Monetary policy with uncertain parameters*

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Abstract

In a simple dynamic macroeconomic model, it is shown that uncertainty about structural parameters does not necessarily lead to more cautious monetary policy, refining the accepted wisdom concerning the effects of parameter uncertainty on optimal policy. In particular, when there is uncertainty about the persistence of inflation, it is optimal for the central bank to respond more aggressively to shocks than if the parameter were known with certainty, since the central bank wants to avoid bad outcomes in the future. Uncertainty about other parameters, in contrast, acts to dampen the policy response.

Keywords: Optimal monetary policy, parameter uncertainty, Brainard conservatism principle, interest rate smoothing.

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1 Introduction

It is widely accepted that policymakers facing uncertainty about the structure of the economy should be more cautious when implementing policy than if acting under complete certainty (or certainty equivalence). The attractiveness of this result, named the 'Brainard conservatism principle' by Alan Blinder (1997, 1998) after the original analysis of William Brainard (1967), lies in both the simplicity of the original argument and in the underlying intuition: when you are uncertain about the effects of policy, it makes sense for policymakers to move more cautiously in the response to economic shocks.[1]

Recently, Svensson (1997a) has shown this result to hold also in a dynamic macroeconomic model, often used to analyze issues in monetary policy. When there is uncertainty about some of the structural parameters, the optimal policy response to current inflation and output (i.e., the coefficients in the policymaker's reaction function) are shown to get smaller as the amount of uncertainty increases.[2] Due to the complexity of the model with parameter uncertainty, however, Svensson chooses to analyze a special case, where only inflation (and no measure of output) enters the central bank's objective function.

The purpose of the present paper is to analyze the effects of multiplicative parameter uncertainty in a more general setting of the same model, where all structural parameters are allowed to be uncertain, and where the preferences of the central bank in the choice between stabilizing output and inflation are allowed to vary. In addition to the initial response of policy, the time path of policy after a shock is examined.

Surprisingly, the results show that parameter uncertainty does not necessarily dampen the policy response, but may actually make policy more aggressive than under certainty equivalence. In particular, uncertainty about the persistence of inflation increases the optimal reaction function coefficients, whereas uncertainty about other parameters dampens the response. In the special case analyzed by Svensson, when the weight on output stabilization in the central bank's objective function is zero, uncertainty about the persistence of inflation does not affect the policy response. For positive weights on output, however, the policy response is increasing in the variance of the persistence parameter, so policy becomes more

[1] That this principle is well understood and used by central bankers in the practical policy process is made clear by Blinder (1998) and Goodhart (1998).

[2] Similar results have been reached by, e.g., Estrella and Mishkin (1998), Sack (1998a), and Wieland (1998).

aggressive as the amount of uncertainty increases.

This result may seem counter-intuitive at first glance, but is less puzzling after careful examination. The possibility that the inflation rate moves away from target by itself leads the central bank to take precautionary steps, to avoid paying the price of larger interest rate (and output) volatility later. Without any costs of output volatility, this is not important for the central bank, but as the bank cares more about stabilizing output, it gets more important to keep inflation at bay, so as to avoid output fluctuations in later periods. As such, the results are similar to those of Craine (1979), who shows that uncertainty about the impact effect of policy leads to less aggressive policy behavior, but uncertainty about the dynamics of the economy leads to more aggressive policy. Also, Sargent (1998a,b) argues that uncertainty leads to more cautious policy, but that 'caution' could mean that the policymaker tries to avoid bad outcomes in the future by responding more aggressively to shocks today.

Perhaps less surprisingly, when parameter uncertainty does act to dampen the current policy response, it is optimal for the central bank to return to a neutral policy stance later than if all parameters were known with certainty. This is due to the persistence of inflation and output: a smaller initial response leads to larger deviations of the goal variables from target in future periods, so policy needs to be away from neutral for a longer period to get inflation and output back on track. Thus, parameter uncertainty can lead to a smoother policy path in response to shocks, an issue analyzed in more detail by Sack (1998a) and Söderström (1999).

The paper is organized as follows. In Section 2 the theoretical framework is presented, and the optimal policy of the central bank is derived in a dynamic economy with stochastic parameters. Since analytical solutions of the model are difficult, if not impossible, to find, Section 3 presents numerical solutions for different configurations of uncertainty, to establish the effects of parameter uncertainty on the optimal policy response to output and inflation shocks. Finally, the results are discussed and conclusions are drawn in Section 4.

2 The model

2.1 Setup

The basic model used in the analysis is the dynamic aggregate supply-aggregate demand framework developed by Lars Svensson (1997a,b) and used by, for example, Ellingsen and Söderström (1999), Rudebusch and Svensson (1998), and Rudebusch (1998). This model is similar to many other models used for monetary policy analysis, for example by Ball (1997), Cecchetti (1998), Taylor (1994), and Wieland (1998), and consists of two equations relating the output gap (the percentage deviation of output from its 'natural' level) and the inflation rate to each other and to a monetary policy instrument, the short interest rate. Assuming a quadratic objective function for the central bank, one can solve for the optimal decision rule as a function of current output and inflation, similar to a Taylor (1993) rule.

Important features of the model are the inclusion of control lags in the monetary transmission mechanism, and the fact that monetary policy only affects the rate of inflation indirectly, via the output gap. Monetary policy is assumed to affect the output gap with a lag of one period, which in turn affects inflation in the subsequent period.[3] Policymakers thus control the inflation rate with a lag of two periods. In the simplest version, including only one lag,[4] the output gap in period $t+1$ (or rather the deviation of the output gap from its long-run mean), y_{t+1}, is related to past output and the ex-post real interest rate in the previous period, $i_t - \pi_t$, by the IS-relationship

$$y_{t+1} = \alpha_{t+1} y_t - \beta_{t+1}(i_t - \pi_t) + \varepsilon^y_{t+1}, \tag{1}$$

where ε^y_{t+1} is an i.i.d. demand shock with mean zero and constant variance σ^2_y. The rate of inflation between periods t and $t+1$ (or its deviation from the long-run mean), π_{t+1}, depends on past inflation and the output gap in the previous period according to the Phillips curve relation

$$\pi_{t+1} = \delta_{t+1}\pi_t + \gamma_{t+1} y_t + \varepsilon^\pi_{t+1}, \tag{2}$$

where ε^π_{t+1} is an i.i.d. supply shock with zero mean and variance σ^2_π.

