, we know that the commitment solution can be implemented by assigning a price-level target under discretion.

A preview of the results shows that this is almost what we will find. When there is no persistence ($\rho = 0$), the commitment solution can be fully implemented with a price-level target. That is, it is possible to find a $\tilde{\lambda}$ under price-level targeting such that $a^*(\lambda) = \tilde{a}(\tilde{\lambda})$ and $b^*(\lambda) = \tilde{b}(\tilde{\lambda})$.

In some of the experiments considered below, a numerical value for κ is needed. Roberts [11] estimates a version of the model above (eq. 9 p. 979):

$$\pi_t = \pi_{t+1|t} + \kappa x_t + \epsilon_t$$

and finds κ to be in the range of 0.25 to 0.36, depending on the measure of inflation expectations. On the basis of that result, I choose $\kappa = \frac{1}{3}$.

4.1 No persistence ($\rho = 0$)

The main result of the paper is that price-level targeting gives a better trade off between inflation- and output gap variability than inflation targeting. Later, this result will be proved for the case of persistence in the residual process. To gain understanding of this result, we start by first considering the special case of $\rho = 0$. By this assumption, it is possible to find an analytical solution for the price-level targeting case.

Proposition 1 *With no persistence in the residual process, the commitment solution can be implemented by assigning a price-level target with a $\tilde{\lambda}$ different from λ.*

With $\rho = 0$, $a^* = b^*$ and $\tilde{a} = \tilde{b}$, it is enough to prove that we can find a $\tilde{\lambda}$, such that $a^*(\lambda) = \tilde{a}(\tilde{\lambda})$. In fact, the preceding sections already provided the information required to pursue this argument. To recapitulate:

$$\begin{aligned} \lim_{\tilde{\lambda} \to 0} \tilde{a}(\tilde{\lambda}) &= 0 \\ \lim_{\tilde{\lambda} \to \infty} \tilde{a}(\tilde{\lambda}) &= 1 \end{aligned}$$

$$\lim_{\lambda \to 0} a^* (\lambda) = 0$$
$$\lim_{\lambda \to \infty} a^* (\lambda) = 1.$$

Thus, since both the coefficient $\tilde{a}$ from the price-level targeting case (eq. (21)) and the counterpart from the commitment case (eq. (11)) is limited by the interval $[0, 1)$, we know that for a given value of λ, implying a fixed value for $a^* (\lambda)$, it is always possible to find a value of $\tilde{\lambda}$ that sets $\tilde{a}(\tilde{\lambda}) = a^* (\lambda)$. That is, it is always possible to implement the commitment solution for an inflation target by assigning a price-level target *with a different* λ (namely $\tilde{\lambda}$).[10]

4.2 Persistence

With persistence in the residual process, two conditions must be satisfied in order to implement the inflation targeting commitment solution with a price-level target:

$$\tilde{a}(\tilde{\lambda}) = a^* (\lambda)$$
$$\tilde{b}(\tilde{\lambda}) = b^* (\lambda).$$

Figure 1 and 2 give the a and b coefficient values for the different cases, for different values of λ. Examining these figures reveals that the commitment solution cannot be perfectly replicated with a price-level target. This does not mean that an inflation target is preferable, it only suggests that it is not possible to fully replicate the commitment solution through a price-level target when $\rho > 0$. To find out whether the price-level target dominates the inflation target, we will study the policy frontiers for the two cases. For this purpose, both the variance of inflation and the output gap must be recovered under the two regimes.

4.3 Variance results

To summarize, the variance's of inflation under inflation targeting and price-level targeting is given by (12), (18) and (23). Similarly, the output gap variances are given by (13), (19) and (24), respectively.

[10] This requires that both $\tilde{a}(\lambda)$ and $a^*(\lambda)$ are continous functions of λ. For $a^*(\lambda)$, this follows trivially from (9). It is possible to solve (18) explicitly for $\tilde{a}(\lambda)$, and then show that $\tilde{a}(\lambda)$ really is continous in λ.

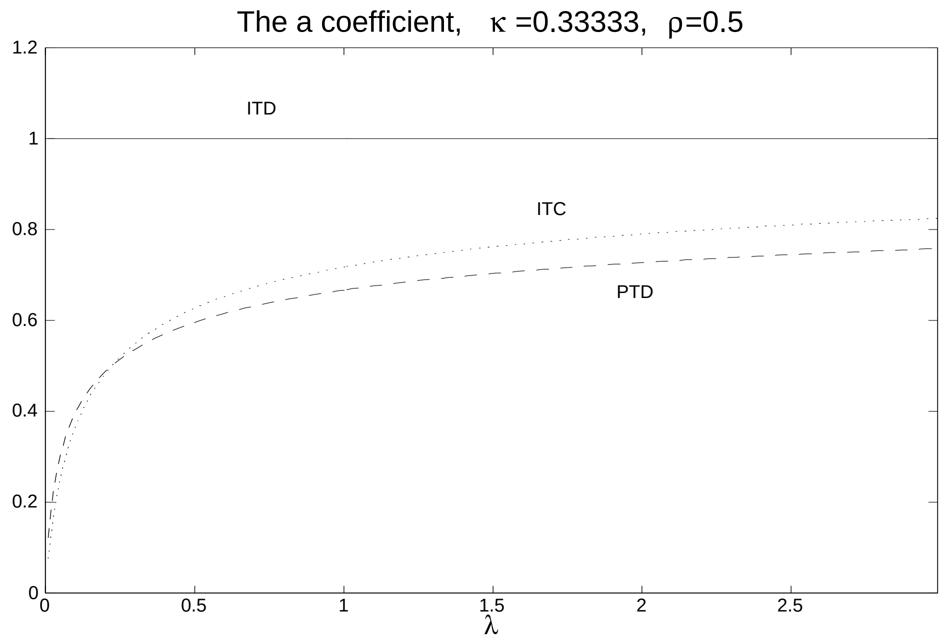

Figure 1:

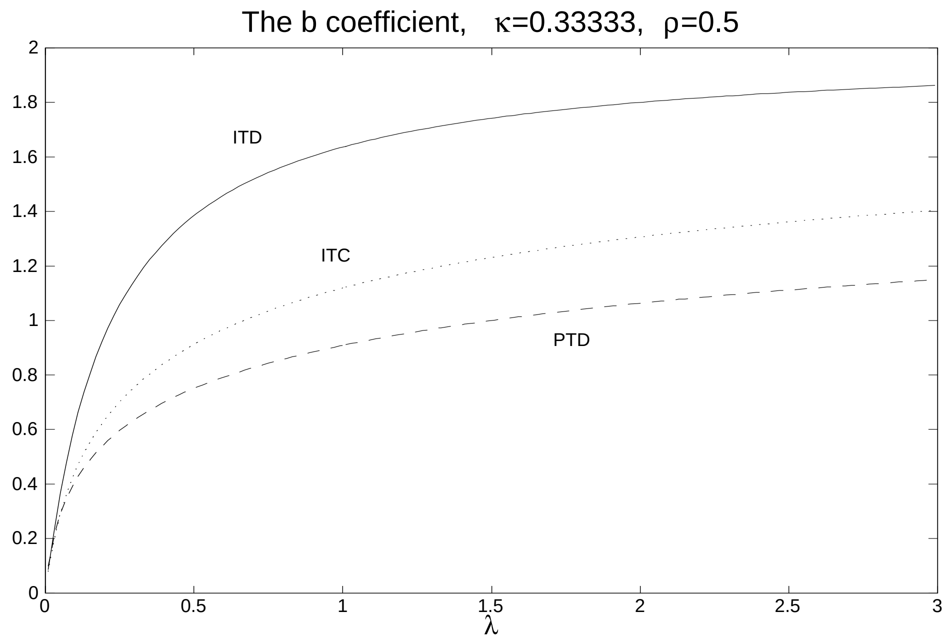

Figure 2:

