

Optimal Monetary Policy in an Operational Medium-Sized DSGE Model: Technical Appendix*

Malin Adolfson
Sveriges Riksbank

Stefan Laséen
Sveriges Riksbank

Jesper Lindé
Sveriges Riksbank

Lars E.O. Svensson
Sveriges Riksbank

First draft: October 2006
This version: August 2008

Abstract

Technical appendix to “Optimal Monetary Policy in an Operational Medium-Sized DSGE Model”

JEL Classification: E52, E58

Keywords: Optimal monetary policy, forecasts, judgment

This technical appendix of Adolfson, Laséen, Lindé, and Svensson [1] contains the detailed specification of the matrices A , B , C , and H in the model, as well as the detailed specification of the measurement equation and the related matrices \bar{D}_0 , \bar{D} , and \bar{D}_s .

1. Definition of the matrices A , B , C , D , and H and the instrument rule $[f_X \ f_x]$

We want to identify the submatrices in the model,

$$X_{t+1} = A_{11}X_t + A_{12}x_t + B_1i_t + C\varepsilon_{t+1},$$

$$Hx_{t+1|t} = A_{22}x_t + A_{21}X_t + B_2i_t.$$

We specify the predetermined variables $X_t = (X_t^{\text{ex}}, X_t^{\text{pd}})'$, the forward-looking variables x_t , the policy instrument i_t , and the shock vector ε_t as follows (the number of the elements in each vector

*All remaining errors are ours. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of Sveriges Riksbank.

is also listed):

$$X_t^{\text{ex}} \equiv \begin{bmatrix} \hat{\epsilon}_t & 1 & \hat{\tau}_t^y & 21 \\ \hat{\epsilon}_{t-1} & 2 & \hat{\tau}_t^c & 22 \\ \hat{\mu}_{zt} & 3 & \hat{\tau}_t^w & 23 \\ \hat{\mu}_{z,t-1} & 4 & \hat{g}_t & 24 \\ \hat{\nu}_t & 5 & \hat{\tau}_{t-1}^k & 25 \\ \hat{\zeta}_t^c & 6 & \hat{\tau}_{t-1}^y & 26 \\ \hat{\zeta}_t^h & 7 & \hat{\tau}_{t-1}^c & 27 \\ \hat{\zeta}_t^q & 8 & \hat{\tau}_{t-1}^w & 28 \\ \hat{\lambda}_t^f & 9 & \hat{g}_{t-1} & 29 \\ \hat{\lambda}_t^{mc} & 10 & \hat{\pi}_t^* & 30 \\ \hat{\lambda}_t^{mi} & 11 & \hat{y}_t^* & 31 \\ \hat{\phi}_t & 12 & \hat{R}_t^* & 32 \\ \hat{\Upsilon}_t & 13 & \hat{\pi}_{t-1}^* & 33 \\ \hat{z}_t^* & 14 & \hat{y}_{t-1}^* & 34 \\ \hat{z}_{t-1}^* & 15 & \hat{R}_{t-1}^* & 35 \\ \hat{\lambda}_{xt} & 16 & \hat{\pi}_{t-2}^* & 36 \\ \hat{\epsilon}_{Rt} & 17 & \hat{y}_{t-2}^* & 37 \\ \hat{\pi}_t^c & 18 & \hat{R}_{t-2}^* & 38 \\ \hat{\pi}_{t-1}^c & 19 & \hat{\pi}_{t-3}^* & 39 \\ \hat{\tau}_t^k & 20 & \hat{y}_{t-3}^* & 40 \\ & & \hat{R}_{t-3}^* & 41 \end{bmatrix}, \quad X_t^{\text{pd}} \equiv \begin{bmatrix} \hat{k}_t & 1 & 42 & \hat{k}_{t-1} & 16 & 57 \\ \hat{m}_t & 2 & 43 & \hat{q}_{t-1} & 17 & 58 \\ \hat{R}_{t-1} & 3 & 44 & \hat{\mu}_{t-1} & 18 & 59 \\ \hat{\pi}_{t-1}^d & 4 & 45 & \hat{a}_{t-1} & 19 & 60 \\ \hat{\pi}_{t-1}^{mc} & 5 & 46 & \hat{\gamma}_{t-1}^{mcd} & 20 & 61 \\ \hat{\pi}_{t-1}^{mi} & 6 & 47 & \hat{\gamma}_{t-1}^{mid} & 21 & 62 \\ \hat{y}_{t-1} & 7 & 48 & \hat{\gamma}_{t-1}^{x*} & 22 & 63 \\ \hat{\pi}_{t-1}^x & 8 & 49 & \hat{x}_{t-1} & 23 & 64 \\ \hat{w}_{t-1} & 9 & 50 & \hat{m}_{t-1}^x & 24 & 65 \\ \hat{c}_{t-1} & 10 & 51 & \hat{k}_{t-1} & 25 & 66 \\ \hat{i}_{t-1} & 11 & 52 & \hat{u}_{t-1} & 26 & 67 \\ \hat{\psi}_{z,t-1} & 12 & 53 & \hat{\pi}_{t-2}^d & 27 & 68 \\ \hat{P}_{k't-1} & 13 & 54 & \hat{\pi}_{t-2}^{mc} & 28 & 69 \\ \Delta \hat{S}_{t-1} & 14 & 55 & \hat{\pi}_{t-3}^d & 29 & 70 \\ \hat{H}_{t-1} & 15 & 56 & \hat{\pi}_{t-3}^{mc} & 30 & 71 \end{bmatrix},$$

$$x_t \equiv \begin{bmatrix} \hat{\pi}_t^d & 1 & \hat{k}_t & 13 \\ \hat{\pi}_t^{mc} & 2 & \hat{q}_t & 14 \\ \hat{\pi}_t^{mi} & 3 & \hat{\mu}_t & 15 \\ \hat{y}_t & 4 & \hat{a}_t & 16 \\ \hat{\pi}_t^x & 5 & \hat{\gamma}_t^{mcd} & 17 \\ \hat{w}_t & 6 & \hat{\gamma}_t^{mid} & 18 \\ \hat{c}_t & 7 & \hat{\gamma}_t^{x*} & 19 \\ \hat{i}_t & 8 & \hat{x}_t & 20 \\ \hat{\psi}_{zt} & 9 & \hat{m}_t^x & 21 \\ \hat{P}_{k't} & 10 & \hat{k}_{t+1} & 22 \\ \Delta \hat{S}_t & 11 & \hat{m}_{t+1} & 23 \\ \hat{H}_t & 12 & & \end{bmatrix}, \quad i_t \equiv \hat{R}_t,$$

$$\epsilon_t \equiv \begin{bmatrix} \epsilon_{\epsilon t} & \epsilon_{z t} & \epsilon_{\nu t} & \epsilon_{\zeta^c t} & \epsilon_{\zeta^h t} & \epsilon_{\zeta^q t} & \epsilon_{\lambda t} & \epsilon_{\lambda^{mc} t} & \epsilon_{\lambda^{mi} t} & \dots \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & \dots \\ \epsilon_{\tilde{\phi} t} & \epsilon_{\Upsilon t} & \epsilon_{\tilde{z}^* t} & \epsilon_{\lambda_{x t}} & \epsilon_{\epsilon_{R t}} & \epsilon_{\hat{\pi} t} & \epsilon'_{\tau t} & \epsilon'_{x^* t} & & \\ 10 & 11 & 12 & 13 & 14 & 15 & 16 : 20 & 21 : 23 & & \end{bmatrix}'.$$

1.1. Predetermined variables

First we specify the exogenous variables and the shocks, then the endogenous predetermined variables.

Row 1 - Stationary technology shock

$$\hat{\epsilon}_{t+1} = \rho_\epsilon \hat{\epsilon}_t + \sigma_{\epsilon_\epsilon} \varepsilon_{\epsilon,t+1}.$$

$$\begin{aligned} \hat{\epsilon}_t & : A_{11}(1,1) = \rho_\epsilon, \\ \varepsilon_{\epsilon,t+1} & : C(1,1) = \sigma_{\epsilon_\epsilon}. \end{aligned}$$

The notation means that the value of the persistence of the technology shock, ρ_ϵ , is placed in the first row and first column of the matrix A_{11} . The notation " $\hat{\epsilon}_t :$ " means that the parameter value pertains to that specific variable.

Row 2 - Lagged stationary technology shock

$$\hat{\epsilon}_t = \hat{\epsilon}_t.$$

$$\hat{\epsilon}_t : A_{11}(2,1) = 1.$$

Row 3 - Permanent technology shock

$$\hat{\mu}_{z,t+1} = \rho_{\mu_z} \hat{\mu}_{zt} + \sigma_{\varepsilon_z} \varepsilon_{z,t+1}.$$

$$\begin{aligned} \hat{\mu}_{zt} & : A_{11}(3,3) = \rho_\epsilon, \\ \varepsilon_{z,t+1} & : C(3,2) = \sigma_{\varepsilon_\epsilon}. \end{aligned}$$

Row 4 - Lagged permanent technology shock

$$\hat{\mu}_{zt} = \hat{\mu}_{zt}.$$

$$\hat{\mu}_{zt} : A_{11}(4,3) = 1.$$

Row 5 - Firm money demand shock

$$\hat{\nu}_{t+1} = \rho_\nu \hat{\nu}_t + \sigma_{\varepsilon_\nu} \varepsilon_{\nu,t+1}.$$

$$\begin{aligned} \hat{\nu}_t & : A_{11}(5,5) = \rho_\nu, \\ \varepsilon_{\nu,t+1} & : C(5,3) = \sigma_{\varepsilon_\nu}. \end{aligned}$$

Row 6 - Consumption preference shock

$$\hat{\zeta}_{t+1}^c = \rho_{\zeta^c} \hat{\zeta}_t^c + \sigma_{\varepsilon_{\zeta^c}} \varepsilon_{\zeta^c,t+1}.$$

$$\begin{aligned} \hat{\zeta}_t^c & : A_{11}(6,6) = \rho_{\zeta^c}, \\ \varepsilon_{\zeta^c,t+1} & : C(6,4) = \sigma_{\varepsilon_{\zeta^c}}. \end{aligned}$$

Row 7 - Labor supply shock

$$\hat{\zeta}_{t+1}^h = \rho_{\zeta^h} \hat{\zeta}_t^h + \sigma_{\varepsilon_{\zeta^h}} \varepsilon_{\zeta^h, t+1}.$$

$$\begin{aligned} \hat{\zeta}_t^h & : A_{11}(7, 7) = \rho_{\zeta^h}, \\ \varepsilon_{\zeta^h, t+1} & : C(7, 5) = \sigma_{\varepsilon_{\zeta^h}}. \end{aligned}$$

Row 8 - Household money demand shock

$$\hat{\zeta}_{t+1}^q = \rho_{\zeta^q} \hat{\zeta}_t^q + \sigma_{\varepsilon_{\zeta^q}} \varepsilon_{\zeta^q, t+1}.$$

$$\begin{aligned} \hat{\zeta}_t^q & : A_{11}(8, 8) = \rho_{\zeta^q}, \\ \varepsilon_{\zeta^q, t+1} & : C(8, 6) = \sigma_{\varepsilon_{\zeta^q}}. \end{aligned}$$

Row 9 - Markup shock - domestic firms

$$\hat{\lambda}_{t+1}^d = \rho_{\lambda} \hat{\lambda}_t^d + \sigma_{\varepsilon_{\lambda}} \varepsilon_{\lambda, t+1}.$$

$$\begin{aligned} \hat{\lambda}_t^d & : A_{11}(9, 9) = \rho_{\lambda}, \\ \varepsilon_{\lambda, t+1} & : C(9, 7) = \sigma_{\varepsilon_{\lambda}}. \end{aligned}$$

Row 10 - Shock to substitution elasticity between domestic and foreign consumption goods

$$\hat{\lambda}_{t+1}^{mc} = \rho_{\lambda^{mc}} \hat{\lambda}_t^{mc} + \sigma_{\varepsilon_{\lambda^{mc}}} \varepsilon_{\lambda^{mc}, t+1}.$$

$$\begin{aligned} \hat{\lambda}_t^{mc} & : A_{11}(10, 10) = \rho_{\lambda^{mc}}, \\ \varepsilon_{\lambda^{mc}, t+1} & : C(10, 8) = \sigma_{\varepsilon_{\lambda^{mc}}}. \end{aligned}$$

Row 11 - Shock to substitution elasticity between domestic and foreign investment goods

$$\hat{\lambda}_{t+1}^{mi} = \rho_{\lambda^{mi}} \hat{\lambda}_t^{mi} + \sigma_{\varepsilon_{\lambda^{mi}}} \varepsilon_{\lambda^{mi}, t+1}.$$

$$\begin{aligned} \hat{\lambda}_t^{mi} & : A_{11}(11, 11) = \rho_{\lambda^{mi}}, \\ \varepsilon_{\lambda^{mi}, t+1} & : C(11, 9) = \sigma_{\varepsilon_{\lambda^{mi}}}. \end{aligned}$$

Row 12 - Risk premium shock

$$\hat{\phi}_{t+1} = \rho_{\tilde{\phi}} \hat{\phi}_t + \sigma_{\varepsilon_{\tilde{\phi}}} \varepsilon_{\tilde{\phi}, t+1}.$$

$$\begin{aligned} \hat{\phi}_t & : A_{11}(12, 12) = \rho_{\tilde{\phi}}, \\ \varepsilon_{\tilde{\phi}, t+1} & : C(12, 10) = \sigma_{\varepsilon_{\tilde{\phi}}}. \end{aligned}$$

Row 13 - Investment specific technology shock

$$\hat{Y}_{t+1} = \rho_Y \hat{Y}_t + \sigma_{\varepsilon_Y} \varepsilon_{Y,t+1}.$$

$$\begin{aligned} \hat{Y}_t &: A_{11}(13, 13) = \rho_Y, \\ \varepsilon_{Y,t+1} &: C(13, 11) = \sigma_{\varepsilon_Y}. \end{aligned}$$

Row 14 - Relative (asymmetric) technology shock (between foreign and domestic)

$$\hat{z}_{t+1}^* = \rho_{z^*} \hat{z}_t^* + \sigma_{\varepsilon_{z^*}} \varepsilon_{z^*,t+1}.$$

$$\begin{aligned} \hat{z}_t^* &: A_{11}(14, 14) = \rho_{z^*}, \\ \varepsilon_{z^*,t+1} &: C(14, 12) = \sigma_{\varepsilon_{z^*}}. \end{aligned}$$

Row 15 - Lagged relative (asymmetric) technology shock (between foreign and domestic)

$$\begin{aligned} \hat{z}_t^* &= \hat{z}_t^*. \\ \hat{z}_t^* &: A_{11}(15, 14) = 1. \end{aligned}$$

Row 16 - Markup shock - exporting firms

$$\hat{\lambda}_{x,t+1} = \rho_{\lambda_x} \hat{\lambda}_{xt} + \sigma_{\varepsilon_{\lambda_x}} \varepsilon_{\lambda_x,t+1}.$$

$$\begin{aligned} \hat{\lambda}_{xt} &: A_{11}(16, 16) = \rho_{\lambda_x}, \\ \varepsilon_{\lambda_x,t+1} &: C(16, 13) = \sigma_{\varepsilon_{\lambda_x}}. \end{aligned}$$

Row 17 - Monetary policy shock

$$\varepsilon_{R,t+1} = \rho_{\varepsilon_R} \varepsilon_{Rt} + \sigma_{\varepsilon_{\varepsilon_R}} \varepsilon_{\varepsilon_R,t+1}.$$

$$\begin{aligned} \varepsilon_{Rt} &: A_{11}(17, 17) = \rho_{\varepsilon_R}, \\ \varepsilon_{\varepsilon_R,t+1} &: C(17, 14) = \sigma_{\varepsilon_{\varepsilon_R}}. \end{aligned}$$

Row 18 - Inflation target

$$\hat{\pi}_{t+1}^c = \rho_{\hat{\pi}^c} \hat{\pi}_t^c + \sigma_{\varepsilon_{\hat{\pi}^c}} \varepsilon_{\hat{\pi}^c,t+1}.$$

$$\begin{aligned} \hat{\pi}_t^c &: A_{11}(18, 18) = \rho_{\hat{\pi}^c}, \\ \varepsilon_{\hat{\pi}^c,t+1} &: C(18, 15) = \sigma_{\varepsilon_{\hat{\pi}^c}}. \end{aligned}$$

Row 19 - Lagged inflation target

$$\begin{aligned} \hat{\pi}_t^c &= \hat{\pi}_t^c. \\ \hat{\pi}_t^c &: A_{11}(19, 18) = 1. \end{aligned}$$

