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## Monetary Policy with Incomplete Exchange Rate Pass-Through

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# Monetary Policy with Incomplete Exchange Rate Pass-Through

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## Abstract

The central bank's optimal reaction to foreign and domestic shocks is analyzed in an inflation targeting model allowing for incomplete exchange rate pass-through. Limited pass-through is incorporated through nominal rigidities in an aggregate supply-aggregate demand model derived from some microfoundations. Three main results are obtained. First, the results suggest that the interest rate response to foreign shocks is smaller when pass-through is low. Second, the inflation–output variability trade-off becomes more favourable as pass-through decreases. Third, lower pass-through, that is larger nominal rigidity, leads to higher exchange rate volatility. With exogenous nominal price stickiness, part of the required relative price adjustment is provided through larger movements in the endogenously determined exchange rate.

**Keywords:** Exchange rate pass-through, exchange rate volatility, inflation targeting, monetary policy, small open economy

**JEL:** E52, E58, F41

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# 1 Introduction

In order for a central bank to conduct monetary policy efficiently, and to respond adequately to different shocks, it is absolutely essential to understand the workings of the economy and the transmission mechanisms of monetary policy. In a small open economy, the exchange rate implies an additional transmission channel for monetary policy apart from the standard aggregate demand channel. Consumer price (CPI) inflation is affected directly by changes in the exchange rate through the effect on import prices. Inflation is also affected indirectly through aggregate demand. Exchange rate changes typically affect the relative price between domestic and foreign goods, thereby influencing aggregate demand. Aggregate demand in turn affects inflation through the aggregate supply (or Phillips curve) relation. In an open economy, inflation is thus greatly influenced by how its determinants adjust to exchange rate movements.

Prior work on open economy inflation targeting models has predominantly included open economy aspects by incorporating a foreign good, and thereby the real exchange rate, based on the assumption of a complete and immediate effect of exchange rate movements on import prices (see e.g. Galí and Monacelli (1999), McCallum and Nelson (1999), and Svensson (2000)).<sup>1</sup> However, the empirical evidence for, large and small, open economies seems to suggest that there are systematic deviations from the law of one price, and that the exchange rate pass-through is incomplete both for export and import prices (see e.g. Adolfson (2001), Alexius and Vredin (1999), and Naug and Nymoén (1996)).

Consider a foreign firm selling goods to the domestic market and setting its price in the domestic (buyer's) currency. If prices denoted in domestic currency are sticky, as a consequence of firms facing costs of changing prices, the domestic currency (import) price will not be fully altered even if exchange rate changes affect marginal cost. This implies that import prices do not move immediately and in a one-to-one relation with the exchange rate (i.e. incomplete exchange rate pass-through).<sup>2</sup> Nominal rigidities thus imply that exchange rate movements have a minor immediate effect on consumer price inflation. In addition, nominal rigidities imply that expectations about future exchange rates as well as expectations about future inflation are important for the inflation–output relation.

Given that the exchange rate pass-through is allowed to be incomplete, the effect of exchange rate movements on CPI inflation is expected to be more limited in the short run but prolonged. In this case,

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<sup>1</sup> An exception is Monacelli (1999) who permits local importers to price discriminate, thereby allowing an incomplete pass-through. Leitemo (2000) obtains a limited and gradual pass-through via an error correction mechanism for the import prices, making them adjust sluggishly to exchange rate fluctuations. Also Batini and Haldane (1999), and Bharucha and Kent (1998) allow for a limited pass-through but only as sensitivity checks to their full-pass through models.

<sup>2</sup> A related cause for incomplete exchange rate pass-through is pricing to market, which implies deliberate price discrimination, where the destination specific markup may be adjusted to absorb part of an exchange rate movement. This does, however, require either a specific functional form of the demand curve (i.e. less convex than the constant elasticity case) or strategic pricing, which is not analyzed in this paper.

a pass-through adjusted aggregate supply relation will perhaps imply a different optimal monetary policy response to a shock, compared to a full pass-through Phillips curve. Monacelli's (1999) results suggest that the performance of monetary policy (in terms of inflation stabilization) can be improved by a simple instrument rule including a direct feedback from the nominal exchange rate (compared to a rule where the interest rate reacts solely to inflation and output). An explicit interest rate response to changes in the exchange rate reduces the volatility of inflation because of the direct control of the exchange rate channel feeding into inflation.

The purpose of this paper is to examine the monetary policy implications of allowing an incomplete pass-through in an inflation targeting framework. The optimal policy responses to both domestic and foreign shocks are analyzed under various assumptions about the degree of pass-through. The optimal policy reaction is directly derived from the central bank's loss function, in contrast to a Taylor rule (i.e. a simple instrumental rule linking the nominal interest to, for example, inflation and output). Moreover, the concept of exchange rate pass-through is studied in detail. A microfounded small open economy aggregate supply–aggregate demand model, adjusted for incomplete (and gradual) exchange rate pass-through, is derived and used in the analysis. The paper deals with questions such as; is the optimal policy response dependent on the degree of pass-through? How is the trade-off between inflation and output variability affected by the degree of pass-through? Further, how is the degree of pass-through and exchange rate volatility related?

Three main results are obtained in the paper. First, the results show how the monetary policy response, to both foreign and domestic shocks, depends on the degree of pass-through. In contrast to the complete pass-through case, the exchange rate channel has less impact when pass-through is low, which for example implies that foreign shocks require smaller interest rate adjustments. Second, incomplete pass-through implies less conflict between inflation and output variability because of the lower exposure to, exogenous as well as policy induced, exchange rate fluctuations. This moves the trade-off frontier closer to the origin as pass-through decreases. Third, the results suggest that the volatility of the nominal exchange rate increases as pass-through decreases. A low pass-through is, in this model, induced by a large exogenous import price stickiness, which in turn implies that prices can not costlessly absorb a country specific shock. The required relative price adjustment is therefore generated through larger movements in the endogenously determined exchange rate. Lastly, the exchange rate pass-through is, in turn, varying with the degree of shock persistence. Transitory movements influence expectations about future prices and exchange rates less, and accordingly yield lower pass-through.

In Section 2 the aggregate supply–aggregate demand model, adjusted for incomplete pass-through, is derived and parameterized, and the central bank's loss function is set up. Section 3 contains the

optimal policy responses, and its implications, to various foreign and domestic shocks under different degrees of pass-through. Conclusions are provided in Section 4.

## 2 The model

The theoretical specification is a forward-looking aggregate supply–aggregate demand model modified to allow for incomplete exchange rate pass-through. To determine the effect of incomplete pass-through in this small open economy, the commodity market is primarily considered. Supply and demand relations are explicitly derived from the agents’ optimization problems. A local currency pricing framework is considered, along the lines of for example Betts and Devereux (2000), where the elasticity of demand is assumed to be constant (and thus independent of the exchange rate, the competitors’ prices and other underlying conditions of competition).<sup>3</sup> Markets are segmented such that different prices can be charged in different markets. However, the constant elastic demand implies that there will not be any ‘genuine’ pricing to market, or deliberate price discrimination, in terms of a varying markup that responds to exchange rate changes (see Bergin and Feenstra (1999)). Incomplete pass-through occurs in this paper due to nominal rigidities, arising from convex costs of adjusting prices (Rotemberg (1982)), and there are no long-run deviations from the law of one price.<sup>4</sup>

### 2.1 Aggregate supply

Consider an open economy with consumption of two different types of goods; domestic ( $C_t^D$ ) and foreign import goods ( $C_t^M$ ) supplied by domestic and foreign producers, respectively. The domestic economy is assumed to be small such that conditions in the rest of the world (the foreign economy) are exogenously given. The producers sell their goods in both the domestic and foreign markets. The foreign market outcome is however not explicitly modeled why the setting, in all its essentials, is a one market–two goods framework.

The two product categories are imperfect substitutes, thus rendering the domestic and foreign producers some market power when setting their prices (i.e. each representative producer supplies a differentiated product  $i \in \{D, M\}$ ). However, because of physical costs of changing the price (menu

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<sup>3</sup> Note that Friberg (1998) shows that a sufficient condition for an exporter to set price in the local currency under uncertainty is that demand is not too convex in the local currency price (i.e. less convex than the constant elasticity case). Most studies nevertheless disregard this problem and assume a local currency pricing framework in combination with the constant elastic substitution function (CES) for convenience reasons (see e.g. Betts and Devereux (2000), Devereux and Engel (1998), Monacelli (1999), and Tille (1998)).

<sup>4</sup> The term pricing to market has been commonly used also in such a setting, somewhat misleadingly since that framework only produces an exchange rate driven price discrimination in the presence of nominal rigidities. On the other hand, different degrees of nominal price stickiness across destination markets do imply deviations from the law of one price (as well as inducing different degrees of pass-through). Once the producers are free to adjust their prices, the law of one price will though be re-established. For a discussion of these matters, and a survey of recent research on open economy dynamic general equilibrium models, see Lane (1999).

costs) and the producers' concern for a stable price path (reputation), due to imperfectly informed consumers and brand-switching costs in the domestic market, there is a negative effect of changing domestic currency prices. It is assumed that these costs of adjusting the price are quadratic.<sup>5</sup> The notation throughout the paper is as follows, lower case letters represent logarithmic values, a hat denotes flexible prices (i.e. prices charged in the absence of adjustment costs), a superindex denotes whether it is domestic or imported goods that are considered, and variables belonging to the foreign market are represented by an asterisk. An asterisk thus labels a price denoted in foreign currency. The monopolistic producers minimize the cost of being away from the optimal price chosen in the absence of adjustment costs:<sup>6</sup>

$$(1) \quad \min_{\{p_{t+s}^i\}_{s=0}^{\infty}} E_t \sum_{s=0}^{\infty} \mathbf{b}^s \left[ (p_{t+s}^i - \hat{p}_{t+s}^i)^2 + \mathbf{g}_i (p_{t+s}^i - p_{t+s-1}^i)^2 \right] \quad i \in \{D, M\},$$

where  $E_t$  denotes the rational expectations as of period  $t$ ,  $\mathbf{b}$  is a discount factor, and  $p_t^i$  is the price (denoted in the buyer's currency) of good  $i$  while  $\hat{p}_t^i$  is the equilibrium price charged in the absence of adjustment costs.  $\mathbf{g}$  is a parameter measuring the ratio of costs of changing the price to costs of being away from the equilibrium price, such that  $\mathbf{g}$  equal to zero implies a fully flexible environment. The first order condition yields

$$(2) \quad \mathbf{p}_t^i = \mathbf{b} E_t \mathbf{p}_{t+1}^i + \frac{1}{\mathbf{g}_i} (\hat{p}_t^i - p_t^i),$$

where  $\mathbf{p}_t^i = p_t^i - p_{t-1}^i$  denotes domestic currency inflation of good  $i$ . This specification, with convex costs of adjustment, will thus lead to gradual changes in the individual (and aggregate) prices implying that the producers alter the price charged in this (and every) period, in the direction of the expected optimal price in future periods.<sup>7</sup>

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<sup>5</sup> The specific adjustment-cost technology is not modelled explicitly, why these costs can arise in any currency. That the price stickiness occurs in the currency of the buyer is however a necessary assumption for obtaining an incomplete pass-through.

<sup>6</sup> This is the dual problem of maximizing the present discounted profits in the absence of adjustment costs, subtracting the cost of deviating from this equilibrium price and subtracting the cost of changing prices:

$$\max_{\{p_{t+s}^i\}_{s=0}^{\infty}} E_t \sum_{s=0}^{\infty} \mathbf{b}^s \left[ \Pi(\hat{p}_{t+s}^i) - (p_{t+s}^i - \hat{p}_{t+s}^i)^2 - \mathbf{g}_i (p_{t+s}^i - p_{t+s-1}^i)^2 \right].$$

In reality the adjustment costs ( $\mathbf{g}_i$ ) are presumably endogenously related to, for example, the functional form of the demand curve, and the rate of inflation. However, the adjustment costs are assumed to be constant to make the equilibrium tractable. For the underlying structure to the approximation of the producer's optimization problem, see Rotemberg (1982).

<sup>7</sup> The purpose of the nominal rigidities in this paper, apart from incorporating an incomplete exchange rate pass-through, is to introduce forward-looking behaviour in the aggregate supply relation. Using the Calvo (1983) formulation (see e.g. Monacelli (1999) and Svensson (2000)) renders staggeredness in the *individual* prices, in contrast to Rotemberg (1982). The two formulations though yield a similar behaviour, or path, of *aggregate* prices (Roberts (1995)). Besides, the Calvo

### 2.1.1 Imported products

The import good's price charged in the domestic market is here established in two steps. First, the flexible import price, in the absence of any nominal rigidities, is determined. Secondly, this flexible price is combined with the adjustment costs the foreign producer is actually facing, and accordingly the optimal 'sticky' import price is resolved.

The imported foreign product's equilibrium flexible price ( $\hat{p}_t^M$ ) is, by assumption, the price charged by a profit maximizing foreign firm, in an imperfectly competitive framework, setting the price in the buyer's currency. The consumers' aggregate demand follows a constant elasticity of substitution (CES) function (see Appendix A), such that the foreign producer sets his price as a constant markup over marginal cost (which is assumed to be equal across the producer's two different destination markets). The foreign producer faces the following optimization problem in a flexible price environment:<sup>8</sup>

$$(3) \quad \begin{aligned} & \max_{\hat{P}_t^M, \hat{P}_t^{M*}} \hat{P}_t^M C_t^M + \hat{P}_t^{M*} E_t C_t^{M*} - H_M(C_t^M + C_t^{M*}, P_t^{Z*}) E_t \\ & \text{s.t.} \quad C_t^M + C_t^{M*} = \mathbf{k}_M \left( \frac{\hat{P}_t^M}{\hat{P}_t} \right)^{-h} C_t + \left( \frac{\hat{P}_t^{M*}}{\hat{P}_t^*} \right)^{-h} C_t^*, \end{aligned}$$

where  $\hat{P}_t^M$  (denoted in domestic currency) and  $\hat{P}_t^{M*}$  (denoted in foreign currency) are the prices charged in the domestic and foreign markets, respectively.  $C_t^M$  is the demand for the foreign good in the domestic market, and  $C_t^{M*}$  is the demand for the foreign good in the foreign market.  $E_t$  is the exchange rate (domestic currency per unit of foreign currency),  $H_M$  is the foreign producer's total cost function, and  $P_t^{Z*}$  is the price of inputs (denoted in foreign currency).  $\mathbf{k}_M$  is the domestic import share of consumption,  $h$  is the (positive) constant price elasticity of demand,  $\hat{P}_t$  is the aggregate price index in the domestic market, and  $C_t$  is the aggregate domestic consumption (a star denotes the foreign market counterparts). The foreign economy is large in the respect that the foreign import share is assumed to be negligible in the foreign aggregate price index. This implies that the world market price for domestic import goods is equal to the foreign aggregate price level ( $\hat{P}_t^{M*} = \hat{P}_t^*$ ). The producer's profit maximization yields the following first order condition with respect to the price charged in the domestic market:

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representation has an exogenously given price adjustment-probability, which is fixed and independent of both the size of the deviation from the equilibrium flex price and the size of for example an exchange rate shock.

