Inflation forecasts with uncertainty intervals

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Since December 1997, the Riksbank's Inflation Report has contained an inflation forecast with uncertainty intervals. These intervals illustrate the assessment of the Riksbank of whether uncertainty in the forecast is greater or less compared with previous forecasts, and whether it is more probable that the main scenario of the forecast is an underestimate or an overestimate of future inflation. The uncertainty intervals together with a probability distribution for the inflation forecast are derived from an assessment of the uncertainty of those factors considered to affect inflation.

In January 1993, the Governing Board of the Riksbank adopted an explicit inflation target, stating that the annual change in the consumer price index is to be held at 2 per cent ±1 percentage point.[1] Since monetary policy is judged to have a full effect on inflation with a lag of one to two years, the Riksbank should base its monetary policy on an assessment of future inflation. This is done with an inflation forecast based, in principle, on all relevant economic information available to the Bank at the time of preparing the forecast. It is also based on the assumption that the repo rate will remain unchanged during the forecast period. The Riksbank presents its view of the inflation outlook in the Inflation Report, published since 1993 (quarterly since 1996).

Since December 1997, the Inflation Report has contained an explicit inflation forecast with uncertainty intervals. Having uncertainty intervals is useful for several reasons. Firstly, the intervals illustrate the fact that the forecast is uncer-

* We would like to thank Claes Berg, Hans Lindberg, Hans Lindblad, Pernilla Meyersson, Staffan Viotti and Anders Vredin for valuable comments. A detailed technical description of a method for calculating uncertainty intervals is available in Blix and Sellin (1998).

[1] See Bäckström (1998) for a detailed discussion of the inflation target.

tain. This uncertainty concerns the shocks that will affect the economy during the forecast period as well as the economic relationships, that is, how factors such as the demand in the economy influences inflation.[2] Secondly, the intervals allow the Riksbank to communicate its judgement of uncertainty at any given forecast horizon, for example whether an inflation figure lower than the forecast in the main scenario is more likely than an inflation figure outcome above this forecast.[3] Thirdly, when uncertainty is specifically illustrated in the form of uncertainty intervals, this creates favourable conditions for a more systematic discussion within the Riksbank regarding the assessment of uncertainty in inflation forecasts. This discussion focuses on the sources of inflation uncertainty and their quantitative significance. Lastly, uncertainty in itself in the inflation assessment can influence the formation of monetary policy. A high level of uncertainty may constitute a reason to have a more cautious policy stance in order to avoid unnecessary large and sudden movements in interest rates.

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In the preparation of the Inflation Report of the Riksbank, an initial assessment of risk and uncertainty is done at the Economics Department. This then functions as the basis for the uncertainty analysis of the Executive Board which is presented in the Inflation Report. A somewhat different approach for producing uncertainty intervals for the inflation forecast is used by the Bank of England,[4] which bases its uncertainty assessments on the inflation risks identified by the Bank's highest decision-making body, the Monetary Policy Committee (MPC). These risks are then described in more detail and accounted for by the Bank's economists. The approach used by the Riksbank, on the other hand, can be described as a 'bottom-up' approach, since the initial assessment is conducted by the Economics Department. In order to implement this kind of approach, we have developed a method that differs significantly in several respects from that used by the Bank of England. This method and the way in which it is used is described in this article.

The traditional statistical approach to producing uncertainty intervals would involve first constructing a model intended for inflation forecasts. In a linear multivariate model, the exogenous shocks would be assumed to be normally distrib-

[2] For a description of these relationships, see Hörngren (1995).

[3] The inflation forecast in the main scenario refers to the most likely development for inflation during the next two years.

[4] See Britton, Fisher and Whitley (1998) for a detailed description.

uted. This would in turn mean that the endogenous variables (inflation among them) would be normally distributed as well. Deriving uncertainty intervals in this kind of model is a well known and, from a methodological standpoint, straightforward statistical problem.[5]

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The Riksbank, however, does not use the approach described above for several reasons. Firstly, the Riksbank does not use any one specific model for making inflation forecasts. Secondly, the standard approach does not allow specific information relevant to the particular forecast period to be used. Thirdly, subjective judgements have proved to be important in making good forecasts. We therefore prefer an approach that is as explicit and rigorous as possible in incorporating subjective judgements about uncertainty into the forecast.

