Empirical Estimation and the Quarterly Projection Model: An Example Focusing on the External Sector

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In this paper, we offer one possible way to estimate a key feature of the Bank of Canada's main macroeconomic model, the Quarterly Projection Model or QPM. The key feature which is the focus of this study is the so-called "short-run equilibrium values" or SREQs which link the dynamic portion of QPM to its steady state. Our estimation is motivated by an unsatisfying feature of the current version of the SREQs. That is, they are produced using a mechanical filter which does not capture the influence of movements in other variables on the filtered time series. In other words, the current SREQs are exogenous. The estimation approach detailed in this paper attempts to make the SREQs endogenous with respect to fluctuations in key economic variables. The first part of this paper demonstrates how we are able to rewrite the external sector of QPM in a form that allows empirical estimation based on cointegration analysis. The second part of the paper then considers the implications for QPM of estimated, endogenous SREQs via both impulse response functions and stochastic simulations. In this latter part of the paper, we also present what we believe are novel approaches for estimating stochastic shocks for calibrated macroeconomic models.

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1.0 Introduction

The Quarterly Projection Model or QPM is currently the main macroeconomic model used at the Bank of Canada.[1] QPM was developed in part to bridge the gulf between macroeconomic models that are used for policy analysis and models that are used purely for projection purposes. Being able to use a model for both projection and policy analysis has been regarded at the Bank of Canada as a goal of model building, owing to the model's role in unifying current analysis, research and policy advice within an integrated framework. In order to achieve its twin objectives, however, QPM uses calibration rather than direct estimation to fix parameter values. Calibration allows model builders to establish general properties, as opposed to specific parameter values, from the data thereby simultaneously achieving a measure of success in both projection and policy analysis. Part of the calibration of QPM extends the notion of the steady state, which is unambiguous in the context of artificial simulations, into the world of real data, where matters are less clear cut. It is precisely this extension which is the focus of the current paper.

In this paper, we study the implications of empirical estimation for the model properties of QPM. In particular, we use the external sector of QPM as an example and present an empirical methodology for estimating the "short-run equilibrium values" or SREQs and then examine the implications of this change for overall model properties. Currently, SREQs, which loosely speaking link the dynamic model to its steady state, are determined solely by the application of a two-sided moving average filter which implies that they are invariant to fluctuations in other economic variables. This fact leads to some unappealing features when filtered SREQs are used for projection or post-historical data purposes. We discuss these features and SREQs, in general, in the following section (Section 2).

The remainder of the paper is organized as follows. Section 3 describes the structure of the external sector in QPM while Section 4 demonstrates how a system of structural equations may be re-written as estimable reduced-form equations. The SREQs estimation strategy is briefly described in Section 5 while the empirical results are presented in Section 6. Sections 7 and 8 then compare the two version of QPM (current

1. See Black et al. (1994) and Coletti et al. (1996) for complete descriptions of the QPM system.

and estimated) using impulse response functions and stochastic simulations. In Section 8, we also describe what we believe is a novel approach to estimating shocks for calibrated macroeconomic models. Section 9 concludes.

2.0 The Role and Implementation of SREQs

QPM differs in significant ways from previous models built and maintained at the Bank of Canada. Dynamic macroeconomic models have often been built without a formal steady state (long-run equilibrium). Since the existence of a well-defined steady state was judged to be critical for dealing with the many contemporary policy questions that require a medium- to long-term perspective, model builders included this feature in QPM. Indeed, not only is there a steady state within QPM, but QPM can be divided into two distinct parts, its steady state (SSQPM) and its dynamic structure.

The steady state is the final, static equilibrium (or “terminal conditions”) upon which the economy is conjectured to settle down, once: (i) all shocks are over; (ii) all adjustment costs have been borne; (iii) all stock-flow dynamics have been completed; and (iv) all expectations are realized. Notionally, there are no further shocks to the economy at the steady state. It is easy to conceptualize a steady state, where all variables grow at constant, predetermined growth rates, inflation is constant and equal to its targeted rate, and relative prices are fixed. The steady state is the long-run anchor for the economy. The dynamic model, on the other hand, traces out the evolution of the economy from its initial conditions to its steady state as described by SSQPM.

The real world is different than this idealized steady state. It is easy to conceptualize a world for policy analysis purposes where no shocks exist other than the ones the modeller specifically includes for consideration. In other work, however, one must come to grips with the historical experience of a multitude of shocks which are constantly hitting the economy. These shocks occur at different frequencies. Some shocks are temporary in nature occurring at business cycle frequency or higher. Examples are exogenous disturbances to aggregate demand (preference shocks) and temporary shifts in investor confidence in Canadian dollar denominated assets -- exchange rate shocks. These types of shocks are addressed exclusively by the dynamic structure of the model since they are assumed to be relatively short-lived. Other shocks can occur at lower frequencies -- the trend component of economic data -- and are permanent or near permanent in nature. Examples of these types of shocks are technology shocks, shocks to the rate of time preference, demographic shocks and fiscal / monetary policy regime shifts.

One possible interpretation of the trends in the historical data is that the economy is constantly adjusting toward a series of short-run equilibriums that need not have been constant. To capture this phenomenon QPM uses the concept of the "short-run equilibrium values" or SREQs. As a practical matter, SREQs, by capturing the lower-frequency effects of violations in the four conditions of a steady state, prevent the dynamics of QPM from jumping to the steady state in an inordinately short period of time. This, in turn, ensures that the shock terms in the dynamic model necessary to replicate history are stationary.[2] In practice, SREQ paths are determined by detrending the historical definition of the variable in question and then forcing the trend to converge onto the steady-state value within some predetermined time period. The detrending is done using a version of the Hodrick-Prescott filter that splines the SREQ to its steady-state value. The date at which the SREQ variables converge on their steady-state values is a function of certain calibration requirements of the dynamic model including an *a priori* condition that shocks imply some changes in the steady state and the dynamic paths of endogenous variables be clearly seen as converging on their steady-state counterparts over the medium term.

While the application of SREQs allows us to link dynamic and steady-state QPM and slows down adjustment to what may be considered more reasonable, there are a number of unappealing features of the current methodology. The first obvious shortcoming of the current methodology is the arbitrariness of certain steady-state assumptions. This problem becomes particularly acute in the case of those variables which are unobservable by nature like the consumer's rate of time preference and for variables in which the data are at odds with the simplified theory behind the model.

Second, the exogenous SREQ paths which rely on univariate filters are fairly arbitrary in nature especially over the projection or post-historical data period where they are essentially a spline from the end of historical data to the steady state. For

2. Only in the very long-run do the SREQs converge to the steady state.

example, our estimate of the SREQ for real world commodity prices comes from an HP filter of actual commodity prices. This is not the case, however, over the projection period. The extended HP filter simply uses a spline to join the historical estimate of the commodity price SREQ to the assumed steady-state value, regardless of the assumed exogenous path for commodity prices in the future. This implies that a temporary but highly persistent shock will have no effect on the path of the SREQ

Third, the long-run reduced-form economic relationships in the model may or may not be consistent with those gleaned from more detailed empirical work. The mechanistic, univariate determination of the exogenous SREQs, especially those which are endogenous in an economic sense, poses a problem. Consider, for instance, the case of the SREQ for the terms of trade and real exchange rate. Economic theory suggests that an important economic relationship between these two variables exist. However, since both theses series are treated as exogenous and constructed with a univariate filter there is nothing that ensures that the respective SREQs share a common trend.[3]

Owing to these and other problems, we have explored a number of alternative ways to calculate SREQs. Notwithstanding the clear need for something like the concept of SREQs, there are conceivably a number of methods to compute them. Notionally, the set of SREQs in QPM is a number of cointegrating vectors bound together by non-linear restrictions. In principle this is an estimable problem although the theoretical richness and highly simultaneous nature of QPM makes direct estimation impossible, even with the latest techniques. Reliable estimates of even a single cointegrating vector with post-war macro data are often quite difficult to obtain; estimating several with cross-equation restrictions is a very difficult task. In the next sections we outline a proposed strategy which allows us to simplify this problem and bridge the gap between QPM's structure/ properties and empirical estimates of various key equilibrium relationships. We demonstrate our proposed methodology using the external sector of QPM.

3. Kozicki (1999) outlines a methodology to detrend multiple time series under common trend restrictions based on a modification of the simple univariate HP filter. Unfortunately Kozicki's methodology cannot be easily amended to include the level restrictions required to use the filter over the projection period.

3.0 Structure of the External Sector in SSQPM[4]

This section outlines briefly the current structure of the external sector. In an attempt to ensure consistency between the structure of SSQPM and the system of reduced-form equations, we begin by describing the most salient features of the current external sector.

SSQPM begins with a standard representative agent who chooses consumption and holdings of financial assets so as to maximize the discounted sum of expected utility over her lifetime. Given the level of financial wealth that emerges from the consumer's problem, as well as the level of government debt and desired capital stock, we can solve residually for net foreign asset (NFA) holdings.[5] The stock of net foreign assets as a share of output, in turn, imposes a condition on the nominal current account share through the balance of payments identity. Finally, we can express the current account share, $cbal\$_t / y\$_t$, in terms of interest on net foreign assets, $r\$_t \cdot nfa\$_{t-1}$, trade shares, x_t and m_t, and their corresponding relative prices, px_t and pm_t, *viz.*,[6]

$$\frac{cbal\$_t}{y\$_t} = (px_t x_t - pm_t m_t) - \left(\frac{r\$_t \cdot nfa\$_{t-1}}{y\$_t}\right) \tag{1}$$

Currently, the equation for the export share, lx_t, is given by

$$lx_t = \alpha_x + lz_t - \beta(ly_t - ly_t^{ROW}) \tag{2}$$

where lz_t denotes the real effective exchange rate (G6 basis) and ly_t^{ROW} is rest-of-the-world real output.[7] The last term in the equation arises from the almost small open economy assumption. Specifically, the size of Canada's economy relative to that of the rest-of-the-world matters for the determination of our export share (β=0.6). The parameter α_x is a so-called "cal" term and serves two related purposes in SSQPM. The first is to ensure that all behavioural, steady-state relationships in the model hold at each point in time. In this sense, the cal term serves the same purpose as the residual in a regression equation. It is

4. The mnemonics used in this paper follow directly from those in QPM. Also, variables proceeded by *l* denote logs.
5. For a detailed exposition of the wealth accounts in SSQPM, see Black et al. (1994).
6. Quantities are shares of real output and relative prices are deflated by the GDP deflator. Variables with '\$' are nominal.
7. The real effective exchange rate is defined such that an increase represents a depreciation.

interesting to note that in many cases the cal term contains a trend, implying that many of the simple relationships in SSQPM are not cointegrating equations. The second role of the cal term is to allow “recalibrations” of SSQPM. Recalibrations effectively permit the user to change the steady-state value of one or more variables while leaving the others unchanged. For instance, one could facilitate a real depreciation of the steady-state exchange rate without causing a reduction to the import shares. In this role, the cal term is more akin to the constant in a regression equation.

Total imports are simply the sum of the import components of consumption, investment and “other” (which subsumes government and exports). These shares are modelled as

$$lm_{i,t} = \alpha_i - lz_t \tag{3}$$

where lm_i represents the i^{th} share and α_i is the corresponding calibration term. So, for the two variables, lx_t and lm_t, SSQPM imposes exchange rate elasticities of one and negative one respectively.

The determination of relative prices is somewhat more tedious and some algebra is required in order to place them in their reduced form. Beginning with the relative price of exports, lpx_t

$$lpx_t = \alpha_{px} + \delta lpcx_t + (1-\delta)lmc_t + \zeta(ly_t^{ROW} - ly_t) \tag{4}$$

where $lpcx_t$ denotes the relative price of Canadian exports and is given by

$$lpcx_t = \lambda lpcom_t + (1-\lambda)lpman_t \tag{5}$$

$lpman_t$ is the relative price of manufactured good in Canada and is set equal to the real exchange rate. The term $lpcom_t$ is the relative price of commodities in Canadian dollars and is defined as

$$lpcom_t = lpcomrow_t + lz_t \tag{6}$$

where $lpcomrow_t$ is the relative price of world commodities expressed in G6 terms. Finally, marginal cost, lmc_t, is given as follows

$$lmc_t = -lti_t - \left(\frac{1-\eta}{\eta}\right) lpx_t \tag{7}$$

where lti_t is the aggregate indirect tax rate.

