The IRB approach in the Basel Committee's proposal for new capital adequacy rules: some simulation-based illustrations

By Tor Jacobson, Jesper Lindé and Kasper Roszbach
Research department.

The Internal Ratings Based (IRB) approach for determining banks' capital adequacy is one of the cornerstones of the Basel Committee's proposed revision of the Basel Accord for banking regulation. This article presents the ideas behind the IRB approach and its fundamental features, and discusses the consequences of a number of its components for the banks' capital adequacy requirements. Using a simulation-based analysis, we will illustrate the relationship between IRB-determined capital and the risks inherent in a loan portfolio in a dynamic perspective assuming different macroeconomic developments.[1] We will also examine the effect of the number of risk classes that banks use and of different risk profiles of their credit portfolios.

The Basel Committee's regulation

In 1988, the Basel Committee introduced capital adequacy regulation that has been adopted by more than 100 countries.

In 1988, the Basel Committee introduced regulation stipulating how a bank's minimum acceptable capital base is to be calculated, i.e. the size of the capital that banks are required to hold as a buffer against future losses on their assets (e.g. the credits in their loan portfolios). From having originally been intended for internationally active G10 banks, the Basel Accord has now been adopted in over 100 countries. The regulation has also been adopted in gen-

The authors would like to thank Sonja Daltung, Kerstin Mitlid, Staffan Viotti and Anders Vredin for valuable comments on earlier drafts.

[1] The conclusions in this article are based in part on a paper entitled *Capital Charges under Basel II: Corporate Credit Risk Modelling and the Macro Economy* by Carling, Jacobson, Lindé & Roszbach (2002), in which the IRB approach is evaluated with the aid of data on a business loan portfolio at a major Swedish bank for the period 1994–2000.

eral by all banks, not exclusively those that operate internationally. The purpose of the Accord was, and still is, to promote security and stability in the bank sector. In recent years, supervisory authorities have expressed increasing concern over the erosion of the effectiveness of the Accord. The banks have devised methods of capital arbitrage that circumvent the capital adequacy rules and that lead to a mismatch between the risks accepted and the buffer capital the banks are obliged to hold.[2] In order to curb this development, the Basel Committee and its extensive hierarchy of working groups have drawn up a proposal for a revision of the 1988 rules. The proposed Accord is considerably more far-reaching and specifies the principles governing the activities of the banks and the supervisory authorities.[3]

Insufficient risk dependence and the possibilities of arbitrage are important reasons behind the Basel Committee's revision of the capital adequacy rules.

Insufficient risk dependence and the possibilities of arbitrage in the present regulations are important reasons behind the Basel Committee's revision of the capital adequacy rules. Its revision can also be seen as a natural consequence of the rapid developments in recent years in credit risk management and credit risk measurement and the banks' greater readiness and ability to quantify credit risk. Current methods of measuring credit risk, in a broad sense, increasingly resemble the market risk models that supervisory authorities have long been allowing banks to use to determine the level of the buffer capital for risk-exposed currency holdings and securities. Early on in its revision of the rules, the Committee discussed the possibility of allowing the corresponding use of credit risk models in determining the buffer capital for credit losses. Since no generally accepted methodology for validating, or evaluating, credit risk models has yet been established, it was decided to formulate the new rules to permit them to be transitional until full-scale credit risk modelling can be used as the basis for capital determination.[4] In practice, this means that the rules will be a compromise solution in which credit risk models are allowed, indirectly, to serve as the basis for determining buffer capital via the banks' internal risk classification systems. The Basel Committee also stresses the

[2] Jackson et al. (1999) contains a review of the extensive empirical research into the effect of the existing capital adequacy requirements on bank behaviour.

[3] The basic principles for the planned capital adequacy Accord were outlined in January 2001. The proposal is available on the Bank for International Settlements' website (www.bis.org/publ/bcbsca.htm).

[4] The difficulties associated with evaluating credit risk models are due partly to the use of such models not yet being widespread, and partly to the fact that the banks that use these models have not done so for a very long time and thus have not had the time to accumulate the large amounts of data the models require. As the actual event – the failure to pay interest or amortizations on a loan – is a relatively infrequent occurrence, data need to be collected for some considerable period of time.

importance of designing the new rules in a way that gives banks an incentive to further develop quantitative methods of handling credit risk.

The proposal for the new rules is based on three pillars.

The proposal is based on three pillars. The first pillar consists of rules for determining the buffer capital the banks must hold to cover credit and other losses that the banks incur. The second pillar consists of the supervisory process of scrutinizing the banks' internal procedures for deciding their capital base, taking risk profile into account. The purpose of the third pillar is to increase the transparency of banks' risk profiles for market players through disclosure requirements. The idea is to amplify the disciplinary effect of the market that implies, for example, that a bank with a high risk propensity is correctly recognized as such by the market and therefore, all else being equal, has to pay more for its financing.

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The buffer capital currently required by the capital adequacy rules shall be at least 8 per cent of a bank's total risk-exposed assets. Depending on type, an asset's value is multiplied by a "risk-weight" before the sum is calculated. In the current rules, the loans in a credit portfolio are partly risk-differentiated; loans to other banks have, for example, a risk weight of 20 per cent, while business loans have a risk weight of 100 per cent. This means that loans to other banks are covered by an actual buffer capital of 1.6 per cent (20 per cent of the 8 per cent that is normally referred to as the absolute capital adequacy requirement); loans to banks are thus considered less risky than loans to businesses. However, the current rules impose a limit on such risk differentiation, so that two similarly sized business loan portfolios, for instance, also need to have the same buffer capital, regardless of each portfolio's actual credit risk profile. The new rules will take risk differentiation in the calculation of asset value considerably further. The constant risk weight for business loans has been replaced by a variable weight, so that businesses with a high credit rating and a low probability of default (PD) are thus assigned a low risk weight, and vice versa.

The first pillar proposes two main alternatives for determining risk weights. The first, the standard approach, should be applicable by all banks.

The first pillar proposes two main alternatives for determining the risk weights by which the risk-exposed assets are to be multiplied. The first, the "standard" approach, is designed to be applicable by all banks. This alternative means that the loans in a portfolio will be divided into a relatively small number of risk classes, although more than in the current regulation.[5,6] The loans in any given risk class are assumed to be homogeneous in terms of risk. The supervisory authority assigns a risk weight to each risk class that is based on an external credit evaluation of the counterparty risk that is typical of loans in this risk class. The buffer capital can then be calculated in a number of simple steps. First, the total value of the loans in each of the risk classes is worked out and multiplied by the relevant risk weight. The risk-weighted assets thus obtained from each of the risk classes are then added together. The buffer capital requirement is then 8 per cent of this sum.

The second alternative is designed with larger and more sophisticated banks in mind.

The second alternative is designed with larger and more sophisticated banks in mind. The IRB approach differs from the standard approach primarily in that it is based on internal rather than external information. The idea behind the IRB approach is to use the information that is collected and processed in the banks' own counterparty evaluations. Since it is part of a bank's normal business to make professional assessments and evaluations of counterparty risk, it should be possible to use such evaluations to determine a risk-differentiated capital base. As a bank's internal risk classification system is a systematic compilation of its credit risk assessments, this in practice makes it a suitable point of departure. Analogously with the standard approach, the loans in each internal rating category or risk class are assumed to be homogeneous in terms of risk. The risk weight for the IRB approach, i.e. the factor calculated for each risk class and by which the sum of all loans in a specific risk class is to be multiplied to obtain the risk-weighted capital base, is calculated by the bank itself. An average PD is then calculated for the risk class on the basis of historical data for the loans in any given risk class over a particular time horizon. Using a formula provided by the supervisory authority, the banks then convert the PDs of the different risk classes into risk weights. The product of the risk weight, the exposure at the time of default (the nominal loan less any collateral is normally used) and the 8 per cent absolute capital adequacy requirement, summated over all loans in the portfolio, gives the bank's buffer capital, exactly as in the standard approach. The current proposal gives the banks the option of deciding for themselves at which of the two levels of complexity they will apply the IRB approach. The more complex method requires the bank to be able to

[5] Risk differentiation in the standard approach will increase in relation to the current rules, partly since there will be more risk-weight classes for the loan categories that are already risk-weighted, and partly since more types of loans, such as credits to business and private borrowers, will be risk-weighted.

[6] A definition of a defaulting loan that is widely used by Swedish banks is a loan for which payment of interest or amortisation is 60 days late.

compile internally data on loss given defaults (LGD) and the exposure at the time of default, while the simple method only expects the bank to be able to produce estimations of the PD.

It is a fair guess that most banks will use probability of default calculations based on historical data for the rating classes in their own portfolio.

We aim to show in this article that despite the detailed nature of the proposal, there still remain a number of important yet unresolved issues concerning the practical application of the IRB approach. We will be examining the consequences of different ways of making the important calculation of the average PD, which in turn results in a risk weight for each rating class. The Basel Committee proposes three basic methods that the banks can use to calculate these probabilities: average external rating of counterparty risk; average estimated probabilities obtained from a credit risk model; and calculations based on historical data or the rating classes in the bank's own portfolio. Our guess is that this last approach will be preferred by most banks. For this reason, this is the method we will illustrate here. We will examine the effects of the different ways in which historical data can be used. These differences can relate to the amount of data used (i.e. which time horizon has been applied) and the choice of method for estimating the PDs. These issues are relevant, regardless of the basic method employed by the banks, i.e. they are also relevant in the case of an external rating or a credit risk model. Another issue that we will discuss is how often we can expect a given capital buffer to prove inadequate in relation to the measured portfolio credit risk. The answer to this question will, of course, depend upon how the buffer capital is calculated and the choice of risk level and risk horizon for the portfolio, however defined. This is relevant to the principles governing the calculation of the capital base as given in the current Basel Accord. Nonetheless, we have opted to study capital adequacy arrived at using the IRB approach. At present, Swedish banks are in the process of adapting their businesses to the new regulations, which in itself justifies an examination of the IRB approach. Therefore, we will be illustrating how the capital as determined by the IRB method varies depending on the method used to estimate the PD in each risk class. We will also be looking at the extent to which the IRB capital covers the loan portfolio's risk of default. We will also be demonstrating the importance of taking explicit account of macroeconomic conditions when making these assessments.

