## The real interest rate and monetary policy

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In recent years, Knut Wicksell's theory on the interest rate gap and its effect on inflation have come into focus. By interest rate gap is meant the difference between the actual real interest rate and the real interest rate that would apply if prices were flexible, often called the "natural" real interest rate. The purpose of this article is to discuss the determinants of the natural real interest rate in a dynamic general equilibrium model. In the long run, households' time preference, economic growth and capital income tax determine the natural real interest rate. The short-run adjustment after a productivity shock and various fiscal policy shocks is also illustrated.

## What determines the "natural interest rate"?

The conventional view is that a large part of the changes in the real interest rate are explained by monetary policy. A key issue in the understanding of monetary policy is how a change in a nominal interest rate, the so-called repo rate, affects the real interest rate. The conventional view is that monetary policy, due to price rigidities,

explains a large part of the short-term changes in the real interest rate. This view is confirmed by the fact that productivity shocks and fiscal policy shocks in simple dynamic general equilibrium models give rise to relatively small changes in the real interest rate.

A related issue is how changes in the real interest rate affect the general price level and the rate of inflation in the economy. Knut Wicksell was one of the first economists to shed light on this issue. In Wicksell's view, the price level is deter-

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mined by a so-called interest rate gap. The interest rate gap is defined as the difference between the actual real interest rate and the "natural" real interest rate. The actual real interest rate can deviate from the natural rate because of price rigidities and is thus affected

by monetary policy. The natural interest rate, on the other hand, is not affected by monetary policy since it is defined as the interest rate that would apply if prices were flexible.

Wicksell's framework can briefly be described in the following way: An increase in the natural interest rate (given the actual interest rate) implies that the firms' profits

increase due to an increase in the return on their capital.<sup>1</sup> This leads to an increased demand for labour and thus, eventually, increased wages. The increased wages increase households' demand for consumer goods, which pushes up prices. That is, if the natural interest rate exceeds the actual interest rate, the price level will increase. Correspondingly, the price level will fall if the natural rate is lower than the actual rate. To maintain a stable price level, a central bank must prevent the occurrence of an interest rate gap.<sup>2</sup>

In recent years, the so-called Wicksellian approach has come into focus due to a couple of essays by Woodford<sup>3</sup> and Neiss and Nelson.<sup>4</sup> Woodford shows how Wicksell's

approach can be incorporated in a dynamic general equilibrium model with price rigidities. Neiss and Nelson's paper has attracted interest since it does not only show that the interest rate gap is a good inflation indicator but also that it is relatively easy to measure. In particular, it is easier to measure than the output gap.<sup>5</sup> This is due to the fluctuations in natural GDP being relatively large while they are small in the natural interest rate. This means that simple filtration methods, such as the HP filter, can be used to calculate the natural interest rate and

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The interest rate gap is a good inflation indicator and is relatively easy to measure.

<sup>&</sup>lt;sup>1</sup> Unless otherwise stated the interest rate refers to the real interest rate.

<sup>&</sup>lt;sup>2</sup> See Siven (1998) and the references included in that work for an accessible and more detailed description of Wicksell's framework.

<sup>3</sup> Woodford (2000).

<sup>4</sup> Neiss & Nelson (2001).

<sup>&</sup>lt;sup>5</sup> The output gap is defined as the difference between actual GDP and "natural" GDP. The definition of natural GDP usually varies although it is defined as the output that would apply if prices were fully flexible in this article.

thus the interest rate gap. However, they are not so useful for calculating the output gap.

The interest rate gap and the level of the natural interest rate are of key importance for the conduct of monetary policy. According to Wicksell among others, the interest rate gap and thus the determination of the natural interest rate is of key importance for the conduct of monetary policy. The purpose of this article is therefore to dis-

cuss what determines the natural interest rate. The discussion is based on a real dynamic general equilibrium model where the interest rate can be interpreted as the natural interest rate. The effects on the interest rate of a productivity shock and a number of fiscal policy shocks are illustrated. The fact that productivity shocks and other real shocks seem to have small effects on the interest rate is also highlighted.