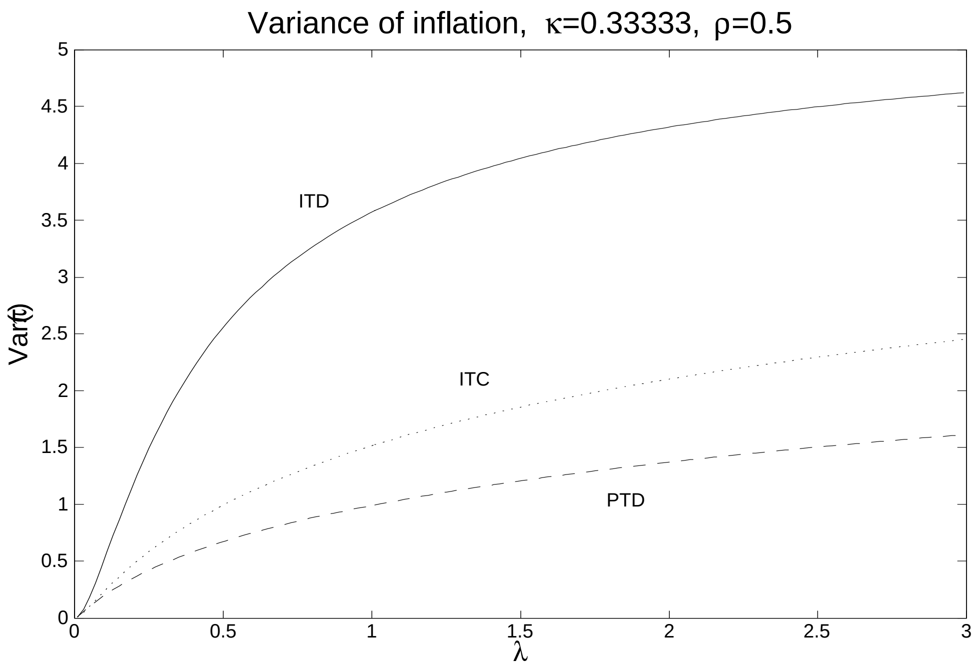

Figure 3:

	$\text{Var}(\pi_t)$	$\text{Var}(x_t)$
ITC	$(e^*)^2\,\sigma_u^2$	$(f^*)^2\,\sigma_u^2$
ITD	$\hat{b}^2\sigma_u^2$	$\hat{d}^2\sigma_u^2$
PTD	$\tilde{e}^2\sigma_u^2$	$\tilde{f}^2\sigma_u^2$

Comparing the two cases is more clear-cut in Svensson [17], since there is no difference in output gap variability. In the forward-looking case, this is not true. Both output gap and the inflation variability will differ under the two regimes and thus, it is hard to judge the result by only inspecting the equations.

To interpret previous findings in the literature, examine the variance plots in figures 3 and 4.

From these figures, it is tempting to make the conclusion that price-level targeting generates higher output-gap variability than does inflation targeting. This conclusion is reached by fixing λ and vertically examining figure 4. This leads Kiley [9] to the conclusion that a price-level target is worse than an inflation target, since it generates a higher variability of the output gap (he makes a comparison with Svensson [17] who finds

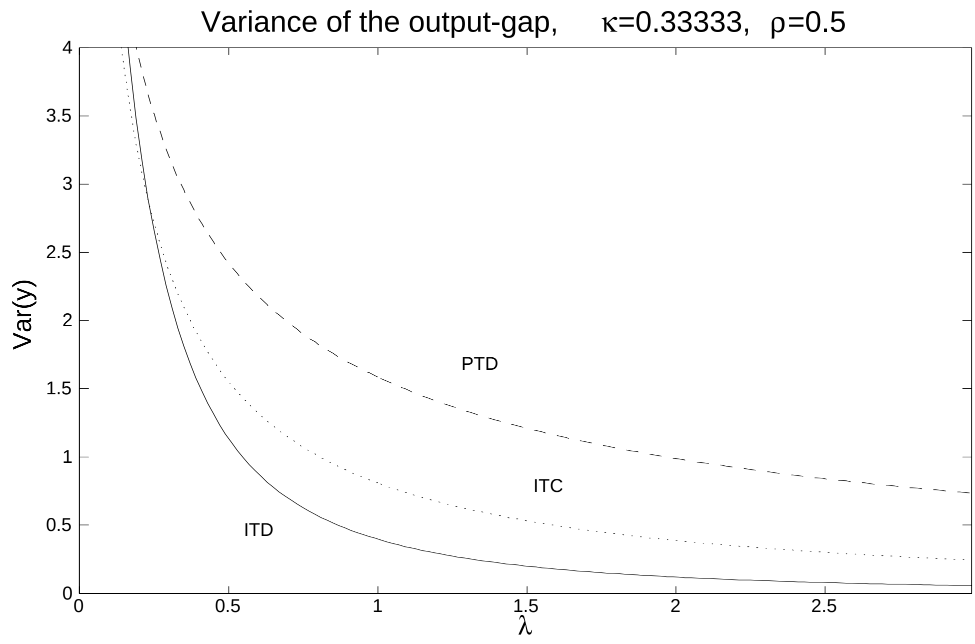

Figure 4:

that (given some conditions) a price-level target gives the same variability of the output gap as does an inflation target, but a lower variability of inflation and thus concludes a price-level target to be preferable). However, the same experiment in figure 3 reveals that the variance of inflation is lower with a price-level target that with an inflation target. It thus seems inconclusive which is the better.

This paper suggests, however, that it is more instructive to read the figure horizontally. A given variance of inflation resulting from a particular value of λ can always be implemented by assigning a different value of $\tilde{\lambda}$ under a price-level target. It is not obvious that both the variance of inflation and the output gap under price-level targeting can be reduced compared to inflation targeting (both under discretion) simultaneously by inspecting the figures. To evaluate this, the next section plots efficiency frontiers.

