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Simple Forward- and Backward-  
looking Models: A View from a  
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# The Empirical Relevance of Simple Forward- and Backward-looking Models: A View from a Dynamic General Equilibrium Model

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## Abstract

Recent research have provided evidence that backward-looking models fit the data well while purely forward-looking models seem to be inconsistent with data. Consequently, many recent papers in the monetary policy rule literature have used “hybrid” models, which contain both backward- and forward-looking components. In this paper, I demonstrate that a dynamic general equilibrium model with flexible prices and forward-looking properties cannot account for the empirical findings, i.e. that backward-looking behavior seems more important than forward-looking behavior, and that backward-looking models fit the data better than purely forward-looking models. The results also show that the equilibrium model cannot replicate the estimated high weight on backward-looking behavior on US data for the hybrid model.

**Keywords:** Monetary policy rules; New Keynesian Phillips-curves; Rational expectations IS-curves; Backward-looking models; Dynamic general equilibrium models; Lucas critique.

**JEL Classification Numbers:** E52, C52, C22.

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# 1 Introduction

Since John B. Taylor (1993) discovered that the Federal Reserve's setting of the Federal Funds rate could be well approximated by a simple rule, the Taylor-rule, a huge literature on the performance of different monetary policy rules has emerged; see Taylor (1999b). One possible explanation for this renewed interest is that researchers wanted to explore if the Taylor-rule has good properties in macroeconomic models, and if it is possible to find better rules that can be implemented instead by the Federal Reserve. To some extent, the increased interest by researchers in this topic can also be explained by the fact that recent empirical work has shown that monetary policy seems to have important real effects, see e.g. Christiano, Eichenbaum and Evans (1996, 1999) and Leeper, Sims and Zha (1996).

In connection with the emerging literature on monetary policy rules, the relevance of the Lucas (1976) critique has received increased attention. The plausible reason for this is the extensive use of backward-looking models in monetary policy analysis; see e.g. Ball (1999), Svensson (1997), Rudebusch and Svensson (1999) and Taylor (1999a). In this class of models, with weak microfoundations, the structure of the model economy is assumed to be unaffected by changes in economic policy (i.e. the monetary policy rule). Now, if the Lucas critique is quantitatively important, this type of policy experiments may produce seriously misleading results. In particular this can be a problem with this literature since the policy experiments considered are very large in nature; the effects of alternative monetary policy rules on, for example, inflation and output variability are computed over an infinite horizon.<sup>1</sup> One line of defense for backward-looking models that has been used is that the Lucas critique, although generally acknowledged in the literature as a very important issue, does not seem to be empirically relevant for many important economic relationships; see the survey by Ericsson and Irons (1995). Moreover, backward-looking models seem to be good approximations of reality, whereas many equilibrium models with stronger microfoundations do not seem to incorporate dynamics that fit the data (see e.g. Fuhrer, 2000). For instance, Rudebusch and Svensson (1999) find that their model estimated on US data (1961 – 1996) does not exhibit parameter instability over time, although it is generally concluded that the Federal Reserve changed the conduct of monetary policy during that period.<sup>2</sup> Lindé (2001b), however, casts some doubts on these results by suggesting that the power of the statistical tests for discovering parameter instability in the Rudebusch and Svensson (1999) model can be very low.<sup>3</sup>

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<sup>1</sup> Even if one acknowledges that people are uncertain about how the economy works and how monetary policy is conducted, this type of experiments would certainly be discovered after a while and thus be subject to the Lucas critique when agents reoptimize. For more traditional policy experiments, such as an evaluation of the effects of a temporary increase in the interest rate (with 0.5 percent for 1 – 2 years, say), results in Leeper and Zha (1999) suggest that the Lucas critique may not be empirically relevant.

<sup>2</sup> See e.g. Rotemberg and Woodford (1997) and Clarida, Galí and Gertler (1998).

<sup>3</sup> The basic argument by Lindé (2001a) is that tests for parameter stability in univariate reduced-form models

In order to work with models based on microfoundations that incorporate forward-looking behavior, and thus are less vulnerable to the Lucas critique, the most recent work in the literature on monetary policy rules has used forward-looking rational expectations models with optimizing agents. In particular, the pure forward-looking IS- and Phillips-curves derived by Roberts (1995), Woodford (1996) and McCallum and Nelson (1999) have been used extensively; see e.g. Clarida, Galí and Gertler (1999) and the references therein. One big problem with the purely forward-looking Phillips- and IS-curves (or AS- and AD-curves, respectively) - as demonstrated by Fuhrer and Moore (1995), Fuhrer (1997), Estrella and Fuhrer (1999) and Estrella and Fuhrer (2000) - is that they seem to be at odds with the data. Using US data for the period 1966 – 1997, Estrella and Fuhrer (1999) show that forward-looking AS- and AD-curves cannot replicate the actual behavior of output and inflation, and that they are subject to parameter instability in connection with monetary regime shifts. Since monetary policy is generally viewed as having mostly short-run real effects on the economy, the inability of purely forward-looking models to mimic the short-run dynamics of the data has severe implications for their usefulness in monetary policy analysis (as emphasized by Estrella and Fuhrer, 2000).

The inconsistencies between purely forward-looking models and the data have led many researchers, see e.g. the references provided in Clarida, Galí and Gertler (1999), to use “hybrid” New Keynesian Phillips- and IS-curves, which include both backward- and forward-looking elements, as a new workhorse model in monetary policy analysis. As noted by Galí and Gertler (1999) and Roberts (2001), the motivation for including inertia is largely empirical, but is often justified theoretically with an assumption that a fixed proportion of firms has backward-looking price setting behavior. Empirically, when estimating the hybrid models on data, backward-looking behavior also seems more important than forward-looking behavior, see e.g. Lindé (2001c), Rudebusch (2000) (and the references therein) and Roberts (2001). This type of models inherits the good properties of the backward-looking models and produces dynamics that are consistent with the data.<sup>4</sup>

The task of this paper is to examine if an equilibrium model with flexible prices and forward-looking properties can replicate the empirical observations, first, that backward-looking models fit the data better than simple forward-looking models, second, that forward-looking models

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have low power in small-samples, because they cannot correctly separate out the effects on the reduced-form parameters of changes in monetary policy rule from other shocks that hit the economy simultaneously.

<sup>4</sup> Galí and Gertler (1999), argue that the purely forward-looking Phillips-curve provides a good approximation to the dynamics of inflation if the output gap is replaced with real marginal costs, although they find that the degree of backward-looking behavior is highly significant in the “hybrid” Phillips-curve they estimate. Rudd and Whelan (2001), however, show that the estimation method (GMM, instrumental variables to compute a proxy for expected inflation) employed by Galí and Gertler induces a strong positive bias for the estimated degree of forward-looking if the inflation rate is highly autocorrelated (which is the case empirically), which is consistent with Rudebusch (2001)/Lindé (2001c) findings that the degree of forward-looking is about 0.30 rather than 0.75 (the estimate obtained by Galí and Gertler) when using a survey measure for expected inflation/the Full Information Maximum Likelihood estimation method instead, respectively.

are more unstable than backward-looking models when there is a monetary regime shift, and third, that backward-looking behavior are more important than forward-looking behavior in the hybrid model. If it can, then these empirical observations are not sufficient for motivating the extensive use of hybrid models and dismiss equilibrium models with flexible prices in monetary policy analysis.

My approach is to set up a slightly modified version of Cooley and Hansen's (1995) real business cycle model with money. The modification is that the model here includes government expenditures and a Taylor inspired policy rule for nominal money growth similar to the rule analyzed by McCallum (1984, 1988).<sup>5</sup> The policy rule for nominal money growth is then estimated on U.S. data for the recent periods in office of Federal Reserve's chairmen Arthur Burns, Paul Volcker, and Alan Greenspan. According to Judd and Rudebusch (1998), the conduct of monetary policy has varied systematically between these periods.<sup>6</sup> By calibrating the equilibrium model with the estimated monetary policy regimes, I study the properties of the reduced-form parameters in the Rudebusch and Svensson (1999) model and the forward-looking model studied by Estrella and Fuhrer (1999) by means of simple Monte Carlo experiments on simulated data from the equilibrium model. I also examine the properties of a nested model, consisting of "hybrid New Keynesian" Phillips- and IS-curves with both backward- and forward-looking elements very similar to the ones estimated by Galí and Gertler (1999), Rudebusch (2000) and Roberts (1997, 2001).

The main results in the paper are as follows. Although the underlying datagenerating process is a dynamic general equilibrium model with forward-looking properties and flexible prices, the fit of the backward- and forward-looking models is very similar. The parameters in both the backward- and forward-looking models also exhibit considerable parameter instability when there is a monetary regime shift, thus making both models sensitive to the Lucas (1976) critique.<sup>7</sup> However, it is shown that with recursive monetary regime shifts from Burns to Volcker and then to Greenspan in samples of the same size as in reality, it not likely that this parameter instability will be detected for neither the backward- nor the forward-looking model using the same statistical test for parameter instability as Rudebusch and Svensson (1999) and Estrella and Fuhrer (1999). When I estimate the hybrid model on simulated data, the estimated weight on forward-looking behavior is much higher than estimates obtained on US data. Since these results are in sharp contrast to those obtained on US data by Estrella and Fuhrer (1999), this

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<sup>5</sup> It is shown in the paper that the rule for nominal money growth can be rewritten as a standard Taylor-type rule in the nominal interest rate.

<sup>6</sup> Judd and Rudebusch (1998) start out by noting that there is instability in the Fed reaction function. They then find support for the hypothesis that the Fed monetary policy rule has varied systematically with the different periods in office of Fed chairmen Burns, Volcker, and Greenspan. As in their analysis, the period with chairman Miller is omitted here because of his very short tenure.

<sup>7</sup> The analysis suggests two important factors behind these results; flawed measures of expected inflation output and model misspecification.

paper raises doubts about the usefulness of simple forward-looking models for evaluating the welfare-effects of different monetary policy rules and warrants further research on the role of sticky prices and wages, and other frictions on the real side of the economy in equilibrium models.