Substituting (5) - (7) into (4) and solving for the relative price of exports yields

$$lpx_t = \Phi_0 + \Phi_1 lpcomrow_t + \Phi_2 lz_t + \Phi_3 lti_t + \Phi_4 (ly_t^{ROW} - ly_t) \tag{8}$$

Thus it is possible to write the relative price of exports in terms of real world commodity prices, the real effective exchange rate, indirect taxes and relative domestic output. All else the same, higher real commodity prices or a weaker real exchange rate will tend to raise the Canadian dollar relative price of exports. Higher aggregate indirect taxes will tend to reduce the *relative* price of exports since this change will not be reflected in the tax rate for exports.[8] Therefore, any change to the aggregate tax rate is assumed to be a change to the relative rate. The last term in equation (8) again reflects the assumption that Canada is an almost small open economy. *Ceteris paribus*, an increase in the size of the Canadian economy, relative to that of the rest of the world, will bring about a reduction in export prices. The current values of these parameters are given in Table 1.

Turning to the relative price of imports, we begin with;

$$lpm_t = \alpha_{pm} + \theta lz_t + (1-\theta) lmc_t - lti_t + \mu\theta lpcomrow_t \tag{9}$$

Substituting (7) into (9) and solving out the relative price of exports yields

$$lpm_t = \Gamma_{0pm} + \Gamma_1 lz_t + \Gamma_2 lti_t + \Gamma_3 lpcomprow_t + \Gamma_4 (ly_t^{ROW} - ly_t) \tag{10}$$

Note that aggregate indirect taxes enter (8) and (9) through the marginal cost equation, (7).[9] By setting $\delta = \theta = 1$ we effectively eliminate the influence of indirect

8. The indirect tax rate on exports has been permanently set to zero in SSQPM. There are no systematic export taxes in Canada.

taxes (or domestic cost conditions) on the terms of trade. However, in doing so Φ_2 and Γ_1 each collapse to unity. This restriction implies that, *ceteris paribus*, devaluations cannot affect the terms of trade in the long run. Moreover, it implies that real exchange rate movements feed one-for-one into export and import prices. This restriction is tested in the empirical section of our paper.

Letting $\Omega \equiv [\Theta \quad \theta\ \mu\ \alpha_{pm}]$ we can see that identification of Ω amounts to recovering $[\theta \quad \mu \quad \alpha_{pm}]$ from the reduced form (10). In summary, we can see that it should be possible, given estimates of equations (2), (3), (8) and (10) to recover the parameters embodied in the relevant structural equations. This should not, however, be interpreted as meaning that we are able to estimate SSQPM. The term Ω should be thought of as merely a subset of the true parameter set embodied in SSQPM. For instance, if we were to begin expanding equations (8) and (10) around z_t, we would find ourselves modelling export and import prices as a function of every exogenous variable in the model and the corresponding set of structural parameters would not be identified. This empirical strategy can be best thought of as a reasonable compromise between a single-equation reduced-form specification and direct estimation of a structural, simultaneous-equation, general-equilibrium model such as SSQPM, the purpose being to provide the researcher with some guidance as to the magnitude of some of the reduced-form relationships embodied in the structural model.

4.0 System Specification

This section demonstrates how a system of structural equations may be written as estimable reduced forms. As described in Section 3.1, equations (2), (3), (8) and (10) represent a set of relatively parsimonious equations which illustrate the principal determinants of the current account balance in SSQPM. This section goes on to describe modifications to this four variable system required for the purpose of estimation. In addition, we replace the export equation with an equation for the real exchange rate.

9. The aggregate indirect tax rate does remain however in the import equation. This reflects the fact that in SSQPM we model the price of imports relative to price at factor cost. For the purpose of this exposition, as well as estimation, we model import prices relative to market prices.

4.1 Real Effective Exchange Rate

The real exchange rate plays a crucial role in SSQPM as it ultimately ensures consistency between the current account and the NFA position. Specifically, the real exchange rate adjusts to maintain a current account that is consistent with keeping the NFA position at its desired steady state. For instance, in the event of a positive terms-of-trade shock that leaves NFA unchanged, the real exchange rate will appreciate to cause a reduction in net exports which just offsets the increase in the terms of trade.

In the past, particularly when there has been substantial uncertainty surrounding the steady-state level of the real exchange rate, we have relied upon one or more empirical models for guidance. Recently, the bilateral exchange rate equation developed by Amano and van Norden (1995) has been used to assess the validity of the current long-run value of the real exchange rate as well as the near term profile of the actual exchange rate. Since empirical exchange rate models are already used, having an explicit exchange rate equation as part of the system could be of potential benefit.

As subsection 3.1 outlines, one of the primary goals of this project is to derive, where possible, an empirical specification that is consistent with the structure of SSQPM. That said, we should remain cognizant of the stock-flow nature of the model when specifying an equation for the real exchange rate. Specifically, one must consider the role of Canada's NFA position (or *nfa)* in the determination of the SREQ for the real exchange rate, z^*.[10] While most of the exchange rate literature tends to focus on the terms of trade and productivity differentials (between traded and non-traded goods across countries), positing a role for *nfa* is not unprecedented. Faruqee (1994), for instance, finds evidence of a long-run relationship between productivity, NFA and the real exchange rates for Japan and the United States. Formal exclusion tests seem to support the belief that *nfa* does play an important role in the determination of z.

Unlike *nfa*, there is an extensive literature which examines the role of the terms of trade in the determination of z.[11] Recently, Amano and van Norden (1995) found evidence of a long-run statistical relationship between energy and non-energy terms of trade and the real bilateral exchange rate for Canada from 1973 to 1992.[12] Therefore, we

10. Henceforth, variables denoted with '*' should be interpreted as SREQs.

11. See for example, Amano and van Norden (1998a,b) for empirical evidence linking the exogenous component of the terms of trade to real exchange rates.

model z^* as a function of non-energy commodity and real G6$ oil prices, denoted *bcne* and *ener*, respectively. We also include the relative price of computers and peripheral equipment expressed in G6 terms, denoted *comp,* to capture the downward trend in import prices over recent history. We do not, however, model the terms of trade as a ratio as in Amano and van Norden (1995), rather we allow them to enter the equation separately. Formally, the exchange rate SREQ is given as

$$lz_t^* = \beta_0 + \beta_1 nfa_t + \beta_2 lbcne_t + \beta_3 lener_t + \beta_4 lcomp_t \tag{11}$$

The last three terms are hoped to capture the low frequency movements in the G6$ terms of trade over the sample period. It is worth noting that the entire set of regressors in equation (11) may be viewed as exogenous determinates of z^*. It seems highly improbable that z^* is a major determinant of *bcne*, *ener* or *comp*. While both z and *nfa* are modelled as endogenous variables in SSQPM, *nfa* is taken to be exogenous for the determination of z. Consequently, we normalize on z^* and interpret (11) as an exchange rate equation. The importance of these assumptions will become clear when we begin discussing estimation issues in Section 5.0.

4.2 Imports

In modeling the import share of output, we begin with an unrestricted specification whereby output appears as a regressor and we then test the adequacy of the restricted share model. This is done in part because recent literature (see Amano and Wirjanto (1997) for an example with Canadian data) clearly rejects the unit output elasticity implied by a share equation. In an effort to estimate an elasticity more consistent with the balanced growth assumption embodied in SSQPM, we include a variable, *trade*, that is intended to proxy for the degree of trade liberalization among G6 nations. The variable *trade* is defined as G6 export activity divided by G6 output. A variant of this proxy was introduced by Beenstock and Warburton (1982). Using U.K. data, they find this variable dominates a simple time trend both in terms of explanatory power as well as the implied output elasticity.

12. Energy and non-energy prices divided by the price of imports.

In addition to output, we include the relative price of imports, *pm*, instead of the real exchange rate as the relevant price variable. We believe that this divergence from the structure of SSQPM is justified on two counts. First and foremost, the relative price of imports would seem to be the relevant variable for determining import activity. While the real exchange rate appears to be a major determinant of *pm*, the two variables alone are not cointegrated. This suggests that while *z* does influence *pm*, it alone cannot explain the low frequency movements in import prices. The second reason for including *pm* is that preliminary results using *z* suggest an implausibly large elasticity, about -2.5.

Putting it all together, the import activity equation is given as[13]

$$lm^*_t = \beta_5 + \beta_6 ly_t + \beta_7 lpm_t + \beta_8 ltrade_t \tag{12}$$

4.3. Relative Export and Import Prices

As subsection 3.1 outlines, *px* and *pm* are currently modelled (in SSQPM) as a function of the real exchange rate, commodity prices, indirect taxes and the domestic-foreign output differential. One of the features of our proposed methodology for generating SREQs is that it explicitly takes account of the time-series properties of the data. Specifically, the method of estimating these relationship hinges on the belief that these data exhibit a specific form of non-stationary behaviour, i.e. they are unit-root processes. However, standard unit-root tests reject non-stationarity for $(ly^*_t - ly_t)$ so we exclude it from the long-run relationship. It is, however, retained in the short-run component of the model.

Finally, we add the computer price series, *comp* to both equations. We expect that it will either be insignificant in the export equation or have an extremely small coefficient. Note also that we have replaced *pcomrow* with *bcne* and *ener* in both equations;

$$lpx^*_t = \beta_9 + \beta_{10} lz_t + \beta_{11} lbcne_t + \beta_{12} lcomp_t + \beta_{13} lener_t \tag{13}$$

$$lpm^*_t = \beta_{14} + \beta_{15} lz_t + \beta_{16} lbcne_t + \beta_{17} lcomp_t + \beta_{18} lener_t \tag{14}$$

13. Note here that *lm* denotes the log of the level of imports rather than the import share of output.

4.4 Net foreign assets-to-GDP ratio

Economic accounting tells us that a country's level of net international indebtedness is equal to either the accumulation of past current account or capital account balances. A significant discrepancy, however, exists due largely to accounting differences and different data sources used to calculate these measures. Since QPM relies on the link between NFA and the current account in its stock-flow accounting we have decided to calculate the NFA position by accumulating the current account.

In modelling the determinants of NFA we consider several key theoretical relationships. First, as suggested by the Yaari-Blanchard-Weil model (Yaari 1965; Blanchard 1985; Weil 1985), which forms the basis of SSQPM, a relationship between the level of government debt and NFA exists in a small open economy. If birth rates are positive, Ricardian equivalence does not hold and the choice between financing government expenditures through taxation or issuing bonds has real effects. In SSQPM a one percentage point increase in the government debt-to-GDP ratio reduces the NFA-to-GDP ratio by about 1.05 percentage points. To control for the effects of changes in government debt in the rest-of-the-world we have constructed a variable, *debt,* which is defined as the difference between Canada's debt-to-GDP ratio and the G6 debt-to-GDP measure.

As previously mentioned, SSQPM is a representative agent model, and as such does not take into account the heterogeneity of consumers. A key difference amongst individuals which is thought to determine their consumption / savings behaviour is their age. The principal theoretical link between demographic trends and aggregate savings is the life-cycle model of consumption behaviour (Ando and Modigliani, 1963). According to this theory, private agents save during the middle (or working) years of their lifetimes, having borrowed during the early years while looking forward to continued consumption in retirement during the later years. In such a world, agents with the largest stock of assets should be those in their retirement years. To capture potential demographic effects we have constructed the variable, *rdem,* which following Masson et al. (1994) is defined as the difference in the Canada-G6 old-age dependency ratios. More specifically, the dependency ratio is defined as the ratio of persons in the economy over 65 years of age relative to those in the working population (between 15 and 64 years of age). The long-run NFA equation is therefore given as

$$nfa_t^* = \beta_{19} + \beta_{20} debt_t + \beta_{21} rdem_t \quad (15)$$

5.0 Estimation Strategy

As mentioned in Section 2, we propose to replace the existing set of SREQs, derived from modified Hodrick-Prescott filters, with a set of cointegrating equations. These equations can then be used over both history and the projection to identify the SREQs relevant to the dynamic model, QPM. For cointegration to exist among a set of variables, integration of some common order greater than zero is necessary. Thus, we test the order of integration of the variables using the augmented Dickey-Fuller (ADF), Phillips-Perron (PP) and the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) unit-root tests. The first two test have nonstationary as the null whereas the latter takes stationarity as its null. Based on these test results, we conclude that the levels of each series contains exactly one unit root.[14] All subsequent tests are based on this assumption.

To test for cointegration we use the full-information maximum-likelihood estimator suggested by Johansen (1988) and Johansen and Juselius (1990), hereafter JJ, to determine the long-run relationship(s) and dynamic adjustment process simultaneously. This methodology has four advantages which we believe to be relevant to this project.[15] First, it allows us to test for the number of independent cointegrating relationships. This feature is of considerable utility when we are attempting to model many variables. In principle, with *p* variables, we could have up to *p-1* such relationships. In the event that there exists more than one vector, restrictions may be imposed to “identify” the individual parameters. Second, the test procedure is invariant to normalization which is useful if we are unsure of how many potentially endogenous variables there are in the equation(s). Third, once the number of cointegrating vectors is established, we are permitted to test for weak exogenity and long-run exclusion of each variable. Finally, as a systems estimator, it allows for formulation and testing of cross-equation restrictions within the long-run structure.

