Targeting inflation over the short, medium and long term*

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Abstract

A central bank pursuing the policy of inflation targeting aims to keep inflation as close as possible to a pre-announced value. But which 'inflation' should this be? Quarterly, annual, biennial? In theoretical models it is typically inflation during one period. We analyze how changing the period over which the inflation rate is defined – i.e. changing central bank preferences – affects optimal monetary policy. It is shown that when targeting inflation is the sole objective of the central bank, more aggressive monetary policy results; but when output stabilization is also a concern, a 'longer-term view' typically leads to a more cautious conduct of monetary policy and less variability in output. The conditions under which inflation targeting in effect becomes price level targeting are also examined.

Key words: Inflation targeting, price level targeting, optimal monetary policy.

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1 Introduction

Theoretical models of optimal monetary policy under inflation targeting typically assume that the central bank has a loss function defined over inflation and output (and possibly other variables). But which 'inflation' is this? Quarterly, annual, biennial or some other measure? Resolving this issue would raise fundamental questions about the welfare costs of inflation. And while our understanding of these costs remain scant, models of inflation targeting simply assume which 'inflation' it is the central bank dislikes, most often the 'one-period' inflation rate. Yet, obviously, this assumption regarding central bank preferences will affect conclusions regarding the design of optimal monetary policy. The purpose of this paper is to analyze how these conclusions change as central bank preferences, in regard to how inflation is defined, are altered.

This question is not just of theoretical interest, there is also a practical side to it. While most inflation targeting central banks aim to stabilize the *annual* inflation rate, there is (at least) one exception – the Reserve Bank of Australia aims to stabilize inflation "over the cycle" (Reserve Bank of Australia (1996)). In its very first *Monthly Bulletin*, the European Central Bank, although not a "pure" inflation targeter, stated that it will ensure that "price stability is maintained over the medium term" and "... a medium-term orientation of monetary policy is important to permit a gradualist and measured response" (ECB (1999), p.47).[1] The analysis in this paper hopes to shed some light on the effects of, and the rationale behind, this policy.

Thus, does the way in which "inflation" is defined matter for optimal monetary policy and for the macroeconomy? In particular, does taking what may be denoted as "a longer-term view" imply a more cautious conduct of monetary policy? A reasonable first guess is perhaps that indeed it should. Extending the period over which inflation is defined can be thought of as extending the window of a moving average, and a shock to inflation in one period will matter less and less to the moving average as the window in enlarged. And if it is this moving average that enters the central bank's loss function, a less aggressive response should perhaps be expected than if the one-period inflation rate is the target variable.[2]

[1] Quotes of this nature are typically used as indicators of 'flexible' inflation targeting, i.e. when central banks also have an explicit dislike for output variation. We propose an alternative (perhaps complementary) interpretation.

[2] This initial intuition rhymes well with results obtained in Williams (1999) where dif-

In this regard, we offer a potentially new (albeit partial) explanation of the observed discrepancy between the predictions given by theoretical models of optimal monetary policy and real-world central bank behavior. Typically, the theoretical models give interest rate responses that are much too volatile when compared with actual policy (see e.g. Rudebusch (1999)). In response, other models have been constructed to produce more gradual, i.e. realistic behavior where either the central bank's preferences are altered or the structure of the economy. Examples of the former are models that postulate a dislike for output variability or, more directly, interest-rate variability. Examples of the latter include Sack (1998) and Söderström (1999a, 1999b) who analyze the implications of lags in the transmission mechanism and parameter uncertainty on optimal monetary policy.

In this paper, we examine the implications for optimal monetary policy of extending the period over which inflation is measured in a basic model of inflation targeting due to Svensson (1997, 1999). To preview some of the results, the analysis shows under some general conditions extending the measurement period has the same effect on optimal monetary policy as increasing the relative weight on output stabilization - a smoother path of policy and output is the result. Alternatively, given a specific weight on output stabilization, we obtain smoother paths of the policy instrument without having to introduce explicit costs from interest-rate variability.

The outline of the remainder of the paper is as follows. Section 2 presents the model of the economy and describes the loss function of the central bank. In Section 3 numerical methods are used to explore the implications for optimal monetary policy of using different definitions of inflation. Section 4 contains a discussion of the implications for the price level of extending the measurement period. Section 5 summarizes the main results and concludes. Appendix A contains a short presentation of the original Svensson (1997) model, while Appendix B contains some details concerning the state-space representation.

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ferent simple reaction functions are evaluated in the FRB/US macroeconometric model. It is found that "policy should in general respond to a much 'smoother' measure of inflation, specifically, the growth rate of prices over the last three years. This measure of inflation evidently filters out the high frequency noise in the inflation process, leaving policy to react to sustained movements in inflation or 'core' inflation. By reacting to a smooth inflation measure, policy implicitly purchases a reduction in output and funds rate variability at the cost of some high frequency variability in inflation" (p.11).

2 A simple model of the economy and central bank behavior

Consider a highly simplified model of a closed economy (this is a simplification of the model analyzed in Svensson (1997, 1999a)):

$$\pi_{1,t+1} = \pi_{1,t} + \alpha_y y_t + \varepsilon_{p,t+1} \quad (1)$$

$$y_{t+1} = \beta_y y_t - \beta_r (i_t - \pi_{1,t}) + \varepsilon_{y,t+1}. \quad (2)$$

The notation is as follows. $\pi_{1,t}$ represents the one-period inflation rate at time t, i.e. $\pi_{1,t} \equiv p_t - p_{t-1}$, p_t being the (log) price level in period t. y_t is the output gap (deviation of output from the natural rate), and i_t is the (repo) interest rate in period t. The coefficients are assumed to fulfill the conditions $\alpha_y > 0$, $\beta_r > 0$, and $0 < \beta_y < 1$. The first equation may thus be interpreted as a traditional Phillips curve, which relates the one-period inflation rate to the output gap and lagged (one-period) inflation. The second equation represents an aggregate demand relation, where the output gap is determined by its own lag and by the lagged real interest rate.[3] The timing of the variables is crucial for the subsequent analysis in this paper; as written now, equations (1)–(2) imply that a change in i_t affects output after one period and one-period inflation after two. We will refer to this as the two-period *control lag*.

Introduce now a central bank that wishes to stabilize (i) inflation around a constant target, π^*, and, possibly, (ii) output around the natural rate. If inflation targeting is the only concern then the central bank is said to pursue *strict inflation targeting* in the terminology of Svensson (1999a). If the central bank also cares about the variability of output, the central bank pursues *flexible inflation targeting*. Formally, the bank acts so as to minimize

$$\min_{\{i_\tau\}_{\tau=t}^{\infty}} \mathrm{E}_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} L(\overline{\pi}_{j,\tau}, y_\tau), \quad (3)$$

[3] Alternatively, one could start from the formulation

$$y_{t+1} = \tilde{\beta}_y y_t - \beta_r \left(i_t - \pi_{1,t+1|t}\right) + \varepsilon_{y,t+1},$$

where the subindex $t+1 \mid t$ is short-hand representing the conditional expectation at time t. For example, $\pi_{1,t+1|t} \equiv E\{\pi_{1,t+1} \mid t\}$. Since, by equation (1), $\pi_{1,t+1|t} = \pi_{1,t} + \alpha_y y_t$, we get (2) where $\beta_y \equiv \tilde{\beta}_y + \beta_r \alpha_y$.

where δ is a discount factor fulfilling $0 < \delta < 1$, the period loss function is defined as

$$L(\overline{\pi}_{j,t}, y_t) \equiv \frac{1}{2}\left[(\overline{\pi}_{j,t} - \pi^*)^2 + \lambda\, y_t^2\right],$$

where λ is the relative weight on output stabilization, and the *target inflation rate* $\overline{\pi}_{j,t}$ is defined as

$$\begin{aligned} \overline{\pi}_{j,t} &\equiv \frac{1}{j}\sum_{s=0}^{j-1} \pi_{1,t-s} \\ &= \frac{1}{j}\left(p_t - p_{t-j}\right). \end{aligned} \tag{4}$$

This j-period moving average permits us to vary the time-span over which 'inflation' is calculated. For example, if the equations (1)–(2) represent an annual model of the economy, then $\overline{\pi}_{1,t}$ is of course simply the annual inflation rate, $\overline{\pi}_{2,t}$ $(=\frac{1}{2}(p_t - p_{t-2}))$ is the average two-year inflation rate (measured on an annual basis), while $\overline{\pi}_{4,t}$ is the average four-year inflation rate.[4] Varying j from 1 and upwards corresponds to the central bank "taking a longer-term view" as discussed in the introduction. The parameter j, being the measurement period for the target variable, will occasionally be referred to as the *window*.[5]

Appendix A discusses some properties of this model when $j = 1$, which is the case solved analytically in Svensson (1997, 1999a).