Row 20–29 - Fiscal VAR(2)

$$\tau_{t+1} = \tilde{\Theta}_1 \tau_t + \tilde{\Theta}_2 \tau_{t-1} + e_{\tau,t+1}, \quad (1.1)$$

where $\tilde{\Theta}_1 = \Theta_0^{-1} \Theta_1$, $\tilde{\Theta}_2 = \Theta_0^{-1} \Theta_2$ and $e_{\tau t} = \Theta_0^{-1} S_\tau \varepsilon_{\tau t} \sim N(0, \Sigma_\tau)$. Rewrite (1.1) as VAR(1):

$$\begin{bmatrix} \hat{\tau}_{t+1} \\ \hat{\tau}_t \end{bmatrix} = \begin{bmatrix} \tilde{\Theta}_1 & \tilde{\Theta}_2 \\ I & 0 \end{bmatrix} \begin{bmatrix} \hat{\tau}_t \\ \hat{\tau}_{t-1} \end{bmatrix} + \begin{bmatrix} e_{\tau,t+1} \\ 0 \end{bmatrix}$$

$$\begin{aligned} \hat{\tau}_t & : A_{11}(20 : 24, 20 : 24) = \tilde{\Theta}_1, \\ \hat{\tau}_{t-1} & : A_{11}(20 : 24, 25 : 29) = \tilde{\Theta}_2, \\ \hat{\tau}_t & : A_{11}(25 : 29, 20 : 24) = I, \\ \varepsilon_{\tau,t+1} & : C(20 : 24, 16 : 20) = \text{chol} \left(\Theta_0^{-1} S_\tau (\Theta_0^{-1} S_\tau)' \right)'. \end{aligned}$$

Row 30–41 - Foreign VAR(4)

$$X_{t+1}^* = \tilde{\Phi}_1 X_t^* + \tilde{\Phi}_2 X_{t-1}^* + \tilde{\Phi}_3 X_{t-2}^* + \tilde{\Phi}_4 X_{t-3}^* + e_{X^*,t+1}, \quad (1.2)$$

where $\tilde{\Phi}_1 = \Phi_0^{-1} \Phi_1$, $\tilde{\Phi}_2 = \Phi_0^{-1} \Phi_2$, etc., and $e_{X^* t} = \Phi_0^{-1} S_{x^*} \varepsilon_{x^* t} \sim N(0, \Sigma_{x^*})$. Rewrite (1.2) as VAR(1):

$$\begin{bmatrix} X_{t+1}^* \\ X_t^* \\ X_{t-1}^* \\ X_{t-2}^* \end{bmatrix} = \begin{bmatrix} \tilde{\Phi}_1 & \tilde{\Phi}_2 & \tilde{\Phi}_3 & \tilde{\Phi}_4 \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix} \begin{bmatrix} X_t^* \\ X_{t-1}^* \\ X_{t-2}^* \\ X_{t-3}^* \end{bmatrix} + \begin{bmatrix} e_{X^*,t+1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} X_t^* & : A_{11}(30 : 32, 30 : 32) = \tilde{\Phi}_1, \\ X_{t-1}^* & : A_{11}(30 : 32, 33 : 35) = \tilde{\Phi}_2, \\ X_{t-2}^* & : A_{11}(30 : 32, 36 : 38) = \tilde{\Phi}_3, \\ X_{t-3}^* & : A_{11}(30 : 32, 39 : 41) = \tilde{\Phi}_4, \\ X_t^* & : A_{11}(33 : 35, 30 : 32) = I, \\ X_{t-1}^* & : A_{11}(36 : 38, 33 : 35) = I, \\ X_{t-2}^* & : A_{11}(39 : 41, 36 : 38) = I, \\ \varepsilon_{x^*,t+1} & : C(30 : 32, 21 : 23) = \text{chol} \left(\Phi_0^{-1} S_{x^*} (\Phi_0^{-1} S_{x^*})' \right)'. \end{aligned}$$

Row 42 - Physical capital stock (note that \hat{k}_{t+1} is predetermined variable no. 42 in period $t + 1$ and forward-looking variable no. 22 in period t):

$$\begin{aligned} \hat{k}_{t+1} & = \hat{k}_{t+1}, \\ \hat{k}_t & : A_{12}(42, 22) = 1. \end{aligned}$$

Row 43 - Financial assets (note that \hat{m}_{t+1} is predetermined variable no. 43 in period $t + 1$ and forward-looking variable no. 23 in period t):

$$\begin{aligned} \hat{m}_{t+1} & = \hat{m}_{t+1}, \\ \hat{m}_{t+1} & : A_{12}(43, 23) = 1. \end{aligned}$$

Row 44–66 - Lagged forward-looking variables

$$\widehat{R}_t : B_1(44, 1) = 1$$

$$\widehat{\pi}_t^d : A_{12}(45, 1) = 1$$

$$\widehat{\pi}_t^{mc} : A_{12}(46, 2) = 1$$

$$\widehat{\pi}_t^{mi} : A_{12}(47, 3) = 1$$

$$\widehat{y}_t : A_{12}(48, 4) = 1$$

$$\widehat{\pi}_t^x : A_{12}(49, 5) = 1$$

$$\widehat{w}_t : A_{12}(50, 6) = 1$$

$$\widehat{c}_t : A_{12}(51, 7) = 1$$

$$\widehat{v}_t : A_{12}(52, 8) = 1$$

$$\widehat{\psi}_{zt} : A_{12}(53, 9) = 1$$

$$\widehat{P}_{k't} : A_{12}(54, 10) = 1$$

$$\Delta \widehat{S}_t : A_{12}(55, 11) = 1$$

$$\widehat{H}_t : A_{12}(56, 12) = 1$$

$$\widehat{u}_t : A_{12}(57, 13) = 1$$

$$\widehat{q}_t : A_{12}(58, 14) = 1$$

$$\widehat{\mu}_t : A_{12}(59, 15) = 1$$

$$\widehat{a}_t : A_{12}(60, 16) = 1$$

$$\widehat{\gamma}_t^{mcd} : A_{12}(61, 17) = 1$$

$$\widehat{\gamma}_t^{mid} : A_{12}(62, 18) = 1$$

$$\widehat{\gamma}_t^{x*} : A_{12}(63, 19) = 1$$

$$\widehat{x}_t : A_{12}(64, 20) = 1$$

$$\widehat{mc}_t^x : A_{12}(65, 21) = 1$$

Row 66 - Lagged physical capital stock

$$\widehat{k}_{t-1} : A_{11}(66, 42) = 1$$

Row 67 - Lagged capacity utilization

$$\widehat{u}_{t-1} \equiv \widehat{k}_{t-1} - \widehat{\bar{k}}_{t-1}$$

$$\widehat{k}_{t-1} : A_{11}(67, 57) = 1$$

$$\widehat{\bar{k}}_{t-1} : A_{11}(67, 66) = -1$$

Rows 68-71 - Lagged inflation

$$\begin{aligned}\hat{\pi}_{t-1}^d & : A_{11} (68, 45) = 1 \\ \hat{\pi}_{t-1}^{mc} & : A_{11} (69, 46) = 1 \\ \hat{\pi}_{t-2}^d & : A_{11} (70, 68) = 1 \\ \hat{\pi}_{t-2}^{mc} & : A_{11} (71, 69) = 1\end{aligned}$$

1.2. Forward-looking variables

Row 1 - Domestic inflation

$$\begin{aligned}\hat{\pi}_t^d - \hat{\pi}_t^c & = \frac{\beta}{1 + \kappa_d \beta} \left(\hat{\pi}_{t+1|t}^d - \rho_{\bar{\pi}^c} \hat{\pi}_t^c \right) + \frac{\kappa_d}{1 + \kappa_d \beta} \left(\hat{\pi}_{t-1}^d - \hat{\pi}_t^c \right) \\ & \quad - \frac{\kappa_d \beta (1 - \rho_{\bar{\pi}^c})}{1 + \kappa_d \beta} \hat{\pi}_t^c + \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \left(\widehat{mc}_t^d + \hat{\lambda}_t^d \right), \\ \widehat{mc}_t^d & \equiv \alpha \left(\hat{\mu}_{zt} + \hat{H}_t - \hat{k}_t \right) + \hat{w}_t + \hat{R}_t^f - \hat{\varepsilon}_t, \\ \hat{R}_t^f & = \frac{\nu R}{v(R-1) + 1} \hat{R}_{t-1} + \frac{\nu(R-1)}{v(R-1) + 1} \hat{\nu}_t\end{aligned}$$

$$\widehat{mc}_t^d \equiv \alpha \hat{H}_t - \alpha \hat{k}_t + \hat{w}_t + \frac{\nu R}{v(R-1) + 1} \hat{R}_{t-1} + \frac{\nu(R-1)}{v(R-1) + 1} \hat{\nu}_t - \hat{\varepsilon}_t + \alpha \hat{\mu}_{zt}$$

Rewrite the above equations as:

$$\begin{aligned}-\frac{\beta}{1 + \kappa_d \beta} \hat{\pi}_{t+1|t}^d & = -\hat{\pi}_t^d + \frac{\kappa_d}{1 + \kappa_d \beta} \hat{\pi}_{t-1}^d + \\ & \quad + \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \alpha \hat{H}_t - \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \alpha \hat{k}_t \\ & \quad + \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \hat{w}_t + \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \frac{\nu R}{v(R-1) + 1} \hat{R}_{t-1} + \\ & \quad + \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \frac{\nu(R-1)}{v(R-1) + 1} \hat{\nu}_t + \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \alpha \hat{\mu}_{zt} \\ & \quad - \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \hat{\varepsilon}_t + \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \hat{\lambda}_t^d \\ & \quad - \frac{(1 - \kappa_d)(\beta \rho_{\bar{\pi}^c} - 1)}{1 + \kappa_d \beta} \hat{\pi}_t^c.\end{aligned}$$

$$\hat{\pi}_{t+1|t}^d : H(1,1) = -\frac{\beta}{1 + \kappa_d \beta}$$

$$\hat{\pi}_t^d : A_{22}(1,1) = -1$$

$$\hat{\pi}_{t-1}^d : A_{21}(1,45) = \frac{\kappa_d}{1 + \kappa_d \beta}$$

$$\hat{\pi}_t^c : A_{21}(1,18) = 1 - \frac{\beta}{1 + \kappa_d \beta} \rho_{\bar{\pi}^c} - \frac{\kappa_d}{1 + \kappa_d \beta} - \frac{\kappa_d \beta (1 - \rho_{\bar{\pi}^c})}{1 + \kappa_d \beta}$$

$$\hat{\lambda}_t^d : \left[A_{21}(1,9) = \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \right]$$

$$\hat{\lambda}_t^d : A_{21}(1,9) = 1 \quad (\text{rescaled})$$

Marginal cost :

$$\hat{w}_t : A_{22}(1,6) = \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)}$$

$$\hat{H}_t : A_{22}(1,12) = \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \alpha$$

$$\hat{k}_t : A_{22}(1,13) = -\frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \alpha$$

$$\hat{\varepsilon}_t : A_{21}(1,1) = -\frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)}$$

$$\hat{\mu}_{zt} : A_{21}(1,3) = \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \alpha$$

$$\hat{\nu}_t : A_{21}(1,5) = \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \frac{\nu(R-1)}{v(R-1) + 1}$$

$$\hat{R}_{t-1} : A_{21}(1,44) = \frac{(1 - \xi_d)(1 - \beta \xi_d)}{\xi_d (1 + \kappa_d \beta)} \frac{\nu R}{v(R-1) + 1}$$

Row 2 - Imported consumption inflation

$$\begin{aligned} \hat{\pi}_t^{mc} - \hat{\pi}_t^c &= \frac{\beta}{1 + \kappa_{mc} \beta} \left(\hat{\pi}_{t+1|t}^{mc} - \rho_{\bar{\pi}^c} \hat{\pi}_t^c \right) + \frac{\kappa_{mc}}{1 + \kappa_{mc} \beta} \left(\hat{\pi}_{t-1}^{mc} - \hat{\pi}_t^c \right) \\ &\quad - \frac{\kappa_{mc} \beta (1 - \rho_{\bar{\pi}^c})}{1 + \kappa_{mc} \beta} \hat{\pi}_t^c + \frac{(1 - \xi_{mc})(1 - \beta \xi_{mc})}{\xi_{mc} (1 + \kappa_{mc} \beta)} \left(\hat{m}_t^{mc} + \hat{\lambda}_t^{mc} \right), \end{aligned}$$

$$\hat{m}_t^{mc} \equiv -\hat{m}_t^x - \hat{\gamma}_t^{x*} - \hat{\gamma}_t^{mcd}$$

$$\begin{aligned}
\hat{\pi}_{t+1|t}^{mc} & : H(2,2) = -\frac{\beta}{1 + \kappa_{mc}\beta} \\
\hat{\pi}_t^{mc} & : A_{22}(2,2) = -1 \\
\hat{\pi}_{t-1}^{mc} & : A_{21}(2,46) = \frac{\kappa_{mc}}{1 + \kappa_{mc}\beta} \\
\hat{\pi}_t^c & : A_{21}(2,18) = 1 - \frac{\beta}{1 + \kappa_{mc}\beta} \rho_{\bar{\pi}^c} - \frac{\kappa_{mc}}{1 + \kappa_{mc}\beta} - \frac{\kappa_{mc}\beta(1 - \rho_{\bar{\pi}^c})}{1 + \kappa_{mc}\beta} \\
& \left[\hat{\lambda}_t^{mc} : A_{21}(2,10) = \frac{(1 - \xi_{mc})(1 - \beta\xi_{mc})}{\xi_{mc}(1 + \kappa_{mc}\beta)} \right] \\
\hat{\lambda}_t^{mc} & : A_{21}(2,10) = 1 \text{ (rescaled)}
\end{aligned}$$

Marginal cost :

$$\begin{aligned}
\widehat{\text{mc}}_t^x & : A_{22}(2,21) = -\frac{(1 - \xi_{mc})(1 - \beta\xi_{mc})}{\xi_{mc}(1 + \kappa_{mc}\beta)} \\
\hat{\gamma}_t^{x*} & : A_{22}(2,19) = -\frac{(1 - \xi_{mc})(1 - \beta\xi_{mc})}{\xi_{mc}(1 + \kappa_{mc}\beta)} \\
\hat{\gamma}_t^{mcd} & : A_{22}(2,17) = -\frac{(1 - \xi_{mc})(1 - \beta\xi_{mc})}{\xi_{mc}(1 + \kappa_{mc}\beta)}
\end{aligned}$$

Row 3 - Imported investment inflation

$$\begin{aligned}
\hat{\pi}_t^{mi} - \hat{\pi}_t^c & = \frac{\beta}{1 + \kappa_{mi}\beta} \left(\hat{\pi}_{t+1|t}^{mi} - \rho_{\bar{\pi}^c} \hat{\pi}_t^c \right) + \frac{\kappa_{mi}}{1 + \kappa_{mi}\beta} \left(\hat{\pi}_{t-1}^{mi} - \hat{\pi}_t^c \right) \\
& - \frac{\kappa_{mi}\beta(1 - \rho_{\bar{\pi}^c})}{1 + \kappa_{mi}\beta} \hat{\pi}_t^c + \frac{(1 - \xi_{mi})(1 - \beta\xi_{mi})}{\xi_{mi}(1 + \kappa_{mi}\beta)} \left(\widehat{\text{mc}}_t^{mi} + \hat{\lambda}_t^{mi} \right), \\
\widehat{\text{mc}}_t^{mi} & \equiv -\widehat{\text{mc}}_t^x - \hat{\gamma}_t^{x*} - \hat{\gamma}_t^{mi,d}
\end{aligned}$$

$$\begin{aligned}
\hat{\pi}_{t+1|t}^{mi} & : H(3,3) = -\frac{\beta}{1 + \kappa_{mi}\beta} \\
\hat{\pi}_t^{mi} & : A_{22}(3,3) = -1 \\
\hat{\pi}_{t-1}^{mi} & : A_{21}(3,47) = \frac{\kappa_{mi}}{1 + \kappa_{mi}\beta}
\end{aligned}$$