In addition to our formal tests for cointegration, we also examine the econometric

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14. For the sake of brevity, we do not report the unit-root test statistics. They are, however, available from the authors upon request.
15. This should not be interpreted as meaning that the JJ estimator dominates all single-equation methods in all circumstances. It has been demonstrated repeatedly that this is not the case.

properties of each equation to assess their forecasting behaviour and temporal stability.[16] In particular, we use: (i) LM and ARCH tests to test for the presence of serial correlation and autoregressive conditional heteroscedasticity in the estimated residuals; (ii) prediction failure and a formal test proposed by Johansen and Juselius (1992) to assess the degree of instability in the parameter vector; (iii) Theil's U-statistic to examine the ability of the error-correction model (ECM) to outperform a random-walk model at different horizons in out-of-sample forecasting exercises; and (iv) recursive estimation to determine whether our evidence of cointegration is temporally stable or whether it is sensitive to the sample period under consideration. Finally, in cases where we find evidence of only one cointegrating vector, we explore the robustness of the parameter estimates by re-estimating the model within single-equation non-linear ECM framework described in Phillips and Loretan (1991). For the sake of brevity, we only report model diagnostic results if they point to a problem with our specification.

6.0 Empirical Results

6.1 Real Effective Exchange Rate

We begin by testing for the presence and uniqueness of a cointegrating relationship between the variables given in equation (11) using the JJ methodology mentioned above. Table 3 reports the trace and λ_{max} test statistics.[17] Based on these test statistics, we conclude that there exists a single cointegrating vector which is identified up to a scalar. While it is true that there exists strong evidence of cointegration, the entire vector of variables may not be needed in order to establish the long-run relationship. Thus we consider the possibility of redundant variables in the cointegrating space. Table 4 shows tests for exclusion of variables from the cointegrating equation, which is distributed as a $\chi^2(q)$ under the null conditional on the chosen rank $(\Pi) = q = 1$.[18] We can easily reject exclusion for each variable from the long run-relationship except the relative price of oil. While we acknowledge that other specifications have found a role for oil/ energy prices, there is evidence that this relationship is somewhat sample sensitive.

On the basis of the above test we elect to omit *lener* from the specification and

16. The results and further details about the actual tests are available from the authors upon request.
17. These tests both take $rank(\Pi) \leq q$ as the null hypothesis but differ in their corresponding alternatives, the former taking $q + 1$ and the latter taking $\geq q + 1$.
18. All of the tests which follow are conditional on the chosen rank.

proceed to re-test for *rank* (Π). We once again see strong evidence in favour of one cointegrating vector and somewhat weaker evidence in favour of a second (Table 5). Given the discrepancy between the two statistics, we set $q = 1$. It should be noted that there exists some Monte Carlo evidence to suggest that use of the Johansen procedure in small samples may lead to over rejection of the null. Therefore, in addition to testing for the value of q using *rank*(Π), we also perform unit-root tests on the equilibrium errors from the cointegrating equation. The ADF test statistic is -4.2 which allows us to reject the null of no cointegration at the 5 per cent level. Given that residual-based tests tend to exhibit low power for local alternatives, we are confident that the series are cointegrated.

As mentioned in Section 4, it seems reasonable to assume that the right-hand-side variables in equation (11) are weakly exogenous for the estimation of the long-run parameter vector, with the possible exception of *nfa*. Since these are testable hypotheses within the JJ framework we now turn to the matrix of loading (speed of adjustment) parameters. Table 6 provides results of the tests of weak exogeneity, which again has the $\chi^2(q)$ distribution under the null. Based on these results, we conclude that non-energy commodity and computer prices are weakly exogenous and the real exchange rate is clearly endogenous. Unfortunately, this test would also seem to suggest that NFA is endogenous. Consequently, *nfa* responds, albeit very slowly, to exchange rate disequilibria.[19] Based on this evidence alone, we are unconvinced that this is indeed the case and test the sensitivity of the model to the status of *nfa*.[20] The results are almost invariant to the state of *nfa* and we consequently select the restricted model (*nfa*, *lbcne* and *lcomp* exogenous). After normalizing on *z*, we arrive at the following long-run relationship

$$lz_t{}^* = \beta_0 + \beta_1 nfa_t + \beta_2 lbcne_t + \beta_3 lcomp_t \qquad (16)$$

Table 7 gives the identified long-run parameters for equation (16). *Ceteris paribus*, higher non-energy commodity prices or a stronger NFA position will lead to a real appreciation of the exchange rate whereas higher computer prices will bring about a small depreciation. The signs of both β_1 and β_2 are consistent with that of SSQPM. The

19. To the extent that we can interpret this as an “exchange rate” equation, given that there appears to be more than one endogenous variable.
20. In SSQPM, both *nfa* and *z* are endogenous variables. However, there is a one way causal link going from *nfa* to *z*.

magnitudes, however, are somewhat different. For instance, the elasticity of commodity prices is currently about -0.1, compared with -0.8 in equation (16). It is worth noting, however, that this result is fairly similar to those reported in other recent studies. For instance, Amano and van Norden (1995) and Lafrance and van Norden (1995) report elasticities of about -0.8 and -0.5, respectively. Thus, the empirical evidence would tend to suggest that the equilibrium real exchange rate should have depreciated by considerably more than 1.5 per cent following the recent 15 per cent decline in commodity prices. However, it must be noted that the relevant variable in equation (16) is non-energy commodity prices only whereas in SSQPM it is the total. Therefore the relevant shock is not 15 per cent (total), but rather about 7 per cent (*bcne* only). Nevertheless, as we see later, the implications for the equilibrium exchange rate are quite different from the current calibration of QPM.

It is more difficult to pin down an elasticity for the NFA position in SSQPM since both are endogenous. Hence, the elasticity will vary according to the source of the shock. The estimated elasticity is about half of that reported in Faruqee (1994) for the United States (-1.5) but much larger than the -0.1 reported by Gagnon (1996) for a panel of 20 industrialized countries. Our estimate, therefore, appears to fall in the middle of these two extremes, but its standard error suggests that there is a considerable amount of uncertainty about its precision.

A comparison of the equilibrium errors (actual minus SREQ) from the current HP filter and the cointegration equation shows that a greater amplitude of disequilibrium comes from the estimated equation. In fact, the standard deviation of the errors is about twice as large. The degree of persistence of the shocks, however, is about the same between the two models. The basic story coming from the long-run equation is that over history the size of the cyclical component has been much bigger than suggested by the filter, but the degree of persistence is about the same. Nevertheless, the magnitude of disequilibria remains 10 per cent or less over 70 per cent of the historical sample.

In regard to model properties, the only notable drawback of the estimated ECM appears to be its inability to outperform a random walk at the one-year frequency.[21] This is somewhat disappointing given that the equation developed in Amano and van Norden

21. This conclusion is based on Theil's U-statistic and data from 1986Q1 onwards.

(1995) outperforms a random walk at all forecasting horizons. The fact that this statistic increases through the first year, however, appears to be unique to the chosen starting date. In addition, the ECM is able to perform better than a random walk over other forecasting horizons, *viz*., 1 through 15 quarters.

6.2 Imports

Turning to the import equation, we begin with the unrestricted specification given by equation (12) and then proceed to test for the presence and uniqueness of a cointegrating relationship using the JJ estimator. The system was estimated from 1969Q1 to 1998Q2 with 2 lags (3 lags in levels). Because the real exchange rate does not appear in this specification we extended the starting date back to 1969Q1 (rather than the end of Bretton Woods, approximately 1973Q1). However, the results are essentially invariant to the choice of starting date. Looking at Table 8 we see somewhat weaker evidence of cointegration compared to the exchange rate equation. Nevertheless, the trace statistic does indicate one non-zero root at the 5 per cent significance level.[22]

After setting q=1 and normalizing on *lm* we observe an unrestricted output elasticity of 1.0 so we proceed to reformulate the model in terms of the import share of output, *m_y*. Using the new specification, tests are performed for both weak exogeneity and exclusion. Tables 9 indicates that we can easily reject the null hypothesis of long-run exclusion for all 3 variables at the 5 per cent level. The test for weak exogeneity (Table 10) indicates that the relative price of imports and G6 trade variable are both exogenous. The former result would be consistent with Canada facing a perfectly elastic world supply curve. Strictly speaking, import quantities and prices are both endogenous in SSQPM. However, the assumption of Canada being a price taker on world markets remains consistent with the structure of SSQPM. Nevertheless, we proceed to check the sensitivity of the results to the status of *lpm*, and conclude that the distinction is unimportant to our results.[23] This exogeneity assumption allows us to estimate a single-equation ECM. Normalized coefficient estimates for both estimation methodologies are provided in Table 11.[24] The above results indicate the estimated elasticities are very similar across the

22. The static Engle and Granger cointegration test procedure was also performed using {*lm,ly,lpm,ltrade*}. Results indicate that the null hypothesis of no cointegration can be rejected at 10 per cent. This result holds regardless of whether we model the level of imports or the import share.

23. Results that follow are based on the assumption of weak exogeneity for *lpm*.

24. No lags were included in the least squares ECM whereas 2 lags were selected for the FIML ECM.

two estimators. The price elasticity of -.68 is fairly similar to that estimated by Amano and Wirjanto (1997) for Canada (about -0.5). Also noteworthy is the fact that the single-equation estimates do not allow us to reject the hypothesis of $\beta_7 = -1$. This restriction also cannot be rejected at the 10 per cent level by the FIML estimates.

In this case, the errors from the cointegration model are fairly similar to those of the HP filter over much of the sample. Moreover, differences in the variances of the errors are much smaller than with the real exchange rate equation. However, there is a significant difference in the degree of persistence, that is, temporary deviations from long-run equilibrium last somewhat longer in the estimated model. So whereas the filter and equation were similar in persistence but differed in amplitude for the real exchange rate, the opposite appears to be true for the import-output ratio.

As with the real exchange rate equation, we subject the import share equation to a number of diagnostic checks in an effort to uncover evidence of model misspecification. One result of note is the path of the eigenvalues. After analyzing the evolution of the largest root of Π through time, it appears that the evidence supporting cointegration is somewhat sample sensitive. This is not surprising given that there is fairly weak evidence of this for the entire sample. Nevertheless, we can reject no cointegration at 5 per cent for the last few years of the sample. The other model diagnostic test results imply no other problems with the estimated equation.

6.3 Relative Export and Import Prices

In the preceding two subsections, we estimated each equation separately. This stems primarily from the fact that there is not a great deal of overlap in terms of the set of regressors. However, in the case of *px* and *pm* there exists a common set of potential "explanatory" variables. Thus there may be efficiency gains to estimating both equations within a single VECM. An additional benefit of modelling cointegrating equations jointly is that the dynamics are contained within a single model. Hence, such a model could be easily used to assess the dynamic path to equilibrium currently embodied in QPM.[25] Finally, simultaneous estimation permits the researcher to test (and impose) cross-equation restrictions. This facility will be useful in modelling *px* and *pm*.

25. Future work will show impulse responses for permanent shocks to the VECM. We do not currently have access to software capable of expressing a partial VECM in its identified VMA(∞) representation.

Although it is possible, in principal, to estimate all four equations within a single model, the dimension of the VAR relative to the sample size would be so great as to offset any potential increases in efficiency. Furthermore, as the dimension of the cointegrating space increases, the issue of identification becomes progressively more difficult. We have, however, experimented with combining the two relative prices with the exchange rate. The results indicate that while there are some small differences in parameter estimates, we cannot reject the hypothesis that any of the equations estimated separately lie in the space spanned by the 3 vectors estimated jointly. Unfortunately, modelling more than one vector raises the difficult issue of identification. In this instance, however, there exists a set of economically plausible restrictions which are believed to be generically identifying restrictions. This raises the question of whether or not to include oil prices, given that they are excluded from the real exchange rate equation. Despite the fact that formal exclusion tests suggest that they belong in the above specification, we doubt such a variable could have an economic relationship with the terms of trade, but not with the exchange rate. Moreover, the evidence of cointegration does not change when we exclude *lener* from the VECM. This would seem to suggest a contradiction between the two tests, since we find the same number of cointegrating vectors at the 1 per cent significance level, regardless of whether or not we include this variable. On the basis of this fact, combined with our judgement, we elect to omit *lener* from the price equations.