4.4 Efficiency frontiers

A frontier plots all combinations of output gap variance and inflation variance attainable for different values of the preference parameter λ. Since there is a tension between these

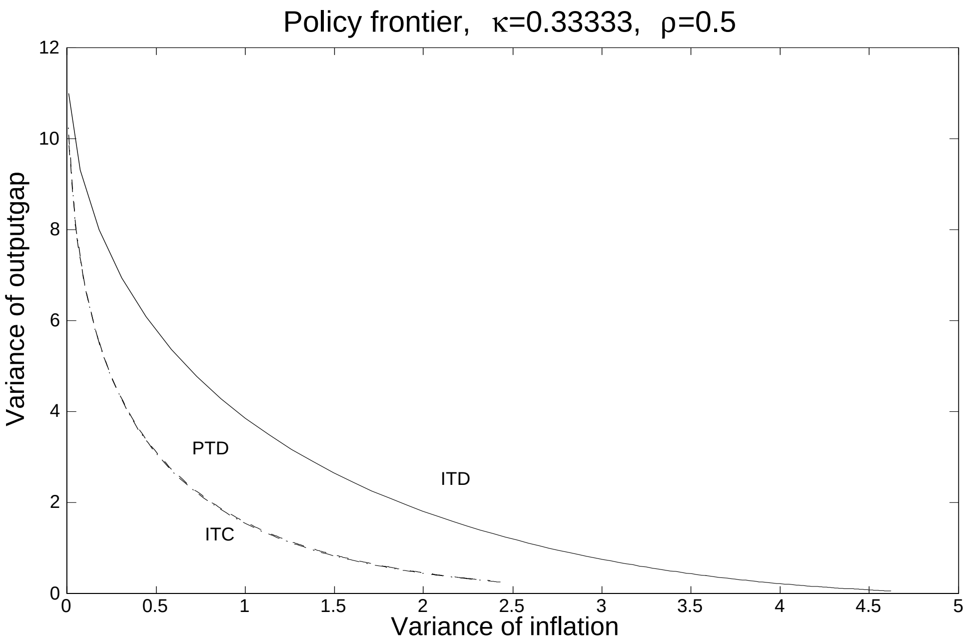

Figure 5:

variances, there will always be a trade-off of increased inflation variability in order to reduce output gap variability. Technically, the frontiers are constructed by fixing κ and then plotting inflation variance and output-gap variance for different values of λ. With $\beta \rightarrow 1$, the slope of the efficiency frontier is equal to λ.

Proposition 2 *Price-level targeting gives a better inflation- output-gap variance trade-off than inflation targeting.*

Figure 5 reveals that price-level targeting dominates inflation targeting (when the central bank acts under discretion) since the frontiers never cross. Thus, using a price-level target is preferable in the absence of commitment. Note that the variance frontiers for inflation targeting and price-level targeting almost coincide. If persistence is increased to almost one, there is a more pronounced difference in the two cases, but the price-level target still dominates the inflation target.[11] From an economic point of view, a price-level target adds credibility in the following sense: with an inflation target, a temporary increase

[11] For very high values of ρ, there will be a discrepancy between the commitment case and the price level targeting case. However, the price level target will still dominate the inflation target under discretion.

in inflation is disregarded in the next period. With a price-level target, this is no longer true. Instead, a temporary increase in inflation must sooner or later be counteracted by a reduction in inflation below target. This has been used as an argument against using price-level targets, and the claim is that it would increase volatility of inflation. However, with forward-looking agents, the anticipated reduction that must take place in the future reduces inflationary expectations and thus helps the central bank fight inflation. These results are related to one of the findings in Smets [15], who shows that it often pays to give the central bank a price level objective even if society cares about inflation stabilization.

4.5 Central bank conservatism and the interpretation of $\tilde{\lambda}$

Now, we will return to the claim that there is a role for a conservative central banker with discretion and inflation targeting. Note that the form of the loss-function (4) means that we can describe social preferences in terms of linear indifference curves, with slope -$\frac{1}{\lambda}$. Thus, given that we assume the same institutional setup for the inflation targeting case (that it is possible to assign a λ different from the true value) as we did for the price-level targeting case, the optimal solution requires finding the point where the indifference curve is tangent to the policy frontier (remember that loss is growing in north-east direction in figure 5). Remember that the two equations (19) and (18) defines the variances. Next, let (18) implicitly define $\hat{\lambda}(\mathrm{Var}\,(\pi_t))$. Then we can write

$$\begin{aligned}\frac{d\mathrm{Var}\,(x_t)}{d\mathrm{Var}\,(\pi_t)} &= \frac{\frac{d\mathrm{Var}(x_t)}{d\lambda}}{\frac{d\mathrm{Var}(\pi_t)}{d\lambda}} \\ &= -\frac{\frac{2(1-\beta\rho)\kappa^2}{\left(\kappa^2+\hat{\lambda}(1-\beta\rho)\right)^3}}{\frac{2\hat{\lambda}\kappa^2}{\left(\kappa^2+\hat{\lambda}(1-\beta\rho)\right)^3}} \\ &= -\frac{(1-\beta\rho)}{\hat{\lambda}}.\end{aligned}$$

To find the point of tangency, we set

$$\begin{aligned}-\frac{1}{\lambda} &= -\frac{(1-\beta\rho)}{\hat{\lambda}} \\ \hat{\lambda} &= (1-\beta\rho)\,\lambda. \qquad (25)\end{aligned}$$

With $\rho > 0$ we conclude that $\hat{\lambda} < \lambda$ and thus that government should appoint a more conservative central banker.[12] This result can be interpreted somewhat similar to the price-level targeting case. Note that it is the persistence that drives down the optimal value for $\hat{\lambda}$. The reason is that a more conservative central banker will guarantee that inflationary shocks are countered harder. Thus inflationary expectations are reduced (since future values u_{t+i} can be forecasted, to some extent), which helps the central bank to stabilize current inflation.

For the case of price-level targeting, things are slightly more complicated, due to two reasons. First, it is hard to find an analytical solution for the slope of the variance frontier. Second, there is an interpretation problem with respect to comparing $\tilde{\lambda}$ and λ.

In principle, we could differentiate (24) and (23) to get the slope of the frontier for the price-level targeting case. However, since $\tilde{e}$ and $\tilde{f}$ are nontrivial functions of $\tilde{a}$ and $\tilde{b}$, which themselves are functions of $\tilde{\lambda}$, this is not an easy task. An alternative approach is to examine the optimal values of $\tilde{\lambda}$ for some reasonable parameter values.

For consistency, we retain the assumption that $\kappa = \frac{1}{3}$ and $\rho = \frac{1}{2}$. Figure 6 plots this for different values of the true λ. For example, the lowest line corresponds to a true value of 0.2. Two pieces of information can be extracted from this figure. First, there is a unique value of the assigned $\tilde{\lambda}$ that minimizes loss for each true value of λ (for example, when $\lambda = 0.2$, the optimal $\tilde{\lambda}$ is 0.29). Second, the shape of the loss-function is flattening with the size of λ. For a large λ, this means that it is the price-level target per se, and not so much the precise level of $\tilde{\lambda}$, that matters. Using the optimal values in figure 6 (of course, with a lot more values for the true λ), it possible to map λ into $\tilde{\lambda}$, or simply $\tilde{\lambda}(\lambda)$. This figure would then reveal which value of $\tilde{\lambda}$ that should be used as a function of the true λ. However, before doing this we must examine an issue of importance for interpreting the result.