Marginal cost :

$$\begin{aligned}
\hat{\gamma}_t^{mid} & : A_{22}(3,18) = -\frac{(1 - \xi_{mi})(1 - \beta\xi_{mi})}{\xi_{mi}(1 + \kappa_{mi}\beta)} \\
\hat{\gamma}_t^{x*} & : A_{22}(3,19) = -\frac{(1 - \xi_{mi})(1 - \beta\xi_{mi})}{\xi_{mi}(1 + \kappa_{mi}\beta)} \\
\widehat{\text{mc}}_t^x & : A_{22}(3,21) = -\frac{(1 - \xi_{mi})(1 - \beta\xi_{mi})}{\xi_{mi}(1 + \kappa_{mi}\beta)} \\
& \left[\hat{\lambda}_t^{mi} : A_{21}(3,11) = \frac{(1 - \xi_{mi})(1 - \beta\xi_{mi})}{\xi_{mi}(1 + \kappa_{mi}\beta)} \right] \\
\hat{\lambda}_t^{mi} & : A_{21}(3,11) = 1 \text{ (rescaled)} \\
\hat{\pi}_t^c & : A_{21}(3,18) = 1 - \frac{\beta}{1 + \kappa_{mi}\beta} \rho_{\bar{\pi}^c} - \frac{\kappa_{mi}}{1 + \kappa_{mi}\beta} - \frac{\kappa_{mi}\beta(1 - \rho_{\bar{\pi}^c})}{1 + \kappa_{mi}\beta}
\end{aligned}$$

Row 4 - Output gap

$$0 = -\hat{y}_t + \lambda_f \hat{\epsilon}_t + \lambda_f \alpha \hat{k}_t - \lambda_f \alpha \hat{\mu}_{zt} + \lambda_f (1 - \alpha) \hat{H}_t$$

$$\begin{aligned} \hat{y}_t & : A_{22}(4, 4) = -1 \\ \hat{H}_t & : A_{22}(4, 12) = \lambda_f (1 - \alpha) \\ \hat{k}_t & : A_{22}(4, 13) = \lambda_f \alpha \\ \\ \hat{\epsilon}_t & : A_{21}(4, 1) = \lambda_f \\ \hat{\mu}_{zt} & : A_{21}(4, 3) = -\lambda_f \alpha \end{aligned}$$

Row 5 - Export price inflation

$$\begin{aligned} \hat{\pi}_t^x - \hat{\pi}_t^c & = \frac{\beta}{1 + \kappa_x \beta} \left(\hat{\pi}_{t+1|t}^x - \rho_{\bar{\pi}^c} \hat{\pi}_t^c \right) + \frac{\kappa_x}{1 + \kappa_x \beta} \left(\hat{\pi}_{t-1}^x - \hat{\pi}_t^c \right) \\ & \quad - \frac{\kappa_x \beta (1 - \rho_{\bar{\pi}^c})}{1 + \kappa_x \beta} \hat{\pi}_t^c + \frac{(1 - \xi_x)(1 - \beta \xi_x)}{\xi_x (1 + \kappa_x \beta)} \left(\widehat{mc}_t^x + \hat{\lambda}_t^x \right), \end{aligned}$$

$$\begin{aligned} \hat{\pi}_{t+1|t}^x & : H(5, 5) = -\frac{\beta}{1 + \kappa_x \beta} \\ \hat{\pi}_t^x & : A_{22}(5, 5) = -1 \\ \hat{\pi}_{t-1}^x & : A_{21}(5, 49) = \frac{\kappa_x}{1 + \kappa_x \beta} \end{aligned}$$

Marginal cost :

$$\begin{aligned} \widehat{mc}_t^x & : A_{22}(5, 21) = \frac{(1 - \xi_x)(1 - \beta \xi_x)}{\xi_x (1 + \kappa_x \beta)} \\ & \quad \left[\hat{\lambda}_t^x : A_{21}(5, 16) = \frac{(1 - \xi_x)(1 - \beta \xi_x)}{\xi_x (1 + \kappa_x \beta)} \right] \\ \hat{\lambda}_t^x & : A_{21}(5, 16) = 1 \text{ (rescaled)} \\ \hat{\pi}_t^c & : A_{21}(5, 18) = 1 - \frac{\beta}{1 + \kappa_x \beta} \rho_{\bar{\pi}^c} - \frac{\kappa_x}{1 + \kappa_x \beta} - \frac{\kappa_x \beta (1 - \rho_{\bar{\pi}^c})}{1 + \kappa_x \beta} \end{aligned}$$

Row 6 - Real wage

$$\begin{aligned} 0 & = E_t \left\{ \begin{aligned} & \eta_0 \hat{w}_{t-1} + \eta_1 \hat{w}_t + \eta_2 \hat{w}_{t+1} + \eta_3 \left(\hat{\pi}_t^d - \hat{\pi}_t^c \right) + \eta_4 \left(\hat{\pi}_{t+1}^d - \rho_{\widehat{\pi}^c} \hat{\pi}_t^c \right) \\ & \quad + \eta_5 \left(\hat{\pi}_{t-1}^c - \hat{\pi}_t^c \right) + \eta_6 \left(\hat{\pi}_t^c - \rho_{\widehat{\pi}^c} \hat{\pi}_t^c \right) \\ & \quad + \eta_7 \hat{\psi}_{zt}^\tau + \eta_8 \hat{H}_t + \eta_9 \hat{\tau}_t^y + \eta_{10} \hat{\tau}_t^w + \eta_{11} \hat{\zeta}_t^h \end{aligned} \right\} \\ \hat{\pi}_t^c & = (1 - \omega_c) \left(\gamma^{cd} \right)^{-(1 - \eta_c)} \hat{\pi}_t^d + \omega_c \left(\gamma^{cmc} \right)^{-(1 - \eta_c)} \widehat{\pi}_t^{mc} \end{aligned}$$

where

$$\eta \equiv \begin{bmatrix} b_w \xi_w \\ \sigma_L \lambda_w - b_w (1 + \beta \xi_w^2) \\ b_w \beta \xi_w \\ -b_w \xi_w \\ b_w \beta \xi_w \\ b_w \xi_w \kappa_w \\ -b_w \beta \xi_w \kappa_w \\ 1 - \lambda_w \\ -(1 - \lambda_w) \sigma_L \\ -(1 - \lambda_w) \frac{\tau^y}{(1 - \tau^y)} \\ -(1 - \lambda_w) \frac{\tau^w}{(1 + \tau^w)} \\ -(1 - \lambda_w) \end{bmatrix} \equiv \begin{bmatrix} \eta_0 \\ \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \\ \eta_7 \\ \eta_8 \\ \eta_9 \\ \eta_{10} \\ \eta_{11} \end{bmatrix}. \quad (1.3)$$

Collect terms and use the definition of $\hat{\pi}_t^c$:

$$\begin{aligned} \eta_2 \hat{w}_{t+1|t} + \eta_4 \hat{\pi}_{t+1|t}^d &= -\eta_0 \hat{w}_{t-1} - \eta_1 \hat{w}_t - \left[\eta_3 + \eta_6 \left((1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \right) \right] \hat{\pi}_t^d \\ &\quad - \eta_6 \omega_c (\gamma^{cmc})^{-(1-\eta_c)} \hat{\pi}_t^{mc} \\ &\quad - \eta_5 \left((1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \right) \hat{\pi}_{t-1}^d - \eta_5 \left((\omega_c) (\gamma^{cmc})^{-(1-\eta_c)} \right) \hat{\pi}_{t-1}^{mc} \\ &\quad - \eta_7 \hat{\psi}_{zt}^\tau - \eta_8 \hat{H}_t - \eta_9 \hat{\tau}_t^y - \eta_{10} \hat{\tau}_t^w - \eta_{11} \hat{\zeta}_t^h + [\eta_3 + \rho_{\hat{\pi}^c} \eta_4 + \eta_5 + \rho_{\hat{\pi}^c} \eta_6] \hat{\pi}_t^c \end{aligned}$$

$$\begin{aligned}\hat{\pi}_{t+1|t}^d & : H(6, 1) = \eta_4 \\ \hat{w}_{t+1|t} & : H(6, 6) = \eta_2\end{aligned}$$

$$\begin{aligned}\hat{\pi}_t^d & : A_{22}(6, 1) = - \left[\eta_3 + \eta_6 \left((1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \right) \right] \\ \hat{\pi}_t^{mc} & : A_{22}(6, 2) = -\eta_6 \omega_c (\gamma^{cmc})^{-(1-\eta_c)} \\ \hat{w}_t & : A_{22}(6, 6) = -\eta_1 \\ \hat{\psi}_{zt}^\tau & : A_{22}(6, 9) = -\eta_7 \\ \hat{H}_t & : A_{22}(6, 12) = -\eta_8\end{aligned}$$

Rescaling the labor supply shock so its coefficient is of the same magnitude as \hat{w}_t

$$\begin{aligned}\left[\hat{\zeta}_t^h : A_{21}(6, 7) = -\eta_{11} \right] \\ \hat{\zeta}_t^h & : \text{if } \eta_{11} \begin{cases} > 0 \Rightarrow A_{21}(6, 7) = \eta_1 \\ < 0 \Rightarrow A_{21}(6, 7) = -\eta_1 \\ = 0 \Rightarrow A_{21}(6, 7) = \eta_{11} \end{cases} \\ \hat{\pi}_t^c & : A_{21}(6, 18) = (\eta_3 + \rho_{\hat{\pi}^c} \eta_4 + \eta_5 + \rho_{\hat{\pi}^c} \eta_6) \\ \hat{\tau}_t^y & : A_{21}(6, 21) = -\eta_9 \\ \hat{\tau}_t^w & : A_{21}(6, 23) = -\eta_{10} \\ \hat{\pi}_{t-1}^d & : A_{21}(6, 45) = -\eta_5 (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \\ \hat{\pi}_{t-1}^{mc} & : A_{21}(6, 46) = -\eta_5 \omega_c (\gamma^{cmc})^{-(1-\eta_c)} \\ \hat{w}_{t-1} & : A_{21}(6, 50) = -\eta_0\end{aligned}$$

Row 7 - Consumption

$$E_t \left[\begin{aligned} & -b\beta\mu_z \hat{c}_{t+1} + (\mu_z^2 + b^2\beta) \hat{c}_t - b\mu_z \hat{c}_{t-1} + b\mu_z (\hat{\mu}_{zt} - \beta \hat{\mu}_{z,t+1}) + \\ & + (\mu_z - b\beta) (\mu_z - b) \hat{\psi}_{zt} + \frac{\tau^c}{1+\tau^c} (\mu_z - b\beta) (\mu_z - b) \hat{\tau}_t^c + (\mu_z - b\beta) (\mu_z - b) \hat{\gamma}_t^{cd} \\ & - (\mu_z - b) (\mu_z \hat{\zeta}_t^c - b\beta \hat{\zeta}_{t+1}^c) \end{aligned} \right] = 0$$

$$\begin{aligned}\hat{\gamma}_t^{cd} & = \omega_c (\gamma^{cmc})^{-(1-\eta_c)} \hat{\gamma}_t^{mcd} \\ \hat{\mu}_{z,t+1|t} & = \rho_{\mu_z} \hat{\mu}_{zt} \\ \hat{\zeta}_{t+1|t}^c & = \rho_{\zeta^c} \hat{\zeta}_t^c\end{aligned}$$

$$\begin{aligned}b\beta\mu_z \hat{c}_{t+1|t} & = (\mu_z^2 + b^2\beta) \hat{c}_t - b\mu_z \hat{c}_{t-1} + b\mu_z (1 - \beta\rho_{\mu_z}) \hat{\mu}_{zt} \\ & + (\mu_z - b\beta) (\mu_z - b) \hat{\psi}_{zt} + \frac{\tau^c}{1+\tau^c} (\mu_z - b\beta) (\mu_z - b) \hat{\tau}_t^c \\ & + (\mu_z - b\beta) (\mu_z - b) \omega_c (\gamma^{cmc})^{-(1-\eta_c)} \hat{\gamma}_t^{mcd} - (\mu_z - b) (\mu_z - b\beta\rho_{\zeta^c}) \hat{\zeta}_t^c\end{aligned}$$

$$\begin{aligned}
\hat{c}_{t+1|t} &: H(7, 7) = b\beta\mu_z \\
\hat{c}_t &: A_{22}(7, 7) = (\mu_z^2 + b^2\beta) \\
\hat{c}_{t-1} &: A_{21}(7, 51) = -b\mu_z \\
\hat{\psi}_{zt} &: A_{22}(7, 9) = (\mu_z - b\beta)(\mu_z - b) \\
\hat{\gamma}_t^{mcd} &: A_{22}(7, 17) = (\mu_z - b\beta)(\mu_z - b)\omega_c(\gamma^{cmc})^{-(1-\eta_c)} \\
\hat{\mu}_{zt} &: A_{21}(7, 3) = b\mu_z(1 - \beta\rho_{\mu_z}) \\
&\text{Rescaling of } \hat{\zeta}_t^c \text{ to give it the same coefficient as } \hat{c}_t \\
&\left[\hat{\zeta}_t^c : A_{21}(7, 6) = -(\mu_z - b)(\mu_z - b\beta\rho_{\zeta^c}) \right] \\
\hat{\zeta}_t^c &: A_{21}(7, 6) = -(\mu_z^2 + b^2\beta) \\
\hat{\tau}_t^c &: A_{21}(7, 22) = \frac{\tau^c}{1 + \tau^c}(\mu_z - b\beta)(\mu_z - b)
\end{aligned}$$

Row 8 - Investment

$$E_t \left\{ \hat{P}_{kt} + \hat{Y}_t - \hat{\gamma}_t^{i,d} - \mu_z^2 S''(\mu_z) \left[(\hat{i}_t - \hat{i}_{t-1}) - \beta(\hat{i}_{t+1} - \hat{i}_t) + \hat{\mu}_{zt} - \beta\hat{\mu}_{z,t+1} \right] \right\} = 0$$

$$\begin{aligned}
\hat{\gamma}_t^{i,d} &= \omega_i (\gamma^{i,mi})^{-(1-\eta_i)} \hat{\gamma}_t^{mi,d} \\
\hat{\mu}_{z,t+1|t} &= \rho_{\mu_z} \hat{\mu}_{zt}
\end{aligned}$$

$$\begin{aligned}
\mu_z^2 S''(\mu_z) \beta \hat{i}_{t+1|t} &= \mu_z^2 S''(\mu_z) (1 + \beta) \hat{i}_t - \mu_z^2 S''(\mu_z) \hat{i}_{t-1} - \hat{P}_{kt} \\
&\quad + \omega_i (\gamma^{i,mi})^{-(1-\eta_i)} \hat{\gamma}_t^{mi,d} + \mu_z^2 S''(\mu_z) (1 - \beta\rho_{\mu_z}) \hat{\mu}_{zt} - \hat{Y}_t
\end{aligned}$$

$$\begin{aligned}
\hat{i}_{t+1|t} &: H(8, 8) = \mu_z^2 S''(\mu_z) \beta \\
\hat{i}_t &: A_{22}(8, 8) = \mu_z^2 S''(\mu_z) (1 + \beta) \\
\hat{i}_{t-1} &: A_{21}(8, 52) = -\mu_z^2 S''(\mu_z)
\end{aligned}$$

$$\begin{aligned}
\hat{P}_{kt} &: A_{22}(8, 10) = -1 \\
\hat{\gamma}_t^{mi,d} &: A_{22}(8, 18) = \omega_i (\gamma^{i,mi})^{-(1-\eta_i)}
\end{aligned}$$

$$\begin{aligned}
\hat{\mu}_{zt} &: A_{21}(8, 3) = \mu_z^2 S''(\mu_z) (1 - \beta\rho_{\mu_z}) \\
&\text{Rescaling of } \hat{Y}_t \text{ to give it the same coefficient as } \hat{i}_t \\
\hat{Y}_t &: A_{21}(8, 13) = -\mu_z^2 S''(\mu_z) (1 + \beta)
\end{aligned}$$