This article is structured as follows. We begin by giving a brief outline of how the main scenario inflation forecasts are prepared. We will then describe how assessments of uncertainty and risk in the most important macro variables for future inflation can be quantified. Finally, we will discuss how these assessments can be aggregated to give an overall picture of the inflation forecast distribution.

The inflation forecast

For a foreign trade dependent economy, such as the Swedish, the international development of growth and inflation is highly important. Forecasts of developments in the economies of Sweden's major trading partners are therefore a natural starting point in making forecasts for the Swedish economy. A further significant international factor is exchange rates. Changes in exchange rates are significant in that they influence Swedish exports and imports. Furthermore, exchange rates have a more direct effect on inflation via import prices in Swedish kronor. Exchange rate movements that are consistent with economic activity in general are therefore an important ingredient in inflation forecasts. In Alexius and Lindberg (1996), several different models are used to obtain an understanding for the level of the long-term exchange rate. It can be more difficult to assess how long the adjustment to this equilibrium exchange rate will take.[6]

The greater part of the work in the inflation forecast is to undertake an

[5] In a non-linear multivariate model, it is necessary to use simulations to calculate uncertainty intervals. This is a time-consuming but well-known procedure.

[6] See also "The krona's long-term path" in Inflation Report 1998:3, pages 26–28.

assessment of the future demand and supply situation in the Swedish economy.[7] Inflation pressure is likely to build up if the total demand in the economy exceeds the long-term production capacity. A number of indicators can be used to assess inflation pressure, such as industrial capacity utilisation, the labour market situation and the output gap. This latter indicator measures the difference between the actual output and the sustainable supply that the economy is capable of producing, known as the potential output. The potential output cannot be observed but can be estimated using several different models, as described in Apel, Hansen and Lindberg (1996) and Apel and Jansson (1999). Inflation expectations are also significant for assessing future inflation. If the decision-makers in the economy expect higher inflation, this can in itself lead to inflationary price and wage increases. For this reason, surveys concerning the inflation expectations of different actors are of interest when making inflation forecasts. Inflation expectations can also be measured on the basis of pricing on the money market, as described in Svensson (1993).

It can be seen from this overview that a large number of indicators and models are used in preparing inflation forecasts. Lastly, an overall assessment of all these factors has to be made and the most probable path for inflation – the main scenario – is presented in the Inflation Report of the Riksbank. It should be noted that the main scenario gives the most probable inflation development under the assumption that the repo rate will remain unchanged. This assumption is made mainly for pedagogical reasons in order to clarify whether the repo rate needs to be raised or lowered. A consequence of this assumption of an unchanged repo rate is that the inflation forecast of the Riksbank is not directly comparable with inflation forecasts prepared by other actors, since these generally assume some form of monetary policy reaction from the Riksbank.

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The statistical measure best corresponding to the forecast in the main scenario is the mode, since this represents the most probable outcome in the distribution (or rather the value that corresponds to the peak of the distribution). In a standard Gaussian distribution, the most common measures of central tendency – the mean, the mode and the median – coincide, but this is not the case for most

[7] The factors judged to be most significant for inflation are discussed in more detail in the facts box at the beginning of Chapter 2 in the Inflation Reports.

other distributions.[8] In the distribution used by the Riksbank (discussed below), the measures *may* coincide but *need not* do so. An advantage of using the mode is that it does not take into account the risk of extreme events. This is also a weakness since the mode does not use information from the entire distribution.[9]

The main issue is not which measure of central tendency is chosen but rather whether the forecast distribution is reasonable.

Finally, it should be noted that different measures of central tendency are simply different ways of summarising information about the distribution in a more easily interpreted and standardised manner. The main issue is not which measure of central tendency is chosen but rather whether the forecast distribution is reasonable. Given that the distribution is reasonable, it will contain more information than the above-mentioned measures. Part of this information can be gauged by using additional measures, such as variance, skewness and kurtosis.[10] For example, if the distribution shows unusually large variance – which can be interpreted such that the uncertainty is greater than usual – then this is useful information for formulating monetary policy. The September 1998 Inflation Report stated that situations that are exceptionally difficult to assess constituted in themselves an argument in favour of not changing the repo rate.