In the model presented here, there are two important modifications to the original Svensson framework: the persistence parameter of the inflation process, δ_{t+1}, is allowed to take values different from unity; and the parameters of the model are stochastic, and therefore time-varying. When the central bank sets its interest rate instrument at time t, it is assumed to know all realizations of the parameters up to and including period t, but it does not know their future realizations, and thus

[3]In the simple one-lag model used here, one period can be thought of as equal to one year. The short interest rate could then be interpreted as the central bank's interest rate instrument, e.g., the federal funds rate target in the U.S., assumed to be held constant for a year at a time. See Svensson (1997a).

[4]Rudebusch and Svensson (1998), Rudebusch (1998), and Söderström (1999) use a version of the model including four lags in each relationship, and estimate it on quarterly U.S. data. Söderström (1999) also formally tests the restrictions imposed by Svensson (1997a,b).

cannot be certain about the effects of policy on the economy.[5] For simplicity, assume that each parameter is given by a constant mean plus a random shock. Thus, for example, the persistence parameter of the output process, α_{t+1}, is given by

$$\alpha_{t+1} = \alpha + \nu^{\alpha}_{t+1}, \tag{3}$$

where ν^{j}_{t+1}, $j = \alpha, \beta, \gamma, \delta$, are i.i.d. shocks with mean zero and constant variance σ^2_j. The parameters are assumed to be independent of each other and of the structural shocks ε^{π}_t and ε^{y}_t.[6] Furthermore, the realizations of the parameters are drawn from the same distribution in each period, so issues of learning and experimentation are disregarded in the analysis.[7]

2.2 Optimal policy

To determine the optimal path for the interest rate over the entire future, contingent on the development of the economy, the central bank is assumed to minimize the expected discounted sum of future values of a loss function, which is quadratic in output and inflation deviations from target (here normalized to zero). Thus, the central bank solves the optimization problem

$$\min_{\{i_{t+\tau}\}_{\tau=0}^{\infty}} E_t \sum_{\tau=0}^{\infty} \phi^{\tau} L(y_{t+\tau}, \pi_{t+\tau}), \tag{4}$$

subject to (1)–(3), where in each period the loss function $L(y_t, \pi_t)$ is given by

$$L(y_t, \pi_t) = \pi_t^2 + \lambda y_t^2, \tag{5}$$

[5] That policymakers do not have complete information about the structural parameters in an economy is clearly not an unrealistic assumption. Holly and Hughes Hallett (1989) point to three reasons why a model's parameters may be seen as stochastic: (1) they are genuinely random; (2) they are really fixed, but are impossible to estimate precisely, due to the sampling variability in a finite data set; and (3) they vary according to some well-defined but imperfectly known scheme, e.g., because the model is a linearization around a trajectory of uncertain exogenous variables. Blinder (1997, 1998), Goodhart (1998), and Poole (1998) all stress the relevance of uncertainty for practical monetary policy.

[6] The assumption of independence is convenient for the derivation of optimal policy, and may be realistic if the model equations (1) and (2) are interpreted as structural relationships. If, on the other hand, one interprets the model as reduced-form relations derived from microeconomic foundations, the parameters might well be correlated if they are derived from the same micro relations.

[7] See Sack (1998b) or Wieland (1998) for similar models of monetary policy including learning and experimentation; or Balvers and Cosimano (1994), Başar and Salmon (1990), and Bertocchi and Spagat (1993) for models in slightly different contexts.

and where ϕ is the central bank's discount factor.[8] The parameter $\lambda \geq 0$ specifies the relative weight of output stabilization to inflation fighting, and is assumed to be known and constant.[9] In the simple case when parameters are non-stochastic, it is relatively straightforward to find an analytical solution for the optimization problem (4), as shown by Svensson (1997a,b). When parameters are stochastic, however, finding an analytical solution is prohibitively difficult, so I shall here focus on numerical solutions.[10]

To solve the central bank's optimization problem it is convenient to rewrite the model (1)–(2) in state-space form as

$$x_{t+1} = A_{t+1} x_t + B_{t+1} i_t + \varepsilon_{t+1}, \tag{6}$$

where $x_{t+1} = [\ y_{t+1} \quad \pi_{t+1}\ ]'$ is a state vector, and $\varepsilon_{t+1} = [\ \varepsilon^y_{t+1} \quad \varepsilon^\pi_{t+1}\ ]'$ is a vector of structural shocks. The parameter matrices A_{t+1} and B_{t+1} are then stochastic with means

$$A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}, \quad B = \begin{bmatrix} -\beta \\ 0 \end{bmatrix}, \tag{7}$$

and variance-covariance matrices

$$\Sigma_A = \begin{bmatrix} \sigma^2_\alpha & 0 & 0 & 0 \\ 0 & \sigma^2_\beta & 0 & 0 \\ 0 & 0 & \sigma^2_\gamma & 0 \\ 0 & 0 & 0 & \sigma^2_\delta \end{bmatrix}, \quad \Sigma_B = \begin{bmatrix} \sigma^2_\beta & 0 \\ 0 & 0 \end{bmatrix}, \quad \Sigma_{AB} = \begin{bmatrix} 0 & 0 \\ -\sigma^2_\beta & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{8}$$

Using the state-space formulation, the central bank's optimization problem can be written as the control problem

$$J(x_t) = \min_{i_t} \left[x_t' Q x_t + \phi E_t J(x_{t+1}) \right], \tag{9}$$

[8] The quadratic specification of the objective function is very common in the literature. Some authors, e.g., Rudebusch and Svensson (1998) and Rudebusch (1998), include an interest rate smoothing objective in the loss function to capture the apparent preference of central banks for small persistent changes in the instrument. As shown by Sack (1998a) and Söderström (1999), however, such an *ad hoc* smoothing objective is not necessary to mimic policy behavior in the U.S., at least not in an unrestricted VAR framework.

[9] Typically, λ is positive also in regimes of inflation targeting, since central banks want to stabilize short-term fluctuations in output even when their main goal is price stability. See Svensson (1998) for a discussion of 'strict' versus 'flexible' inflation targeting, and Fischer (1996) for a critique of central banks' tendency to only acknowledge price stability and not output stabilization as the goal of monetary policy.

[10] Svensson (1997a) analytically solves a very simple case of parameter uncertainty, where δ_{t+1} is non-stochastic and always equal to unity, and where $\lambda = 0$. Since the most interesting results are obtained when $\lambda > 0$ and δ_{t+1} is stochastic, I shall not follow his route.

subject to (6), where Q is a (2×2) preference matrix of the central bank, with λ and 1 on the diagonal and zeros elsewhere. The loss function will in this framework be quadratic, so

$$J(x_{t+1}) = x'_{t+1} V x_{t+1} + w, \tag{10}$$

where the matrix V remains to be determined.