The structure of the paper is as follows. In the next section, I introduce the monetary equilibrium model, and indicate how to compute the equilibrium. Estimation and calibration issues are addressed in Section 3. In Section 4, I present the backward- and forward-looking models that I study. Next, in Section 5, results of the Monte Carlo simulations are reported. Some concluding remarks and tentative implications are provided in Section 6.

## 2 The equilibrium model

In this section, I describe and solve a slightly modified version of Cooley and Hansen's (1989, 1995) monetary equilibrium business cycle model. The model is a standard real business cycle model with some additional features. A stochastic nominal money supply interacts with a cash-in-advance technology and one-period nominal wage contracts, which creates short run real effects of nominal money supply shocks. As in Cooley and Hansen (1995), one period is one quarter.<sup>8</sup>

The difference between the model in this paper and the one in Cooley and Hansen (1995) is that the central bank is here assumed to use a policy rule when it decides on the nominal money supply growth in each period similar to that suggested by McCallum (1984, 1988). More specifically, the growth rate in nominal money supply in period  $t$  is assumed to follow a Taylor inspired rule and depend on the output gap, the difference between actual and targeted inflation rate (hereafter named inflation gap), an uncontrollable shock, and the growth rate in nominal money in period  $t - 1$ . This specification is intended to capture the real world phenomenon that central banks use money supply to affect inflation and output gaps, although they act gradually and do not have perfect control of the process. It is shown that this monetary policy rule for nominal money growth can be rewritten as a Taylor-rule for the nominal interest rate.

In the model I abstract from population and technological growth and represent all variables in per capita terms.

Finally, a notational comment; in the following, capital letters denote economy wide averages which the agent takes as given and small letters individual specific values which the agent internalizes.

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<sup>8</sup> I would like to emphasize that the qualitative aspects of the results in the paper are not at all dependent on whether I calibrate the model to match quarterly or yearly data.

## 2.1 An equilibrium monetary business cycle model

Infinitely many identical infinitely lived agents maximize expected utility with preferences summarized by

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, h_t), \\ u(c_{1t}, c_{2t}, h_t) \equiv \alpha \ln(c_{1t}) + (1 - \alpha) \ln(c_{2t}) - \gamma h_t \end{aligned} \quad (1)$$

where  $c_{1t}$  is consumption of the “cash good” in period  $t$ ,  $c_{2t}$  is consumption of the “credit good,” and  $h_t$  is the share of available time spent in employment which enters linearly in (1) because of the “indivisible labor” assumption (see Hansen, 1985). In (1),  $\beta$  is the subjective discount factor,  $\gamma$  the disutility the agent gets from working, while  $\alpha$  reflects the trade-off between consumption of the cash and credit goods.

The flow budget constraint facing the agent is

$$c_{1t} + c_{2t} + i_t + \frac{m_{t+1}}{P_t} + \frac{b_{t+1}}{P_t} = \left( \frac{W_t^c}{P_t} \right) h_t + R_t^K k_t + \frac{m_t}{P_t} + (1 + R_{t-1}) \frac{b_t}{P_t} + \frac{TR_t}{P_t} \quad (2)$$

where  $i_t$  denotes the agent’s investment,  $m_{t+1}$  and  $b_{t+1}$  the agent’s holdings of nominal money and government bonds at the end of period  $t$ ,  $P_t$  the aggregate price level,  $W_t^c$  the contracted nominal wage,  $R_t^K$  the gross real return on the capital stock  $k_t$ ,  $R_{t-1}$  the nominal interest rate on government bonds between periods  $t - 1$  and  $t$ , and  $TR_t$  nominal lump-sum transfers (or taxes if negative) from the government.

The agent has the following cash-in-advance constraint for the cash-good  $c_{1t}$ ,

$$P_t c_{1t} = m_t + (1 + R_{t-1}) b_t + TR_t - b_{t+1} \quad (3)$$

which always holds with equality since the nominal interest rate will always be positive in this model.

The government’s budget constraint is

$$P_t G_t + TR_t = M_{t+1} - M_t + B_{t+1} - (1 + R_{t-1}) B_t \quad (4)$$

where  $G$  is exogenous public consumption expenditures, and  $M$  and  $B$  aggregate nominal money supply and government bonds. As in Cooley and Hansen (1995), I will assume that  $B_t = 0$  for  $t \geq 0$  and only use it to compute the nominal interest rate in the economy. It can be shown that the nominal interest rate in equilibrium is given by

$$R_t = \frac{\alpha}{1 - \alpha} \frac{C_{2t}}{C_{1t}} - 1 \quad (5)$$

where  $C_{1t}$  and  $C_{2t}$  are aggregate consumption of the cash and credit goods, respectively.

Government consumption,  $G$ , in (4) is assumed to be generated by the following stationary AR(1)-process,

$$\ln G_{t+1} = \left(1 - \rho^{\ln G}\right) \ln \bar{G} + \rho^{\ln G} \ln G_t + \varepsilon_{t+1}^{\ln G}, \quad 0 < \rho^{\ln G} < 1, \quad \varepsilon^{\ln G} \sim i.i.d. N\left(0, \sigma_{\ln G}^2\right). \quad (6)$$

Aggregate nominal money supply is assumed to evolve according to

$$M_{t+1} = e^{\mu_t} M_t \quad (7)$$

where the growth rate in nominal money supply in period  $t$ , defined as  $\Delta \ln M_{t+1}$  and denoted  $\mu_t$ , is assumed to be determined by

$$\begin{aligned} \mu_t &= \eta \mu_{t-1} - \lambda_\pi (\pi_t - \pi^*) - \lambda_Y (\ln Y_t - \ln Y^*) + \xi_t, \quad 0 < \eta < 1, \\ \xi &\sim i.i.d. \text{ Log Normal}, \quad \mathbb{E}[\xi] = (1 - \eta) \bar{\mu}, \quad \text{Var}(\xi) = \sigma_\xi^2 \end{aligned} \quad (8)$$

where  $\pi_t$  is defined as  $\ln P_t - \ln P_{t-1}$ , and  $\lambda_\pi$  and  $\lambda_Y$  measure how the central bank reacts to deviations in the inflation ( $\pi_t - \pi^*$ ) and the output gap ( $\ln Y_t - \ln Y^*$ ), respectively.<sup>9</sup> The implicit assumption underlying the specification in (8) is that the central bank tries to stabilize inflation and/or output, and one might think of (8) as an implementable monetary policy rule for a central bank which has been attached a conventional quadratic loss function in the inflation and output gaps. For simplicity, we will also set  $\pi^*$  and  $\ln Y^*$  in (8) equal to steady state nominal money supply growth ( $\bar{\mu}$ ) and log of output ( $\ln \bar{Y}$ ), respectively. The error term,  $\xi$ , can be thought of as policy shocks from the perspective of the private sector. By introducing the persistence component  $\eta \mu_{t-1}$ , it is also assumed that the central bank reacts gradually to shocks which hit the economy.

The policy rule in (8) is not optimal. One important reason for choosing it nevertheless, is that it is possible to derive a standard Taylor-type rule (see Taylor, 1993 and 1999a) for the nominal interest rate as an equilibrium relationship within the equilibrium model given the functional form of (8). Log-linearizing (5), (3) and (19), and substituting these equations into (8), it is possible to derive

$$R_t = -\frac{\lambda_\pi \pi^* + \lambda_Y \ln Y^*}{\left(1 - \bar{P}\bar{G}\right) \kappa_3} + \frac{1 + \lambda_\pi - \eta L}{\left(1 - \bar{P}\bar{G}\right) \kappa_3} \pi_t + \frac{\lambda_Y + \bar{P}\bar{G} (1 - \eta L) (1 - L) \frac{\bar{Y}}{\bar{C}}}{\left(1 - \bar{P}\bar{G}\right) \kappa_3} \ln Y_t + (1 + \eta - \eta L) R_{t-1} + \varepsilon_t^R \quad (9)$$

where  $\varepsilon_t^R \equiv \left[ \frac{-\xi_t + \left(\frac{\bar{C}-\bar{G}}{\bar{C}}\right) \bar{P}\bar{G}(1-\eta L)(1-L) \ln G_t - \delta \frac{\bar{K}}{\bar{C}} \bar{P}\bar{G}(1-\eta L)(1-L) \ln I_t}{\left(1 - \bar{P}\bar{G}\right) \kappa_3} \right]$ ,  $\kappa_3 = \frac{\bar{C}-\bar{C}_1}{\bar{C}} > 0$  (bar denotes steady state values) and  $L$  is the lag operator.<sup>10</sup>

<sup>9</sup> Although we assume that  $\xi$  is log normally distributed, we require that  $\xi$  has mean  $(1 - \eta) \bar{\mu}$ , and variance  $\sigma_\xi^2$  as seen in (8). By using that  $\mathbb{E}[\xi] = e^{\mathbb{E}[\ln \xi] + \frac{1}{2} \text{Var}(\ln \xi)}$  and that  $\text{Var}(\xi) = \mathbb{E}\{(\xi - \mathbb{E}[\xi])^2\} = \mathbb{E}[\xi^2] - [(1 - \eta) \bar{\mu}]^2 = e^{2\mathbb{E}[\ln \xi] + \text{Var}(\ln \xi)} - [(1 - \eta) \bar{\mu}]^2$  since  $\xi$  is log-normally distributed, one can pin down the mean and the variance for  $\ln \xi$  as  $-\frac{1}{2} \ln(\sigma_\xi^2 + [(1 - \eta) \bar{\mu}]^2) + 2 \ln((1 - \eta) \bar{\mu})$  and  $\ln(\sigma_\xi^2 + [(1 - \eta) \bar{\mu}]^2) - 2 \ln((1 - \eta) \bar{\mu})$  respectively.

<sup>10</sup> Note that a problem with interpreting (9) as a Taylor-type rule is that the residual is correlated with the arguments. However, this feature has sometimes also been acknowledged when estimating Taylor-type rules on real-world data, see e.g. Clarida, Gali and Gertler (2000) and McCallum and Nelson (1999).