Uncertainty assessment and asymmetric risk assessment

The uncertainty within a factor may vary over time. The more important the factor is for inflation, the more it should affect the inflation forecast distribution.

The main scenario inflation forecast prepared by the Riksbank is the path of inflation which is considered the most probable for the coming two years. It is based on an assessment of the most probable development of a number of factors that are likely to affect inflation, such as aggregate demand and output in the economy, import prices and wages.

[8] See Appendix for definitions of the different measures.

[9] The mode, the mean and the median may, however, can be misleading measures of central tendency if the distribution has more than one "peak". In this case, the mode selects only one of the peaks and disregards information about other peaks. With regard to the mean and the median, the situation is no better, since they risk selecting the least probable outcome. This problem does not occur, however, in the distribution used by the Riksbank, since this distribution has only one peak.

[10] Skewness is a measure (approximate) of whether or not the distribution is symmetric. A standard Gaussian distribution is an example of a symmetric distribution that consequently has no skewness. Kurtosis is a measure (approximate) of the risk of extreme events occurring (that is, how wide the tails of the distribution are). Kurtosis is often discussed when the distribution is compared with a standard Gaussian distribution. The common expression "excess kurtosis" means that the tails in the distribution are fatter than in a standard Gaussian distribution.

However, the development of these important factors for inflation is naturally uncertain. At a specific point in time, this uncertainty may be greater or less than the level of uncertainty which has historically been characteristic for a particular factor. This should be reflected in the distribution of the inflation forecast. The more important a particular factor is for inflation, the more importance should be attached to it when assessing the total uncertainty of the inflation forecast.

A factor may have asymmetric risks. The more important the factor is for inflation, the more the asymmetry should affect the inflation forecast distribution.

In addition to the forecast having greater or less uncertainty than normal, there may sometimes be reason to presume that the forecast in the main scenario has more likely underestimated rather than overestimated future inflation. If this is the case, we say that the forecast carries an upside risk. Correspondingly, the forecast is said to carry a downside risk if it is judged to be more likely that the main scenario is an overestimate of future inflation. This kind of asymmetry in the forecast distribution for individual factors that affect inflation should be reflected in the form of asymmetry in the inflation forecast distribution. Just as in the case of the uncertainty as discussed above, the more important the particular factor is for inflation, the more importance should be attached to the asymmetric risk (asymmetry in the inflation forecast distribution). As a measure of asymmetry – or skewness – in the forecast distribution, we use the difference between the mode and the mean.[11]

In practice, assessments of uncertainty and asymmetric risk are carried out as follows: economists at the Bank prepare forecasts for twelve months and twenty-four months ahead for those factors that form their particular areas of expertise. For each forecast horizon, they then specify whether the assessments are more (or less) uncertain than the historical norm. In Table 1, economist Y.Y. has stated that he estimates the level of uncertainty in factor 1 for 1999 to be 90 per cent of what has historically been considered normal. Let us assume, for example, that factor 1 concerns wage formation and that several significant labour market agreements have recently been entered into. This means that the uncertainty is assessed to be less than normal. If, instead, a turning point in the business cycle is ahead, this could entail greater uncertainty since turning points are notoriously hard to forecast. Another example which could lead to greater uncertainty is if a political election is due to occur during the forecast period. For the year 2000, Y.Y. has stated the uncertainty of factor 1 to be 1.00, which means that the uncertainty is neither greater nor

[11] This measure is proportional to the third central moment, which is normally used to derive a measure of skewness in a probability distribution. See Appendix for a more detailed description.

less than normal. The economists also state whether they consider the upside risk in the forecast to be greater or less than 50 per cent. An economist stating an upside risk of, for example, 55 per cent thus considers that the forecast carries a slight upside risk, whilst a forecaster stating an upside risk of 50 per cent considers that the forecast is equally likely to be an overestimate as an underestimate. If an economist reports an upside risk of less than 50 per cent, as shown by Z.Z. in Table 1 for 1999, it implies that the forecast carries a downside risk.

Table 1. Uncertainty assessment and asymmetric risk assessment

	Uncertainty		Asymmetric risk		Economist
	1999	2000	1999	2000	
Factor 1	0.90	1.00	50	55	Y.Y
Factor 2	1.00	1.10	45	50	Z.Z.