2.1 State-space representation and solution

As soon as $j > 1$ it is no longer possible to obtain analytical solutions. Instead, we cast the problem in terms of a standard optimal regulator problem, and use well-established tools to solve for the optimal reaction function. A basic state-space representation of equations (1) and (2) is

$$\begin{bmatrix} \pi_{1,t+1} \\ y_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & \alpha_y \\ \beta_r & \beta_y \end{bmatrix} \begin{bmatrix} \pi_{1,t} \\ y_t \end{bmatrix} + \begin{bmatrix} 0 \\ -\beta_r \end{bmatrix} i_t + \begin{bmatrix} \varepsilon_{p,t+1} \\ \varepsilon_{y,t+1} \end{bmatrix},$$

[4] If instead equations (1)–(2) represent a quarterly model, then $\overline{\pi}_{4,t}$ is annual inflation measured at a quarterly rate (unless $\pi_{1,t} \equiv 4(p_t - p_{t-1})$, in which case $\overline{\pi}_{4,t}$ is annual inflation measured at an annual rate).

[5] Haldane (1997) refers to this as the *periodicity* of inflation.

or simply

$$x_{t+1} = Ax_t + Bi_t + \varepsilon_{t+1},$$

where x_t is the state vector. In analyzing the effects of having different windows j on monetary policy and on the economy, it is however useful to introduce additional state variables. Doing so, it is then straight-forward to present the central bank's objective (which will include not only the current one-period inflation rate, but also lagged ones) as a quadratic form in the state vector. Expand the vector x_t by inserting a $(j-1) \times 1$ column vector $\widetilde{\pi}_t$ of lagged inflation rates (i.e. $\widetilde{\pi}_t \equiv \left[\pi_{1,t-1} \;\; \pi_{1,t-2} \; ... \; \pi_{1,t-(j-1)}\right]'$):

$$\widetilde{x}_t \equiv \left[\begin{array}{ccc} \pi_{1,t} & \widetilde{\pi}_t' & y_t \end{array} \right]'.$$

The state vector in this extended set-up, $\widetilde{x}_t$, will be a $(2+j-1) \times 1$ vector. The extended state-space representation is hence

$$\widetilde{x}_{t+1} = A_j \widetilde{x}_t + B_j i_t + \varepsilon_{j,t+1}, \tag{5}$$

where the matrices A_j and B_j have been adapted to conform to the new state vector, and $\varepsilon_{j,t+1}$ is simply the old ε_{t+1} with some additional zeros in the appropriate places. The objective of the central bank is correspondingly expressed in terms of the extended state-vector:

$$\min_{\{i_\tau\}_{\tau=t}^{\infty}} \quad \mathrm{E}_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} \, \widetilde{x}_\tau' \, Q_j \, \widetilde{x}_\tau, \tag{6}$$

where the matrix Q_j is of dimension $(2+j-1) \times (2+j-1)$ with elements

$$Q_j \equiv \left[\begin{array}{ccccc} \frac{1}{q_j} & \frac{1}{q_j} & \cdots & \frac{1}{q_j} & \\ \frac{1}{q_j} & \frac{1}{q_j} & \cdots & \frac{1}{q_j} & \\ \vdots & \vdots & \ddots & \ddots & 0_{j\times 1} \\ \frac{1}{q_j} & \frac{1}{q_j} & \cdots & \frac{1}{q_j} & \\ & & 0_{1\times j} & & \lambda \end{array} \right],$$

and where $q_j = j^2$. Appendix B contains an example ($j = 4$).[6]

[6]Alternatively, we may express the central bank objective as

$$\min_{\{i_\tau\}_{\tau=t}^{\infty}} \quad E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} \, Y_\tau' \, K \, Y_\tau,$$

Minimizing (6) subject to the constraint (5) results in an optimal reaction function of the form[7]

$$i_t = F_j \widetilde{x}_t, \tag{7}$$

where the vector F_j has $(2+j-1)$ elements. Substituting the optimal reaction function into equation (5) gives the reduced form of the model:

$$\begin{aligned} \widetilde{x}_{t+1} &= A_j \widetilde{x}_t + B_j F_j \widetilde{x}_t + \varepsilon_{j,t+1} \\ &= (A_j + B_j F_j) \widetilde{x}_t + \varepsilon_{j,t+1} \\ &\equiv M_j \widetilde{x}_t + \varepsilon_{j,t+1}. \end{aligned} \tag{8}$$

where the matrix M_j is of dimension $(2+j-1) \times (2+j-1)$.

3 Optimal monetary policy with different measurement periods

This section reports the results on optimal monetary policy for different values of j (and of λ) derived using numerical methods. The numbers assigned to the parameters in equations (1) and (2) have been taken from Orphanides and Wieland (1999), who estimate these equations on annual data 1976 - 1998 for the Euro area. The numbers are presented in Table 1 below.

Table 1: Parameter values

α_y	β_y	β_r	Var(ε_p)	Var(ε_y)
0.34	0.77	0.40	0.88	0.71

Our interest here is to see the implications of changing the measurement period j in the loss function, equation (3). The benchmark against which we compare is the case of $j = 1$ (i.e. the original model due to Svensson 1997), and the main question we pose is whether a 'longer-term view' leads to less aggressive monetary policy as compared to this benchmark. As indicators of the 'aggressiveness' of policy we look at i) the dynamic response of policy

where Y_t contains the goal variables (i.e. $\overline{\pi}_{j,t}$ and y_t), being formed as $Y_t \equiv G'_{xj}\widetilde{x}_t$ and K is simply $\begin{bmatrix} 1 & 0 \\ 0 & \lambda \end{bmatrix}$. Naturally we have $Q_j = G'_{xj}\widetilde{x}_t G_{xj}$.

[7] In this model with no forward-looking variables, the solutions under discretion and under commitment coincide.

(i.e. the impulse responses); ii) the immediate response of policy (from the reaction function); and iii) the variability of policy (the unconditional standard deviation of the change in the instrument). We will also look at the effect on other variables, most notably output and the one-period inflation rate.

Thus: does a 'longer-term view' lead to less aggressive monetary policy? The answer varies with the value of λ, and we can identify three ranges.

First range, $\lambda = 0$

In the case of strict inflation targeting ($\lambda = 0$) the answer is always no. The aggressive policy characteristic of strict inflation targeting in the baseline case of $j = 1$ (see Appendix A) is in fact increased and perpetuated. In Figure 1 the dynamic response of policy following a unit shock to the aggregate supply equation is shown.[8] The first column (corresponding to strict inflation targeting, $\lambda = 0$) shows how monetary policy becomes strongly cyclical. The reason for this is the behavior of (one-period) inflation, which also becomes cyclical as the measurement period is extended – this is shown in Figure 2. Consider first the case of $j = 1$ (the top left-hand graph). Due to the two-period control lag, a shock to inflation at time t cannot be eliminated until $t + 2$. On the other hand, with no concern for output stabilization, inflation can then be completely eradicated, and inflation is pushed back to (the normalized value of) zero. But as soon as $j > 1$, the central bank must begin to compensate for past deviations from target — bygones are no longer bygones. When $j = 2$, the central bank will look at $\overline{\pi}_{2,t+2}$, i.e the average inflation over $t + 1$ and $t + 2$. Since inflation is 1 percent in $t + 1$ (the first half of its window), inflation in the second half (i.e. $t+2$) must be pushed to *minus* 1 percent. Furthermore, at $t+3$, inflation must be brought up to *plus* 1 percent, in order to keep the average over two periods (now $t+2$ and $t+3$) equal to zero. This oscillating pattern (+1,-1,+1,-1,...) goes on indefinitely. If $j = 3$ a different pattern emerges; the central bank now looks at $\overline{\pi}_{3,t+2}$, i.e. average inflation over t, $t+1$ and $t+2$. Since inflation is 1 percent in the first third of its window, as well as in the second, inflation in the remaining period (i.e. $t + 2$) must be brought down to *minus 2* percent. Moving the 3-period window ahead one period at a time, $t + 3$ and $t + 4$ inflation must be +1 percent, while $t + 5$ inflation must again be minus 2 percent, always in order

[8] Impulse-responses with respect to shocks to the demand equation are not reported in this paper, since these do not give additional insights.

to keep the 3-period average equal to zero. This pattern (+1,+1,-2,+1,+1,-2...) continues to infinity. For larger values of j the sequence (+1,+1,-2) is interrupted by sequences of zeros that become longer as j increases.[9] Thus, optimal monetary policy with $\lambda = 0$ will, with regular intervals, first become more expansive (to increase inflation to 1 percent), then more restrictive (first to halt the increase in inflation and then to reverse it), then more expansive again (so that inflation returns to zero) and finally restrictive to push the output gap back to its natural level (see e.g the graphs in the lower left-hand corners of Figures 1 and 2, corresponding to $j = 20$).