The production function is assumed to have constant returns to scale and be of Cobb-Douglas type

$$Y_t = e^{\ln Z_t} K_t^\theta H_t^{1-\theta} \quad (10)$$

where  $K_t$  and  $H_t$  are aggregate (average) capital stock and hours worked, respectively, and  $Z_t$  the technology level which is assumed to follow a stationary AR(1)-process (in natural logs)

$$\ln Z_{t+1} = \rho^{\ln Z} \ln Z_t + \varepsilon_{t+1}^{\ln Z}, \quad \varepsilon^{\ln Z} \sim i.i.d. N(0, \sigma_{\ln Z}^2). \quad (11)$$

Individual and aggregate investment in period  $t$  produces productive capital in period  $t + 1$  according to

$$k_{t+1} = (1 - \delta) k_t + i_t \quad (12)$$

and

$$K_{t+1} = (1 - \delta) K_t + I_t \quad (13)$$

where  $\delta$  is the rate of capital depreciation.

The perfect competition zero profit maximizing conditions for the representative firm are

$$W_t^c = (1 - \theta) e^{\ln Z_t} \left( \frac{K_t}{H_t} \right)^\theta P_t \quad (14)$$

and

$$R_t^K = \theta e^{\ln Z_t} \left( \frac{K_t}{H_t} \right)^{\theta-1}. \quad (15)$$

The nominal wage  $W_t^c$  is assumed to be set at the end of period  $t - 1$  (see Cooley and Hansen (1995) for further details on the nominal wage arrangement) as

$$\ln W_t^c = \ln(1 - \theta) + E_{t-1} \ln Z_t + \theta (K_t - E_{t-1} H_t) + E_{t-1} P_t \quad (16)$$

where  $E_{t-1}$  denotes the conditional expectations operator on all relevant information in period  $t - 1$ .<sup>11</sup> Moreover, households are assumed to transfer to the firms the right to choose aggregate hours worked in period  $t$ ,  $H_t$ , to equate the marginal product of labor to the contracted wage rate. If we combine (14) and (16) in natural logarithms, using (11) below, we obtain

$$\ln H_t = E_{t-1} \ln H_t + \frac{1}{\theta} (\ln P_t - E_{t-1} \ln P_t) + \frac{1}{\theta} \varepsilon_t^{\ln Z}. \quad (17)$$

Similarly, one realizes that the natural logarithm of  $h_t$  for an agent in equilibrium is given by

$$\ln h_t = E_{t-1} \ln H_t + \frac{1}{\theta} (\ln P_t - E_{t-1} \ln P_t) + \frac{1}{\theta} \varepsilon_t^{\ln Z}. \quad (18)$$

The aggregate resource constraint

$$Y_t = C_{1t} + C_{2t} + I_t + G_t \equiv C_t + I_t + G_t \quad (19)$$

also holds in every period where  $C_t$  is total consumption.

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<sup>11</sup> Note that  $\ln K_t$  is known at the end of period  $t - 1$  through the equilibrium decision rules (see Appendix A).

## 2.2 Equilibrium in the model

The equilibrium in the model consists of a set of decision rules for the agents  $\ln k_{t+1} = k(\mathbf{S}_t, \ln k_t, \ln \hat{m}_t)$ ,  $\ln \hat{m}_{t+1} = \hat{m}(\mathbf{S}_t, \ln k_t, \ln \hat{m}_t)$  and  $\ln h_t = h(\mathbf{S}_t, \ln k_t, \ln \hat{m}_t)$ , and a set of aggregate decision rules  $\ln K_{t+1} = K(\mathbf{S}_t)$ ,  $\ln H_t = H(\mathbf{S}_t)$ ,  $\ln \hat{P}_t = \hat{P}(\mathbf{S}_t)$  where  $\mathbf{S}_t = \left[ \ln Z_{t-1}, \varepsilon_t^{\ln Z}, \mu_{t-1}, \xi_t, \ln G_t, \ln K_t, \ln \hat{P}_{t-1} \right]'$  such that; (i) agents maximize utility, (ii) firms maximize profits, and (iii), individual decision rules are consistent with aggregate outcomes. Equilibrium condition (iii) implies that  $k(\mathbf{S}_t, \ln K_t, 1) = K(\mathbf{S}_t)$ ,  $\hat{m}(\mathbf{S}_t, \ln K_t, 1) = 1$ , and  $h(\mathbf{S}_t, \ln K_t, 1) = H(\mathbf{S}_t)$  for all  $\mathbf{S}_t$ .

In Appendix A, I describe how to compute the equilibrium in this model.

## 3 Estimation and calibration

The parameters in the equilibrium model are determined in two ways. About half of the parameters ( $\eta, \bar{\mu}, \sigma_{\xi}^2, \lambda_{\pi}, \lambda_Y, \rho^{\ln G}, \sigma_{\ln G}^2$  and  $\bar{g} \equiv \frac{\bar{G}}{Y}$ ) are estimated on U.S. data 1960-1997 with instrumental variables (IV) and ordinary least squares (OLS). The other half of the parameters ( $\alpha, \beta, \delta, \gamma, \theta, \rho^{\ln Z}$  and  $\sigma_{\ln Z}^2$ ) are adapted from Cooley and Hansen (1995), and chosen so that the model's steady state properties are consistent with U.S. growth facts.

To estimate the parameters  $\eta, \bar{\mu}, \sigma_{\xi}^2, \lambda_{\pi}$ , and  $\lambda_Y$  in the monetary policy rule (8) for different Fed chairmen periods, I collected quarterly data on real gross national product per capita in natural logarithms ( $\ln Y_t$ ), growth rate in nominal money supply ( $\mu_t$ ) and the inflation rate in the consumer price index ( $\pi_t$ ). To compute measures of  $\ln Y_t - \ln Y^*$  and  $\pi_t - \pi^*$ , I simply filtered the series for output and inflation rate with the Hodrick-Prescott (H-P) filter (see Hodrick and Prescott, 1997).<sup>12</sup> It is standard to use H-P filtered output as measure of the output gap, but it is less clear how to compute an appropriate measure of  $\pi^*$  from historical data as discussed by Judd and Rudebusch (1998).<sup>13</sup> Since the model does not distinguish between money controlled by the Fed (the monetary base, M0) and money used in private transactions (M2), I compromise between them and use M1 as a measure of money as in Cooley and Hansen (1989, 1995). The reason for estimating with IV rather than Ordinary Least Squares (OLS), is that OLS is likely to be a biased and inconsistent estimator due to the fact that we may have contemporaneous correlation between the error term and the regressors in (8). In terms of the theoretical model used in this paper, there will, via the equilibrium decision rules, be a positive correlation between the error term  $\xi_t$  and the regressors  $\pi_t$  and  $\ln Y_t$  in (8). As instruments in the estimation, I therefore use  $(\ln Y - \ln Y^*)_{t-1}$ ,  $\mu_{t-1}$  and  $(\pi - \pi^*)_{t-1}$  which are uncorrelated with the error term

<sup>12</sup> I use the common value 1600 (quarterly data) for the smoothness coefficient  $\lambda$  in the H-P filter. See Appendix B for a detailed description of the raw data and data transformations.

<sup>13</sup> Although my approach regarding  $\pi - \pi^*$  appears to be as good as any other considerable alternative (see Judd and Rudebusch), I have nevertheless experimented with other measures (such as the average inflation rate during a given chairmen's term), but it did not have any impact on the conclusions drawn in the paper.

$\xi_t$  in (8). In addition to that, the estimated  $\lambda_\pi$  and  $\lambda_Y$  will be correlated in general, why inference must be conducted with great care. One final problem when estimating the monetary policy rule on revised data for the inflation rate and the output gap rather than using real-time data is that the estimates may be biased and inconsistent because of measurement errors. However, Figure 3 in Orphanides (2000) indicates that the correlations between the current and the real-time inflation rate and output gap are very high (close to 1), although there is a big level difference between the current and real-time output gap. Therefore, for estimation purposes the use of real-time or current (revised) data are not likely to be of decisive importance.

**Table 1a: IV estimation results for the monetary policy rule (8).**

Estimation period	Estimation output								
	$\hat{\eta}$	$\hat{\lambda}_\pi$	$\hat{\lambda}_Y$	$\hat{\sigma}_\xi$	$\bar{R}^2$	D-W	B-G $\chi^2(4)$	J-B	$T$
Whole sample	0.931 (0.033)	0.181 (0.093)	0.083 (0.092)	0.0138	0.89	1.39	33.33 (0.000)	0.400 (0.819)	112
Burns	0.515 (0.151)	0.182 (0.087)	-0.166 (0.148)	0.0073	0.73	1.96	14.17 (0.007)	0.715 (0.699)	33
Volcker	0.717 (0.116)	0.377 (0.203)	-0.137 (0.249)	0.0158	0.73	1.59	11.86 (0.019)	1.289 (0.525)	32
Greenspan	0.919 (0.054)	0.532 (0.540)	0.013 (0.262)	0.0153	0.91	0.98	17.37 (0.002)	1.249 (0.536)	42

Note: Standard errors in parentheses for  $\hat{\eta}$ ,  $\hat{\lambda}_\pi$  and  $\hat{\lambda}_Y$ , and  $p$ -values in parentheses for the Breusch-Godfrey autocorrelation test (null hypothesis no autocorrelation up to 4 lags) and the Jarque-Bera normality test (null hypothesis normally distributed residuals). A constant,  $(\ln Y - \ln Y^*)_{t-1}$ ,  $\mu_{t-1}$  and  $(\pi - \pi^*)_{t-1}$  have been used as instruments.  $T$  denotes the number of observations in the regressions.

I estimate the monetary policy rule (8) with IV for the whole sample period (1970Q1 – 1997Q4), for chairman Burns’ office period (1970Q1 – 1978Q1), chairman Volckers’ office period (1979Q3 – 1987Q2), chairman Greenspan’s office period (1987Q3 – 1997Q4), and omit chairman Miller as in Judd and Rudebusch (1998) because of his short tenure. The results of the estimations are reported in Table 1a (a constant is included in the regressions but is omitted from the table).