Everyone involved in the forecasting meets to discuss their assessments and bring them into line.

How is consistency among uncertainty and risk assessments achieved? All subjective assessments expressed as a percentage must be followed by an explanation in economic terms. This is in order to create a basis for discussions about consistency in terms of the overall picture of uncertainty. Everyone involved in the forecasting meets to discuss their assessments and bring them into line. The approach using assessments of inflation uncertainty and asymmetric risk entails that these meetings are a rewarding forum for focused discussion among those involved in preparing the inflation forecast. During such a discussion, it would be conceivable that the original assessments of, for example, asymmetric risks for factors 1 and 2 in Table 1 would be brought into line with one another if the asymmetric risks for these two factors are likely to be similar. This may, for example, cause Y.Y. to revise his/her figures to 47 for 1999 and to 53 for the year 2000. At a later stage, the Executive Board of the Riksbank also becomes involved in the discussion of the inflation forecast and the uncertainty assessments surrounding it. A discussion of different assessments of the uncertainty can then be conducted in concrete terms concerning the factors that affect inflation, and the final assessment of the Riksbank is gradually formulated.

For each factor that is important for inflation, we thus now have a forecast in the main scenario, an assessment of whether the uncertainty is greater or less than historically characteristic for the factor, and an assessment of whether the forecast in the main scenario is more likely to be an overestimate or an underestimate. All of this information then has to somehow be summarised and aggregated to produce an assessment of uncertainty and asymmetric risk in the inflation forecast. This is described in the following section, which is of more methodological and technical character; it describes how a special probability distribution for inflation forecasts can be designed so that it reflects the subjective judgements that have been made about uncertainty and asymmetric risk in the factors that are important for future inflation.

Probability distribution for inflation forecasts

The forecast distribution for inflation that is used by the Riksbank allows skewness.

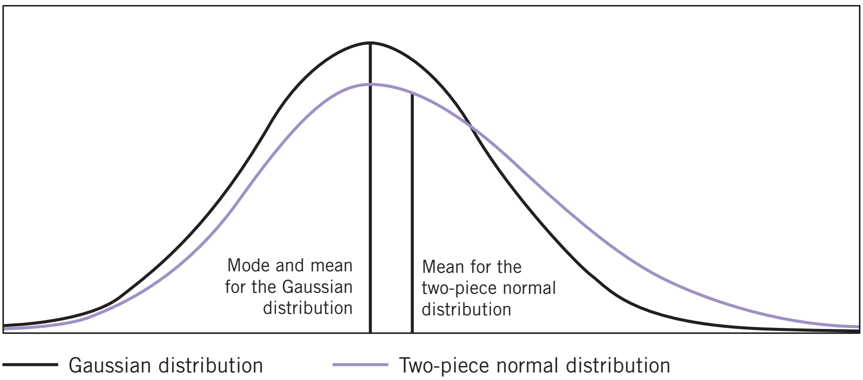
A comprehensive description of the forecast for a given factor together with the uncertainty and asymmetric risk in the forecast can be shown in a probability distribution that allows skewness. An example of such a distribution is the two-piece normal distribution.[12] The name is derived from the fact that the distribution to the left of the mode is proportional to a Gaussian distribution with a specific standard deviation, whilst the distribution to the right of the mode is proportional to a Gaussian distribution with another standard deviation. If both standard deviations are the same, then the two-piece normal distribution coincides with a Gaussian. The two-piece normal distribution is completely defined with three parameters, the mode and the two standard deviations mentioned above. This can be compared with the standard Gaussian distribution which can be fully defined using two parameters, the mean (which coincides with the mode and the median) and the standard deviation.[13]

We use the subjective judgements of uncertainty and asymmetric risk to increase or decrease the "standard deviation parameters" in the two-piece normal distribution. For example, if we have 10 per cent greater uncertainty than normal and 55 per cent upside risk, this gives an approximate[14] two-piece normal distribution, the right side of which is proportional to a standard Gaussian with the standard deviation 1.10(55/45)S, whilst the left side is proportional to a standard Gaussian with the standard deviation 1.10(45/55)S, where S is the standard deviation estimated from historical data. The historical standard deviation is calculated on the basis of historical errors in the Bank's inflation forecasts, adjusted to account for the assumption of a constant repo rate.[15] Figure 1 shows the two-piece normal distribution together with a Gaussian distribution with the same