The cyclical movement in the interest rate is of course mirrored in a cyclical pattern in output – see the first column of Figure 3 which shows the response of output. Furthermore, the cyclical pattern in the interest rate, output and one-period inflation mean that the unconditional standard deviation of these variables become unbounded (see Table 2, top panel).

Second range, λ positive but very small

For positive, but very small values of λ the answer to our question varies with j. For medium-sized windows (values of j from around 2 to 8 or 12, roughly) policy actually becomes *more* aggressive, and output more volatile. In Figure 5 the initial responses of monetary policy are shown. The two top curves, corresponding to $\lambda = 0.01$ and $\lambda = 0.1$, respectively, reveal a larger monetary policy response as j increases. Figure 1, second and third columns, show that the following dynamic response is more volatile. Figure 6 contains the unconditional standard deviations of the first difference of policy[10]: it increases for small j. Consequently, output becomes more volatile (Figure 3 contains the impulse responses, while Figure 7 displays the unconditional standard deviations).

However, with a larger window (values of j from around 8 or 12 and onwards) *less* aggressive monetary policy results. The initial responses in Figure 5 are now milder than when $j = 1$, and the dynamic responses in Figure 1 (second and third columns) are more drawn out; together this may be termed a more cautious monetary policy. Furthermore, the drop in output is much smaller, although the response is more prolonged. This is most likely

[9] For example, when $j = 8$ the sequence is $(1, 1, -2, 0, 0, 0, 0, 0, 1, 1, -2, 0, 0, 0, 0, 0, 1, ...)$. See Figure 2, third graph from top, first column.

[10] See also Table 2 which, however, for reasons of space does not extend to as high values of j as Figure 6 does.

why the unconditional standard deviations in Figure 7 only revert to their original level (i.e. they do not fall below it).

Why does the weight on output stabilization affect how the response of optimal monetary changes to increasing j? Why do we need to distinguish between very small and larger values of λ when evaluating the effect of longer measurement periods? The intuition behind this is the following. There are two opposing forces that come into play when j is increased, one which makes policy more aggressive, the other less. The first force, which in itself makes policy more aggressive, is due to the longer memory implied by a larger window. As explained above, policy must react also to past deviations from target, leading to larger response coefficients. However, this effect is increasing in j only up to a certain point.[11] The second mechanism, having a mitigating effect on policy, is that as j increases, a given shock will matter less to the j-period average $\overline{\pi}_{j,t}$ (see its definition in equation (4)). Put differently, a given observation of $\pi_{1,t+2}$ will be associated with decreasing values of $\overline{\pi}_{j,t+2}$ as j increases. This gives the central bank more room for manoeuvre which, with positive weights on λ, it will exploit by not raising interest rates as much. This second effect is increasing in j for all values of j, and will thus eventually dominate over the first, regardless of λ.[12] It is also increasing in λ, meaning that when λ is sufficiently large, it dominates for all values of j. This brings us to the third, and final, range.

Third range, larger λ

For larger values of λ the answer is yes, regardless of j. As just mentioned, the second effect from increasing j, with a mitigating effect on policy, dominates for all values of j — the fact that a given shock matters less to the average $\overline{\pi}_{j,t}$ as j increases will lead to more cautious monetary policy. See e.g Figure 5 where the initial response of monetary policy falls for all values of j when $\lambda = 1$. The dynamic responses in Figure 1 (fourth and fifth columns) remain largely unchanged for the medium-range values of j, while for large values of j they show a smoother path for optimal monetary policy. Also, the

[11] Remember, when $\lambda = 0$ and $j = 2$, the optimal response is to push one-period inflation to -1 percent; when $j = 3$ one-period inflation should be -2 percent, which also is the case for higher values of j (with a longer control lag, this "cut-off value" of j would be larger). Now, with positive λ, optimal policy is not quite as extreme, but the basic pattern remains.

[12] In Figure 5 , the second effect takes over at approximately $j = 13$ in the case of $\lambda = 0.01$, and $j = 7$ in the case of $\lambda = 0.1$.

unconditional standard deviations of policy in Figure 6 fall. As for output, the drop is less severe the larger the value of j. The dynamic response of output is also less volatile – see the fourth and fifth columns of Figure 3. However, the unconditional standard deviations are hardly affected – see Figure 7.

Summary

To sum up, for given (strictly positive) values of λ, large values of j do indeed lead to a less 'aggressive' monetary policy, especially in terms of the initial response but also the subsequent dynamic development of the interest rate. The effect on output is also a 'smoothing' one: the drop in output is not as large, while the following dynamic response is more drawn out. However, the unconditional standard deviations remain about the same, in contrast with those of policy, which fall.

More generally, these results indicate that in this class of models of optimal monetary policy there are two ways of reducing the variability of the policy instrument. First, and as is well-known, one may assign a higher value to λ (look at the top row in Figure 1, but note that the scales are different in each column): as λ increases, the policy response becomes smoother.[13] The second is increasing j. For example, compare the third and fourth columns of Figure 1. Increasing λ from 0.1 to 0.5 (for given $j = 1$) has roughly the same effect as, for given λ, increasing j from 1 to, say, 20. The same holds true for the behavior of output (see Figure 3, but note that the scales are different in each column). Hence, when the weight on output stabilization is strictly positive, *there is an observational equivalence between increasing λ (for given j) and increasing j (for given λ), in terms of the dynamic response of the interest rate and of output.*

4 Implications for the price level

The analysis above showed that the increased memory that comes with longer measurement periods leads, under strict inflation targeting, to a cyclical behavior in one-period inflation. In this section we discuss the behavior of the price level as the measurement period j is extended.

[13] Graph A1 in Appendix A provides a summary.

Consider Figure 8. Here the response of the price level following a unit shock to aggregate supply is traced out for different combinations of j and λ (hence, Figure 8 builds on Figure 2). The initial value of the price level has been normalized to 1, and due to the two-period control lag it always rises to 3 following the shock. The following development, however, depends on j and λ. In the first column, corresponding to $\lambda = 0$, the top graph shows the familiar case of $j = 1$. The central bank now acts to completely eliminate inflation in $t + 2$; thus the price level remains at 3. When $j > 1$ the central bank must compensate for previous deviations from target (see Figure 2). This, in turn, implies that the price level will jump between 1 (its original value), 2 and 3 indefinitely. However, as j increases, the price level remains at the original level for longer periods of time.

Obviously there is an analogy with *price level targeting* here. Extending the pattern shown in the first column of Figure 8, one can see that as j approaches infinity the price level returns to and remains at 1, i.e. *as j increases we approach the case of price level targeting.* As is easily understood, this implies a reduced long-term variability in the price level. On the other hand, there is increased short-term variability of one-period inflation, the interest rate and of output, as was seen in the impulse-responses in Figures 1, 2 and 3. In fact, the one-period inflation rate $\pi_{1,t}$ and the output gap y_t become non-stationary with infinite variances. In this regard, this model yields results that are well in accord with an old, established view on the relative merits of price level targeting and inflation targeting.[14]

That there will be an analogy is perhaps most easily seen by looking at the definition of $\overline{\pi}_{j,t}$ in equation (4) and letting $j \rightarrow \infty$. However, this analogy is not perfect: it holds only when $\lambda = 0$. As soon as $\lambda > 0$ there will always be some base-drift in the price level. Consider Figure 8 again, and now look at the second column (corresponding to $\lambda = 0.01$). When $j = 1$ (top graph), the price level settles at a value slightly higher than 3 (since one-period inflation rate is brought down to zero *almost*, but not quite, as soon as it is possible for the central bank to do so). When $j > 1$, one-period inflation will oscillate around zero, but eventually settle down to zero (see Figure 2). Hence the price level will exhibit a partial reversal of the original base-drift.[15]

[14] However, see Svensson (1999b) and Vestin (1999) for models that do not imply this trade-off.

[15] In fact, when $\lambda = 0$ the one-period inflation rate and price level never settle down as soon as $j > 1$; they in fact do as soon as λ takes on a strictly positive value.