<sup>8</sup> Whether the producer maximizes profits in his own currency, or in the buyers' currency, does not affect the first order conditions, given the constant elastic demand.

$$(4) \quad \hat{P}_t^M = \left( \frac{\mathbf{h}}{\mathbf{h}-1} \right) \underbrace{MC^*(C_t^M + C_t^{M*}, P_t^{Z*})}_{\hat{p}_t^{M*} = \hat{p}_t^*} E_t,$$

where  $MC^*(.)$  denotes foreign currency marginal cost, and  $\mathbf{h}$ , the price elasticity of demand, determines the constant markup. The producer's equilibrium price in the domestic market is simply the price charged in the foreign market ( $\hat{P}_t^*$ ) corrected for the exchange rate ( $E_t$ ), which in logarithms can be expressed as  $\hat{p}_t^M = \hat{p}_t^* + e_t$  (where  $\hat{p}_t^*$  captures the marginal cost (denoted in foreign currency) and the constant markup). Given that the foreign producer faces identical demand elasticities across the two destinations he is selling to, there are no incentives for the exporter to deviate from the law of one price (i.e. no deliberate price discrimination) although markets are segmented. This implies that prices (denoted in the producer's own currency), in a flexible price setting, would be the same in both markets, and maintained equal irrespectively of any exchange rate changes.<sup>9</sup> The markup is constant, which in this case would imply that any exchange rate movement is completely reflected in the local currency price, that is a complete pass-through. Hence, with entirely flexible prices, the long run inflation rate of import goods would be the imported foreign inflation rate corrected for changes in the nominal exchange rate,  $\hat{p}_t^M = \hat{p}_t^* + (e_t - e_{t-1})$ . However, due to nominal rigidities in the domestic market, there may be short run deviations from the full pass-through equilibrium price, and deviations from the law of one price could occur.

The foreign producer faces price adjustment costs in the domestic market, such that the price actually charged differs from the price that would prevail in a flexible price setting (i.e.  $p_t^M \neq \hat{p}_t^M$ ).<sup>10</sup> Inserting the equilibrium price ( $\hat{p}_t^M = \hat{p}_t^* + e_t$ ) into the adjustment cost minimization in equation (2) yields the following relation:

$$(5) \quad \mathbf{p}_t^M = \mathbf{b} E_t \mathbf{p}_{t+1}^M + \frac{1}{\mathbf{g}_M} (\hat{p}_t^* + e_t - p_t^M),$$

where  $\mathbf{p}_t^M = p_t^M - p_{t-1}^M$  denotes domestic currency inflation of import goods. The present price change of imports is dependent on the expectation about future import price changes, and the contemporaneous difference between the foreign producer's equilibrium price (in the absence of price rigidities) and the price actually charged. Exchange rate movements will, due to the costly price

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<sup>9</sup> An identical foreign currency price across destinations could also arise if the product is homogenous without any possibilities for price discrimination, such that the world market price is taken as given.

<sup>10</sup> For simplicity, the price setting in the foreign market is assumed to be completely flexible (i.e.  $p_t^* = \hat{p}_t^*$ ). The foreign producer is thus, in contrast to the domestic market, able to charge the equilibrium flex price there.



adjustment, create a wedge between the price charged in the domestic market ( $p_t^M$ ) and the price charged in the foreign market ( $\hat{p}_t^*$ ). Consequently, the last term captures deviations from the law of one price. The degree of pass-through is highly dependent on  $\mathbf{g}_M$  (i.e. the ratio of costs of changing the price to costs of staying away from the equilibrium price). As  $\mathbf{g}_M$  increases, i.e. a greater nominal rigidity, pass-through decreases. Price discrimination is thus generated by these nominal rigidities. In this framework price stickiness can render price differentials across destinations, and deviations from the law of one price, both ex ante and ex post an exchange rate change, depending on whether the change is expected to be permanent or transitory, or is entirely unexpected.<sup>11</sup>

### 2.1.2 Domestically produced products

Now consider the domestically produced goods sold in the domestic and foreign markets. Given the constant markup, implied by the CES function, the exchange rate will affect the price charged in the domestic market only through its effect on the domestic producer's marginal costs (i.e. via its effect on imported intermediate inputs). The domestic producer faces an optimization problem equivalent to the foreign producer's (see also equation (A8) in Appendix A), which yields a standard monopolist's first order condition with respect to the price charged in the domestic market:

$$(6) \quad \hat{P}_t^D = \left( \frac{\mathbf{h}}{\mathbf{h}-1} \right) MC(Y_t, P_t^Z),$$

where  $\mathbf{h}$  is the (positive) constant price elasticity of demand and the elasticity of substitution between domestic and foreign goods.  $MC(\cdot)$  is marginal cost,  $Y_t = C_t^D + C_t^{D*}$  is the demand for domestic products (domestic and foreign demand), and  $P_t^Z$  is the price of inputs (denoted in the domestic currency). There are decreasing returns to scale such that the cost function is convex in quantity produced. Furthermore, marginal costs are also affected by exchange rate movements, via their effect on the price of imported inputs.

The price of domestic products on the foreign market will just be the price on the domestic market corrected for the exchange rate ( $P_t^{D*} = P_t^D / E_t$ ), given identical demand elasticities (see equation

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<sup>11</sup> The producer sets his price based on exchange rate expectations, due to the quadratic adjustment costs, which can be seen explicitly by solving equation (5) forward (following Rotemberg (1982)):

$$p_t^M = r_1 p_{t-1}^M + 1/(r_2 \mathbf{g}_M \mathbf{b}) E_t \sum_{s=0}^{\infty} (1/r_2)^s (\hat{p}_{t+s}^* + e_{t+s}),$$

where  $r_1$  and  $r_2$  are the stable and unstable roots, respectively.

(A8) in Appendix A). For simplicity, the domestic producer is thus assumed to follow the law of one price such that there is a full pass-through to her export market.<sup>12</sup>

Taking logarithms of equation (6), and using a first order Taylor approximation around steady-state, yields the following expression for the equilibrium flexible price:

$$(7) \quad \hat{p}_t^D = \mathbf{x}_y y_t + \mathbf{x}_z p_t^Z,$$

where  $y_t$  is total demand for domestically produced goods, and  $\mathbf{x}_x$  is a constant measuring how marginal cost is affected by variable  $X$  in steady-state. Recognizing that the input price partly consists of imported products as well as domestically produced goods (see equation A7), it evolves according to  $p_t^Z = (1 - \mathbf{k}_W) p_t^D + \mathbf{k}_W p_t^M$ , where  $p_t^D$  is the price of domestically produced goods,  $p_t^M$  is the price of imported products (denoted in the buyer's currency), and  $\mathbf{k}_W$  is the share of imported inputs. Furthermore, assume for simplicity that the input prices enter multiplicatively in the cost function (i.e.  $\mathbf{x}_z = 1$ ). Assuming that the domestic producers also face quadratic price adjustment costs, equation (2) holds. Using the above, and inserting (7) into equation (2), yields

$$(8) \quad p_t^D = \mathbf{b} E_t p_{t+1}^D + \frac{1}{\mathbf{g}_D} (\mathbf{x}_y y_t + \mathbf{k}_W (p_t^M - p_t^D)) + \mathbf{e}_t^P,$$

where inflation of domestically produced goods ( $p_t^D$ ) responds positively to both aggregate output and the relative price of imports ( $p_t^M - p_t^D$ ), the latter which can be interpreted as a direct real exchange rate effect on domestic inflation.<sup>13</sup> Recall, however, that the foreign price ( $p_t^M$ ) is subject to a limited pass-through, via the nominal rigidities in the foreign producer's optimization problem so that  $p_t^M \neq \hat{p}_t^* + e_t$  in the short run. As mentioned above, an increase in the price of import goods, induced by an exchange rate depreciation, feeds back into domestic prices directly through the marginal cost (originating in intermediate foreign inputs) but also indirectly through the effect of relative price changes on aggregate demand. In addition, expectations about future inflation will be modified when the exchange rate changes. An explicit shock to domestic inflation,  $\mathbf{e}_t^P$ , has been added as an iid zero mean disturbance. This supply shock enters through shocks to the marginal cost, and consists of a productivity disturbance or a cost push shock (e.g. in terms of wages).

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<sup>12</sup> Differing rates of nominal price stickiness across the domestic producer's two markets would, analogous to the foreign producer, render an incomplete exchange rate pass-through also in domestic exports. It is straightforward to extend the model to allow for this.

<sup>13</sup> See e.g. Hallsten (1999) for empirical evidence, where both the output gap and the real exchange rate enters positively in the supply relation, however with a lag.

The foreign and domestic producers may face different costs of changing their prices such that  $g \neq g_M$ , for example originating in some bias for one good over the other. Differing nominal price stickiness implies that the effect of an exchange rate movement on the relative price between foreign and domestic goods could be even stronger. Hence, this could possibly render consequences for the optimal policy as well as the inflation–output variability trade-off (see e.g. Walsh (1999)). Moreover, the exposure to the transmission channels of monetary policy differs between the domestic and foreign producers. In the model used in this paper, domestic inflation responds to interest rate changes through the aggregate demand channel and (to some degree) through the exchange rate channel, while inflation of imported products only responds to exchange rate changes (see equations (8) and (5), respectively).<sup>14</sup>

Total, or CPI, inflation is by assumption composed of a convex combination of domestic inflation ( $p_t^D$ ) and import goods' inflation ( $p_t^M$ ), following  $p_t = (1 - k_M)p_t^D + k_M p_t^M$ , where  $k_M$  denotes the import share of aggregate consumption. This is an approximation motivated by the underlying CES function, and a log-linearization of the corresponding aggregate price index (see Appendix A). Combining equation (5) and (8) yields

$$(9) \quad p_t = \mathbf{b} E_t p_{t+1} + \mathbf{a}_Y y_t + \mathbf{a}_W (p_t^M - p_t^D) + \mathbf{a}_M (\hat{p}_t^* + e_t - p_t^M) + (1 - k_M) e_t^P,$$

where  $\mathbf{a}_Y = (1 - k_M) \mathbf{x}_y / g_D$ ,  $\mathbf{a}_W = (1 - k_M) \mathbf{k}_W / g_D$ , and  $\mathbf{a}_M = \mathbf{k}_M / g_M$ . Note that  $e_t^P$  only captures disturbances specific to the domestic market that do not originate in shocks to the inflation of import goods. The underlying foreign inflation shock, and disturbances to the exchange rate, will however feed into the aggregate supply relation implicitly, through the variables  $\hat{p}_t^*$  and  $e_t$ , respectively.

The short-run deviations from the law of one price, captured in the last term ( $\hat{p}_t^* + e_t - p_t^M$ ), is the major difference between the aggregate supply relation in equation (9) and a standard open economy supply relation (see e.g. Svensson (2000)). As a result exchange rate movements have an incomplete pass-through effect on total inflation.<sup>15</sup>

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<sup>14</sup> In the case of a 'genuine' pricing to market model, where the markup endogenously responds to exchange rate movements, there would be an effect of aggregate demand also on the price of imports (see e.g. Naug and Nymoen (1996)).

<sup>15</sup> A complete pass-through supply relation is retrieved by assuming a fully flexible environment,  $g_M = 0$ , such that  $p_t^M = \hat{p}_t^* + e_t$  holds also in the short run, which implies that the last term in equation (9) would disappear. In addition, the last term seemingly vanishes if the domestic currency inflation of import goods does not contribute directly to total (CPI) inflation (i.e.  $k_M = 0$ ). However, if import goods enter domestic production as intermediate inputs ( $k_W > 0$ ), the price of imports, which is subject to a limited pass-through, affects marginal costs and consequently also domestic inflation. The difference between the equilibrium price and the price actually charged (i.e.  $\hat{p}_t^* + e_t - p_t^M$ ) is still fundamental for the foreign producer's price setting (see equation (5)), why the corresponding aggregate supply curve accordingly must be different from a full pass-through relation.

## 2.2 Aggregate demand

Aggregate domestic consumption ( $c_t$ ) consists of consumption of domestically produced goods ( $c_t^D$ ) and consumption of imported goods ( $c_t^M$ ) following a CES function (see equation (A1) in Appendix A). From the CES function follows that the domestic consumption of domestic goods must be given by (see also equation (A3) in Appendix A)

$$(10) \quad c_t^D = c_t - \mathbf{h}(p_t^D - p_t) = c_t + \mathbf{h}\mathbf{k}_M(p_t^M - p_t^D),$$

using  $p_t = (1 - \mathbf{k}_M)p_t^D + \mathbf{k}_M p_t^M$  (which is a log-linearization of equation (A2) in Appendix A).

The representative domestic consumer's intertemporal utility function is assumed to take the form<sup>16</sup>

$$(11) \quad \begin{aligned} & \max_{\{C_{t+s}, B_{t+s}, B_{t+s}^*\}_{s=0}^{\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} \mathbf{b}^s U(C_{t+s}^D, C_{t+s}^M) = \mathbb{E}_t \sum_{s=0}^{\infty} \mathbf{b}^s \frac{(C_{t+s})^{1-\frac{1}{\mathbf{s}}} - 1}{1-\frac{1}{\mathbf{s}}} \\ \text{s.t.} \quad & P_t C_t + \frac{1}{1+I_t} B_t + \frac{1}{(1+I_t^*)(1+f_t)} B_t^* E_t = \Pi(Y_t, P_t^D) + B_{t-1} + B_{t-1}^* E_t, \end{aligned}$$

where  $C_t$  is the CES aggregate of consumption of domestic and imported goods.  $\mathbf{s}$  is the constant intertemporal elasticity of substitution,  $B_t$  denotes (end of period  $t$ ) bond holdings denominated in domestic currency units, and  $I_t$  is the nominal domestic interest rate implying that  $1/(1+I_t)$  is the price of a domestic bond.  $B_t^*$  represents (the domestic consumers') foreign currency bond holdings, which are sold to a risk-adjusted price,  $1/[(1+I_t^*)(1+f_t)]$ , where  $f_t$  is a risk premium that will reflect temporary deviations from uncovered interest rate parity (see McCallum and Nelson (1999)), and  $I_t^*$  is the nominal foreign interest rate.  $\Pi(Y_t, P_t^D)$  are profits from production with  $Y_t$  denoting aggregate output (which is equal to total demand for domestically produced goods). Government transfers and money holdings are disregarded, and the domestic producers are assumed to follow the law of one price on their export markets implying a complete pass-through.<sup>17</sup> The first order condition with respect to consumption implies an Euler equation of the form

$$(12) \quad c_t = \mathbb{E}_t c_{t+1} - \mathbf{s}(i_t - \mathbb{E}_t p_{t+1}),$$

where  $i_t$  is the (log) short term nominal interest rate, which is assumed to be the monetary policy instrument. Inserting (10) into (12) yields

<sup>16</sup> Assuming the function to be separable in consumption and leisure, the marginal utility of consumption is only dependent on the level of consumption, thereby making any disutility of production (or labor) superfluous for the purposes here.