[12] See Appendix for a brief description of the two-piece normal distribution and John (1982) for a more detailed description. This distribution should not be confused with a Gaussian bivariate distribution, which is a joint distribution for two Gaussian variables.
[13] See Appendix for a more detailed description of the relevant probability distributions.
[14] See Blix and Sellin (1998) for the exact formulae.
[15] Since the inflation forecast is made on the basis of a technical assumption that the repo rate will be held constant, no account has been taken of the fact that changes in interest rates will affect demand in the economy and thereby inflation.

mode and with the standard deviation S=1. It can be seen that there is upward skewness in the two-piece normal distribution, since the mean is higher than the mode, which means that there is a higher probability of an outcome above the forecast than below the forecast. The standard deviation for the two-piece normal distribution is a function of the two "standard deviation parameters" and is greater than the previous standard deviation S, which reflects both the 10 per cent higher uncertainty and the asymmetric risk.

Figure 1. Gaussian distribution and two-piece normal distribution



The inflation forecast distribution should be skewed if any of the factors is skewed.

No straightforward method exists for aggregating the distributions for the forecasts in respect of the different factors to produce a single distribution for the inflation forecast. If we assume, for example, a linear relationship between the macro variables and inflation, we could in principle derive the forecast distribution for inflation. Unfortunately, this approach is not feasible, since if several two-piece normal variables are aggregated – in contrast to when the variables have standard Gaussian distribution –, this does not result in any known distribution. However, a reasonable assumption would be that such an aggregation would result in a skewed distribution if the distribution of any of the factors is skewed.[16] We thus use a two-piece normal distribution as an approximation of the unknown distribution function for the inflation forecast.

A remaining key issue concerns how the assessments should be aggregated to form an overall picture of the distribution of the inflation forecast. We have chosen to *assume* that the skewness in the distribution is the sum of the skewness in

[16] A situation whereby different skewnesses cancel each other out when they are aggregated is possible.

the macro variables, weighted according to their importance for future inflation. If, for example, factor 1 has a skewness of –0.1 percentage point, its net contribution to the overall inflation skewness will be –0.1 percentage point multiplied by the weight of the factor, and so on for all other factors. The construction of these calculations is described in detail in Blix and Sellin (1998). Note that with this method, no assessments of uncertainty and asymmetric risk are required in the inflation forecast distribution but rather these are *derived* from corresponding assessments for the different factors which may be assumed to affect inflation. We have thus developed a method that connects uncertainty and risk assessments in the macro variables to the probability distribution for the inflation forecast.

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Whilst the assumptions we have made above result in a statistical approximation, it is important to stress that they are principally based on macro theory. This is perhaps best illustrated by some examples which show that the assumptions are *qualitatively* reasonable. If we first consider the benchmark case in which there is no skewness in any of the macro variables, then our assumption will imply that there will not be any skewness in the distribution for the inflation forecast either. We consider this to be a reasonable property to use as a starting point for the relationship between the macro variables and inflation. If, on the other hand, the estimated consumption carries a downside risk (negative skewness), the assumption will mean that the inflation distribution will also carry a downside risk, since the weight for consumption is typically positive. What happens if we suppose an upside risk in another variable, such as wage trend? Whether or not the sum of consumption skewness and wage skewness results in positive or negative skewness in the inflation forecast distribution depends on two things: the extent of the skewness and their relative importance (weight) for future inflation. This example can also be used to judge whether the inflation skewness is *quantitatively* reasonable.

How are the weights derived? A noteworthy feature of our method is that the weights reflect the importance of the macro variables for inflation. The weights used by the Riksbank are derived from a macroeconomic model that uses changes in the respective macro variables to calculate the effects of inflation twelve and twenty-four months ahead.