When parameters are non-stochastic, so that there is only additive uncertainty in the linear-quadratic model, it is well known that optimal policy is certainty-equivalent, that is, only the expected value of the state vector x_{t+1} matters for optimal policy. In that case, the expected value of the value function (10) is simply

$$E_t J(x_{t+1}) = (E_t x_{t+1})' V (E_t x_{t+1}) + w. \tag{11}$$

When parameters are stochastic, however, certainty equivalence no longer holds, since the variance of the vector x_{t+1} also matters for policy. In mathematical terms, the difference from the certainty equivalence case is that the expected value of the value function is now

$$E_t J(x_{t+1}) = (E_t x_{t+1})' V (E_t x_{t+1}) + \mathrm{tr}(V \Sigma_{t+1|t}) + w, \tag{12}$$

where $\Sigma_{t+1|t}$ is the variance-covariance matrix of x_{t+1}, evaluated at time t, and the notation 'tr' denotes the trace operator. Consequently, the variance-covariance matrix of x_{t+1}, containing the parameter variances, will affect the optimal policy rule.

Appendix A shows that the optimal decision rule for the central bank is to set the short interest rate as a linear function of the state vector in each period, that is

$$i_t = f x_t, \tag{13}$$

where

$$f = -\left[B' \left(V + V'\right) B + 2v_{11} \Sigma_B^{11}\right]^{-1} \left[B' \left(V + V'\right) A + 2v_{11} {\Sigma_{AB}^{11}}'\right]. \tag{14}$$

Here Σ_{AB}^{ij} denotes the covariance matrix of the ith row of A_{t+1} with the jth row of B_{t+1}, and v_{ij} denotes element (i, j) of the matrix V, which is given by iterating on the Ricatti equation

$$\begin{aligned} V &= Q + \phi(A + Bf)' V (A + Bf) \\ &+ \phi v_{11} \left(\Sigma_A^{11} + 2\Sigma_{AB}^{11} f + f' \Sigma_B^{11} f\right) + \phi v_{22} \Sigma_A^{22}. \end{aligned} \tag{15}$$

As is clear from equations (14) and (15), the optimal policy rule depends on the variances (and covariances) of the parameters in the economy, so certainty equivalence ceases to hold when the parameters are stochastic. To obtain an analytical solution for this problem, one would need to solve equation (15) for the fixed-point value of V. For some simple configurations, for example, in the non-stochastic case, this is manageable (although tedious), since the system of equations obtained is relatively straightforward to solve. In this setup of multiplicative parameter uncertainty, however, the system of equations is highly non-linear and far too complicated to yield a usable solution. Therefore I proceed by numerical methods to analyze the optimal behavior of the central bank in this setting.

3 The effects of parameter uncertainty on optimal policy

Having derived the optimal policy rule (13) for the central bank, this section will analyze how the rule, and the resulting path of the short interest rate, depends on the degree of uncertainty in the economy. I therefore choose some values for the mean parameters α, β, γ, and δ, and for the discount factor ϕ, and then examine how optimal policy behaves for different configurations of the parameter variances $\sigma_\alpha^2, \sigma_\beta^2, \sigma_\gamma^2$, and σ_δ^2, and of the preference parameter λ.

Shocks to output and inflation in equations (1) and (2) will affect monetary policy on two different, but related, levels. First, there is an initial effect, as policy is adjusted to respond to current shocks. This effect is given by the vector f in the decision rule (13). Second, there is a dynamic effect of shocks, since these will not be completely offset in the initial period, but will partly be transmitted to subsequent periods through the dynamics of the economy. Thus policy will also need to respond to past shocks, as these remain in the economy. I will distinguish between these two effects, and begin by analyzing the initial response of policy in Section 3.1, followed by an analysis of the dynamic response over time in Section 3.2.

The exact parameter values used for this numerical exercise are chosen so as to best illustrate the results, but are also consistent with empirical studies of the monetary transmission mechanism in the U.S. The reported results will not depend on the exact configuration of parameter values, but hold for many different plausible and implausible configurations.

The mean of the persistence parameter of the output gap, α_{t+1}, is given a value of 0.85, taken from Cooley and Hansen (1995, Table 7.1). This value is the autocorrelation coefficient of the observed detrended output process, and as such would

Table 1: Numerical values of parameter means and variances

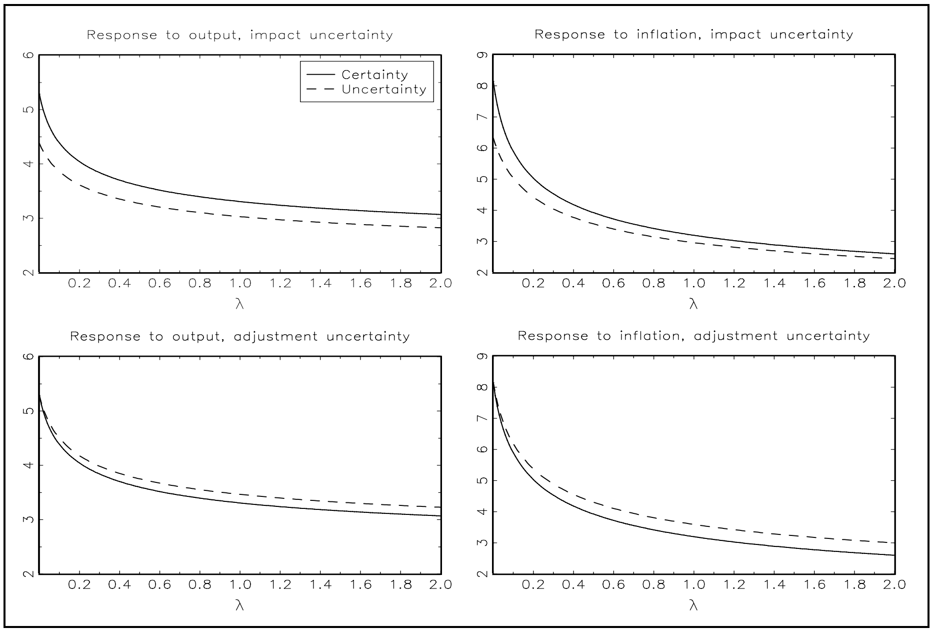
Stochastic parameters				Non-stochastic parameters	
	Mean	Variance			Value
α_{t+1}	0.85	$\{0.01, \ 0.00, \ 0.01, \ 0.01\}$		ϕ	0.95
β_{t+1}	0.35	$\{0.01, \ 0.00, \ 0.01, \ 0.01\}$		λ	[0,2]
γ_{t+1}	0.4	$\{0.01, \ 0.00, \ 0.01, \ 0.01\}$			
δ_{t+1}	1.0	$\{0.00, \ 0.10, \ 0.10, \ 0.20\}$			