In the long run, the interest rate is determined by households' time preference, economic growth and the capital income tax. In order to understand what determines the interest rate, it is convenient to first study the long-run determinants and then the shortrun adjustment of the interest rate to different types of shocks. In the long run, house-

holds' time preference, the growth rate and capital income tax determine the interest rate. In the short run, the interest rate is determined in an interaction with all of the other variables of the model. Important factors are, however, productivity, capital intensity and the growth of consumption.<sup>6</sup>

The next section contains an account of the dynamic general equilibrium model. Thereafter follows a discussion of what determines the interest rate in the long run and an illustration of the short-run adjustment of the interest rate to various types of shocks. We study the adjustment of a productivity shock, a public consumption shock, a labour income tax shock and a capital income tax shock. Finally, some concluding comments are made.

## The dynamic general equilibrium model

The purpose of the model is to illustrate the basic mechanisms that determine the interest rate in a simple and consistent way. In order to conduct a meaningful discussion on the determinants of the interest rate, a conceptual framework or model is required. The purpose of the model is to illustrate the basic mechanisms that determine the interest

<sup>6</sup> Unless otherwise stated, productivity refers to total factor productivity.

rate in a simple and consistent way. We perform the analysis in a dynamic general equilibrium model based on the neo-classical growth model. The equilibrium model is in principle the simplest possible. There is no heterogeneity among households or firms. Neither is there any "asymmetric" information, i.e. all households and firms have access to the same information. These assumptions simplify the discussion considerably without losing the basic mechanisms. Since the interest rate is a "price" that affects the allocation of consumption and leisure over time, it is necessary to have a dynamic model.<sup>7</sup> The modelling approach is based on the following principles:

- Economic outcomes do not occur arbitrarily but are the result of rational households and firms that maximise utility and profit, respectively.
- Households and firms are assumed to be "price takers" on every market, i.e. their individual behaviour does not affect prices. The economy is in equilibrium when prices are such that supply and demand in every market are equally large.
- Households' and firms' decisions are consistent with one another; i.e. it is a general equilibrium model.
- The long-run determinants of the economy and the short-run dynamic adjustment to different types of shocks are explained in one and the same model.

The model is similar to the one presented by Jonsson and Klein.<sup>8</sup> The main reason for using that model is that it has proven to be empirically relevant. It is not only consistent

with a number of stylised facts on long-run growth, it can also explain a large part of the short-run fluctuations in Swedish data.

A non-technical description of the model is presented here while the appendix provides a more detailed and formal description. Households choose consumption and leisure in order to maximise their "utility" given that The main reason for using Jonsson and Klein's model is that it has proven to be empirically relevant.

Households choose consumption and leisure in order to maximise their "utility" given that their budget and time constraints are fulfilled.

their budget and time constraints are fulfilled. When they choose a level of con-

<sup>&</sup>lt;sup>7</sup> In a dynamic model the analysis is more complicated since the income effect of a shock depends on the length of the shock. A short-lived shock has a relatively small income effect while a persistent shock has a relatively large income effect.

<sup>&</sup>lt;sup>8</sup> Jonsson & Klein (1995). The difference lies in the specification of the tax system, Jonsson and Klein also include a consumption tax.

sumption, a balance is struck between the utility of consuming more today and the utility of saving and thus obtaining a higher future level of consumption.

Utility is assumed to increase with the level of consumption and leisure. However, the *marginal utility* is declining, which means that the marginal value of any good decreases as more of that good is consumed. The economic significance of this is that households wish to smooth consumption over time. The model accordingly incorporates a form of the permanent income hypothesis.<sup>9</sup>

Households must comply with their budget and time constraints in each period. The revenues in the budget constraint consist of labour and capital incomes, which are both subject to tax. A certain part of the tax is returned to the households in the form of transfers. Their expenditure consists of consumption and saving (i.e. investments in the economy's capital stock). Time is allocated between work and leisure.

Households have a subjective discount rate or a so-called time preference.