The Durbin-Watson (D-W) and Breusch-Godfrey (B-G) statistics indicate the presence of positive autocorrelation in the regressions, suggesting difficulties in interpreting the significance levels of the estimates of  $\eta$ ,  $\lambda_\pi$  and  $\lambda_Y$ . However, use of the asymptotic  $\chi^2$ -distribution for the Breusch-Godfrey test is very likely to yield an oversized test (i.e., an exaggerated probability of rejecting a true null hypothesis of no autocorrelation) for sample sizes as small as the present ones. Simulated small-sample adjusted  $p$ -values for the Breusch-Godfrey test confirm the size problem and result in a non-significant autocorrelation effect.<sup>14</sup> Not surprisingly, we get the highest estimate of  $\lambda_\pi$  during chairman Greenspan’s office period, and the lowest for chairman Burns.

A quick examination of the parameter estimates and standard errors in Table 1a might give the impression - in contrast to the findings by Judd and Rudebusch (1998) - that the

<sup>14</sup> See Lindé (2001a) for further details.

different estimated policy rules are not statistically significant across regimes (subsamples) and the whole sample period. Although the estimated policy rules produce non-local dynamics in the equilibrium model (which can be easily demonstrated), it is not appropriate if the results of the paper are hanging on the effects of changes in coefficients which only seem large because they are imprecisely estimated. To examine this issue in greater detail, I therefore conducted a Wald stability test.<sup>15</sup> Table 1b reports the results.

**Table 1b: Wald stability test results for the monetary policy rule (8).**

Estimated rule	Alternative rule				Test period	$T$
	Whole Sample	Burns	Volcker	Greenspan		
Whole sample	N.C.	177.77 (0.000)	49.81 (0.000)	17.31 (0.001)	1970Q1 – 1997Q4	112
Burns	7.93 (0.048)	N.C.	18.88 (0.000)	47.15 (0.000)	1970Q1 – 1978Q1	33
Volcker	3.40 (0.334)	8.30 (0.040)	N.C.	7.45 (0.059)	1979Q3 – 1987Q2	32
Greenspan	0.44 (0.933)	91.24 (0.000)	23.75 (0.000)	N.C.	1987Q3 – 1997Q4	42

Note: N.C. stands for not computed.  $p$ -values (i.e. the lowest asymptotic significance level for which the null hypothesis of parameter stability can be rejected) for the test statistic in parentheses.  $T$  denotes the number of observations in the regressions in each test period. Under the null hypothesis of unchanged parameters, the Wald-statistic follows the  $\chi^2$ -distribution with degrees of freedom equal to the number of parameter restrictions being tested (here, 3).

The results in Table 1b clearly indicate that the changes between the estimated monetary policy rules in Table 1a are significant, confirming the results in Judd and Rudebusch (1998) for the estimated money growth rule. Only in two cases, during Volcker’s and Greenspan’s office periods, we cannot reject the null hypothesis that the estimated policy rules for these periods are equal to the policy rule estimated for the whole sample period on reasonable significance levels. However, it is clear that these non-rejections are only a small sample problem, since we can strongly reject the null that the rules estimated in the Volcker and Greenspan sub-samples are valid description of policy behavior for the whole sample period.

To examine if these parameter changes are in line with typical experiments conducted with interest rate rules in the monetary policy literature, we can insert the estimates of  $\eta$ ,  $\lambda_\pi$  and  $\lambda_Y$  into the implied Taylor-type rule for the nominal interest rate in (9). It is then possible to verify that the resulting parameter changes in the Taylor-type rule for the nominal interest rate are well in line with typical parameter experiments considered in the interest rate rule literature. Also, when I calibrate the equilibrium model with the estimated monetary policy rules in Table 1a, the implied relative volatilities of the inflation rate and the output gap conform reasonably well to what we see in the data.<sup>16</sup>

Based on the estimations in Lindé (2001a), I set  $\rho^{\ln G} = 0.80$  and  $\sigma_{\ln G} = 0.0098444$  in (6).

<sup>15</sup> The Wald test allows for autocorrelation and unequal variance in the residuals. An  $F$ -test, which assumes that the residuals are white noise produced very similar results.

<sup>16</sup> For instance, the relative inflation rate and output gap volatilities in the data for Burns’ and Greenspan’s office periods are around 2.2 and 1.6, respectively, whereas the corresponding ratios generated by the equilibrium model (using the estimated Burns and Greenspan policy rules, respectively) are approximately 2.4 and 1.4.

To compute values for  $\bar{\mu}$  and  $\bar{g}$ , I took averages of quarterly nominal money growth and the ratio of government expenditures to gross national product to get 0.01310 and 0.21038 respectively.

$\gamma$  is calibrated in the same way as in Cooley and Hansen (1995) and set so that hours worked as share of available time in steady state,  $\bar{H}$ , equals 0.30, which implies setting  $\gamma = 3.404$  (see Lindé 2001a for further details). The remaining parameters are directly taken from Cooley and Hansen;  $\alpha$  is set to 0.84,  $\beta$  is set to 0.989,  $\delta$  is set to 0.019,  $\theta$  is set to 0.40 and  $\rho^{\ln Z}$  and  $\sigma_{\ln Z}$  are set to 0.95 and 0.00721 respectively.

## 4 Investigated models

In this section, I present the models that I have chosen to study. I also describe in detail how the variables in the models are measured.

### 4.1 The backward-looking model

In this Section, I will briefly present the backward-looking model that I have chosen to study - the Rudebusch and Svensson (1999) model. The Rudebusch and Svensson model, which draws on the theoretical model presented by Svensson (1997), is intended to be a good approximation of reality. It contains much richer dynamics than the simple Svensson model by allowing for four lags of inflation in the AS curve and two lags of output in the AD curve, since it is intended that the model should be empirically acceptable.

It consists of aggregate supply (AS) and aggregate demand (AD) equations relating the output gap (the percentage deviation of output from its steady state level) and the inflation rate to each other and a monetary policy instrument, the (short-run) interest rate. Formally, the model economy is described by the following equations

$$\begin{aligned}\pi_t &= \sum_{j=1}^4 \alpha_{\pi,j} \pi_{t-j} + \alpha_y y_{t-1} + \varepsilon_t^\pi, \\ y_t &= \beta_{y,1} y_{t-1} + \beta_{y,2} y_{t-2} + \beta_r \sum_{j=1}^4 \frac{1}{4} (i - \pi)_{t-j} + \varepsilon_t^y.\end{aligned}\tag{20}$$

In (20), the first equation is the AS curve (or Phillips curve), where the (annualized) inflation rate  $\pi$  depends on past inflation rates, the output gap in the previous period and an exogenous supply shock  $\varepsilon_t^\pi$  (i.i.d. with zero mean and variance  $\sigma_\pi^2$ ). The second equation in (20) is the AD curve, where the output gap  $y_t$  is related to past output gaps  $y_{t-1}$  and  $y_{t-2}$ , the average ex post real interest rate in the four previous periods,  $\sum_{j=1}^4 \frac{1}{4} (i - \pi)_{t-j}$ , and an exogenous demand shock  $\varepsilon_t^y$  (i.i.d. with zero mean and constant variance). The central bank, which is assumed to control the nominal interest rate  $i_t$ , thus affects the inflation rate with a two period lag. The monetary

transmission mechanism is via output to the inflation rate. In the Rudebusch and Svensson framework, the sum of the estimated  $\alpha_{\pi,j}$ 's is restricted to equal 1 to get an accelerationist Phillips curve where long-run monetary neutrality holds.

Rudebusch and Svensson estimate (20) on quarterly US data for the sample period 1961Q1 to 1996Q2. They cannot reject the hypothesis that  $\sum_{j=1}^4 \alpha_{\pi,j}$  equals 1 so they maintain that assumption throughout their analysis. But in the model framework here - when we have the equilibrium model as a data generating process - this restriction will only be fulfilled (i.e. not rejected) for the “Whole sample” estimated policy rule, so I therefore only imposed it for this regime.<sup>17</sup>

Rudebusch and Svensson measure the inflation rate  $\pi_t$ , the output gap  $y_t$  and the ex post real interest rate  $(i - \pi)_t$  in the following way. To measure  $\pi_t$ , they compute  $400(\ln P_t - \ln P_{t-1})$  where  $P$  is the quarterly chain-weighted GDP price index.  $y_t$  is measured as the percentage gap between real output and potential (steady state) output  $100((Y_t - Y_t^*)/Y_t^*)$ . The *ex post* real interest rate (in period  $t$ ) included in the AD curve is measured as  $\frac{1}{4} \sum_{j=1}^4 (i - \pi)_{t-j}$  where  $i$  is the average quarterly federal funds rate (in the equilibrium model, the annualized nominal interest rate  $i_t$  is computed as 400 times the one period nominal interest rate on government bonds) and  $\pi$  is the inflation rate defined previously. All variables are then demeaned prior to estimation of the model economy; hence no constants are included in the regressions. In all the estimations throughout this paper, all the variables are measured precisely in the same way as Rudebusch and Svensson.

## 4.2 The purely forward-looking model

In this Section, I will briefly present the forward-looking model that I have chosen to study, which contains an aggregate supply (AS) due Roberts (1995) and an aggregate demand (AD) curve due to McCallum and Nelson (1999). Both these equations were derived in an optimizing rational expectations framework with utility maximizing agents. Very similar specifications have also been derived by Woodford (1996) and Rotemberg and Woodford (1997) and this is presently the workhorse model in monetary policy analysis, see e.g. Clarida, Galí and Gertler (1999) and the references therein.