tend to overestimate the true persistence of the output gap, unaffected by active stabilization policy. To the parameter β_{t+1}, the elasticity of output with respect to the real interest rate, a mean value of 0.35 is assigned, taken from Fuhrer's (1994, Table 3) estimate of output's sensitivity to the long real interest rate for the U.S. from 1966 to 1993. The mean of the persistence parameter of the Phillips curve, δ_{t+1}, is assigned a value of unity, leading to a standard accelerationist Phillips curve, on average. Finally, for γ_{t+1}, the inflation rate's sensitivity to the output gap, I assign a mean value of 0.4, which is approximately what Romer (1996, Table 2) finds for the U.S. economy for the period 1952–73, and which is also consistent with the correlation coefficient reported by Cooley and Hansen (1995, Table 7.1).

Since uncertainty concerning $\alpha_{t+1}, \beta_{t+1}$, and γ_{t+1} has similar effects on policy, but uncertainty concerning δ_{t+1} has very different implications, the analysis will concentrate on three different configurations of uncertainty: (1) when $\alpha_{t+1}, \beta_{t+1}$, and γ_{t+1} are stochastic, but δ_{t+1} is not (so $\sigma_\alpha^2 = \sigma_\beta^2 = \sigma_\gamma^2 = 0.01$ and $\sigma_\delta^2 = 0$); (2) when δ_{t+1} is stochastic, but $\alpha_{t+1}, \beta_{t+1}$, and γ_{t+1} are constant ($\sigma_\alpha^2 = \sigma_\beta^2 = \sigma_\gamma^2 = 0$ and $\sigma_\delta^2 = 0.1$); and (3) when all four parameters are stochastic ($\sigma_\alpha^2 = \sigma_\beta^2 = \sigma_\gamma^2 = 0.01$ and $\sigma_\delta^2 = 0.1$ and 0.2). For simplicity, I shall call the first of these the case of 'impact uncertainty,' since the parameters $\alpha_{t+1}, \beta_{t+1}$, and γ_{t+1} are all part of the direct impact of policy on output and inflation (via output). The second is a case of 'adjustment uncertainty,' since the parameter δ_{t+1} mainly determines the adjustment dynamics of the model; and the third case is a combination of impact and adjustment uncertainty. In each case, optimal policy will be compared to the certainty equivalence case, when all parameters are constant and equal to their means. The actual degree of uncertainty assigned through the parameter variances is chosen to make clear the effects of parameter uncertainty on policy. The qualitative results remain irrespective of the actual size of the parameter variances. The resulting values for the means and variances of the stochastic parameters are given in the left-hand panel of Table 1.

As shown in the right-hand panel of Table 1, the discount factor ϕ is assigned a

Figure 1: Initial response to output and inflation, impact uncertainty and adjustment uncertainty



value of 0.95, implying a discount rate of 5% per period. Finally, since the effects of uncertainty on policy depend crucially on the value of the preference parameter λ, this will be allowed to take values varying from 0, that is, 'strict inflation targeting,' to 2, with a larger weight on stabilizing output than on fighting inflation.

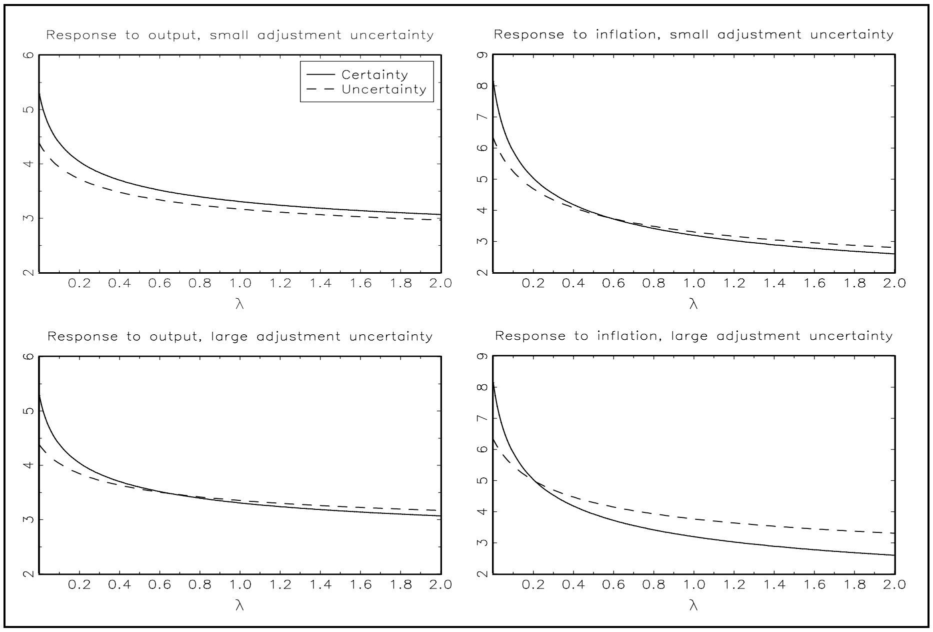
3.1 The initial policy response

The two top graphs of Figure 1 show the initial policy response to current output and inflation shocks for different values of λ in the case of certainty and in the case of impact uncertainty, that is, when there is some uncertainty about α_{t+1}, β_{t+1}, and γ_{t+1}, but δ_{t+1} is non-stochastic. The left-hand graph shows the response to output (or demand shocks) and the right-hand graph the response to inflation (supply shocks), with the solid line representing the certainty case, and the dashed line representing the response under uncertainty.

For the case of impact uncertainty, the response coincides well with the accepted wisdom formalized by Brainard (1967) and stressed by Blinder (1997, 1998). When there is uncertainty about α_{t+1}, β_{t+1}, and/or γ_{t+1}, it is optimal for the central bank to be more cautious and respond less fiercely to any shocks to output and inflation.[11]

[11] In his original analysis, Brainard (1967) shows that large covariances between the instrument and exogenous variables may overturn his conservatism result (see also Blinder, 1998). Since all parameters and shocks are assumed independent here, such situations are not considered.