Table 12 shows the trace and λ_{max} test statistics for the 5 variable unrestricted VECM with 3 lags in differences. These tests indicate that the null hypothesis of no cointegration can be rejected at 1 and 5 per cent levels, respectively. Moreover, they suggest that the null of at most one vector can also be rejected. On this basis, we conclude that this system contains two independent sets of stationary combinations. Conditional on a rank of 2, we can now proceed to test for exclusion and weak exogeneity. As Table 13 illustrates, we can easily reject long-run exclusion for each variable in the system. Turning to the test for weak exogeneity we observe one rather strange result. Table 14 suggests that the relative price of exports is exogenous. In this instance, we are forced to impose our priors regarding the structure of the system and leave *lpx* endogenous. Assuming exogeneity for computer prices and non-energy commodity prices, however, is consistent with the test results.

Finally, allowing the exchange rate to remain endogenous is consistent with the structure of SSQPM. While our specification has a recursive rather than truly simultaneous structure, these data are jointly determined within SSQPM. As such, we prefer to model variables which are determined simultaneously within SSQPM as endogenous within our specification, even if this is not literally true. One implication of this decision, however, is that our interpretation of these equations as prices equations is independent of the statistical model. We could just as easily normalize one (or both) equation(s) on *z* and then refer to them as exchange rate equations.[26]

At this point, we have defined only the cointegration space through a set of 2 basis vectors. We now proceed to impose formal restrictions on these vectors in an effort to identify the individual parameters. Since these restrictions will be just identifying, we have no means of testing them. Nevertheless, we exclude the price of exports from the import price equation and vice-versa. Formally, we impose one zero restriction to a different element in each vector. After excluding energy prices as well as imposing the above restrictions, we are left with the following specification

$$lpx^*_t = \beta_9 + \beta_{10} lz_t + \beta_{11} lbcne_t + \beta_{12} lcomp_t \quad (17)$$

$$lpm^*_t = \beta_{13} + \beta_{14} lz_t + \beta_{15} lbcne_t + \beta_{16} lcomp_t \quad (18)$$

and Table 15 presents the empirical results.

Based on this specification, the equilibrium errors for *px* and *pm* appear to be more volatile than those from the HP filter but the degree of persistence is quite similar. More interesting, however, is the negative correlation between the two measures for both prices. Turning to the SREQs we again observe that the variance of the estimated SREQ exceeds that of both the filter and the actual series.

The Johansen stability test suggests some instability in the late 1980's which appears to dissipate over the 1990's. Also, we observe forecast failure for *px* in 3 quarters and *pm* in only 1. In an attempt to better understand the source of instability, we also look at the evolution of the eigenvalues and found no evidence to suggest any instability in the roots. Moreover, we can reject that either of the roots is zero for every sub-sample over the

26. Of course we could also do this within SSQPM.

past 10 years. Nevertheless, the instability parameter vector remains of some concern given that these equations are to be used for future projections and policy analysis. Owing to the added complexity of modelling two cointegrating vectors in a system of equations, we do not yet have the full dynamic simulations or Theil's-U results.

6.4 The Net Foreign Asset-to-GDP Ratio

Due largely to constraints imposed by data availability, estimation of a NFA-to-GDP SREQ proceeded differently than the other equations in this paper. Both the domestic and G6 government debt are reported on an annual frequency and the demographic ratios suffer from the fact that the underlying census data are not even available on a yearly basis (and hence involve some interpolation to come up with annual estimates). We considered interpolating the data to a quarterly frequency but examination showed that interpolation induced spurious dynamics into the data (particularly in first differences). This constraint combined with limitations on the availability of government debt data for Japan, France and the United Kingdom limits the analysis to annual data over the 1970 to 1997 period. Since degrees of freedom are a serious consideration our analysis is limited to single-equation tests. In principle, the economic exogeneity of the debt and demographic data justify the use of single-equation methods.

The most satisfactory model, from an empirical perspective is described by equation (20) below. The NFA to GDP ratio is modelled a function of the Canadian debt-to-GDP ratio relative to the G6 measure (*debt*), and the Canadian old-age dependency ratio relative to the G6 measure (*rdem*). The evidence in support of cointegration is weak (ADF -3.3 and PP -2.4) but this may be due in part to the low power of the tests with the null of no cointegration. In contrast, the KPSS test (0.08) is unable to reject the hypothesis of cointegration at the 90 per cent level. Visual inspection of the residuals is also supportive of the cointegration hypothesis.

$$nfa_t^* = \beta_{19} + \beta_{20} debt_t + \beta_{21} rdem_t \qquad (19)$$

The parameter estimates, shown in Table 16, imply that a higher domestic debt ratio lowers, and an older population raises, the net foreign asset ratio in the long-run as suggested in subsection 3.2.4. Our estimate of the relationship between government debt and the net foreign asset position β_{20}, is about -0.6, significantly lower than the -1.05 contained in SSQPM. This estimate is broadly consistent with estimates for the United States of -0.8 and Japan -0.5 reported in Masson et al. (1994). The estimated coefficient, β_{21}, on the demographic variable is large relative to those in Masson et al. for the United States and Japan of 0.7 and 1.8, respectively.

6.5 Exports and the Current Account Balance

In Section 3.0 we list a set of equations which define the trade block in SSQPM. Each of these equations is then given the interpretation as determining a particular endogenous variable in the system. While this interpretation is somewhat arbitrary when there is more than one endogenous variable in the equation, it remains useful for the purpose of describing the structure of the model. As such, we define equations for the current account, trade prices and quantities. The real exchange rate is then determined so as to provide trade prices (and quantities) which will yield a current account that is consistent with the net foreign asset position. As noted in Section 4.0, however, our system of equations is structured slightly differently in that we explicitly model the exchange rate as a function of exogenous variables. Consequently, we must choose another variable to be determined "residually". We have opted to make export volumes the residual in this system. In doing so, we can only hope that the remaining equations will imply an export to output SREQ, x^*, that is sensible. Specifically, the equilibrium errors from the export to output SREQ path must have a mean of zero and be approximately stationary. Clearly, this is a lot to expect from a set of trade equations which are effectively unrelated to the net foreign asset SREQ.

Before determining x^*, we must first calculate the current account to output SREQ, *cbal**, which is defined simply as;

$$cbal^* \equiv \frac{\Delta(nfa^* \cdot y^*)}{y^*} \qquad (20)$$

Based on equation (21), we can the compute the equilibrium errors and then compare them to those derived from the HP filter.[27] The series are fairly similar except in the late

27. The term equilibrium error has a somewhat different meaning in this context because we are dealing with a series, *cbal*, that is stationary in levels.

1980's and early 1990's. This stems from the fact that the SREQ defined by equation (21) does not follow the cyclical pattern in the actual data nearly as closely as the filter does. This is primarily due to the choice of smoothing parameter for the filter.

With *cbal** now defined, we can use equation (1) to solve for *x**.[28] Surprisingly, the equilibrium errors for the export share (*x* - *x**) appear reasonable with mean of almost zero and time-series properties which tend to revert back to its mean, although formal tests cannot reject a unit root.[29] It is also interesting to note that equilibrium errors for exports correlate quite highly with those of the real exchange rate, particularly over the early 1990's. During this period, the real exchange rate was significantly stronger than it's long run determinants would suggest. Indeed, it appears to have been generated by a widening of the Canada-US short term interest rate differential. At the same time, exports were much weaker than would be implied by the *x** derived from equation (6). Even more interesting is the fact that this phenomenon is not captured to nearly the same extent by the univariate HP filter, which suggests that it is not present in the original data. From this, one could conclude that exports would have been significantly stronger over this period, were it not for the appreciation in the real exchange rate. The fact that we do not observe a depression in *x** over this period could be due to certain mitigating factors such as the Canada-U.S. Free Trade Agreement and / or a strong US economy.

7.0 Comparisons Using Impulse Response Functions

At this point, we consider the impact of the new system of estimated SREQ equations on the behaviour of QPM under three temporary shocks. It must also be recognized that the dynamic equations in QPM were not recalibrated so these results should not be interpreted as final but as an illustration of the key differences between the two methods for calculating SREQs. In the following we consider: (i) an exchange rate shock where the impulse response functions (IRFs) across the two types of SREQs are similar; (ii) a commodity price shock where the IRFs are grossly different; and (iii) a government spending shock which highlights the effect of movements in the government debt on the external sector. All graphs show the shock minus control response of QPM

28. In this exercise, we use the current *r$** derived from the HP filter. In the future, however, this will be replaced by some form of equation.

29. Because *x** is generated via an identity, there remains some question as to how many "regressors" are used to construct it. Using N = 2, or more generally greater than N

with (solid line) and without (broken line) the estimated SREQ equations.

7.1 Nominal Exchange Rate Shock

We consider the impact of a 1 per cent appreciation of the nominal exchange rate which lasts for approximately one year (Figure 1). In QPM, such a shock causes a temporary reduction in inflation as a consequence of lower direct passthrough and, because a shock to the nominal exchange rate also feeds into the real exchange rate in the short run, lower aggregate demand stemming from weaker net exports. This fall in inflation prompts a reduction in short-term nominal interest rates which provides a boost to consumption and helps to offset some of the exchange rate appreciation.

7.1.1 How Does the Shock Affect the SREQs?

(i) Immediately following the shock output falls as net exports respond to a stronger exchange rate. However, in the very short run, the government debt falls by less than output, hence we witness a small increase in the debt-to-output ratio. This, in turn, causes a small weakening of nfa_t*.

(ii) As consumption responds positively to lower borrowing costs we see a small decline in net foreign assets. This weakening of nfa_t causes a depreciation of z_t* .

(iii) The appreciation of the real exchange rate causes a reduction in both px_t* and pm_t*.

(iv) The appreciation of the nominal exchange rate causes a reduction in the relative price of imports with causes a slight increase in m_t* .

7.1.2 How do the SREQs Affect Key Behavioural Variables?

(i) Weaker desired net foreign asset holding (in addition to that generated by lower interest rates) causes a small consumption boom. This helps to mitigate the impact of lower net exports on output and hence inflation.

(ii) The depreciation of z_t* , which peaks in 2002, results in a weaker expected real exchange rate. This offsets the negative impact of the original shock on net exports. Beginning in 2001, net exports are stronger than with the current version of QPM. Stronger net exports combined with stronger consumption mitigates the fall in output to about 66 per cent as much as in the presence of a positive exchange rate shock. It is interesting to note here that export activity is stronger despite the fact that the actual real exchange rate is essentially unchanged (compared with the old model). This stems from the fact that the expectations of the real exchange rate are not perfectly model consistent, rather they have a weight on the SREQ.

On balance we see that with the new model exchange rate shocks have a smaller effect on output, albeit with about a one year lag, and hence inflation returns to control faster. Thus the new SREQ equations appear to have an equilibrating effect for exchange rate shocks. Intuitively, this stems from the fact a small proportion of the shock is now being interpreted as permanent. Hence, both the actual *and* desired NFA holdings fall. Consequently, we get a stronger positive reaction by consumers.

Neither of the preceding shocks show any really dramatic changes in the behaviour of QPM. Nevertheless, because these types of shocks occur so frequently in the quarterly projection exercise, understanding their effects remains useful. The next two shocks, however, will demonstrate that the behaviour of QPM can be quite different.

7.2 Persistent Commodity Price Shock

Previously, we have argued that the implementation of the new SREQ equations would help to eliminate the current dichotomy between permanent and nearly permanent shocks in QPM. Recall that under the current system, SREQs do not respond to shocks unless they are believed to be permanent in nature. Consequently, we should expect to see important differences emerge between the two models as shocks become more persistent. For this reason, we have elected to contrast the models under a 10% real commodity price shock with a half-life of 3.5 years (Figure 4).

In QPM, commodity prices influence the nominal price of *both* imports and exports. Consequently, a positive commodity price shock feeds directly into the CPIXFE via the price of consumer imports which increases, despite an appreciation of the exchange rate. Hence, the monetary authority is required to tighten policy in an effort to return inflation to control. In doing so, output is driven below potential for an extended period of time. Part of this decline is achieved via a reduction in net exports (mostly through a decline in exports). Because expected output is lower, capital accumulation slows and potential output remains below control for the duration of the shock.

7.2.1 How Does the Shock Affect the SREQs?

(i) Higher commodity prices cause an appreciation of z_t* .

(ii) Higher commodity prices cause both px_t* and pm_t* to increase. However, px_t* responds by more and consequently the terms-of-trade SREQ increases.

(iii) Higher relative import prices lower m_t* .

(iv) Lower government debt-to-output ratio drives up nfa_t*.