The model economy is given by the equations

$$\begin{aligned}\pi_t &= E_t \pi_{t+1} + \alpha_y y_t + \varepsilon_t^\pi, \\ y_t &= E_t y_{t+1} + \beta_r (i_t - E_t \pi_{t+1}) + \varepsilon_t^y.\end{aligned}\tag{21}$$

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<sup>17</sup> By computing a simple  $F$ -test on the same sample size as in the whole sample period (quarterly data between 1970Q1 – 1997Q4 implies  $T = 112$ ), I received an average  $p$ -value of 0.366 for the “Whole sample” estimated policy rule, and around 0.035 for the Burns, Volcker and Greenspan policy rules.

In (21), the first equation is the AS-curve where the inflation rate  $\pi$  in period  $t$  is determined by the as of  $t$  expected inflation rate in period  $t + 1$ ,  $E_t\pi_{t+1}$ , and on the output gap  $y$  in  $t$ , and an exogenous supply shock  $\varepsilon_t^\pi$ .<sup>18</sup> In the AD-curve, which is given by the second equation, the output gap in period  $t$ ,  $y_t$ , depends on the as of  $t$  conditional expectation of  $y$  in period  $t + 1$ , and the *ex ante* real interest rate,  $i_t - E_t\pi_{t+1}$ , and an exogenous demand shock  $\varepsilon_t^y$ .

The empirical properties of this model have been studied by Estrella and Fuhrer (1999). Estrella and Fuhrer estimate the model in (21) by substituting actual future inflation/output for expected inflation/output with GMM/2SLS using instrumental variables for all regressors. By doing so, they add a rational expectations error in the residuals  $\varepsilon_t^\pi$  and  $\varepsilon_t^y$ . I employ the same procedure, using four lags of inflation, money growth, the output-gap and the nominal interest rate as instruments (i.e.  $\pi_{t-1}, \dots, i_{t-4}$ ). No instruments in period  $t$  were used in order to avoid contemporaneous correlations with the residuals  $\varepsilon_t^\pi$  and  $\varepsilon_t^y$ . Note that in order for the instruments to be valid, it is required that  $\varepsilon_t^\pi$  and  $\varepsilon_t^y$  are not autocorrelated, a condition that seems to be met when the model is estimated on simulated data (see Table 3a).

### 4.3 The hybrid model

This model nests both of the models presented above by including both forward- and backward-looking components. It has been used extensively in the most recent literature on monetary policy rules. The theoretical and empirical reasons for including backward-looking components in (21) are discussed by e.g. Clarida, Galí and Gertler (1999), Rudebusch (2000) and Roberts (2001). This type of model has the following form:

$$\begin{aligned}\pi_t &= \omega_\pi E_t\pi_{t+1} + (1 - \omega_\pi) \left( \sum_{j=1}^4 \alpha_{\pi,j} \pi_{t-j} \right) + \alpha_y y_t + \varepsilon_t^\pi, \\ y_t &= \omega_y E_t y_{t+1} + (1 - \omega_y) (\beta_y y_{t-1} + (1 - \beta_y) y_{t-2}) + \beta_r (i_{t-1} - E_{t-1}\pi_t) + \varepsilon_t^y.\end{aligned}\tag{22}$$

The degree of forward-looking behavior is then determined by the choice of  $\omega_\pi$  and  $\omega_y$  in the model. As in Rudebusch (2000), I impose the restriction that  $\sum_{j=1}^4 \alpha_{\pi,j} = 1$  in the AS-equation since this restriction cannot be rejected when estimating (22) on simulated data. The same restriction is imposed in the AD-equation as well. As is evident from (22), I have also imposed the restriction that the coefficients on the forward- and backward looking components sum to unity. Rudebusch (2000) also makes this assumption; Galí and Gertler (1999) present results with and without this assumption imposed. As in Rudebusch (2000), I lag the real interest rate one period in the AD-equation. As in the previous subsection, (22) is estimated with GMM/2SLS using the same set of instruments as described there.

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<sup>18</sup> In the Woodford (1996) and the Rotemberg and Woodford (1997) derivations of the AS-curve in (21),  $E_t\pi_{t+1}$  is multiplied with the household discount factor  $\beta$ . Since  $\beta$  is very close to one (often assumed to be 0.99 on quarterly data), one can view the AS-curve here as an approximation of their AS-curve.

Rudebusch (2000) obtains a point estimate of  $\omega_\pi$  around 0.30 (however, he notes explicitly that there is some uncertainty about the appropriate value of this parameter and assesses that a reasonable interval for it is between 0.00 – 0.60). Rudebusch, however, does not estimate  $\omega_y$  nor discuss any empirical reasonable value of it except that he mentions that the results in Fuhrer (2000) imply that  $\omega_y$  should be considerably lower than one.

## 5 Estimation results for investigated models on simulated data

In this section, I present the results of some Monte-Carlo experiments designed to examine the econometric properties of the investigated models. Of particular interest is to examine whether policy changes like those that have been observed in practice, imply that the parameters in the models (20)-(22) changes significantly. I will discuss this from both a statistical as well as an economic point of view. Although the concept of statistical significance perhaps not is the optimal metric to judge the importance of the parameter changes that are likely to occur when changing the monetary regime, it is the only widely adopted concept in economics. In the first subsection, I will present and motivate how the experiments have been carried out.

### 5.1 Testing strategy

To investigate if, and how much, the parameters in the models change when there is a monetary policy regime shift, I have estimated the models (20) with OLS and (21) and (22) with GMM (see the end of Section 4.2) on simulated data from the equilibrium model calibrated with the estimated monetary policy rules in Table 1a. Throughout the analysis, the equilibrium model's steady state will remain unchanged. Each sample in the  $N$  simulations is  $T$  periods. The parameters reported in the tables are averages of the  $N$  simulations.

To investigate if the Lucas critique is of significant importance, i.e. that there are significant parameter changes in the investigated models due to the monetary regime shift, I have applied the following procedure (following Lindé 2001a):

1. Simulate the equilibrium model for  $T$  periods under the assumption that the monetary policy rule changes completely unexpectedly after  $T/2$  periods from one regime to another (for example, from Burns to Volcker or Burns to Greenspan).<sup>19</sup>
2. Estimate the models (20) and (21) on the first  $1, \dots, T/2$  observations in the simulated sample. Denote the estimated parameter vectors  $\hat{\beta}_{BL}$  and  $\hat{\beta}_{FL}$ , respectively.

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<sup>19</sup> The simulations are made in the GAUSS programming language, using the random number generator RDND with RDNDSEED set to  $159425 + iter$  for  $iter = 1, 2, \dots, N$ . To get a stochastic initial state in each simulation, the model is simulated for  $T + 100$  periods, where the first 100 are then discarded in all the estimations.



3. Estimate the models (20) and (21) on the last  $T/2 + 1, \dots, T$  observations in the simulated sample. Denote the estimated parameter vectors  $\hat{\alpha}_{BL}$  and  $\hat{\alpha}_{FL}$ , respectively.
4. Use a version of the  $F$ -test, often called the Chow breakpoint test, to examine if the null hypotheses  $\alpha_{BL} = \beta_{BL}$  and  $\alpha_{FL} = \beta_{FL}$  are rejected at the 5 percent significance level.
5. Repeat Steps 1 - 4 many ( $N$ ) times to compute probabilities for how often the null hypotheses are rejected for the given significance level.
6. To get correct significance levels, Steps 1 - 5 above are carried out twice. In the first round, small-sample critical values are computed under the (true) null hypotheses  $H_0 : \alpha_{BL} = \beta_{BL}$  and  $H_0 : \alpha_{FL} = \beta_{FL}$  (that is, compute the distribution of  $F$ -statistics when there has been no regime shift). In the second round, these adjusted critical values ensure a correct size in the  $F$ -testing for regime shifts.

If the computed probabilities in Step 5 (in the second round) of rejecting parameter stability are lower/higher than the given significance levels, the Lucas critique is/is not relevant for the models in a statistical sense.

The critical assumptions in steps 1 – 6 are clearly made in step 1 - 3, and I would like to briefly comment on them. First, I have chosen to change monetary policy regime in the middle of the sample. The motivation behind this choice is that it gives the highest possible power in the testing. Secondly, I have chosen to model the once and for all change in monetary policy regime as a completely unexpected shift in the estimated monetary policy rule where I let the economy bring the state vector from the last period in the previous regime (period  $T/2$ ) to the first period in the new regime (period  $T/2 + 1$ ). The assumptions made in Step 2 and 3 imply that the breakpoint date is known to econometrician. By this procedure, I implicitly assume a first order Markov chain for the different monetary policy regimes where I let the diagonal elements in the transition matrix approach unity. The second and third assumptions are very convenient since they allow me to use the same decision rules for the first  $T/2$  periods and then change to new decision rules in the beginning of period  $T/2 + 1$  for the remaining  $T/2$  periods.

## 5.2 Results for the backward-looking model

The results for the backward-looking model in (20) for the estimated monetary policy rules in Table 1a for sample size  $T = 200$  (corresponding to 50 years of quarterly data), are provided in Tables 2a and 2b. In Table 2b, I also provide results for the monetary policy rule (8).<sup>20</sup>

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<sup>20</sup> To be able to generate reliable small sample critical values under the null (when there is no regime shift) in the first round, the model has been simulated  $N = 50,000$  times. Note that the probabilities in the diagonal (when there is no regime shift) equal 0.05 (exactly) at the 5 percent significance level since the same shock realizations have been used in the second round.