Figure 2: Initial response to output and inflation, combinations of impact and adjustment uncertainty



Increasing the variance of either $\alpha_{t+1}, \beta_{t+1}$, or γ_{t+1} will weaken the optimal response of the central bank, and in the limit, as the variances tend to infinity, the optimal response is to do nothing.[12]

However, in the case of adjustment uncertainty, when the persistence parameter of inflation, δ_{t+1}, is stochastic, but $\alpha_{t+1}, \beta_{t+1}$, and γ_{t+1} are constant, the effects of uncertainty on policy are dramatically altered, as seen in the two bottom graphs of Figure 1. Now, when $\lambda = 0$, so that the central bank cares only about stabilizing inflation, uncertainty about δ_{t+1} does not affect the optimal response to output or inflation. When $\lambda > 0$, however, the pattern goes against the Brainard conservatism principle: the optimal policy under parameter uncertainty is *more* aggressive than under certainty equivalence, so that the initial central bank response is stronger, not weaker.

Finally, consider the case when there is uncertainty about all four parameters, shown in Figure 2. Now we have two different possibilities: when λ is low, optimal policy under uncertainty is more cautious than under certainty, since the uncertainty about $\alpha_{t+1}, \beta_{t+1}$, and γ_{t+1} dampens the response, but the uncertainty about δ_{t+1} has no or little effect. As λ increases, the uncertainty about δ_{t+1} starts to affect the response positively, and eventually the response under uncertainty is stronger than

[12]The special case analyzed by Svensson (1997a), when $\lambda = 0$ and δ_{t+1} is non-stochastic, is represented along the vertical axes of the top graphs of Figure 1.

under certainty. For a given λ, whether the initial response is more or less aggressive under uncertainty depends on the relative variances of $\alpha_{t+1}, \beta_{t+1}$, and γ_{t+1} on the one hand and δ_{t+1} on the other. When the degree of adjustment uncertainty is relatively small ($\sigma_\delta^2 = 0.1$) in the top graphs of Figure 2, the response to inflation shocks is larger under uncertainty for $\lambda \geq 0.58$, whereas the response to output shocks is always smaller under uncertainty.[13] When adjustment uncertainty gets relatively more important, however, in the lower part of Figure 2 (where $\sigma_\delta^2 = 0.2$), policy is more likely to be more aggressive under uncertainty; the corresponding cutoff values are now $\lambda \geq 0.66$ for output shocks and $\lambda \geq 0.22$ for inflation shocks .

Since the above results may be counterintuitive at first glance, they may need some further explanation. The model used here differs from that of Brainard (1967) in two respects: it is dynamic rather than static, and it incorporates uncertainty concerning not only the impact effect of policy, but also concerning the dynamic development of the economy. Craine (1979) comes to a similar conclusion, using a dynamic model with one target variable, encompassing the Brainard result as a special case. In the formulation of Holly and Hughes Hallett (1989), let z_t be the target variable, p_t a policy variable, and e_t an exogenous variable, and let them be related by the equation

$$z_t = a_t z_{t-1} + b_t p_t + c_t e_t + \varepsilon_t^z. \tag{16}$$

Using a quadratic objective function, impact uncertainty (concerning b_t) can be shown to lead to less aggressive policy in response to shocks, but adjustment uncertainty (concerning a_t) leads to more aggressive policy. Naturally, since dynamics are necessary to model adjustment uncertainty, a dynamic formulation is crucial for the latter result.

In the Holly and Hughes Hallett setup, it is straightforward to separate impact from adjustment uncertainty, but in the Svensson model, this separation is less clear-cut. The analysis above shows that the Craine (1979) result is valid also in the Svensson setup, and the results of Holly and Hughes Hallet (1989) imply that this does not depend crucially on the assumptions that policy affects one target variable only via the other.

Craine's result can be understood by realizing that adjustment uncertainty implies that shocks hitting the economy eventually lead to fluctuations so large that the discounted sum of the variance of target variables is unbounded. In the two-target setup, where policy affects inflation only via output, this intuition needs to

[13]For these parameter values, this is true for all λ at least up to 50,000.

be modified. Now, 'adjustment uncertainty' leads to potentially large variability in one of the target variables, the inflation rate. As long as $\lambda < +\infty$, the central bank is concerned that inflation might move away from target by itself, that is, that $\delta_{t+\tau} > 1$. If this happens, the bank must adjust the interest rate to move inflation closer to target, which in turn will move output away from target. If $\lambda = 0$, the cost of the extra output variability is zero, so the central bank would gladly adjust output to keep inflation at bay. If $\lambda > 0$, however, the extra adjustment of the interest rate and output if inflation moves away is costly. Since its loss function is quadratic, the central bank does not want to take the bet that inflation stays under control, so instead the optimal policy is to move more aggressively in response to any shock, to minimize the expected cost of future adjustment.[14] Consequently, when the central bank is uncertain about the workings of the economy, it may be optimal to respond more aggressively to shocks, so as to avoid bad outcomes in the future.[15]