Rogoff found that assigning a λ less than the "true" value reduced the inflation bias. He interpreted this as appointing a conservative central banker. Therefore, it is tempting

[12] This confirms Result 7, p. 1680 in Clarida, Gali and Gertler [3], which they derive by examining a restricted form of commitment and noticing that this solution looks identical to the case of discretion, but with a lower value of λ.

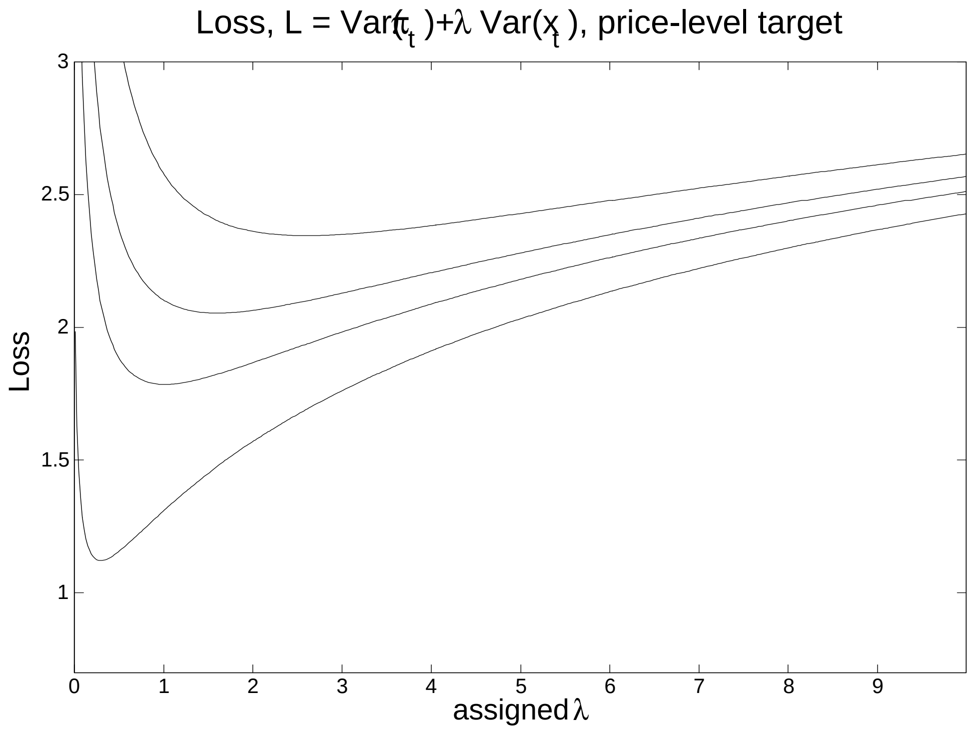
Loss, L = Var(π_t)+λ Var(x_t), price-level target
Loss
3
2.5
2
1.5
1
0
1
2
3
4
5
6
7
8
9
assigned λ

Figure 6:

to draw the conclusion that finding $\tilde{\lambda} > \lambda$ means that a *less* conservative central banker should be appointed. This is a premature conclusion, however. The reason is that λ and $\tilde{\lambda}$ have different interpretations as can be seen from comparing the two loss-functions, for convenience reproduced in terms of variances.

$$\mathrm{E}\left[L_t\right] = \mathrm{Var}\left(\pi_t\right) + \lambda \mathrm{Var}\left(y_t\right)$$

$$\mathrm{E}\left[L_t\right] = \mathrm{Var}\left(p_t\right) + \tilde{\lambda} \mathrm{Var}\left(y_t\right).$$

In the first loss-function, λ can be interpreted as the relative weight placed on the variability of the output gap, compared to the variability of *inflation.* In the case of price-level targeting, $\tilde{\lambda}$ measures the relative weight placed on output-gap variability, compared to the variability of *the price level.* Since the two weights have different benchmarks, not much is gained from comparing their absolute values. However, since the inflation rate is tightly linked to the price level, it is possible to interpret the relative size of the two weights. To recapitulate, the price level, it's variance and the variance of inflation are given by

$$p_t = \tilde{a} p_{t-1} + \tilde{b} u_t.$$

$$\mathrm{Var}\left(p_t\right) = \frac{\left(1 + \tilde{a}\rho\right) \tilde{b}^2}{\left(1 - \tilde{a}^2\right)\left(1 - \tilde{a}\rho\right)} \sigma_u^2$$

$$\mathrm{Var}\left(\pi_t\right) = \frac{2\left(1 - \rho\right) \tilde{b}^2}{\left(1 - \tilde{a}\rho\right)\left(1 + \tilde{a}\right)} \sigma_u^2.$$

Thus, by taking the ratio we get

$$\frac{\mathrm{Var}\left(\pi_t\right)}{\mathrm{Var}\left(p_t\right)} = \frac{2\left(1 - \rho\right)\left(1 - \tilde{a}\right)}{1 + \tilde{a}\rho},$$

or,

$$\mathrm{Var}\left(p_t\right) = \frac{1 + \tilde{a}\rho}{2\left(1 - \rho\right)\left(1 - \tilde{a}\right)} \mathrm{Var}\left(\pi_t\right).$$

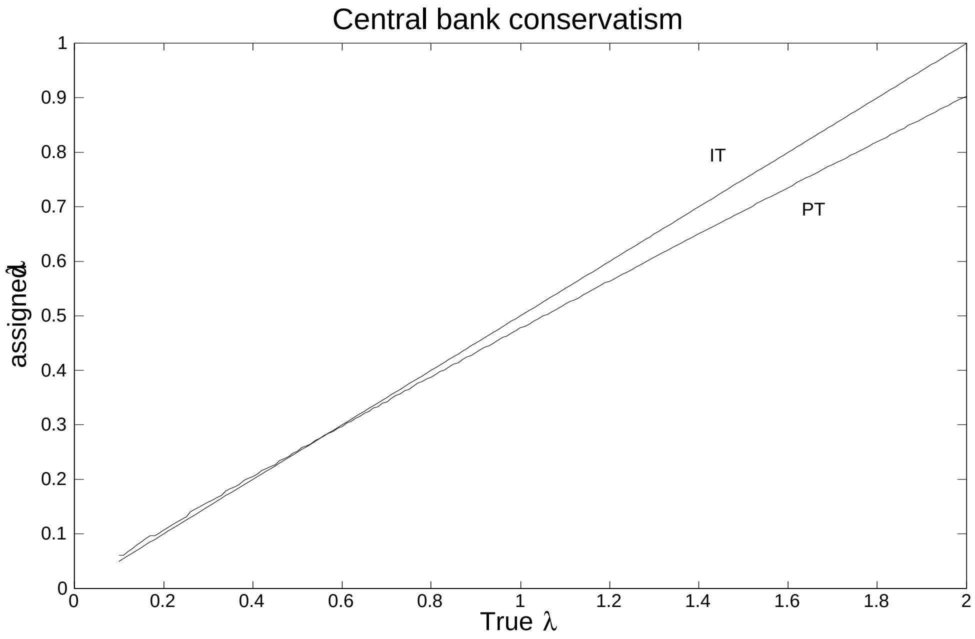

Figure 7:

In order to get a comparable weight (i.e. to have the same normalization in both loss-functions) the following equation must be satisfied:

$$\lambda = \frac{2\,(1-\rho)\left(1-\tilde{a}\left(\tilde{\lambda}\right)\right)}{1+\tilde{a}\left(\tilde{\lambda}\right)\rho}\tilde{\lambda}. \tag{26}$$

That is, $\lambda > \frac{2(1-\rho)\left(1-\tilde{a}\left(\tilde{\lambda}\right)\right)}{1+\tilde{a}\left(\tilde{\lambda}\right)\rho}\tilde{\lambda}$ implies that the central bank should be more weight-conservative. To understand this result, we take the numerical values from figure 6, and multiply them with the appripriate scale factor in (26). This give us $\tilde{\lambda}$ values converted to have the same normalization (variance of inflation) as the inflation targeting case. Also, we use (25) to get the optimal values for the inflation targeting case. Figure (7) presents both these graphs in order to decide on the conservatism matter.The inflation targeting case, represented by the straight line (since the coefficient in (25) is just a constant) intersect the price-level targeting case. Thus, for the parameter values used in the example, for $\lambda > 0.55$, the price-level target is accompanied by a slightly more conservative central-banker, while a $\lambda < 0.55$ implies a slightly less. However, the difference is small (a result robust for other parameter values), especially in light of the above discussion that the loss-function is flat for large values of λ. The conclusion is that conservatism is equally important for price-level targeting as it is for inflation targeting.

5 Conclusions

The main result of the paper is that when forward-looking price setting is important and the central bank acts under discretion, a price-level target may dominate an inflation target. The result can be interpreted in line with Rogoff's classic result that by strategically delegating a loss-function different from society's (in Rogoff's case, a more "conservative" central banker in the sense that $\lambda_{\text{banker}} < \lambda_{\text{society}}$), a better outcome can occur. In this case, there is a "two-dimensional" assignment. First, the inflation target is replaced by a price-level target. Second, a different value of λ is assigned.

In the previous literature the effects of a price-level target have been subject to a misinterpretation. It has been recognized that *for a given value of* λ, a price-level target generates more variability of the output gap than does an inflation target. It has also been recognized, however, that inflation variability is lower under the price-level target. Conventional wisdom explained this result by claiming that in the price-level targeting case, a positive shock must be countered by a monetary tightening at a later stage, which will induce more volatility of the output gap than in the inflation targeting case, where bygones are treated as bygones. The point of this paper is that this is not the most interesting comparison. If we instead examine the policy frontiers, it becomes clear that the price-level target dominates the inflation target, since a better outcome can always be implemented by assigning a different λ in the price-level targeting case. Also, the paper shows that a proper interpretation of the optimal assigned values of λ reveals that the central banker should be about as conservative with a price-level target as he should be with an inflation target. This means that the result does not hinge upon the possibility to "select λ", it is rather the possibility to delegate a price-level target that is important.

The institutional setup assumed in this paper is that the government is able to delegate a target different from society's to an otherwise independent central bank. It might be argued that some other target can give an even better outcome than a price-level target.

It might also be argued that the institutional setup becomes more implausible the more the assigned target deviates from that of society. The point of this paper is to propose a target that is fairly close to the targets presently used by central banks, and one that would be easy to communicate to the public.

With no persistence, the commitment solution of the inflation targeting case can be implemented. With persistence, this is not true. However, it is still always the case (in the model examined, that is) that a price-level target generates a better outcome than the inflation target, and is almost as good as the commitment solution. With price-level targeting, the private sector expects the central bank to counter an above average inflation (normalized to zero in this paper) with a below average inflation somewhere in the future. In other words, a positive shock to inflation reduces the expected future inflation and thus lowers the current amount of intervention.

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A Finding c and d

To get the c and d coefficients when the price level follows

$$p_t = ap_{t-1} + bu_t$$

use the stationary version of (48) in (49) and simplify to get

$$\begin{aligned} x_t &= \frac{1-a\beta}{\kappa}\left(bp_{t-1} + bu_t - p_{t-1}\right) - \frac{1-\beta b\left(1-\rho\right)}{\kappa}u_t \\ &= -\frac{\left(1-a\beta\right)\left(1-a\right)}{\kappa}p_{t-1} - \frac{1-b\left[1+\beta\left(1-\rho-a\right)\right]}{\kappa}u_t \\ &= -cp_{t-1} - du_t \end{aligned}$$

B Variance calculations

This appendix calculates the variance for inflation and the output gap, given a model implying an evolution of the price level of the form

$$p_t = ap_{t-1} + bu_t \tag{27}$$

$$y_t = cp_{t-1} + du_t. \tag{28}$$

Subtracting p_{t-1} from (27) gives

$$\pi_t = -\left(1-a\right)p_{t-1} + bu_t$$

$$\text{Var}\left(\pi_t\right) = \left(1-a\right)^2 \text{Var}\left(p_{t-1}\right) + b^2\text{Var}\left(u_t\right) - 2\left(1-a\right)b\text{Cov}\left(p_{t-1}, u_t\right) \tag{29}$$

$$\text{Var}\left(p_t\right) = a^2\text{Var}\left(p_{t-1}\right) + b^2\text{Var}\left(u_t\right) + 2ab\text{Cov}\left(p_{t-1}, u_t\right) \tag{30}$$

$$\begin{aligned} \text{Cov}\left(p_{t-1}, u_t\right) &= \text{Cov}\left(ap_{t-2} + bu_{t-1}, \rho u_{t-1} + \varepsilon_t\right) \\ &= a\rho\text{Cov}\left(p_{t-2}, u_{t-1}\right) + b\rho\text{Var}\left(u_{t-1}\right) \\ &= \frac{b\rho}{1-a\rho}\sigma_u^2 \end{aligned} \tag{31}$$

Using stationary and substituting (31) into (30) yields

$$\begin{aligned} \left(1-a^2\right) \mathrm{Var}\left(p_t\right) &= b^2\sigma_u^2 + \frac{2ab^2\rho}{1-a\rho}\sigma_u^2 \\ \mathrm{Var}\left(p_t\right) &= \frac{b^2\left(1+a\rho\right)}{\left(1-a^2\right)\left(1-a\rho\right)}\sigma_u^2. \end{aligned} \tag{32}$$