Our results suggest that the choice of method for calculating the average historical default risk for the rating classes is very important; the longer the period for which the default risk is calculated, the lower the capital adequacy require-

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ment; the longer the future period during which one assumes that the buffer capital has to provide coverage, the weaker the co-variance with future credit risk and the greater the risk of the buffer capital proving inadequate. The extent of this effect also hinges on the choice of method for calculating the default risk. Macroeconomic conditions are of importance for the design of the new capital adequacy system. A bank's business cycle sensitivity also has a major impact on the co-variation between the buffer capital and the portfolio's credit risk, and therefore also on the probability that the capital buffer will prove inadequate. Our results are not in contradiction with the fears that a strong co-variation between buffer capital and credit risk in the new Accord will increase the chances of procyclicality effects (i.e. the undesirable intensification of the business cycle).

Methodology

In this section we describe and justify each step in our analysis. Appendix A provides the technical details of the calculations. This section is written in such a way that it can be read independently of the appendix. The appendix is intended mainly for readers who are interested in applying the model themselves or redoing (parts of) the calculations.

DATA GENERATION

Our method is based on an analysis of simulated data.

Our method is quantitative but not empirical, since it is based on an analysis of simulated data. There are several reasons for this. Most importantly, actual data, to the extent we would need, are impossible to extract. Not only do we intend our analysis to cover bank loan portfolios over a long period, we also wish to analyse the characteristics of 1,000 portfolios. Another reason is that we would like to generate data with a strong and controllable correlation with macroeconomic developments to enable us to study the effects over a business cycle. The obvious drawback to a simulation approach is that the results depend upon how realistic the simulation model actually is. In the following section, we will describe the structure of the data-generating model. This model enables us to generate time-series over different periods for hypothetical loan portfolios that consist of a large number of loans. We have also generated data describing the macroeconomic situation (the output gap, i.e. the difference between estimated potential GDP and actual GDP) in order to examine how the business cycle affects the banks' IRB-determined capital base, particularly with respect to variations in the portfolios' riskiness.

The business loans in the simulated bank loan portfolios are distributed into ten credit risk or rating classes. Although we have chosen the number of classes relatively arbitrarily, the number is within the limits that most banks work with. The structure of the portfolio, i.e. the risk classes' risk profile and proportion of the total portfolio, is characterised by so called "transition matrices". The elements of a transition matrix give the probability that a counterparty (i.e. a business) will migrate from one rating class to another, or remain in the same class. They thus describe the counterparties' movement through the portfolio's different rating classes over a given period of time (e.g. from start to one year ahead).

There are good reasons to assume that changes in the business cycle give rise to different levels in credit worthiness for companies.

There are good reasons to assume that changes in the business cycle, all else being equal, give rise to different levels in a company's credit worthiness. For example, we expect relatively few companies to be forced into bankruptcy during an economic phase with high demand, or to put it another way, that relatively few credits in a bank's portfolio default. In our analysis, which is "dynamic" in the sense that we follow the development of a portfolio over several periods, we will be working with three different transition matrices, which ought to provide a fair description of migrations in a portfolio during good, normal and bad macroeconomic conditions. To prevent the simulated business loans in the bank's portfolio from "jumping" too much between different states of nature, we will smooth the migrations between the transition matrices of the different states. This smoothing process is governed by the state of the business cycle at the time.[7]

Transition matrices

The loan portfolios are characterised by transition matrices.

As previously mentioned, the loan portfolios are characterised by transition matrices. This comes about in the following way. Assume that our hypothetical bank, just as banks are meant to do under the new Basel Accord, assigns all its borrowers (counterparties) a quarterly credit worthiness

[7] We use the output gap, that is the difference between the actual real GDP and an estimated potential GDP, to approximate the business cycle. The potential GDP is what the economy could produce if all its resources were being utilised to the full.

appraisal or credit rating as part of its lending activities. This means that the loans in the portfolio are distributed over a number (in this case ten) of rating classes. If the bank does this for a series of quarters and systematically records the data relating to the migration of these loans between rating classes, it can, using this information, estimate a transition matrix (TM):

$$TM = \begin{bmatrix} p_{11} & p_{12} & \cdot & \cdot & \cdot & p_{1r} & p_{1d} \\ p_{21} & p_{22} & \cdot & \cdot & \cdot & p_{2r} & p_{2d} \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ p_{r1} & p_{r2} & \cdot & \cdot & \cdot & p_{rr} & p_{rd} \end{bmatrix},$$

where r is the number of non-defaulting rating classes and d denotes the rating class in which the defaulting loans are placed. If the bank, as in our case, uses ten non-defaulting rating classes, the TM contains ten rows and eleven columns. The element p_{24} denotes an estimated probability that a loan placed in rating class 2 at time t will have migrated to rating class 4 at time $t + 1$; the element p_{11} represents the probability that loans in the rating class with the highest credit rating will still be in that class in the next period, while p_{rd} is the probability that the worst rated loans will default during the period t to $t + 1$. It might be worth noting that the sum of the probabilities in any one row (with respect to one rating class) is 1.

There is reason to expect that the probabilities in the transition matrix are not constant throughout a business cycle.

As already mentioned, there is reason to expect that the probabilities in the transition matrix are not constant throughout a business cycle. During a boom period, the default risk should decrease and in a recession a transition matrix with higher probabilities of borrower downgrading will reflect a bank loan portfolio more accurately. This business cycle dependency is something we take into account by assuming three economic situations: a kind of normal economic situation, a boom period and a slump, each of which are characterised by their own transition matrix: TM_{normal}, TM_{high} and TM_{low}. The transition matrix TM_{normal} has the following appearance:[8]

[8] TM_{high} and TM_{low} are shown in Appendix A.

$$TM_{\text{normal}} = \begin{bmatrix} .90 & .04 & .03 & .02 & .01 & .00 & .00 & .00 & .00 & .00 & .00 \\ .01 & .90 & .02 & .02 & .01 & .01 & .01 & .01 & .01 & .00 & .00 \\ .00 & .01 & .89 & .03 & .02 & .02 & .01 & .01 & .01 & .00 & .00 \\ .00 & .01 & .02 & .85 & .03 & .03 & .015 & .02 & .01 & .01 & .005 \\ .00 & .005 & .01 & .025 & .82 & .04 & .03 & .03 & .025 & .01 & .005 \\ .01 & .02 & .00 & .03 & .05 & .80 & .04 & .02 & .01 & .01 & .01 \\ .00 & .02 & .02 & .03 & .04 & .05 & .75 & .04 & .02 & .02 & .01 \\ .00 & .00 & .01 & .01 & .02 & .04 & .08 & .75 & .05 & .025 & .015 \\ .00 & .00 & .00 & .01 & .02 & .03 & .06 & .12 & .70 & .04 & .02 \\ .00 & .00 & .00 & .00 & .01 & .02 & .02 & .06 & .15 & .70 & .04 \end{bmatrix}.$$

The numeric values of the matrix elements are determined with the use of empirical data.

The numerical values of the matrix elements have not been calculated and are therefore, to a certain extent, arbitrary. In the case of TM_{normal} we have used empirical data for the business loan portfolio of a major Swedish bank.[9] We have, however, allowed the exact values in the matrix to deviate slightly from actual data in order to obtain a smoother reduction of the probabilities when moving along a row in the matrix away from the diagonal elements.[10] TM_{high} and TM_{low} are fair, if somewhat arbitrary, adjustments of TM_{normal}, and have been given such values that the portfolios' default risks, both on average and during boom and recession periods, give rise to credit losses that roughly correspond to the actual credit losses incurred by the Swedish bank sector (see Figure 3). In the next section, we will describe in more detail just how the prevailing economic situation changes the characterisation of the portfolio with the help of these matrices.

The business cycle

Macroeconomic conditions are approximated with a time-series of the Swedish output gap.

As pointed out, we approximate macroeconomic conditions with a time-series of quarterly observations of the Swedish output gap.[11] A recession is characterised by a nega-

[9] See Carling, Jacobson, Lindé & Roszbach (2002) for a detailed description of this data, in particular the characteristics of the transition matrices that are estimated for this loan portfolio.

[10] The probability of migrating from one arbitrary class to another is in fact less than the probability of remaining in the same risk class. For example, the following should apply: $p15<p14<p13<p12<p11$. In the case of a real loan portfolio, it may be that the risk of default in the risk classes does not increase monotonously, so that this inequality does not hold. Changes may also come about in the definitions of the rating classes, causing problems in estimating the transition matrices.

[11] See footnote 1 for an explanation.

tive output gap with unutilised resources, while a positive output gap, i.e. when GDP is higher than the trend, is associated with a boom. Since GDP's trend is not observable, a time-series for the output gap is estimated using data from observable variables, typically those that are related to actual GDP. We have chosen to use a vector autoregressive time-series model (VAR), in which foreign and Swedish GDPs, inflation and interest rates, along with Swedish credit losses, the repo rate, the real exchange rate and import prices are the variables included. For further details, see Appendix A.

Figure 1. Actual output gap according to the VAR model and an approximation of this output gap according to the AR(5) model

Per cent

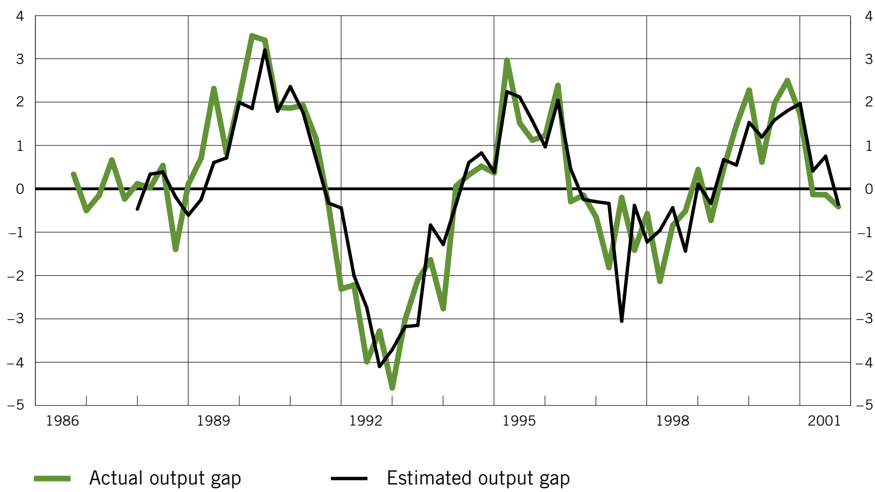
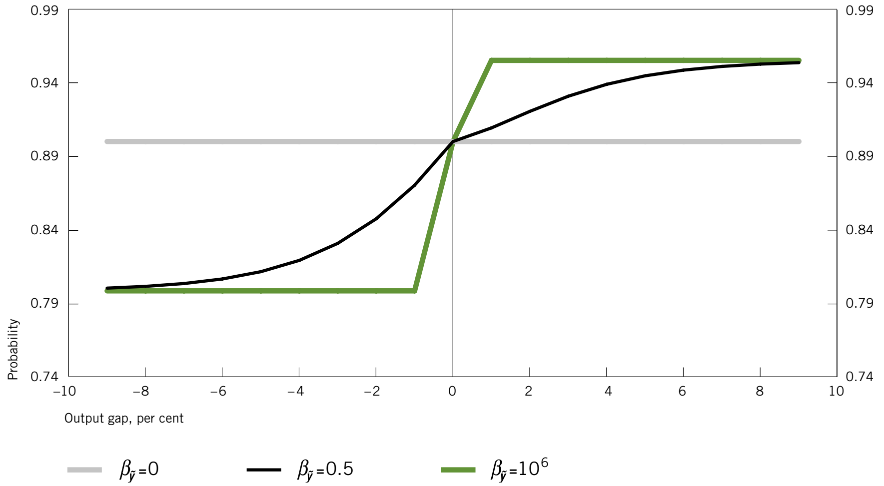


