Are There Price Bubbles in the Swedish Equity Market?*

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Abstract

Analyzing the behavior of Swedish equity prices and their fundamental values over almost a century of data, it seems as if the the actual price follows the computed fundamental price quite well. However there are periods characterized by substantial deviations from the fundamental price, indicating the presence of bubbles in equity prices.

Keywords: price bubbles, stock market crashes, regime shifts.
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1 Introduction

In early 1997 prices on the New York Stock Exchange and some other exchanges had risen for several years without a major setback. This inevitably led some observers to reminisce about the 1987 stock market crash. Testifying before the Senate Banking Committee on February 26, the Chairman of the Federal Reserve Board Alan Greenspan stated

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> "It's not markets that are irrational. It's people who become irrationally exuberant on occasion and take actions that induce what economists like to call bubbles, which eventually burst."
> Norris (1997)

It is unusual for a central banker to express an opinion about the valuation of stocks and the chairman's statement was seen as rather controversial by many market participants and other commentators.

The existence or non-existence of price bubbles has also caused some controversy in the academic literature. The existence of irrational bubbles seems to be getting less support, while there are still some advocates of the existence of bubbles in a rational expectations framework. Deviations of asset prices from fundamental values can also be explained in models of imperfect information aggregation. See Kleidon (1995) for a review of these and other models that could potentially explain a stock market crash. In this paper we will look at the possible existence of rational price bubbles in the Swedish stock market, using almost a century of data.

The paper is organized as follows. In the next section we discuss the theory of rational bubbles quite generally. This is followed by a presentation of a specific rational expectations model with price bubbles in section 3. The data used are presented in section 4. The model is tested on both monthly and annual data in section 5. Section 6 concludes.

2 Theory on Rational Bubbles

In this section we give a brief introduction to the theory of stock prices and discuss the literature on rational bubbles. A more specific derivation linking the theory and the empirical model used in this paper is given in section 3.

The fundamental value, P_t^*, of a stock can in general be defined as the expected value of the sum of the discounted future dividends,

$$P_t^* = \lim_{j \to \infty} E\left[\sum_{j=1}^{J}\left(\prod_{k=1}^{J} m_{t+k}\right) D_{t+j} \mid \Omega_t\right], \tag{1}$$

where D_t is the dividend in period t, E is the expectations operator, m_t is the (possibly stochastic) discount factor at time t and Ω_t is the information set available to the investor at time t. To assure that only discounted dividends contribute to P_t^* the following transversality condition must hold,

$$\lim_{j \to \infty} \prod_{j=1}^{J} m_{t+j} P_{t+J}^* = 0. \tag{2}$$

If stock prices are determined in a rational expectations equilibrium, the price of the stock is the discounted value of the price plus dividend in the next period,

$$P_t = E\left[m_{t+1}\left(P_{t+1} + D_{t+1}\right) \mid \Omega_t\right]. \tag{3}$$

Solving equation (3) recursively gives an infinite number of possible solutions of the form

$$P_t = P_t^* + B_t, \tag{4}$$

i.e., the price of the stock in time t is determined by the fundamental value, P_t^*, defined above and, possibly, a rational bubble,

$$B_t = E\left[\prod_{j=1}^{J \to \infty} m_{t+1} P_{t+J} \mid \Omega_t\right]. \tag{5}$$

For the bubble to be a viable outcome it must be expected to continue to expand in the next period,

$$B_t = E\left[m_{t+1} B_{t+1} \mid \Omega_t\right]. \tag{6}$$

In the literature there are numerous specifications of the bubble process. We will discuss some of the more cited work here. For simplicity assume that the discount factor is a constant, $m_t = 1/(1+R)$. The fundamental value in this case only depends on expected future dividends and the rational bubble satisfies,

$$B_t = E\left[\frac{B_{t+1}}{1+R} \mid \Omega_t\right]. \tag{7}$$

The simplest possible bubble process would be a deterministic one, for example,

$$B_t = \left(\frac{1}{1+R}\right)^t B_0, \tag{8}$$

where $B_0 > 0$ is a constant. This type of deviation from fundamentals does not appear to be likely since it would imply that the stock price would diverge explosively forever from B_0. A more realistic model would allow for a stochastic bubble process. This was first proposed by Blanchard and Watson (1982) who assume two states of nature, one where the bubble survives and one where it collapses. More explicitly consider the following process as an example,