Row 9 - Lagrange multiplier (first order condition with respect to m_{t+1})

$$\begin{aligned} E_t \left[-\mu \hat{\psi}_{zt} + \mu \hat{\psi}_{z,t+1} - \mu \hat{\pi}_{t+1}^d - \mu \hat{\mu}_{z,t+1} + \left(\mu - \beta \tau^k \right) \hat{R}_t + \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \hat{\tau}_{t+1}^k \right] &= 0 \\ \hat{\mu}_{z,t+1|t} &= \rho_{\mu_z} \hat{\mu}_{zt} \end{aligned}$$

$$\begin{aligned} \mu \hat{\psi}_{z,t+1|t} - \mu \hat{\pi}_{t+1|t}^d &= \mu \hat{\psi}_{zt} - \left(\mu - \beta \tau^k \right) \hat{R}_t + \mu \hat{\mu}_{z,t+1|t} - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \hat{\tau}_{t+1|t}^k \\ \hat{\mu}_{z,t+1|t} &= \rho_{\mu_z} \hat{\mu}_{zt} \end{aligned}$$

$$\begin{aligned} 0 &= -\mu \hat{\psi}_{zt} + \mu \hat{\psi}_{z,t+1|t} - \mu \hat{\mu}_{z,t+1|t} + \left(\mu - \beta \tau^k \right) \hat{R}_t - \mu \hat{\pi}_{t+1|t}^d + \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \hat{\tau}_{t+1|t}^k \\ \hat{\mu}_{z,t+1|t} &= \rho_{\mu_z} \hat{\mu}_{zt} \\ \hat{\tau}_{t+1|t}^k &= \tilde{\Theta}_1(1, 1) \hat{\tau}_t^k + \tilde{\Theta}_1(1, 2) \hat{\tau}_t^y + \tilde{\Theta}_1(1, 3) \hat{\tau}_t^c + \tilde{\Theta}_1(1, 4) \hat{\tau}_t^w + \tilde{\Theta}_1(1, 5) \hat{g}_t \\ &\quad + \tilde{\Theta}_2(1, 1) \hat{\tau}_{t-1}^k + \tilde{\Theta}_2(1, 2) \hat{\tau}_{t-1}^y + \tilde{\Theta}_2(1, 3) \hat{\tau}_{t-1}^c + \tilde{\Theta}_2(1, 4) \hat{\tau}_{t-1}^w + \tilde{\Theta}_2(1, 5) \hat{g}_{t-1} \end{aligned}$$

$$\begin{aligned} \mu \hat{\psi}_{z,t+1|t} - \mu \hat{\pi}_{t+1|t}^d &= \mu \hat{\psi}_{zt} + \mu \rho_{\mu_z} \hat{\mu}_{zt} - \left(\mu - \beta \tau^k \right) \hat{R}_t \\ &\quad - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_1(1, 1) \hat{\tau}_t^k - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_1(1, 2) \hat{\tau}_t^y \\ &\quad - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_1(1, 3) \hat{\tau}_t^c - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_1(1, 4) \hat{\tau}_t^w \\ &\quad - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_1(1, 5) \hat{g}_t - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_2(1, 1) \hat{\tau}_{t-1}^k \\ &\quad - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_2(1, 2) \hat{\tau}_{t-1}^y - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_2(1, 3) \hat{\tau}_{t-1}^c \\ &\quad - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_2(1, 4) \hat{\tau}_{t-1}^w - \frac{\tau^k}{1 - \tau^k} (\beta - \mu) \tilde{\Theta}_2(1, 5) \hat{g}_{t-1} \end{aligned}$$

$$\hat{\pi}_{t+1|t}^d : H(9, 1) = -\mu$$

$$\hat{\psi}_{z,t+1|t} : H(9, 9) = \mu$$

$$\hat{\psi}_{zt} : A_{22}(9, 9) = \mu$$

$$\hat{R}_t : B_2(9, 1) = -\left(\mu - \beta \tau^k \right)$$

$$\begin{aligned}
\hat{\mu}_{zt} & : A_{21}(9, 3) = \mu\rho\mu_z \\
\hat{\tau}_t^k & : A_{21}(9, 20) = -\frac{\tau^k}{1-\tau^k}(\beta-\mu)\tilde{\Theta}_1(1, 1) \\
\hat{\tau}_t^y & : A_{21}(9, 21) = -\frac{\tau^k}{1-\tau^k}(\beta-\mu)\tilde{\Theta}_1(1, 2) \\
\hat{\tau}_t^c & : A_{21}(9, 22) = -\frac{\tau^k}{1-\tau^k}(\beta-\mu)\tilde{\Theta}_1(1, 3) \\
\hat{\tau}_t^w & : A_{21}(9, 23) = -\frac{\tau^k}{1-\tau^k}(\beta-\mu)\tilde{\Theta}_1(1, 4) \\
\hat{g}_t & : A_{21}(9, 24) = -\frac{\tau^k}{1-\tau^k}(\beta-\mu)\tilde{\Theta}_1(1, 5) \\
\hat{\tau}_{t-1}^k & : A_{21}(9, 25) = -\frac{\tau^k}{1-\tau^k}(\beta-\mu)\tilde{\Theta}_2(1, 1) \\
\hat{\tau}_{t-1}^y & : A_{21}(9, 26) = -\frac{\tau^k}{1-\tau^k}(\beta-\mu)\tilde{\Theta}_2(1, 2) \\
\hat{\tau}_{t-1}^c & : A_{21}(9, 27) = -\frac{\tau^k}{1-\tau^k}(\beta-\mu)\tilde{\Theta}_2(1, 3) \\
\hat{\tau}_{t-1}^w & : A_{21}(9, 28) = -\frac{\tau^k}{1-\tau^k}(\beta-\mu)\tilde{\Theta}_2(1, 4) \\
\hat{g}_{t-1} & : A_{21}(9, 29) = -\frac{\tau^k}{1-\tau^k}(\beta-\mu)\tilde{\Theta}_2(1, 5)
\end{aligned}$$

Row 10 - Price of capital

$$\begin{aligned}
0 & = \hat{\psi}_{zt} + \hat{\mu}_{z,t+1|t} - \hat{\psi}_{z,t+1|t} - \frac{\beta(1-\tilde{\delta})}{\mu_z}\hat{P}_{k,t+1|t} + \hat{P}_{kt} - \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\hat{r}_{t+1|t}^k \\
& \quad + \frac{\tau^k}{1-\tau^k}\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}\hat{\tau}_{t+1|t}^k, \\
\hat{r}_{t+1|t}^k & = \rho\mu_z\hat{\mu}_{zt} + \hat{w}_{t+1|t} + \frac{\nu R}{v(R-1)+1}\hat{R}_t + \frac{\nu(R-1)}{v(R-1)+1}\rho\nu\hat{\nu}_t + \hat{H}_{t+1|t} - \hat{k}_{t+1|t} \\
\hat{\tau}_{t+1|t}^k & = \tilde{\Theta}_1(1, 1)\hat{\tau}_t^k + \tilde{\Theta}_1(1, 2)\hat{\tau}_t^y + \tilde{\Theta}_1(1, 3)\hat{\tau}_t^c + \tilde{\Theta}_1(1, 4)\hat{\tau}_t^w + \tilde{\Theta}_1(1, 5)\hat{g}_t \\
& \quad + \tilde{\Theta}_2(1, 1)\hat{\tau}_{t-1}^k + \tilde{\Theta}_2(1, 2)\hat{\tau}_{t-1}^y + \tilde{\Theta}_2(1, 3)\hat{\tau}_{t-1}^c + \tilde{\Theta}_2(1, 4)\hat{\tau}_{t-1}^w + \tilde{\Theta}_2(1, 5)\hat{g}_{t-1}
\end{aligned}$$

$$\begin{aligned}
& \hat{\psi}_{z,t+1|t} + \frac{\beta(1-\tilde{\delta})}{\mu_z} \hat{P}_{k,t+1|t} + \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \hat{w}_{t+1|t} + \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \hat{H}_{t+1|t} \\
& - \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \hat{k}_{t+1|t} \\
= & \hat{\psi}_{zt} + \left[\rho_{\mu_z} - \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \rho_{\mu_z} \right] \hat{\mu}_{zt} + \hat{P}_{kt} - \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \frac{\nu R}{v(R-1)+1} \hat{R}_t \\
& - \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \frac{\nu(R-1)}{v(R-1)+1} \rho_{\nu} \hat{\nu}_t \\
& + \frac{\tau^k}{1-\tau^k} \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \tilde{\Theta}_1(1,1) \hat{\tau}_t^k + \frac{\tau^k}{1-\tau^k} \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \tilde{\Theta}_1(1,2) \hat{\tau}_t^y \\
& + \frac{\tau^k}{1-\tau^k} \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \tilde{\Theta}_1(1,3) \hat{\tau}_t^c + \frac{\tau^k}{1-\tau^k} \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \tilde{\Theta}_1(1,4) \hat{\tau}_t^w \\
& + \frac{\tau^k}{1-\tau^k} \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \tilde{\Theta}_1(1,5) \hat{g}_t + \frac{\tau^k}{1-\tau^k} \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \tilde{\Theta}_2(1,1) \hat{\tau}_{t-1}^k \\
& + \frac{\tau^k}{1-\tau^k} \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \tilde{\Theta}_2(1,2) \hat{\tau}_{t-1}^y + \frac{\tau^k}{1-\tau^k} \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \tilde{\Theta}_2(1,3) \hat{\tau}_{t-1}^c \\
& + \frac{\tau^k}{1-\tau^k} \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \tilde{\Theta}_2(1,4) \hat{\tau}_{t-1}^w + \frac{\tau^k}{1-\tau^k} \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z} \tilde{\Theta}_2(1,5) \hat{g}_{t-1}
\end{aligned}$$

$$\hat{w}_{t+1|t} : H(10,6) = \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}$$

$$\hat{\psi}_{z,t+1|t} : H(10,9) = 1$$

$$\hat{P}_{k,t+1|t} : H(10,10) = \frac{\beta(1-\tilde{\delta})}{\mu_z}$$

$$\hat{H}_{t+1|t} : H(10,12) = \frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}$$

$$\hat{u}_{t+1|t} : H(10,13) = -\frac{\mu_z - \beta(1-\tilde{\delta})}{\mu_z}$$

$$\hat{\psi}_{zt} : A_{22}(10,9) = 1$$

$$\hat{\psi}_{zt} : A_{22}(10,9) = 1$$

$$\hat{P}_{kt} : A_{22}(10,10) = 1$$

$$\begin{aligned}
\hat{R}_t &: B_2(10, 1) = -\frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \frac{\nu R}{v(R-1) + 1} \\
\hat{\mu}_{zt} &: A_{21}(10, 3) = \rho_{\mu_z} - \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \rho_{\mu_z} \\
\hat{\nu}_t &: A_{21}(10, 5) = -\frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \frac{\nu(R-1)}{v(R-1) + 1} \rho_{\nu} \\
\hat{\tau}_t^k &: A_{21}(10, 20) = \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \tilde{\Theta}_1(1, 1) \\
\hat{\tau}_t^y &: A_{21}(10, 21) = \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \tilde{\Theta}_1(1, 2) \\
\hat{\tau}_t^c &: A_{21}(10, 22) = \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \tilde{\Theta}_1(1, 3) \\
\hat{\tau}_t^w &: A_{21}(10, 23) = \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \tilde{\Theta}_1(1, 4) \\
\hat{g}_t &: A_{21}(10, 24) = \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \tilde{\Theta}_1(1, 5) \\
\hat{\tau}_{t-1}^k &: A_{21}(10, 25) = \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \tilde{\Theta}_2(1, 1) \\
\hat{\tau}_{t-1}^y &: A_{21}(10, 26) = \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \tilde{\Theta}_2(1, 2) \\
\hat{\tau}_{t-1}^c &: A_{21}(10, 27) = \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \tilde{\Theta}_2(1, 3) \\
\hat{\tau}_{t-1}^w &: A_{21}(10, 28) = \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \tilde{\Theta}_2(1, 4) \\
\hat{g}_{t-1} &: A_{21}(10, 29) = \frac{\tau^k}{1 - \tau^k} \frac{\mu_z - \beta(1 - \tilde{\delta})}{\mu_z} \tilde{\Theta}_2(1, 5)
\end{aligned}$$

Row 11 - Change in nominal exchange rate (UIP)

$$(1 - \tilde{\phi}_s) E_t \Delta \hat{S}_{t+1} = \tilde{\phi}_s \Delta \hat{S}_t + (\hat{R}_t - \hat{R}_t^*) + \tilde{\phi}_a \hat{a}_t - \hat{\phi}_t$$

$$\Delta \hat{S}_{t+1} : H(11, 11) = (1 - \tilde{\phi}_s)$$

$$\Delta \hat{S}_t : A_{22}(11, 11) = \tilde{\phi}_s$$

$$\hat{a}_t : A_{22}(11, 16) = \tilde{\phi}_a$$

$$\hat{R}_t : B_2(11, 1) = 1$$

$$\hat{\phi}_t : A_{21}(11, 12) = -1$$

$$\hat{R}_t^* : A_{21}(11, 32) = -1$$

Row 12 - Hours worked (aggregate resource constraint)

$$(1 - \omega_c) \left(\gamma^{cd} \right)^{\eta_c} \frac{c}{y} \left(\hat{c}_t + \eta_c \hat{\gamma}_t^{cd} \right) + (1 - \omega_i) \left(\gamma^{id} \right)^{\eta_i} \frac{\tilde{i}}{y} \left(\hat{i}_t + \eta_i \hat{\gamma}_t^{id} \right) + \frac{g}{y} \hat{g}_t + \frac{y^*}{y} \left(\hat{y}_t^* - \eta_f \hat{\gamma}_t^{x*} + \hat{z}_t^* \right)$$

$$= \lambda_d \left[\hat{\varepsilon}_t + \alpha \left(\hat{k}_t - \hat{\mu}_{zt} \right) + (1 - \alpha) \hat{H}_t \right] - \left(1 - \tau^k \right) r^k \frac{\bar{k}}{y \mu_z} \left(\hat{k}_t - \bar{k}_t \right)$$

$$0 = (1 - \omega_c) \left(\gamma^{cd} \right)^{\eta_c} \frac{c}{y} \left(\hat{c}_t + \eta_c \hat{\gamma}_t^{cd} \right) + (1 - \omega_i) \left(\gamma^{id} \right)^{\eta_i} \frac{\tilde{i}}{y} \left(\hat{i}_t + \eta_i \hat{\gamma}_t^{id} \right) + \frac{g}{y} \hat{g}_t + \frac{y^*}{y} \left(\hat{y}_t^* - \eta_f \hat{\gamma}_t^{x*} + \hat{z}_t^* \right)$$

$$- \lambda_d \left[\hat{\varepsilon}_t + \alpha \left(\hat{k}_t - \hat{\mu}_{zt} \right) + (1 - \alpha) \hat{H}_t \right] + \left(1 - \tau^k \right) r^k \frac{\bar{k}}{y \mu_z} \left(\hat{k}_t - \bar{k}_t \right)$$

$$\hat{\gamma}_t^{cd} = \omega_c \left(\gamma^{cmc} \right)^{-(1-\eta_c)} \hat{\gamma}_t^{mcd}$$

$$\hat{\gamma}_t^{i,d} = \omega_i \left(\gamma^{i,mi} \right)^{-(1-\eta_i)} \hat{\gamma}_t^{mi,d}$$

$$0 = (1 - \omega_c) \left(\gamma^{cd} \right)^{\eta_c} \frac{c}{y} \hat{c}_t + (1 - \omega_c) \left(\gamma^{cd} \right)^{\eta_c} \frac{c}{y} \eta_c \omega_c \left(\gamma^{cmc} \right)^{-(1-\eta_c)} \hat{\gamma}_t^{mcd} + (1 - \omega_i) \left(\gamma^{id} \right)^{\eta_i} \frac{\tilde{i}}{y} \hat{i}_t$$

$$+ (1 - \omega_i) \left(\gamma^{id} \right)^{\eta_i} \frac{\tilde{i}}{y} \eta_i \omega_i \left(\gamma^{i,mi} \right)^{-(1-\eta_i)} \hat{\gamma}_t^{mi,d} - \frac{y^*}{y} \eta_f \hat{\gamma}_t^{x*} + \frac{g}{y} \hat{g}_t + \frac{y^*}{y} \hat{y}_t^* + \frac{y^*}{y} \hat{z}_t^*$$

$$- \lambda_d \hat{\varepsilon}_t + \lambda_d \alpha \hat{\mu}_{zt} - \lambda_d (1 - \alpha) \hat{H}_t - \left[\lambda_d \alpha - \left(1 - \tau^k \right) r^k \frac{\bar{k}}{y \mu_z} \right] \hat{k}_t - \left(1 - \tau^k \right) r^k \frac{\bar{k}}{y \mu_z} \bar{k}_t$$

$$\hat{c}_t : A_{22} (12, 7) = (1 - \omega_c) \left(\gamma^{cd} \right)^{\eta_c} \frac{c}{y}$$

$$\hat{i}_t : A_{22} (12, 8) = (1 - \omega_i) \left(\gamma^{id} \right)^{\eta_i} \frac{\tilde{i}}{y}$$

$$\hat{H}_t : A_{22} (12, 12) = -\lambda_d (1 - \alpha)$$

$$\hat{k}_t : A_{22} (12, 13) = - \left[\lambda_d \alpha - \left(1 - \tau^k \right) r^k \frac{\bar{k}}{y \mu_z} \right]$$

$$\hat{\gamma}_t^{mcd} : A_{22} (12, 17) = (1 - \omega_c) \left(\gamma^{cd} \right)^{\eta_c} \frac{c}{y} \eta_c \omega_c \left(\gamma^{cmc} \right)^{-(1-\eta_c)}$$

$$\hat{\gamma}_t^{mi,d} : A_{22} (12, 18) = (1 - \omega_i) \left(\gamma^{id} \right)^{\eta_i} \frac{\tilde{i}}{y} \eta_i \omega_i \left(\gamma^{i,mi} \right)^{-(1-\eta_i)}$$

$$\hat{\gamma}_t^{x*} : A_{22} (12, 19) = -\frac{y^*}{y} \eta_f$$

$$\hat{\varepsilon}_t : A_{21} (12, 1) = -\lambda_d$$

$$\hat{\mu}_{zt} : A_{21} (12, 3) = \lambda_d \alpha$$

$$\hat{z}_t^* : A_{21} (12, 14) = \frac{y^*}{y}$$

$$\hat{g}_t : A_{21} (12, 24) = \frac{g}{y}$$

$$\hat{y}_t^* : A_{21} (12, 31) = \frac{y^*}{y}$$

$$\bar{k}_t : A_{21} (12, 42) = - \left(1 - \tau^k \right) r^k \frac{\bar{k}}{y \mu_z}$$