Figure 1 shows that the output gap was strongly positive at the end of the 1980s, only to drop dramatically from the beginning of the 1990s to the end of 1992, when the Riksbank floated the krona. Note that as the estimated output gap is level-adjusted to zero on average during this period, the percentage points on the axes should be interpreted as deviations from the average economic conditions during the sample period.

For the sake of simplicity, we assume that the probabilities in the transition matrices depend solely on the output gap.

For the sake of simplicity, we assume in our analysis that the probabilities in the transition matrices (which we will need to simulate bank data that fluctuate with the business cycle) depend solely on the output gap. This means that we are approximating the process that generates the values for the

Figure 2. The element$_{p\,1,\,1}$ in the transition matrix as a function of the output gap for different values of the parameter $\beta_{\tilde{y}}$
Per cent



output gap with a simple autoregressive model. The advantage of this simplification is that we do not need to model the remaining macroeconomic variables from the VAR model when generating our portfolio data. The results of the estimation (see Appendix A) show that an auto-regression in which the output gap in the current period is a function of five previous realisations (an AR(5) model) provides a reliable statistical approximation. As can be seen from Figure 2, the AR(5) model not only follows the trend in the output gap in accordance with the VAR model but also takes up the short term variations.

The transition matrix as a function of the business cycle

The parameters of the model have been chosen in such a way that both the business cycle and credit losses display a behaviour pattern which resembles that of the Swedish banking sector during the 1990s.

Here we describe how we make the probabilities of the transition matrix change over time with the movements of the business cycle. The transition matrix in each quarter will be limited by the extremities of the boom and recession periods (TM_{high} and TM_{low}). With the normal situation as given by the TM_{normal} matrix (page 43) as a starting point, the transition matrix at any given point in time is determined by macroeconomic conditions. A positive (negative) output gap, or a boom or slump period, will thus give a lower (higher) PD in the transition matrix and increased (decreased) probabilities of upgrading, and vice versa for downgrading. We can also determine the value of a parameter β_y that governs the rate at which the transition matrix moves towards the peak and trough of a business cycle.[12] Figure 2 illustrates the effects on the portfolio's cyclical sensitivity of choosing different values for the parameter β_y. Our goal is to decide on a level for β_y that ensures that the credit losses in our simulated portfolio exhibit a behaviour pattern that resembles the losses incurred by the Swedish bank sector during the 1990s. The figure shows that a portfolio with $\beta_y = 0$ will be completely cyclically insensitive; the probability that a company in rating class 1 in period t will be in the same class in the next period (i.e. $t + 1$) is constant (0.9) and thus independent of the output gap. A high value for β_y, on the other hand, means that the probability shifts greatly between the upper and the lower parameters when the output gap is not zero, thereby creating a cyclically sensitive portfolio. Only for relatively small positive values for β_y, like 0.5 (the assumed value in our analysis), is there a smooth transition in probabilities, and therefore "normal" cyclical sensitivity.

The construction and simulation of a hypothetical portfolio over time

As we are working with simulated data, we have no natural starting point for distributing the companies into the portfolio's different rating classes. Our analysis therefore needs a set of starting values – in other words, an initial distribution. In the appendix, we describe how the TM_{normal} transition matrix can be used to calculate such a distribution of companies in the portfolio. Since the TM_{normal} transition matrix is based on data from a Swedish bank, the initial distribution that we thus obtain will resemble what is observable in the data. Hereafter we will refer to this initial distribution as the steady state distribution.

The data simulation takes place as follows:

1. In the first period ($t = 1$), we allocate companies to all rating classes. The number of companies in each class is determined by multiplying the steady state proportion by the total number of companies. We have set this figure at

[12] This means that we can control the cyclical sensitivity of the loan portfolio by selecting the value of β_y. It should be noted that in practice β_y is slightly different for each rating class, which could mean that the sensitivity of the various rating classes to the macro-economy differs slightly. In this article, we have used the values 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.6, 0.6, 0.8, and 0.8 for rating classes 1 through 10, which means that companies in rating classes 7 to 10 are assumed to be slightly more cyclically sensitive than companies in classes 1 to 6. The appendix provides more details about the calibration of the parameters.

10,000, and each company is given a company registration number $i = 1,\ldots,$ 10,000 and a loan amount.[13]

2. We then calculate the output gap $\tilde{y}_t$ with the aid of the AR(5) model and generate the accompanying transition matrix for the period in question. The matrix is used to calculate the distribution of existing companies in the next quarter, $t + 1$.[14]
3. The new distribution in period $t + 1$ consists not only of the companies that were there in the previous period. We also assume that the bank grants loans to new companies in each period. We assume that the *distribution* of these new loans is the same as in the steady state distribution. We further assume that the *number* of new companies to which the bank grants loans in each period is constant and equal to the average number of companies that default, multiplied by the number of companies in the portfolio at $t = 1$ (i.e. 10,000). This means that if the output gap was 0 in all periods $t = 1,\ldots, T$, both the number and distribution of companies in the portfolio would be constant over time. It is assumed that the new companies are not able to default in the same period as they are granted credit.
4. For each period $t = 1,\ldots, T$, we record the following information on all "active" companies in the bank's portfolio: the company's registration number, the period, the risk class allocated, default (= 0 if the company is active, 1 if it defaults in this period) and size of loan. This information can then be used to calculate the capital adequacy requirements under Basel II and the portfolio's Value-at-Risk (VaR).

In this article, we use $N = 1{,}000$ hypothetical loan portfolios and assume that the banks keep information on their portfolios for $t = 1,\ldots, 40$ quarters, i.e. 10 years. Note that the data generation process is the same for all the N different simulated portfolios insofar as they are characterised by the same transition matrix. On the other hand, each portfolio is exposed to a unique series of macroeconomic condi-

[13] For the sake of simplicity, we assume that all the companies have loans of the same value. One could also let the companies have different loans by drawing loan amounts at random from a distribution with a mean of 1 and a standard deviation equal to what can be observed in actual bank data. The problem with the latter approach is that one would be assuming that over time the amount of the loan is independent of the companies and rating classes in the portfolio. It would be worthwhile to study the co-variation of the loan amount with rating class over time for the loan portfolios of Sweden's "big four" banks.

[14] For each company, i, we generate a random number, $p_{i,t}$ from a uniform probability distribution (which assumes a value in the interval [0.1]). If this company i is in rating class l during period t it is given rating class k in the next period, i.e. period $t + 1$, when the condition $\tilde{p}_{i,t} < \Sigma p_{lk,t}$ is satisfied for the lowest possible value of $k = 1,\ldots, r$, and where the probabilities $p_{lk,t}$ are obtained from the transition matrix. It should be noted that if the condition is not satisfied for $k = r$, this means that company i defaults during the period in question.

tions and all loans to unique, idiosyncratic risks. We also assume that each company in each period has a company-specific risk of defaulting.[15]

Figure 3 shows how well our simulation approach can match the actual credit losses to non-financial companies (as a percentage of total lending) that were incurred by the four major banks from 1990 Q1 to 1996 Q4.[16] As can be seen, the values predicted by our model correspond closely to the actual values. The results of the model's simulation can therefore be assumed to provide a fair, albeit stylised, picture of how the credit losses in the Swedish bank sector are affected by macroeconomic changes.

Figure 3. Actual and simulated credit losses for non-financial companies

Per cent

Note. Credit losses are expressed as a percentage of total lending. Simulated credit losses are generated in accordance with steps 1-4 of the dynamic portfolio model method and by applying the actual output gap (see Figure 1).

[15] We have described the way we model the company-specific risk in detail in Appendix A.

[16] To calculate the credit losses of the "big four" banks as a proportion of their lending to non-financial enterprises, we have used the following information: The total credit losses of the "big four" banks have been taken from Sveriges Riksbank's Financial Stability Reports (see, for example, Figure 1:9 in Financial Stability Report (1999). A compilation of what proportion of these total credit losses were loans to non-financial enterprises is provided by Dahlheim, Lind & Nedersjö (1993) for 1991 and 1992. For 1993 through 1996 we have obtained corresponding figures from the Swedish Financial Supervising Authority. For 1990 Q4 we assumed that the credit loss ratio was the same as for the whole of 1991. Information on the total volume of credit granted to non-financial enterprises has been obtained from Jan-Olof Elldin at the Riksbank's Financial Statistics Department (now the Monetary Policy Department).

VaR and IRB capital

The analyses of the simulated loan portfolios are made using two variables: VaR, which gives the portfolio risk, and IRB capital, which indicates the capital adequacy of the portfolio, calculated in the manner specified by the new Basel Accord.

In this context, VaR is a measure of the credit risk to which a loan portfolio is exposed for a given time horizon. More specifically, we define the VaR as the amount, expressed in kronor, that the bank risks losing within a given period of time, with a maximum probability of Z. An alternative way of expressing this is to say that there is a probability of $1 - Z$ that the bank's loss will not exceed the VaR amount in kronor during the given period of time, j. In practice, it is normal to select a time horizon, j, of one year and a probability, Z, in the interval [0.001–0.01].

Value-at-Risk is a measure of the credit risk in a loan portfolio for a given time horizon.