Finally, households have a subjective discount rate or a so-called time preference,  $\beta$ , which indicates how they value future consumption and leisure relative to consumption and

leisure today. Their discounting of future consumption and leisure can be described in the following way,

$$U(c_t, l_t) + \beta U(c_{t+1}, l_{t+1}) + \beta^2 U(c_{t+2}, l_{t+2}) + \dots$$

where  $U(c_t, l_t)$  denotes the households' so-called utility function, which consists of consumption,  $c_t$ , and leisure,  $l_t$ , at a particular time t. If  $\beta$  is less than 1 households are "impatient", which means that they value consumption and leisure today higher than in the future. If  $\beta$  is larger than 1 households are "patient" and they value future consumption and leisure higher. In most studies  $\beta$  is usually less than 1 even if there are arguments for it being close to 1.<sup>10</sup>

Firms choose their capital stock and labour force in order to maximise profits in each period. The firms in the economy produce a product that can be used for private consumption, investments and public consumption. They choose their capital stock and labour force in

order to maximise profits in each period. Since there are a large number of firms, the individual firm does not affect the interest rate or the wage rate, i.e. these are taken as given.

<sup>&</sup>lt;sup>9</sup> Friedman (1957).

<sup>&</sup>lt;sup>10</sup> Frank Ramsey assumed in an article in 1928 that  $\beta$  should be equal to 1 with the motivation that it would be "ethically indefensible" to set a lower value on the utility of future generations than the present.

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The public sector is assumed to "purchase" a certain part of the output and finance this by taxing households' capital and labour incomes. A part of these incomes are returned to households in the form of transfers while the rest are used for public consumption.

What determines the interest rate in this economy? Since there are a large number of households and firms, their individual decisions do not affect the interest rate. At the aggregate level, it is the collective saving and

At the aggregate level, the collective saving and investment decisions by households and firms determine the interest rate.

investment decisions by households and firms that determine the interest rate. That is, it is the total supply and demand for capital that determines the interest rate. Somewhat simplified, it can be described as follows:

- Firms choose capital so that the marginal product of capital will be equal to the interest rate. The marginal product depends partly on productivity and partly on capital intensity (capital in relation to labour). A high productivity increases the marginal product of capital and therefore the interest rate. The interest rate also increases when capital is a scarce resource in relation to labour. In other words, high productivity and low capital intensity tend to increase the interest rate.
- Households' choice of consumption and saving, and also their supply of labour, depends on the interest rate. An increase in the interest rate implies that households would like to reduce consumption now (i.e. increase saving) and consume more in the future. A high interest rate is therefore associated with a high *consumption growth*. The supply of labour depends on the development of wages but also on the interest rate. A high interest rate makes it profitable to increase the supply of labour and save the extra income.

# What determines the interest rate in the long run?

The long-run determinants of the interest rate are relatively simple to derive. It is namely possible to derive a simple equation in terms of the model's parameters and exogenous variables. The long run is defined as the hypothetical state when the effects of any shocks have vanished and the variables in the model are independent of time. This is usually called "steady state" in the literature. These longrun determinants are shown in the following equation,

$$r = \left(\frac{1}{\beta}\gamma^{1-\alpha(1-\sigma)} - 1\right)\frac{1}{1-\tau^k},$$

where *r* denotes the interest rate,  $\gamma$  denotes the growth rate of the economy,  $\alpha$  denotes the weight given by households to consumption in relation torelative leisure in the utility function,  $1/\sigma$  is a measure of households' intertemporal substitution willingness. A high  $\sigma$  means that households' willingness to substitute consumption over time is low and they would like to have a smooth consumption path, finally,  $\tau^k$  denotes the capital income tax.<sup>11</sup>

According to the above equation, the interest rate is determined in the long run by three factors. Firstly, there is a pure *time preference factor*.

According to the above equation, the interest rate is determined by three factors in the long run. Firstly, there is a pure *time preference factor*,  $\beta$ . Assume that households are "impatient" and value present consumption more than future consumption. If households are then to

be willing to save and postpone their consumption, they will want to be compensated for doing so. This compensation consists of receiving interest on their savings. The more impatient households are, the higher the interest rate will be.