The level at which the price level ultimately settles depends not only on j, but as seen also on λ. This can be seen in the remaining columns of Figure 8 or in Table 3 which documents the value of the price level as $t \to \infty$ for different combinations of λ and j. Remember, the price level starts at 1, increases to 3 (due to the two-period control lag) and then evolves in a way which depends on λ and j. For $\lambda = 0$ (first column), the price level settles down only asymptotically (i.e. as $j \to \infty$) at the starting level of 1 (hence the analogy with price level targeting). When $\lambda = 0.01$ there is a *partial* reversal of the initial base drift as soon as $j > 1$, i.e. the price level settles at a value less than 3. Note that the degree of reversal increases with j (i.e. looking down the second column, the value at which the price level settles drops significantly) but is never complete. However, this pattern does not hold true for larger values of λ (see remaining columns). As λ is increased, the effect of changing j becomes smaller and smaller. For example, when $\lambda = 1$, there is hardly any effect at all on the final price level or the size of the base drift of increasing j.

Thus, the somewhat surprising result is that the intuitive equivalence between price level targeting and inflation targeting with a very large window holds only when $\lambda = 0$. With higher values of λ, there will never be a complete return to the initial price level, since the cost of doing so (in output terms) is deemed to be too high.[16]

5 Summary and conclusions

The policy of inflation targeting, in theory and in practice, most often focuses on stabilizing the *annual* rate of inflation. Yet it is not obvious, from a theoretical point of view, why this measure, as opposed to inflation measured over shorter or longer periods, is what should enter the central bank's loss function. In this paper we investigate the implications of varying the j-period window over which the inflation rate is measured. As discussed in the introduction, the initial intuition concerning such variations is that extending the window ought to lead to a more smooth conduct of monetary policy, in the sense of lower response coefficients in the optimal reaction function and a more muted dynamic response of the interest rate.

[16] In the price-level targeting model of Svensson (1999b) the reversal back to the original price level following a shock will always be complete, but will take longer with higher values of λ.

The analysis of this paper shows that the effect of increasing j on the conduct of monetary policy hinges on the relative weight assigned to output stabilization, λ, and we identify three ranges. 1) In the case of strict inflation targeting, i.e. when no weight is given to output stabilization, monetary policy in fact becomes *more* volatile. The intuition behind this result is that with a 'longer-term view', policy makers acquire a longer memory — optimal policy must compensate for past deviations from target, past deviations that would be 'by-gones' with shorter definition horizons. 2) In the case with a positive weight on output stabilization (flexible inflation targeting), but when this weight is very small, we must distinguish between a medium-sized window (meaning that j is from approximately 2 to 8 or 12) and a larger window (values of j from around 8 or 12 and up). In the former case, optimal monetary policy also becomes more volatile, while in the latter it becomes more cautious. There are two opposing forces here. The first, making policy more aggressive, is due to the longer memory just mentioned. But this effect is increasing in j only up to a point. The second effect, with a mitigating effect, is due to the simple fact that a single shock will matter less to the j-period average when j is increased. This effect is increasing in j for all values of j, meaning that it eventually dominates. 3) Finally, for larger values of λ, the initial intuition is also borne out – monetary policy does become more smooth, even in the medium range for j, since the cost of correcting for past deviations from target is deemed too high. The initial response of monetary policy following a shock is milder and the subsequent dynamic behavior is less volatile as the j-period window is extended. It was shown that *in terms of the dynamic response of the interest rate and of output, there is an observational equivalence between increasing λ (for given j) and increasing j (for given λ).*[17]

Another conclusion from the analysis is that, as j approaches infinity and for small values of λ an analogy exists with *price level targeting*. As is easily understood, price level targeting implies zero base drift in the price level, while inflation targeting implies a positive base drift. Furthermore, the 'conventional wisdom' regarding the relative merits of price level targeting and inflation targeting claims that the choice involves a trade-off between low-frequency price level variability and high-frequency inflation variability.[18] In

[17] Hence a central banker who for some reason wants to obtain less volatility in the policy instrument, but does not want to increase λ, can instead extend the window over which 'inflation' is measured.

[18] See, however, Svensson (1999b) and Vestin (1999) for models yielding results at odds

this model, the analogy is at its closest for $\lambda = 0$: then the price level returns to, and stays at, its initial level following a shock to inflation, i.e there is no base drift. Also, the unconditional variance of one-period inflation becomes unbounded.[19] For very small values of λ, the analogy is less precise, but nonetheless some elements of price level targeting remain. First of all, the size of the base drift is reduced by increasing j. Furthermore, the unconditional variance of one-period inflation increases somewhat. But for larger values of λ, the analogy disappears. The size of the base drift (and the unconditional variance of one-period inflation) is largely unaffected by increasing j. Thus, the somewhat surprising result is that the intuitive equivalence between price level targeting on the one hand, and inflation targeting with a very long measurement period on the other, holds only when $\lambda = 0$.

In closing, let us note another reason why central banks may wish to target inflation measured over longer periods. Figure 4 shows how target inflation $\overline{\pi}_{j,t}$ evolves following a shock to aggregate supply, for different values of λ and j. Naturally, the larger the j, the smoother the behavior of the target variable. In this sense a central bank employing a longer window may appear to be more 'successful', ex post, than one that uses a shorter window. While this is a matter of speculation, it is difficult to rule out the possibility of such considerations completely, especially in situations where the credibility of the inflation target is a concern.

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with this established view.

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Appendix A: The Svensson (1997) model

The case of $j = 1$ is analyzed in Svensson (1997, 1999a) and we briefly present the results here in order to cast them in terms of our notation. The first-order condition for minimizing (3) subject to (1)-(2) is

$$\pi_{1,t+2|t} = \pi^* + c\left(\pi_{1,t+1|t} - \pi^*\right), \tag{9}$$

where

$$c \equiv \frac{\lambda}{\lambda + \delta\,\alpha_y^2\,k},$$

and

$$k \equiv \frac{1}{2}\left(1 - \frac{\lambda\,(1-\delta)}{\delta\,\alpha_y^2} + \sqrt{\left(1 + \frac{\lambda\,(1-\delta)}{\delta\,\alpha_y^2}\right)^2 + \frac{4\lambda}{\alpha_y^2}}\right) \geq 1.$$

Consider first the case of strict inflation targeting ($\lambda = 0 \rightarrow c = 0$). Then the central bank should set its instrument so that the first inflation that can be affected is equal to the target (on an expected basis). Since we have a two-year control lag, it will be expected inflation two periods hence. In effect, the conditional inflation forecast becomes an intermediate target.

For flexible inflation targeting (i.e., $\lambda > 0$) the central bank should only gradually adjust its inflation forecast to the target, since this will reduce output fluctuations. The higher the weight on output stabilization, the slower the adjustment of the inflation forecast to the target (c is increasing in λ; Svensson (1997), p. 1132). See Figure 9 for an illustration of this.

The central bank's optimal reaction function may be derived by, first, noting that expected inflation two periods into the future is

$$\pi_{1,t+2|t} = (1 + \alpha_y\beta_r)\pi_{1,t} + \alpha_y(1 + \beta_y)y_t - \alpha_y\beta_r i_t, \tag{10}$$

(which follows from equations (1)-(2)) and, second, substituting this into the first-order condition, equation (9). Solving for i_t we thus get

$$i_t = \pi_{1,t} + \frac{1-c}{\alpha_y\beta_r}\left(\pi_{1,t} - \pi^*\right) + \frac{1 - c + \beta_y}{\beta_r} y_t,$$

which is on a 'Taylor-rule'- like form. Hence the repo rate is increasing in the excess of current inflation over the target, and in the output gap. For later reference we can also state the reaction function as:

$$i_t = \frac{1 + \alpha_y \beta_r - c}{\alpha_y \beta_r} \pi_{1,t} + \frac{1 - c + \beta_y}{\beta_r} y_t, \tag{11}$$

(where for simplicity $\pi^* = 0$). Hence a positive weight on output stabilization (leading to $c > 0$) leads to smaller coefficients in the reaction function, i.e. concern about real variability leads to 'smoother' policy.

Illustration using assumed parameter values

Using the parameter values from Orphanides and Wieland (1999) we now illustrate some properties of this base-line model. Impulse-responses following a shock to the aggregate supply equation are displayed in Figure 9 (the response of one-period inflation, output and the instrument) for six different values of λ ranging between 0 and 1. In the case of strict inflation targeting inflation returns to target as soon as it is possible for the central bank to force it to do so. This quick return to target requires very dramatic changes in the interest rate and in output. When some weight is assigned to output stabilization, the picture changes. Now inflation is only gradually returned to target – as was shown in equation (9) – and the greater the weight on output stabilization, the slower is this return. Hence, the required interest rate changes are much smaller, and output is much smoother.