There are two more parameters we need to determine to be able to calculate VaR: the forecasting horizon, j, and the number of possible scenarios of the future, F, for each of the N portfolios for which we have generated data. We use $j = 1, 2, 3, 4$ quarters and $F = 1{,}000$. In each of the N portfolios we begin with the structure of the portfolio and the macroeconomic conditions in the final period, T. We then simulate 1,000 macroeconomic scenarios for the periods $T + 1, T + 2, \ldots, T + j$, and for each scenario we record information on the aggregate proportion of the portfolio that is defaulted for the coming $j = 1, 2, 3, 4$ quarters. Using this information we can then calculate, for each portfolio, the VaR at the 95 per cent level, $j = 1, 2, 3, 4$ quarters ahead in time, as the 95^{th} percentile in the distribution for the aggregated losses in the F different future scenarios.[17]

To calculate VaR we need to determine two additional parameters: the forecasting horizon and the number of possible future scenarios for each of the portfolios.

The loan portfolio's IRB capital is calculated in a number of steps. Firstly, we calculate the average risk of default for each rating class. This is a key component, since fluctuations in the probability of default (PD) over time have a direct effect on the IRB capital. According to the draft Basel Accord, the PD shall be characterised as a long-term estimate based on data for at least five years and covering a complete business cycle. This idea is based on the view

The loan portfolio's IRB capital is calculated in a number of steps. Firstly, we calculate the average risk of default for each rating class.

[17] Details about how VaR is calculated are provided in Appendix B.

that changes in portfolio risk shall be reflected in transitions by counterparties from one risk class to another; in other words, a bank's internal rating system shall be fully effective. If this is true, then there is some point in using a constant, characterising PD that is not allowed to vary during the course of a business cycle. In practice, it will be necessary to arrive at some sort of compromise between two conflicting approaches. On the one hand, the PD should vary over time so that it can reflect changes in credit risk that the internal credit system fails to pick up. On the other hand, it is important to avoid exaggerated, short-term instability in estimates of the PD, since this will cause unnecessarily wide fluctuations in the IRB capital, in the sense that they do not correspond to changes in the actual portfolio risk. We will look at differences in IRB capital for a range of horizons in the risk-weight estimates, namely 1, 4, 8, 20 and 40 quarters.

Given the estimated PDs for all of a bank's rating classes, the next step is to calculate the IRB risk weights as a function of these PDs.

Given the estimated PDs for all of a bank's rating classes, the next step is to calculate the IRB risk weights as a function of these PDs. The risk weights are then multiplied by the exposed assets (loans) in all rating classes and then summated to arrive at a total exposure. These exposed assets in turn serve as the denominator in the calculation of the portfolio's capital adequacy ratio.[18]

Some question marks, for example concerning the number of rating classes, still remain over the Basel Committee's proposal regarding the IRB approach.

Even though the Basel Committee's proposal for the IRB approach is specific in many respects, several question marks still remain. One of these relates to the number of rating classes banks should use. Variation in the capital adequacy requirement can arise, through several variables and parameters in the risk-weight formula. A higher PD in any rating class will, for example, raise the capital adequacy requirement for this rating class. Moreover, changes in transition frequencies between rating classes will affect the distribution of counterparts over the classes, and consequently the risk weight of these classes in the aggregated IRB capital. This means that changes in the number of rating classes and the boundaries between them can also affect the capital adequacy requirement. A reduction in the number of classes will shift the distribution of all loans over the classes and probably reduce the level of transition activity. But this also influences PDs that are associated with the risk classes, as the riskiest

[18] The risk weight function is discussed in Appendix C. We do not discuss the properties of the risk weight function or its derivation in this report. What can be mentioned, however, is that the function has been estimated by the Board of Governors using data on US bonds. The suitability of the function itself can thus also be a matter for discussion.

(safest) class will be aggregated with safer (riskier) classes. The significance of these effects is therefore an important empirical question.

Estimation of probabilities of default

Method A estimates the PD for a rating class over the most recent quarter.

The results show the amount of IRB capital calculated using two basic methods. Using Method A, the bank estimates the PD (and thus the risk weight) $f_{i,t}$, for rating class i for quarter s by calculating the probability of default between the immediately preceding quarter, $s-1$, and quarter s. If we call the single-period PD between quarter $s-1$ and quarter s $d_{i,s-1}$, then $f_{i,t}$ is determined as the average of $d_{i,s-h}$ for a number of horizons, h, backwards in time. In other words, $f_{i,t} = (1/\mathrm{h})\sum_1^h d_{i,s-1}$. If the bank wishes the characterising probability to be based on for example, three years of historical data, it calculates the twelve single-period probabilities and then uses the average of these twelve figures.

Method B starts with the companies in a rating class *h* quarters earlier and calculates what proportion of them has defaulted until today.

The other method, Method B, of calculating the risk of default, $f_{i,t}$, for a rating class, i, for a given horizon h back in time, starts with the companies that were in the rating class h periods earlier, and then calculates what proportion of them defaulted between $t-h$ and t. In other words, with Method B we calculate only one probability for the entire period. Both methods are reasonable, but it is fairly evident that the former method, A, makes use of more information than the latter does. It is relevant to consider Method B since a bank that has not recorded historical data for *each* quarter back in time could still produce the profile of the portfolio at a particular time in the past and then evaluate it on the basis of its current profile. We have chosen to show results for horizons h = 1, 4, 8, 20 and 40 quarters. Given the numerous mergers in the banking sector, and the introduction of internal rating systems in most banks in recent years, it is hardly likely that any Swedish bank has access to data going back for more than 40 quarters.

Another question that is closely related to the main topic is that of the risk horizon of the IRB capital, in other words, how far into the future the buffer capital should provide cover for the portfolio's credit risks. Since, as far as we are aware there is no given answer to this question, we show the results for risk horizons of one to four periods (quarters). These describe how well the IRB capital covers the portfolio risk, defined as its VaR.

Results

In this section, we give a numerical illustration of the IRB approach for calculating a bank's capital base. We use a dynamic perspective and an explicit connection to the effects of the business cycle. More specifically, using simulations, we will examine the amount of IRB capital in relation to the loan portfolio's credit risk, as measured in terms of its VaR. The overriding purpose of the simulations is to estimate how often the capital base, determined using the IRB formula, fails to provide adequate cover for the credit losses incurred by the portfolio. The opposite problem – an excessively large capital base in relation to the portfolio risk – is naturally of equal interest. As we are using admittedly realistic but nonetheless simulated portfolios and transition matrices, the results should not be interpreted literally but rather seen as illustrations of the qualitative character of the effects.[19]

Using Method A, the average amount of IRB capital is the same for a range of time horizons, while with Method B it falls as the horizons become longer.

The results of a few simple experiments are presented in Table 1. It should be noted that all the figures in the table are shown in relation to the portfolio value. They also relate to the average of the 1,000 simulated portfolios. We can immediately see that with Method A the average amount of IRB capital is the same for each of the time horizons h. However, using Method B, it is lower for the longer time horizons. We can also note that the shorter the time horizon h the bank uses, the wider the variations in the capital requirement. This is a natural implication of our method, where we have assumed that it is the underlying macroeconomic conditions that determine the risk of default. If the bank uses data for a long period of time – one that includes several business cycles – the booms and slumps will tend to offset each other. If, on the other hand, the bank only uses data for the previous few quarters, or even only one quarter, there will naturally be wider fluctuations in the capital adequacy requirement, as the estimation period could have been a boom or a trough. It should also be noted that since Method A and Method B are identical when $h = 1$, the results for this time horizon are the same.

Our simulated portfolios imply a higher average capital adequacy level than with the existing set of rules.

We may also note that our simulated portfolios imply a higher average capital adequacy level than with the present set of rules (around 11 per cent instead of the present 8 per cent).

[19] The default risk in our simulated data generally matches the empirical distribution on a portfolio level, but is only approximately matched for individual risk classes.

Table 1. Results of the simulation of the amount of IRB capital and Value-at-Risk (VaR) for the portfolio method

	Amount of IRB capital as a proportion of the value of the portfolio using information *h* quarters backwards in time				
Method of calculation	$h = 1$	$h = 4$	$h = 8$	$h = 20$	$h = 40$
A	0.111	0.113	0.114	0.112	0.111
	(0.043)	(0.042)	(0.039)	(0.026)	(0.023)
B	0.111	0.114	0.111	0.099	0.087
	(0.043)	(0.044)	(0.040)	(0.019)	(0.012)

	Average VaR *j* quarters ahead			
VaR percentile	$j = 1$	$j = 2$	$j = 3$	$j = 4$
95 %	0.020	0.039	0.059	0.080
99 %	0.023	0.044	0.066	0.090

	99th percentile for the amount of IRB capital				
Method of calculation	$h = 1$	$h = 4$	$h = 8$	$h = 20$	$h = 40$
A	0.223	0.221	0.213	0.178	0.169
B	0.223	0.227	0.208	0.144	0.115

Note: The number of simulated portfolios is 1,000. At first, these have been simulated 40 periods, after which the amount of IRB capital has been calculated. After that each and every one of these 1,000 portfolios has been simulated 1,000 times for four more periods with different macroeconomic results. This is done in order to calculate a measure of future credit risk (VAR) for every portfolio. The figures within the parentheses are the standard deviation of the amount of IRB capital.

However, this result should be interpreted with some caution as it could be an effect of the probabilities in the chosen transition matrices being typical of banks with a higher risk propensity than is actually the case. Empirical results for a business loan profile in Carling, Jacobson, Lindé and Roszbach (2002) suggest that the capital requirement based on the IRB approach could vary widely depending on the phase in the business cycle. In other words, in a recession it could exceed the absolute capital adequacy requirement of 8 per cent, while in good times it could fall to very low levels.[20] This result is consistent with the Basel Committee's endeavour to design its new Accord in such a way that the capital adequacy requirement more accurately reflects a portfolio's credit risk. However, what is a reasonable or desirable variation in a bank's capital base due to changes in macroeconomic conditions remains an open question.

In Table 1, we have also illustrated the average of the 95% and 99% VaR estimates of the portfolio risk for a range of forecasting horizons (the coming 1, 2,

[20] These empirical results are calculated on the assumption that neither the bank's portfolio nor credit policy changes as a result of the new regulation.

The Value-at-Risk exposure increases almost linearly with the forecasting horizon.