$$B_{t+1} = \left(\frac{1+R}{q}\right) B_t + \varepsilon_{t+1}, \qquad \text{with probability } q, \tag{9a}$$

$$B_{t+1} = \varepsilon_{t+1}, \qquad \text{with probability } 1-q, \tag{9b}$$

where $E(\varepsilon_t) = 0$. This model introduces a constant probability, 1-q, that a bubble will burst in any one period. If the bubble bursts, the stock price returns to its intrinsic value. However the shock term ensures that a bubble can regenerate itself and thus deviations from fundamentals can reoccur after a crash. Observing how markets behave in real life one might find it somewhat unsatisfactory to have a constant probability of a crash in each period. It seems plausible that the probability of a crash is higher during certain periods, for example if prices appear to be very far from a value based on dividends or reported earnings. Indeed, Blanchard and Watson (1982) suggest that the probability q can be made stochastic, depending on the length of time the bubble has survived or the spread between the price and the fundamental value.

Rational investors know that the bubble will burst eventually so why do they stay in the market? Blanchard and Watson (1982) argue that even though they know that the bubble *will* crash they do not know *when*. Hence in each period if investors expect the bubble to continue to grow, they may hold on to their investment to the next period, hoping for even larger capital gains. The growth rate of the bubble will compensate for taking the risk that the bubble will burst. Following Blanchard and Watson (1982) several articles have studied more in depth under what conditions bubbles in asset prices are consistent with rational expectations (see for example Tirole (1982, 1985) and Diba and Grossman (1987, 1988)).

The rational bubbles above are formulated as totally separated from fundamentals. Recently a new strand of the literature considers the link between bubbles and fundamentals. Froot and Obstfeld (1991) introduce what they refer to as intrinsic bubbles. These bubbles are deterministic non-linear functions of the asset's fundamentals and have several appealing features. For instance deviations from fundamentals can persist over long periods of time so that stable economic fundamentals can be associated with persistent, stable mispricing. Also asset prices can overreact to "news" about fundamentals. Further extensions along these lines are made in Ikeda and Shibata (1992). As in Froot and Obstfeldt (1991), Ikeda and Shibata let the bubble be a function of the fundamentals but with the important extension that they allow fundamentals to be stochastic.

As mentioned earlier, in Blanchard and Watson (1982) the probability that the bubble will collapse is exogenously given. van Norden and Schaller (1993) van Norden (1996) follow Blanchard and Watson's suggestion of endogenizing the probability. They do this by letting the probability that the bubble will collapse be a function of the extent of the mispricing. The larger the bubble the greater is the probability that it will collapse. The empirical properties of testing for bubbles in this framework has recently been investigated by van Norden and Vigfusson (1998). We will make use of this type of model in the next section.

3 A Regime-Switching Model of Asset Prices and Bubbles

We use the model developed in van Norden and Schaller (1993) and van Norden (1996). This is basically an extension of the Lucas (1978) asset pricing model to include price bubbles. After having imposed the market clearing condition that all dividends be consumed in the period they are paid out the first-order condition, with Constant Relative Risk Aversion (CRRA) utility, can be restated as the following equilibrium condition:

$$P_t D_t^{\gamma} = \beta E_t \left[D_{t+1}^{\gamma} \left(P_{t+1} + D_{t+1} \right) \right], \tag{10}$$

where P_t and D_t are the real price and real dividend of the stock respectively, γ is the coefficient of relative risk aversion, and β is the discount factor. From this difference equation we can derive the fundamental price,

$$P_t^* = D_t^{-\gamma} \sum_{s=1}^{\infty} \beta^s E_t D_{t+s}^{1+\gamma}. \tag{11}$$

Next we assume a dividend process of the form

$$D_{t+1} = D_t e^{\alpha + \varepsilon_{t+1}}, \tag{12}$$

where $\alpha > 0$ and $\varepsilon_t \sim N(0, \sigma^2)$. Thus, dividends follow a random walk with drift α and standard deviation σ. This model can be shown to have the solution

$$P_t^* = \rho D_t, \tag{13}$$

where

$$\rho = \frac{\beta e^{\alpha(1+\gamma)+(1+\gamma)^2\sigma^2/2}}{1 - \beta e^{\alpha(1+\gamma)+(1+\gamma)^2\sigma^2/2}}. \tag{14}$$

Hence, we get a very convenient solution where the fundamental price is a linear function of todays dividend. This is the result of assuming CRRA and a simple dividend process of the type in (12).