Secondly, the interest rate depends on *economic growth*.

Secondly, the interest rate depends on *economic growth*,  $\gamma$ . In an expanding economy, future consumption opportunities are greater than

at present. Since the marginal utility of consumption is declining, households value an increase in consumption today higher than one in the future. They will therefore be willing to pay a premium or interest to be able to increase their consumption today. How high an interest rate they are willing to pay depends on their substitution willingness,  $1/\sigma$ , and the weight of consumption in the utility function,  $\alpha$ . A low substitution willingness implies that they are prepared to pay a high interest rate to be able to increase consumption today.

## Thirdly, the interest rate depends on the *capital income tax*.

Thirdly, the interest rate depends on the *capital income tax*,  $\tau^k$ . Households' consumption decisions depend on the return of capital

after tax. If the tax rate on capital incomes increases, households will require a higher return to compensate for the tax loss. An increase in the tax rate on capital income will therefore give a higher interest rate. It is worth noting that the labour income tax does not affect the interest rate in the long run.

<sup>&</sup>lt;sup>11</sup> See also appendix for a discussion on the parameters' economic significance.

## Some simulation examples

This section shows the adjustment of the interest rate to a number of shocks over time. First the effects of a productivity shock are shown and then the effects of different fiscal

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policy shocks.<sup>12</sup> The purpose of the simulations is to qualitatively illustrate the determinants of the interest rate.

In a dynamic model, the adjustment of the economy depends on the duration of the shock, and we therefore present simulations with both short-lived and persistent shocks. The short-lived shock is 1 per cent in the first period and thereafter zero. The persistent shock is also 1 per cent initially but is auto-correlated with a factor of 0.95. This means that productivity, in for instance a productivity shock, declines in a smooth path towards zero during a number of periods.

The adjustment paths in the diagrams are shown as percentage deviations from their long-run values. The parameter values that are used in the simulations are given in the appendix together with the formal description of the model.

#### A productivity shock

Diagram 1 shows the dynamic paths of the interest rate, the wage rate, consumption, investment and the supply of labour to a short-lived productivity shock. In period one, the increase in productivity leads to an increase in the marginal product of labour and thus wages. The higher wages affect the supply of labour by income and substitution effects. The income effect, which tends to reduce the supply of labour, will be small, since the productivity shock is short-lived. The supply of labour therefore increases and together with the increased productivity, this entails a strong increase in production.

Households choose how much of the increased production they wish to consume and save. Since the marginal utility of consumption is decreasing, households wish to smooth consumption over time. A large part of the increase in production is therefore saved.<sup>13</sup>

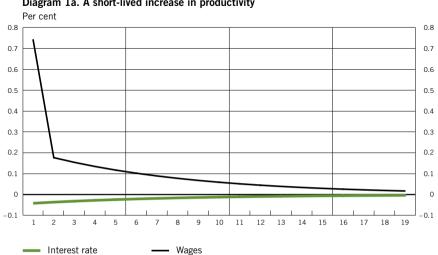
In period two, productivity returns to its long-run level. The only difference to the time before the productivity shock is that households have accumulated a

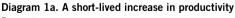
<sup>&</sup>lt;sup>12</sup> It is not uncommon for economists to categorise shocks as so-called "supply shocks" and "demand shocks". In a dynamic general equilibrium model, a distinction of this kind is rather meaningless or even misleading. A productivity shock affects for instance the supply of goods, since the production opportunities are changed, but it also affects demand since incomes are changed.

<sup>&</sup>lt;sup>13</sup> The relatively high fluctuations in investment that can be observed in the data, which Keynes related to "animal spirits", thus arise naturally in an economy where households wish to smooth consumption over time.

larger capital stock. This is used to enjoy more consumption and leisure over a number of periods.

What happens to the interest rate? The interest rate is below its long-run level throughout the entire period of adjustment. This is due to the higher capital stock giving rise to a higher capital intensity, which pushes down the interest rate. The interest rate determines households' allocation of consumption over time.