3 and 4 quarters). It should be observed that VaR is independent of the method of calculation, and that the portfolio risk does not depend on the method used to determine the capital requirement. Table 1 shows that the risk exposure increases almost linearly with the forecasting horizon. The average 95% VaR for one year ahead is 8 per cent, which may be compared with some 2 per cent for the next quarter. The linear increase in the risk over time can be explained in part by the fact that the proportion of companies defaulting on their loans each quarter remains roughly constant in the long term. The more pronounced linearity for the 95% VaR than for the 99% VaR is probably due to the fact that extremely unfavourable outcomes – which are reflected in the 99% but not the 95% VaR – do not occur at regular intervals. The rate of increase in the average 99% VaR will therefore be less regular than for the 95% VaR.

Finally, in the lower section of the table we show the 99th percentile of the distribution of the IRB capital for the 1,000 simulated portfolios. The higher standard deviation in the IRB capital for $h = 1$ than for $h = 40$, which we observed earlier, is reflected here in the form of a markedly higher percentile value for $h = 1$ than for $h = 40$. The distribution of IRB capital of the 1,000 portfolios thus acquires a longer "tail" the shorter the historical period used for calculating the capital. We should also note how the results with Method A differ from those obtained with Method B. Whereas the two methods generate broadly equally large values when h is less than 20, the 99th percentile is considerably greater with Method A than with Method B for $h = 20$ and $h = 40$. With system A, in other words, as a consequence of the (historically) extremely poor macroeconomic conditions included in the calculations, the IRB capital will be considerably higher than with system B.

Which method, A or B, implies the highest correlation between IRB capital and VaR?

This observation brings us directly to the question of which of the methods, A or B, gives the highest correlation between the IRB capital and VaR? The objective of the new Accord is to make banks' capital bases sensitive to risk; in other words, the greater the credit risk the larger the IRB capital. The upper section of Table 2 reflects the co-variation between the various measures of IRB capital and portfolio risk, defined as the 99% VaR. As expected, the co-variation between the buffer capital and credit risk is stronger the shorter the horizon used for calculating the risk of default. For the shortest horizon, $h = 1$, the IRB capital is determined solely by the losses incurred during the immediately preceding quarter. The portfolio risk is driven by changes in the output gap. As this is autocorrelated (observations in the series co-vary with earlier observations) the latest observations in the sample incorporate more information than an average observation. A short horizon therefore gives a closer match between the buffer capital and the portfolio risk than when information from several periods is used. That Method A generates a closer correlation than Method B for all values of h above 1 is also in accordance with our expectations, since Method B only uses information from two quarters to estimate the PD, $f_{i,t}$. Method B thus disregards more recent information about the prevailing state of the economy. The difference between the two methods becomes even more marked as the horizon, h, becomes longer. Our conclusion is that in this respect Method A is a better tool than Method B; to what extent this is so depends in practice on the autocorrelation with macroeconomic conditions (output gap), the cyclical sensitivity of loan portfolios, and h. It is worth noting that if a portfolio's credit risk is exclusively idiosyncratic in character, and is not affected by changes in the macroeconomy, a higher h could very well result in a higher correlation between risk and buffer capital.

Table 2. Interaction between capital adequacy requirements, Value-at-Risk and macroeconomic conditions based on simulated data

	Correlation between IRB capital and VaR at the 99 per cent level					
Quarters ahead	Method A			Method B		
j	h = 1	h = 8	h = 40	h = 1	h = 8	h = 40
1	0,90	0,69	0,67	0,90	0,63	0,54
2	0,89	0,61	0,59	0,89	0,54	0,47
3	0,84	0,51	0,51	0,84	0,44	0,40
4	0,80	0,42	0,43	0,80	0,35	0,33

	Probability that IRB capital will fall short of VaR at the 95 per cent level					
Quarters ahead	Method A			Method B		
j	h = 1	h = 20	h = 40	h = 1	h = 20	h = 40
1	0,00	0,00	0,00	0,00	0,00	0,00
2	0,00	0,00	0,00	0,00	0,00	0,00
3	0,00	0,00	0,00	0,00	0,04	0,13
4	0,11	0,16	0,16	0,11	0,30	0,40

	Correlation between the capital adequacy requirements in period *t* and macroeconomic developments, *t*+1, *t*+2, *t*+3 och *t*+4					
	Method A			Method B		
Quarter	h = 1	h = 8	h = 20	h = 1	h = 8	h = 20
t	–0,89	–0,60	–0,53	–0,89	–0,54	–0,37
t+1	–0,74	–0,38	–0,34	–0,74	–0,31	–0,23
t+2	–0,60	–0,15	–0,15	–0,60	–0,08	–0,08
t+3	–0,35	0,12	0,08	–0,35	0,19	0,09
t+4	–0,17	0,37	0,30	–0,17	0,43	0,25

Note: See Table 1.

What is the probability that the buffer capital will fall short of the losses incurred on the portfolio in the future?

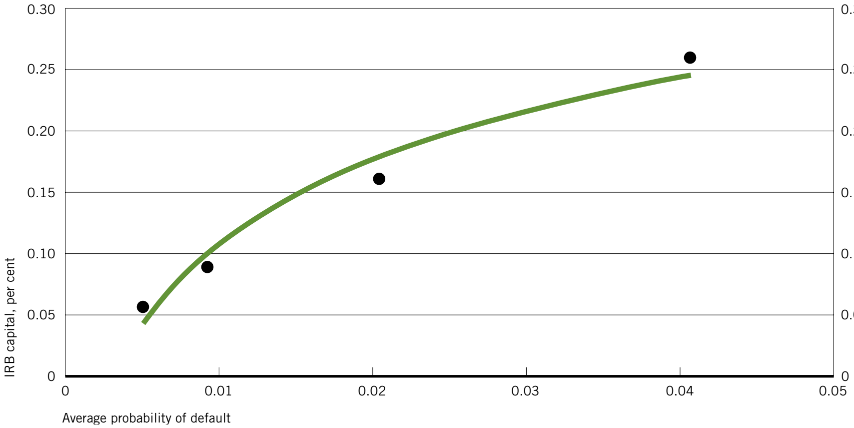
The next question is: What is the probability that the capital adequacy requirement, and thus the buffer capital, will fall short of future portfolio losses? The risk that the bank itself will default or be compelled to sell assets to cover the losses incurred can be measured in terms of the probability that the IRB capital will fall short of some critical VaR percentile. We can see in the second panel in Table 2 that the risk that the buffer capital will not be adequate to cover the 95% VaR is almost non-existent for forecasting horizons of up to an including six months. For longer horizons and higher values for h, there is a significant risk that the buffer capital will not be adequate, especially if Method B is used.

A large capital base is not an end in itself, but a means of enabling supervisory authorities to compel banks to ensure their survival in the event of unfavourable outcomes.

A large capital base is not an end in itself, but a means of enabling supervisory authorities to compel banks to ensure their survival in the event of unfavourable outcomes. Admittedly, an exaggeratedly high capital adequacy requirement could give rise to other problems. For example, it might encourage banks to devote their energy to devising means of circumventing the rules instead of concentrating on their core business, that of assessing and pricing risk. Our analysis shows that the IRB capital can be inadequate in the event of a deep recession. Is this acceptable? To put it another way, how often should this be allowed to happen? The point at issue is to decide on the correct level for the absolute capital adequacy requirement of 8 per cent. This is the level used in the current regulatory system, and it has its origins in the capital ratios that well-run banks adhered to when the current rules were drawn up a decade or so ago. It has been decided that the new Accord should retain the 8 per cent level, and an attempt has been made to modify the IRB approach so that it prevents the capital adequacy requirement on average from falling below the level in the existing set of rules. When the new Accord was being worked out, priority was not given to identifying the socially optimal level for a bank's capital base. This topic also lies outside the scope of this article. In the absence of such an analysis, it is difficult to determine whether the results in Table 2 are reasonable in terms of their social welfare.

A much discussed problem that can arise if buffer capital is based on risk is that of its "procyclicality". A close co-variation between buffer capital and risk, which will result when h is low, can mean that banks will need to add financial assets to their reserves precisely when the macroeconomy stance is weak. This

Figure 4. Average probability of default and IRB capital



Note. Figure 4 shows how the average IRB capital (as a proportion of the portfolio) varies with the portfolio's average risk of default when macroeconomic conditions are normal ($\beta\tilde{y} = 0$).

A much discussed problem associated with having a risk-based buffer capital is that of procyclicality.

could result in a credit crunch that would aggravate the recession. The reverse also applies: the lower level of risk during a boom could lead to a low capital adequacy requirement that would release capital for a credit expansion. This in turn would reinforce the boom. This is illustrated in the lower section of Table 2: a small h and j in combination with Method A, which generates a close relationship between buffer capital and risk, is broadly associated with the closest correlation between the capital adequacy requirement and macroeconomic conditions. Empirically, the quantitative significance of this effect has, however, only been supported to a limited extent.

Finally, in Figure 4 we show a tentative relationship between the average IRB capital and the portfolio's average PD under normal macroeconomic conditions, $\tilde{y}_t = 0$. The figure shows that the capital does not appear to increase linearly with the risk, but that the capital/risk ratio is falling. This means, at the risk of taking things to extremes, that the IRB approach gives banks an incentive to increase their portfolios' riskiness rather than reduce it, as a higher risk does not require proportionately more capital.

To obtain a better understanding of the relevance of the macroeconomy for the effects of the new Basel Accord, we have examined what effect the cyclical sensitivity of loan portfolios has on the conclusions to be drawn from Table 1 and Table 2. The cyclical sensitivity in the simulations is determined by two groups of parameters, the three different transition matrices, and the number of degrees of freedom that control the rate at which the portfolio shifts from the normal situation to a recession or a boom. Table 3 and Table 4 illustrate simulations for banks with portfolios that are perfectly insensitive to cyclical fluctuations ($\beta_{\tilde{y}} = 0$) and banks that are assumed to be heavily exposed to cyclical fluctuations ($\beta = 10^8$). The portfolios of both types of bank have the same average default risk. All our calculations have been made using estimation Method A, since it appears more useful than Method B in the light of the earlier results.

Just as in Table 1, the average VaR at a 99 per cent level doubles for every quarter by which the forecasting horizon is extended. This is true of both types of bank, but the VaR is roughly twice as high for the cyclical sensitive ones. Table 3 also illustrates how cyclical sensitivity influences the average capital adequacy requirement, which in the case of cyclically sensitive banks is some 25 per cent higher with a standard deviation that is greater by a factor of 10 to 25, almost regardless of the horizon h considered. These differences are also reflected in differences in the 99th percentile of the banks' IRB capital. The maximum loss the bank will be exposed to at the 1 per cent level is around 125 per cent higher in the case of cyclically sensitive banks for all horizons h.