As stated in the previous section, a price bubble is defined as the deviation from fundamental price,

$$B_t = P_t - P_t^*. \tag{15}$$

Since both prices on the right-hand side must fulfil (10), so does the price bubble. There are two states of nature, one in which the bubble survives to the next period and one in which it (partially) collapses. We will refer to these as states S and C respectively. The probability of a bubble surviving to the next period is assumed to be a function of the relative size of the bubble, $q(b_t)$. The larger the bubble the smaller the probability that it will survive, $q' < 0$.[1] The expected value of the bubble conditional upon a collapse is assumed to be

$$E\left[B_{t+1} \mid C\right] = u\left(b_t\right) P_t, \tag{16}$$

where $u(0) = 0$ and $0 < u' < 1$. The expected value of the bubble conditional upon survival can then be shown to be

$$E\left[B_{t+1} \mid S\right] = \frac{1}{q\left(b_t\right)} \left[\beta^{-1} e^{\gamma(\alpha+\gamma\sigma^2/2)} B_t - \left(1 - q\left(b_t\right)\right) u\left(b_t\right) P_t\right]. \tag{17}$$

The model is linearized to yield the following three-equation empirical model for the return $R_t = (P_t + D_t)/P_{t-1}$,[2]

$$(R_{t+1} \mid S) = \beta_{S0} + \beta_{Sb} b_t + \eta_{St}, \qquad \eta_{St} \sim N(0, \sigma_S^2), \tag{18a}$$

$$(R_{t+1} \mid C) = \beta_{C0} + \beta_{Cb} b_t + \eta_{Ct}, \qquad \eta_{Ct} \sim N(0, \sigma_C^2), \tag{18b}$$

[1] We use Lagrange's notation in letting a prime denote the first derivative.

[2] For a complete derivation of the model see vanNorden and Schaller (1993).

$$q_t = \Phi(\beta_{q0} + \beta_{qb} |b_t|), \tag{18c}$$

where Φ is the standard normal cumulative density function. The functional form for q has been chosen to ensure that $0 < q < 1$.

4 The Data

Dividend payments in Sweden are heavily concentrated to the three months of March, April, and May. This detracts somewhat from the value of using a monthly time series. Nevertheless, we use a monthly index of Swedish stock returns constructed by Frennberg and Hansson (1992a, 1992b) and later updated.[3] However, we also conduct the analysis using annual data as a robustness check on our results.

The Frennberg-Hansson data set includes a value-weighted index that includes dividends and one that excludes dividends. It also includes a dividend series. In this study we use the dividend series and the index with dividends excluded. The standard procedure of using the sum of the last twelve months' dividends was used to handle the seasonality problem. In Frennberg and Hansson (1992b) an annual dividend index is reported. With the help of this index we could compute the dividend for the base year 1918 (which was not reported in Frennberg and Hansson (1992a)) and update the dividend series to June 1998. The price indices are constructed along the same principles as in the standard work by Ibbotson and Sinquefield (1989) and span the period from January 1919 to June 1998. An index of consumer prices, which is also part of the Frennberg-Hansson data set, was used to compute real returns.

5 Empirical Results

In order to test the model we need a measure of the fundamental price. According to (13) the fundamental price is a linear function of this years dividend, $P_t^* = \rho D_t$, where ρ is given in (14). From that expression we see that to determine a value for ρ we need values for each of the parameters α, σ, β, and γ. The first two we can obtain by estimating them from the (log of) the dividend process in (12) by simple OLS. The discount factor β would

[3] We are grateful to Björn Hansson for providing us with the updated data set.

usually be assumed to lie between 0.99 and 1.0. There is no consensus on the magnitude of the relative risk aversion coefficient γ, but most economists seem to consider the value of around 10 typically arrived at in CCAPM studies to be too high. Rather than guessing a value for γ (and β), we use the average price-dividend ratio for the whole sample as a proxy for ρ and ask whether this leads to plausible combinations of values for the other two parameters for which we have no readily available information: γ and β. These combinations of values for γ and β are plotted in Figure 1. Given that we think $\beta > 0.995$ is reasonable we end up with a relative risk aversion coefficient between 0 and 8.5. These values do not seem unreasonable to us. Hence, we use the average price dividend ratio for the whole sample period as a proxy for ρ and derive a fundamental price according to (13). The fundamental price series, which is simply the level-adjusted dividend series, is plotted together with the actual price series in Figure 2. The deflated series, which are used to compute real returns, are shown in Figure 3.