Row 13 - Capital-services flow

$$\begin{aligned}\hat{u}_t &= \frac{1}{\sigma_a} \hat{r}_t^k - \frac{1}{\sigma_a} \frac{\tau^k}{(1 - \tau^k)} \hat{\tau}_t^k, \\ \hat{u}_t &\equiv \hat{k}_t - \widehat{\bar{k}}_t, \\ \hat{r}_t^k &= \hat{\mu}_{zt} + \widehat{w}_t + \hat{R}_t^f + \hat{H}_t - \hat{k}_t.\end{aligned}$$

$$0 = -(1 + \sigma_a) \hat{k}_t + \sigma_a \widehat{\bar{k}}_t + \hat{\mu}_{zt} + \widehat{w}_t + \hat{R}_t^f + \hat{H}_t - \frac{\tau^k}{(1 - \tau^k)} \hat{\tau}_t^k$$

$$0 = -(1 + \sigma_a) \hat{k}_t + \sigma_a \widehat{\bar{k}}_t + \widehat{w}_t + \hat{H}_t + \frac{\nu R}{v(R - 1) + 1} \hat{R}_{t-1} + \hat{\mu}_{zt} + \frac{\nu(R - 1)}{v(R - 1) + 1} \hat{\nu}_t - \frac{\tau^k}{(1 - \tau^k)} \hat{\tau}_t^k$$

$$\begin{aligned}\widehat{w}_t &: A_{22}(13, 6) = 1 \\ \hat{H}_t &: A_{22}(13, 12) = 1 \\ \hat{k}_t &: A_{22}(13, 13) = -(1 + \sigma_a)\end{aligned}$$

$$\begin{aligned}\hat{\mu}_{zt} &: A_{21}(13, 3) = 1 \\ \hat{\nu}_t &: A_{21}(13, 5) = \frac{\nu(R - 1)}{v(R - 1) + 1} \\ \hat{\tau}_t^k &: A_{21}(13, 20) = -\frac{\tau^k}{(1 - \tau^k)} \\ \widehat{\bar{k}}_t &: A_{21}(13, 42) = \sigma_a \\ \hat{R}_{t=1} &: A_{21}(13, 44) = \frac{\nu R}{v(R - 1) + 1}\end{aligned}$$

Row 14 - Real money balances

$$\hat{q}_t = \frac{1}{\sigma_q} \left[\hat{\zeta}_t^q + \frac{\tau^k}{1 - \tau^k} \hat{\tau}_t^k - \hat{\psi}_{zt} - \frac{R}{R - 1} \hat{R}_{t-1} \right],$$

$$0 = -\sigma_q \hat{q}_t - \hat{\psi}_{zt} - \frac{R}{R - 1} \hat{R}_{t-1} + \hat{\zeta}_t^q + \frac{\tau^k}{1 - \tau^k} \hat{\tau}_t^k,$$

$$\begin{aligned}\hat{\psi}_{zt} &: A_{22}(14, 9) = -1 \\ \hat{q}_t &: A_{22}(14, 14) = -\sigma_q\end{aligned}$$

$$\begin{aligned}\hat{\zeta}_t^q &: A_{21}(14, 8) = 1 \\ \hat{\tau}_t^k &: A_{21}(14, 20) = \frac{\tau^k}{1 - \tau^k} \\ \hat{R}_{t-1} &: A_{21}(14, 44) = -\frac{R}{R - 1}\end{aligned}$$

Row 15 - Loan market clearing

$$\nu\bar{w}H \left(\hat{\nu}_t + \hat{w}_t + \hat{H}_t \right) = \frac{\mu\bar{m}}{\pi^d\mu_z} \left(\hat{\mu}_t + \hat{m}_t - \hat{\pi}_t^d - \hat{\mu}_{zt} \right) - q\hat{q}_t,$$

$$0 = -\nu\bar{w}H\hat{\nu}_t - \nu\bar{w}H\hat{w}_t - \nu\bar{w}H\hat{H}_t + \frac{\mu\bar{m}}{\pi^d\mu_z}\hat{\mu}_t + \frac{\mu\bar{m}}{\pi^d\mu_z}\hat{m}_t - \frac{\mu\bar{m}}{\pi^d\mu_z}\hat{\pi}_t^d - \frac{\mu\bar{m}}{\pi^d\mu_z}\hat{\mu}_{zt} - q\hat{q}_t,$$

$$\hat{\pi}_t^d : A_{22}(15, 1) = -\frac{\mu\bar{m}}{\pi^d\mu_z}$$

$$\hat{w}_t : A_{22}(15, 6) = -\nu\bar{w}H$$

$$\hat{H}_t : A_{22}(15, 12) = -\nu\bar{w}H$$

$$\hat{q}_t : A_{22}(15, 14) = -q$$

$$\hat{\mu}_t : A_{22}(15, 15) = \frac{\mu\bar{m}}{\pi^d\mu_z}$$

$$\hat{\nu}_t : A_{21}(15, 5) = -\nu\bar{w}H$$

$$\hat{\mu}_{zt} : A_{21}(15, 3) = -\frac{\mu\bar{m}}{\pi^d\mu_z}$$

$$\hat{m}_t : A_{21}(15, 43) = \frac{\mu\bar{m}}{\pi^d\mu_z}$$

Row 16 - Net foreign assets

$$\begin{aligned} \hat{a}_t &= -y^*\widehat{mc}_t^x - \eta_f y_t^* \hat{\gamma}_t^{x*} + y^* \hat{y}_t^* + y^* \hat{z}_t^* + (c^m + \tilde{i}^m) \hat{\gamma}_t^f \\ &\quad - c^m \hat{c}_t + c^m \eta_c (1 - \omega_c) \left(\gamma^{cd} \right)^{-(1-\eta_c)} \hat{\gamma}_t^{mcd} \\ &\quad - \tilde{i}^m \hat{i}_t + \tilde{i}^m \eta_i (1 - \omega_i) \left(\gamma^{id} \right)^{-(1-\eta_i)} \hat{\gamma}_t^{mid} + \frac{R}{\pi\mu_z} \hat{a}_{t-1}, \\ \hat{\gamma}_t^f &= \widehat{mc}_t^x + \hat{\gamma}_t^{x,*} \end{aligned}$$

$$\begin{aligned} 0 &= -\hat{a}_t - [y^* - (c^m + \tilde{i}^m)] \widehat{mc}_t^x - [\eta_f y_t^* - (c^m + \tilde{i}^m)] \hat{\gamma}_t^{x,*} + y^* \hat{y}_t^* + y^* \hat{z}_t^* \\ &\quad - c^m \hat{c}_t + c^m \eta_c (1 - \omega_c) \left(\gamma^{cd} \right)^{-(1-\eta_c)} \hat{\gamma}_t^{mcd} \\ &\quad - \tilde{i}^m \hat{i}_t + \tilde{i}^m \eta_i (1 - \omega_i) \left(\gamma^{id} \right)^{-(1-\eta_i)} \hat{\gamma}_t^{mid} + \frac{R}{\pi\mu_z} \hat{a}_{t-1}, \end{aligned}$$

$$\begin{aligned}
\hat{c}_t & : A_{22}(16, 7) = -c^m \\
\hat{i}_t & : A_{22}(16, 8) = -\tilde{i}^m \\
\hat{a}_t & : A_{22}(16, 16) = -1 \\
\hat{\gamma}_t^{mcd} & : A_{22}(16, 17) = c^m \eta_c (1 - \omega_c) \left(\gamma^{cd} \right)^{-(1-\eta_c)} \\
\hat{\gamma}_t^{mid} & : A_{22}(16, 18) = \tilde{i}^m \eta_i (1 - \omega_i) \left(\gamma^{id} \right)^{-(1-\eta_i)} \\
\hat{\gamma}_t^{x*} & : A_{22}(16, 19) = -[\eta_f y_t^* + (c^m + \tilde{i}^m)] \\
\widehat{mc}_t^x & : A_{22}(16, 21) = -[y^* + (c^m + \tilde{i}^m)] \\
\hat{z}_t^* & : A_{21}(16, 14) = y^* \\
\hat{y}_t^* & : A_{21}(16, 31) = y^* \\
\hat{a}_{t-1} & : A_{21}(16, 60) = \frac{R}{\pi \mu_z}
\end{aligned}$$

Row 17 - Relative price - imported cons vs. domestic

$$0 = -\hat{\gamma}_t^{mcd} + \hat{\gamma}_{t-1}^{mcd} + \hat{\pi}_t^{mc} - \hat{\pi}_t^d$$

$$\begin{aligned}
\hat{\pi}_t^d & : A_{22}(17, 1) = -1 \\
\hat{\pi}_t^{mc} & : A_{22}(17, 2) = 1 \\
\hat{\gamma}_t^{mcd} & : A_{22}(17, 17) = -1
\end{aligned}$$

$$\hat{\gamma}_{t-1}^{mcd} : A_{21}(17, 61) = 1$$

Row 18 - Relative price - imported investment vs. domestic

$$0 = -\hat{\gamma}_t^{mid} + \hat{\gamma}_{t-1}^{mid} + \hat{\pi}_t^{mi} - \hat{\pi}_t^d$$

$$\begin{aligned}
\hat{\pi}_t^d & : A_{22}(18, 1) = -1 \\
\hat{\pi}_t^{mi} & : A_{22}(18, 3) = 1 \\
\hat{\gamma}_t^{mid} & : A_{22}(18, 18) = -1
\end{aligned}$$

$$\hat{\gamma}_{t-1}^{mid} : A_{21}(18, 62) = 1$$

Row 19 - Relative price - export vs. foreign

$$0 = -\hat{\gamma}_t^{x*} + \hat{\gamma}_{t-1}^{x*} + \hat{\pi}_t^x - \hat{\pi}_t^*$$

$$\begin{aligned}
\hat{\pi}_t^x & : A_{22}(19, 5) = 1 \\
\hat{\gamma}_t^{x*} & : A_{22}(19, 19) = -1 \\
\hat{\pi}_t^* & : A_{21}(19, 30) = -1 \\
\hat{\gamma}_{t-1}^{x*} & : A_{21}(19, 63) = 1
\end{aligned}$$

Row 20 - Real exchange rate

$$\begin{aligned}\tilde{\hat{x}}_t &= -\omega_c(\gamma^{cmc})^{-(1-\eta_c)}\hat{\gamma}_t^{mcd} - \hat{\gamma}_t^{x,*} - \widehat{\text{mc}}_t^x \\ 0 &= -\tilde{\hat{x}}_t - \omega_c(\gamma^{cmc})^{-(1-\eta_c)}\hat{\gamma}_t^{mcd} - \hat{\gamma}_t^{x,*} - \widehat{\text{mc}}_t^x\end{aligned}$$

$$\begin{aligned}\widehat{\text{mc}}_t^x &= \widehat{\text{mc}}_{t-1}^x + \hat{\pi}_t^d - \hat{\pi}_t^x - \Delta\widehat{S}_t \\ \hat{\gamma}_t^{x,*} &= \hat{\gamma}_{t-1}^{x,*} + \hat{\pi}_t^x - \hat{\pi}_t^* \\ \hat{\gamma}_t^{mcd} &= \hat{\gamma}_{t-1}^{mcd} + \hat{\pi}_t^{mc} - \hat{\pi}_t^d\end{aligned}$$

$$\begin{aligned}\hat{\gamma}_t^{mcd} &: A_{22}(20,17) = -\omega_c(\gamma^{cmc})^{-(1-\eta_c)} \\ \hat{\gamma}_t^{x,*} &: A_{22}(20,19) = -1 \\ \tilde{\hat{x}}_t &: A_{22}(20,20) = -1 \\ \widehat{\text{mc}}_t^x &: A_{22}(20,21) = -1\end{aligned}$$

Row 21 - Marginal cost export

$$0 = -\widehat{\text{mc}}_t^x + \widehat{\text{mc}}_{t-1}^x + \hat{\pi}_t^d - \hat{\pi}_t^x - \Delta\widehat{S}_t$$

$$\begin{aligned}\hat{\pi}_t^d &: A_{22}(21,1) = 1 \\ \hat{\pi}_t^x &: A_{22}(21,5) = -1 \\ \Delta\widehat{S}_t &: A_{22}(21,11) = -1 \\ \widehat{\text{mc}}_t^x &: A_{22}(21,21) = -1\end{aligned}$$

$$\widehat{\text{mc}}_{t-1}^x : A_{21}(21,65) = 1$$

Row 22 - Physical capital stock (note that \widehat{k}_{t+1} is predetermined variable no. 42 in period $t+1$ and forward-looking variable no. 22 in period t):

$$0 = -\widehat{k}_{t+1} + (1-\tilde{\delta})\frac{1}{\mu_z}\widehat{k}_t - (1-\tilde{\delta})\frac{1}{\mu_z}\hat{\mu}_{zt} + \left[1 - (1-\tilde{\delta})\frac{1}{\mu_z}\right]\hat{Y}_t + \left[1 - (1-\tilde{\delta})\frac{1}{\mu_z}\right]\hat{v}_t,$$

$$\begin{aligned}\widehat{k}_{t+1} &: A_{22}(22,22) = -1, \\ \widehat{k}_t &: A_{21}(22,42) = (1-\tilde{\delta})\frac{1}{\mu_z}, \\ \hat{\mu}_{zt} &: A_{21}(22,3) = -(1-\tilde{\delta})\frac{1}{\mu_z}, \\ \hat{Y}_t &: \left[A_{21}(22,13) = 1 - (1-\tilde{\delta})\frac{1}{\mu_z}\right], \\ \hat{Y}_t &: A_{21}(22,13) = \tilde{S}''\mu_z^2(1+\beta)\left(1 - (1-\tilde{\delta})\frac{1}{\mu_z}\right) \quad (\text{rescaled shock!}) \\ \hat{v}_t &: A_{22}(22,8) = 1 - (1-\tilde{\delta})\frac{1}{\mu_z}.\end{aligned}$$

Row 23 - Financial assets (note that \widehat{m}_{t+1} is predetermined variable no. 43 in period $t+1$ and forward-looking variable no. 23 in period t)

$$0 = -\widehat{m}_{t+1} + \widehat{\mu}_t - \widehat{\mu}_{zt} - \widehat{\pi}_t^d + \widehat{m}_t$$

$$\widehat{m}_{t+1} : A_{22}(23, 23) = -1$$

$$\widehat{\mu}_{zt} : A_{21}(23, 3) = -1$$

$$\widehat{m}_t : A_{21}(23, 43) = 1$$

$$\widehat{\pi}_t^d : A_{22}(23, 1) = -1$$

$$\widehat{\mu}_t : A_{22}(23, 15) = 1$$

1.3. Specifying D

We also specify the target variables, for convenience including some variables of interest which may have zero weight in the loss function. They are the quarterly and four-quarterly CPI inflation gaps relative to steady-state inflation, which equals the official inflation target; the output gap relative to steady-state output (we may also use the gap relative to potential output, meaning the flexprice and flexwage output level); the quarterly interest rate; the quarterly interest-rate differential; and the real exchange rate. The inflation and interest rates are all measured at an annual rate. Then the vector of target variables, Y_t , and the corresponding matrix D satisfy

$$Y_t \equiv [4\widehat{\pi}_t^{cpi}, \overline{\widehat{\pi}}_t^{cpi}, \widehat{y}_t, 4\widehat{R}_t, 4(\widehat{R}_t - \widehat{R}_{t-1}), \widehat{x}_t]' \equiv D \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix},$$

where

$$\overline{\widehat{\pi}}_t^{cpi} = \widehat{\pi}_t^{cpi} + \widehat{\pi}_{t-1}^{cpi} + \widehat{\pi}_{t-2}^{cpi} + \widehat{\pi}_{t-3}^{cpi}.$$