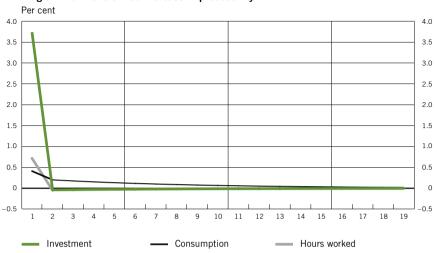


Diagram 1b. A short-lived increase in productivity

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The low interest rate implies that households prefer to have a falling consumption profile throughout the adjustment.

The results of a persistent productivity shock are shown in Diagram 2. It is basically the same mechanisms that affect economic development as in the case of a short-lived shock. The marginal product of labour and thus wages increases initially, which implies that households increase the supply of labour. The increase will not be as great in this case since the income effect is greater.

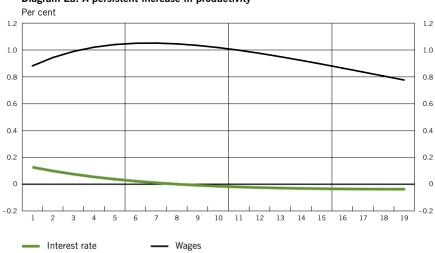
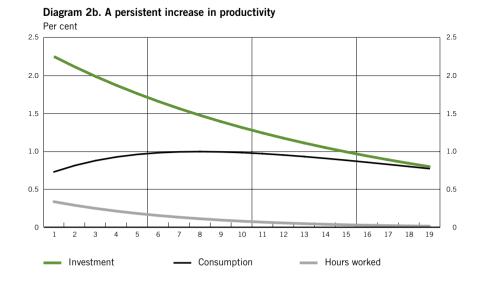


Diagram 2a. A persistent increase in productivity



The interest rate follows a qualitatively different path compared to the shortlived productivity shock. Initially, the interest rate is higher than its long-run level, which is due to the high productivity. The large investments lead to a gradual increase of the capital stock. This, together with the slowly decreasing productivity, gradually pushes the interest rate down.

Productivity shocks have relatively small effects on the interest rate in this model

These simulations illustrate that productivity shocks have relatively small effects on the interest rate in this model. In the case of a short-lived productivity shock, the interest

rate only falls by 0.05 per cent and in the case of a persistent productivity shock, it increases by about 0.15 per cent. This is to be compared with the effects on consumption and in particular investments. In the case of a short-lived productivity shock, investments increase by over 3.5 per cent. It is basically these mechanisms that give rise to Neiss and Nelson's conclusion that the interest rate gap is easier to measure than the output gap.

#### FISCAL POLICY SHOCKS

This section illustrates the effect on the interest rate of shocks to public consumption, labour income tax and capital income tax.

An increase in public consumption leads to a negative income effect due to the reduction of transfers. This means that the supply of labour increases and investments are reduced. These effects reduce capital intensity, which pushes the interest rate upwards. This applies both to short-lived and persistent increases in public consumption. Quantitatively, the interest rate increase is stronger if the shock is persistent since the income effect is greater. Diagram 3 illustrates the dynamic path of a short-lived increase in public consumption.

The effect on the interest rate of an increase in labour income tax is ambiguous. As in the case of a productivity shock, the adjustment depends on the duration of the shock.

An increased tax rate on labour income affects the supply of labour and thus output negatively. Households reduce both consumption and investment. Whether this leads to an increase or reduction in the interest rate depends on the strength of the reduction in the supply of labour relative to investment.