Three distinct subperiods can be distinguished in Figure 2. Up until a few years after the second world war the actual price consistently lies below the fundamental price. In the post-war period up until the early 1980's the actual price fluctuates around the fundamental. In the 1980's and 90's the actual price lies above the fundamental price. There have been three partial collapses of the price bubble during this latter period. The third of these brought the price back to its fundamental value. However, in 1992-93 the two price series drastically part company when the fundamental price drops significantly while the actual price rises. Profits were depressed after the severe recession in 1989-91, but equity prices started rising soon after Sweden abandoned the fixed exchange rate in November 1992 in expectation of higher future profits. It does not seem likely that the stock market has been undervalued from 1919 to 1945. It is more likely that we have overestimated the fundamental price for the earlier part of the sample. The wide gap at the end of the sample could likewise be due to the fundamental price being underestimated. A model that predicts systematic mispricing of over 50 percent for several years at a time does not seem credible. From Figure 2 it is clear that the period up until 1945 is charactereized by a lower dividend growth rate (change in the logarithm of the fundamental price) than the rest of the period studied, especially the latter part of the sample. This could explain the over- and underestimation respectively of the fundamental price, since the price-dividend ratio is a function of the dividend growth rate (cf. expression (14)).

The model we have been looking at so far assumes that the price-dividend ratio (and the dividend growt rate) is constant. As we have just noted this assumption could be called into question. This is even more evident from Figure 4, especially considering the latter part of the sample. It is possible to distinguish between three periods with different price-dividend ratios. Up until 1945 the price-dividend ratio lies between 150 and 300. After that we observe three peaks above 400 while the troughs during the post-war period do not fall below 200, which they did before 1945. In early 1983 there is a dramatic increase in the price-dividend ratio, which now also varies considerably over time. The average monthly dividend growth rates during the three periods identified above are $\alpha_{1920:1-1945:12} = -0.0016$, $\alpha_{1946:1-1982:12} = 0.0009$, and $\alpha_{1983:1-1998:7} = 0.0084$. In Figure 5 we can see the stable period 1946:1-1982:12 surrounded by the more volatile periods, with a markedly higher dividend growth rate in the latter period. We note the extremely large shocks to the dividend process following the time when the Swedish krona was allowed to float in 1992. The high volatility after 1992 together with the higher growth rate of dividends could explain the gap between actual and fundamental prices towards the end of the period studied.

A fundamental price based on different ρ's for the three different periods identified above ($\rho_{1920:1-1945:12} = 235$, $\rho_{1946:1-1982:12} = 302$, and $\rho_{1983:1-1998:7} = 558$) is shown in Figure 6. The actual price tracks the fundamental price quite well in this model with exogenously given price-dividend regime shifts. The magnitudes of the fluctuations around the fundamental are sometimes quite substantial - upwards of 30 percent. This could be an indication that bubbles are a significant feature of our equity price series.

We will next formally estimate and test the rational bubble model presented in Section 3 above.[4] As explained in that section, much of the analysis centers on the relative size of the bubble, $b_t = B_t/P_t$, which is depicted in Figure 7. The empirical model (18) has some testable implications. First, the larger the bubble is the smaller will be the probability that it will survive to the next period, thus the model implies that we should find $\beta_{qb} < 0$. Second, we expect to find $\beta_{Cb} < 0$, since a larger bubble implies a larger capital loss in case of a collapse. Finally, $\beta_{Sb} > \beta_{Cb}$ to compensate investors for the

[4]The GAUSS programs made available by the Bank of Canada on its WWW home page were used to estimate and test the model. See van Norden and Vigfusson (1996) for a guide to these programs.

probability that the bubble will collapse. van Norden (1996) also suggests a test to ascertain whether the switching regime model motivated by the possible existence of bubbles can be reduced to a simple mixture of normals model, where we switch between two normal distributions with a constant probability, i.e. imposing the restriction $\beta_{Sb} = \beta_{Cb} = \beta_{qb} = 0$ on (18). In the presence of bubbles the switching regression should reject this restriction. Since the mixture of normals model is nested by (18), the hypothesis can be tested using an ordinary likelihood ratio test.