<sup>17</sup> Changing any of these assumptions would not alter the consumers' Euler equation.

$$(13) \quad c_t^D = E_t c_{t+1}^D - \mathbf{h} \mathbf{k}_M (E_t \mathbf{p}_{t+1}^M - E_t \mathbf{p}_{t+1}^D) - \mathbf{s} (i_t - E_t \mathbf{p}_{t+1}).$$

Since the domestic goods are traded, aggregate demand ( $Y_t$ ) for domestically produced goods is given by the sum of domestic demand ( $C_t^D$ ) and foreign demand ( $C_t^{D*}$ ). A log-linear approximation around steady-state yields;  $y_t = (1 - \mathbf{k}_D)c_t^D + \mathbf{k}_D c_t^{D*}$ , where  $\mathbf{k}_D$  is the export (steady-state) share of total demand for the domestic good. The foreign demand for domestic goods follows,  $c_t^{D*} = -\mathbf{h}(p_t^D - e_t - \hat{p}_t^*) + a_y^* y_t^*$ , where  $a_y^*$  denotes the income elasticity of foreign consumption (see also equation (A8) in Appendix A). Inserting this and equation (13) into the aggregate demand relation implies

$$(14) \quad \begin{aligned} y_t &= E_t y_{t+1} + (1 - \mathbf{k}_D) \left( -\mathbf{k}_M \mathbf{h} E_t (\mathbf{p}_{t+1}^M - \mathbf{p}_{t+1}^D) - \mathbf{s} (i_t - E_t \mathbf{p}_{t+1}) \right) - \mathbf{k}_D (E_t c_{t+1}^{D*} - c_t^{D*}) + \mathbf{e}_t^y \\ &= E_t y_{t+1} - \mathbf{b}_q E_t (\mathbf{p}_{t+1}^M - \mathbf{p}_{t+1}^D) - \mathbf{b}_i (i_t - E_t \mathbf{p}_{t+1}) + \mathbf{b}_e (E_t \mathbf{p}_{t+1}^D - (E_t e_{t+1} - e_t) - E_t \hat{p}_{t+1}^*) \\ &\quad - \mathbf{b}_y^* (E_t y_{t+1}^* - y_t^*) + \mathbf{e}_t^y, \end{aligned}$$

where  $\mathbf{b}_q = \mathbf{k}_M \mathbf{h} (1 - \mathbf{k}_D)$ ,  $\mathbf{b}_i = \mathbf{s} (1 - \mathbf{k}_D)$ ,  $\mathbf{b}_e = \mathbf{k}_D \mathbf{h}$ , and  $\mathbf{b}_y^* = \mathbf{k}_D a_y^*$ .  $\mathbf{e}_t^y$  has been added as an iid zero mean disturbance to domestic demand. This could be motivated by a shock to domestic preferences that shifts aggregate demand. The difference between this demand relation and a full pass-through demand curve is the implicit deviation from the law of one price (i.e.  $p_t^M \neq \hat{p}_t^* + e_t$ ), which makes the relative price of imports ( $p_t^M - p_t^D$ ) diverge from the (inverse) relative price of exports ( $p_t^D - e_t - \hat{p}_t^*$ ). Due to different degrees of price stickiness across markets, an exchange rate change affects demand differently in the domestic and foreign markets. At first glance, some of the coefficients' signs might appear surprising. For example, a future expected depreciation affects today's output negatively. However, some intertemporal substitution is at hand, but also expectations about future output and about future inflation rates change why the effect on today's output is ambiguous. Recall furthermore that the complete model consists of a simultaneous system of equations why analyzing separate coefficients in one equation might be of limited interest.<sup>18</sup>

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<sup>18</sup> Note that the relative price *level* affects the intratemporal allocation between consumption of imports and domestic goods, while the *change* in relative price affects the intertemporal consumption decision (see equations (10) and (13), respectively). However, observe additionally that all difference terms disappear when solving equation (14) forward;

$$y_t = \mathbf{b}_q (p_t^M - p_t^D) - \mathbf{b}_i \sum_{s=0}^{\infty} E_t (i_{t+s} - p_{t+s+1}) - \mathbf{b}_e (p_t^D - e_t - \hat{p}_t^*) + \mathbf{b}_y^* y_t^* + \sum_{s=0}^{\infty} E_t \mathbf{e}_{t+s}^y,$$

using the appropriate transversality conditions.

### 2.3 Parity and foreign conditions

Combining the first order conditions for domestic and foreign currency bond holdings (see equation (11)), assuming perfect capital mobility, implies that the exchange rate fulfills a modified uncovered interest rate parity (UIP) condition:

$$(15) \quad i_t - i_t^* = E_t e_{t+1} - e_t + \mathbf{f}_t,$$

where  $\mathbf{f}_t$  is the risk premium that creates deviations from UIP. Since anything that influences this interest rate differential also affects the exchange rate (such as foreign interest rate disturbances, e.g. originating in foreign inflation and output shocks, or disturbances to the domestic interest rate), it is hard to distinguish a ‘genuine’ exchange rate shock that is not a reaction to some other underlying disturbance in the model. A shock to the risk premium could however be interpreted as capturing an autonomous disturbance to expectations about future exchange rate changes (e.g. due to some exogenous change in perceived risk) resulting in a ‘pure’ exchange rate shock.

Moreover, the domestic economy is assumed to be small relative to the rest of the world so that foreign output (working as a demand shifter), and inflation of foreign products (that is the change in marginal cost; see equation (4)) are taken as exogenously given. These variables are assumed to follow AR(1) processes:

$$(16) \quad y_{t+1}^* = \mathbf{r}_y^* y_t^* + u_{t+1}^{y^*},$$

$$(17) \quad \hat{\mathbf{p}}_{t+1}^* = \mathbf{r}_p^* \hat{\mathbf{p}}_t^* + u_{t+1}^{p^*},$$

where the coefficients are non-negative and less than unity. The shocks are uncorrelated zero mean iid disturbances with variance  $\mathbf{s}_{y^*}^2$  and  $\mathbf{s}_{p^*}^2$ , respectively. The foreign interest rate is assumed to follow a simple Taylor rule with some persistence added, that is a linear function of foreign inflation, output and the lagged interest rate (see e.g. Clarida et al. (1998)):

$$(18) \quad i_t^* = (1 - \mathbf{r}_i^*) (b_p^* \hat{\mathbf{p}}_t^* + b_y^* y_t^*) + \mathbf{r}_i^* i_{t-1}^* + u_t^{i^*},$$

where the coefficients are constant and positive, and  $\mathbf{r}_i^*$  specifies the degree of interest rate smoothing.  $u_t^{i^*}$  is a zero mean iid shock, with variance  $\mathbf{s}_{i^*}^2$ , capturing foreign monetary policy disturbances.

The exogenous shocks, to domestic inflation and output (added to equations (8) and (14)) and to the exchange rate (i.e. the risk premium shock in equation (15)), are assumed to follow

$$(19a) \quad \mathbf{e}_{t+1}^p = \mathbf{t}_p \mathbf{e}_t^p + \mathbf{u}_{t+1}^p,$$

$$(19b) \quad \mathbf{e}_{t+1}^y = \mathbf{t}_y \mathbf{e}_t^y + \mathbf{u}_{t+1}^y,$$

$$(19c) \quad \mathbf{f}_{t+1} = \mathbf{t}_f \mathbf{f}_t + \mathbf{u}_{t+1}^f,$$

where the disturbances are zero mean iid shocks with variance  $\mathbf{s}_p^2$ ,  $\mathbf{s}_y^2$ , and  $\mathbf{s}_f^2$ , respectively. All coefficients are positive and less than one. The shocks that enter the economy are thus not permanent, but persistent. Since there are no backward looking components in the aggregate supply or the aggregate demand curves, the persistence in inflation and output is thus entirely due to the serially correlated exogenous shocks.<sup>19</sup>

## 2.4 The central bank's loss function

The central bank's objective is to stabilize both inflation and output (as in e.g. Svensson (2000)). It chooses a path for the policy instrument, the short-term interest rate  $i_t$ , in order to minimize its intertemporal loss function, which is quadratic in the deviations of inflation and output from their constant targets (here normalized to zero for simplicity).<sup>20</sup> The central bank assumes the output target to be equal to the natural output level such that there is no inflation bias (i.e. no deviation of average inflation from the constant inflation target). The central bank's optimization problem is

$$(20a) \quad \min_{\{i_{t+s}\}_{s=0}^{\infty}} E_t \sum_{s=0}^{\infty} \mathbf{d}^s L_{t+s},$$

$$(20b) \quad L_t = \left[ \mathbf{p}_t^2 + \mathbf{I} (y_t)^2 + \mathbf{n}_i (i_t - i_{t-1})^2 \right],$$

where  $L_t$  is the period loss function.  $\mathbf{I}$  measures the relative weight on output stabilization,  $\mathbf{n}_i$  corresponds to the weight on interest rate smoothing, and  $\mathbf{d}$  is a discount factor.  $\mathbf{I} > 0$  implies that the central bank does not immediately force the inflation rate back to the long run target after a given

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<sup>19</sup> Endogenous persistence in the inflation of foreign and domestic goods could though be justified by simply assigning an adjustment cost to the speed of changing price (i.e. including a cost of changing inflation in equation (1)). However, Hallsten (1999) shows that this backward looking component in the inflation relation lacks significance for Swedish data. For the aggregate demand relation, the consumer's utility function does not motivate any endogenous persistence without an assumption of either some sort of adjustment costs or habit formation (see e.g. Svensson (2000), and McCallum and Nelson (1999)).

<sup>20</sup> Note that the Rotemberg (1982), as well as the Calvo (1983), pricing framework per se implies that monetary policy can affect the mean of output. However, the objective function of the central bank stabilizes the model. This yields a stationary inflation rate, implying that the policy maker does not influence the average output level.

shock, but adjusts the instrument less and hence gradually brings the inflation rate into line with the targeted level. The higher  $I$  is, the slower is the adjustment of the inflation rate.

Even if the central bank's objective only is to stabilize inflation and output, interest rate smoothing can be optimal due to data and model uncertainty, i.e. measurement errors in the data, and uncertainty about the economic structure and the transmission of monetary policy, respectively, (see e.g. Sack and Wieland (1999)). Another motivation is the central bank's concern for financial stability. Furthermore, with forward-looking behaviour, a gradual and persistent adjustment of the short interest rate induces expectations about future interest rate changes. This implies a greater effect on the long rates and thereby also yielding a substantial impact on aggregate demand, without sizeable short interest rate variability (Woodford (1999)). In that case, interest rate smoothing ( $n_i > 0$ ) can be interpreted as a way to bring the discretionary outcome closer to the outcome under commitment, where expectations of future policy matters.

The state-space representation of the model (i.e. equations (5), (8), (9) and (14)–(19); see also Appendix B) follows

$$(21) \quad \tilde{A}_0 \begin{pmatrix} x_{1,t+1} \\ \mathbb{E}_t x_{2,t+1} \end{pmatrix} = \tilde{A} \begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix} + \tilde{B} i_t + \tilde{\mathbf{u}}_{t+1},$$

$$x_{1,t} = (i_{t-1} \quad y_t^* \quad i_t^* \quad \hat{p}_t^* \quad \mathbf{e}_t^p \quad \mathbf{e}_t^f \quad \mathbf{e}_t^y \quad (p_{t-1}^M - p_{t-1}^D) \quad (\hat{p}_{t-1}^* + e_{t-1} - p_{t-1}^M))',$$

$$x_{2,t} = (y_t \quad \mathbf{p}_t^D \quad \mathbf{p}_t^M \quad \Delta e_t)',$$

$$\tilde{\mathbf{u}}_{t+1} = (0 \quad u_{t+1}^{y^*} \quad u_{t+1}^{i^*} \quad u_{t+1}^{p^*} \quad \mathbf{u}_{t+1}^p \quad \mathbf{u}_{t+1}^f \quad \mathbf{u}_{t+1}^y \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)',$$

where  $x_{1,t}$  is a  $9 \times 1$  vector of predetermined state variables,  $x_{2,t}$  is a  $4 \times 1$  vector of forward-looking variables, and  $\tilde{\mathbf{u}}_{t+1}$  is a  $13 \times 1$  vector of disturbances. This implies that the intertemporal control problem can be expressed as a stochastic linear quadratic regulator problem:

$$(22) \quad J(x_t) = \min_{i_t} \left\{ x_t' \tilde{Q} x_t + \mathbf{d}' \mathbb{E}_t J(x_{t+1}) \right\}.$$

Consider the discretionary case where the policy maker reoptimizes every period, taking expectations about future policy outcomes as given (i.e. independent of the current choice of the short interest rate). The central bank thus lacks commitment mechanisms. Since the objective function is quadratic and the constraint linear, the value function in the Bellman equation will be quadratic in the predetermined state variables,  $x_{1,t}' V_t x_{1,t} + \mathbf{w}_t$ , and the forward-looking variables will be a linear function of the



predetermined variables,  $x_{2,t} = Hx_{1,t}$ , (see e.g. Söderlind (1999)). In that case, the central bank's optimal reaction function will be to set the short interest rate as a linear function of the predetermined state variables (see Appendix B):<sup>21</sup>

$$(23) \quad i_t = -Fx_{1,t},$$

where the policy reaction coefficients ( $F$ ) are determined by iterating on the value function. The optimal policy is certainty equivalent such that it is independent of the distribution of the disturbances, and thus only the expected value of the state variables matter (see e.g. Currie and Levine (1993)).

A policy change in the nominal interest rate is transmitted into the model economy through two channels, via the real interest rate and via the exchange rate, which is affected by all (nominal) interest rate changes. Both channels affect aggregate demand, and thereby influence inflation indirectly, but the exchange rate also affects inflation directly through changes in the price of imports (see equations (14) and (9), respectively). The degree of pass-through will hence influence how monetary policy is transmitted via the exchange rate channel.

The model is solved using numerical methods, and consequently it needs to be parameterized. The values for the model parameters shown in Table 1 are selected along the lines of Svensson (2000), without any attempt to calibrate or estimate the model.

**Table 1: Parameterization**

Central bank loss function	Supply relations	Demand relation	Foreign Taylor rule	Shock persistence	Shock variance
$I = 0.5$	<i>price stickiness:</i>	<i>consumer utility function:</i>	$b_p^* = 1.5$	$t_p = 0.8$	$s_p^2 = 1$
$n_i = 0.1$	$g_M = \{0.01, 0.5, 2, 100\}$	$h = 6$	$b_y^* = 0.5$	$t_r = 0.8$	$s_y^2 = 1$
$d = 0.99$	$g_D = 10$	$s = 0.5$	$r_i^* = 0.8$	$t_y = 0.8$	$s_f^2 = 1$
	<i>foreign influence:</i>	$b = 0.99$			$s_{y^*}^2 = 1$
	$k_M = 0.3$	<i>foreign economy elements:</i>			$s_{p^*}^2 = 1$
	$k_W = 0.1$	$k_D = 0.3$			$s_{r^*}^2 = 1$
	<i>Production function:</i>	$a_y^* = 0.9$			
	$x_y = 0.8$	$r_y^* = 0.8$			
	<i>producer discount rate:</i>	$r_p^* = 0.8$			
	$b = 0.99$				

<sup>21</sup> In the commitment case, the optimal policy additionally depends on the shadow prices of the forward-looking variables.

These parameters imply a discount factor that yields an annual interest rate of 4% (assuming a quarterly model), a price elasticity of demand that generates 20 % markup over marginal cost, an import share consisting of 30 % of total consumption, and an export share of 30 % of aggregate demand. The rate of nominal rigidity ( $g_M$ ), or import price stickiness, is chosen such that the degree of pass-through captures the standard open economy case of almost full pass-through, two intermediate cases of incomplete pass-through, and one case with approximately no pass-through, hence basically approaching a closed economy setting. The intermediate pass-through cases appear most relevant for small open economies since the empirical evidence typically suggest that the degree of exchange rate pass-through is in the range of 20–80% (see e.g. Menon (1996) for a survey of the empirical literature).<sup>22</sup> The domestic price stickiness ( $g_D$ ) in turn, is chosen to obtain a reasonable output elasticity in the aggregate supply relation.

### 3 Policy responses under different degrees of pass-through

The optimal discretionary policy is examined, under different degrees of pass-through, in terms of reaction functions, the inflation–output variability trade-off, and the overall variation in some key variables. To describe the effects of incomplete exchange rate pass-through on the central bank’s reaction, and its implications, simple impulse response exercises are carried out when foreign and domestic disturbances hit the economy.