In the event of a short-lived tax increase, the effect on investment will be relatively strong, which will reduce capital intensity and increase the interest rate. If the increase is persistent, on the other hand, the effect will be less strong initially. This makes capital intensity increase to start with and the interest rate falls. The

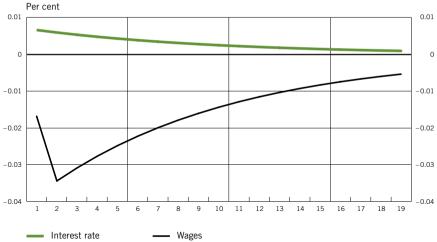
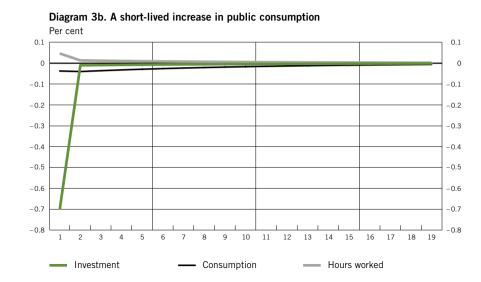
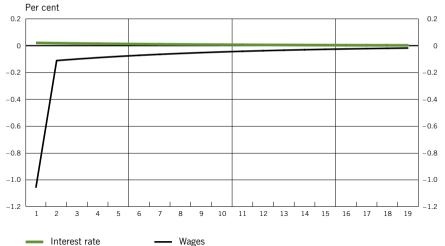


Diagram 3a. A short-lived increase in public consumption

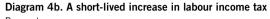


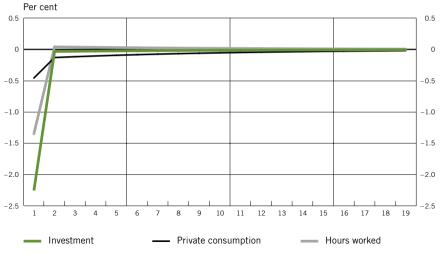
effects of a short-lived and a persistent increase of the labour income tax are illustrated in Diagrams 4 and 5, respectively.

The capital stock has to be determined one period in advance and cannot be changed in the current period. This means that a short-lived increase in capital income tax has no effect on the capital stock. However, a persistent increase of the tax rate will affect the capital stock and the interest rate. The direct effect of a persistent increase of the tax rate is a reduction of the interest rate after tax. This makes it more advantageous to increase consumption and reduce investment.



#### Diagram 4a. A short-lived increase in labour income tax





The lower interest rate also tends to reduce the supply of labour. The reduced investment entails a lower capital intensity, which pushes up the interest rate. This is illustrated in Diagram 6.

These simulations illustrate that fiscal policy shocks also tend to have small effects on the interest rate. These simulations illustrate for fiscal policy shocks what we found earlier for the productivity shock, i.e. they tend to have small effects on the interest rate. In quantitative

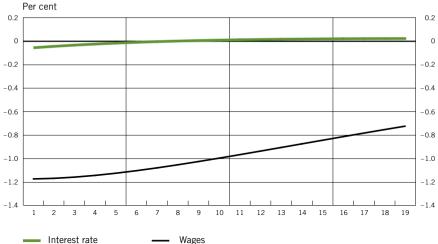
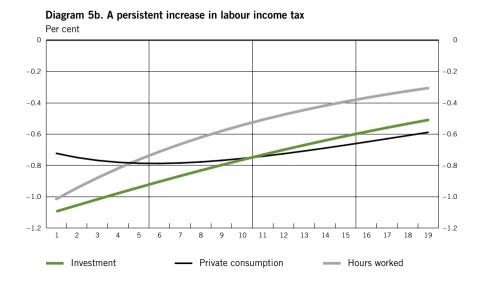


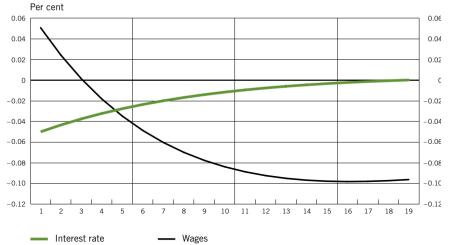
Diagram 5a. A persistent increase in labour income tax



terms, the effect on the interest rate of the fiscal policy shocks is even somewhat less than that of the productivity shock.