See also the first column of Table 2, which shows the unconditional standard deviations of inflation and output when *j=1*. Looking down the column, we see that the standard deviation of inflation rises, and the standard deviation of output falls, as more weight is placed on output stabilization.

Finally, see Figure 5 which contains the initial policy responses for different λ and j. The intercept of each curve corresponds to $j = 1$. Here, again, we see that optimal policy becomes 'smoother' as λ increases – as shown in equation (11).

Appendix B: Statespace representation, an example

As an example, consider the case of $j = 4$. The objective of the central bank is to minimize the squared deviations of the four-period average inflation rate $\overline{\pi}_{4,t}$ from the target π^*, where $\overline{\pi}_{4,t} \equiv \frac{1}{4}\left(\pi_{1,t} + \pi_{1,t-1} + \pi_{1,t-2} + \pi_{1,t-3}\right)$. The extended state vector then is

$$\widetilde{x}_t \equiv \begin{bmatrix} \pi_{1,t} & \pi_{1,t-1} & \pi_{1,t-2} & \pi_{1,t-3} & y_t \end{bmatrix}'$$

and the state-space formulation is

$$\begin{aligned} \widetilde{x}_{t+1} &= A_4 \widetilde{x}_t + B_4 i_t + \varepsilon_{4,t+1} \\ &= \begin{bmatrix} 1 & 0 & 0 & 0 & \alpha_y \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \beta_r & 0 & 0 & 0 & \beta_y \end{bmatrix} \begin{bmatrix} \pi_{1,t} \\ \pi_{1,t-1} \\ \pi_{1,t-2} \\ \pi_{1,t-3} \\ y_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\beta_r \end{bmatrix} i_t + \begin{bmatrix} \varepsilon_{p,t+1} \\ 0 \\ 0 \\ 0 \\ \varepsilon_{y,t+1} \end{bmatrix} \end{aligned}$$

(In A_4, three rows/columns have been inserted after the first row/column, in B_4 three rows have been inserted after the first row, and filled with the appropriate elements.) The central bank's objective is

$$\min \ \widetilde{x}_t' \, Q_4 \, \widetilde{x}_t$$

where

$$Q_4 \equiv \begin{bmatrix} \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & 0 \\ \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & 0 \\ \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & 0 \\ \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & 0 \\ 0 & 0 & 0 & 0 & \lambda \end{bmatrix}$$

Figure 1: Monetary policy response following unit shock to aggregate supply. Different combinations of λ and the measurement period j.

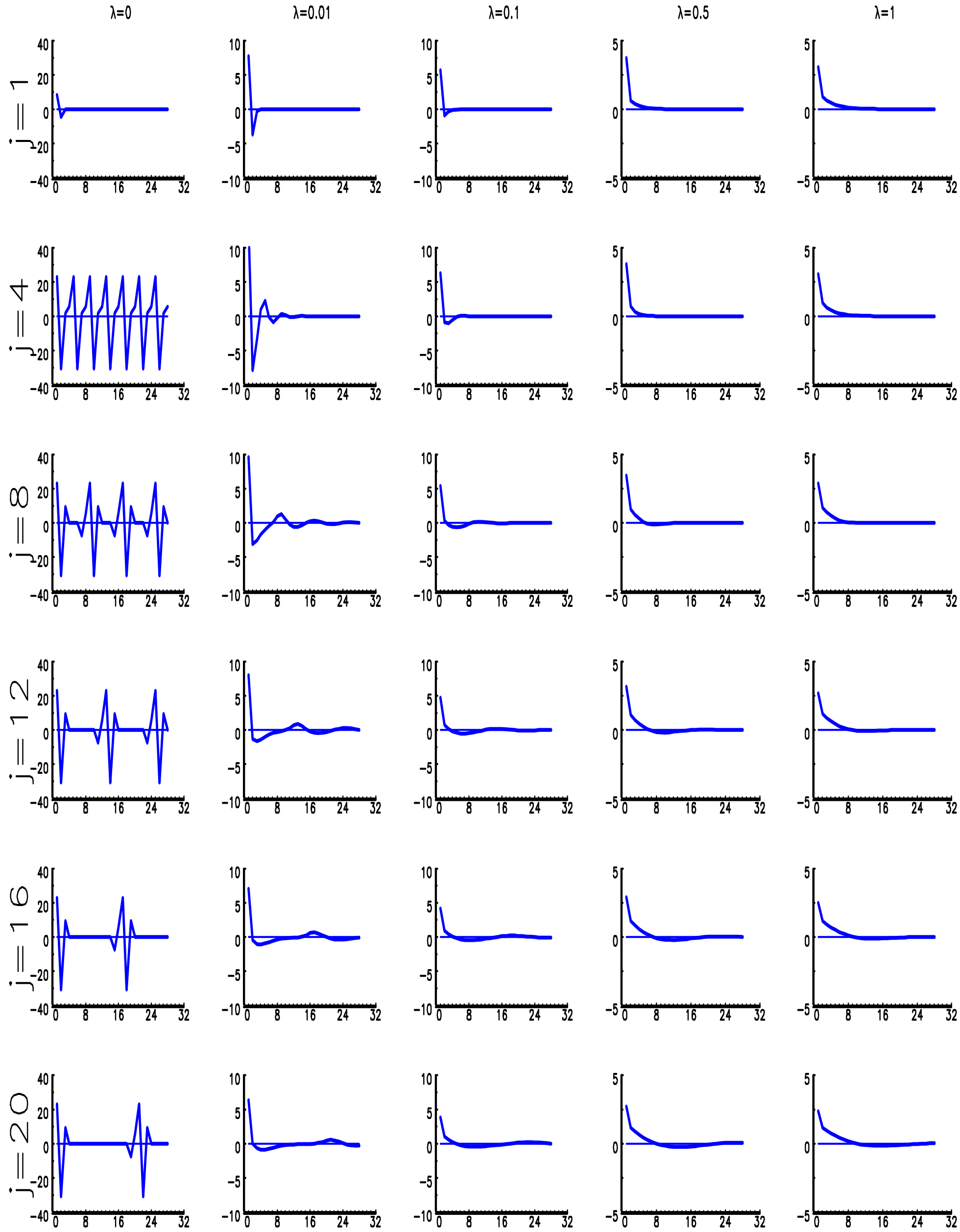


Figure 2: Response of one-period inflation following unit shock to aggregate supply.
Different combinations of λ and the measurement period j.