First, it is of interest to examine whether the changes in the parameters are economically important. To shed light on this issue, Table 3 reports the OLS estimation results of the model (20) on simulated data generated by the equilibrium model calibrated with the estimated monetary policy rules in Table 1a.<sup>21</sup>

**Table 2a: OLS estimation of the Rudebusch and Svensson (1999) model in (20) for different regimes on simulated data.**

Estimation output for the AS-curve										
Estimated policy rule	$\alpha_{\pi,1}$	$\alpha_{\pi,2}$	$\alpha_{\pi,3}$	$\alpha_{\pi,4}$	$\alpha_y$	$\bar{R}^2$	DW	$\hat{\sigma}$	$\chi^2(1)$ <i>p</i> -value	$\chi^2(4)$ <i>p</i> -value
Whole S	0.559	0.293	0.129	0.019	0.052	0.77	1.99	3.46	0.715	0.796
Burns	0.062	0.133	0.062	0.041	0.496	0.37	2.03	4.47	0.894	0.921
Volcker	0.136	0.140	0.051	0.022	0.410	0.35	2.01	5.39	0.943	0.956
Greenspan	0.174	0.077	0.042	0.022	-0.003	0.10	2.00	2.65	0.882	0.866

  

Estimation output for the AD-curve									
Estimated policy rule	$\beta_{y,1}$	$\beta_{y,2}$	$\beta_r$	$\bar{R}^2$	DW	$\hat{\sigma}$	$\chi^2(1)$ <i>p</i> -value	$\chi^2(4)$ <i>p</i> -value	
Whole S	0.824	0.099	-0.015	0.81	2.01	2.24	0.936	0.856	
Burns	0.474	0.332	0.017	0.51	2.12	2.83	0.452	0.487	
Volcker	0.476	0.327	-0.041	0.50	2.11	3.32	0.512	0.504	
Greenspan	0.694	0.214	-0.014	0.76	2.03	2.00	0.895	0.870	

Note:  $\hat{\sigma}$  denotes the standard error of regression in percentage units. The *p*-values measure the likelihood that the computed test statistics are insignificant at the 5 percent significance level for the Breusch-Godfrey  $\chi^2$ -test (null hypotheses no autocorrelation up to first and/or fourth order). DW denotes the Durbin-Watson statistic. All the statistics reported are averages of  $N = 50000$  simulations of sample size  $T = 200$ .

From Table 2a, we see that the estimated parameters in the model are heavily affected by changes in the monetary policy rule. In particular this is true for the AS-curve; the output parameter varies from about  $-0.003$  to  $0.50$  and the real interest rate coefficient in the AD-curve, although low in general, also alters in sign.

From an econometric point of view, the estimated equations often pass (in about 80 percent on average) statistical tests for autocorrelation, as indicated by the Breusch-Godfrey statistics for autocorrelation. The adjusted r-squares are also satisfactory in most cases, in particular for the AD-equation, the exception being the low adjusted r-square for the AS curve for the Greenspan regime. The reason for this is the estimated high value for  $\lambda_\pi$  during the Greenspan regime, which drives down the autocorrelation (and the volatility) of the inflation rate. Consequently, all in all, an econometrician who estimated this model from the data would not reject it for statistical reasons in most cases.

<sup>21</sup> I have not been able to solve analytically for the reduced-form parameters in the models (20), (21) and (22) as functions of the monetary policy rule parameters  $\eta$ ,  $\lambda_\pi$  and  $\lambda_Y$  (and the other variables and parameters in the equilibrium model). Consequently, we have to estimate them on simulated data to quantify the importance of the monetary regime shifts. I expanded the number of simulations until the coefficients converged in mean down to five digits, which required slightly less than 50000 simulations for  $T = 200$ .

Table 2b examines if the parameter changes reported in Table 2a also are important according to the metric of statistical significance using the procedure described in the previous subsection.<sup>22</sup> I show results only for the 5 percent significance level, but the results are qualitatively unaffected by choice of significance level. As in Table 2a, the results are for  $T = 200$ . If the sample size in each simulation is decreased/increased to 100/400, the probabilities become lower/higher.

**Table 2b:  $F$ -test probabilities for rejecting the null hypothesis of parameter stability in the Rudebusch and Svensson (1999) model in (20) at the 5 percent significance level.**

	Comparison regime			
	Whole sample	Burns	Volcker	Greenspan
Benchmark regime	<i>The aggregate supply function; <math>H_0 : \alpha_{AS} = \beta_{AS}</math></i>			
Whole sample	0.050	0.766	0.653	0.576
Burns	0.562	0.050	0.054	0.341
Volcker	0.499	0.056	0.050	0.286
Greenspan	0.594	0.576	0.427	0.050
Benchmark regime	<i>The aggregate demand function; <math>H_0 : \alpha_{AD} = \beta_{AD}</math></i>			
Whole sample	0.050	0.262	0.216	0.065
Burns	0.168	0.050	0.060	0.074
Volcker	0.109	0.044	0.050	0.050
Greenspan	0.084	0.200	0.161	0.050
Benchmark regime	<i>Either AS- or AD-curve; <math>H_0 : \alpha_{AS} = \beta_{AS}</math> and <math>\alpha_{AD} = \beta_{AD}</math></i>			
Whole sample	N.C.	0.821	0.717	0.600
Burns	0.653	N.C.	0.082	0.394
Volcker	0.566	0.079	N.C.	0.324
Greenspan	0.627	0.642	0.501	N.C.

Note: N.C. is shorthand notation for not computed. The Chow (1960) statistic underlying the computation of the probabilities is defined as  $\frac{(\hat{\sigma}_T^2 - \frac{T_1}{T} \hat{\sigma}_{T_1}^2 - \frac{T_2}{T} \hat{\sigma}_{T_2}^2)/k}{(\frac{T_1}{T} \hat{\sigma}_{T_1}^2 + \frac{T_2}{T} \hat{\sigma}_{T_2}^2)/(T-2k)}$  and it follows the  $F$ -distribution with  $k, T - 2k$  degrees of freedom where  $k$  is the number of parameter restrictions that are being tested,  $T \equiv T_1 + T_2$  is the total number of observations (here  $T_1 = T_2 = \frac{T}{2}$ ) and  $\hat{\sigma}_T^2$ ,  $\hat{\sigma}_{T_1}^2$ , and  $\hat{\sigma}_{T_2}^2$  denote the estimated standard error of the regression during both monetary regimes, the first monetary regime, and the second monetary regime respectively. The small sample critical values are generated under the null hypothesis in a first round of  $N = 50,000$  simulations, and the probabilities reported in the table are then computed from a second round of simulations (again,  $N = 50,000$ ) where the small sample critical values are used in the testing.

As seen in Table 2b, the probabilities of rejecting the null hypothesis of parameter stability between regimes are clearly higher than the given significance levels in most cases. Thus, the parameter changes reported in Table 2a are also significant from a statistical point of view. For both the AS- and AD-curves, we see that the probabilities of rejecting parameter stability are found to be low between the Burns and Volcker regimes and vice versa (0.056 and 0.054, and

<sup>22</sup> Note that when testing parameter stability in Table 2b, I do not impose the natural rate hypothesis in the AS-curve estimated on data generated by the ‘‘Whole sample’’ policy rule (although this hypothesis cannot be rejected on average for this regime) so that this restriction does not affect the results in Table 2b.

0.044 and 0.060, respectively), indicating that the Lucas critique is not quantitatively important in these cases in a statistically significant way on this sample size. This is quite natural since we can see in Table 1a that the estimated monetary policy rules for Burns and Volcker are most similar (low  $\eta$ , negative  $\lambda_Y$ ) qualitatively. In general, we also find that the probabilities are lower for the AD-curve than for the AS-curve, implying that the AD-curve is less sensitive to the Lucas critique than the AS-curve in the backward-looking model. Looking at Table 2a, this is not surprising given the big changes for the output parameter in the AS-equation. However, the most interesting hypothesis to test - because both the AS- and AD-curves are used in policy analysis - is the null hypothesis of instability in either the AS- or the AD-curve. In Table 2b, the results for this hypothesis clearly indicate that the parameters in the Rudebusch and Svensson model as a whole are not exogenous to the parameters in the monetary policy rule using an equilibrium model as a datagenerating process. Thus, we conclude that the Lucas critique applies strongly to this model.

At first glance, the clear indications of parameter instability reported in Table 2b when there is monetary regime shift may seem inconsistent with the findings in Rudebusch and Svensson (1999) and Estrella and Fuhrer (1999) that the backward-looking model does not exhibit parameter instability although it is generally acknowledged that the Federal Reserve has changed the conduct of monetary policy during the sample period. Therefore, I considered the following experiment. I changed the estimated policy rules recursively in the model in the order Burns -> Volcker -> Greenspan, setting the number of periods for each estimated policy rule equal to the number of periods in the data (38, 32 and 42 respectively, see Table 1a). To initiate the Burns regime, I simulated the “Whole sample” estimated policy for 4 periods (the number of lags in the AS-curve). It is then possible to estimate the (20) model for the same number of periods as the whole sample (1970Q1 – 1997Q4 =>  $T = 112$ ). When I change the monetary policy rule, the same assumptions carefully explained in Section 5.1 are maintained. Those assumptions (first, changes in the monetary policy rule are completely unexpected to agents in the model, second, agents learn directly about the shift in the monetary policy rule and the nature of the new rule, and third, the new rule is fully credible and expected to last forever) ensures the highest possibilities of detecting the monetary regime shifts on the simulated data. Two stability tests are then applied on the simulated data, a Chow-test as in Table 2b for multiple breakpoint the same dates as when there actually has been a regime shift (assuming that the econometrician has perfect knowledge when the shifts occurred), and the Andrews (1993) test (used by Rudebusch and Svensson and Estrella and Fuhrer) for structural stability over all possible breakpoints in the middle 40 percent of the sample (covering the changes in the monetary policy rules from Burns -> Volcker -> Greenspan). The probabilities of detecting structural instability in the AS-

and AD-curves in the backward-looking model are 0.44 and 0.35 for the Chow test and 0.15 and 0.04 for the Andrews test, respectively. Since these probabilities are so low, despite an upper bound due to strong assumptions aimed at making these probabilities as high as possible, we can conclude that the high probabilities in Table 2b are not necessarily inconsistent with the evidence of stability reported on real world data, since the power of the stability tests to discover the regime shifts seems low.

### 5.3 Results for the purely forward-looking model

The results for the same exercises as in the previous subsection are reported in Tables 3a and 3b for the purely forward-looking model in (21).