The co-variation between the IRB capital and the 99 per cent VaR depends very closely on the bank's sensitivity to changes in macroeconomic conditions. Table 4 shows that a greater sensitivity to variations in the output gap leads to a

Table 3. Comparison of a relatively cyclical sensitive portfolio with a relatively less sensitive portfolio with respect to the amount of IRB capital and Value-at-Risk

	Amount of IRB capital as a proportion of the value of the portfolio using information h quarters backwards in time				
Business cycle sensitivity	$h = 1$	$h = 4$	$h = 8$	$h = 20$	$h = 40$
$\beta_{\tilde{y}} = 0$	0.105	0.105	0.105	0.105	0.105
	(.008)	(.005)	(.003)	(.002)	(.002)
$\beta_{\tilde{y}} = 10^6$	0.129	0.135	0.137	0.133	0.132
	(.080)	(.077)	(.071)	(.051)	(.046)
	Average VaR at the 99 per cent level j quarters ahead				
	$j = 1$	$j = 2$	$j = 3$	$j = 4$	
$\beta_{\tilde{y}} = 0$	0.016	0.030	0.044	0.057	
$\beta_{\tilde{y}} = 10^6$	0.032	0.063	0.095	0.127	
	99th percentile for the amount of IRB capital				
	$h = 1$	$h = 4$	$h = 8$	$h = 20$	$h = 40$
$\beta_{\tilde{y}} = 0$	0.121	0.115	0.113	0.111	0.109
$\beta_{\tilde{y}} = 10^6$	0.280	0.279	0.278	0.254	0.238

Note: See Table 1.

higher correlation between the buffer capital, on the one hand, and macroeconomic conditions and the VaR, on the other. It should be noted that even if a bank is fully protected against macroeconomic fluctuations, the IRB capital and the VaR will co-vary to some extent; by exactly how much will depend on the forecasting horizon.

Table 4. Interaction between capital adequacy requirements, Value-at-Risk and macroeconomic conditions: a comparison of a relatively cyclical sensitive portfolio to a relatively less sensitive

	Correlation between IRB capital and VaR at the 99 per cent level					
Quarters ahead	$\beta_{\tilde{y}} = 0$			$\beta_{\tilde{y}} = 10^6$		
j	$h = 1$	$h = 8$	$h = 40$	$h = 1$	$h = 8$	$h = 40$
1	0.40	0.41	0.37	0.87	0.82	0.81
2	0.38	0.37	0.33	0.86	0.78	0.78
3	0.32	0.35	0.30	0.85	0.74	0.75
4	0.29	0.30	0.32	0.84	0.70	0.72

	Probability that the IRB capital will fall short of VaR at the 95 per cent level					
Quarters ahead	$\beta_{\tilde{y}} = 0$			$\beta_{\tilde{y}} = 10^6$		
j	$h = 1$	$h = 20$	$h = 40$	$h = 1$	$h = 20$	$h = 40$
1	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.12	0.01	0.00
3	0.00	0.00	0.00	0.28	0.10	0.06
4	0.00	0.00	0.00	0.38	0.40	0.41

	Correlation between the capital adequacy requirements in period *t* and macroeconomic conditions at *t*, *t*+1, *t*+2, *t*+3 och *t*+4					
	$\beta_{\tilde{y}} = 0$			$\beta_{\tilde{y}} = 10^6$		
Quarter	$h = 1$	$h = 8$	$h = 20$	$h = 1$	$h = 8$	$h = 20$
t	0.02	–0.01	–0.01	–0.77	–0.55	–0.49
t+1	–0.01	0.01	–0.02	–0.63	–0.35	–0.32
t+2	–0.00	0.02	0.00	–0.50	–0.15	–0.15
t+3	0.01	0.04	0.02	–0.28	0.11	0.06
t+4	0.01	0.01	0.01	–0.10	0.33	0.26

Note: See Table 1.

The risk that a cyclically sensitive bank will default turns out to be far greater than with the bank in Table 1 and Table 2, for example, especially when h is small and forecasting horizons are long. With a forecasting horizon of one year, which may be regarded as a reasonable period in the IRB context, the probability of the buffer capital falling below the 95 per cent VaR is approximately 0.4, regardless of the h chosen. This may be compared with 0.11–0.16 (for A, but 0.11–0.40 for B) for the bank in Tables 1 and 2. The perfectly insensitive bank has a risk of 0.0 for all h and all forecasting horizons. The reason for this is that a model for calculating the capital adequacy requirement that only takes account of economic conditions during the latest quarter disregards important information about the ensuing development of the business cycle. Shortcomings of this type will naturally have a greater impact on a bank's capital ratios if it is sensitive to the business cycle. A bank that is entirely shielded from macro fluctuations will base its buffer capital on an estimate of the constant, steady state default risk (at portfolio level). As the risk of default does not fluctuate at all, the buffer capital will be adequate under almost any circumstances whatsoever.

The results suggest that the number of classes as such does not necessarily affect capital adequacy requirements.

As a final experiment, we have investigated whether the number of risk classes used by a bank affects the level of IRB capital and its co-variation with the VaR. The underlying hypothesis is that fewer classes should lead to wider variations in their risk profiles. As we have already observed, the non-linearity of the IRB approach's risk-weight function prompts the question of what effect the number of classes (naturally in combination with the size of the portfolio) will have on the capital adequacy requirement. The results of a number of modest experiments suggest that the number of risk classes as such has no effect on the capital adequacy requirement. However, the question of the number of classes is to be considered in combination with the bank's ability to classify its corporate clients into the given rating classes (see Carling, Jacobson, Lindé & Roszbach (2002)). If there is a growing tendency for corporate clients to be wrongly classified by the bank, a reduction in the number of classes could give rise to wider fluctuations in risk weights and consequently also in the IRB capital.

Summary and conclusions

The main principle underlying the Basel proposal is to make buffer capital significantly more dependent on risk than in the existing system.

In January 2001, the Basel Committee published its proposals for new capital adequacy rules for banks. The main principle in its proposals is that the buffer capital should be significantly more dependent on risk than in the existing system. Currently, in the case of business loans, for example, a bank is expected to maintain 8 per cent of its exposure as a buffer to cover credit losses. Some types of collateral accepted by banks can reduce the capital requirement, but broadly speaking the requirement is constant and independent of counterparty risk. Under the new regime, banks would be given greater responsibility for calculating both capital at risk and the required amount of buffer capital. The proposal includes two alternative systems: a standard method and a more sophisticated, IRB approach whereby the banks would need to introduce or improve upon each of their internal rating systems for classifying the counterparties in their loan portfolios. Quantified risk characteristics in these rating classes are used to calculate risk weights, which in turn determine how much capital the bank needs to hold in reserve for each krona it has lent.

In this article we have looked at what effect the characteristics of the proposed IRB approach would have by simulating a large number of business loan portfolios. These simulations have provided new information about what consequences the new Accord will have for banks and how these effects will vary depending on the final wording of the Accord adopted by the supervisory authorities.

The longer the period for which the probability of default is calculated, the lower the capital adequacy requirement.

One conclusion is that the distribution of the IRB capital over the 1,000 portfolios in our analysis depends on the method used to calculate the average actual, historical risk of default for each rating class: the longer the period for which the risk of default is calculated, the lower the capital adequacy requirement. The shift in the distribution is reflected in both the mean value and the variance.

The longer the forecasting horizon, the higher the probability that the buffer capital will fall short of the credit losses.

On top of this, it turns out that the probability that the buffer capital will fall short of the credit losses increases as the forecasting horizon lengthens. The longer the time horizon it is assumed the IRB capital has to cover, the weaker the co-variation with the future credit risk and the greater the risk of the buffer capital proving inadequate. This characteristic is due to the way the portfolio risk, defined as the VaR, increases linearly with the forecasting horizon. For the portfolios we have examined, the probability varies between 0 and as much as 0.4 for forecasting horizons up to one year. The magnitude of this effect also depends on the preferred method of calculation, since this has an influence on the correlation between the IRB capital and the VaR.

The bank's business cycle sensitivity has a considerable effect on the correlation between the IRB capital and the portfolio's VaR.

Macroeconomic conditions, not unexpectedly, also play an important role in the way the new capital adequacy system ought to be designed. Our results suggest that a bank's business cycle sensitivity has a marked influence on the correlation between the IRB capital and the portfolio's VaR and thus also on the probability that the buffer capital will turn out to be inadequate.

Variations in the cyclical sensitivity of banks can multiply the variance in the IRB capital many times over. Given otherwise identical capital adequacy systems, the risk that a bank's buffer capital will not be enough to cover its credit losses will double or even triple. How important the selection of calculation method is in this context will in turn depend on the interaction, if any, between the forecasting horizon for the risk of default and the serial correlation in the output gap.

The new Basel Accord could reinforce cyclical fluctuations more than current regulation.

The results of our analysis are not inconsistent with the results of other studies examining whether or not the new Basel Accord will involve greater risk of procyclical effects. A strong co-variation between buffer capital and credit risk, which some of the possible methods of calculation can generate, could very well mean that banks will need to build up their financial reserves precisely when macroeconomic conditions are weak. The new Accord would thus have a tendency to reinforce cyclical fluctuations rather more than the existing system of capital adequacy rules.

To sum up, therefore, we may note that even if the basic characteristics of the new Accord have now been settled, quite some work remains to be done on the details for the practical application of the Accord by banks. How factors such as the forecasting horizon, method of calculation, and consideration of the business cycle sensitivity of banks are finally incorporated into the Accord will be of significance to the amount of IRB capital and its variance. Decisions on these factors could have far-reaching consequences for the functioning of the banking system and the economy in general. Close attention is already being paid to problems associated with inadequate capital reserves and the possibility that the Accord will reinforce cyclical effects. The need for such attention will certainly not decline in the future; the final wording of the Accord, however, will not be affected to any significant extent by such analyses.

References

Carling, K., Jacobson, T., Lindé, J. & Roszbach, K., (2002), *Capital Charges under Basel II: Corporate Credit Risk Modelling and the Macro Economy*, Sveriges Riksbank Working Paper Series, No. 142.

Dahlheim, B., Lind, G. & Nedersjö, A.K., (1992), Bank Results in Sweden and Other Nordic Countries, *Sveriges Riksbank Economic Review*, No. 2, pp. 21–30.