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The calculations thus result in an inflation forecast distribution reflecting the subjective uncertainty and also the upside and downside risks in the forecasts for the different factors that contribute to inflation. If the distribution has a high degree of skewness (the upside or downside risk is considerable), this may be cause to revise the original forecast in the main scenario. It is ultimately a question of judgement as to whether or not an extreme event, such as a large fall in demand due to a worsening of the Asian crisis, will affect the main scenario.

Calculating uncertainty intervals

The starting point for calculating uncertainty intervals for the inflation forecast is the two probability distributions calculated for the inflation forecast for twelve and twenty-four months ahead. We will use the forecast in Inflation Report 1998:4 as an example, since this provides a good illustration of the skewness permitted in the distribution used by the Riksbank. Figure 2 shows the probability distribution for the inflation forecast for a twelve-month and twenty-four-month horizon, December 1999 and December 2000 respectively. The broken line shows the inflation forecast in the main scenario (the mode). We have indicated the uncertainty interval within which inflation was judged to lie with 90 per cent probability. This means that the risk of the inflation outcome being above the upper limit is 5 per cent and the risk of an outcome below the lower limit is 5 per cent.[17]

The uncertainty assessment in Inflation Report 1998:4 judged more probable that the inflation forecast was an overestimate of future inflation rather than an underestimate.

The mean of the distribution for the inflation forecast for twelve months ahead is 0.1 percentage point lower than the mode forecast in the main scenario, whilst it is slightly less than 0.2 percentage points lower for the twenty-four-month forecast. Both distributions are thus characterised by negative skewness. This reflects the downside risk in the inflation forecasts for both twelve and twenty-four months ahead. From the uncertainty assessment, it was thus judged more probable that the inflation forecast was an overestimate of future inflation rather than an underestimate. The causes of the skewness can be traced to skewness in the distributions of some of the factors that contribute to inflation. Inflation Report 1998:4 summarises the reasons for the skewness as follows: "All in all, the inflation assessment has a downside risk in the form of an international slowdown that is more marked and protracted." (page 42).

[17] Wallis (1999) criticises the Bank of England for not having symmetric intervals. The implication of this is that, in contrast to the uncertainty intervals used by the Riksbank, the probabilities of the intervals being too high or too low are not the same. This could, for example, lead to intervals of 3 per cent and 7 per cent respectively. See also The Economist (1999).

Figure 2 a. 90 per cent uncertainty interval for CPI forecast for December 1999

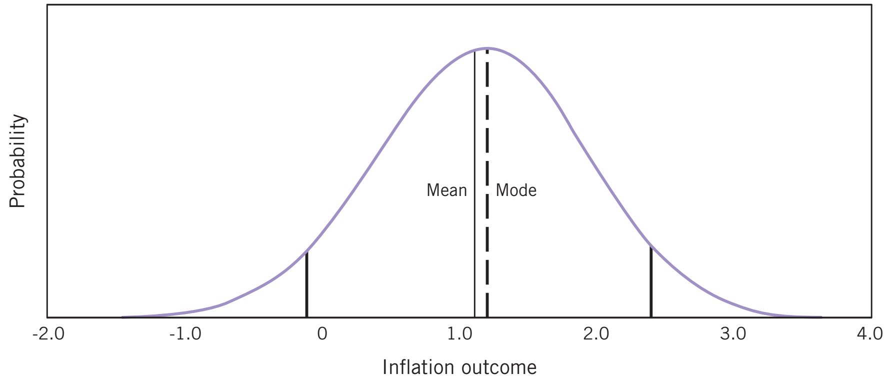
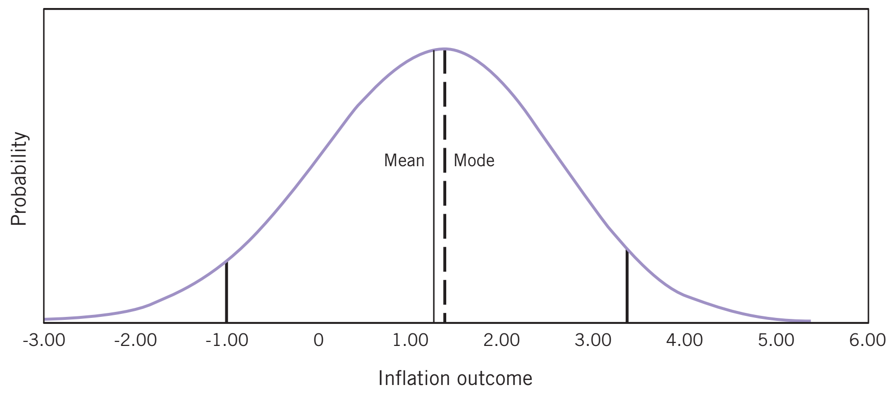
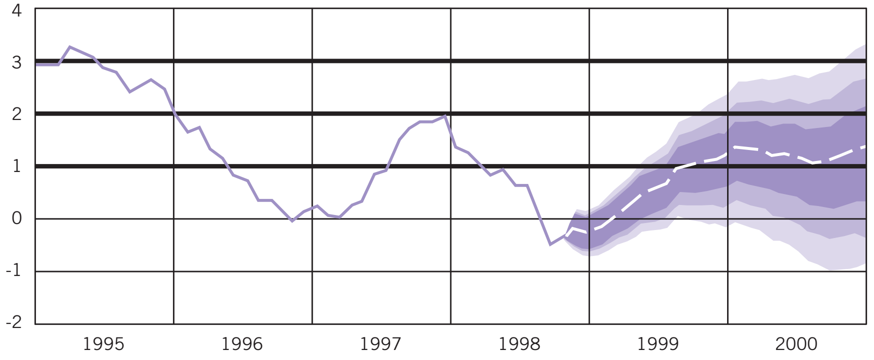