7.2.2 How Do the SREQs Affect the Key Behavioural Variables?

(i) While each of the changes described above exert some influence on the model, the first clearly dominates. In fact, the monetary authority now sees a completely different picture in terms of the response of inflation to the commodity price shock. This stems from the influence of z_t* on the real and nominal exchange rates. Basically, the stronger nominal exchange rate appreciation causes the price of consumer imports to falls. Furthermore, a significantly stronger expected real exchange rate places additional downward pressure on net exports.

(ii) The increase to the terms-of-trade SREQ generates higher consumption and lower net exports. Both features are consistent with the current version QPM, but only when the shock is permanent. Near permanent shocks currently generate a decline in consumption over the medium term.

(iii) A lower m_t* contributes to weaker net exports. However, this effect is quite small.

(iv) A stronger desired net foreign asset position leads to lower consumption

On balance, effect (ii) dominates (iv) and hence we witness a significant rise in consumption beginning immediately after the shock. Thus the addition of endogenous SREQs has significantly increased the “wealth” channel said to exist in QPM. Again, this arises because the model now better recognizes the duration of the shock. In addition, effects (i), (ii), and (iii) lead to a much more significant decline in net exports, which is necessary in order to maintain roughly the same actual net foreign asset position in the presence of a positive terms of trade shock.

7.3 A Temporary Increase in Government Spending

Here we consider the impact of a 1 per cent reduction in government spending which lasts for about 2 years (Figure 3). Initially, the reduction to g lowers output below potential and consequently inflation falls below target. This prompts a lowering of short-term nominal interest rates by the monetary authority which then leads to a depreciation of the nominal and real exchange rates. Of course because government spending falls with no initial change to taxes, the debt-to-GDP ratio also falls.

7.3.1 How Does the Shock Affect the SREQs?

(i) A lower debt-to-GDP ratio causes an increase in nfa_t*.

(ii) Lower government debt leads to a stronger actual net foreign asset position. Consequently, z_t* appreciates.

(iii) A weaker real exchange rate raises px_t* and pm_t* by the same proportion

(iv) A weaker real exchange rate increases the price of imports which lowers m_t*.

7.3.2 How Do SREQs Affect Key Behavioural Variables of QPM?

(i) The increase in nfa_t* basically reflects the fact that a portion of the reduction in government debt is now being interpreted by agents as being permanent (or at least long lasting). This is causing consumers to temporarily reduce spending in a fashion similar to when they observe permanent government debt shock.[30] Of course, this temporary reduction in consumer spending is further aggravating the original shock, thereby requiring a more aggressive loosening in monetary policy.

(ii) Since the actual net foreign asset position also increases in response to lower debt, z_t* appreciates which puts upward pressure on the actual (and expected) real and nominal exchange rates. This offsets, partially, the stimulative effects of looser policy and hence requires further loosening. Stated otherwise, the exchange rate does not depreciate by as much as it would in the absence of a change to z_t*.

(iii) and (iv) As previously mentioned, only changes to the terms-of-trade SREQ affects the behaviour of QPM. Finally, the reduction to m_t* mitigates some of the increase

30. Although the magnitude and duration of the reduction to consumption is much smaller.

to net exports.

8.0 Comparisons Using Stochastic Simulations

In this penultimate section, we compare the properties of the two models by examining temporal correlations generated via stochastic simulations. Stochastic simulations, however, require a method for generating shocks. In the case of estimated models such as FRB-US, the distribution of shocks is usually based on the distribution of the estimated residuals corresponding to various equations in the model. For calibrated models such as QPM, there is no equally obvious method; making the process of generating a set of shocks a more difficult task. Therefore, before discussing the temporal correlation results, we outline, in the next subsections, our methodologies for generating both rest-of-the-world and domestic shocks.[31]

8.1 VAR and Rest-of-World Shocks

In QPM, the rest-of-the-world (ROW) variables are completely exogenous, implying that we need a methodology which captures both the persistence and covariance of ROW variables. To this end, we estimate and then simulate stochastically a VAR representing loosely the rest of the world. More specifically, a VAR is estimated using four key QPM ROW variables (i.e., world commodity prices, price level, output gap and short-term interest rate). The estimated VAR is then simulated stochastically (using the estimated orthogonalized variance-covariance matrix) and the resulting dynamic paths are used to represent the exogenous ROW variables in QPM. This method allows us to include data-measured persistence and covariance into the ROW shocks.

The variables in our ROW VAR are the quarterly growth rate in world commodity prices (or $\Delta lpcomrow$ in QPM parlance), the quarter-over-quarter G-6 GDP inflation rate ($\Delta lprow$), the G-6 output gap ($lyrow\_gap$) and a measure of a G-6 short-term nominal interest rate ($r1row$). The VAR is estimated over the 1973Q1 to 1998Q1 sample period and is identified by a Wold causal ordering of commodity price inflation, GDP deflator inflation, output gap and short-term interest rate.[32] To get a sense of the dynamic behaviour of the ROW variables within the VAR, we examine the resulting impulse

31. For more detailed explanations of the shock estimation methodologies see: Amano and Pioro (1999) and Amano et al. (1999).

response functions (IRFs). Figure 4 displays the 20-period IRFs of each variable along with one standard error intervals.[33] The main diagonal IRFs represent the variable that is being shocked while each column traces out the responses to a particular innovation. Column 1, therefore, corresponds to the dynamic response of the variables to an one-standard deviation world commodity price shock.

Overall, the IRFs conform broadly to what we would expect. Consider, for instance, column 3 which displays the IRFs to a one standard deviation (0.6 per cent) innovation in the G-6 output gap. The innovation induces world commodity prices to increase by 0.3 per cent and GDP inflation to rise gradually over 6 quarters. As a consequence of higher inflation and a positive output gap, the short-term interest rate rises by about 0.6 per cent within one year. Other impulse responses appear equally plausible except for that corresponding to the G-6 interest rate shock. In this case, a positive innovation to short-term interest rates leads to a short-lived (2 to 3 periods) increase in prices and output, suggesting we may be omitting an inflation indicator variable which is in the information set of the monetary authorities but not included in our VAR (Christiano and Eichenbaum 1995). While it would be useful to explore a larger dimensional VAR, the limited number of foreign variables in QPM prevents us from doing so. An alternative and viable approach to resolving the price and output puzzle is to relax our recursive-causal ordering. Along this margin, we examined many sets of IRFs based on different exclusion restrictions and were unable to find a set of restrictions which resolved the price and output puzzles without introducing other problems.[34] Finally, in order to get a sense of how well the simulated data approximates the actual, we report, in Table 17, some summary statistics from the data. In short, the simulated data appear to capture reasonably well both the volatility and persistence of the actual data.

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32. The optimal data-based lag choice for our VAR is 3. However, we use 2 lags since this VAR offered us a smoother IRF without a loss in their general shape.
33. The confidence intervals are generated using Monte Carlo integration. We sample antithetically, instead of randomly, from the posterior density. Geweke (1988) finds the antithetic approach to be more efficient than the random approach.
34. It should be noted that the volatility and persistence of the artificial data are reasonably robust to alternative VAR models. For instance, we considered a VAR with exclusion restrictions which gave us an almost diagonal reduced-form variance-covariance matrix (the polar case to our basecase VAR) and did not find this alternative VAR to change grossly the moments of the simulated data.

8.2 Estimation by Simulation and Domestic Shocks

Our application of the estimation-by-simulation approach starts with a simple AR(1) representation of innovations which are then re-parameterized until QPM produces standard deviations and autocorrelation coefficients of economic variables which match approximately (within a 95 per cent confidence interval) those in the historical data. Loosely speaking, we re-parameterize the persistence (ρ) and variance (σ_u^2) of the following shock process,

$$\varepsilon_t = \rho\varepsilon_{t-1} + u_t;\ u_t \sim iid(0,\sigma_u^2)$$

until we match approximately the population moments of the artificial (or model-generated) data with the empirical sample moments, that is $\arg min\left\|\hat{W}_T - \tilde{W}_T(\tilde{\rho},\tilde{\sigma}_u^2)\right\|$ where $\hat{W}_T$ are the moments under consideration calculated from empirical data and $\tilde{W}_T(\tilde{\rho},\tilde{\sigma}_u^2)$ are QPM simulated moments based on a specific parameterization of ρ and σ_u^2.[35] This approach, known as estimation by simulation, can be thought of as informal method of moment estimation where we estimate the free parameters so as to set the distance between moments of the artificial and empirical data within some confidence interval (for a formal treatment of estimation by simulation, see McFadden 1989).[36] Obviously the inferences drawn from the matching depend on the variables and moments used. Moments must exist for the comparison to be meaningful, so often the data must be transformed to induce stationarity. Moreover, Singleton (1988) and Cogley and Nason (1995) show that the detrending method (or spectral bandwidth) considered in calculating moments may itself have a large effect on conclusions so in an effort to control for such problems we attempt to match the standard deviations and autocorrelation coefficients based on three methods of detrending: first difference, fourth difference and Hodrick-Prescott filtering.

Preliminary research suggested that attaching shock terms to all the behavioural equations in QPM and following the estimation-by-simulation method would lead to

35. One potential problem with informal moment-matching is that parameters may be selected even if they are not identifiable. In other words, we are forcing innovations to be some AR(1) process even though their true representation may not. Another potential concern is that the parameterization of the shocks may not be unique; that is, another set of parameterizations could lead us to similar population moments.

36. Estimation by simulation is closely related to the approach of calibrating model parameters so as to match a statistic generated by the model with that in data. Kydland and Prescott (1982), for instance, calibrate the coefficient of relative risk aversion in their real business-cycle model by matching the variance of detrended output.

excessive volatility in many of the important macroeconomic aggregates (such as output). We, therefore, attempt to estimate a relatively small (but important) number of shocks. More specifically, we attempt to match the volatility and persistence of consumption, investment, exports, imports, GDP price deflator and real G6 exchange rate. Admittedly our choices are somewhat arbitrary but we believe the innovations terms we include capture the key macroeconomic shocks hitting the Canadian economy.

For the initial magnitude of the shocks, we start with *iid* shocks parameterized to match the standard deviation of empirical residuals from an AR(k) model for each of the variables under consideration. The estimation period is 1973Q1 to 1998Q1. With these initial shocks we simulate stochastically our two versions of QPM. For each experiment, QPM is run over 109 quarters with the first 8 quarters omitted from analysis; this leaves 101 quarters which corresponds to the length of the historical sample over which we calculate the moments to be matched. Each complete experiment is based on 200 successful (stable) replications. The distributions for variables of interest are built up by averaging across time and across replications.

Since we are unable to match any of the sample moments under consideration with the simple *iid* shocks, we re-parameterized the shocks (that is, ρ and σ_u^2) until the standard deviation and autocorrelation coefficient from the simulated data match approximately those from the historical data.[37] The persistence and volatility of these shocks are presented in Table 18 and a couple of interesting results emerge. First, there appears to be no role for nominal shocks, that is, the GDP deflator shock is set to zero. This likely reflects the excessive volatility of output generating unwarranted variability in inflation via the Phillips curve relationship rather than there being no role for a price shock term.[38] Second, the magnitude of the estimated shocks across the two models (required to match some properties of the data) is very similar. In fact, the shocks for imports, exports and the exchange rate are identical, suggesting that the estimated SREQs have only little effect on the stochastic properties of QPM's external sector. The estimated stochastic shock for investment is an exception. In this case, QPM with estimated SREQs requires substantially more volatility in the investment shock to match the properties of

37. Complete results are available from the authors upon request.

38. For example, these stochastic simulations generate a standard deviation of output of about 8.5 per cent whereas that in the data is about 3.4.

the historical data.

8.3 Temporal Correlations

In this section, we use temporal correlations to determine how much the implementation of the estimated SREQs changes the stochastic properties of QPM. To preview the results, the inclusion of estimated SREQs does very little to the temporal correlations under consideration but the correlations by themselves are interesting from the viewpoint of model evaluation. Since the focus of our study has been with the external sector of QPM, we focus on three temporal correlations associated with the open-economy linkages of QPM: (i) output and imports; (ii) price of commodities and the terms of trade; and (iii) nominal exchange rate and CPI inflation.[39]

To give a sense of our idea, Figure 5 presents correlations between eight leads and lags of the terms of trade and the price of commodities. The upper panel presents the data in first differences, the middle panel in fourth differences and the lower panel in HP detrended. The vertical axis corresponds to the magnitude of correlation between the terms of trade and price of commodities while the horizontal axis corresponds to the lags and leads of the terms of trade, with negative integers representing leads and positive lags. The solid line in each panel represents temporal correlations (found in the data 1973Q1 to 1998Q1 sample period) whereas the dashed lines represent correlations generated from artificial data. From the Figure, it is apparent that both models generate dynamic correlations that capture broad movements between the terms of trade and the price of commodities similar to those found in the data. In particular, both models exhibit an inverted-V shaped, positive relationship between the terms of trade and price of commodities. We are, however, unable to replicate closely the magnitude of the first-difference correlation. The frequency zero, empirical correlation between the terms of trade and the price of commodities peaks at around 0.4 whereas those based on the artificial data range peak at around 0.75.