3.2 The time path of policy

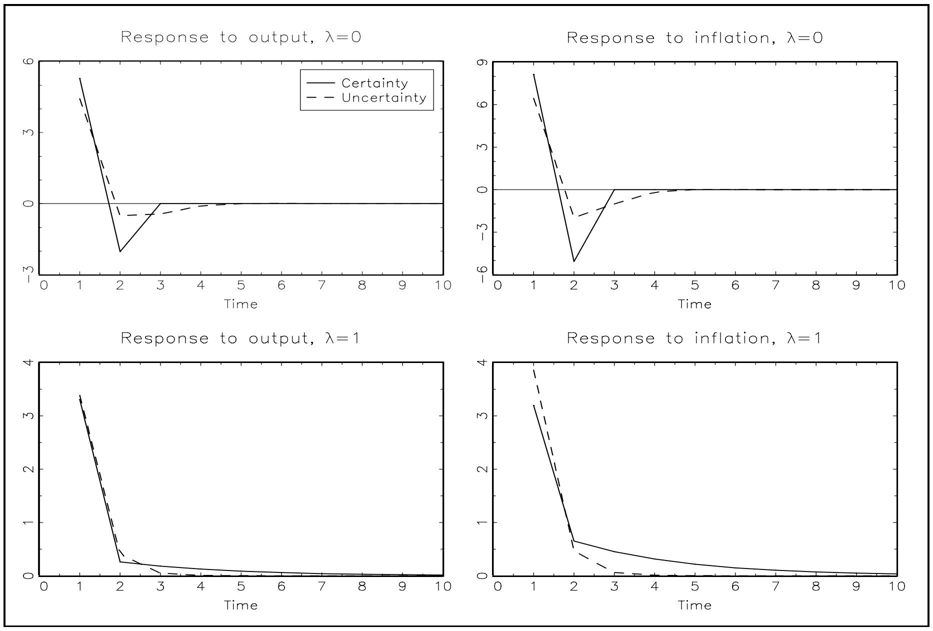
The introduction of multiplicative parameter uncertainty also has interesting implications for the dynamic response of monetary policy, that is, the response of policy to past shocks to output and inflation. Figures 3 and 4 show the response of monetary policy to supply and demand shocks over the first ten periods following a shock, for $\lambda = 0$ and $\lambda = 1$. Figure 3 illustrates the case where there is both impact and adjustment uncertainty, with $\sigma_\alpha^2 = \sigma_\beta^2 = \sigma_\gamma^2 = 0.01$, and $\sigma_\delta^2 = 0.2$, and Figure 4 illustrates the case of impact uncertainty only, with $\sigma_\alpha^2 = \sigma_\beta^2 = \sigma_\gamma^2 = 0.05$, and $\sigma_\delta^2 = 0$.

As noted by Ellingsen and Söderström (1999), in the simple Svensson model under certainty equivalence, the response of monetary policy over time varies substantially with the preference parameter λ. In particular, for small values of λ, the optimal policy response to an inflationary shock under certain parameter configurations is to raise the interest rate instrument in the first period, but then lower it below the initial level and move gradually back to neutral policy. This is shown by

[14] It should be noted that the qualitative effects of uncertainty concerning δ_{t+1} do not hinge on its mean value being equal to unity. For smaller values of the mean, uncertainty still makes policy more aggressive, although quantitatively the effects get smaller as the probability of a realization above unity gets small.

[15] Onetski and Stock (1998) and Sargent (1998a) use robust control theory, where the policymaker chooses policy to minimize the risk of bad outcomes under model uncertainty, to show that particular configurations of uncertainty lead to more aggressive policy than under certainty equivalence. Intuitively, 'cautious' policy can also mean that bad future outcomes are avoided by acting more aggressively today.

Figure 3: Policy response over time, combination of impact and adjustment uncertaint

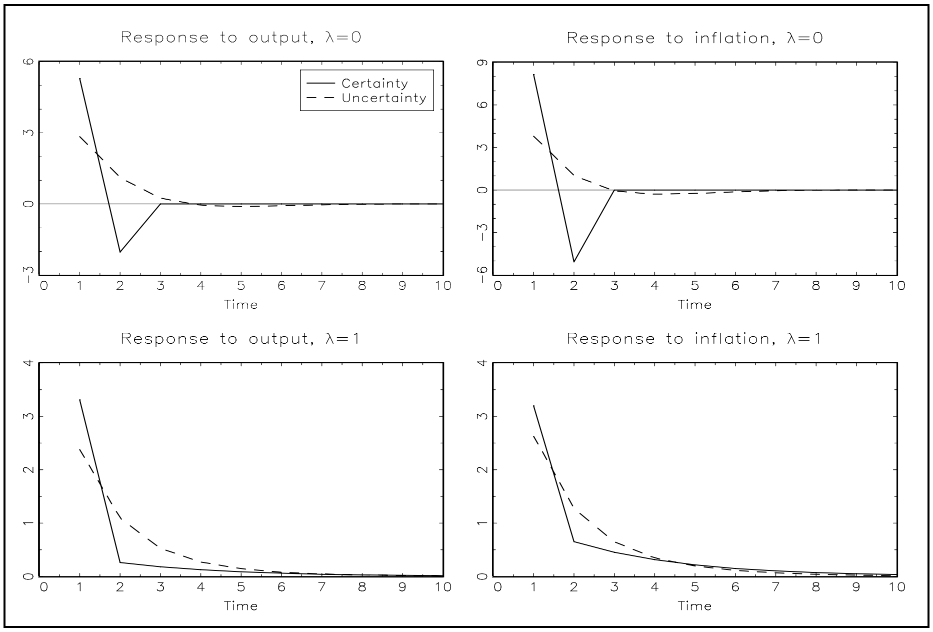


the solid lines in the top two graphs of Figures 3 and 4.

When parameters are uncertain, this behavior can be mitigated or magnified, depending on whether the initial response is dampened or strengthened. When, as in the bottom graphs of Figure 3, uncertainty about δ_{t+1} dominates (since $\lambda = 1$), so that the initial policy response is more aggressive under uncertainty, policy in later periods is closer to neutral, since the strong initial move has neutralized a larger part of the shock. If, on the other hand, uncertainty about $\alpha_{t+1}, \beta_{t+1}$, and γ_{t+1} dominates, as in Figure 4, so that the policy response is initially dampened, policy stays away from neutral longer, to compensate for the weaker initial response.

Thus, as is clear from Figure 4, parameter uncertainty can lead to smoother paths of the interest rate than under certainty equivalence, without introducing an explicit smoothing objective into the central bank's loss function. Casual observation suggests that central banks tend to respond to shocks by first slowly moving the interest rate in one direction, and then gradually moving back to a more neutral stance. When parameters are certain, the model suggests a large initial move, and then a quick return to the original level, unless λ is very large. Under certain configurations of parameter uncertainty, however, the central bank behaves in a more gradual way: although the initial response is always the largest, it is more modest under these cases of uncertainty, and the policy move is drawn out longer over time. In particular, the tendency of the bank to 'whipsaw' the market by

Figure 4: Policy response over time, impact uncertainty



creating large swings in the interest rate is mitigated.[16]

4 Concluding remarks

The purpose of this paper has been to illustrate how uncertainty about parameters in a dynamic macroeconomic model can lead the central bank to pursue *more* aggressive monetary policy, providing a counterexample to the results of Brainard (1967). When a policymaker is uncertain about the adjustment dynamics of the economy—in the context of this paper, the persistence parameter of the inflation process—he might find it optimal to move more aggressively in response to shocks, so as to avoid bad outcomes in the future. Uncertainty about the impact effect of policy still leads to less aggressive policy, in accordance with Brainard's original analysis.