Studying the results in Table 1 all coefficients in the regression, except the first two are significant at the one percent level. The coefficient of the absolute bubble has the expected sign, $\beta_{qb} < 0$, giving support to the predicted relation between the relative size of the bubble and being in the surviving state. β_{Cb} has the wrong sign and is significantly different from zero. We find that β_{Sb} is larger than β_{Cb}, as it should be, but this difference is not statistically different from zero according to the Wald test. Finally, the mixture of normals model is strongly rejected by the data. As shown in Table 2, using annual data gives a similar picture to the monthly data. There is some support for the bubble model and we still reject the mixture of normals model at any reasonable significance level.

6 Conclusions

The objective of this paper is to analyse whether Swedish equity prices are characterized by the existence of rational bubbles. Looking at the behavior of equity prices and their fundamental values over almost a century of data, it seems as if the the actual price follows the computed fundamental price rather well. However there are periods characterized by substantial deviations from the fundamental price, indicating the presence of bubbles in Swedish equity prices.

To formally test for the existence of rational bubbles we adapt a switching regime approach suggested by van Norden (1996). The results are somewhat mixed, and some of the theoretical implications fail to find support in the data. Still strong support is found for the conjecture that the size of deviations from the fundamental price matters, the larger the bubble the greater is the probability that it will collapse.

In this paper we stay close to standard specifications in the literature and use dividends as a proxy for fundamentals under the assumption that dividends follow a random walk process. It might be fruitful in future research to explore different specifications of the dividend process.

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Table 1. Estimates of the regime shift model 1920:1-1998:6

Parameters	Estimates	t-value
β_{S0}	0.0135	1.142
β_{Sb}	0.067	1.868
β_{C0}	0.0081	5.619
β_{Cb}	0.0462	4.252
β_{q0}	1.7684	6.111
β_{qb}	-3.4571	-3.623
σ_S	0.0905	7.829
σ_C	0.0365	21.978

Wald	$\beta_{Sb} = \beta_{Cb}$	0.27284
		(0.42716)
LR	$\beta_{Sb} = \beta_{Cb} = \beta_{qb} = 0$	54.256
		(4.6511e-11)

Table 2. Estimates of the regime shift model 1919-1997 (Annual)

Parameters	Estimates	t-value
β_{S0}	0.154	4.531
β_{Sb}	0.6684	5.369
β_{C0}	-0.0293	-0.99
β_{Cb}	0.5803	3.9
β_{q0}	-0.7599	-1.428
β_{qb}	-0.0051	-0.002
σ_S	0.1654	9.523
σ_C	0.0636	2.892

Wald	$\beta_{Sb} = \beta_{Cb}$	0.17938
		(0.6719)
LR	$\beta_{Sb} = \beta_{Cb} = \beta_{qb} = 0$	34.525
		(5.81e-07)

Figure 1. Combinations of beta and gamma for given parameter values:

α=0.001175, σ=0.019678, ρ=330.08

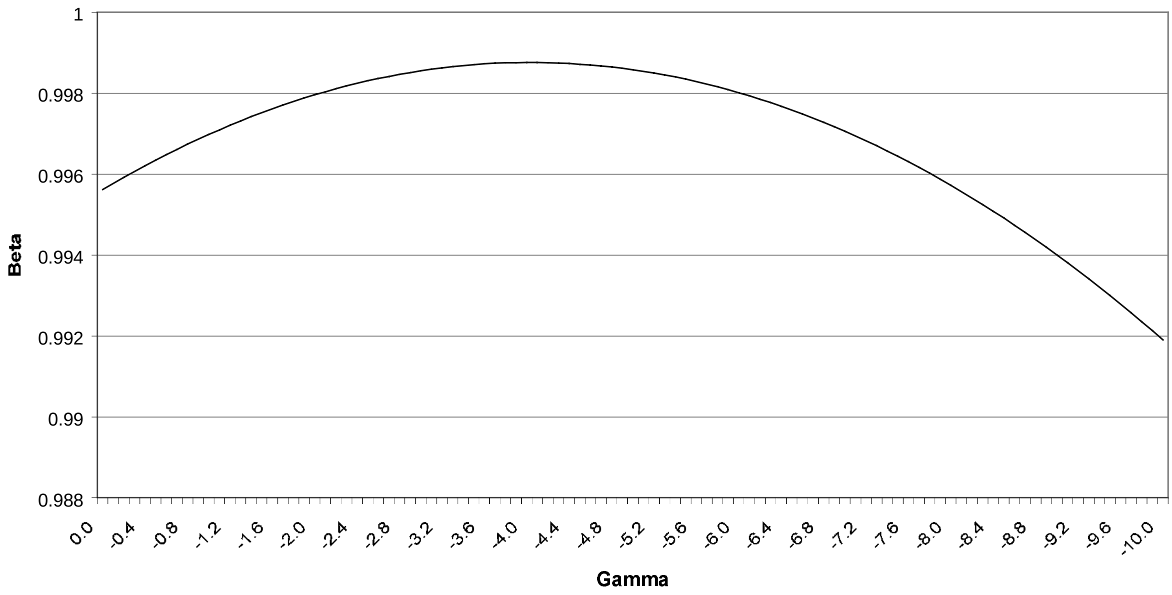

Figure 2. Actual and Fundamental Equity Prices (in logs)

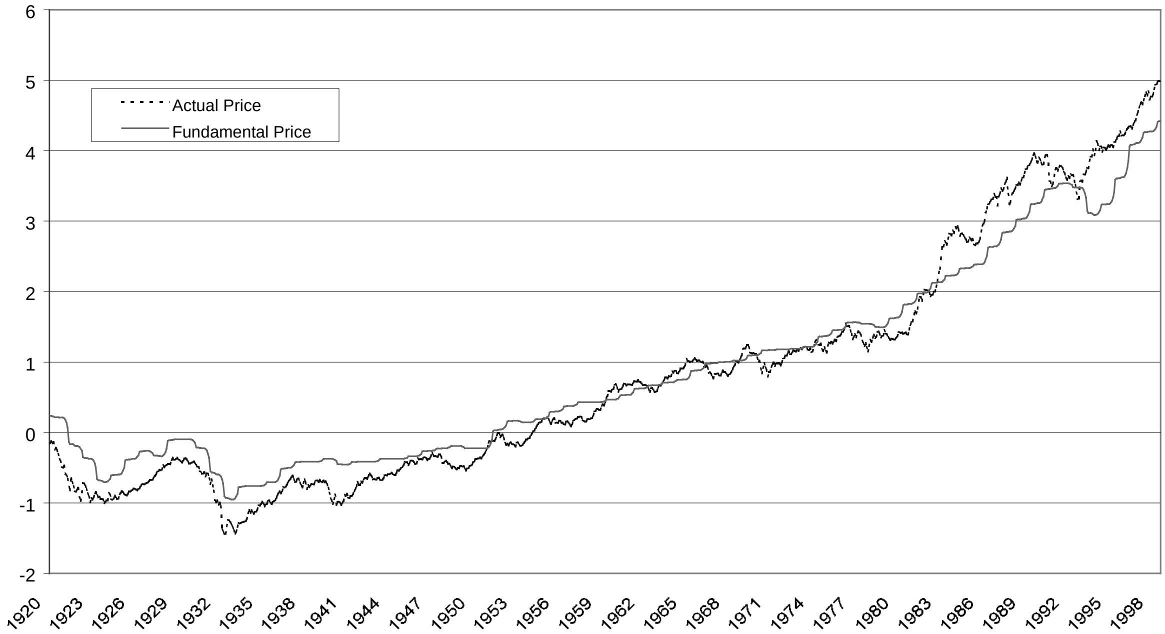

Figure 3. Real Actual and Fundamental Equity Prices (in logs)