#### 3.1 Exchange rate pass-through

The term ‘exchange rate pass-through’ is generally used to characterize the percentage change in import prices caused by an unidentified shock to the exchange rate. However, the degree of pass-through possibly depends on whether this exchange rate movement is caused by a ‘genuine’ exchange rate shock, or whether some other disturbance to the economy generates an implicit exchange rate change. If, for example, different shocks have different degrees of persistence this will affect the resulting degree of pass-through. The pass-through is also dependent on whether one defines it as partial—only measuring the direct effect on the price relation, excluding the effect on other variables—or total—determining the entire effect an exchange rate change causes, working through every interaction of the price determination.<sup>23</sup> Moreover, in the model used here incomplete pass-

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<sup>22</sup> Even though all pass-through coefficients resulting from the chosen  $g_M$ ’s seem reasonable, some reservation could possibly be raised regarding the largest structural parameter,  $g_M = 100$ . Roberts (1995) reports estimates of 0.25 to 0.36 for  $(b/g)$ , where  $b$  denotes the supply elasticity of the product, using survey data for the US. A structural parameter of  $g_M = 100$ , would in that case require a supply elasticity of 25-36, which seems rather far-fetched.

<sup>23</sup> Apart from the exchange rate, the import price in this model is dependent only on the exogenously given foreign price ( $\hat{p}_i^*$ ), which is not affected by any exchange rate changes. However, a movement in the exchange rate might additionally

through is caused by nominal rigidities, why the pass-through additionally depends on the structural parameter governing the price stickiness (i.e.  $g_M$ ).

**Table 2:** Partial and total pass-through under different degrees of price stickiness;  
(contemporaneous responses,  $\Delta p_t^M / \Delta e_t$  )

Structural price stickiness $g_M$	Partial pass-through, <i>unspecified</i> $\Delta e_t$ $\frac{1}{1+g_M}$	Total pass-through derived from a:				
		<i>risk premium shock</i> $u_t^f$	<i>foreign inflation shock</i> $u_t^{i*}$	<i>foreign demand shock</i> $u_t^{e*}$	<i>domestic cost-push shock</i> $u_t^p$	<i>domestic demand shock</i> $u_t^y$
0.01	0.99	0.987	-0.624	0.991	0.998	0.989
0.5	0.66	0.675	-0.556	0.742	0.929	0.682
2	0.33	0.402	-0.461	0.498	0.811	0.411
100	0.01	0.034	-0.098	0.065	0.237	0.035

Note: The partial pass-through, caused by an unspecified exchange rate movement, is constructed from equation (5) by solving for the price level of import goods, assuming that the expectations of future inflation is zero (i.e. the partial derivative with respect to the exchange rate). The total pass-through is derived from simulations of the entire model under identified shocks to the system.

The degree of pass-through is thus contingent upon a row of factors and assumptions. In Table 2 the resulting contemporaneous pass-through, for different choices of the structural import price stickiness ( $g_M$ ), and alternative sources of exchange rate movements, are displayed. Pass-throughs caused by identified structural shocks, as well as initiated by unspecified exchange rate movements, are presented. The latter, ‘unspecified’, (partial) pass-through category is derived directly from the price setting relation in equation (5), while the ‘identified’ (total) pass-through characterization is derived from simulations of the entire model and the actual responses, of the import price level and the exchange rate, when different types of shocks enter the economy. As the nominal rigidity increases (larger  $g_M$ ), the exchange rate pass-through becomes smaller, no matter what type of shock hits the economy (see Table 2). Consequently, although the import price stickiness parameter ( $g_M$ ) directly determines only the partial pass-through, the relation between the nominal rigidity and the total (as well as partial) pass-through is monotonic. The unspecified, or partial, pass-through is somewhat smaller than the total pass-through caused by identified shocks. This difference is most evident when the nominal rigidity is large, or equivalently when pass-through is small. Risk premium shocks (i.e. exchange rate shocks) and domestic demand shocks imply a total pass-through fairly similar to the unspecified partial pass-through, while domestic cost push shocks, in contrast, appear to render a

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affect expectations about future import prices, why the partial and total pass-through need not be equivalent;  
 $\partial p_t^M / \partial e_t \neq dp_t^M / de_t = \partial p_t^M / \partial e_t + (\partial p_t^M / \partial p_{t+1}^M)(dp_{t+1}^M / de_t)$ , see equation (5).

much larger total pass-through. The reason is that the domestic cost push disturbance is the most costly shock in terms of the size and persistence of the inflationary impulse. Since the price adjustment is gradual this also induces higher expectations about future inflation, which in turn increases the total exchange rate pass-through. Further, the total pass-through is negative for the foreign cost push shock since it yields an import price increase at the same time as the foreign policy reaction induces an exchange rate appreciation.

It is believed that price setters respond differently to temporary and permanent exchange rate movements (see e.g. Froot and Klemperer (1989)). Transitory exchange rate movements are expected to result in low pass-through, or no pass-through at all, while permanent exchange rate changes are believed to yield larger import price responses. In the model used here this can be examined using different degrees of persistence in the risk premium disturbance ( $t_f$ ). A highly persistent risk premium shock will induce larger and more persistent exchange rate movements. Figure 1 displays the total exchange rate pass-through under varying assumptions about the risk premium persistence. The total pass-through seems to be increasing in the degree of persistence, irrespectively of the amount of price stickiness. Consequently, transitory exchange rate changes yield lower import price responses than more persistent movements in the exchange rate do. This occurs because the costly price adjustment implies gradual price changes based on expectations about the future development of the exchange rate (see Footnote 11). With transitory exchange rate movements, the incentives for a gradual price adjustment, towards the new short-lived equilibrium price, will thus be smaller.<sup>24</sup> However, the range in which the degree of pass-through fluctuates, appears to be predominantly contingent upon the degree of import price stickiness. With low nominal rigidity also a temporary exchange rate change induces quite large pass-through, while the opposite is true for large price stickiness and permanent exchange rate changes (cf. Figures 1a and 1d).

### **3.2 Pass-through and exchange rate volatility**

Prior empirical literature has related evidence of incomplete exchange rate pass-through to, for example, the considerable amount of observed short-run nominal exchange rate volatility and costs of changing prices. In contrast to these studies, where exchange rate movements are treated as exogenous, the nominal exchange rate is in this model determined endogenously. Moreover, by here treating the degree of price stickiness as exogenously given, the causality between pass-through and the nominal exchange rate volatility can be examined more closely.

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<sup>24</sup> Note, however, that also the initial exchange rate movement becomes smaller when the risk premium persistence is low. A smaller exchange rate movement implies, per se, that the price adjustment is relatively more expensive, since the deviation from the equilibrium flex price is small, thereby inducing lower pass-through.

Given the central bank's objective of targeting inflation (as well as some output and interest rate stabilization) the price levels, of domestic and foreign goods, are not controlled in this type of model.<sup>25</sup> As long as the inflation rate is stabilized the price levels are assumed to be irrelevant to the policy maker, whatever they happen to be. Consequently, the exchange rate level is non-stationary within this system. The interest rate parity condition governing the exchange rate's development in equation (15) only pins down the expected exchange rate *change*, disregarding the *level* of the exchange rate. Consider for example a positive risk premium shock. Although the shock eventually expires, it depreciates the exchange rate permanently (see Figure 2d). The law of one price for imported goods is satisfied in the long run, which implies that a cointegrating relation between the foreign and domestic prices determines the limit of the exchange rate. The exchange rate must balance the difference between the import price denoted in domestic currency ( $p_t^M$ ), that is permanently affected (i.e. raised) by the shock, and the foreign currency price ( $\hat{p}_t^*$ ), which is exogenously determined and thus not reacting to the risk premium shock.<sup>26</sup> Consequently the exchange rate stays permanently depreciated, and as this new steady-state level of the exchange rate does not induce any further inflationary impulses the policy maker must be indifferent to such a movement.

In addition, the exchange rate responds more to the risk premium shock when the degree of pass-through is low, and the initial depreciation is hence larger. For all shocks except foreign cost push shocks, the common feature seems to be that the initial movement in the exchange rate becomes larger as pass-through decreases (see e.g. Figure 4d). Furthermore, the unconditional variance of the exchange rate difference clearly increases when pass-through decreases (see Table 3). The volatility in the exchange rate is thus negatively related to the degree of pass-through. The reason is that the prices of foreign and domestic products are *exogenously* rigid in this model and can not costlessly absorb a shock. This implies that a large exchange rate response is required when there is a country specific shock, in order to create the necessary adjustment of the relative price between foreign and domestic products.<sup>27</sup> Hence, since the exchange rate is *endogenously* determined the structural pass-through parameter ( $g_M$ ) will affect the development of the exchange rate. In that case, as pass-through decreases, that is as the nominal rigidity increases, the exchange rate is expected to fluctuate more because part of the relative price adjustment is accomplished through the exchange rate (as in Betts and Devereux (2000)).<sup>28</sup>

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<sup>25</sup> This is thus not specific to the setting used here, but typical for all inflation targeting models.

<sup>26</sup> The law of one price states in this model that,  $(\hat{p}^* + e - p^M) \sim I(0)$ . This cointegrating relation enters through equation (5), given that  $p^M \sim I(0)$ . Together with the unaltered foreign currency price this implies that  $\lim_{S \rightarrow \infty} E_t e_{t+S} = \lim_{S \rightarrow \infty} E_t p_{t+S}^M$ .

<sup>27</sup> Given the quadratic adjustment costs, a larger exchange rate movement implies that the producer is further away from the equilibrium flex price, which makes the price adjustment relatively cheaper (see equation (1)).

<sup>28</sup> Still, the relative price of imports moves less as the nominal rigidity increases (see e.g. Figure 2g). However, if the exchange rate did not fluctuate more in those cases, the relative price adjustment would perhaps be even smaller.

**Table 3:** Unconditional variances

Partial pass-through	$\text{var}(p)$	$\text{var}(p^D)$	$\text{var}(p^M)$	$\text{var}(y)$	$\text{var}(\Delta e)$	$\text{var}(p^M - p^D)$	$\text{var}(i)$
0.99	54.649	54.717	56.685	2.088	58.556	9.662	41.585
0.66	54.415	54.926	54.637	1.745	60.505	8.824	40.984
0.33	53.17	54.621	50.824	1.393	63.229	9.803	40.255
0.01	24.996	34.731	11.342	0.246	66.778	379.86	34.611

Note: See Appendix B for the variance–covariance matrix, and calculation of the asymptotic variances.

A larger degree of nominal import price stickiness (i.e. lower pass-through) does not necessarily imply a larger real volatility in this model. Rather, the unconditional variance of output appears to be increasing in the degree of pass-through (see Table 3). The reason behind this somewhat surprising result is probably the fact that output is directly affected by the degree of pass-through. Larger price rigidity induces lower pass-through, which in turn implies less exposure to foreign shocks through the exchange rate channel. Consequently, output is less sensitive to such disturbances when pass-through is low. Besides, if prices are less affected by the shock, the smaller is the required output adjustment.<sup>29</sup> Moreover, this induces fewer interest rate adjustments overall, and the interest rate volatility is therefore increasing in the degree of pass-through.

### 3.3 Policy reaction function

The policy reaction function is explicitly derived from the central bank’s objective function in equation (20). This implies that the policy maker does not restrict her use of information, but acts optimally and responds directly to inflationary impulses, like for example a risk premium shock. This yields a policy rule, or reaction function, that does not resemble a simple Taylor rule. A simple Taylor rule implies that the central bank reacts on certain variables, like inflation and output, in contrast to responding to direct shocks. Consequently, the Taylor policy maker adjusts the interest rate merely indirectly to a disturbance, as it is reflected in inflation or output.

Consider once more a shock to the exchange rate through a positive risk premium disturbance. As expected, less price stickiness (larger pass-through) results in stronger inflationary impulses, and thereby also larger interest rate responses (see Figures 2a and 2c). This follows from the central bank’s reaction function, where the response coefficient on the risk premium shock becomes larger as pass-

<sup>29</sup> Then again, this result is consistent with the findings of De Long and Summers (1986). They show that an increased price flexibility may increase the steady-state variance of output as long as the current price level and expectations about future price changes affect output in different directions.

through increases (see Table 4). Hence, this implies the largest policy adjustment when the effect of the exchange rate shock is the greatest, that is, in the full pass-through case. Recall, on the other hand, that the exchange rate volatility decreases as pass-through increases. However, since the policy maker reacts on the risk premium ( $f_t$ ), and not on the exchange rate ( $e_t$ ) per se, this does not affect the interest rate response directly.

Note furthermore that the reaction on the lagged interest rate is decreasing in the degree of pass-through (see Table 4). This reflects that a smaller pass-through requires a more persistent interest rate response, since the effect of an exchange rate movement is more prolonged in this case. Nevertheless, the interest rate persistence is also implicitly incorporated in the reaction function through the persistence in other variables, and the policy maker's response to these variables. Take for example the lagged relative price of imports ( $p_{t-1}^M - p_{t-1}^D$ ), which enters with a positive coefficient. Since a high lagged relative price indicates that the lagged inflation, and thereby also the lagged interest rate, is large, some of the interest rate smoothing is induced in this way.

**Table 4:** Reaction function of the policy maker; coefficients in  $-F(i_t = -Fx_{1,t})$

Partial pass-through	$i_{t-1}$	$y_t^*$	$i_t^*$	$\hat{p}_t^*$	$e_t^p$	$e_t^f$	$e_t^y$	$(p_{t-1}^M - p_{t-1}^D)$	$(\hat{p}_{t-1}^* + e_{t-1} - p_{t-1}^M)$
0.99	0.015	0.034	0.913	-0.660	3.523	0.913	0.192	-0.024	0
0.66	0.025	0.025	0.842	-0.655	3.503	0.842	0.360	0.067	0
0.33	0.033	0.022	0.797	-0.647	3.529	0.797	0.460	0.120	0
0.01	0.038	0.029	0.796	-0.623	3.688	0.796	0.446	0.109	0

The reaction coefficients on the risk premium shock, on foreign output, inflation, and interest rate, are all increasing in the degree of pass-through. As pass-through increases the inflationary impulses become larger, which in turn require larger interest rate adjustments. In contrast to these foreign variables, the response coefficients on the domestic demand and cost push disturbances (i.e.  $e_t^y$  and  $e_t^p$ ) are decreasing in the degree of pass-through (see Table 4). The interest rate adjustment to domestic shocks is thus larger when pass-through is low (see e.g. Figure 5c). The reason for this is the effect working through the exchange rate channel, which transmits monetary policy to a greater extent the larger pass-through is. Consider for example a positive domestic demand shock. To counter this demand disturbance the interest rate is raised, implying a concurrent appreciation of the exchange rate, which in turn induces lower inflation of imported goods ( $p^M$ ) (see Figure 5). As pass-through increases the appreciation feeds into import prices to a greater extent. Consequently, this counteracts the demand disturbance more, which requires less adjustment of the interest rate when pass-through is

complete. In this case, the exchange rate thus work as a shock absorber, with larger impact the larger pass-through is. Overall, the flipside is a larger exposure to foreign disturbances when the exchange rate pass-through becomes more complete.<sup>30</sup>

That the direct reaction on the wedge term ( $\hat{p}_{t-1}^* + e_{t-1} - p_{t-1}^M$ ) is zero should not be interpreted as the nonexistence of a policy response to deviations from the law of one price (see Table 4). Rather than responding directly to the wedge term, recall that the central bank reacts to the underlying components, such as foreign inflation ( $\hat{p}_t^*$ ), and the risk premium ( $f_t$ ). Note additionally that the central bank responds to all disturbances, permanent as well as temporary, in the absence of transmission lags of monetary policy.