It should be stressed that the model and the examples used are highly stylized and may not be entirely satisfactory from an empirical point of view, so any serious implications for policy are difficult to estimate. However, the qualitative points obtained from this simple model are also present in the more general empirical framework of Rudebusch and Svensson (1998), and are likely to remain also in models incorporating forward-looking behavior.

[16]This issue of parameter uncertainty leading to more plausible paths of policy is examined more carefully by Sack (1998a) and Söderström (1999). The latter shows, however, that the Svensson model always implies excessive volatility of the policy instrument, whereas optimal policy from an unrestricted VAR model comes very close to mimicking the actual behavior of the Federal Reserve.

It is possible that configurations of uncertainty in the real world are such that the Brainard result is always valid, or to quote Blinder (1998, p. 12), "My intuition tells me that this finding is more general—or at least more wise—in the real world than the mathematics will support." Using the standard errors of econometric parameter estimates as proxies for the degree of uncertainty concerning each parameter in a more complete econometric formulation of the Svensson model, Söderström (1999) shows that in the resulting configuration of variances, uncertainty about $\alpha_{t+1}, \beta_{t+1}$, and γ_{t+1} dominates uncertainty about δ_{t+1}, so parameter uncertainty does act to dampen policy. Also, Rudebusch (1998) argues that multiplicative parameter uncertainty is not a very important source of cautious behavior of the Federal Reserve. Nevertheless, the main point in this paper is that the effects on policy of parameter uncertainty may be less clear-cut than previously recognized. Determining the relevance of this result for actual policy should be an interesting topic for future research.

A Solving the control problem

First, the state vector x_{t+1} has expected value

$$E_t x_{t+1} = Ax_t + Bi_t, \tag{17}$$

and covariance matrix

$$\Sigma_{t+1|t} = \begin{bmatrix} \Sigma^y_{t+1|t} & \Sigma^{y,\pi}_{t+1|t} \\ \Sigma^{\pi,y}_{t+1|t} & \Sigma^{\pi}_{t+1|t} \end{bmatrix}, \tag{18}$$

evaluated at t. Since all parameters are assumed independent, the off-diagonal elements of $\Sigma_{t+1|t}$ are zero. The diagonal elements are

$$\begin{aligned} \Sigma^y_{t+1|t} &= \mathrm{Var}_t[\alpha_{t+1} y_t - \beta_{t+1}(i_t - \pi_t) + \varepsilon^y_{t+1}] \\ &= x_t' \Sigma^{11}_A x_t + 2x_t' \Sigma^{11}_{AB} i_t + i_t' \Sigma^{11}_B i_t + \Sigma^{11}_\varepsilon, \end{aligned} \tag{19}$$

and

$$\begin{aligned} \Sigma^\pi_{t+1|t} &= \mathrm{Var}_t[\delta_{t+1}\pi_t + \gamma_{t+1} y_t + \varepsilon^\pi_{t+1}] \\ &= x_t' \Sigma^{22}_A x_t + \Sigma^{22}_\varepsilon, \end{aligned} \tag{20}$$

where Σ^{ij}_{AB} is the covariance matrix of the ith row of A_{t+1} with the jth row of B_{t+1}, that is,

$$\Sigma^{11}_A = \begin{bmatrix} \sigma^2_\alpha & 0 \\ 0 & \sigma^2_\beta \end{bmatrix}, \quad \Sigma^{22}_A = \begin{bmatrix} \sigma^2_\gamma & 0 \\ 0 & \sigma^2_\delta \end{bmatrix}, \tag{21}$$

$$\Sigma^{11}_B = \sigma^2_\beta, \quad \Sigma^{11}_{AB} = \begin{bmatrix} 0 \\ -\sigma^2_\beta \end{bmatrix}, \tag{22}$$

and

$$\Sigma^{11}_\varepsilon = \sigma^2_y, \quad \Sigma^{22}_\varepsilon = \sigma^2_\pi. \tag{23}$$

The extra term to take into account in equation (12) is then

$$\begin{aligned} \mathrm{tr}(V\Sigma_{t+1|t}) &= v_{11}\left(x_t' \Sigma^{11}_A x_t + 2x_t' \Sigma^{11}_{AB} i_t + i_t' \Sigma^{11}_B i_t + \Sigma^{11}_\varepsilon\right) \\ &+ v_{22}\left(x_t' \Sigma^{22}_A x_t + \Sigma^{22}_\varepsilon\right), \end{aligned} \tag{24}$$

where v_{11} and v_{22} are the diagonal elements of the matrix V.

Using equations (10), (12), and (17) in the control problem (9), we can express the Bellman equation as

$$x_t'Vx_t + w$$
$$= \min_{i_t} \left\{ x_t'Qx_t + \phi\,(Ax_t + Bi_t)'\,V\,(Ax_t + Bi_t) + \phi \mathrm{tr}(V\Sigma_{t+1|t}) + \phi w \right\}, \tag{25}$$

which gives the necessary first-order condition as[17]

$$\phi \left[B'(V+V')Ax_t + B'(V+V')Bi_t + \frac{d\mathrm{tr}(V\Sigma_{t+1|t})}{di_t} \right] = 0, \tag{26}$$

where, from (24),

$$\frac{d\mathrm{tr}(V\Sigma_{t+1|t})}{di_t} = 2v_{11}\left(\Sigma_{AB}^{11\,\prime} x_t + \Sigma_B^{11} i_t \right). \tag{27}$$

Thus we get the optimal policy rule

$$\begin{aligned} i_t &= -\left[B'\,(V+V')\,B + 2v_{11}\Sigma_B^{11} \right]^{-1} \left[B'\,(V+V')\,A + 2v_{11}\Sigma_{AB}^{11\,\prime} \right] x_t \\ &= f x_t. \end{aligned} \tag{28}$$

Finally, using equation (24) and the policy rule (28) in the Bellman equation (25) gives

$$\begin{aligned} x_t'Vx_t + w &= x_t'Qx_t + \phi\left[(Ax_t + Bfx_t)'V(Ax_t + Bfx_t) + w\right] \\ &+ \phi v_{11}\left(x_t'\Sigma_A^{11}x_t + 2x_t'\Sigma_{AB}^{11}fx_t + x_t'f_t'\Sigma_B^{11}fx_t + \Sigma_\varepsilon^{11} \right) \\ &+ \phi v_{22}\left(x_t'\Sigma_A^{22}x_t + \Sigma_\varepsilon^{22} \right) \\ &= x_t' \left[\begin{array}{l} Q + \phi(A+Bf)'V(A+Bf) \\ +\phi v_{11}\left(\Sigma_A^{11} + 2\Sigma_{AB}^{11}f + f_t'\Sigma_B^{11}f\right) + \phi v_{22}\Sigma_A^{22} \end{array} \right] x_t \\ &+ \phi\left[w + v_{11}\Sigma_\varepsilon^{11} + v_{22}\Sigma_\varepsilon^{22} \right], \end{aligned} \tag{29}$$

so the matrix V is determined by

$$\begin{aligned} V &= Q + \phi(A+Bf)'V(A+Bf) \\ &+ \phi v_{11}\left(\Sigma_A^{11} + 2\Sigma_{AB}^{11}f + f'\Sigma_B^{11}f \right) + \phi v_{22}\Sigma_A^{22}. \end{aligned} \tag{30}$$

See also Chow (1975).