Figure 6 presents a similar style temporal correlation between the real exchange rate and leads and lags of exports. In all three panels, the empirical correlations are

39. It should be noted that we considered many other temporal correlations. However, the difference between the two versions of QPM are very small so they are not presented here since, as already mentioned in the text, the goal of this section is to contrast the differences between the two versions of QPM.

generally positive implying that a depreciation of the Canadian dollar leads to an increase in Canadian exports. Moreover, the timing of the peak effect appears similar across the panels; that is, fluctuations of the Canadian dollar and exports lagged about one-year exhibit the strongest correlation. In contrast to these results, the simulated data from both versions of QPM admit a slight negative relationship between real exchange rate movements and lags of exports, suggesting a potential problem with QPM along this margin.

Finally, Figure 7 plots the correlation between movements in the nominal exchange rate and inflation. In this case, the differenced data suggest that a depreciation of the Canadian dollar is related to a fall in inflation. This counterintuitive result likely reflects the fact that we have not controlled for other important variables such as the output gap. Thus, even though the simulated data correlations move in the opposite direction of the actual data, drawing any firm conclusion about the properties of QPM from this figure is difficult. In future work, therefore, we will consider the ability of QPM simulated data to reproduce the results from well-established reduced-form relationships found in the data. This would allow us to control for the effects of “other” variables on the relationship of interest.

9.0 Concluding Remarks

In this paper, we have proposed and implemented a methodology to estimate empirically the external sector SREQs in QPM. As well, we have examined the implications of estimated SREQs for the deterministic and stochastic properties of QPM. The results suggest that our sector-by-sector cointegration approach which exploits the underlying theory of SSQPM and the time-series properties of the data is a viable alternative approach to the application of a mechanical filter. Moreover, we have found that estimated SREQs have strong implications for impulse response functions, and, therefore, projection analysis. On the other hand, the stochastic simulation results imply that the implications of estimated SREQs are small (at least for the margin considered in this paper), suggesting that alternative methods for calculating SREQs may not be that important for policy analysis experiments.

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TABLE 1:
Calibration of Export Price Equation

Reduced-Form Parameters	Structural Forms	SSQPM Values
Φ_0	$\alpha_{px}\left[1-\frac{(\delta-1)(1-\eta)}{\eta}\right]^{-1}$	**1.09**
Φ_1	$\delta\lambda\left[1-\frac{(\delta-1)(1-\eta)}{\eta}\right]^{-1}$	**0.36**
Φ_2	$\delta\left[1-\frac{(\delta-1)(1-\eta)}{\eta}\right]^{-1}$	**0.69**
Φ_3	$(\delta-1)\left[1-\frac{(\delta-1)(1-\eta)}{\eta}\right]^{-1}$	**-0.29**
Φ_4	$\zeta\left[1-\frac{(\delta-1)(1-\eta)}{\eta}\right]^{-1}$	**0.08**

TABLE 2:
Calibration of Import Price Equation

Reduced-Form Parameters	Structural Forms	SSQPM Value
Γ_0	$\alpha_{pm} + \left[\frac{(\theta-1)(1-\eta)}{\delta(\eta-1)+1}\right]\alpha_{px}$	**0.12**
Γ_1	$\theta + \left[\frac{(\theta-1)(1-\eta)}{\delta(\eta-1)+1}\right]\delta$	**0.81**
Γ_2	$(\theta-2) + \left[\frac{(\theta-1)(1-\eta)}{\delta(\eta-1)+1}\right](\delta-1)$	**-1.18**
Γ_3	$\mu\theta + \left[\frac{(\theta-1)(1-\eta)}{\delta(\eta-1)+1}\right]\delta\lambda$	**0.16**
Γ_4	$\left[\frac{(\theta-1)(1-\eta)}{\delta(\eta-1)+1}\right]\zeta$	**-0.001**

TABLE 3:
Cointegration Test for Real Exchange Rate[a]

Eigenvalue	λ_{max} statistic	Trace statistic	H_0: q=
0.37	45.0**	88.7**	0
0.22	25.1	43.7	1
0.11	11.3	18.6	2
0.07	7.2	7.3	3
0.00	0.1	0.1	4

a. Henceforth ** denotes significance at the 1 per cent level and * at the 5 per cent level.

TABLE 4:
Test for Exclusion: LR Test

rank(Π)	$\chi^2(q)$ @5%	*z*	*nfa*	*lbcne*	*lcomp*	*lener*
1	**3.84**	7.58	5.54	16.26	7.25	0.19

TABLE 5:
Cointegration Test for Real Exchange Rate *(Excluding lener)*

Eigenvalue	λ_{max} statistic	Trace statistic	H_0: q=
0.37	44.6**	71.5**	0
0.20	22.2*	26.9	1
0.05	4.6	4.8	2
0.00	0.2	0.2	3

.[40]

TABLE 6:
Test for Weak Exogeneity: LR Test

rank(Π)	$\chi^2(q)$ @5%	*z*	*nfa*	*lbcne*	*lcomp*
1	**3.84**	10.4	5.1	2.8	0.0

TABLE 7:
Cointegrating Equation for the Real Exchange Rate

Parameter	Estimate	Standard Error	t-statistic
β_0	3.11	n / a	n / a
β_1	-0.87	0.26	-3.40**
β_2	-0.76	0.145	-5.26**
β_3	0.06	0.02	3.00**
α [a]	-0.16	0.03	-5.62

a. This parameter does not have the standard distribution under the null.

TABLE 8:
Cointegration Test for Imports

Eigenvalues	λ_{max} statistic	Trace statistic	H_0: q=
0.18	22.2	48.3*	0
0.14	17.4	26.1	1
0.07	8.00	8.7	2
0.01	0.8	0.8	3

40. We also tested the null that {*nfa*, *lbcne*, *lcomp*} are jointly exogenous. Due to the influence of *nfa*, this restriction can be rejected @ 1%.

TABLE 9:
Test for Exclusion: LR Test

rank(Π)	$\chi^2(q)$ @ 5%	*m_y*	*lpm*	*ltrade*
1	**3.84**	14.72	11.20	11.80

TABLE 10:
Test for Weak Exogeneity: LR Test

rank(Π)	$\chi^2(q)$ @ 5%	*m_y*	*lpm*	*ltrade*
1	**3.84**	7.88	2.17	2.86

TABLE 11:
Cointegrating Equation for Imports

	NLLS ECM			**FIML ECM**		
Parameter	Value	S.E.	t-stat	Value	S.E.	t-stat
β_5	0.45	0.24	1.85	0.39	-	-
β_6	1.00	-	-	1.00	-	-
β_7	-0.68	0.21	-3.16**	-.68	0.10	-6.8**
β_8	1.04	0.16	6.70**	1.00	0.10	10.0**
α	-0.14	0.04	-3.27	-0.19	0.05	-3.81

TABLE 12:
Cointegration Test for Relative Trade Prices

Eigenvalues	λ_{max}	Trace	H_0: q=
0.32	37.5*	92.6**	0
0.26	29.6*	55.1**	1
0.17	18.5	26.0	2
0.07	7.4	7.5	3
0.00	0.1	0.1	4

TABLE 13:
Test for Exclusion: LR Test

rank(Π)	$\chi^2(q)$ @5 %	*lpm*	*lpx*	*lz*	*lcomp*	*lbcne*
2	**5.99**	17.6	10.3	11.3	13.2	11.4

TABLE 14:
Test for Weak Exogeneity: LR Test

rank(Π)	$\chi^2(q)$ @5 %	*lpm*	*lpx*	*lz*	*lcomp*	*lbcne*
2	**5.99**	6.0	1.3	10.5	4.3	3.4

TABLE 15:
Cointegrating Equations for p*m* and *px*

Parameter	Value	Standard Error	t-stat
β_9	-5.77	-	-
β_{10}	1.00	-	-
β_{11}	0.97	0.20	4.9**
β_{12}	0.03	0.025	1.2
β_{13}	-3.58	-	-
β_{14}	1.00	-	-
β_{15}	0.5	0.12	4.2**
β_{16}	0.09	0.02	4.5**
α_{11}	-0.08	0.05	-1.6
α_{12}	0.11	0.08	1.4
α_{21}	-0.23	0.1	-2.2
α_{22}	0.14	0.07	2.1

TABLE 16:
Cointegration Equation for NFA

Parameter	Value	Standard Error	t-stat
β_{19}	0.08	0.03	-2.72
β_{20}	-0.62	0.05	-11.44
β_{21}	5.03	0.67	7.47

Table 17:
Summary Statistics from Actual and Simulated ROW Data

Variable	Actual Variance (x1000)	Simulated Variance (x1000)	Actual AR(1) Coefficient	Simulated AR(1) Coefficient
$\Delta lpcomrow$	1.77	1.54	0.38	0.35
$\Delta lprow$	0.04	0.03	0.94	0.91
$lyrow$_gap	0.20	0.20	0.85	0.85
$r1row\$$	0.90	0.90	0.93	0.93

TABLE 18:
Stochastic Shocks Estimated by Simulation

	Original QPM		Estimated QPM	
	ρ	σ_u x 100	ρ	σ_u x 100
Consumption	0.45	1.19	0.45	1.33
Investment	0.90	2.15	0.95	2.77
Export	0.40	2.55	0.40	2.55
Import	0.40	2.75	0.40	2.75
Real Exchange Rate	0.70	1.44	0.70	1.44
GDP Deflator	0.00	0.00	0.00	0.00

FIGURE 1. Exchange Rate Shock (+1%)

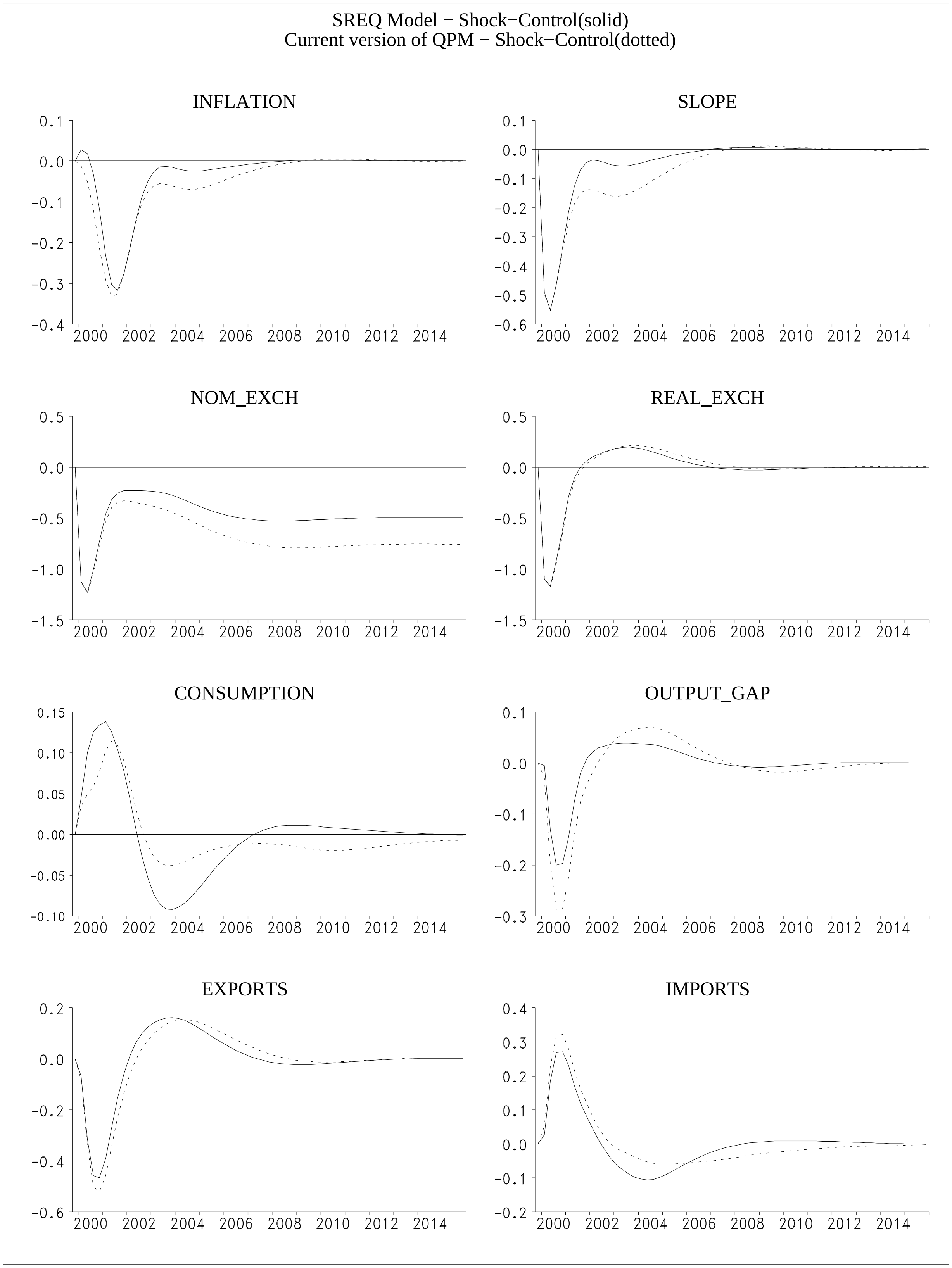


FIGURE 2. Commodity Price Shock (+10%)