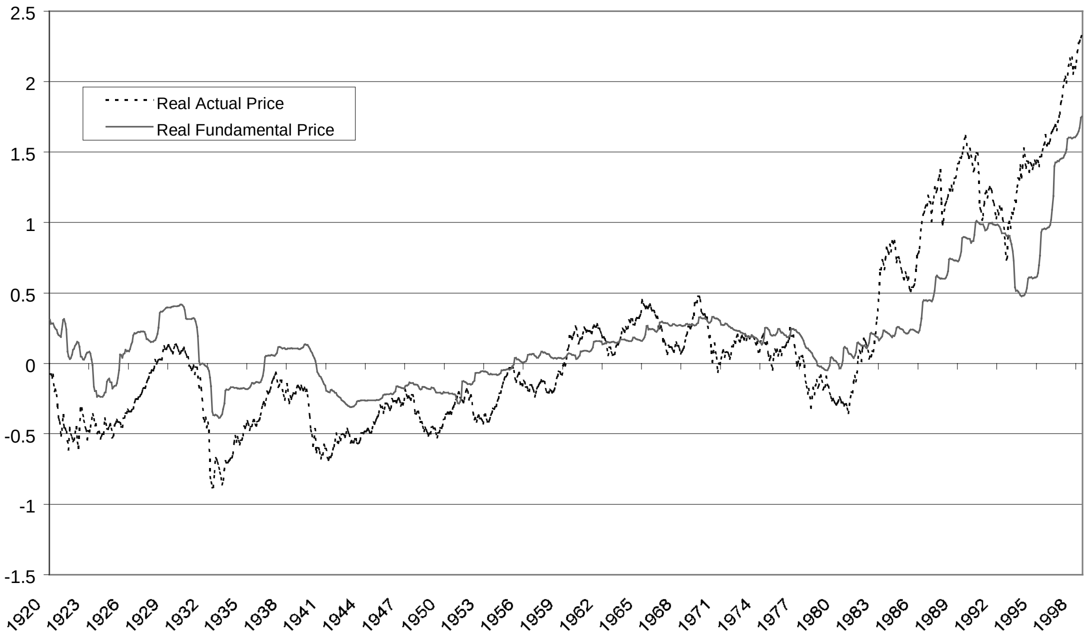

Figure 4. Price-Dividend Ratio

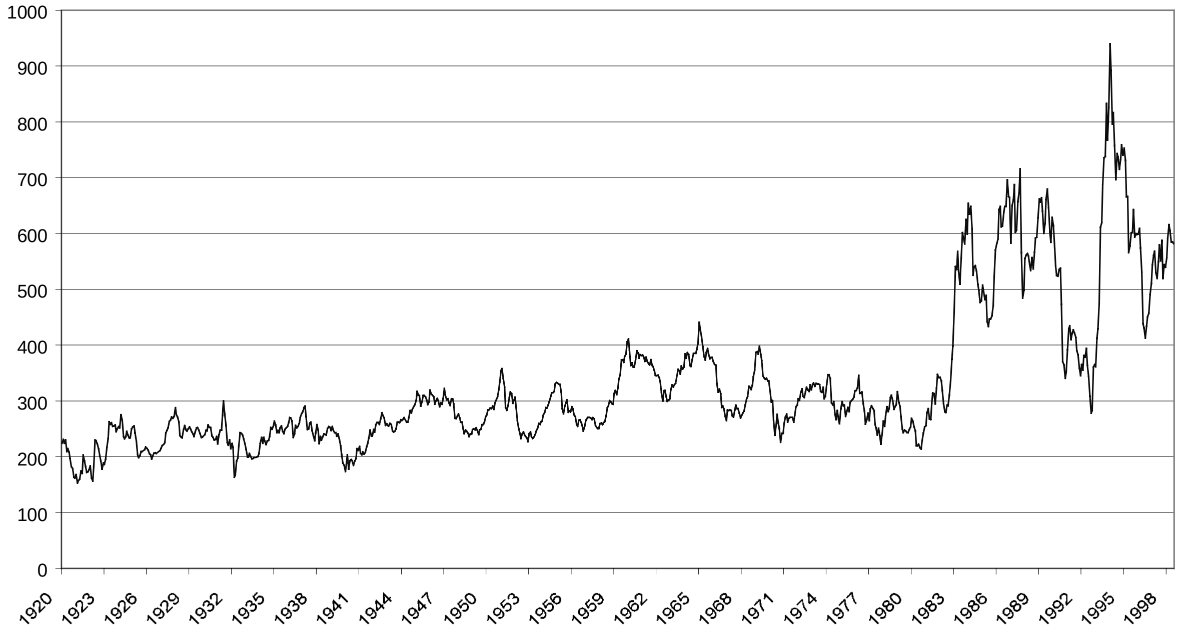

Figure 5. Average Real Dividend Growth Rate (5-Year Moving Average)