### 3.4 Policy trade-offs

In a closed economy the policy maker can, in general, entirely wipe out a domestic demand shock by simply raising the interest rate, without affecting output and inflation.<sup>31</sup> In contrast, for the open economy case here, the policy maker can not counter a demand shock by raising the interest rate without also affecting the exchange rate and thereby inflation. Consequently, the demand shock can not be completely neutralized, and the central bank is forced to trade off output variability for reduced inflation variability (see Walsh (1999) for a discussion of these matters). Hence, the policy maker faces a trade-off not only when the economy is hit by cost push shocks but also by, for example, demand and exchange rate disturbances (see Figure 7). However, exchange rate disturbances, domestic demand shocks, as well as foreign demand and inflation shocks, generate much less variance in both inflation and output, for equal shock magnitudes, compared to the domestic cost push shocks. As the economy is hit by a combination of shocks, most of the unconditional variance in inflation and output therefore originates from the domestic cost push disturbances.<sup>32,33</sup> Nevertheless, the policy maker still faces a trade-off between inflation and output variability for the other shocks, but in a different scale than for the domestic cost push shock. If the variance of, for example, an exchange rate disturbance were larger, the trade-off curve would be located further away from the origin. Still, in the model used here the exchange rate disturbance (or any other of the ‘minor’ shocks) must be outsized by orders of magnitude, so as to generate the same dimension of inflation and output variability as is

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<sup>30</sup> In contrast, for domestic cost push shocks the exchange rate, surprisingly, counteracts the policy maker’s objective of bringing down inflation, since it depreciates and thus adds to the inflationary impulse.

<sup>31</sup> Note that this is not the case if the policy maker’s objective function penalizes interest rate changes.

<sup>32</sup> See Appendix B for the variance-covariance matrix. The variances of all shocks are, by assumption, identical and set to one, making the shock vector equal to;  $\mathbf{u}_0 = (0 \ 1 \ (1+(1-\mathbf{r}_t)(b_y^* + b_p^*)) \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$ . The assumption that all structural shocks have the same variance might be excessively restrictive in ‘reality’. However, the domestic cost push shock is the primary source of inflation and output variability, also when allowing for different variances in the disturbances.

<sup>33</sup> Recall that only the magnitude of the loss function, and the inflation-output trade-off, are affected by the size of the shocks. The policy maker’s reaction function is certainty equivalent and thus independent of the disturbances’ covariance matrix.



caused by a domestic cost push shock. These results consequently imply that cost push shocks are not the only shocks that should be offset by the central bank (in contrast to the closed economy; see Clarida et al. (1999)). Nonetheless the cost push shocks appear to be the most ‘costly’ disturbances, which requires firmer interest rate responses.

How is the inflation–output variance trade-off affected by the exchange rate pass-through? As pass-through decreases, the exchange rate channel becomes less important in transmitting policy, implying that for example a domestic demand shock involves a less severe conflict between stabilizing inflation or output. An interest rate change still yields an exchange rate movement but this movement does not pass-through to prices to the same extent. An open economy with incomplete pass-through thus obtains some of the characteristics of a closed economy setting, and this makes the trade-off between inflation and output variability less considerable. Furthermore, when pass-through is small, autonomous exchange rate disturbances will have less impact on the economy, which makes both inflation and output variances smaller. These effects, running through the exchange rate—the less serious conflict between policy objectives, and the lower exposure to foreign shocks—together imply that also the inflation–output variability trade-off in the face of a combination of shocks, will be located closer to the origin (see Figure 8). For a given output variability the inflation variance is thus smaller when the exchange rate pass-through is low. However, quantitatively the degree of pass-through seems to have a rather small effect on the inflation–output variability frontier. The difference in central bank loss is for example less than 3% between the case with 99% partial pass-through and the case with 33% partial pass-through (not shown).

### 3.5 Robustness issues

How is monetary policy affected by the degree of openness in the economy, and what does this imply in terms of, for example, inflation and output variability? A more open economy implies that the exposure to foreign disturbances increases, and that the exchange rate channel plays a more important role in the monetary policy transmission. Recall that a larger exchange rate pass-through has similar implications, why openness and pass-through are somewhat related questions although their specific mechanisms work differently. In this model the degree of openness is captured through three different parameters, the import and export shares ( $k_M$  and  $k_D$ , respectively), and the share of imported intermediate inputs in production ( $k_W$ ). For instance, increasing the import share of consumption ( $k_M$ ) implies that deviations from the law of one price more significantly affect total (CPI) inflation, as is the case when pass-through becomes larger. However, a more open economy also implies that the real exchange rate directly influences both inflation and output to a greater extent (see e.g. equation (9)).

The results discussed above mostly appear to be qualitatively robust to changing the degree of openness. For example, the exchange rate volatility is decreasing in the degree of pass-through, and the policy reaction to a risk premium disturbance decreases as pass-through decreases, irrespectively of the degree of openness.<sup>34</sup> Note, however, that the size of the reaction coefficients and the resulting variability change as the degree of openness changes (see Tables C1 and C2 in Appendix C). In analogy with the complete pass-through case, a more open economy implies larger policy reactions to foreign disturbances, since their impact on inflation and output is larger in this case. In contrast, the policy response to domestic disturbances appears to decrease when the economy becomes more open. This also affects the variability in the economy. The exchange rate volatility becomes lower as the degree of openness increases. Given that foreign shocks influence, for example, domestic prices to a greater extent in this case, there is less need for exchange rate induced relative price adjustments.<sup>35</sup> Consequently, the variability in nominal and real exchange rates is smaller when the economy is more open. Nonetheless, the output volatility is, in contrast, increasing in the degree of openness. This occurs because the impact of foreign shocks is larger when the economy is more open, with greater influence on prices, which also requires larger adjustments in output. Stabilization is thus provided through output rather than via real exchange rate movements.

## 4 Conclusions

A small open economy aggregate supply–aggregate demand model, allowing for incomplete exchange rate pass-through, has been developed to analyze the effects of limited exchange rate pass-through on monetary policy. Solving for the endogenous policy response to foreign and domestic shocks, indicates that the optimal policy reaction, and its implications, is dependent on the degree of pass-through. Exchange rate pass-through, in turn, is contingent upon many factors and assumptions. In this model incomplete pass-through is incorporated through the exogenously imposed nominal import price stickiness. However, pass-through is additionally dependent on what type of shock enters the economy. The results suggest that transitory exchange rate movements yield lower import price responses than persistent movements do. Consequently, pass-through is increasing in the degree of shock persistence. Nonetheless, the predominant source for incomplete pass-through appears to be the degree of price stickiness, rather than the degree of shock persistence.

In contrast to the full pass-through case, exchange rate movements do not induce large inflationary impulses under incomplete pass-through, when for example foreign disturbances hit the economy. Consequently, the optimal policy reaction to risk premium changes (i.e. exchange rate shocks), foreign

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<sup>34</sup> The results are also robust to changes in other parameters, such as, for example, the degree of substitutability between goods and the amount of exogenous persistence in the disturbances. Changing the variance-covariance matrix, for instance to allow for different variances in the disturbances, neither affects the main results.

<sup>35</sup> Compare with the complete pass-through case, where the same mechanism is at work (see Section 3.2).

interest rate changes, and foreign demand changes, decreases as pass-through becomes lower. The impact on the domestic economy, both on prices and on output, appears to be smaller when pass-through is low, which implies that the short interest rate adjustment will be smaller.

Contrary to this, the optimal response to domestic disturbances, such as demand and cost push shocks, increases as pass-through decreases. With complete pass-through the policy induced exchange rate movement has a larger effect on prices, implying that the exchange rate channel of transmitting policy has a more sizeable influence. Consequently, some of the stabilization is provided through the resultant exchange rate change, and the policy maker does not need to adjust the interest rate as much.

The exchange rate channel of monetary transmission also implies that the policy maker faces a trade-off between inflation and output variability, not only in presence of cost push shocks, but also for demand shocks and foreign disturbances. The trade-off frontier is located closer to the origin as pass-through decreases, because of the lower exposure to foreign shocks and to policy induced exchange rate fluctuations.

In models where exchange rate changes are exogenously given and prices not very responsive to these changes, monetary policy makers can not rely on exchange rates to provide the necessary adjustments to real shocks (Devereux and Engel (2000)). In the model used here, the exchange rate is endogenously determined and its development is dependent, among other things, on the exogenously given import price stickiness. In order to induce the necessary relative price adjustment, the exchange rate response is required to be larger as the price rigidity increases, since nominal prices are sticky and can not costlessly absorb a shock. The results also indicate that the exchange rate volatility increases as the nominal rigidity increases, or in other words, it is decreasing in the degree of pass-through.

## Appendix A

### A.1 The consumers' preferences

#### *Domestic consumers*

The domestic consumption follows a CES function, such that the aggregate consumption index ( $C_t$ ) consists of consumption of imported goods ( $C_t^M$ ) and consumption of domestic goods ( $C_t^D$ ), in the following form:

$$(A1) \quad C_t = \left[ (1 - \mathbf{k}_M)^{\frac{1}{h}} (C_t^D)^{\frac{h-1}{h}} + (\mathbf{k}_M)^{\frac{1}{h}} (C_t^M)^{\frac{h-1}{h}} \right]^{\frac{h}{h-1}},$$

where  $\mathbf{k}_M$  denotes the total import share of consumption in the domestic country. The corresponding aggregate price index (CPI) is

$$(A2) \quad P_t = \left[ (1 - \mathbf{k}_M) (P_t^D)^{1-h} + \mathbf{k}_M (P_t^M)^{1-h} \right]^{\frac{1}{1-h}}.$$

A log-linearization of (A2), taking first differences, yields the following simplified expression for aggregate (CPI) inflation,  $\mathbf{p}_t = (1 - \mathbf{k}_M) \mathbf{p}_t^D + \mathbf{k}_M \mathbf{p}_t^M$ .<sup>36</sup> For simplicity, it is assumed that all products within a category are alike such that issues of substitution between different types of goods within a category are disregarded. Equation (A1) implies that the demand for domestic and imported goods follow, respectively,

$$(A3a) \quad C_t^D = (1 - \mathbf{k}_M) \left( \frac{P_t^D}{P_t} \right)^{-h} C_t,$$

$$(A3b) \quad C_t^M = \mathbf{k}_M \left( \frac{P_t^M}{P_t} \right)^{-h} C_t.$$

The relative consumption allocation is then given by

$$(A4) \quad \frac{C_t^D}{C_t^M} = \frac{(1 - \mathbf{k}_M)}{\mathbf{k}_M} \left( \frac{P_t^D}{P_t^M} \right)^{-h}.$$

### Foreign consumers

The conditions in the foreign economy is assumed to be exogenously given, and the domestic economy is small in that the domestic export good plays a negligible part in the foreign aggregate consumption and price indices. This is modelled through a CES (Dixit-Stiglitz) aggregator over foreign consumption of a continuum (with unit mass) of differentiated goods, assuming the elasticity of substitution between them to be  $h$  (i.e. the same elasticity of substitution as between domestic and foreign goods). This implies that the foreign demand for the domestic export good, and the foreigners' demand for their own good (i.e. domestic import good), respectively, follows

$$(A5a) \quad C_t^{D*} = \left( \frac{P_t^D / E_t}{P_t^*} \right)^{-h} C_t^*,$$

$$(A5b) \quad C_t^{M*} = \left( \frac{P_t^{M*}}{P_t^*} \right)^{-h} C_t^*.$$

### A.2 The domestic producer's optimization problem

The domestic producer is assumed to use a composite (Cobb-Douglas) input ( $Z_t$ ), consisting of domestic intermediate goods ( $Z_t^D$ ) and foreign import goods ( $Z_t^M$ ), in her production ( $Y_t$ ) such that the production function follows

$$(A6) \quad Y_t = (Z_t)^{1-q} = \left[ (Z_t^D)^{1-k_w} (Z_t^M)^{k_w} \right]^{1-q}, \quad 0 \leq q < 1.$$

The compound intermediate input price, in domestic currency units, is given by

$$(A7) \quad P_t^Z = \frac{(P_t^D)^{1-k_w} (P_t^M)^{k_w}}{(1-k_w)^{1-k_w} k_w^{k_w}},$$

where  $k_w$  denotes the share of imported inputs in the domestic production. The profit maximization problem of this firm, in an imperfectly competitive setting with flexible prices, is given by

$$(A8) \quad \begin{aligned} & \max_{\hat{P}_t^D, \hat{P}_t^{D*}, Z_t} \hat{P}_t^D C_t^D + \hat{P}_t^{D*} E_t C_t^{D*} - P_t^Z Z_t \\ \text{s.t.} \quad & Y_t = (Z_t)^{1-q} \geq C_t^D + C_t^{D*} = (1-k_F) \left( \frac{\hat{P}_t^D}{\hat{P}_t} \right)^{-h} C_t + \left( \frac{\hat{P}_t^{D*}}{\hat{P}_t^*} \right)^{-h} C_t^*, \end{aligned}$$

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<sup>36</sup> The share of import goods in CPI inflation is constant for small deviations around the steady-state, and hence equal to  $k_M$ .

where the domestic and foreign aggregate consumption follow CES functions. The producer satisfies the demand for domestic products, which is equal to  $Y_t = C_t^D + C_t^{D*}$  (domestic and foreign demand).  $E_t$  is the exchange rate (domestic currency per unit foreign currency),  $C_t$  is the aggregate (domestic) consumption index consisting of a composite bundle of domestic and foreign goods,  $\hat{P}_t$  is the corresponding price index, and  $k_M$  is the import share of consumption (a star denotes the foreign counterparts). The goods are well differentiated such that the domestic producer disregards her own effect on aggregate prices, as well as takes the competitor's price (i.e. the price of import goods,  $\hat{P}_t^M$ ) as fixed, implying that any strategic interaction is absent.

The first order conditions, with respect to the (flex) prices charged in the domestic and foreign markets ( $\hat{P}_t^D$  and  $\hat{P}_t^{D*}$ , denoted in domestic and foreign currency, respectively) are given by

$$(A9a) \quad \hat{P}_t^D = \left( \frac{h}{h-1} \right) MC(Z_t, P_t^Z),$$

$$(A9b) \quad \hat{P}_t^{D*} = \left( \frac{h}{h-1} \right) MC(Z_t, P_t^Z) \frac{1}{E_t},$$

where  $h$  is the (positive) constant price elasticity of demand, and  $MC$  is marginal cost;  $MC = P_t^Z / \left( (1-q)Z_t^{-q} \right) = P_t^Z / \left( (1-q)Y_t^{\frac{-q}{1-q}} \right)$ . The equilibrium prices of the domestic good, in a flexible price environment, hence consist of a constant and identical markup over marginal costs.

## Appendix B

### B.1 The central banker's optimization problem

The central bank's period loss function can be stated as  $L_t = (z_t' K z_t)$  where  $z_t = (\mathbf{p}_t, y_t, (i_t - i_{t-1}))'$  denotes a vector of goal variables composed of  $z_t = T_x x_t + T_i i_t$ , such that  $T_x$  is a  $3 \times 13$  matrix mapping the goal variables to the state variables,  $T_i$  is a  $3 \times 1$  matrix, and  $K$  is a  $3 \times 3$  diagonal matrix with diagonal  $(1, \mathbf{I}, \mathbf{n}_i)$ ,

$$T_x = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (1 - \mathbf{k}_M) & \mathbf{k}_M & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$T_i = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

This implies that the intertemporal control problem can be expressed as:

$$(B1) \quad \begin{aligned} J(x_t) &= \min_{\{i_{t+s}\}_{s=0}^{\infty}} E_t \sum_{s=0}^{\infty} \mathbf{d}^s (x_{t+s}' \quad i_{t+s}) \begin{pmatrix} Q & U \\ U' & R \end{pmatrix} \begin{pmatrix} x_{t+s} \\ i_{t+s} \end{pmatrix} \\ &= \min_{\{i_{t+s}\}_{s=0}^{\infty}} E_t \sum_{s=0}^{\infty} \mathbf{d}^s (x_{t+s}' Q x_{t+s} + 2x_{t+s}' U i_{t+s} + i_{t+s}' R i_{t+s}) \end{aligned}$$

where  $Q = T_x' K T_x$ ,  $U = T_x' K T_i$  and  $R = T_i' K T_i$ .