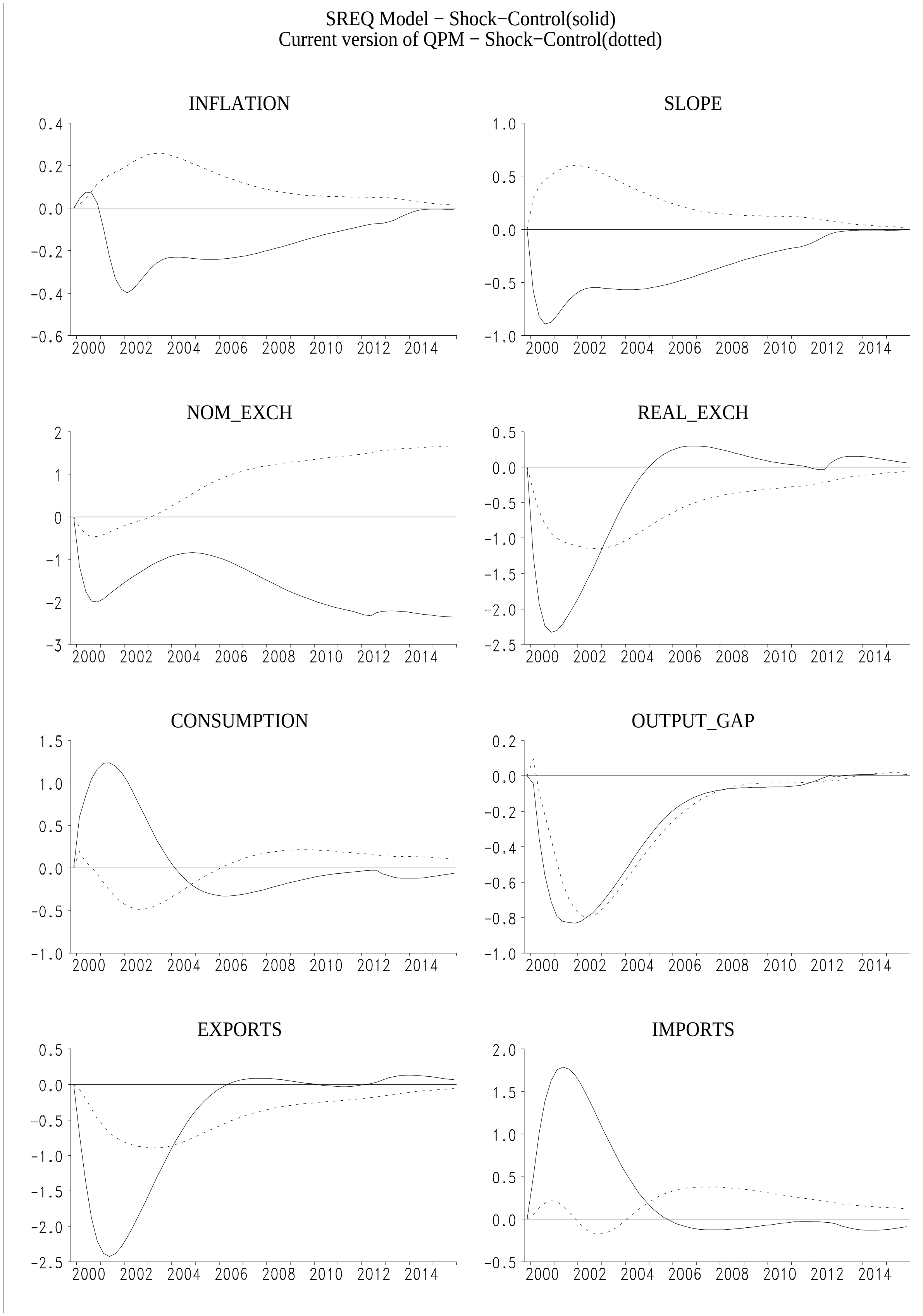


FIGURE 3. Government Spending Shock (-1%)