Figure 2 b. 90 per cent uncertainty interval for CPI forecast for December 2000



An alternative way of presenting uncertainty intervals is by using a fan chart, as shown in Figure 3. This is based on the distributions in Figure 2, which are shown as cross-sections on the fan chart in Figure 3 at the time horizons December 1999 and December 2000. The outer limits of the fan represent the 90 per cent uncertainty intervals. The 75 per cent and 50 per cent uncertainty intervals are also shown together with the path of the forecast in the main scenario (the broken line). The time interval between the figures for December 1999 and December 2000 is interpolated to produce the fan. This is done by adjusting the intervals upwards monthly by a given factor. The factor must be chosen so that the adjustment lies within the given interval limits for December 1999 and December 2000 respectively. The fan chart has several advantages compared with Figure 2. Firstly, only one diagram is required. Secondly, the entire path of the forecast can be seen and also the actual developments up until the forecast dates. The diagram also illustrates how the uncertainty develops in relation to the forecast horizon. The width of the uncertainty intervals above the forecast compared with the width of the intervals below the forecast illustrates where the majority of the uncertainty is estimated to be. If the forecast is not judged to carry any asymmetric risk, the intervals above and below the forecast will be of equal size.

Figure 3. CPI with uncertainty intervals. Annual percentage change



Source: Inflation Report 1998:4

Another way of illustrating the uncertainty intervals for the inflation forecast is to calculate the probability of the inflation outcome being in a given interval. This can easily be done, since we already know the overall probability distribution for the inflation forecast. Table 2 is taken from Inflation Report 1998:4 and shows the probabilities of inflation outcomes below 1 per cent, between 1 and 2 per cent, between 2 and 3 per cent and over 3 per cent for twelve and twenty-four months ahead.

Table 2. 12-month CPI inflation. Percentage probability of different outcomes

Dec.–Dec.	**CPI<1**	**1<CPI<2**	**2<CPI<3**	**CPI>3**	**Total**
1999	42	45	12	1	100
2000	41	30	20	9	100

Source: Inflation Report 1998:4

For the year 2000 the assessment in Inflation Report 1998:4 was that inflation would probably be below the Riksbank target of 2 per cent, under the assumption of unchanged repo rate.

The probability of inflation lying within the tolerance interval for 1999 was thus judged in December 1998 to be 45+12=57 per cent. There was also considered to be a significant probability (42 per cent) that inflation would be below 1 per cent, whilst the probability of inflation exceeding 3 per cent was judged to be negligible (1 per cent). The assessment for the year 2000 was similar, that is, that inflation would probably be below the Riksbank target of 2 per cent, under the assumption of unchanged repo rate.