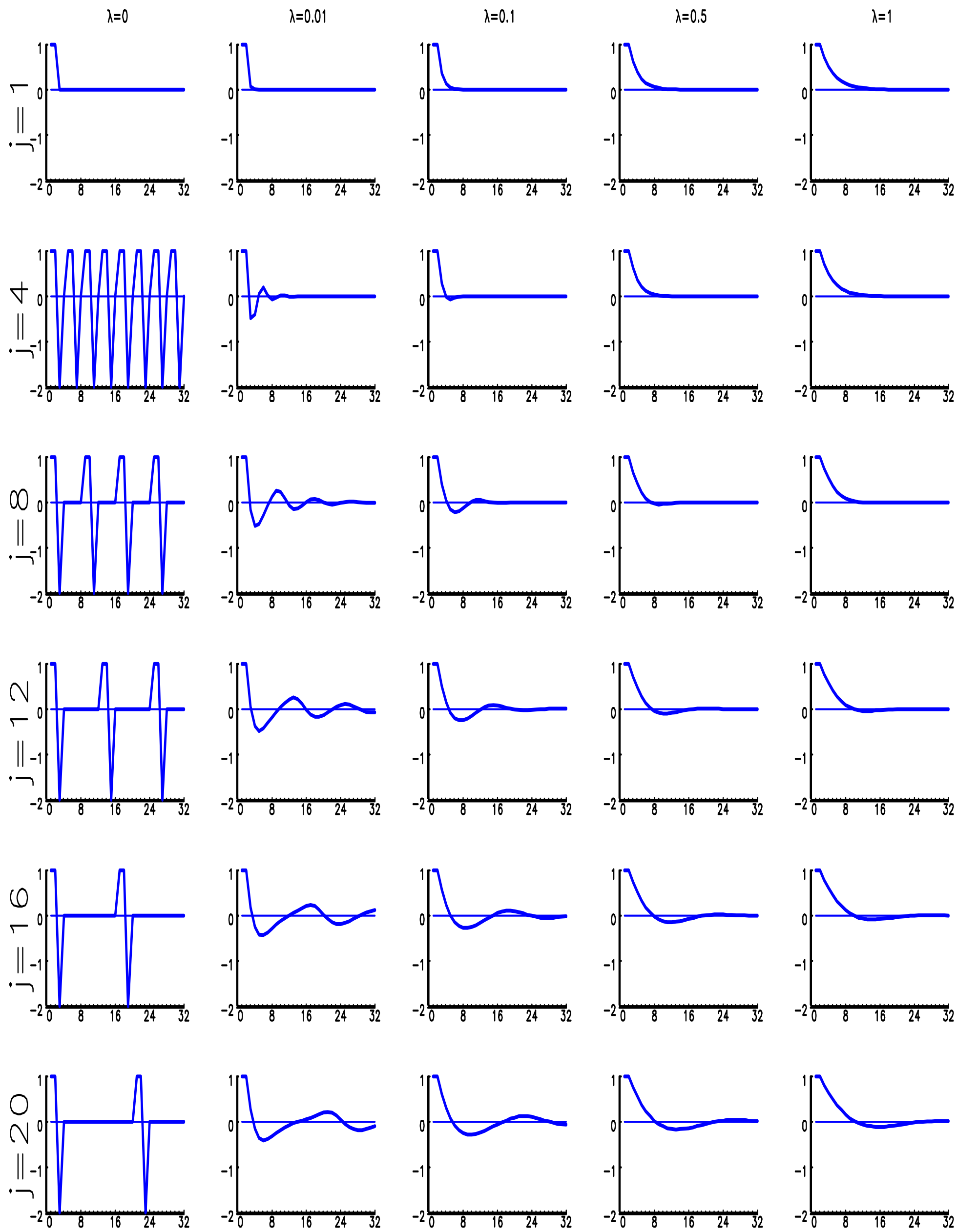


Figure 3: Output response following shock to aggregate supply.
Different combinations of λ and the measurement period j.

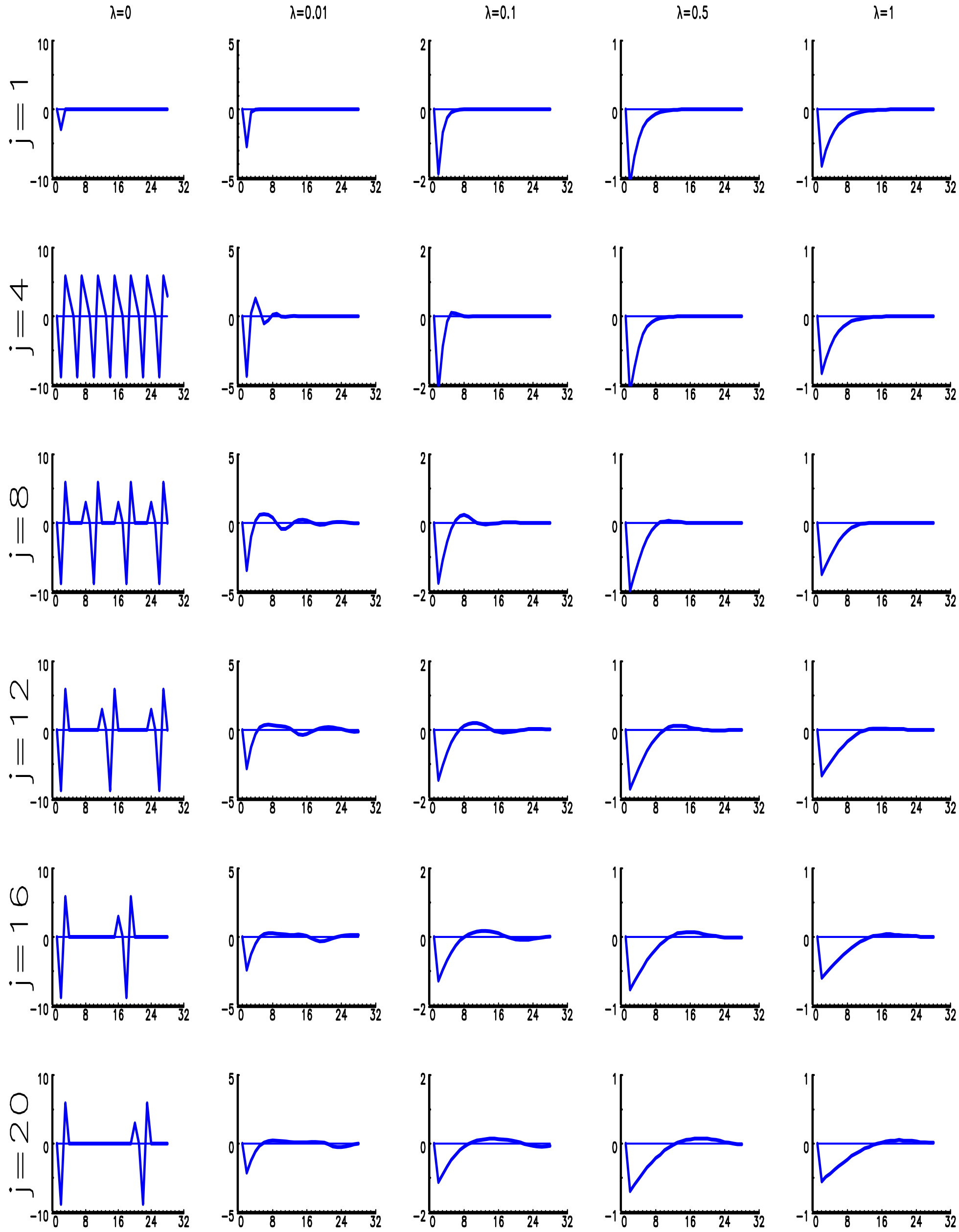


Figure 4: Response of j-period inflation following unit shock to aggregate supply. Different combinations of λ and the measurement period j.