Quarterly CPI inflation gap:

$$4\widehat{\pi}_t^{cpi} = 4[(1 - \omega_c)(\gamma^{cd})^{-(1-\eta_c)}\widehat{\pi}_t^d + \omega_c(\gamma^{cmc})^{-(1-\eta_c)}\widehat{\pi}_t^{mc}], \quad (1.4)$$

$$\widehat{\pi}_t^d : D(1, n_X + 1) = 4(1 - \omega_c)(\gamma^{cd})^{-(1-\eta_c)},$$

$$\widehat{\pi}_t^{mc} : D(1, n_X + 2) = 4\omega_c(\gamma^{cmc})^{-(1-\eta_c)}.$$

Four-quarter CPI inflation gap:

$$\overline{\widehat{\pi}}_t^{cpi} = \widehat{\pi}_t^{cpi} + \widehat{\pi}_{t-1}^{cpi} + \widehat{\pi}_{t-2}^{cpi} + \widehat{\pi}_{t-3}^{cpi}. \quad (1.5)$$

$$\widehat{\pi}_t^d : D(2, n_X + 1) = (1 - \omega_c)(\gamma^{cd})^{-(1-\eta_c)},$$

$$\widehat{\pi}_{t-1}^d : D(2, 45) = (1 - \omega_c)(\gamma^{cd})^{-(1-\eta_c)},$$

$$\widehat{\pi}_{t-2}^d : D(2, 68) = (1 - \omega_c)(\gamma^{cd})^{-(1-\eta_c)},$$

$$\widehat{\pi}_{t-3}^d : D(2, 70) = (1 - \omega_c)(\gamma^{cd})^{-(1-\eta_c)},$$

$$\widehat{\pi}_t^{mc} : D(2, n_X + 2) = \omega_c(\gamma^{cmc})^{-(1-\eta_c)},$$

$$\widehat{\pi}_{t-1}^{mc} : D(2, 46) = \omega_c(\gamma^{cmc})^{-(1-\eta_c)},$$

$$\widehat{\pi}_{t-2}^{mc} : D(2, 69) = \omega_c(\gamma^{cmc})^{-(1-\eta_c)},$$

$$\widehat{\pi}_{t-3}^{mc} : D(2, 71) = \omega_c(\gamma^{cmc})^{-(1-\eta_c)}.$$

Alternatively, we could incorporate the exogenous time-varying inflation target in the inflation gap,¹

$$Y_t \equiv [4(\hat{\pi}_t^{cpi} - \hat{\pi}_t^c), \overline{\hat{\pi}}_t^{cpi} - 4\hat{\pi}_t^c, \hat{y}_t, 4\hat{R}_t, 4(\hat{R}_t - \hat{R}_{t-1}), \hat{x}_t]' \equiv D \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix},$$

in which case

$$\begin{aligned} \hat{\pi}_t^c &: D(1, 18) = -4, \\ \overline{\hat{\pi}}_t^c &: D(2, 18) = -4. \end{aligned}$$

Output gap,

$$\hat{y}_t : D(3, n_X + 4) = 1.$$

Interest rate,

$$\hat{R}_t : D(4, n_X + n_x + n_i) = 4.$$

Interest-rate differential,

$$\begin{aligned} \hat{R}_t &: D(5, n_X + n_x + n_i) = 4, \\ \hat{R}_{t-1} &: D(5, 44) = -4. \end{aligned}$$

Real exchange rate,

$$\hat{x}_t : D(6, n_X + 20) = 1.$$

1.4. Defining f_X and f_x - Interest rate rule

$$i_t = f_X \check{X}_t + f_x \check{x}_t$$

$$\begin{aligned} \hat{R}_t &= \rho_R \hat{R}_{t-1} + (1 - \rho_R) \left[\hat{\pi}_t^c + r_\pi (\hat{\pi}_{t-1}^c - \hat{\pi}_t^c) + r_y \hat{y}_{t-1} + r_x \hat{x}_{t-1} \right] \\ &\quad + r_{\Delta\pi} (\hat{\pi}_t^c - \hat{\pi}_{t-1}^c) + r_{\Delta y} (\hat{y}_t - \hat{y}_{t-1}) + \varepsilon_{Rt} \end{aligned}$$

$$\begin{aligned} \hat{R}_t &= \rho_R \hat{R}_{t-1} + (1 - \rho_R) \hat{\pi}_t^c + (1 - \rho_R) r_\pi \hat{\pi}_{t-1}^c - (1 - \rho_R) r_\pi \hat{\pi}_t^c \\ &\quad + (1 - \rho_R) r_y \hat{y}_{t-1} + (1 - \rho_R) r_x \hat{x}_{t-1} + r_{\Delta\pi} \hat{\pi}_t^c - r_{\Delta\pi} \hat{\pi}_{t-1}^c + r_{\Delta y} \hat{y}_t - r_{\Delta y} \hat{y}_{t-1} + \varepsilon_{Rt} \\ &= \rho_R \hat{R}_{t-1} + (1 - \rho_R) [1 - r_\pi] \hat{\pi}_t^c + r_{\Delta\pi} (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \hat{\pi}_t^d \\ &\quad + r_{\Delta\pi} (\omega_c) (\gamma^{cmc})^{-(1-\eta_c)} \hat{\pi}_t^{mc} \\ &\quad + [(1 - \rho_R) r_\pi - r_{\Delta\pi}] (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \hat{\pi}_{t-1}^d \\ &\quad + [(1 - \rho_R) r_\pi - r_{\Delta\pi}] \omega_c (\gamma^{cmc})^{-(1-\eta_c)} \hat{\pi}_{t-1}^{mc} \\ &\quad + r_{\Delta y} \hat{y}_t + [(1 - \rho_R) r_y - r_{\Delta y}] \hat{y}_{t-1} + (1 - \rho_R) r_x \hat{x}_{t-1} + \varepsilon_{Rt} \end{aligned}$$

¹ Note that one could alternative replace $4\hat{\pi}_t^c$ by $\hat{\pi}_t^c + \hat{\pi}_{t-1}^c + \hat{\pi}_{t-2}^c + \hat{\pi}_{t-3}^c$ and consider the latter the time-varying four-quarter inflation target.

$$\begin{aligned}
\hat{R}_{t-1} &: f_X(1, 44) = \rho_R \\
\hat{\pi}_t^c &: f_X(1, 18) = (1 - \rho_R)[1 - r_\pi] \\
\hat{\pi}_t^d &: f_x(1, 1) = r_{\Delta\pi}(1 - \omega_c)(\gamma^{cd})^{-(1-\eta_c)} \\
\hat{\pi}_t^{mc} &: f_x(1, 2) = r_{\Delta\pi}\omega_c(\gamma^{cmc})^{-(1-\eta_c)} \\
\hat{\pi}_{t-1}^d &: f_X(1, 45) = [(1 - \rho_R)r_\pi - r_{\Delta\pi}](1 - \omega_c)(\gamma^{cd})^{-(1-\eta_c)} \\
\hat{\pi}_{t-1}^{mc} &: f_X(1, 46) = [(1 - \rho_R)r_\pi - r_{\Delta\pi}](\omega_c)(\gamma^{cmc})^{-(1-\eta_c)} \\
\hat{y}_t &: f_x(1, 4) = r_{\Delta y} \\
\hat{y}_t^{flex} &: f_x(1, n_x + 4) = -r_{\Delta y} \\
\hat{y}_{t-1} &: f_X(1, 48) = (1 - \rho_R)r_y - r_{\Delta y} \\
\hat{y}_{t-1}^{flex} &: f_X(1, n_X + 48) = -((1 - \rho_R)r_y - r_{\Delta y}) \\
\hat{x}_{t-1} &: f_X(1, 64) = (1 - \rho_R)r_x \\
\varepsilon_{Rt} &: f_X(1, 17) = 1
\end{aligned}$$

2. Potential output under flexible prices and wages - expanding the state

The flexible price and wage model is parameterized under i) $\xi^d = \xi^{mc} = \xi^{mi} = \xi^x = \xi^w = 0$, ii) setting the four markup as well as the fiscal shocks to zero ($\sigma_{\varepsilon_\lambda} = \sigma_{\varepsilon_{\lambda^{mc}}} = \sigma_{\varepsilon_{\lambda^{mi}}} = \sigma_{\varepsilon_{\lambda^x}} = 0$, and $\Theta_0^{-1}S_\tau = 0_{5 \times 5}$, respectively), and iii) we solve the system under the assumption that $\hat{\pi}^{cpi} = 0$. The flexprice model can be written

$$\begin{bmatrix} X_{t+1} \\ H^f x_{t+1|t} \end{bmatrix} = A^f \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix} + \begin{bmatrix} C \\ 0_{n_x \times n_\varepsilon} \\ 0_{n_i \times n_\varepsilon} \end{bmatrix} \varepsilon_{t+1}, \quad (2.1)$$

where,

$$H^f \equiv \begin{bmatrix} H \\ 0_{1 \times n_x} \end{bmatrix}, \quad A^f \equiv \begin{bmatrix} A_{11} & A_{12} & B_1 \\ A_{21} & A_{22} & B_2 \\ 0_{1 \times n_X} & e_1 & 0 \end{bmatrix},$$

and

$$\begin{aligned}
\hat{\pi}_t^d &: e_1(1, 1) = (1 - \omega_c)(\gamma^{cd})^{-(1-\eta_c)}, \\
\hat{\pi}_t^{mc} &: e_1(1, 2) = \omega_c(\gamma^{cmc})^{-(1-\eta_c)}.
\end{aligned}$$

This implies the following solution

$$\begin{aligned}
X_t^{flex} &= M^{flex} X_{t-1}^{flex} + C^{flex} \varepsilon_t, \\
\tilde{x}_t^{flex} &= \tilde{F}^{flex} \tilde{X}_t^{flex}, \\
&= \tilde{F}^{flex} M^{flex} X_{t-1}^{flex} + \tilde{F}^{flex} C^{flex} \varepsilon_t,
\end{aligned}$$

where $\tilde{x}_t^{flex} = \begin{bmatrix} x_t^{flex} \\ i_t^{flex} \end{bmatrix}$ and $\tilde{F}^{flex} = [F_x^{flex} \quad F_i^{flex}]'$.

We expand the state of the distorted economy with the predetermined state variables in the flexprice model so that $\dot{X}_t = (X', X_t^{flex'})'$, which implies

$$\dot{X}_{t+1} = \dot{A}_{11} \dot{X}_t + \dot{A}_{12} x_t + \dot{B}_1 i_t + \dot{C} \varepsilon_{t+1},$$

$$\dot{H}x_{t+1|t} = \dot{A}_{21}\dot{X}_t + \dot{A}_{22}x_t + \dot{B}_2i_t,$$

where $\dot{A}_{11} = \begin{bmatrix} A_{11} & 0_{nX*nX} \\ 0_{nX*nX} & M^{flex} \end{bmatrix}$, $\dot{A}_{12} = \begin{bmatrix} A_{12} \\ 0_{nX*nX} \end{bmatrix}$,

$$\dot{A}_{21} = \begin{bmatrix} A_{21} & 0_{nX*nX} \end{bmatrix}, \dot{A}_{22} = A_{22}, \dot{B}_1 = \begin{bmatrix} B_1 \\ 0_{nX*ni} \end{bmatrix}, \dot{B}_2 = B_2,$$

$$\dot{C} = \begin{bmatrix} C \\ C^{flex} \end{bmatrix}, \text{ and } \dot{H} = H.$$

We also need to change the target variable matrix to include the flexprice output gap

$$Y_t \equiv [4\hat{\pi}_t^{cpi}, \bar{\pi}_t^{cpi}, \bar{\pi}_t^d, (y_t - y_t^{flex}), 4\hat{R}_t, 4(\hat{R}_t - \hat{R}_{t-1}), \hat{x}_t]' \equiv \dot{D} \begin{bmatrix} X_t \\ X_t^{flex} \\ x_t \\ i_t \end{bmatrix}$$

$$\dot{D} = \begin{bmatrix} D_X & D_{X^{flex}} & D_x & D_i \end{bmatrix}'$$

where D above has been partitioned such that $D = \begin{bmatrix} D_X & D_x & D_i \end{bmatrix}'$ and $D_{X^{flex}}$ is defined so that

$$\hat{y}_t^{flex} : D_{X^{flex}}(4, n_X + 4) = -F_{x,4}^{flex}.$$

3. The measurement equation and the definition of the matrices \bar{D}_0 , \bar{D} , and \bar{D}_s

Let $s_t \equiv (X_t', \Xi_{t-1}', x_t', i_t)'$ and call s_t the state of the economy (although it includes nonpredetermined variables). The solution to the model under optimal policy under commitment can be written in Klein form as

$$s_{t+1} \equiv \begin{bmatrix} \tilde{X}_{t+1} \\ x_{t+1} \\ i_{t+1} \end{bmatrix} = \begin{bmatrix} M & 0 & 0 \\ f_x M & 0 & 0 \\ f_i M & 0 & 0 \end{bmatrix} s_t + \begin{bmatrix} I \\ f_x \\ f_i \end{bmatrix} \tilde{C}\varepsilon_{t+1}.$$

As noted in Adolfson et al. [1, Appendix C], it can also be written in AIM form as

$$s_{t+1} = \bar{B}s_t + \bar{C}\varepsilon_{t+1}, \tag{3.1}$$

where

$$\bar{C} \equiv \Phi\Psi$$

and the innovation $\bar{C}\varepsilon_t$ has the distribution $N(0, Q)$ with $Q \equiv \text{E}[\bar{C}\varepsilon_{t+1}(\bar{C}\varepsilon_{t+1})'] = \bar{C}\bar{C}'$. Consider the state unobservable, let Z_t denote the n_Z -vector of observable variables, and let the measurement equation be

$$Z_t = \bar{D}_0 + \bar{D}_s s_t + \eta_t, \tag{3.2}$$

where η_t is an n_Z -vector of measurement errors with distribution $N(0, \Sigma_\eta)$ (the vector η_t here should not be confused with vector η of coefficients specified in (1.3)). We assume that the matrix R is diagonal with very small elements.² Equations (3.1) and (3.2) correspond to the state-space form for the derivation of the Kalman filter in Hamilton [3].

Note that the matrix \bar{D}_s is related to the matrix $\bar{D} \equiv [\bar{D}_X \ \bar{D}_x \ \bar{D}_i]$ (partitioned conformably with X_t , x_t , and i_t) as

$$\bar{D}_s = [\bar{D}_X \ 0 \ \bar{D}_x \ \bar{D}_i],$$

since s_t includes the vector of predetermined variables Ξ_{t-1} .

² The role of the measurement errors in the estimation of the model and the state of the economy is further discussed in Adolfson et al. [2].

3.1. Difference specification

The vector of observable variables is

$$Z_t = \begin{bmatrix} \pi_t^d & \Delta \ln(W_t/P_t) & \Delta \ln C_t & \Delta \ln I_t & \hat{x}_t & R_t & \hat{H}_t & \dots \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & \dots \\ \Delta y_t & \Delta \ln \tilde{X}_t & \Delta \ln \tilde{M}_t & \pi_t^{cpi} & \pi_t^{def,i} & \Delta \ln Y_t^* & \pi_t^* & R_t^* \\ 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \end{bmatrix}' \quad (3.3)$$

and corresponds to the data used in estimating Ramses.