## Concluding comments

According to Wicksell among others, the interest rate gap is of key importance in the conduct of monetary policy. To maintain a stable price level, the central bank





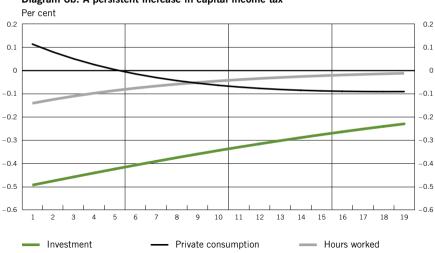


Diagram 6b. A persistent increase in capital income tax

To maintain a stable price level, the central bank must prevent an interest rate gap from arising.

must prevent an interest rate gap from arising. This means that the actual interest rate must be controlled so that it is at the level of the natural interest rate all the time.

In practice, it is difficult to measure the interest rate gap since the natural interest rate cannot be observed. It may therefore be valuable to understand the determinants of the natural interest rate in simple theoretical models. This can be exemplified by the so-called new economy. The new economy has been characterised by a long and high increase in productivity. According to the model, the effect of this should be an initially higher interest rate, which subsequently declines. After a time, it even falls below its long-run level. How should monetary policy react to this? In order not to create an interest rate gap, which can affect inflation, the central bank must initially increase the interest rate and then reduce it apace with changes in the natural interest rate.

Despite it probably being difficult to measure the interest rate gap, it may still be easier to measure this gap than the output gap. This is shown by Neiss and Nelson.<sup>14</sup> The reason for this is that the natural interest

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rate, in contrast to natural GDP, is relatively insensitive to productivity and fiscal policy shocks in simple dynamic general equilibrium models. However, this is not a general result but depends on, among other things, the utility function and capital formation. With another type of utility function and installation costs for capital, Boldrin, Christiano and Fisher<sup>15</sup> and Smets and Wouters<sup>16</sup> show that productivity shocks can very well have strong effects on the natural interest rate.

Smets and Wouter's result moreover indicates that there may be a number of practical problems in using the interest rate gap as an indicator for monetary policy. The interest rate gap is very sensitive to the identification of shocks and different model assumptions.

It can be noted in conclusion that the significance of the interest rate gap for the development of inflation and the conduct of monetary policy is not without objections. However, this also applies to other estab-

The significance of the interest rate gap for inflation development and the conduct of monetary policy is not entirely without objections.

lished concepts such as the output gap and NAIRU, which are both used as monetary policy indicators by central banks and external analysts.<sup>17</sup> There are in other words good reasons to use different models and approaches as a basis for monetary policy decisions.

<sup>14</sup> Neiss & Nelson (2001).

<sup>&</sup>lt;sup>15</sup> Boldrin, Christiano & Fisher (2001).

<sup>16</sup> Smets & Wouters (2001).

<sup>&</sup>lt;sup>17</sup> See Annual Report (1999), for a critical discussion on NAIRU and potential GDP and their significance for monetary policy. An excellent discussion on NAIRU is also contained in Rogerson (1997).

## Appendix

Households maximise the sum of their discounted utility over an "infinite" time horizon such that time and budget constraints are fulfilled. Households preferences,  $\Omega$ , can formally be described in the following way,

$$\Omega = E\left[\sum_{t=0}^{\infty} \beta^{t} U(c_{t}, 1-h_{t})\right],$$

where E denotes the unconditioned expectation operator,  $U(\cdot)$  denotes the utility function,  $c_t$  denotes consumption and  $h_t$  denotes hours worked in period t. For the sake of simplicity, the time endowment is normalised to 1,  $1-h_t$  denotes accordingly leisure. The parameter  $\beta$  denotes households' subjective discount factor or so-called time preference. Households having an infinite planning horizon simplifies the solution of the model but can also be motivated on the basis of altruistic links between different generations. The period utility function has the standard iso-elastic form,

$$U(c_t, 1-h_t) = \frac{\left(c_t^{\alpha}(1-h_t)^{1-\alpha}\right)^{1-\sigma}}{1-\sigma},$$

where the parameter  $\sigma$  is greater than zero and is a measure of households' risk aversion. The inverse of  $\sigma$  is at the same time a measure of households' intertemporal substitution elasticity. A high  $\sigma$  means that households' willingness to substitute consumption over time is low and that they want to have a smooth consumption path. The parameter  $\alpha$  is between zero and 1 and states the relative weight given by households to consumption and leisure respectively.