Using (32) and (31) in (29) gives

$$\begin{aligned} \mathrm{Var}\left(\pi_t\right) &= \left[\frac{\left(1-a\right)^2 b^2\left(1+a\rho\right)}{\left(1-a^2\right)\left(1-a\rho\right)} + b^2 - \frac{2\left(1-a\right)b^2\rho}{1-a\rho}\right]\sigma_u^2 \\ &= \frac{1-a}{1-a\rho}\left[\frac{1+a\rho}{1+a} + \frac{1-a\rho}{1-a} - 2\rho\right]b^2\sigma_u^2 \\ &= \frac{2\left(1-\rho\right)}{\left(1-a\rho\right)\left(1+a\right)}b^2\sigma_u^2. \end{aligned} \tag{33}$$

To calculate the variance of the output gap, it is convenient to use the form (28) and note that this implies

$$\mathrm{Var}\left(x_t\right) = c^2\mathrm{Var}\left(p_{t-1}\right) + d^2\mathrm{Var}\left(u_t\right) + 2cd\mathrm{Cov}\left(p_{t-1}, u_t\right).$$

Substituting (32) and (31) into the above equation gives

$$\begin{aligned} \mathrm{Var}\left(x_t\right) &= c^2\frac{b^2\left(1+a\rho\right)}{\left(1-a^2\right)\left(1-a\rho\right)}\sigma_u^2 + d^2\sigma_u^2 + 2cd\frac{b\rho}{1-a\rho}\sigma_u^2 \\ &= \frac{b^2c^2\left(1+a\rho\right) + d^2\left(1-a^2\right)\left(1-a\rho\right) + 2\rho bcd\left(1-a^2\right)}{\left(1-a^2\right)\left(1-a\rho\right)}\sigma_u^2. \end{aligned} \tag{34}$$

C Inflation targeting, commitment

Following Currie and Levine [4], Woodford [18] and the appendix in Clarida, Gali and Gertler [3], define the Lagrangian

$$\min_{\{x_i\}_{i=t}^{\infty}} \mathrm{E}_t\left\{\sum_{i=0}^{\infty}\frac{\beta^i}{2}\left(\pi_t^2 + \lambda x_t^2\right) + \phi_{t+i}\left(\pi_{t+i} - \kappa x_{t+i} - \beta\pi_{t+i+1} - u_{t+i}\right)\right\}$$

We start by taking the first-order conditions with respect to inflation:

$$\begin{aligned} \frac{\partial}{\partial\pi_{t+i}} &= -\beta^{i-1}\phi_{t+i-1}\beta + \beta^i\left(\pi_{t+i} + \phi_{t+i}\right) = 0 \Leftrightarrow \\ \pi_{t+i} &= \phi_{t+i-1} - \phi_{t+i},\ i > 0 \\ \pi_t &= -\phi_t \text{ (that is, } i = 0) \end{aligned} \tag{35}$$

Next, we take the first-order conditions with respect to x_{t+i}.

$$\begin{aligned}\frac{\partial}{\partial x_{t+i}} &= \lambda x_{t+i} - \phi_{t+i}\kappa = 0 \Leftrightarrow \\ \phi_{t+i} &= \frac{\lambda}{\kappa} x_{t+i},\ i \geq 0. \end{aligned} \tag{36}$$

Combining the first-order conditions by substituting (36) into (35) gives

$$\pi_{t+i} = -\frac{\lambda}{\kappa}\left(x_{t+i} - x_{t+i-1}\right). \tag{37}$$

Substituting (37) into the constraint finally gives a second-order stochastic difference equation for x_t:

$$\begin{aligned}-\frac{\lambda}{\kappa}\left(x_t - x_{t-1}\right) &= \kappa x_t - \frac{\beta\lambda}{\kappa}\left(x_{t+1|t} - x_t\right) + u_t \Leftrightarrow \\ x_t &= a x_{t-1} + a\beta x_{t+1|t} - \frac{\kappa a}{\lambda} u_t,\end{aligned}$$

with $a = \frac{\lambda}{(1+\beta)\lambda+\kappa^2}$. Rewrite the last equation as

$$x_{t+1|t} - \frac{1}{a\beta} x_t + \frac{1}{\beta} x_{t-1} = \frac{\kappa}{\beta\lambda} u_t.$$

Factorizing the right-hand side lead polynomial by solving

$$h^2 - \frac{1}{a\beta} h + \frac{1}{\beta} = 0.$$

Denoting the stable root by δ, we have

$$\delta = \frac{1 - \sqrt{1 - 4\beta a^2}}{2\alpha\beta},$$

and, since $h_1 h_2 = \frac{1}{\beta}$, we have the unstable root $h_2 = \frac{1}{\beta\delta}$. Thus, we can rewrite our equation as

$$\left(1 - \delta L\right)\left(1 - \frac{1}{\beta\delta} L\right) x_{t+1} = \frac{\kappa}{\beta\lambda} u_t.$$

Solving this finally gives

$$\begin{aligned}\left(1 - \delta L\right) x_t &= -\frac{\kappa\delta}{\lambda}\frac{1}{1 - \delta\beta\rho} u_t \\ x_t &= \delta x_{t-1} - \frac{\kappa\delta}{\lambda\left(1 - \delta\beta\rho\right)} u_t.\end{aligned} \tag{38}$$

By substituting (38) into (37), inflation can be recovered as

$$\pi_t = \delta\pi_{t-1} + \frac{\delta}{1-\delta\beta\rho}\left(u_t - u_{t-1}\right).$$

Or, summarizing the maximized policy in terms of the price level as

$$p_t = a^* p_{t-1} + b^* u_t \tag{39}$$

$$b^* = \frac{a^*}{1-\beta a^*\rho}$$

$$a^* = \frac{\left(\lambda\left(1+\beta\right)+\kappa^2\right) - \sqrt{1-4\beta\left(\frac{\lambda}{\lambda(1+\beta)+\kappa^2}\right)^2}}{2\lambda\beta}. \tag{40}$$

D Inflation targeting, discretion

Since no credible promise can be made, and there is no endogenous state variable, the value function in the case of inflation targeting can be written as

$$\begin{aligned} V\left(u_t\right) &= \mathrm{E}_t\left[\min_{x_t}\frac{1}{2}\left(\pi_t^2+\lambda x_t^2\right)+\beta V\left(u_{t+1}\right)\right] \\ &= \gamma_0+\gamma_1 u_t+\frac{\gamma_2}{2}u_t^2, \end{aligned}$$

where the minimization is subject to (1). The control variable x_t will be a linear function of the exogenous variables

$$x_t = -\hat{d}u_t. \tag{41}$$

Also, the forward looking variable (being a linear function of the exogenous variable and the control variable) will have the form

$$\pi_t = \hat{b}u_t. \tag{42}$$

Using the above in (1) gives

$$\begin{aligned} \pi_t &= \kappa x_t + u_t + \beta\hat{b}E_t u_{t+1} \\ &= \kappa x_t + \left(1+\beta\rho\hat{b}\right)u_t. \end{aligned} \tag{43}$$