Financial Stability Report, (1999), No 1, May, Sveriges Riksbank, Stockholm, Sweden.

Jackson, P., Furfine, C., Groeneveld, H., Hancock, D., Jones, D., Perraudin, W., Radecki, L. & Yoneyama, M., (1999), *Capital Requirements and Bank Behaviour: The Impact of the Basel Accord*, Basel Committee on Banking Supervision Working Papers, No. 1.

Jacobson, T. & Lindé, J., (2000), Credit Rating and the Business Cycle: can bankruptcies be forecast?, *Sveriges Riksbank Economic Review*, No. 4, pp. 11–33.

The New Basel Capital Accord: An Explanatory Note, Secretariat of the Basel Committee on Banking Supervision, Bank for International Settlements, (www.bis.org/publ/bcbsca.htm).

Wilson, T., (1997), *Portfolio Credit Risk (I)*, Risk, Vol. 10, No. 9 (September), pp. 111–117.

Wilson, T., (1997), *Portfolio Credit Risk (II)*, Risk, Vol. 10, No. 10 (October), pp. 56–61.

Appendix A: Method of analysis, transition matrices and the business cycle

In this Appendix, we describe in detail the various components of our analysis. We make use throughout of hypothetical bank portfolios consisting of 10,000 business loans per quarter distributed among a number of rating classes. The portfolios are characterised by transition matrices, in which the elements consist of probabilities that a counterparty will migrate from one rating class to another, or probabilities that counterparties will stay in the same class. The transition matrices thus reflect the migration between the rating classes in the portfolio for any given time horizon, from the initial date, say, until one year later. The movements between these transition matrices are determined by the prevailing output gap, which is the measure of the general state of the economy.

Transition matrices

Let us suppose that in its credit operations, our hypothetical bank gives each counterparty a credit rating each quarter. This means that the loans in the portfolio are distributed among a number of rating classes. If the bank does this for a longer series of quarters and systematically records the data relating to the migration of these loans among the rating classes, it can then, using this information, estimate a transition matrix (*TM*):

$$TM = \begin{bmatrix} p_{11} & p_{12} & \cdot & \cdot & \cdot & p_{1r} & p_{1d} \\ p_{21} & p_{22} & \cdot & \cdot & \cdot & p_{2r} & p_{2d} \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ p_{r1} & p_{r2} & \cdot & \cdot & \cdot & p_{rr} & p_{rd} \end{bmatrix},$$

where r is the number of non-defaulting rating classes and d denotes the rating class in which defaulting loans are placed. If the bank uses, say, ten non-defaulting rating classes, *TM* will have ten rows and eleven columns. The element p_{kl} denotes an estimated probability that a loan in rating class k in period t will be moved to rating class l in period $t + 1$. For example, p_{11} represents the probability that loans in the rating class with the highest credit rating will still be in that class in the next period, while p_{rd} indicates the probability that the lowest rated loans will default during the time interval t to $t + 1$. It might be worth noting that *TM* has to be estimated on the assumption that the sum of the probabilities in any one row (in respect of one rating class) is 1.

$$TM_{low} = \begin{bmatrix} .80 & .08 & .05 & .03 & .02 & .01 & .01 & .00 & .00 & .00 & .00 \\ .00 & .80 & .07 & .04 & .03 & .02 & .02 & .015 & .005 & .00 & .00 \\ .00 & .005 & .80 & .05 & .04 & .03 & .02 & .02 & .015 & .01 & .01 \\ .00 & .00 & .02 & .81 & .03 & .02 & .03 & .05 & .01 & .02 & .01 \\ .00 & .00 & .005 & .01 & .79 & .05 & .03 & .04 & .02 & .03 & .025 \\ .00 & .00 & .01 & .02 & .03 & .82 & .03 & .00 & .02 & .03 & .04 \\ .00 & .00 & .005 & .01 & .015 & .02 & .82 & .03 & .03 & .03 & .04 \\ .00 & .00 & .00 & .00 & .01 & .01 & .04 & .79 & .05 & .05 & .05 \\ .00 & .00 & .00 & .00 & .00 & .01 & .02 & .04 & .75 & .09 & .09 \\ .00 & .00 & .00 & .00 & .00 & .00 & .01 & .03 & .06 & .72 & .18 \end{bmatrix}$$

$$TM_{high} = \begin{bmatrix} .95 & .03 & .01 & .01 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\ .03 & .95 & .01 & .01 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\ .02 & .03 & .93 & .02 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\ .01 & .02 & .06 & .88 & .02 & .01 & .00 & .00 & .00 & .00 & .00 \\ .01 & .01 & .05 & .07 & .82 & .025 & .01 & .00 & .00 & .005 & .00 \\ .005 & .015 & .02 & .04 & .08 & .785 & .03 & .02 & .005 & .00 & .00 \\ .00 & .01 & .02 & .03 & .07 & .09 & .72 & .03 & .015 & .01 & .005 \\ .00 & .00 & .02 & .02 & .05 & .07 & .10 & .69 & .02 & .02 & .01 \\ .00 & .00 & .00 & .01 & .03 & .05 & .07 & .14 & .66 & .03 & .01 \\ .00 & .00 & .00 & .00 & .01 & .02 & .02 & .07 & .18 & .68 & .02 \end{bmatrix}$$

As already noted, there is reason to believe that the probabilities in *TM* do not remain constant throughout an economic cycle (see e.g. Wilson (1997)). It is reasonable to suppose that the risk of default can be expected to decline when the economy is strong, and that in a recession a transition matrix with higher probabilities that borrowers' credit ratings will be lowered will more accurately describe a bank's loan portfolio. Using three different transition matrices, we will now explain how general macroeconomic conditions can alter the characterisation of the portfolio. In the case of TM_{normal}, the numerical values of the matrix elements have been determined with the aid of empirical data from a major Swedish bank's business loan portfolio (see Carling, Jacobson, Lindé & Roszbach (2002)). TM_{high} and TM_{low} – which are reasonable, albeit arbitrary, adjustments of TM_{normal} – have been arrived at on the basis of the transition matrices presented in Wilson (1997). TM_{normal} is described in the main text, while TM_{low} and TM_{high} are shown below.

If TM_{low} and TM_{high} are compared with TM_{normal} in the main text, we can see how the whole block of probabilities has shifted to the right (left) in the matrix in TM_{low} (TM_{high}). This means that on average corporate borrowers are running a greater risk of having their credit rating downgraded (upgraded) in a recession (boom), in relation to a normal, more balanced point on the economic cycle.

The business cycle

Macroeconomic developments are approximated using a time-series of quarterly observations of Sweden's output gap. This is expressed as the difference between the actual real GDP and an estimated secular trend in GDP. A direct observation of the secular trend in GDP is not possible, a time-series has to be estimated on the basis of observed data. Numerous estimating methods are described in the literature. We have chosen to use a vector autoregressive time-series model. Let X_t stand for a 9 x 1 column vector with the variables y_t^* (logarithmic foreign GDP, TCW-weighted), π_t^* (foreign inflation, annual rate; in other words $\pi_t^* = \ln(p_t^*/p_{t-4}^*)$, TCW-weighted), R_t^* (foreign interest rate, 3-month duration, TCW-weighted), y_t (logarithmic Swedish GDP at current prices), π_t (annual rate of inflation, measured using GDP deflator), k_t (logarithmic credit losses in Sweden's big four banks), R_t (repo rate and its equivalent before 1 June 1994), Q_t (real effective exchange rate, TCW-weighted), and π_t^{imp} (import price index at producer stage, as defined by Statistics Sweden). The VAR model for X_t can then be expressed as:

$$(1) \qquad X_t = C + \tau T_t + \delta_1 D_{92Q3} + \delta_2 D_{93Q101Q3} + \sum_{j=1}^{2} \Gamma_j X_{t-j} + \varepsilon_t \ ,$$

where C is a constant, T_1 a linear time trend, and D_{92Q3} is a dummy variable that assumes the value 1 in the third quarter of 1992 and equals 0 otherwise. $D_{93Q101Q3}$ is a dummy variable that assumes the value 0 before 1993 and 1 thereafter. We estimate the model (1) on quarterly data for the period between 1986 Q3 and 2001 Q3. As the model incorporates two lags, this means that we use data for the period between 1986 Q1 and 2001 Q3. By simulating the estimated model dynamically for the period 1986 Q3 to 2001 Q3 (using the real values for X, 1986

Q1 to 1986 Q2 as the opening values in the simulation and where $\hat{\varepsilon}_t = 0$), we can obtain a trend that varies with time for the variables in X_t, which we have called $\tilde{X}_t$. The deviation around the trend, designated $\tilde{X}_t$, can then be calculated as $X_t - \tilde{X}_t$. The resultant deviation around the trend for GDP, which we call the output gap in the following, is shown as the green line in Figure 1 (page 44).

Since in our analysis we will, for the sake of simplicity, assume that the probabilities in the transition matrix are solely dependent on the output gap, and thus not on the other variables in the VaR model, we have decided to approximate the data-generation process for the output gap by a simple AR(5) model The estimated model can be expressed using the following formula:

$$\tilde{y}_t = \underset{(0.12)}{0.65}\tilde{y}_{t-1} + \underset{(0.13)}{0.19}\tilde{y}_{t-2} + \underset{(0.13)}{0.05}\tilde{y}_{t-3} + \underset{(0.13)}{0.41}\tilde{y}_{t-4} + \underset{(0.11)}{0.67}\tilde{y}_{t-5} + \hat{\varepsilon}_{y,t}. \tag{2}$$
$$\bar{R}^2 = 0.76,\ \hat{\sigma} = 0.90\%,\ Box - Ljung\ Q(8) = 3.89\ (p - value = 0.87)$$

As we can see from the estimation results, the estimated equation is an acceptable approximation for the output gap. We have plotted the estimated values according to equation 2 (the black line) in Figure 1 (page 44).