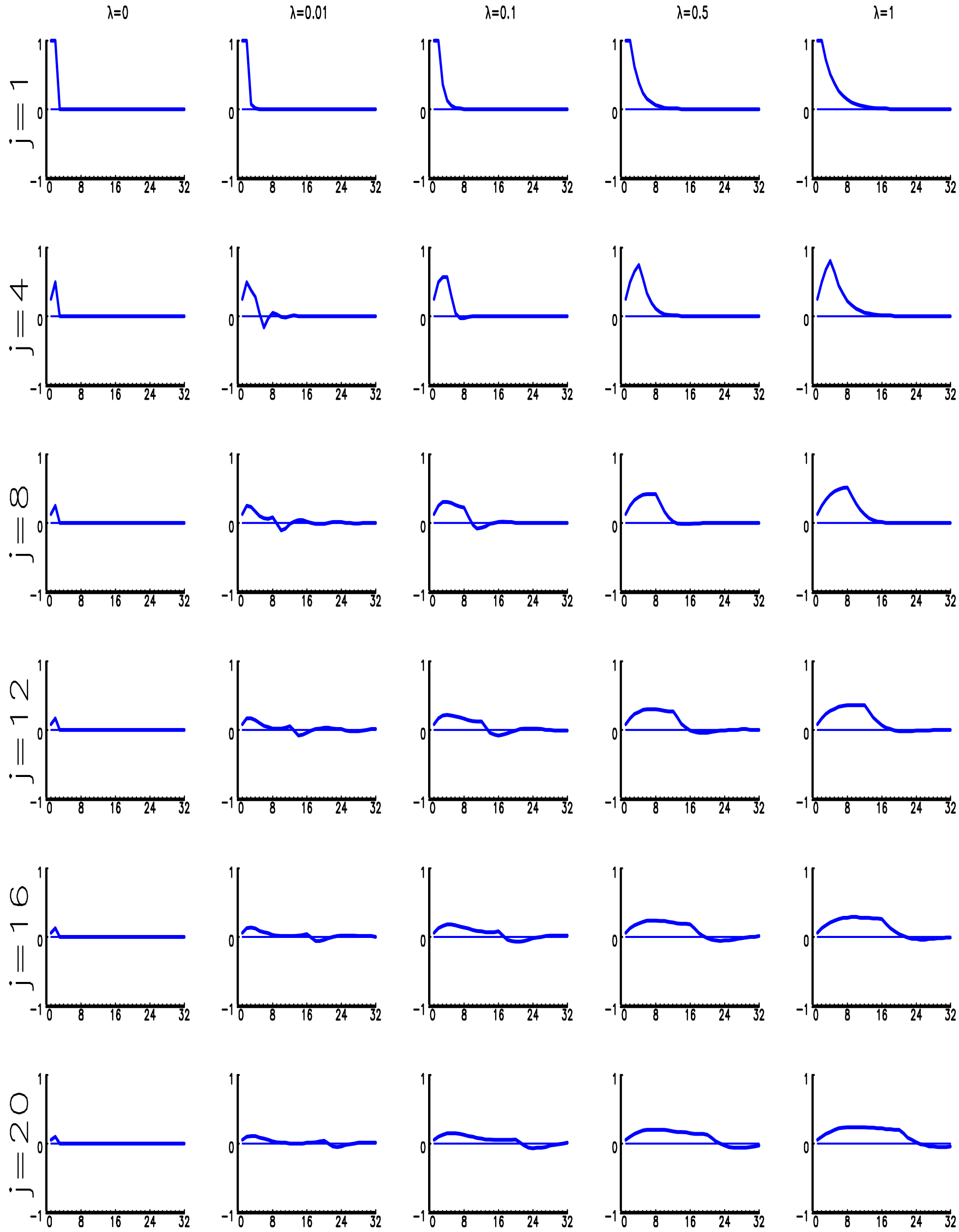


Figure 5: Initial policy response

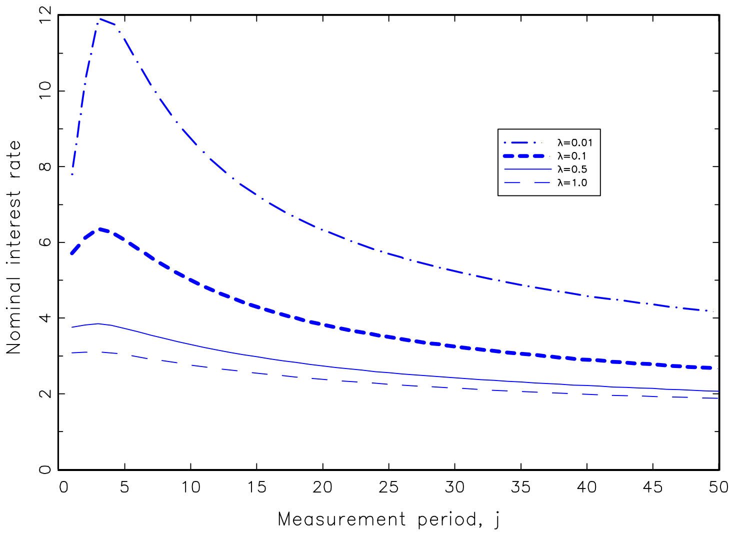


Figure 6: Unconditional standard deviation of change in monetary policy instrument.

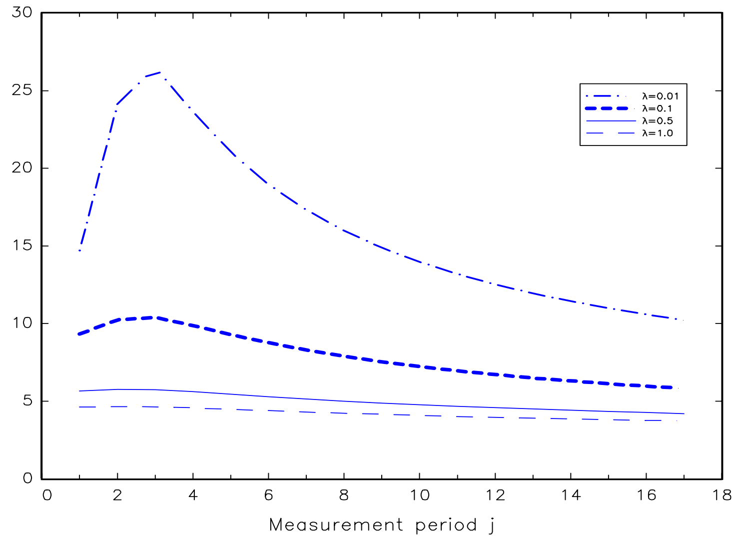


Figure 7: Unconditional standard deviation of output.

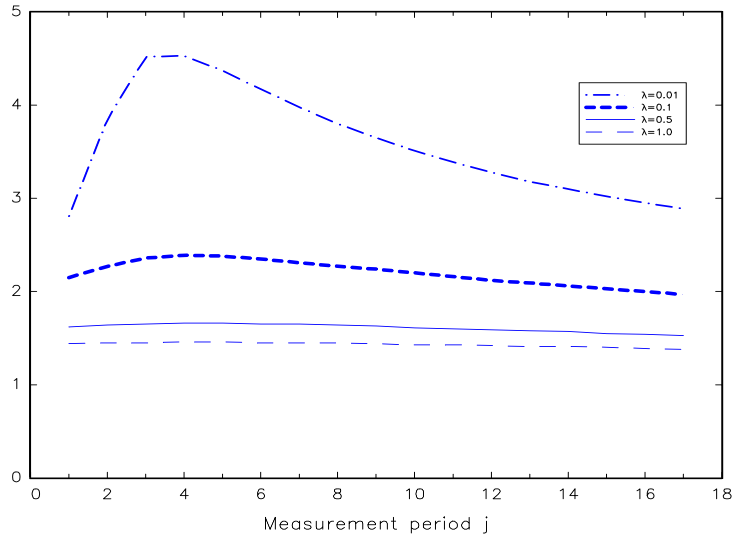


Figure 8: Response on price level following unit shock to aggregate supply. Different combinations of λ and the measurement period j.