Solving the minimization problem results in the following first-order condition:

$$\begin{aligned} 0 &= E_t\left(\pi_t \frac{\partial \pi_t}{\partial x_t} + \lambda x_t\right) \\ &= \pi_t \kappa + \lambda x_t, \end{aligned}$$

or,

$$x_t = -\frac{\kappa}{\lambda}\pi_t. \tag{44}$$

Substituting (44) into (43) gives

$$\begin{aligned} \pi_t &= \kappa\left(-\frac{\kappa}{\lambda}\pi_t\right) + \left(1 + \beta\hat{b}\rho\right)u_t \\ \pi_t &= \frac{\lambda\left(1+\beta\hat{b}\rho\right)}{\lambda+\kappa^2}u_t. \end{aligned} \tag{45}$$

Comparing (42) and (45) it is clear that

$$\hat{b} = \frac{\lambda\left(1+\beta\rho\hat{b}\right)}{\lambda+\kappa^2},$$

or, solving for $\hat{b}$

$$\hat{b} = \frac{\lambda}{\kappa^2 + \lambda\left(1-\beta\rho\right)}.$$

Substituting this into the first-order condition gives

$$x_t = -\hat{d}u_t,$$

where

$$\hat{d} = \frac{\kappa}{\kappa^2 + \lambda\left(1-\beta\rho\right)}.$$

E Price-level targeting, discretion

The value function is given by

$$\begin{aligned} V_t\left(p_{t-1}, u_t\right) &= \mathrm{E}_t\left(\min_{x_t}\left(\frac{1}{2}\left(p_t^2 + \lambda x_t^2\right) + \beta V_{t+1}\left(p_t, u_{t+1}\right)\right)\right) \\ &= \gamma_{0,t} + \gamma_{1,t}p_{t-1} + \frac{1}{2}\gamma_{2,t}p_{t-1}^2 \\ &\quad + \gamma_{3,t}p_{t-1}u_t + \gamma_{4,t}u_t + \frac{1}{2}\gamma_{5,t}u_t^2 \end{aligned} \tag{46}$$

$$u_t = \rho u_{t-1} + \varepsilon_t,$$

where the minimization in the above problem is subject to (1). The guess of the value function will only be used when taking conditional expectations at t of the derivative with respect to p_t, that is:

$$\mathrm{E}_t \left[\frac{\partial}{\partial p_t} V_{t+1}\left(p_t, u_{t+1}\right) \right] = \gamma_{1,t+1} + \gamma_{2,t} p_{t+1} + \gamma_{3,t+1} \mathrm{E}_t u_{t+1}$$

so without loss, $\gamma_{0,t}$, $\gamma_{4,t}$ and $\gamma_{5,t}$ can be set to zero. Note that if there is no persistence in the residual process ($\rho = 0$), $\gamma_{3,t}$ can also be set to zero. Finally, $\gamma_{1,t}$ will only concern a drift in the price level. If x^*, π^* and p_t^* are all set to zero, $\gamma_{1,t}$ will be of no interest and can also be set to zero. It is possible to show that the inflation bias resulting from $x^* > 0$ can be eliminated with a price-level target with a drift. Thus, the guess of loss function can be written

$$V_t\left(p_{t-1}, u_t\right) = \gamma_{1,t} p_{t-1} + \frac{1}{2} \gamma_{2,t} p_{t-1}^2 + \gamma_{3,t} p_{t-1} u_t. \tag{47}$$

The state variable will follow a linear path (the quadratic loss function ensures that the policy instrument x_t is a linear function of the state variables)

$$\begin{aligned} p_{t+1} &= a_{t+1} p_t + b_{t+1} u_{t+1} \\ p_{t+1|t} &= a_{t+1} p_t + b_{t+1} \rho u_t, \end{aligned} \tag{48}$$

where the coefficients remain to be determined.

Rewrite (1) using $\pi_t = p_t - p_{t-1}$:

$$x_t = \frac{1}{\kappa}\left(1+\beta\right) p_t - \frac{\beta}{\kappa} p_{t+1|t} - \frac{1}{\kappa}\left(p_{t-1} + u_t\right).$$

Inserting (48) into the above equation gives

$$x_t = \frac{1}{\kappa}\left[1 + \beta\left(1 - a_{t+1}\right)\right] p_t - \frac{1}{\kappa} p_{t-1} - \frac{1 + \beta \rho b_{t+1}}{\kappa} u_t \tag{49}$$

or,

$$p_t = \frac{\kappa}{1 + \beta\left(1 - a_{t+1}\right)} x_t + \frac{1}{1 + \beta\left(1 - a_{t+1}\right)} p_{t-1} + \frac{1 + \beta \rho b_{t+1}}{1 + \beta\left(1 - a_{t+1}\right)} u_t \tag{50}$$

$$\begin{aligned}\frac{\partial p_t}{\partial x_t} &= \frac{\kappa}{1+\beta(1-a_{t+1})}\\ \frac{\partial V_{t+1}(p_t,u_{t+1})}{\partial p_t} &= \gamma_{2,t+1}p_t+\gamma_{3,t+1}u_{t+1}.\end{aligned}$$

Solving the minimization (46) results in the following first-order conditions:

$$\begin{aligned}0 &= \mathrm{E}_t\left[p_t\frac{\partial p_t}{\partial x_t}+\lambda x_t+\beta\frac{\partial V_{t+1}(p_t,u_{t+1})}{\partial p_t}\frac{\partial p_t}{\partial x_t}\right]\\ &= \mathrm{E}_t\left[\frac{\kappa}{1+\beta(1-a_{t+1})}p_t+\lambda x_t+\beta\frac{\partial V_{t+1}(p_t,u_{t+1})}{\partial p_t}\frac{\partial p_t}{\partial x_t}\right]\\ &= p_t+\frac{\lambda[1+\beta(1-a_{t+1})]}{\kappa}x_t+\beta\gamma_{2,t+1}p_t+\beta\gamma_{3,t+1}\mathrm{E}_t u_{t+1}.\end{aligned}$$

From (2) follows that $\mathrm{E}_t(u_{t+1})=\rho u_t$. Inserting this equation and (49) into the above and simplifying gives

$$\begin{aligned}p_t &= \frac{\lambda(1+\beta(1-a_{t+1}))}{\kappa^2+\lambda(1+\beta(1-a_{t+1}))^2+\beta\kappa^2\gamma_{2,t+1}}p_{t-1}+\\ &\quad \frac{\lambda(1+\beta(1-a_{t+1}))(1+\beta\rho b_{t+1})-\beta\rho\kappa^2\gamma_{3,t+1}}{\kappa^2+\lambda(1+\beta(1-a_{t+1}))^2+\beta\kappa^2\gamma_{2,t+1}}u_t. \qquad (51)\end{aligned}$$