The transition matrix as a function of the output gap

In this section we present our method for modelling the continuous shifting from a characterising transition matrix in one quarter to another matrix in the next quarter. These shifts occur in response to changes in the macroeconomic conditions as time passes. In other words, we want the probabilities in the transition matrix to shift with time as a function of changes in the output gap. Let us call the portfolio-characterising transition matrix in each quarter $TM_{\text{state},t}$. Let us limit this in the extreme positions – TM_{high} and TM_{low} – and use TM_{normal} as an identifier. We arrive at the elements, or probabilities, in $TM_{\text{state},t}$ by using a flexible probability distribution, $\chi^2(df)$. We control the appearance of this distribution by using the parameter df, which indicates the number of degrees of freedom. For example, if the probability mass for the distribution shifts to the right, then df increases. We let df be a function of the output gap, $df_t = df(\tilde{y}_t)$, such that a positive output gap (boom) results in lower probabilities of default in the transition matrix, higher probabilities of a higher credit rating and lower probabilities of the rating being lowered. A negative output gap (recession) results in the opposite, namely higher probabilities of default, lower probabilities of a rating upgrade and higher probabilities of it being lowered. The function $df_t = df(\tilde{y}_t)$ is expressed as:

(3) $$df_t = \overline{df} + \frac{(\underline{df} - \overline{df})}{1+e^{-\beta_{\tilde{y}}\tilde{y}_t}} - \frac{\left(\frac{\overline{df}+\underline{df}}{2} - df\right)}{e^{\text{abs}(\beta_{\tilde{y}}\tilde{y}_t)}}$$

where df, $\overline{df}$ and $\underline{df}$ are the degrees of freedom associated with TM_{normal}, TM_{high} and TM_{low}.[21] $\beta_{\tilde{y}}$ is a vector with parameters that we can use to control the rate of convergence of df_t towards $\overline{df}$ or $\underline{df}$, for each rating category when the economy is moving towards a boom or a recession respectively. In other words, high values for $\beta_{\tilde{y}}$ correspond to a cyclically sensitive loan portfolio, and low values to a less sensitive loan portfolio. Figure 2 (page 45) illustrates how the choice of the effect parameters $\beta_{\tilde{y}}$ influences the probabilities of transition in $TM_{\text{state, t}}$.

Given our three transition matrices, TM_{normal}, TM_{high} and TM_{low}, we can calibrate values for df, $\overline{df}$ and $\underline{df}$ that match the transition probabilities in these matrices.[22] Given df, $\overline{df}$ and $\underline{df}$, the selected effect parameters $\beta_{\tilde{y}}$, and a time-series for the output gap, $\tilde{y}_t$, the degrees of freedom df_t for each quarter can be calculated. Finally, once we know df_t, we can determine a time-series with transition matrices $TM_{\text{state,t}}$ that characterises the loan portfolio throughout its duration.

We have selected $\beta_{\tilde{y}}$ so that the credit losses in our model display a pattern that resembles the losses incurred by the Swedish banking sector during the 1990s (see Figure 3 on page 48).

Construction and simulation of a hypothetical portfolio over time

In this section we describe our algorithm for constructing and simulating a dynamic business loan portfolio. Our first step is to generate an initial distribution of companies in the various $k = 1,\ldots, r$ rating classes, which we do in the following way:

1. Let us assume that there is an equal proportion of companies in each class in the first period t.
2. We use the transition matrix TM_{normal} and calculate the distribution of companies at the beginning of the next period, assuming that macroeconomic conditions are normal, i.e. that the output gap is 0. We can then arrive at the resultant distribution of companies in the portfolio at the beginning of period $t + 1$ by using: $F_{t+1}\,[f_{1,t+1} \cdot \cdot \cdot f_{r,t+1}]$ where $\sum_{k=1}^{r} f_{kt+1} = 1$.

[21] Note the following properties for the function $df(\tilde{y}_t)$; (i) if $\tilde{y}_t = 0$, $df_t = df$, (ii) when $\tilde{y}_t \to \infty$, then $df_t \to \overline{df}$, (iii) when $\tilde{y}_t \to \infty$, then $df_t \to \underline{df}$.

[22] For a full description of how this is done, see "Notes to Jacobson, Lindé and Roszbach", a technical appendix which can be obtained from the authors on request.

3. We repeat step 2 until $F_t = F_{t+1}$, i.e. until the distribution of companies does not change from one quarter to the next, which means that $\sum_{k=1}^{r} f_{k,t+1} - f_{k,t} = 0$. We can then use this distribution, which we call in the following the steady state distribution, as the starting value in all of our simulations.

Simulating a hypothetical business loan portfolio for the bank over the period $t = 1, 2, \ldots, T$ involves the following steps:

1. In the first period, period 1, we allot a number of companies to each rating class by multiplying the steady state distribution by the number of companies in the portfolio, which we have taken to be 10,000.
2. We give each company a company registration number, $i = 1, \ldots, 10{,}000$, and a loan amount. We select a random number $\varepsilon_{y,1}$ and use equation (2) to calculate $\tilde{y}_t$, after which we use equation (3) to generate a relevant transition matrix, $TM_{\text{state},t}$, which is used in turn to calculate the distribution of the companies in the portfolio in the next quarter, $t + 1$.[23]
3. The new distribution in the next period consists not only of the companies that existed in the previous period. We also assume that the bank grants new loans to new companies in each period. We assume that the distribution of these loans to the new companies is the same as in the long-term distribution. We assume that the number of new companies to which the bank grants loans in each quarter is constant and equal to the proportion of companies that defaults in each period in the steady state (when the output gap is 0) multiplied by the number of companies in the initial portfolio (10,000). It is assumed that the new companies do not default during the period in which they are granted their loans. The new companies are given company registration numbers 10,001, 10,002, etc. in the order in which they are added to the portfolio.
4. We repeat steps 2 and 3 until $t = T$.
5. For each period $t = 1, \ldots, T$, we record the following information about all surviving companies in the bank's portfolio: company registration number, time period, rating class, default (variable which has the value 0 if the company survives, 1 if it defaults during the period), loan amount and the macroeconomic conditions for

[23] For each company i we generate a random figure $\tilde{p}_{i,t}$ from a uniform probability distribution (assumes values in the interval [0,1]). If the company i is in the rating class l in time period t it is assigned the rating class k in the next period (i.e. period $t + 1$) when the condition $\tilde{p}_{i,t} < \sum_{k=1} p_{lk,t}$ is fulfilled for the lowest possible value of $k = 1, \ldots, r$ where the probabilities $p_{lk,t}$ are obtained from the transition matrix $TM_{\text{state},t}$. If the inequality is not satisfied for $k = r$ it means that the company i defaults in this period. This means that the company i, apart from the macroeconomic conditions that shift the $p_{lk,t}$ probabilities, also has a company-specific risk of defaulting depending on the outcome of the stocastic variable $\tilde{p}_{i,t}$.

each period. This information can then be used to calculate the capital adequacy requirement according to Basel II along with the portfolio's VaR.

By this means, we can generate data for $n = 1, 2, \ldots, N$ different portfolios ($N = 1{,}000$). Each portfolio contains a total of some 400,000 observations, since we have assumed that the banks record the data on their loan portfolios for 40 quarters (10 years).

Appendix B:
Basel Committee's risk-weight function

According to the Basel Committee's proposal of 16 January 2001, the risk-weight function for a rating class k in quarter t is:

where $LGD_{k,t}$ is the estimated loss given default for rating class k in quarter t, and

$$(B.1) \qquad RW_{k,t} = \min\left\{\left(LGD_{k,t}/50\right) \times BRW_{k,t};\ 12.5 \times LGD_{k,t}\right\}$$

$$(B.2) \qquad BRW_{k,t} = 976.5 \times N\{1.118 \times N^{-1}(PD_{k,t}) + 1.288\} \times \{1+0.047 \times (1 - PD_{k,t})/PD_{k,t}^{0.44}\}$$

where N is a standard normal distribution (with mean 0 and standard deviation 1) and $PD_{k,t}$ is the estimated probability of default for rating class k in quarter t. The following characteristics of the risk-weight function should be noted: (i) the higher a company's *LGD*, given its *BRW*, the greater the risk-weight (*RW*), and (ii) the higher the *PD* for any given risk class, at any given *LGD* level, the higher the *BRW* and thus the risk-weight.

The IRB capital for the bank's portfolio is calculated using formula:

$$(B.3) \qquad IRBcap_t = 0.08 \sum_{k=1}^{r} RW_{k,t} \times Exposure_{k,t},$$

where *Exposure* is the sum of the loans granted in the risk class $k = 1, 2, \ldots, r$ in period t. Formula (B.3) shows quite clearly that the absolute level of 8 per cent plays a crucial role in the new Basel proposal.

In the simulations we have used $LGD_{10} = 0.8$ and $LGD_k = 0.8*(k-1)/9$, $k = 1, 2, \ldots 9$.

Appendix C: Calculation of Value-at-Risk (VaR)

The analyses of the simulated loan portfolios we have described are made using two variables: Value-at-Risk (VaR), which describes the portfolio risk; and IRB capital, which indicates the capital adequacy requirement of the portfolio, calculated in the manner specified by the new Basel Accord.

In this context, VaR measures the loan portfolio's credit risk for a given time horizon. More specifically, we calculate VaR as the amount, expressed in kronor, that the bank risks losing within any given period of time, j, with a maximum probability of Z per cent. An alternative way of expressing this is to say that the probability of the bank's credit losses not exceeding the VaR amount, expressed in kronor, during the time period ending with j is (100-Z) per cent. In practice, banks tend to select a one-year time horizon j and a probability, Z, in the interval [0.001–0.01].

In the analysis the VaR for a given portfolio n during the period T for the time horizons of the coming j = 1, 2, 3, 4 quarters is calculated in the following way:

1. Take the initial structure of portfolio n at time T and the macroeconomic conditions at that time.
2. Simulate the portfolio for the coming $j = 4$ quarters using the method described in Steps 2–5 in Appendix A. The total loss incurred on the portfolio is then calculated for the coming j = 1, 2, 3, 4 quarters. Call the total loss on the portfolio $L_{i,j} = \sum_{s=1}^{j} L_{i,s}$.
3. Repeat Step 2 for various macroeconomic scenarios i = 1, 2, …, 1,000 times for the coming j = 1, 2, 3, 4 quarters. We thus obtain for portfolio n a set, $L_1 \ldots \mathrm{L}_{1000}$, in other words 1,000 different possible portfolio losses for different quarters j into the future. We sort all the possible outcomes, L_i, from the smallest to the largest loss for the coming j = 1, 2, 3, 4 quarters. The VaR at a 95 per cent level on horizon j for this portfolio is then calculated as the 950th largest $L_{i,j}$ for the coming j = 1, 2, 3, 4 quarters.
4. By repeating steps 2–3 for all 1,000 portfolios, we can calculate the VaR for all the portfolios at different levels of significance.