**Table 3a: GMM estimation results of the forward-looking model in (21) for different regimes on simulated data.**

Estimation output for the AS-curve						
Estimated policy rule	$\alpha_y$	$\bar{R}^2$	DW	$\hat{\sigma}$	B-G $\chi^2(1)$ <i>p</i> -value	B-G $\chi^2(4)$ <i>p</i> -value
Whole sample	0.0091	0.74	2.02	3.31	0.874	0.847
Burns	0.0906	0.36	1.92	4.31	0.376	0.743
Volcker	0.0727	0.34	1.87	4.49	0.287	0.562
Greenspan	0.0063	0.09	1.97	2.28	0.855	0.869

  

Estimation output for the AD-curve						
Estimated policy rule	$\beta_r$	$\bar{R}^2$	DW	$\hat{\sigma}$	B-G $\chi^2(1)$ <i>p</i> -value	B-G $\chi^2(4)$ <i>p</i> -value
Whole sample	0.0098	0.81	1.93	1.97	0.942	0.916
Burns	0.1307	0.58	2.08	2.67	0.900	0.895
Volcker	0.1095	0.59	2.08	2.69	0.906	0.908
Greenspan	0.0188	0.76	1.99	1.84	0.903	0.891

Note: See Table 2a. Four lags of inflation, money growth, the output-gap and the nominal interest rate were used as instruments.

From Table 3a we see that there is evidence of parameter instability in the forward-looking model in both the AS- and the AD-curve when there is a monetary policy regime shift, although the instability does not appear to be as pronounced as in the backward-looking model. Since the actual datagenerating process - the equilibrium model - has forward looking properties the different parameters reported in Table 3a is due to model misspecification.

From an econometric point of view, the forward-looking model is not inferiorly specified in comparison with the backward-looking model in terms of fit, autocorrelations etc. These findings are very interesting since they are in sharp contrast to the empirical papers which have demonstrated that backward-looking models outperform forward-looking models on real-world data, see e.g. Estrella and Fuhrer (1999).

In some papers, the authors have chosen to model the residuals in (21) as serially correlated

shocks in order to improve the empirical fit of the model, see e.g. Rotemberg and Woodford (1997, 1999).<sup>23</sup> Estrella and Fuhrer (1999) also allow the residuals in (21) to follow univariate AR(2) processes, but even then the forward-looking model is still outperformed by the backward-looking model in terms of fit and parameter stability on US data. Here, the results in Table 3a imply that allowing the residuals to follow AR(2)-processes does not solve the bad fit problem with the forward-looking model, because univariate first and/or second order autocorrelation is not a problem in the estimations of (21) as indicated by the Durbin-Watson and the Breusch-Godfrey statistics.

**Table 3b: *F*-test probabilities for rejecting the null hypothesis of parameter stability in the forward-looking model (21) at the 5 percent level.**

	Comparison regime			
	Whole sample	Burns	Volcker	Greenspan
Benchmark regime	<i>The aggregate supply function; <math>H_0 : \alpha_{AS} = \beta_{AS}</math></i>			
Whole sample	0.050	0.292	0.255	0.102
Burns	0.252	0.050	0.072	0.229
Volcker	0.170	0.045	0.050	0.141
Greenspan	0.171	0.338	0.289	0.050
Benchmark regime	<i>The aggregate demand function; <math>H_0 : \alpha_{AD} = \beta_{AD}</math></i>			
Whole sample	0.050	0.107	0.095	0.066
Burns	0.054	0.050	0.062	9 0.046
Volcker	0.033	0.039	0.050	0.029
Greenspan	0.052	0.081	0.078	0.050
Benchmark regime	<i>Either AS- or AD-curve; <math>H_0 : \alpha_{AS} = \beta_{AS}</math> and <math>\alpha_{AD} = \beta_{AD}</math></i>			
Whole sample	N.C.	0.357	0.307	0.155
Burns	0.287	N.C.	0.106	0.262
Volcker	0.189	0.069	N.C.	0.159
Greenspan	0.207	0.383	0.326	N.C.

Note: See Table 2b.

Table 3b reports that the parameter instability visualized in Table 3a is also statistically significant for the AS-curve, but not the AD-curve in most cases. The mechanical reason behind this result is the following. Because the parameter in the AD-curve is less tightly estimated than in the AS-curve on average in each simulated sample, it is more difficult for the Chow-test to detect changes in that parameter for this sample size. Basically, the pattern in Table 3b is very similar to that in Table 2b, although the probabilities reported are clearly lower for the forward-

<sup>23</sup> Theoretically, the presence of autocorrelated disturbance terms is motivated differently depending on model setup. In the Clarida, Gali and Gertler (1999) setup, the error term in the Phillips-curve reflects deviations between real marginal costs and the output gap, while the error term in the IS-curve reflects expected changes in the natural output level and government expenditures.

than the backward-looking model. When considering recursive multiple regime shifts (Burns  $\rightarrow$  Volcker  $\rightarrow$  Greenspan) and applying the Andrews (1993) test and the Chow test for multiple breaks on the simple forward-looking model the same way as explained at the end of Section 5.2, the corresponding probabilities are 0.15 and 0.03 in the Chow test and 0.26 and 0.03 in the Andrews test for the AS- and AD-curves, respectively. On average, these probabilities are not higher than the ones for the backward-looking model, which suggests that it should not be more easy to detect structural instability in the forward-looking model than in the backward-looking model. This finding is also in sharp contrast to the findings by Estrella and Fuhrer (1999), who strongly rejected parameter stability in the forward-looking model on US data.

One might be surprised that the pure forward-looking model does not outperform the backward-looking in terms of fit, since the true datagenerating process (DGP) actually is forward-looking. The explanation for this is problems with measuring expected inflation and output-gaps accurately. Here, I used instrumented actual realizations as in Estrella and Fuhrer (1999). But by using the equilibrium model it is, however, possible to compute the true values for the expected inflation rate/output gap analytically.<sup>24</sup> And when the true expectations that come out of the equilibrium model are used instead of instrumented actual values, the forward-looking model fits the data much better than the backward-looking model.<sup>25</sup> However, although (21) then is a very good approximation of the true DGP, the parameters still change significantly when there is monetary regime shift.

#### 5.4 Results for the hybrid model

The results for the hybrid model in (22) for the estimated monetary policy rules in Table 1a are provided in Table 4. Note that I do not apply the statistical testing for the hybrid model (22) because the results will be a linear combination of those for the pure forward- and backward-looking models.

From Table 4 we can immediately see that we still have problems with parameter instability also in the hybrid model, in particular this is true for the AS-curve. Although not reported, this parameter instability is significant for the AS-curve - although to a much lesser extent than in the pure backward- and forward-looking models - but not significant for the AD-curve using  $T = 200$  observations in each simulation.

<sup>24</sup> As noted in Appendix A, the equilibrium decision rules can be written  $\ln K_{t+1} = K(\mathbf{S}_t)$ ,  $\ln H_t = H(\mathbf{S}_t)$ ,  $\ln \hat{P}_t = \hat{P}(\mathbf{S}_t)$  where  $\mathbf{S}_t$  is a  $1 \times 8$  row vector which contains all the aggregate state variables  $\ln Z_{t-1}$ ,  $\varepsilon_t^{\ln Z}$ ,  $\ln G_t$ ,  $\mu_{t-1}$ ,  $\xi_t$ ,  $\ln K_t$ ,  $\ln \hat{P}_{t-1}$ . By definition we have that  $E_t \pi_{t+1} = E_t \ln P_{t+1} - \ln P_t$  where  $\ln P_t = \ln \hat{P}_t + \ln M_{t+1}$  and from the Cobb-Douglas production function it follows that  $E_t \ln Y_{t+1} = E_t \ln Z_{t+1} + \theta \ln K_{t+1} + (1 - \theta) \ln E_t H_{t+1}$ . By plugging in the decision rules and the exogenous stochastic processes for  $\ln Z_t$ ,  $\mu_t$  and  $\ln G_t$  (see equations 7, 8, 11 and 6) in the expressions for  $E_t \pi_{t+1}$  and  $E_t \ln Y_{t+1}$ , it is possible to solve for the expectations analytically.

<sup>25</sup> For instance, the adjusted r-squares increase to 0.92, 0.65, 0.70 and 0.38 for the AS-curve and 0.98, 0.85, 0.90 and 0.97 for the AD-curve for the ‘‘Whole Sample’’, Burns, Volcker and Greenspan regimes, respectively.

**Table 4: GMM estimation results of the hybrid model in (22) for different regimes on simulated data.**

Estimation output for the AS-curve											
Estimated policy rule	$\omega_\pi$	$\alpha_{\pi,1}$	$\alpha_{\pi,2}$	$\alpha_{\pi,3}$	$\alpha_{\pi,4}$	$\alpha_y$	$\bar{R}^2$	DW	$\hat{\sigma}$	$\chi^2(1)$ <i>p</i> -value	$\chi^2(4)$ <i>p</i> -value
Whole S	0.680	0.46	0.25	0.20	0.09	0.010	0.77	2.08	3.24	0.437	0.707
Burns	0.747	0.61	0.20	0.12	0.07	0.082	0.40	2.11	4.23	0.943	0.964
Volcker	0.726	0.64	0.21	0.10	0.05	0.080	0.38	2.09	4.41	0.885	0.933
Greenspan	0.745	0.50	0.23	0.17	0.10	0.024	0.14	2.11	2.24	0.268	0.500

  

Estimation output for the AD-curve									
Estimation policy rule	$\omega_y$	$\beta_y$	$\beta_r$	$\bar{R}^2$	DW	$\hat{\sigma}$	$\chi^2(1)$ <i>p</i> -value	$\chi^2(4)$ <i>p</i> -value	
Whole S	0.633	0.89	-0.026	0.82	2.03	1.95	0.824	0.920	
Burns	0.720	0.65	-0.043	0.58	2.13	2.66	0.324	0.554	
Volcker	0.687	0.67	-0.044	0.60	2.12	2.69	0.420	0.662	
Greenspan	0.629	0.76	-0.023	0.77	2.07	1.82	0.678	0.822	

Note: See Table 3a.