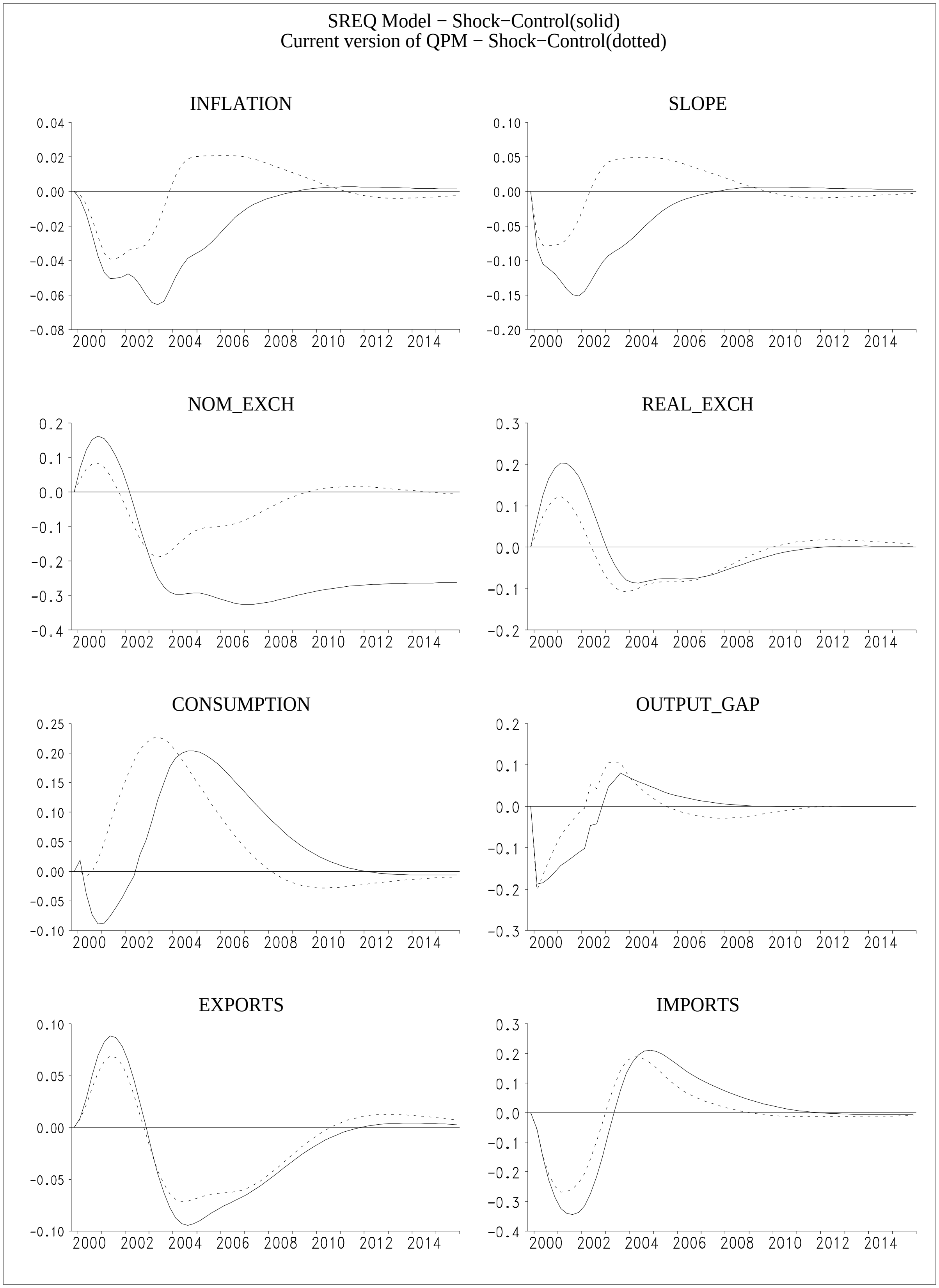


FIGURE 4:
ROW VAR IMPULSE RESPONSE FUNCTIONS

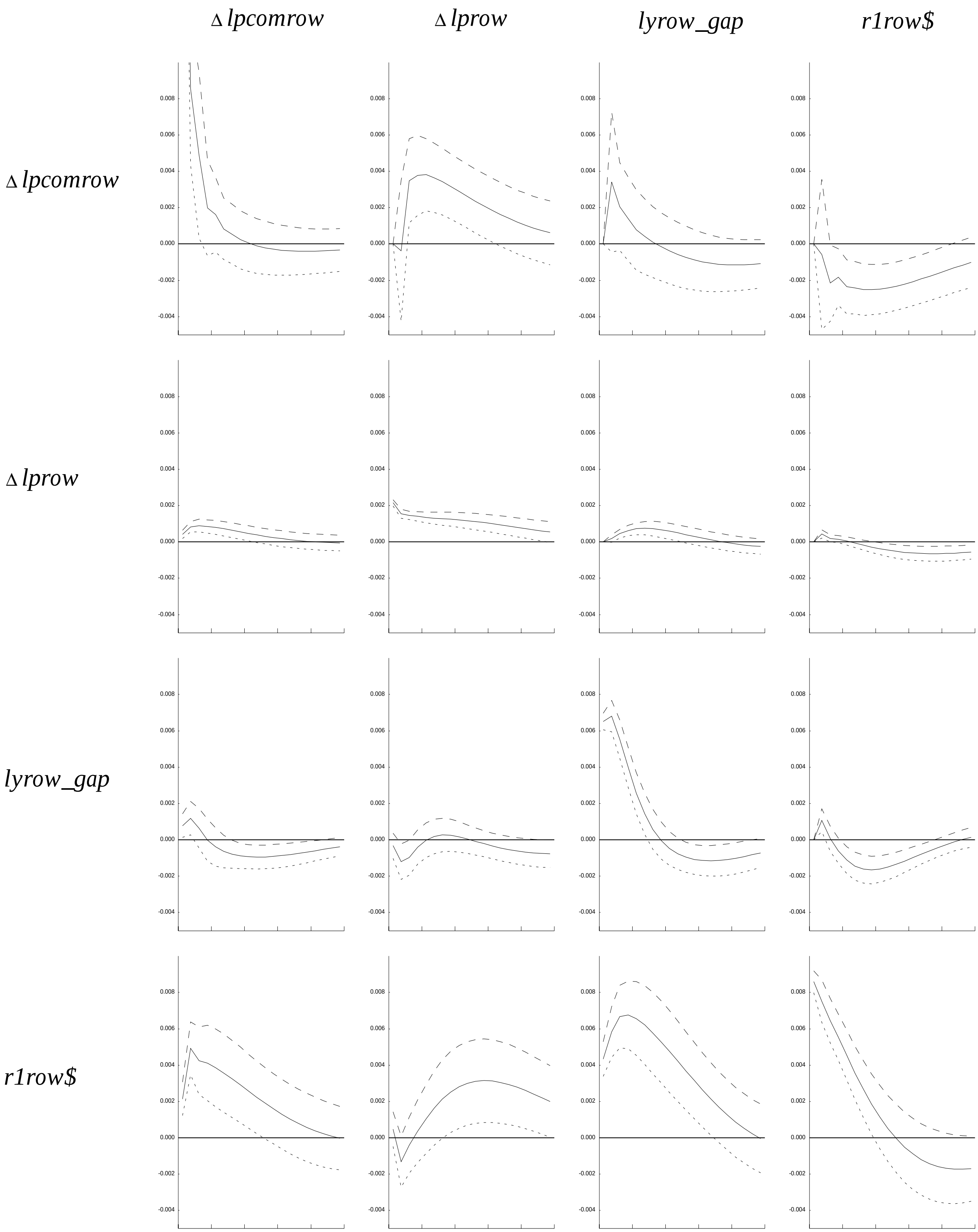


Figure 5

PCOM and TOT

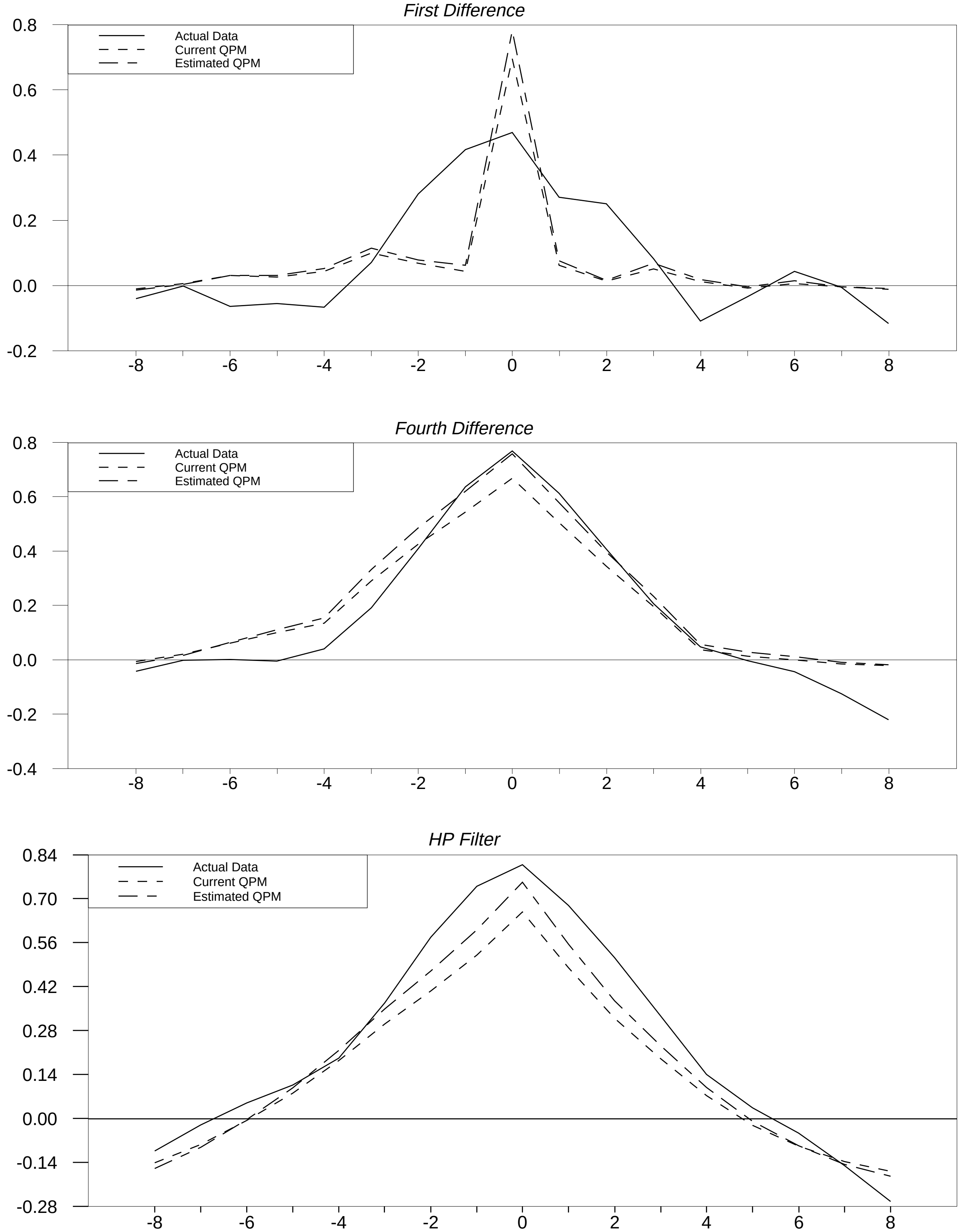
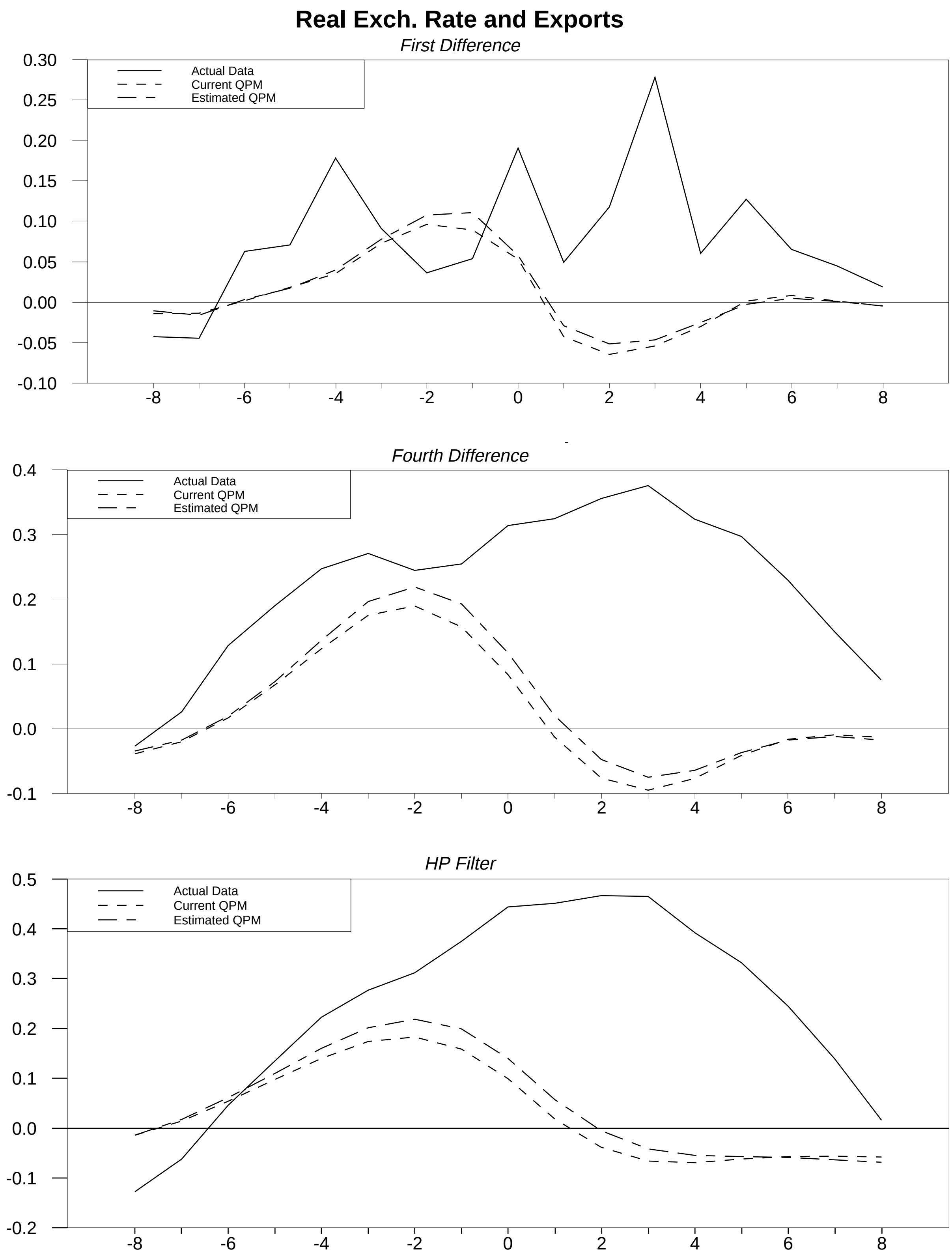
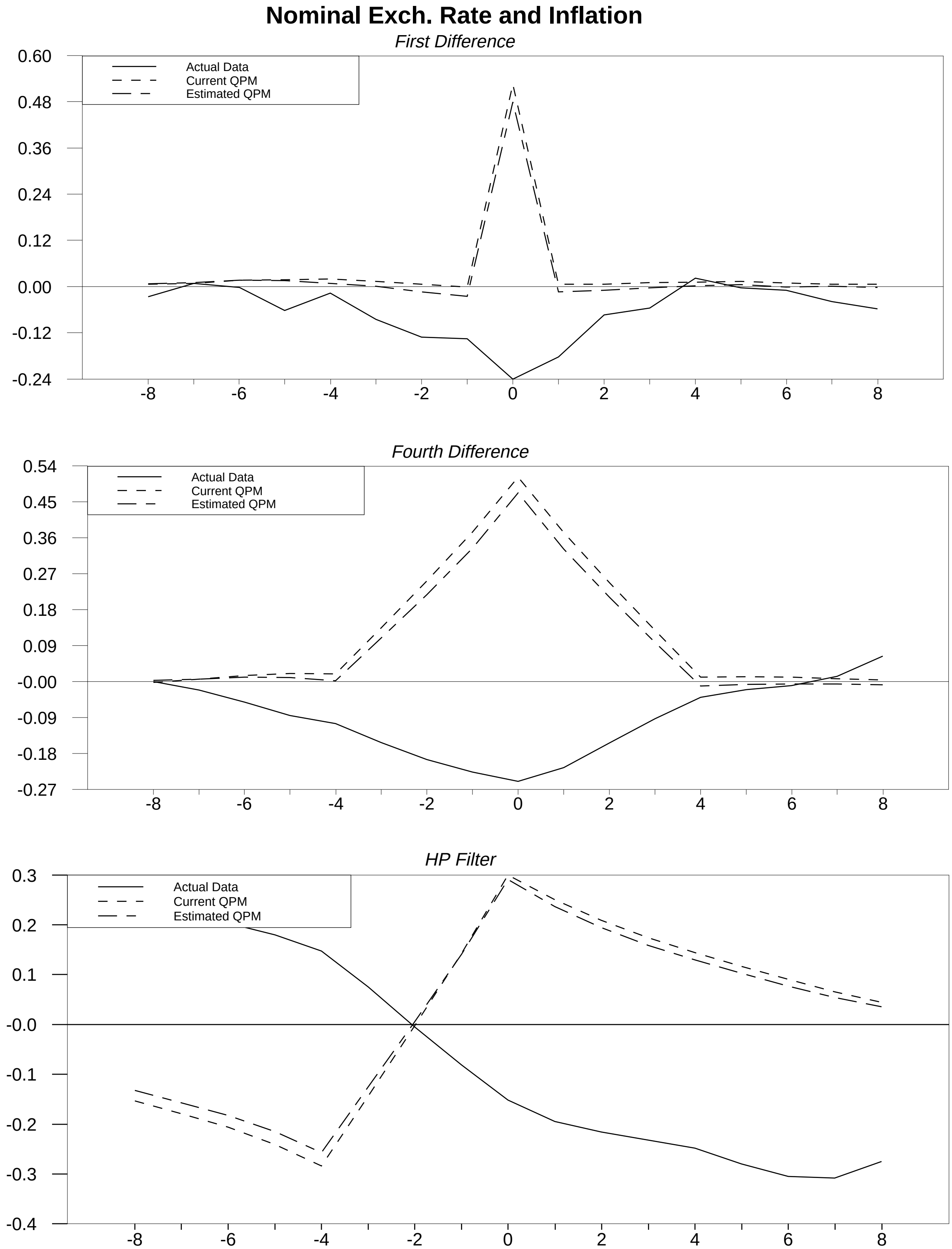


Figure 6



Real Exch. Rate and Exports
First Difference
Actual Data
Current QPM
Estimated QPM
0.30
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-8
-6
-4
-2
0
2
4
6
8
Fourth Difference
Actual Data
Current QPM
Estimated QPM
0.4
0.3
0.2
0.1
0.0
-0.1
-8
-6
-4
-2
0
2
4
6
8
HP Filter
Actual Data
Current QPM
Estimated QPM
0.5
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-8
-6
-4
-2
0
2
4
6
8

Figure 7


Nominal Exch. Rate and Inflation
First Difference
Actual Data
Current QPM
Estimated QPM
0.60
0.48
0.36
0.24
0.12
0.00
-0.12
-0.24
-8
-6
-4
-2
0
2
4
6
8
Fourth Difference
Actual Data
Current QPM
Estimated QPM
0.54
0.45
0.36
0.27
0.18
0.09
-0.00
-0.09
-0.18
-0.27
-8
-6
-4
-2
0
2
4
6
8
HP Filter
Actual Data
Current QPM
Estimated QPM
0.3
0.2
0.1
-0.0
-0.1
-0.2
-0.3
-0.4
-8
-6
-4
-2
0
2
4
6
8