In order to solve the above equation, $\gamma_{2,t+1}$ and $\gamma_{3,t+1}$ must be identified. This can be done by differentiating (47) with respect to

$$V_{p,t}(p_{t-1},u_t)=\gamma_{2,t}p_t+\gamma_{3,t}u_t, \qquad (52)$$

and comparing this to the equivalent expression obtained by using the envelope theorem on (46)

$$\begin{aligned}V_{p,t}(p_{t-1},u_t) &= \mathrm{E}_t\left[\lambda x_t\left(-\frac{1}{\kappa}\right)\right]\\ &= \mathrm{E}_t\left(-\frac{\lambda}{\kappa^2}\{[1+\beta(1-a_{t+1})]p_t-p_{t-1}-(1+\beta\rho b_{t+1})u_t\}\right)\\ &= -\frac{\lambda}{\kappa^2}\{[1+\beta(1-a_{t+1})](b_t u_t+a_t p_{t-1})-p_{t-1}-(1+\beta\rho b_{t+1})u_t\}\\ &= \frac{\lambda}{\kappa^2}(1-[1+\beta(1-a_{t+1})]a_t)p_{t-1}+ \qquad (53)\\ &\quad \frac{\lambda}{\kappa^2}\{(1+\beta\rho b_{t+1})-[1+\beta(1-a_{t+1})]b_t\}u_t \qquad (54)\end{aligned}$$

comparing (52) and (53) it is clear that

$$\gamma_{2,t}=\frac{\lambda}{\kappa^2}\{1-[1+\beta(1-a_{t+1})]a_t\} \qquad (55)$$

$$\gamma_{3,t} = \frac{\lambda}{\kappa^2} \left\{ (1 + \beta \rho b_{t+1}) - \left[1 + \beta (1 - a_{t+1}) \right] b_t \right\}. \tag{56}$$

Using (55) and (56) in (51) gives

$$\begin{aligned} p_t \;=\; & \frac{\lambda \left[1 + \beta (1 - a_{t+1}) \right]}{\kappa^2 + \lambda \left[1 + \beta (1 - a_{t+1}) \right]^2 + \beta \lambda \left\{ 1 - \left[1 + \beta (1 - a_{t+2}) \right] a_{t+1} \right\}} p_{t-1} + \\ & \frac{\lambda \left[1 + \beta (1 - a_{t+1}) \right] + \beta \rho \lambda \left\{ b_{t+1} - (1 + \beta \rho b_{t+2}) + \left[1 + \beta (1 - a_{t+2}) \right] b_{t+1} \right\}}{\kappa^2 + \lambda \left[1 + \beta (1 - a_{t+1}) \right]^2 + \beta \lambda \left\{ 1 - \left[1 + \beta (1 - a_{t+2}) \right] a_{t+1} \right\}} u_t \end{aligned}$$

Finally, comparing this to (48), the following equations must hold:

$$a_t = \frac{\lambda \left[1 + \beta (1 - a_{t+1}) \right]}{\kappa^2 + \lambda \left[1 + \beta (1 - a_{t+1}) \right]^2 + \beta \lambda \left\{ 1 - \left[1 + \beta (1 - a_{t+2}) \right] a_{t+1} \right\}} \tag{57}$$

$$b_t = \frac{\lambda \left[1 + \beta (1 - a_{t+1}) \right] + \beta \rho \lambda \left\{ b_{t+1} - (1 + \beta \rho b_{t+2}) + \left[1 + \beta (1 - a_{t+2}) \right] b_{t+1} \right\}}{\kappa^2 + \lambda \left[1 + \beta (1 - a_{t+1}) \right]^2 + \beta \lambda \left\{ 1 - \left[1 + \beta (1 - a_{t+2}) \right] a_{t+1} \right\}} \tag{58}$$

F Limit calculations

F.1 Inflation targeting, commitment

We have

$$\begin{aligned} \lim_{\lambda \to \infty} a^* (\lambda) &= \lim_{\lambda \to \infty} \frac{\left(\lambda (1 + \beta) + \kappa^2 \right) \left(1 - \sqrt{1 - 4\beta \left(\frac{\lambda}{\lambda (1+\beta) + \kappa^2} \right)^2} \right)}{2 \lambda \beta} \\ &= \lim_{\lambda \to \infty} \frac{\left((1 + \beta) + \frac{\kappa^2}{\lambda} \right) \left(1 - \sqrt{1 - 4\beta \left(\frac{1}{(1+\beta) + \frac{\kappa^2}{\lambda}} \right)^2} \right)}{2\beta} \\ &= \frac{(1 + \beta) \left(1 - \sqrt{1 - 4\beta \left(\frac{1}{(1+\beta)} \right)^2} \right)}{2\beta} \\ &= 1. \end{aligned}$$

Next, the lower limit

$$\begin{aligned}
\lim_{\lambda \to 0} a^*(\lambda) &= \lim_{\lambda \to 0} \frac{\left(\lambda(1+\beta)+\kappa^2\right)\left(1-\sqrt{1-4\beta\left(\frac{\lambda}{\lambda(1+\beta)+\kappa^2}\right)^2}\right)}{2\lambda\beta} \\
&= \lim_{\lambda \to 0} \frac{1-\sqrt{1-4\beta\left(\frac{\lambda}{\lambda(1+\beta)+\kappa^2}\right)^2}}{\frac{2\lambda\beta}{\lambda(1+\beta)+\kappa^2}} \\
&= \lim_{e \to 0} \frac{1-\sqrt{1-4\beta e^2}}{2\beta e}, \text{ with } e = \frac{\lambda}{\lambda(1+\beta)+\kappa^2} \\
&= \lim_{e \to 0} \frac{-\left(1-4\beta e^2\right)^{-\frac{1}{2}}(-8\beta e)}{2\beta} \\
&= 0
\end{aligned}$$

where we have used L'Hôpital's rule.

F.2 Price-level targeting, discretion

$$\begin{aligned}
\lim_{\tilde{\lambda} \to 0} \tilde{a}\left(\tilde{\lambda}\right) &= \lim_{\tilde{\lambda} \to 0} \frac{\omega\tilde{\lambda}}{\kappa^2+\omega^2\tilde{\lambda}+\beta\tilde{\lambda}(1-\omega\tilde{a})} \Leftrightarrow \\
\lim_{\tilde{\lambda} \to 0} \tilde{a}\left(\tilde{\lambda}\right) &= 0
\end{aligned}$$

$$\begin{aligned}
\lim_{\tilde{\lambda} \to \infty} \tilde{a} &= \lim_{\tilde{\lambda} \to \infty} \frac{\omega\tilde{\lambda}}{\kappa^2+\omega^2\tilde{\lambda}+\beta\tilde{\lambda}(1-\omega\tilde{a})} \Leftrightarrow \\
\lim_{\tilde{\lambda} \to \infty} \tilde{a} &= \lim_{\tilde{\lambda} \to \infty} \frac{\omega}{\frac{\kappa^2}{\tilde{\lambda}}+\omega^2+\beta(1-\omega\tilde{a})} \Leftrightarrow \\
\lim_{\tilde{\lambda} \to \infty} \tilde{a} &= \frac{\omega}{\omega^2+\beta\left(1-\omega\lim_{\tilde{\lambda} \to \infty} \tilde{a}\left(\tilde{\lambda}\right)\right)}
\end{aligned}$$

From the last line we see that $\lim_{\tilde{\lambda} \to \infty} \tilde{a} = 1$ is a solution (remember that $\omega = 1+\beta(1-\tilde{a})$.