Next, we need specify how the data corresponds to the model variables:

$$\begin{aligned} \pi_t^d &= 400(\pi - 1)\pi + 4\pi\hat{\pi}_t^d, \\ \Delta \ln(W_t/P_t) &= 100 \ln \mu_z + \Delta \hat{w}_t + \hat{\mu}_{zt}, \\ \Delta \ln C_t &= 100 \ln \mu_z + \Delta \hat{c}_t + \frac{c}{c^d + c^m} \eta_c \frac{c^d}{c} \frac{c^m}{c} \left[(\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1} \right] \Delta \hat{\gamma}_t^{mcd} + \hat{\mu}_{zt}, \\ \Delta \ln I_t &= 100 \ln \mu_z + \Delta \hat{i}_t + \frac{\tilde{i}}{\tilde{i}^d + \tilde{i}^m} \eta_i \frac{\tilde{i}^d}{\tilde{i}} \frac{\tilde{i}^m}{\tilde{i}} \left[(\gamma^{i,mi})^{-1} - (\gamma^{i,d})^{-1} \right] \Delta \hat{\gamma}_t^{mi,d} \\ &\quad + \left(1 - \tau^k\right) r^k \bar{k} \frac{1}{\mu_z} \Delta \hat{u}_t + \hat{\mu}_{zt}, \\ \hat{x}_t &= \hat{x}_t, \\ R_t &= 400(R - 1)R + 4R\hat{R}_t, \\ \hat{H}_t &= \hat{H}_t, \\ \Delta y_t &= 100 \ln \mu_z + \Delta \hat{y}_t + \hat{\mu}_{zt}, \\ \Delta \ln \tilde{X}_t &= 100 \ln \mu_z + \Delta \left[\hat{y}_t^* - \eta_f \hat{\gamma}_t^{x,*} + \hat{z}_t^* \right] + \hat{\mu}_{zt}, \\ \Delta \ln \tilde{M}_t &= 100 \ln \mu_z + \frac{c^m}{c^m + \tilde{i}^m} \Delta \hat{c}_t - \frac{c^m}{c^m + \tilde{i}^m} \eta_c (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \Delta \hat{\gamma}_t^{mcd} \\ &\quad + \frac{\tilde{i}^m}{c^m + \tilde{i}^m} \Delta \hat{i}_t - \frac{\tilde{i}^m}{c^m + \tilde{i}^m} \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \Delta \hat{\gamma}_t^{mi,d} + \hat{\mu}_{zt}, \\ \pi_t^{cpi} &\equiv 400(\pi - 1)\pi + 4\pi\hat{\pi}_t^{cpi} + 4\pi \frac{\tau^c}{1 + \tau^c} \Delta \hat{\tau}_t^c \\ &= 400(\pi - 1)\pi + 4\pi(1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \hat{\pi}_t^d \\ &\quad + 4\pi\omega_c (\gamma^{cmc})^{-(1-\eta_c)} \hat{\pi}_t^{mc} + 4\pi \frac{\tau^c}{1 + \tau^c} \Delta \hat{\tau}_t^c, \\ \pi_t^{def,i} &= 400(\pi - 1)\pi + 4\pi\tilde{i}_{\pi^d} \hat{\pi}_t^d + 4\pi\tilde{i}_{\pi^m} \hat{\pi}_t^{mi} \\ &\quad + 4\pi \left[\left(\tilde{i}_{\pi^d} - \frac{I^d}{I^d + I^m} \right) \eta_i \omega_i (\gamma^{i,mi})^{-(1-\eta_i)} \right. \\ &\quad \left. - \left(\tilde{i}_{\pi^m} - \frac{I^m}{I^d + I^m} \right) \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \right] \Delta \hat{\gamma}_t^{mi,d}, \\ \Delta \ln Y_t^* &= 100 \ln \mu_z + \Delta \hat{y}_t^* + \Delta \hat{z}_t^* + \hat{\mu}_{zt}, \\ \pi_t^* &= 400(\pi - 1)\pi + 4\pi\hat{\pi}_t^*, \\ R_t^* &= 400(R - 1)R + 4R\hat{R}_t^*. \end{aligned} \quad (3.4)$$

Domestic inflation

$$\pi_t^d = 400(\pi - 1)\pi + 4\pi\hat{\pi}_t^d,$$

$$\begin{aligned}\bar{D}_0(1,1) &= 400(\pi - 1)\pi \\ \bar{D}_s(1, n_{\bar{X}} + 1) &= 4\pi\end{aligned}$$

Change in real wage

$$\Delta \ln(W_t/P_t) = 100 \ln \mu_z + \hat{w}_t - \hat{w}_{t-1} + \hat{\mu}_{zt},$$

$$\begin{aligned}\bar{D}_0(2,1) &= 100 \ln \mu_z \\ \hat{w}_t &: \bar{D}_s(2, n_{\bar{X}} + 6) = 1 \\ \hat{w}_{t-1} &: \bar{D}_s(2, 50) = -1 \\ \hat{\mu}_{zt} &: \bar{D}_s(2, 3) = 1\end{aligned}$$

Change in real consumption

$$\begin{aligned}\Delta \ln C_t &= 100 \ln \mu_z + \hat{c}_t - \hat{c}_{t-1} + \frac{c}{c^d + c^m} \eta_c \frac{c^d}{c} \frac{c^m}{c} \left[(\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1} \right] \hat{\gamma}_t^{mcd} \\ &\quad - \frac{c}{c^d + c^m} \eta_c \frac{c^d}{c} \frac{c^m}{c} \left[(\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1} \right] \hat{\gamma}_{t-1}^{mcd} + \hat{\mu}_{zt},\end{aligned}$$

$$\begin{aligned}\bar{D}_0(3,1) &= 100 \ln \mu_z \\ \hat{c}_t &: \bar{D}_s(3, n_{\bar{X}} + 7) = 1 \\ \hat{c}_{t-1} &: \bar{D}_s(3, 51) = -1 \\ \hat{\gamma}_t^{mcd} &: \bar{D}_s(3, n_{\bar{X}} + 17) = \frac{c}{c^d + c^m} \eta_c \frac{c^d}{c} \frac{c^m}{c} \left[(\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1} \right] \\ \hat{\gamma}_{t-1}^{mcd} &: \bar{D}_s(3, 61) = -\frac{c}{c^d + c^m} \eta_c \frac{c^d}{c} \frac{c^m}{c} \left[(\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1} \right] \\ \hat{\mu}_{zt} &: \bar{D}_s(3, 3) = 1\end{aligned}$$

Change in real investment

$$\begin{aligned}\Delta \ln I_t &= 100 \ln \mu_z + \hat{u}_t - \hat{u}_{t-1} + \frac{\tilde{i}}{\tilde{i}^d + \tilde{i}^m} \eta_i \frac{\tilde{i}^d}{\tilde{i}} \frac{\tilde{i}^m}{\tilde{i}} \left[(\gamma^{i,mi})^{-1} - (\gamma^{i,d})^{-1} \right] \hat{\gamma}_t^{mi,d} \\ &\quad - \frac{\tilde{i}}{\tilde{i}^d + \tilde{i}^m} \eta_i \frac{\tilde{i}^d}{\tilde{i}} \frac{\tilde{i}^m}{\tilde{i}} \left[(\gamma^{i,mi})^{-1} - (\gamma^{i,d})^{-1} \right] \hat{\gamma}_{t-1}^{mi,d} \\ &\quad + \left(1 - \tau^k\right) r^k \bar{k} \frac{1}{\mu_z} \hat{k}_t - \left(1 - \tau^k\right) r^k \bar{k} \frac{1}{\mu_z} \hat{k}_t - \left(1 - \tau^k\right) r^k \bar{k} \frac{1}{\mu_z} \hat{u}_{t-1} + \hat{\mu}_{zt}, \\ \hat{u}_t &= \hat{k}_t - \hat{\bar{k}}_t\end{aligned}$$

$$\begin{aligned}
\bar{D}_0(4, 1) &= 100 \ln \mu_z \\
\hat{i}_t &: \bar{D}_s(4, n_{\bar{X}} + 8) = 1 \\
\hat{i}_{t-1} &: \bar{D}_s(4, 52) = -1 \\
\hat{\gamma}_t^{mi,d} &: \bar{D}_s(4, n_{\bar{X}} + 18) = \frac{\tilde{i}}{\tilde{i}^d + \tilde{i}^m} \eta_i \frac{\tilde{i}^d \tilde{i}^m}{\tilde{i}} \left[(\gamma^{i,mi})^{-1} - (\gamma^{i,d})^{-1} \right] \\
\hat{\gamma}_{t-1}^{mi,d} &: \bar{D}_s(4, 62) = -\frac{\tilde{i}}{\tilde{i}^d + \tilde{i}^m} \eta_i \frac{\tilde{i}^d \tilde{i}^m}{\tilde{i}} \left[(\gamma^{i,mi})^{-1} - (\gamma^{i,d})^{-1} \right] \\
\hat{k}_t &: \bar{D}_s(4, n_{\bar{X}} + 13) = (1 - \tau^k) r^k \bar{k} \frac{1}{\mu_z} \\
\hat{\bar{k}}_t &: \bar{D}_s(4, 42) = - (1 - \tau^k) r^k \bar{k} \frac{1}{\mu_z} \\
\hat{u}_{t-1} &: \bar{D}_s(4, 67) = - (1 - \tau^k) r^k \bar{k} \frac{1}{\mu_z} \\
\hat{\mu}_{zt} &: \bar{D}_s(4, 3) = 1
\end{aligned}$$

Real exchange rate

$$\begin{aligned}
\hat{x}_t &= \hat{x}_t \\
\hat{x}_t &: \bar{D}_s(5, n_{\bar{X}} + 20) = 1
\end{aligned}$$

Interest rate

$$\begin{aligned}
R_t &= 400(R - 1)R + 4R\hat{R}_t, \\
\bar{D}_0(6, 1) &= 400(R - 1)R \\
\hat{R}_t &: \bar{D}_s(6, n_{\bar{X}} + n_x + n_i) = 4R
\end{aligned}$$

Hours worked

$$\begin{aligned}
\hat{H}_t &= \hat{H}_t, \\
\hat{H}_t &: \bar{D}_s(7, n_{\bar{X}} + 12) = 1
\end{aligned}$$

Change in output (GDP)

$$\Delta y_t = 100 \ln \mu_z + \hat{y}_t - \hat{y}_{t-1} + \hat{\mu}_{zt},$$

$$\begin{aligned}
\bar{D}_0(8, 1) &= 100 \ln \mu_z \\
\hat{y}_t &: \bar{D}_s(8, n_{\bar{X}} + 4) = 1 \\
\hat{y}_{t-1} &: \bar{D}_s(8, 48) = -1 \\
\hat{\mu}_{zt} &: \bar{D}_s(8, 3) = 1
\end{aligned}$$

Change in real exports

$$\Delta \ln \tilde{X}_t = 100 \ln \mu_z + \hat{y}_t^* - \hat{y}_{t-1}^* - \eta_f \hat{\gamma}_t^{x,*} + \eta_f \hat{\gamma}_{t-1}^{x,*} + \hat{z}_t^* - \hat{z}_{t-1}^* + \hat{\mu}_{zt},$$

$$\begin{aligned} \bar{D}_0(9, 1) &= 100 \ln \mu_z \\ \hat{y}_t^* &: \bar{D}_s(9, 31) = 1 \\ \hat{y}_{t-1}^* &: \bar{D}_s(9, 34) = -1 \\ \hat{\gamma}_t^{x,*} &: \bar{D}_s(9, n_{\tilde{X}} + 19) = -\eta_f \\ \hat{\gamma}_{t-1}^{x,*} &: \bar{D}_s(9, 63) = \eta_f \\ \hat{z}_t^* &: \bar{D}_s(9, 14) = 1 \\ \hat{z}_{t-1}^* &: \bar{D}_s(9, 15) = -1 \\ \hat{\mu}_{zt} &: \bar{D}_s(9, 3) = 1 \end{aligned}$$

Change in real imports

$$\begin{aligned} \Delta \ln \tilde{M}_t &= 100 \ln \mu_z + \frac{c^m}{c^m + \tilde{\gamma}^m} \hat{c}_t - \frac{c^m}{c^m + \tilde{\gamma}^m} \hat{c}_{t-1} - \frac{c^m}{c^m + \tilde{\gamma}^m} \eta_c (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \hat{\gamma}_t^{mcd} \\ &+ \frac{c^m}{c^m + \tilde{\gamma}^m} \eta_c (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \hat{\gamma}_{t-1}^{mcd} + \frac{\tilde{\gamma}^m}{c^m + \tilde{\gamma}^m} \hat{i}_t - \frac{\tilde{\gamma}^m}{c^m + \tilde{\gamma}^m} \hat{i}_{t-1} \\ &- \frac{\tilde{\gamma}^m}{c^m + \tilde{\gamma}^m} \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \hat{\gamma}_t^{mi,d} + \frac{\tilde{\gamma}^m}{c^m + \tilde{\gamma}^m} \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \hat{\gamma}_{t-1}^{mi,d} + \hat{\mu}_{zt}, \end{aligned}$$

$$\begin{aligned} \bar{D}_0(10, 1) &= 100 \ln \mu_z \\ \hat{c}_t &: \bar{D}_s(10, n_{\tilde{X}} + 7) = \frac{c^m}{c^m + \tilde{\gamma}^m} \\ \hat{c}_{t-1} &: \bar{D}_s(10, 51) = -\frac{c^m}{c^m + \tilde{\gamma}^m} \\ \hat{\gamma}_t^{mcd} &: \bar{D}_s(10, n_{\tilde{X}} + 17) = -\frac{c^m}{c^m + \tilde{\gamma}^m} \eta_c (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \\ \hat{\gamma}_{t-1}^{mcd} &: \bar{D}_s(10, 61) = \frac{c^m}{c^m + \tilde{\gamma}^m} \eta_c (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \\ \hat{i}_t &: \bar{D}_s(10, n_{\tilde{X}} + 8) = \frac{\tilde{\gamma}^m}{c^m + \tilde{\gamma}^m} \\ \hat{i}_{t-1} &: \bar{D}_s(10, 52) = -\frac{\tilde{\gamma}^m}{c^m + \tilde{\gamma}^m} \\ \hat{\gamma}_t^{mi,d} &: \bar{D}_s(10, n_{\tilde{X}} + 18) = -\frac{\tilde{\gamma}^m}{c^m + \tilde{\gamma}^m} \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \\ \hat{\gamma}_{t-1}^{mi,d} &: \bar{D}_s(10, 62) = \frac{\tilde{\gamma}^m}{c^m + \tilde{\gamma}^m} \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \\ \hat{\mu}_{zt} &: \bar{D}_s(10, 3) = 1 \end{aligned}$$

CPI inflation

$$\begin{aligned} \pi_t^{cpi} &= 400(\pi - 1)\pi + 4\pi(1 - \omega_c) \hat{\pi}_t^d \\ &+ 4\pi\omega_c \hat{\pi}_t^{mc} + 4\pi \frac{\tau^c}{1 + \tau^c} \hat{\tau}_t^c - 4\pi \frac{\tau^c}{1 + \tau^c} \hat{\tau}_{t-1}^c, \end{aligned}$$

$$\begin{aligned}
\bar{D}_0(11, 1) &= 400(\pi - 1)\pi \\
\hat{\pi}_t^d &: \bar{D}_s(11, n_{\tilde{X}} + 1) = 4\pi(1 - \omega_c) \\
\hat{\pi}_t^{mc} &: \bar{D}_s(11, n_{\tilde{X}} + 2) = 4\pi\omega_c \\
\hat{\tau}_t^c &: \bar{D}_s(11, 22) = 4\pi \frac{\tau^c}{1 + \tau^c} \\
\hat{\tau}_{t-1}^c &: \bar{D}_s(11, 27) = -4\pi \frac{\tau^c}{1 + \tau^c}
\end{aligned}$$