The model, i.e. the system of equations (5), (8), (9), and (14)–(19), can be rewritten in state-space form:

$$(B2) \quad \begin{aligned} \tilde{A}_0 \begin{pmatrix} x_{1,t+1} \\ E_t x_{2,t+1} \end{pmatrix} &= \tilde{A} \begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix} + \tilde{B} i_t + \tilde{\mathbf{u}}_{t+1}, \\ x_{1,t} &= (i_{t-1} \quad y_t^* \quad i_t^* \quad \hat{\mathbf{p}}_t^* \quad \mathbf{e}_t^p \quad \mathbf{e}_t^f \quad \mathbf{e}_t^y \quad (p_{t-1}^M - p_{t-1}^D) \quad (\hat{p}_{t-1}^* + e_{t-1} - p_{t-1}^M))', \\ x_{2,t} &= (y_t \quad \mathbf{p}_t^D \quad \mathbf{p}_t^M \quad \Delta e_t)', \\ \tilde{\mathbf{u}}_{t+1} &= (0 \quad u_{t+1}^{y^*} \quad u_{t+1}^{i^*} \quad u_{t+1}^{p^*} \quad \mathbf{u}_{t+1}^p \quad \mathbf{u}_{t+1}^f \quad \mathbf{u}_{t+1}^y \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)', \end{aligned}$$

where  $x_{1,t}$  is a  $9 \times 1$  vector of predetermined state variables,  $x_{2,t}$  is a  $4 \times 1$  vector of forward-looking variables and  $\tilde{\mathbf{u}}_{t+1}$  is a  $13 \times 1$  vector of disturbances,

$$\tilde{A}_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -b_y^*(1-r_i^*) & 1 & -b_p^*(1-r_i^*) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\tilde{A} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_y^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_i^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_p^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & t_p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & t_f & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & t_y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k_D a_y^*(1-r_y^*) & -k_D h & r_p^* k_D h & 0 & -k_D h & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -\frac{x_t}{g_D} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\tilde{B} = (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ (1-k_D)\mathbf{s} + k_D\mathbf{h} \ 0 \ 0 \ 1)' .$$

The predetermined state vector is defined from stationary variables only, in order to avoid problems with the numerical algorithm used to capture the discretionary solution. If the number of stable eigenvalues of the solution, or transition, matrix (see equation (B13)) equals the number of predetermined variables<sup>37</sup>, the system has a stable solution, which is captured by the numerical algorithm. In contrast, if the state vector contains non-stationary variables, yielding unstable roots, it is unclear whether the algorithm captures the solution to the policy maker's problem. Hence, the system is written such that the non-stationary variables like the price level and the exchange rate, enters the state-space representation only in relative or difference forms.<sup>38</sup>

Premultiplying equation (B2) with  $\tilde{A}_0^{-1}$  yields:

<sup>37</sup> In the case of commitment, this is a necessary condition for a stable solution (Blanchard and Kahn (1980)).

<sup>38</sup> Consequently, the following identities are also used to put up the state space representation;

$$(p_t^M - p_t^D) = (p_{t-1}^M - p_{t-1}^D) + \mathbf{p}_t^M - \mathbf{p}_t^D \text{ and } (\hat{p}_t^* + e_t - p_t^M) = (\hat{p}_{t-1}^* + e_{t-1} - p_{t-1}^M) + \hat{\mathbf{p}}_t^* + \Delta e_t - \mathbf{p}_t^M .$$



$$(B3) \quad \begin{pmatrix} x_{1,t+1} \\ E_t x_{2,t+1} \end{pmatrix} = A \begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix} + B i_t + \mathbf{u}_{t+1},$$

where  $A = \tilde{A}_0^{-1} \tilde{A}$ ,  $B = \tilde{A}_0^{-1} \tilde{B}$  and  $\mathbf{u}_{t+1} = \tilde{A}_0^{-1} \tilde{\mathbf{u}}_{t+1}$ .

In the discretionary case, where the central banker reoptimizes every period, the forward-looking variables can be expressed as a linear function of the predetermined variables,  $x_{2,t} = Hx_{1,t}$ . Using this, partitioning equation (B3), according to the predetermined state variables and forward-looking variables, and taking expectations:

$$(B4) \quad \begin{pmatrix} E_t x_{1,t+1} \\ E_t x_{2,t+1} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} i_t,$$

$$(B5) \quad \begin{pmatrix} I_{13 \times 13} \\ H_{4 \times 13} \end{pmatrix} E_t x_{1,t+1} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} i_t,$$

$$(B6) \quad \begin{pmatrix} E_t x_{1,t+1} \\ x_{2,t} \end{pmatrix} = \begin{pmatrix} I & -A_{12} \\ H & -A_{22} \end{pmatrix}^{-1} \left( \begin{pmatrix} A_{11} \\ A_{21} \end{pmatrix} x_{1,t} + \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} i_t \right),$$

$$(B7) \quad \begin{aligned} x_{2,t} &= (A_{22} - HA_{12})^{-1} (HA_{11} - A_{21}) x_{1,t} + (A_{22} - HA_{12})^{-1} (HB_1 - B_2) i_t \\ &= Dx_{1,t} + Gi_t, \end{aligned}$$

$$(B8) \quad \begin{aligned} E_t x_{1,t+1} &= A_{11} x_{1,t} + A_{12} x_{2,t} + B_1 i_t = / Dx_{1,t} + Gi_t / \\ &= (A_{11} + A_{12} D) x_{1,t} + (B_1 + A_{12} D) i_t \\ &= A^* x_{1,t} + B^* i_t. \end{aligned}$$

Rewriting the central banker's period loss function (see equation (B1)) in terms of the predetermined variables, using equation (B7), yields

$$(B9) \quad \begin{aligned} L_t &= \begin{pmatrix} x'_{1,t} & D' x'_{1,t} + G' i_t \end{pmatrix} \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix} \begin{pmatrix} x_{1,t} \\ Dx_{1,t} + Gi_t \end{pmatrix} + 2 \begin{pmatrix} x_{1,t} \\ Dx_{1,t} + Gi_t \end{pmatrix} U i_t + i_t' R i_t \\ &= x'_{1,t} (Q_{11} + D' Q_{21} + Q_{12} D + D' Q_{22} D) x_{1,t} + 2 x'_{1,t} (Q_{12} G + D' Q_{22} G + U_1 + D' U_2) i_t \\ &\quad + i_t' (G' Q_{22} G + G' U_2 + U_2' G + R) \\ &= x'_{1,t} Q^* x_{1,t} + 2 x'_{1,t} U^* i_t + i_t' R^* i_t. \end{aligned}$$

Using equations (B8) and (B9) implies that the Bellman equation of the optimization problem, considering the discretionary case, can be written

$$\begin{aligned}
(B10) \quad J(x_{1,t}) &= x'_{1,t} V_t x_{1,t} + \mathbf{w}_t \\
&= \min_{i_t} \left\{ L_t + \mathbf{d} E_t \left[ x'_{1,t+1} V_{t+1} x_{1,t+1} + \mathbf{w}_{t+1} \right] \right\} \\
&= \min_{i_t} \left\{ x'_{1,t} Q^* x_{1,t} + 2x'_{1,t} U^* i_t + i'_t R^* i_t + \mathbf{d} E_t \left[ (A^* x_{1,t} + B^* i_t)' V_{t+1} (A^* x_{1,t} + B^* i_t) + \mathbf{w}_{t+1} \right] \right\},
\end{aligned}$$

where  $V_t$  is a negative semidefinite matrix and  $\mathbf{w}_t$  is a scalar, both yet to be determined by iterating on the value function. The first order condition yields,  $2(R^* + \mathbf{d} B^{*'} V_{t+1} B^*) i_t + 2(U^{*'} + \mathbf{d} B^{*'} V_{t+1} A^*) x_{1,t} = 0$ , implying that the interest rate is equal to

$$\begin{aligned}
(B11) \quad i_t &= -(R^* + \mathbf{d} B^{*'} V_{t+1} B^*)^{-1} (U^{*'} + \mathbf{d} B^{*'} V_{t+1} A^*) x_{1,t} \\
&= -F_t x_{1,t}
\end{aligned}$$

The forward-looking variables and the predetermined state variables can, by combining (B11) with (B7) and (B3) respectively, be written as:

$$(B12) \quad x_{2,t} = (D - GF) x_{1,t},$$

$$\begin{aligned}
(B13) \quad x_{1,t+1} &= (A_{11} + A_{12}(D - GF) - B_1 F) x_{1,t} + \mathbf{u}_{t+1} \\
&= M x_{1,t} + \mathbf{u}_{t+1}.
\end{aligned}$$

Using (B11) in equation (B7) and inserting in (B10) yields

$$J(x_{1,t}) = x'_{1,t} (Q^* - U^* F_t - F_t' U^{*'} + F_t' R^* F_t + \mathbf{d} (A^* - B^* F_t)' V_{t+1} (A^* - B^* F_t)) x_{1,t} + \mathbf{d} \mathbf{w}_{t+1},$$

implying that the value function (the so called Ricatti equation) is equal to:

$$(B14) \quad V_t = Q^* - U^* F_t - F_t' U^{*'} + F_t' R^* F_t + \mathbf{d} (A^* - B^* F_t)' V_{t+1} (A^* - B^* F_t).$$

In contrast, if the central banker credibly can commit to a certain policy solution, the monetary policy will also be transmitted through the private agents' expectations about current central bank behaviour. The commitment solution does not require a numerical algorithm (see Söderlind (1999) for the different optimization procedures). The optimal reaction function is in that case determined from a decomposition of the stable eigenvalues from the first order condition of the optimization problem (i.e. the intertemporal loss function (B1) subject to the transition equation (B3)).

## B.2 Variance–Covariance Matrices

The unconditional variance–covariance matrix of the disturbance vector,  $\mathbf{u}_{t+1}$ , is given by

$\Sigma_{\mathbf{u}} = (\Sigma_{\mathbf{u}1} \quad \mathbf{0}_{9 \times 4})$ , where  $\Sigma_{\mathbf{u}1}$  is defined as

$$\Sigma_{\mathbf{u}1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{s}_{y^*}^2 & (1-\mathbf{r}_i^*)b_y^* \mathbf{s}_{y^*}^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (1-\mathbf{r}_i^*)b_y^* \mathbf{s}_{y^*}^2 & \mathbf{s}_{i^*}^2 + (1-\mathbf{r}_i^*)^2 (b_p^{*2} \mathbf{s}_{p^*}^2 + b_y^{*2} \mathbf{s}_{y^*}^2) & (1-\mathbf{r}_i^*)b_p^* \mathbf{s}_{p^*}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (1-\mathbf{r}_i^*)b_p^* \mathbf{s}_{p^*}^2 & \mathbf{s}_{p^*}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{s}_p^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{s}_f^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{s}_y^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The dynamics of the predetermined variables can be written as (B13), implying that the asymptotic unconditional variance–covariance matrix of  $x_1$  is given by

$$(B15) \quad \Sigma_{x1} = M \Sigma_{x1} M' + \Sigma_{\mathbf{u}1},$$

$$(B16) \quad \text{vec}(\Sigma_{x1}) = [I_{n1^2} - (M \otimes M)]^{-1} \text{vec}(\Sigma_{\mathbf{u}1}),$$

using  $\text{vec}(A+B) = \text{vec}(A) + \text{vec}(B)$ , and  $\text{vec}(ABC) = (C' \otimes A)\text{vec}(B)$ , (see Rudebusch and

Svensson (1999)). The variables of interest ( $z_t^g$ ) can be written as a function of the predetermined variables ( $x_{1t}$ ),

$$\begin{aligned} z_{t+1}^g &= T_x^g x_{t+1} + T_i^g i_{t+1} \\ &= \begin{bmatrix} T_{x1}^g & T_{x2}^g \end{bmatrix} \begin{bmatrix} x_{1t+1} \\ x_{2t+1} \end{bmatrix} + T_i^g i_{t+1} \\ &= \begin{bmatrix} T_{x1}^g & T_{x2}^g \end{bmatrix} \begin{bmatrix} x_{1t+1} \\ Hx_{1t+1} \end{bmatrix} - T_i^g Fx_{1t+1} \\ &= Tx_{1t+1}, \end{aligned}$$

implying that the variance–covariance matrix of the interest variables is:

$$(B17) \quad \Sigma_z = T \Sigma_{x1} T'.$$

## Appendix C: Robustness

**Table C1:** Unconditional variances, different degrees of openness

Partial pass-through	$\text{var}(\mathbf{p})$	$\text{var}(\mathbf{p}^D)$	$\text{var}(\mathbf{p}^M)$	$\text{var}(y)$	$\text{var}(\Delta e)$	$\text{var}(p^M - p^D)$	$\text{var}(i)$
$k_M = 0.15, k_D = 0.15, k_W = 0.05$							
0.99	54.998	55.045	62.806	1.841	64.128	30.235	40.638
0.66	54.889	55.188	58.144	1.644	67.698	26.802	39.86
0.33	54.310	55.107	52.7	1.434	72.704	24.458	39.212
0.01	37.194	43.145	15.833	0.552	91.102	477.697	40.176
$k_M = 0.6, k_D = 0.6, k_W = 0.2$							
0.99	53.919	54.047	54.257	2.477	56.614	3.06	41.847
0.66	53.220	54.245	52.833	1.875	57.67	3.05	41.496
0.33	50.275	53.199	48.677	1.295	58.237	5.535	40.463
0.01	10.148	23.597	5.611	0.052	39.411	282.388	23.325

Note: See Appendix B for the variance-covariance matrix, and calculation of the asymptotic variances.