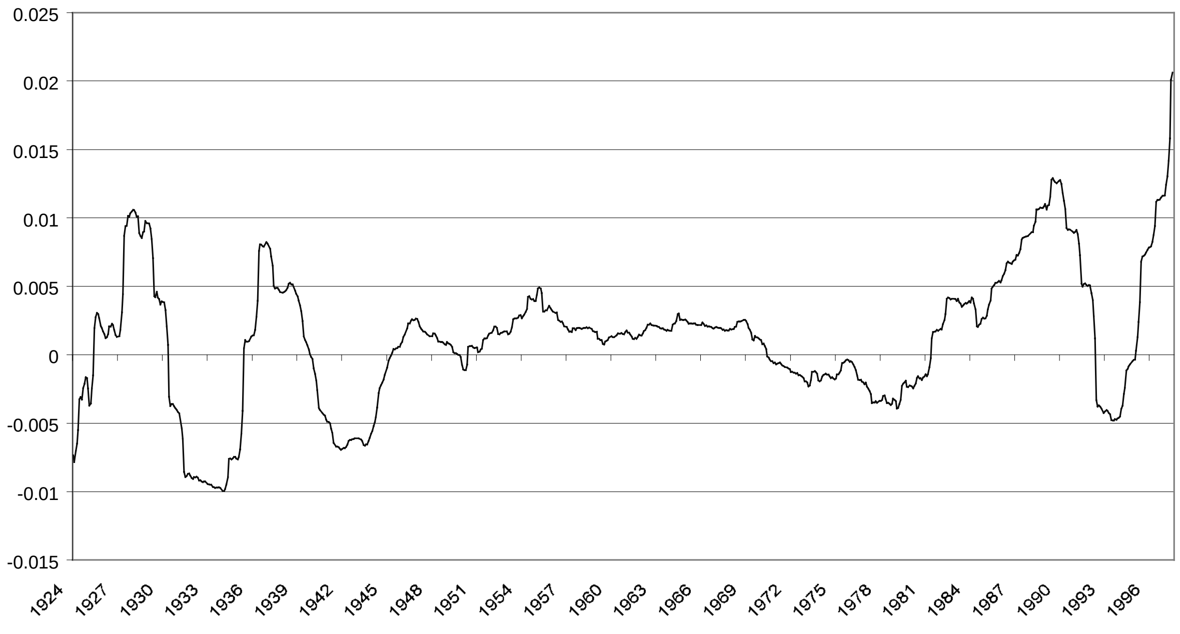

Fig 6. Actual and Fundamental Prices with Different Price-Dividend Regimes: 1920:1-45:12, 1946:1-82:12, and 1983:1-98:6

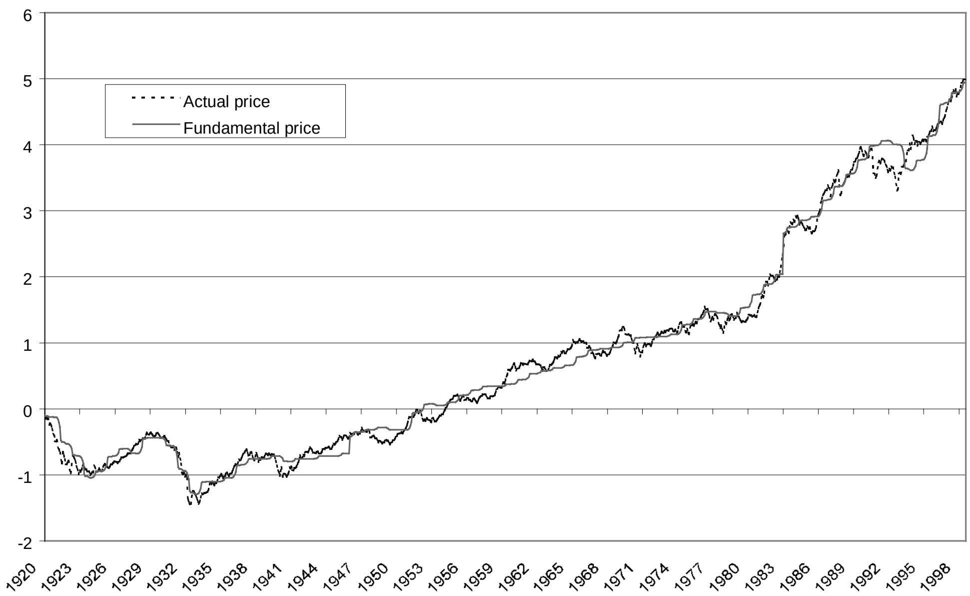

Figure 7. Relative Price Bubble: b=(P-P*)/P