Inflation investment deflator

$$\begin{aligned}
\hat{\pi}_t^{def,i} &= \frac{\tilde{z}^d}{\tilde{z}^d + \frac{\eta^{mi}}{\eta^{mi-1}}\tilde{z}^m} \hat{\pi}_t^d + \frac{c^m}{\left(c^d + \frac{\eta^{mc}}{\eta^{mc-1}}c^m\right)} \frac{\eta^{mc}}{\eta^{mc} - 1} \hat{\pi}_t^{mc} \\
&+ \left[\begin{aligned} &\left(\frac{\tilde{z}^d}{\tilde{z}^d + \frac{\eta^{mi}}{\eta^{mi-1}}\tilde{z}^m} - \frac{\tilde{z}^d}{\tilde{z}^d + \tilde{z}^m} \right) \eta_i \omega_i (\gamma^{i,mi})^{-(1-\eta_i)} \\ &- \left(\frac{\tilde{z}^m}{\left(\tilde{z}^d + \frac{\eta^{mi}}{\eta^{mi-1}}\tilde{z}^m\right)} \frac{\eta^{mi}}{\eta^{mi-1}} - \frac{\tilde{z}^m}{\tilde{z}^d + \tilde{z}^m} \right) \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \end{aligned} \right] \Delta \hat{\gamma}_t^{mcd}. \\
&\Delta \hat{\gamma}_t^{mi,d} = \hat{\pi}_t^{mi} - \hat{\pi}_t^d
\end{aligned}$$

$$\begin{aligned}
\hat{\pi}_t^{def,i} &= 400(\pi - 1)\pi \\
&+ 4\pi \left\{ \begin{aligned} &\frac{\tilde{z}^d}{\tilde{z}^d + \frac{\eta^{mi}}{\eta^{mi-1}}\tilde{z}^m} - \left(\frac{\tilde{z}^d}{\left(\tilde{z}^d + \frac{\eta^{mi}}{\eta^{mi-1}}\tilde{z}^m\right)} - \frac{\tilde{z}^d}{\tilde{z}^d + \tilde{z}^m} \right) \eta_i \omega_i (\gamma^{i,mi})^{-(1-\eta_i)} \\ &+ \left(\frac{\tilde{z}^m}{\left(\tilde{z}^d + \frac{\eta^{mi}}{\eta^{mi-1}}\tilde{z}^m\right)} \frac{\eta^{mi}}{\eta^{mi-1}} - \frac{\tilde{z}^m}{\tilde{z}^d + \tilde{z}^m} \right) \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \end{aligned} \right\} \hat{\pi}_t^d \\
&+ 4\pi \left\{ \begin{aligned} &\frac{\tilde{z}^m}{\left(\tilde{z}^d + \frac{\eta^{mi}}{\eta^{mi-1}}\tilde{z}^m\right)} \frac{\eta^{mi}}{\eta^{mi-1}} + \left(\frac{\tilde{z}^d}{\tilde{z}^d + \frac{\eta^{mi}}{\eta^{mi-1}}\tilde{z}^m} - \frac{\tilde{z}^d}{\tilde{z}^d + \tilde{z}^m} \right) \eta_i \omega_i (\gamma^{i,mi})^{-(1-\eta_i)} \\ &- \left(\frac{\tilde{z}^m}{\left(\tilde{z}^d + \frac{\eta^{mi}}{\eta^{mi-1}}\tilde{z}^m\right)} \frac{\eta^{mi}}{\eta^{mi-1}} - \frac{\tilde{z}^m}{\tilde{z}^d + \tilde{z}^m} \right) \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \end{aligned} \right\} \hat{\pi}_t^{mi}.
\end{aligned}$$

$$\bar{D}_0(12, 1) = 400(\pi - 1)\pi$$

$$\begin{aligned}
\hat{\pi}_t^d &: \bar{D}_s(12, n_{\tilde{X}} + 1) = 4\pi \left\{ \begin{aligned} &\frac{\tilde{z}^d}{\left(\tilde{z}^d + \frac{\eta^{mi}}{\eta^{mi-1}}\tilde{z}^m\right)} \\ &- \left(\frac{\tilde{z}^d}{\left(\tilde{z}^d + \frac{\eta^{mi}}{\eta^{mi-1}}\tilde{z}^m\right)} - \frac{\tilde{z}^d}{\tilde{z}^d + \tilde{z}^m} \right) \eta_i \omega_i (\gamma^{i,mi})^{-(1-\eta_i)} \\ &+ \left(\frac{\tilde{z}^m}{\left(\tilde{z}^d + \frac{\eta^{mi}}{\eta^{mi-1}}\tilde{z}^m\right)} \frac{\eta^{mi}}{\eta^{mi-1}} - \frac{\tilde{z}^m}{\tilde{z}^d + \tilde{z}^m} \right) \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \end{aligned} \right\} \\
\hat{\pi}_t^{mi} &: \bar{D}_s(12, n_{\tilde{X}} + 3) = 4\pi \left\{ \begin{aligned} &\frac{\tilde{z}^m}{\left(\tilde{z}^d + \frac{\eta^{mi}}{\eta^{mi-1}}\tilde{z}^m\right)} \frac{\eta^{mi}}{\eta^{mi-1}} \\ &+ \left(\frac{\tilde{z}^d}{\tilde{z}^d + \frac{\eta^{mi}}{\eta^{mi-1}}\tilde{z}^m} - \frac{\tilde{z}^d}{\tilde{z}^d + \tilde{z}^m} \right) \eta_i \omega_i (\gamma^{i,mi})^{-(1-\eta_i)} \\ &- \left(\frac{\tilde{z}^m}{\left(\tilde{z}^d + \frac{\eta^{mi}}{\eta^{mi-1}}\tilde{z}^m\right)} \frac{\eta^{mi}}{\eta^{mi-1}} - \frac{\tilde{z}^m}{\tilde{z}^d + \tilde{z}^m} \right) \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \end{aligned} \right\}
\end{aligned}$$

Foreign output

$$\Delta \ln Y_t^* = 100 \ln \mu_z + \hat{y}_t^* - \hat{y}_{t-1}^* + \hat{z}_t^* - \hat{z}_{t-1}^* + \hat{\mu}_{zt},$$

$$\begin{aligned} \bar{D}_0(13, 1) &= 100 \ln \mu_z \\ \hat{\mu}_{zt} &: \bar{D}_s(13, 3) = 1 \\ \hat{z}_t^* &: \bar{D}_s(13, 14) = 1 \\ \hat{z}_{t-1}^* &: \bar{D}_s(13, 15) = -1 \\ \hat{y}_t^* &: \bar{D}_s(13, 31) = 1 \\ \hat{y}_{t-1}^* &: \bar{D}_s(13, 34) = -1 \end{aligned}$$

Foreign inflation

$$\pi_t^* = 400(\pi - 1)\pi + 4\pi\hat{\pi}_t^*,$$

$$\begin{aligned} \bar{D}_0(14, 1) &= 400(\pi - 1)\pi \\ \hat{\pi}_t^* &: \bar{D}_s(14, 30) = 4\pi \end{aligned}$$

Foreign interest rate

$$R_t^* = 400(R - 1)R + 4R\hat{R}_t^*.$$

$$\begin{aligned} \bar{D}_0(15, 1) &= 400(R - 1)R \\ \hat{R}_t^* &: \bar{D}_s(15, 32) = 4R \end{aligned}$$

3.2. Cointegration specification

The vector of observable variables is

$$Z_t = \begin{bmatrix} \pi_t^d & \ln(W_t/P_t) - \ln Y_t & \ln C_t - \ln Y_t & \ln I_t - \ln Y_t & \hat{x}_t & R_t & \hat{H}_t & \dots \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & \dots \\ \Delta \ln Y_t & \ln \tilde{X}_t - \ln Y_t & \ln \tilde{M}_t - \ln Y_t & \pi_t^{cpi} & \pi_t^{def,i} & \ln Y_t^* - \ln Y_t & \pi_t^* & R_t^* \\ 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \end{bmatrix}' \quad (3.5)$$

and corresponds to the data used in estimating Ramses. However, it should be noted that in the current version of [1] we are only using the difference specification in equation (3.3).

Next, we need to specify how the cointegrated data corresponds to the model variables (see the section above for the specification of the non-cointegrated variables):

$$\begin{aligned} \ln(W_t/P_t) - \ln Y_t &= \hat{w}_t - \hat{y}_t, \\ \ln C_t - \ln Y_t &= \hat{c}_t + \frac{c}{c^d + c^m} \eta_c \frac{c^d}{c} \frac{c^m}{c} \left[(\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1} \right] \hat{\gamma}_t^{mcd} - \hat{y}_t, \\ \ln I_t - \ln Y_t &= \hat{i}_t + \frac{\tilde{i}}{\tilde{i}^d + \tilde{i}^m} \eta_i \frac{\tilde{i}^d}{\tilde{i}} \frac{\tilde{i}^m}{\tilde{i}} \left[(\gamma^{i,mi})^{-1} - (\gamma^{i,d})^{-1} \right] \hat{\gamma}_t^{mi,d} + \left(1 - \tau^k \right) r^k \bar{k} \frac{1}{\mu_z} \hat{u}_t - \hat{y}_t, \\ \ln \tilde{X}_t - \ln Y_t &= \hat{y}_t^* - \eta_f \hat{\gamma}_t^{x,*} + \hat{z}_t^* - \hat{y}_t, \\ \ln \tilde{M}_t - \ln Y_t &= \frac{c^m}{c^m + \tilde{i}^m} \hat{c}_t - \frac{c^m}{c^m + \tilde{i}^m} \eta_c (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \hat{\gamma}_t^{mcd} \\ &\quad + \frac{\tilde{i}^m}{c^m + \tilde{i}^m} \hat{i}_t - \frac{\tilde{i}^m}{c^m + \tilde{i}^m} \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \hat{\gamma}_t^{mi,d} - \hat{y}_t, \\ \ln Y_t^* - \ln Y_t &= \hat{y}_t^* + \hat{z}_t^* - \hat{y}_t. \end{aligned} \quad (3.6)$$

Real wage

$$\ln(W_t/P_t) = \widehat{w}_t - \widehat{y}_t,$$

$$\begin{aligned}\widehat{w}_t &: \bar{D}_s(2, n_{\tilde{X}} + 6) = 1, \\ y_t &: \bar{D}_s(2, n_{\tilde{X}} + 4) = -1.\end{aligned}$$

Real consumption

$$\ln C_t - y_t = \widehat{c}_t + \frac{c}{c^d + c^m} \eta_c \frac{c^d}{c} \frac{c^m}{c} \left[(\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1} \right] \widehat{\gamma}_t^{mcd} - y_t,$$

$$\begin{aligned}\widehat{c}_t &: \bar{D}_s(3, n_{\tilde{X}} + 7) = 1, \\ \widehat{\gamma}_t^{mcd} &: \bar{D}_s(3, n_{\tilde{X}} + 17) = \frac{c}{c^d + c^m} \eta_c \frac{c^d}{c} \frac{c^m}{c} \left[(\gamma^{cmc})^{-1} - (\gamma^{cd})^{-1} \right], \\ \widehat{y}_t &: \bar{D}_s(3, n_{\tilde{X}} + 4) = -1.\end{aligned}$$

Real investment

$$\begin{aligned}\ln I_t - \ln Y_t &= \widehat{i}_t + \frac{\tilde{i}}{\tilde{i}^d + \tilde{i}^m} \eta_i \frac{\tilde{i}^d}{\tilde{i}} \frac{\tilde{i}^m}{\tilde{i}} \left[(\gamma^{i,mi})^{-1} - (\gamma^{i,d})^{-1} \right] \widehat{\gamma}_t^{mi,d} + (1 - \tau^k) r^k \bar{k} \frac{1}{\mu_z} \widehat{k}_t - (1 - \tau^k) r^k \bar{k} \frac{1}{\mu_z} \widehat{\bar{k}}_t - \widehat{y}_t \\ \widehat{u}_t &= \widehat{k}_t - \widehat{\bar{k}}_t,\end{aligned}$$

$$\begin{aligned}\widehat{i}_t &: \bar{D}_s(4, n_{\tilde{X}} + 8) = 1, \\ \widehat{\gamma}_t^{mi,d} &: \bar{D}_s(4, n_{\tilde{X}} + 18) = \frac{\tilde{i}}{\tilde{i}^d + \tilde{i}^m} \eta_i \frac{\tilde{i}^d}{\tilde{i}} \frac{\tilde{i}^m}{\tilde{i}} \left[(\gamma^{i,mi})^{-1} - (\gamma^{i,d})^{-1} \right], \\ \widehat{k}_t &: \bar{D}_s(4, n_{\tilde{X}} + 13) = (1 - \tau^k) r^k \bar{k} \frac{1}{\mu_z}, \\ \widehat{\bar{k}}_t &: \bar{D}_s(4, 42) = - (1 - \tau^k) r^k \bar{k} \frac{1}{\mu_z}, \\ \widehat{y}_t &: \bar{D}_s(4, n_{\tilde{X}} + 4) = -1.\end{aligned}$$

Real exports

$$\ln \tilde{X}_t - \ln Y_t = \widehat{y}_t^* - \eta_f \widehat{\gamma}_t^{x,*} + \widehat{z}_t^* - \widehat{y}_t,$$

$$\begin{aligned}\widehat{y}_t^* &: \bar{D}_s(9, 31) = 1, \\ \widehat{\gamma}_t^{x,*} &: \bar{D}_s(9, n_{\tilde{X}} + 19) = -\eta_f, \\ \widehat{z}_t^* &: \bar{D}_s(9, 14) = 1, \\ \widehat{y}_t &: \bar{D}_s(9, n_{\tilde{X}} + 4) = -1.\end{aligned}$$

Real imports

$$\begin{aligned}\ln \tilde{M}_t - \ln Y_t &= \frac{c^m}{c^m + \tilde{\gamma}^m} \hat{c}_t - \frac{c^m}{c^m + \tilde{\gamma}^m} \eta_c (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)} \hat{\gamma}_t^{mcd} \\ &\quad + \frac{\tilde{\gamma}^m}{c^m + \tilde{\gamma}^m} \hat{i}_t - \frac{\tilde{\gamma}^m}{c^m + \tilde{\gamma}^m} \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)} \hat{\gamma}_t^{mi,d} - \hat{y}_t, \\ \hat{c}_t &: \bar{D}_s(10, n_{\tilde{X}} + 7) = \frac{c^m}{c^m + \tilde{\gamma}^m}, \\ \hat{\gamma}_t^{mcd} &: \bar{D}_s(10, n_{\tilde{X}} + 17) = -\frac{c^m}{c^m + \tilde{\gamma}^m} \eta_c (1 - \omega_c) (\gamma^{cd})^{-(1-\eta_c)}, \\ \hat{i}_t &: \bar{D}_s(10, n_{\tilde{X}} + 8) = \frac{\tilde{\gamma}^m}{c^m + \tilde{\gamma}^m}, \\ \hat{\gamma}_t^{mi,d} &: \bar{D}_s(10, n_{\tilde{X}} + 18) = -\frac{\tilde{\gamma}^m}{c^m + \tilde{\gamma}^m} \eta_i (1 - \omega_i) (\gamma^{i,d})^{-(1-\eta_i)}, \\ \hat{y}_t &: \bar{D}_s(10, n_{\tilde{X}} + 4) = -1.\end{aligned}$$

Foreign output

$$\ln Y_t^* - \ln Y_t = \hat{y}_t^* + \hat{z}_t^* - \hat{y}_t,$$

$$\begin{aligned}\hat{z}_t^* &: \bar{D}_s(13, 14) = 1, \\ \hat{y}_t^* &: \bar{D}_s(13, 31) = 1, \\ \hat{y}_t &: \bar{D}_s(13, n_{\tilde{X}} + 4) = -1.\end{aligned}$$

3.3. Smoothed estimates

The Kalman filter calculates a forecast of the state vector s_t , $s_{t|t-1}$, as a linear function of previous observations,

$$s_{t|t-1} \equiv \mathbb{E}[s_t | \mathcal{Z}_{t-1}], \quad (3.7)$$

where $\mathcal{Z}_{t-1} \equiv (Z_{t-1}, Z_{t-2}, \dots, Z_1)$. The matrix $P_{t|t-1}$ represents the MSE of this forecast

$$P_{t|t-1} \equiv \mathbb{E} \left[(s_t - s_{t|t-1}) (s_t - s_{t|t-1})' | \mathcal{Z}_{t-1} \right].$$

The key equations of the Kalman filter are

$$s_{t|t} = s_{t|t-1} + P_{t|t-1} \bar{D}'_s (\bar{D}_s P_{t|t-1} \bar{D}'_s + \Sigma_\eta)^{-1} (Z_t - \bar{D}_0 - \bar{D}_s s_{t|t-1}), \quad (3.8)$$

$$s_{t+1|t} = \bar{B} s_{t|t}, \quad (3.9)$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1} \bar{D}'_s (\bar{D}_s P_{t|t-1} \bar{D}'_s + \Sigma_\eta)^{-1} \bar{D}_s P_{t|t-1}, \quad (3.10)$$

$$P_{t+1|t} = \bar{B} P_{t|t} \bar{B}' + Q. \quad (3.11)$$

The smoothed estimates of s_t , $s_{t|T}$ for $t \leq T$, are denoted

$$s_{t|T} \equiv \mathbb{E}[s_t | \mathcal{Z}_T]. \quad (3.12)$$

The sequence of smoothed estimates $\{s_{t|T}\}_{t=1}^T$ can be calculated by first calculating the sequences $\{s_{t|t}\}_{t=1}^T$, $\{s_{t+1|t}\}_{t=1}^{T-1}$, $\{P_{t|t}\}_{t=1}^T$ and $\{P_{t+1|t}\}_{t=1}^{T-1}$ and then calculate

$$s_{t|T} = s_{t|t} + P_{t|t} \bar{B}' P_{t+1|t}^{-1} (s_{t+1|T} - s_{t+1|t}) \quad (3.13)$$

by iterating backward through the sample for $t = T - 1, T - 2, \dots, 1$ (Hamilton [3, chapt. 13.6]).

References

- [1] Adolfson, Malin, Stefan Laséen, Jesper Lindé, and Lars E.O. Svensson (2008), “Optimal Monetary Policy in an Operational Medium-Sized DSGE Model,” Sveriges Riksbank Working Paper Series No. 225.
- [2] Adolfson, Malin, Stefan Laséen, Jesper Lindé, and Mattias Villani (2006), “Evaluating an Estimated New Keynesian Small Open Economy Model”, working paper, www.riksbank.se, *Journal of Economic Dynamics and Control*, forthcoming .
- [3] Hamilton, James D. (1994), *Time Series Analysis*, Princeton University Press.