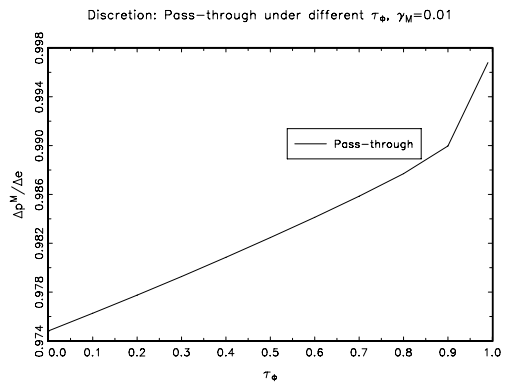
**Table C2:** Policy reaction function, different degrees of openness; coefficients in  $-F(i_t = -Fx_{i,t})$

Partial pass-through	$i_{t-1}$	$y_t^*$	$i_t^*$	$\hat{p}_t^*$	$e_t^p$	$e_t^f$	$e_t^y$	$(p_{t-1}^M - p_{t-1}^D)$	$(\hat{p}_{t-1}^* + e_{t-1} - p_{t-1}^M)$
$k_M = 0.15, k_D = 0.15, k_W = 0.05$									
0.99	0.038	0.037	0.793	-0.555	3.460	0.793	0.373	-0.016	0
0.66	0.058	0.013	0.67	-0.547	3.409	0.67	0.606	0.063	0
0.33	0.072	0.000	0.594	-0.534	3.413	0.594	0.741	0.108	0
0.01	0.080	0.021	0.603	-0.475	3.812	0.603	0.696	0.078	0
$k_M = 0.6, k_D = 0.6, k_W = 0.2$									
0.99	0.006	0.024	0.978	-0.734	3.525	0.978	0.071	-0.037	0
0.66	0.01	0.028	0.954	-0.732	3.521	0.954	0.166	0.042	0
0.33	0.012	0.031	0.940	-0.729	3.525	0.940	0.22	0.086	0
0.01	0.014	0.031	0.932	-0.734	2.954	0.932	0.248	0.127	0

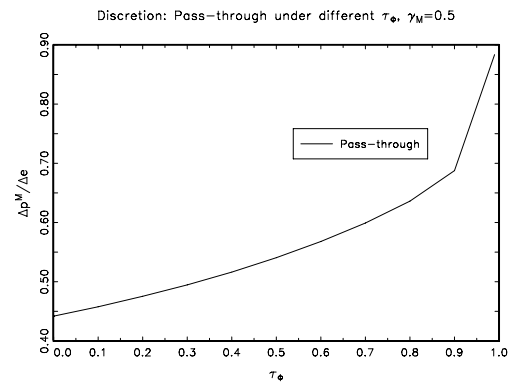
**Figure 1:** Exchange rate pass-through under varying degrees of risk premium persistence ( $\tau_\phi$ )

Note: Range of variation,  $\tau_\phi = [0, 0.99]$

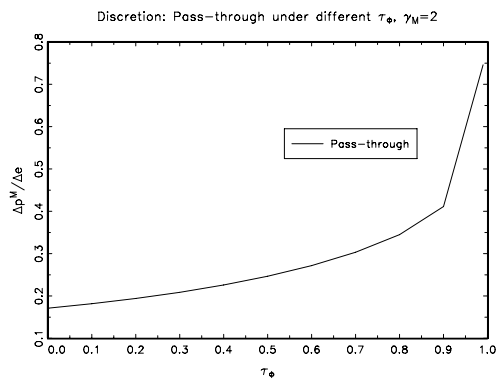
a)  $\gamma_M = 0.01$



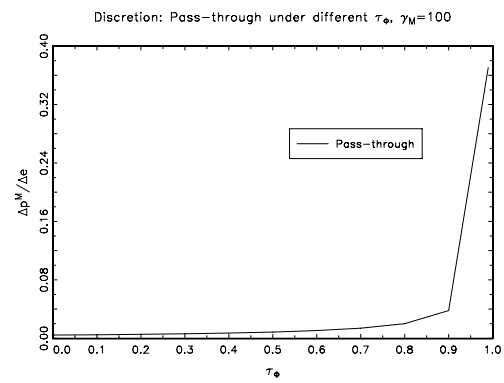
b)  $\gamma_M = 0.5$



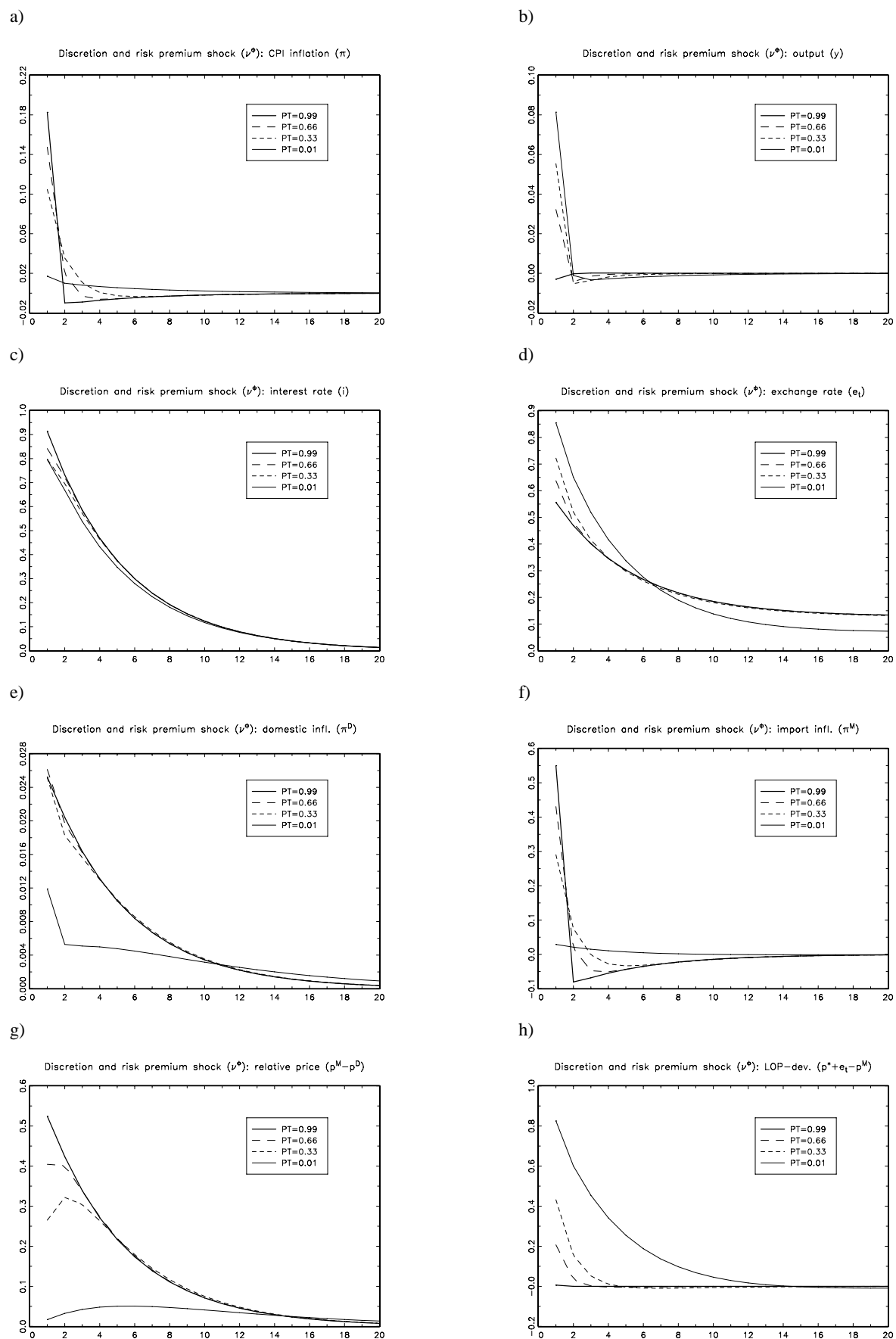
c)  $\gamma_M = 2$



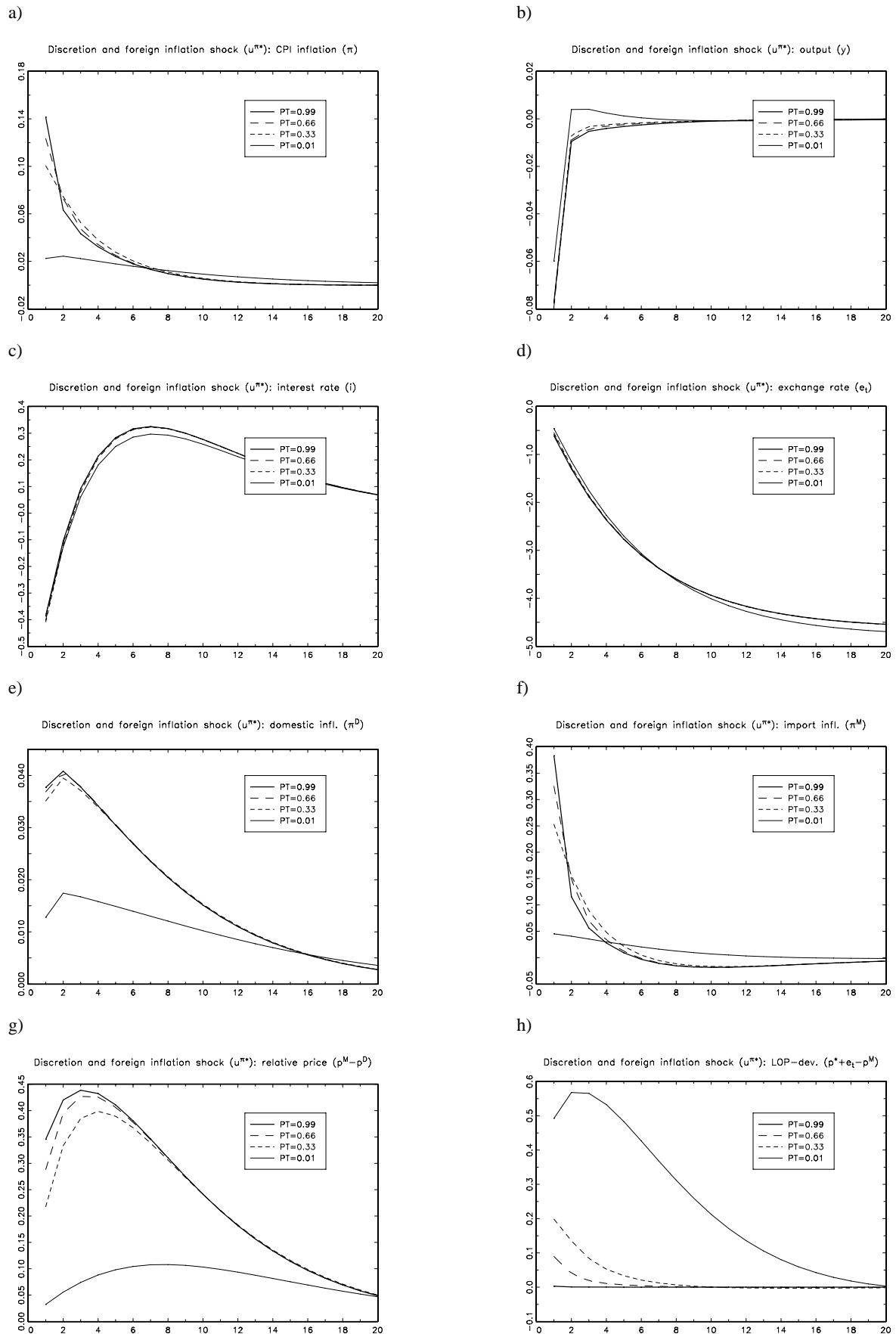
d)  $\gamma_M = 100$



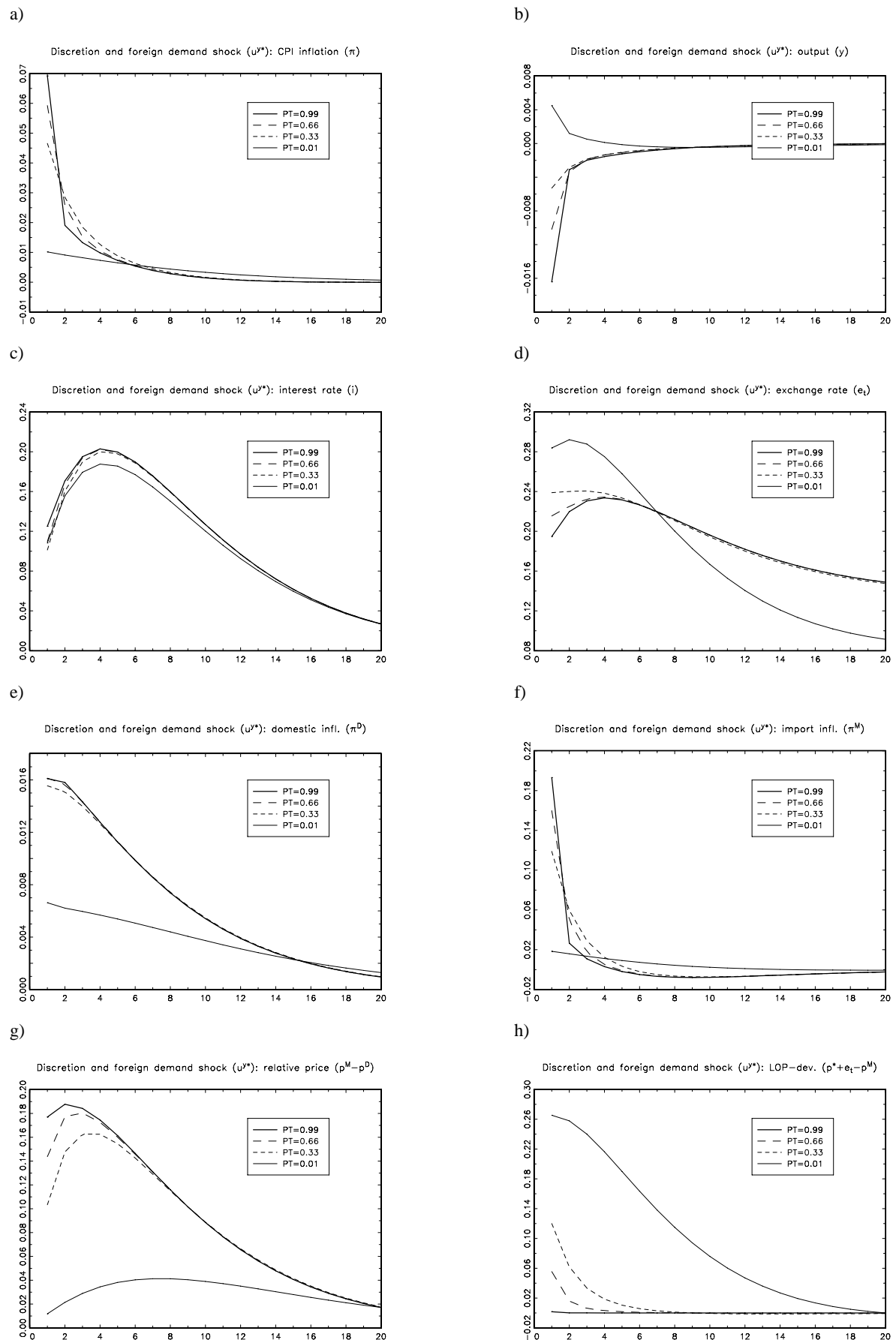
**Figure 2:** Impulse responses under different degrees of pass-through, risk premium shock ( $v^\phi = 1$ )



**Figure 3:** Impulse responses under different degrees of pass-through, foreign inflation shock ( $u^{\pi^*} = 1$ )



**Figure 4:** Impulse responses under different degrees of pass-through, foreign demand shock ( $u^{y^*} = 1$ )

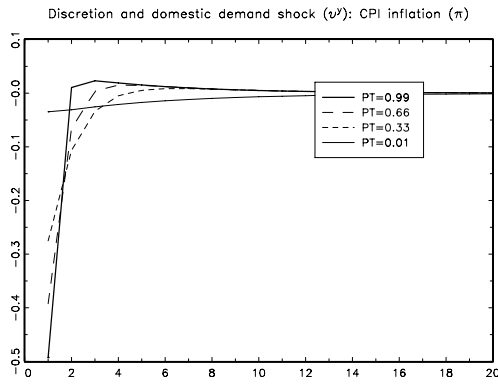




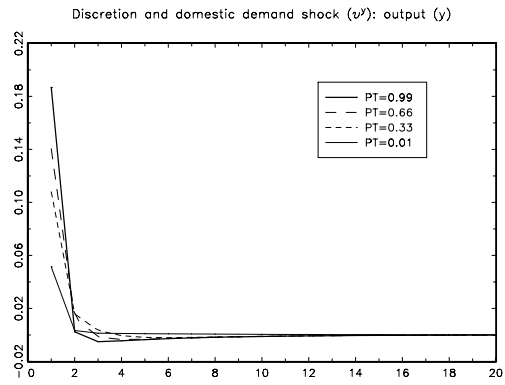
**Figure 5:** Impulse responses under different degrees of pass-through, domestic demand shock