[17]Use the rules $\partial x'Ax/\partial x = (A + A')x$, $\partial y'Bz/\partial y = Bz$, and $\partial y'Bz/\partial z = B'y$, see, e.g., Ljungqvist and Sargent (1997). Note also that V is not necessarily symmetric in this setup with multiplicative uncertainty.

References

Ball, Laurence, "Efficient rules for monetary policy," Working Paper No. 5952, National Bureau of Economic Research, March 1997.

Balvers, Ronald J. and Thomas F. Cosimano, "Inflation variability and gradualist monetary policy," *Review of Economic Studies* 61 (4), October 1994, 721–738.

Başar, Tamer and Mark Salmon, "Credibility and the value of information transmission in a model of monetary policy and inflation," *Journal of Economic Dynamics and Control* 14 (1), February 1990, 97–116.

Bertocchi, Graziella and Michael Spagat, "Learning, experimentation, and monetary policy," *Journal of Monetary Economics* 32 (1), August 1993, 169–183.

Blinder, Alan S., "What central bankers could learn from academics—and vice versa," *Journal of Economic Perspectives* 11 (2), Spring 1997, 3–19.

———, *Central banking in theory and practice*, The MIT Press, Cambridge, Mass., 1998.

Brainard, William, "Uncertainty and the effectiveness of policy," *American Economic Review* 57 (2), May 1967, 411–425.

Cecchetti, Stephen G., "Policy rules and targets: Framing the central banker's problem," Federal Reserve Bank of New York *Economic Policy Review* 4 (2), June 1998.

Chow, Gregory C., *Analysis and control of dynamic economic systems*, John Wiley & Sons, New York, 1975.

Cooley, Thomas F. and Gary D. Hansen, "Money and the business cycle," in Cooley, Thomas F. (ed.), *Frontiers of business cycle research*, Princeton University Press, 1995.

Craine, Roger, "Optimal monetary policy with uncertainty," *Journal of Economic Dynamics and Control* 1 (1), February 1979, 59–83.

Ellingsen, Tore and Ulf Söderström, "Monetary policy and market interest rates," Working Paper No. 56, Sveriges Riksbank, August 1998.

Estrella, Arturo and Frederic Mishkin, "Rethinking the role of NAIRU in monetary policy: Implications of model formulation and uncertainty," Working Paper No. 6518, National Bureau of Economic Research, April 1998.

Fischer, Stanley, "Why are central banks pursuing long-run price stability?," in *Achieving price stability*, Federal Reserve Bank of Kansas City, 1996.

Fuhrer, Jeffrey C., "Optimal monetary policy and the sacrifice ratio," in Fuhrer, Jeffrey C. (ed.), *Goals, guidelines, and constraints facing monetary policymakers*, Federal Reserve Bank of Boston, 1994.

Goodhart, Charles A. E., "Central bankers and uncertainty," mimeo, London School of Economics, 1998.

Holly, Sean and Andrew Hughes Hallett, *Optimal control, expectations and uncertainty*, Cambridge University Press, 1989.

Ljungqvist, Lars and Thomas J. Sargent, *Recursive macroeconomics*, manuscript, November 1997.

Onatski, Alexei and James H. Stock, "Robust monetary policy under model uncertainty in a small model of the U.S. economy," mimeo, Harvard University, December 1998.

Poole, William, "A policymaker confronts uncertainty," Federal Reserve Bank of St. Louis *Review* 80 (5), September/October 1998, 3–8.

Romer, Christina D., "Inflation and the growth of output," Working Paper No. 5575, National Bureau of Economic Research, May 1996.

Rudebusch, Glenn D., "Is the Fed too timid? Monetary policy in an uncertain world," mimeo, Federal Reserve Bank of San Francisco, October, 1998.

Rudebusch, Glenn D. and Lars E. O. Svensson, "Policy rules for inflation targeting," Working Paper No. 6512, National Bureau of Economic Research, April 1998.

Sack, Brian, "Does the Fed act gradually? A VAR analysis," Finance and Economics Discussion Paper No. 1998-17, Board of Governors of the Federal Reserve System, March 1998(a).

———, "Uncertainty, learning, and gradual monetary policy," Finance and Economics Discussion Paper No. 1998-34, Board of Governors of the Federal Reserve System, August 1998(b).

Sargent, Thomas J., "Discussion of 'Policy rules for open economies' by Laurence Ball," mimeo, University of Chicago, April 1998(a).

———, "Central banking in theory and practice: A book review," mimeo, Stanford University, August 1998(b).

Söderström, Ulf, "Should central banks be more aggressive?," Working Paper, Sveriges Riksbank, May 1999.

Svensson, Lars E. O., "Inflation targeting: Some extensions," Working Paper No. 5962, National Bureau of Economic Research, March 1997(a).

———, "Inflation forecast targeting: Implementing and monitoring inflation targets," *European Economic Review* 41 (6), June 1997(b), 1111–1146.

———, "Inflation targeting in an open economy: Strict or flexible inflation targeting?," *Victoria Economic Commentaries* 15 (1), March 1998.

Taylor, John B., "Discretion versus policy rules in practice," *Carnegie-Rochester Conference Series on Public Policy* 39, December 1993, 195–214.

———, "The inflation/output variability trade-off revisited," in Fuhrer, Jeffrey C. (ed.), *Goals, guidelines, and constraints facing monetary policymakers*, Federal Reserve Bank of Boston, 1994.

Wieland, Volker, "Monetary policy and uncertainty about the natural unemployment rate," Finance and Economics Discussion Paper No. 1998-22, Board of Governors of the Federal Reserve System, April 1998.