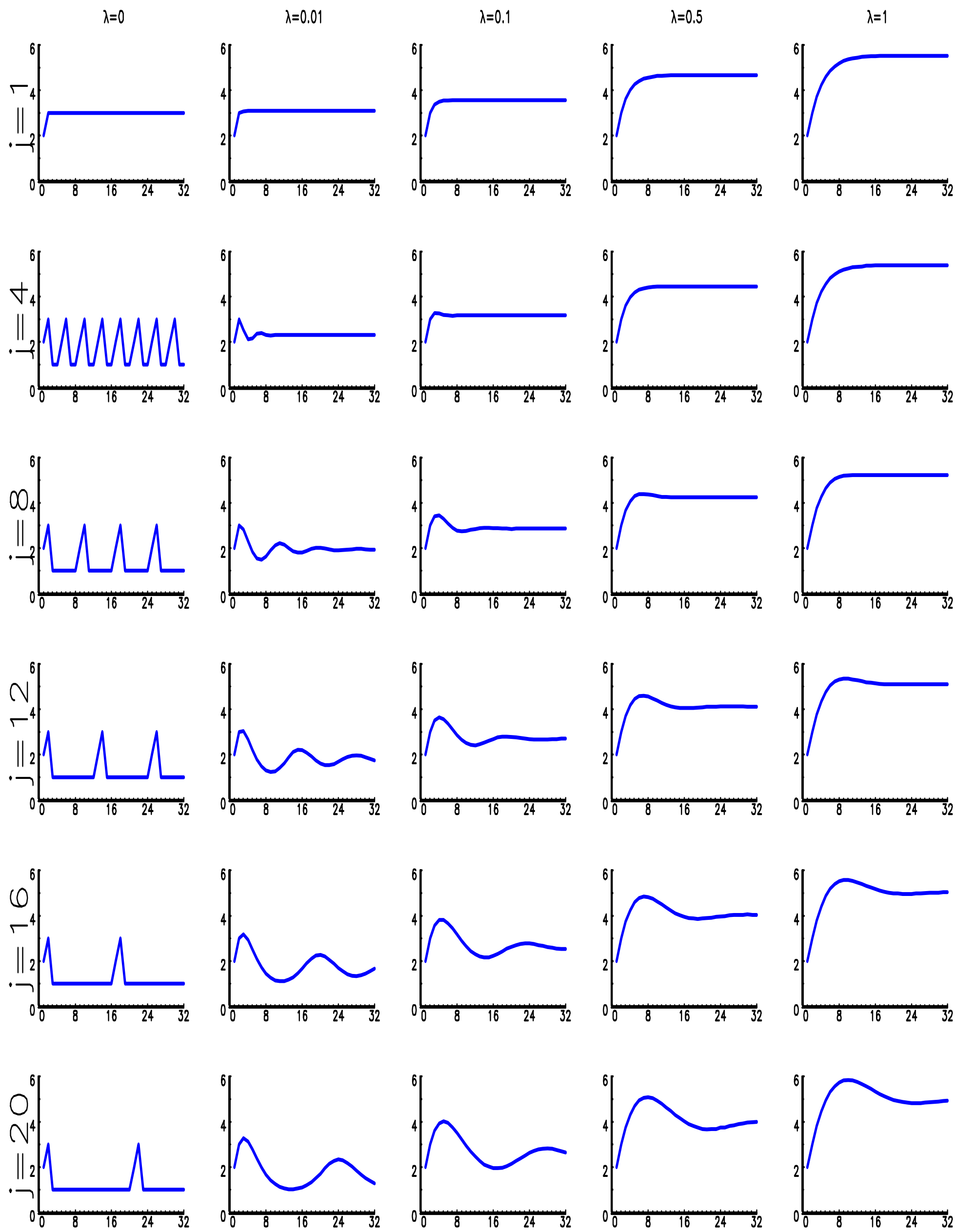
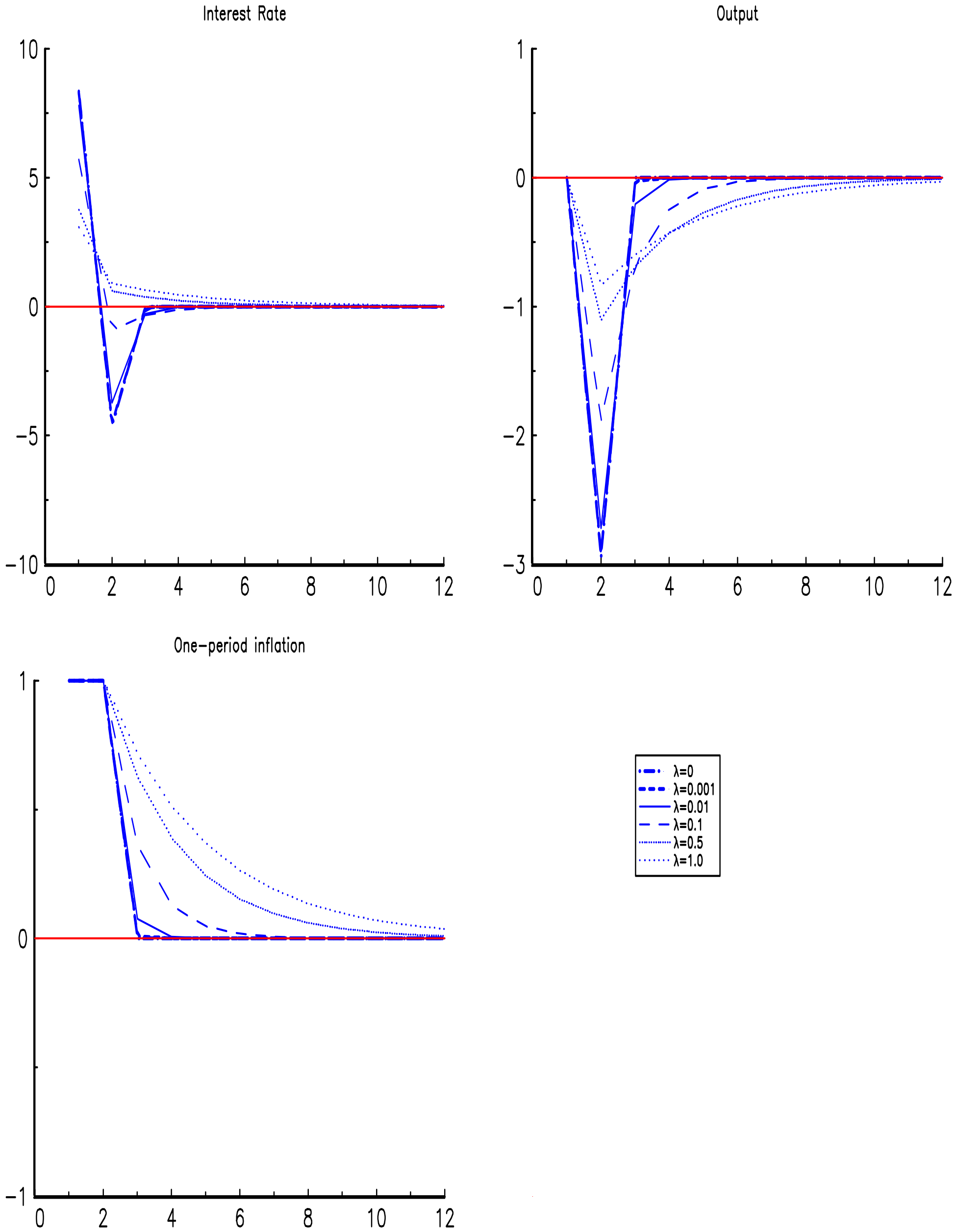


Figure 9: (Appendix) Bench-mark case, $j = 1$.
Impulse-responses with respect to unit shock to aggregate supply.



Tables

Table 2. Unconditional standard deviations

Length of window, j	*1*	*2*	*3*	*4*	-	*8*	-	*12*	-	*16*
$\boldsymbol{\lambda} = \mathbf{0.0}$										
$\overline{\pi}_{j,t}$	1.36	1.06	0.71	0.53		0.26		0.18		0.13
y_t	3.01	∞	∞	∞		∞		∞		∞
Δi_t	16.46	∞	∞	∞		∞		∞		∞
$\pi_{1,t}$	1.36	∞	∞	∞		∞		∞		∞
i_t	9.81	∞	∞	∞		∞		∞		∞
$\boldsymbol{\lambda} = \mathbf{0.01}$										
$\overline{\pi}_{j,t}$	1.36	1.12	0.88	0.72		0.43		0.33		0.28
y_t	2.81	3.85	4.52	4.53		3.80		3.28		2.95
Δi_t	14.71	24.16	26.48	23.62		15.99		12.53		10.59
$\pi_{1,t}$	1.36	1.38	1.45	1.50		1.61		1.67		1.71
i_t	8.93	13.62	15.54	14.64		10.75		8.78		7.66
$\boldsymbol{\lambda} = \mathbf{0.1}$										
$\overline{\pi}_{j,t}$	1.41	1.25	1.10	0.98		0.70		0.57		0.49
y_t	2.15	2.27	2.36	2.39		2.27		2.12		2.00
Δi_t	9.33	10.24	10.40	9.88		7.90		6.72		5.97
$\pi_{1,t}$	1.41	1.39	1.38	1.39		1.46		1.54		1.62
i_t	6.20	6.67	6.86	6.73		5.85		5.25		4.86
$\boldsymbol{\lambda} = \mathbf{0.5}$										
$\overline{\pi}_{j,t}$	1.57	1.46	1.36	1.28		1.04		0.90		0.80
y_t	1.62	1.64	1.65	1.66		1.64		1.59		1.54
Δi_t	5.67	5.76	5.75	5.61		5.01		4.58		4.28
$\pi_{1,t}$	1.57	1.56	1.55	1.55		1.58		1.63		1.70
i_t	4.34	4.38	4.40	4.36		4.14		3.96		3.83
$\boldsymbol{\lambda} = \mathbf{1.0}$										
$\overline{\pi}_{j,t}$	1.69	1.60	1.52	1.45		1.23		1.09		1.00
y_t	1.44	1.45	1.45	1.46		1.45		1.42		1.39
Δi_t	4.63	4.66	4.65	4.57		4.23		3.97		3.78
$\pi_{1,t}$	1.69	1.69	1.68	1.68		1.70		1.74		1.80
i_t	3.85	3.87	3.87	3.84		3.73		3.64		3.58

Table 3. The size of the base drift:
convergence of price level (at t=200).
(Initial price level = 1.)

Window	$\lambda = 0.0$	$\lambda = 0.01$	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 1$
$j = 1$	3.0	3.08	3.56	4.67	5.53
$j = 2$	–	2.79	3.44	4.61	5.50
$j = 3$	–	2.51	3.29	4.53	5.44
$j = 4$	–	2.31	3.17	4.46	5.39
$j = 5$	–	2.18	3.07	4.40	5.34
$j = 6$	–	2.08	2.99	4.34	5.30
$j = 7$	–	2.00	2.92	4.29	5.26
$j = 8$	–	1.94	2.86	4.25	5.22
$j = 9$	–	1.89	2.81	4.21	5.19
$j = 10$	–	1.85	2.77	4.17	5.16
$j = 11$	–	1.82	2.73	4.14	5.14
$j = 12$	–	1.79	2.70	4.11	5.11
$j = 13$	–	1.76	2.67	4.08	5.09
$j = 14$	–	1.74	2.64	4.06	5.07
$j = 15$	–	1.72	2.61	4.04	5.05
$j = 16$	–	1.70	2.59	4.02	5.03
$j = 17$	–	1.69	2.57	4.00	5.01
$j = 18$	–	1.67	2.55	3.98	5.00
$\vdots$	$\vdots$				
$j \to \infty$	1.0				