The parameters on the forward-looking components in the AS- and AD-curves are in both cases around 0.7, indicating that forward-looking behavior is more important than backward-looking behavior for the estimated hybrid model, when the underlying DGP is a dynamic general equilibrium model with forward-looking properties. Again, the estimation results differ sharply to those reported on US data by e.g. Rudebusch (2000) (his point estimate is 0.29 for  $\omega_\pi$ ) using a survey measure of inflation expectations and Lindé (2001c) who estimate the hybrid model with FIML and obtains estimates for  $\omega_\pi$  and  $\omega_y$  around 0.3 whereas Roberts (2001) reports an average estimate for  $\omega_\pi$  of about 0.4.

## 6 Concluding remarks

In this paper, I have tried to shed some new light on the “empirical puzzles” why backward-looking models seem to fit US data well and have stable parameters, while some purely forward-looking models seem inconsistent with the data in terms of fit, parameter stability and lack of inflation inertia as demonstrated by e.g. Estrella and Fuhrer (1999, 2000). Using a dynamic general equilibrium model with flexible prices and forward-looking properties, I have shown that according to the equilibrium model we should not expect the backward-looking model to fit the US data much better and have more stable parameters than the purely forward-looking model. Recent papers that have estimated hybrid AS- and AD-equations, which nest the purely forward- and backward-looking models by including both forward- and backward-looking elements, suggest that backward-looking behavior seems more important than forward-looking behavior. The estimation results on simulated data from the dynamic general equilibrium model



point in the opposite direction.

Consequently, the analysis in this paper provides an additional piece of evidence against the use of purely forward-looking models and equilibrium models with flexible prices and without any frictions on real side of the economy (such as habit persistence, adjustment costs to capital and variable capital utilization, see e.g. Christiano, Eichenbaum and Evans, 2001) in monetary policy analysis. In a recent paper, however, Dittmar, Gavin and Kydland (2001), suggest that an equilibrium model with flexible prices can generate inflation persistence if the central bank operates via an interest rate rule rather than a money supply rule. Based on the results in this paper, it would be interesting to check if their model can also account for the results in Estrella and Fuhrer (1999) which the equilibrium model used in this paper could not.

## Appendix A Computation of equilibrium

In order to make all variables in the equilibrium model converge to a (constant) steady state, I transform the nominal variables by dividing  $m_{t+1}$  and  $P_t$  with  $M_{t+1}$ , and  $m_t$  with  $M_t$ . If we introduce the notation

$$\hat{m}_{t+s} \equiv \frac{m_{t+s}}{M_{t+s}} \text{ and } \hat{P}_{t+s} \equiv \frac{P_{t+s}}{M_{t+s+1}}$$

and use the transformations to rewrite the equations (2), (3), (4), (8), (17) and (18), the representative agent's optimization problem can, following Hansen and Prescott (1995), be expressed as the recursive dynamic programming problem:

$$V(\mathbf{S}_t, \hat{m}_t, k_t) \equiv \max_{\{\hat{m}_{t+1}, h_t, k_{t+1}\}} [\alpha \ln(c_{1t}) + (1 - \alpha) \ln(c_{2t}) - \gamma h_t + \beta E_t V(\mathbf{S}_{t+1}, \hat{m}_{t+1}, k_{t+1})] \\ \text{s.t. (10), (6), (11),} \quad (\text{A.1})$$

$$\begin{aligned} c_{1t} &= \frac{\hat{m}_t + e^{\mu_t} - 1}{e^{\mu_t} \hat{P}_t} - G_t, \\ c_{2t} &= (1 - \theta) e^{\ln Z_t} \left( \frac{K_t}{H_t} \right)^\theta h_t + (1 + R_t^K - \delta) k_t - k_{t+1} - \frac{\hat{m}_{t+1}}{\hat{P}_t}, \\ \mu_t &= \frac{\eta}{1 + \lambda_\pi} \mu_{t-1} - \frac{\lambda_\pi}{1 + \lambda_\pi} \left( \ln \hat{P}_t - \ln \hat{P}_{t-1} - \pi^* \right) - \frac{\lambda_Y}{1 + \lambda_\pi} (\ln Y_t - \ln Y^*) + \frac{1}{1 + \lambda_\pi} \xi_t, \\ H_t - E_{t-1} \ln H_t &= \frac{1}{\theta(1 + \lambda_\pi) + (1 - \theta)\lambda_Y} \left( \ln \hat{P}_t - E_{t-1} \ln \hat{P}_t \right) + \frac{1 + \lambda_\pi - \lambda_Y}{\theta(1 + \lambda_\pi) + (1 - \theta)\lambda_Y} \varepsilon_t^{\ln Z} + \frac{\xi_t - (1 - \eta)\bar{\mu}}{\theta(1 + \lambda_\pi) + (1 - \theta)\lambda_Y}, \\ h_t - E_{t-1} \ln H_t &= \frac{1}{\theta(1 + \lambda_\pi) + (1 - \theta)\lambda_Y} \left( \ln \hat{P}_t - E_{t-1} \ln \hat{P}_t \right) + \frac{1 + \lambda_\pi - \lambda_Y}{\theta(1 + \lambda_\pi) + (1 - \theta)\lambda_Y} \varepsilon_t^{\ln Z} + \frac{\xi_t - (1 - \eta)\bar{\mu}}{\theta(1 + \lambda_\pi) + (1 - \theta)\lambda_Y}, \\ \ln K_{t+1} &= K(\mathbf{S}_t), \ln H_t = H(\mathbf{S}_t), \ln \hat{P}_t = \hat{P}(\mathbf{S}_t). \end{aligned}$$

In (A.1),  $\mathbf{S}_t$  is a  $1 \times 8$  row vector which contains all the aggregate state variables  $\ln Z_{t-1}$ ,  $\varepsilon_t^{\ln Z}$ ,  $\ln G_t$ ,  $\mu_{t-1}$ ,  $\xi_t$ ,  $\ln K_t$ ,  $\ln \hat{P}_{t-1}$  and a constant term. If  $\lambda_\pi = 0$ , then  $\ln \hat{P}_{t-1}$  vanishes in  $\mathbf{S}_t$ .<sup>26</sup> In maximization of (A.1), the agent takes the economy-wide aggregate (average) variables as given. The functions  $K$ ,  $\hat{P}$  and  $H$  describe the relationship perceived by agents between the aggregate decision variables and the state of the economy. As the solution to the problem in (A.1), we have the agent's decision rules  $\ln k_{t+1} = k(\mathbf{S}_t, \ln k_t, \ln \hat{m}_t)$ ,  $\ln \hat{m}_{t+1} = \hat{m}(\mathbf{S}_t, \ln k_t, \ln \hat{m}_t)$  and  $\ln h_t = h(\mathbf{S}_t, \ln k_t, \ln \hat{m}_t)$ . The competitive equilibrium is obtained when the individual and average decision rules coincide for  $\ln k_t = \ln K_t$  and  $\ln \hat{m}_{t+1} = \ln \hat{m}_t = 0$ .

Since it is impossible to derive the decision rules analytically, I have used the same method as Cooley and Hansen (1995) and computed the decision rules numerically by approximating the original problem with a second order Taylor expansion around the constant steady state values in the nominal-growth adjusted economy. As a consequence of this approximation, the method produces linear decision rules (in natural logarithms for  $K_{t+1}$ ,  $H_t$  and  $\hat{P}_t$ ). The algorithm utilized is described in detail in Hansen and Prescott (1995).

<sup>26</sup> Note that the household budget constraint on line 4 in (A.1) incorporates the fact that the contracted nominal wage divided by the price level equals the equilibrium marginal product of labor since firms unilaterally determine hours worked in period  $t$ .

## Appendix B Data sources and definitions

In this appendix, I provide the sources of the data collected in Table B.1 below.

**Table B.1: The data set.**

Variables	Sample period	Source
GNP	1960Q1-1997Q4	FRED database, Federal Reserve Bank of St. Louis
GEC	1960Q1-1997Q4	FRED database, Federal Reserve Bank of St. Louis
M1	1959Q1-1997Q4	FRED database, Federal Reserve Bank of St. Louis
POP	1960-1996	OECD Main Economic Indicators
CPI	1959Q1-1997Q4	FRED database, Federal Reserve Bank of St. Louis

Note: All real macroeconomic variables are measured in 1992 billion U.S. dollars. Abbreviations; GNP denotes real (fixed, seasonally adjusted) gross national product; GEC real (chained, seasonally adjusted) government consumption and investment; M1 (not seasonally adjusted) nominal money supply 1; CPI (not seasonally adjusted) consumer price index; POP average U.S. population (for 1997, POP is set equal to average gross growth rate times the value for 1996).

The transformations made to generate the variables used in Tables 1a and 1b are displayed in Table B.2.

**Table B.2: Generation of composite quarterly data series.**

Variable	Sample period	Calculation formula
$\ln Y$	1960Q1-1997Q1	$\ln(\text{GNP}/\text{POP})$
$\mu$	1960Q1-1997Q4	$\ln(\text{M1}_t/\text{M1}_{t-4})$
$\pi$	1960Q1-1997Q4	$\ln(\text{CPI}_t/\text{CPI}_{t-4})$
$\ln G$	1960Q1-1997Q4	$\ln(\text{GEC}/\text{POP})$

Note: To get measures of  $\ln Y - \ln Y^*$ ,  $\ln G$  and  $\pi - \pi^*$ ,  $\ln Y$ ,  $\ln G$  and  $\pi$  are then subject to Hodrick-Prescott filtering with the smoothness coefficient  $\lambda$  set to 1600.

To compute measures of the ratio of government expenditures to output and the growth rate in nominal money supply in steady state,  $\bar{g}$  and  $\bar{\mu}$  respectively, I computed the sums  $\frac{1}{152} \sum_{t=1960Q1}^{1997Q4} (\text{GEC}_t/\text{GNP}_t)$  and  $\frac{1}{152} \sum_{t=1960Q1}^{1997Q4} (1 + \mu_t)^{\frac{1}{4}} - 1$ .

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