Summary

In this article, we have presented a new method for deriving a probability distribution for the inflation forecast. This reflects the subjective judgements that have been made regarding uncertainty and asymmetric risk in the factors that are judged to affect inflation. We have also outlined how this method is applied in practice.

The analysis is based on two sorts of assessments. Firstly, an assessment is made of whether uncertainty in the forecast is greater or less than the uncertainty that has historically characterised the particular factor. Secondly, an assessment is undertaken regarding whether there is a greater risk of the forecast in the main scenario underestimating or overestimating future inflation. These assessments are represented in a two-piece normal probability distribution for each factor. The probability distributions are then aggregated to form an overall probability distribution for the inflation forecast, which is used to calculate the uncertainty intervals around the main scenario. This aggregation is carried out using weights to represent the importance of the macro variables for inflation.

The initial work on uncertainty assessments is carried out by the Economics Department and coincides with the preparation of the main scenario. It is natural for economists with expertise concerning a particular factor to make assessments for that specific factor. In order to ensure consistency, the economists meet to bring their assessments into line with each other. The uncertainty and risk assessments made by the Economics Department then function as the basis for discussion by the Executive Board. At this stage, the different assessments of uncertainty can be discussed in very concrete terms, focusing on the factors that are important for inflation. The inflation assessment of the Executive Board may result in the main scenario and the inflation forecast distribution being revised. The final inflation assessment is presented in the Inflation Report of the Riksbank and enables the Riksbank to communicate in a pedagogical manner its view of uncertainty and upside and downside risks for the inflation forecast in the main scenario.

Appendix

Definitions of common measures of central tendency

Let the distribution be defined as $f(x)$. The mean is defined as $\int_{-\infty}^{\infty} xf(x)dx$; the median is the value μ such that $\int_{-\infty}^{\mu} f(x)dx = 0.5$ and the mode is the value μ such that $\left.\frac{\partial f}{\partial x}\right|_{x=\mu} = 0$.

Two-piece normal distribution

A standard Gaussian distribution is defined as

$$f(x;\mu,\sigma) = (2\pi\sigma^2)^{-1/2}\exp\left\{-\frac{1}{2\sigma^2}(x-\mu)^2\right\}.$$

The two-piece normal distribution consists of two standard Gaussian distributions having the same μ but different standard deviations:

$$f(x;\mu,\sigma_1,\sigma_2) = \begin{cases} C\exp\left\{-\frac{1}{2\sigma_1^2}(x-\mu)^2\right\} \text{ for } x \le \mu \\ C\exp\left\{-\frac{1}{2\sigma_2^2}(x-\mu)^2\right\} \text{ for } x > \mu, \end{cases}$$

where $C = k(\sigma_1 + \sigma_2)^{-1}$, $k = \sqrt{2/\pi}$ and μ is the mode. C is an integration constant required to scale the standard Gaussian distributions when they are aggregated.

The probability of outcomes between L_1 and L_2 for the two-piece normal distribution is derived in John (1982) and is

$$pr[L_1 \le x \le L_2] = \int_{L_1}^{L_2} f(x)dx = \frac{2\sigma}{(\sigma_1+\sigma_2)}\left[\Phi\left(\frac{L_2-\mu}{\sigma}\right) - \Phi\left(\frac{L_1-\mu}{\sigma}\right)\right],$$

where $\Phi(.)$ is the standard normal cumulative distribution and

$$\begin{cases} \sigma = \sigma_1 \text{ if } L_1 \le L_2 \le \mu \\ \sigma = \sigma_2 \text{ if } \mu \le L_1 \le L_2 \end{cases}.$$

The variance is given by

$$var(x) = (1 - k^2)(\sigma_2 - \sigma_1)^2 + \sigma_1\sigma_2$$

and the skewness (the third central moment) is given by

$$E[(x - \mu)^3] = k(\sigma_2 - \sigma_1)[(2k^2 - 1)(\sigma_2 - \sigma_1)^2 + \sigma_1\sigma_2],$$

which is proportional to $k(\sigma_2 - \sigma_1)$ since $2k^2 - 1 > 0$. Therefore, we will use the simpler expression

$$\gamma \cong \tilde{\mu} - \mu = k(\sigma_2 - \sigma_1)$$

to describe the skewness. The advantage of this is that γ is exactly the difference between the mean and the mode.

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