The specific function form has the following characteristics. Households prefer a consumption basket with more consumption, more leisure or both. This is a reasonable assumption at the aggregate level since it is likely that the average household in Sweden would like to consume more if this were possible. Households want to have diversity in the consumption basket, i.e. they want to have both consumption and leisure in the consumption basket. They also prefer a smooth level of the consumption basket over time. Finally, consumption and leisure are normal goods in the following sense: If income increases, they both want to increase consumption and have more leisure. At the aggregated level, this is probably a reasonable assumption even if there may be examples of specific goods that are not normal in this sense.

Public consumption does not formally generate any utility for households.

However, we would have exactly the same allocation of resources if public consumption were incorporated in an additive way in the utility function. The only thing that would be affected is the level of households' utility.

Households' budget constraint is given by,

$$c_{t} + i_{t} = \left( (1 - \tau_{t}^{k})r_{t} + \tau_{t}^{k}\delta \right)k_{t} + (1 - \tau_{t}^{h})w_{t}h_{t} + tr_{t}$$

The left side consists of households' expenditure and the right side of their income after taxes and transfers. The notation is as follows,  $i_t$  denotes investment,  $k_t$  denotes capital stock,  $r_t$  denotes the rental rate of capital,  $w_t$  denotes the wage rate,  $\tau_t^k$  denotes capital income tax,  $\tau_t^h$  denotes the labour income tax,  $tr_t$  denotes transfers and  $\delta$  is a parameter that denotes the rate of depreciation of capital. The formulation of the budget constraint implies that depreciation is modelled as tax deductible.

The capital stock evolves according to,

$$k_{t+1} = i_t + (1 - \delta)k_t,$$

That is, in the next period the capital stock is equal to current investment plus the current capital stock minus the depreciation of the current period's capital stock.

Firms solve the following maximisation problem,

$$\max_{\{k_t,h_t\}}[y_t - r_t k_t - w_t h_t],$$

where  $y_t$  denotes firms' income. The expenditure consists of costs for capital and labour. The firms' production function that combines labour and capital is of the standard Cobb-Douglas type,

$$y_t = z_t k_t^{\theta} (\gamma^t h_t)^{1-\theta},$$

where  $\theta$  (0< $\theta$ <1) denotes capital's share of income,  $z_t$  denotes total factor productivity (exogenously given) and  $\gamma^t$  ( $\gamma$ >1) denotes the long-run growth of the economy.

The restriction on  $\theta$  implies "constant returns to scale", that is, if capital and labour increase by a certain factor  $\lambda$ , total output will also increase by the same factor  $\lambda$ . The economic significance of this is that small and large firms are equal-

ly efficient. It is hardly likely that this is the case for each particular firm and industry. However, it can still be a reasonable assumption at an aggregate level.

The public sector simply fulfils its budget constraint each period. It consists of exogenous objectives for public consumption and taxes on households' capital and labour incomes. Transfers are determined residually (and hence endogenously), formally,

 $TR_t + G_t = \tau_t^h H_t + \tau_t^k (r_t - \delta) K_t,$ 

where  $G_t$  denotes public consumption and large letters indicate that they are aggregated variables.

Finally, the aggregate resource constraint must be fulfilled, that is,

$$Y_t = C_t + I_t + G_t.$$

In order to carry out the simulations, values must be set for the parameters in the model. This can be done in a number of different ways. Usually, the parameters are estimated by econometric methods. Since the simulations are only used for illustrative purposes, the parameters are set at what can be judged to be standard values in the literature. Table 1 shows the parameter values used in the simulations.

Table 1. Parameter values in the simulation examples

σ	δ	$\theta$	α	β	γ
2.00	0.10	0.36	0.33	0.99	1.02

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