( $v^y = 1$ )

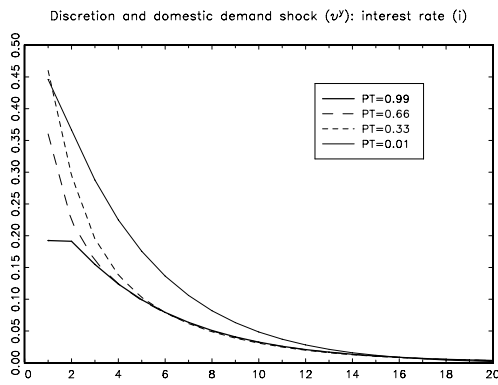
a)



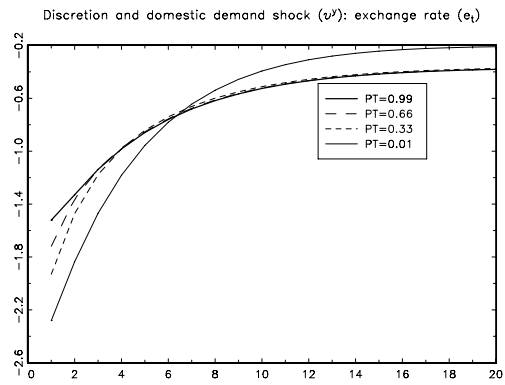
b)



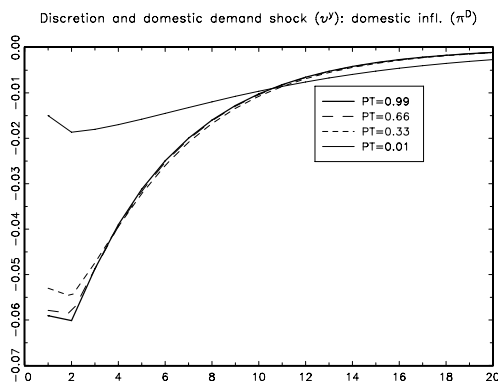
c)



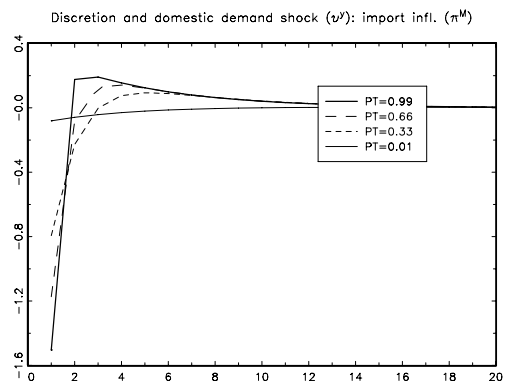
d)



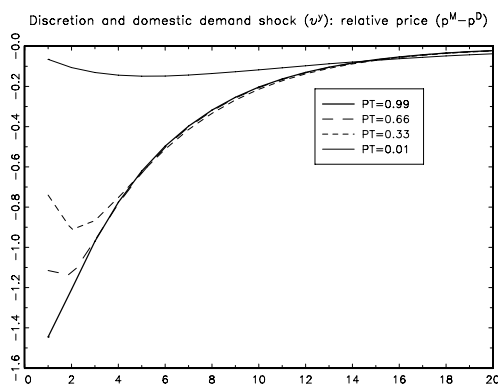
e)



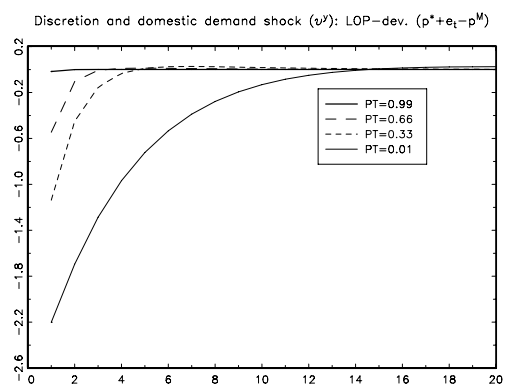
f)



g)



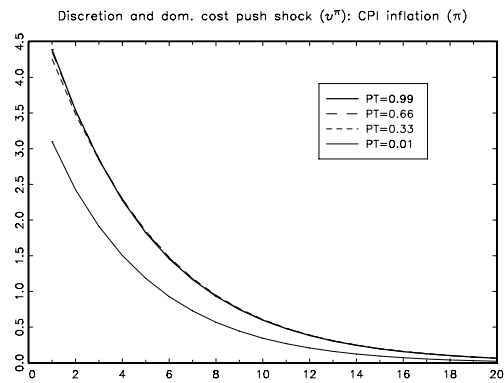
h)



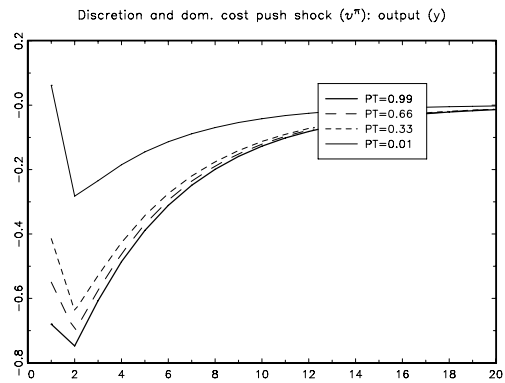
**Figure 6:** Impulse responses under different degrees of pass-through, domestic cost push shock

$(v^\pi = 1)$

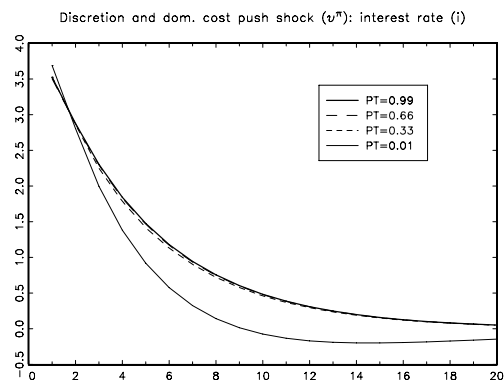
a)



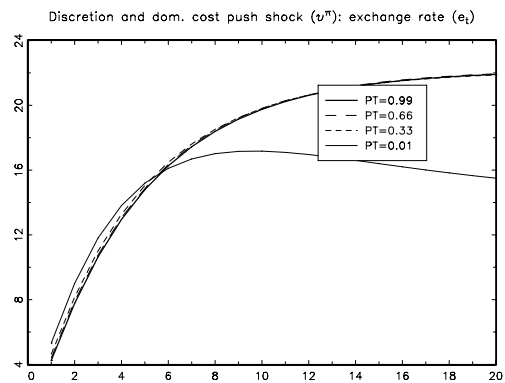
b)



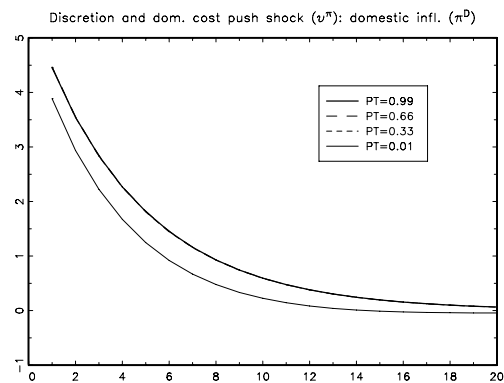
c)



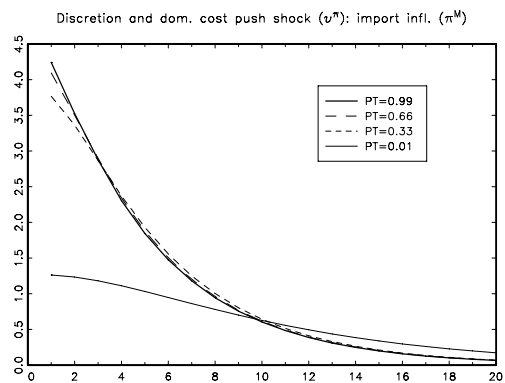
d)



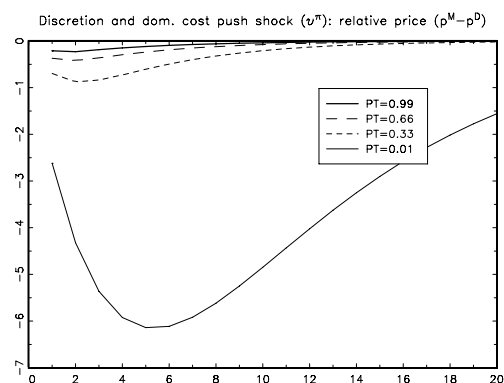
e)



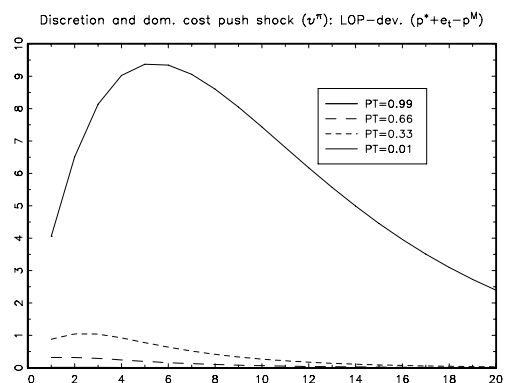
f)



g)

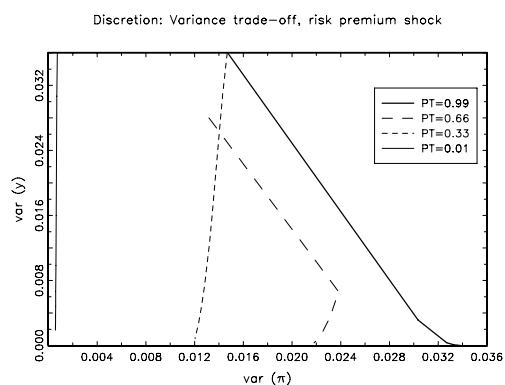


h)

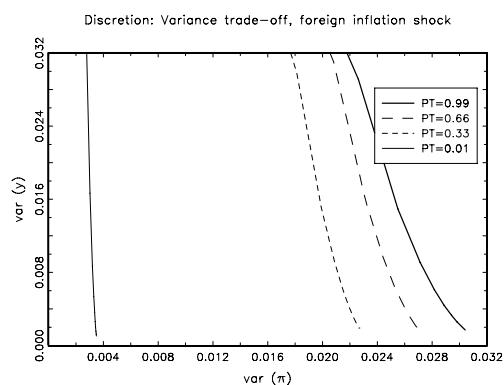


**Figure 7:** Inflation–output variability trade-off under different degrees of pass-through, individual shocks

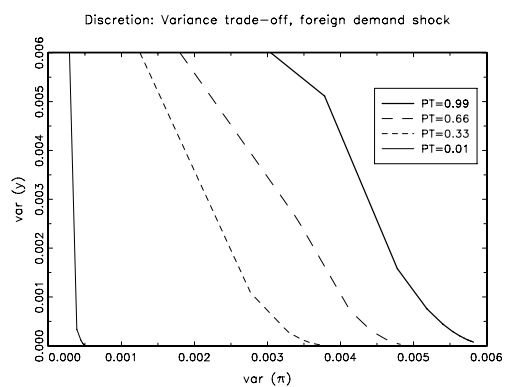
a) Risk premium shock ( $v_i^\phi$ )



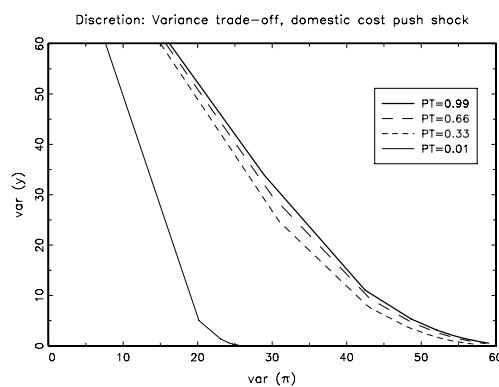
b) Foreign inflation shock ( $u_i^{\pi^*}$ )



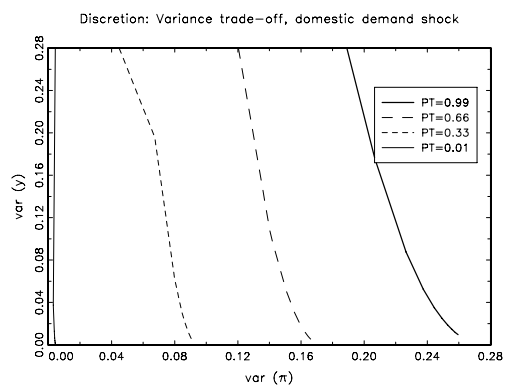
c) Foreign demand shock ( $u_i^{y^*}$ )



d) Domestic cost push shock ( $v_i^x$ )

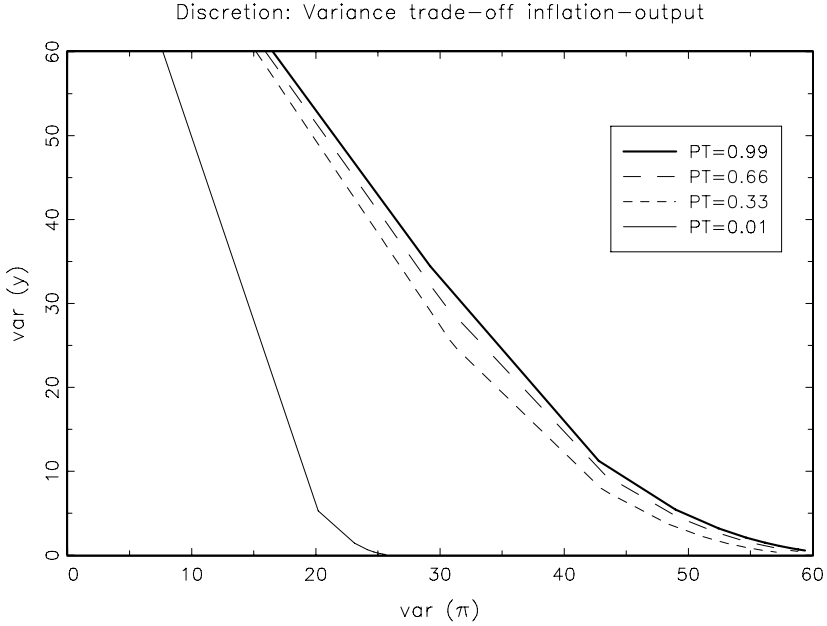


e) Domestic demand shock ( $v_i^y$ )



Note: The degree of output stabilization is altered such that  $\lambda = [0,1]$ , step 0.1. The y-axis is truncated to circumvent the extreme case of  $\lambda = 0$ , which makes the trade-off frontier skewed.

**Figure 8:** Inflation–output variability trade-off under different degrees of pass-through



Note: The degree of output stabilization is altered such that  $\lambda = [0,1]$ , step 0.1. The y-axis is truncated to circumvent the extreme case of  $\lambda = 0$ , which makes the trade-off frontier skewed.

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