Small Structural Models for Monetary Policy Analysis. Sveriges Riksbank June 6-7 2003.

Discussion of

Priors from General Equilibrium Models for VARs
by
Del Negro and Schorfheide

Discussant

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THE GENERAL IDEA

Aspect	DSGE	(S)VAR
Foundations in economic theory	Yes	Loosely
Policy invariant parameters	Yes	No
Parametrization	Sparse	Over-parametrized
Empirical validity	Poor	Substantial
Forecasting performance	Poor	Satisfactory (Bayesian)

Have the cake and eat it too: Flexible VAR models with DSGE based prior

> '... loosening the straightjacket of theory without shedding it completely' (Ingram and Whiteman, JME1994)

> '... take the DSGE model restrictions seriously without imposing them dogmatically' (Del Negro and Schorfheide, 2003).

The hallmark of macroeconomic forecasting [...] will be a marriage of the best of the nonstructural and structural approaches (Diebold, JEP 1998).

MODEL

Theoretical model: Small new Keynesian Dynamic Stochastic General Equilibrium (DSGE) model with parameter vector θ.

Empirical / encompassing model: Unrestricted VAR model

$$y_t = \Phi y_{t-1} + \epsilon_t, \qquad \epsilon_t \overset{iid}{\sim} N(0, \Sigma_u).$$

Minnesota prior: center over $\Phi = I_p$, i.e. multivariate random walk.

Del Negro-Schorfheide prior: center over **DSGE-VAR model**:

$$y_t = \Phi^*(\theta) y_{t-1} + \epsilon_t, \quad \epsilon_t \overset{iid}{\sim} N[0, \Sigma_u^*(\theta)],$$

where the *restriction functions* $\Phi^*(\theta)$ and $\Sigma_u^*(\theta)$ are defined as

$$\begin{aligned} \Phi^*(\theta) &= [E_\theta(y_{t-1}y_{t-1}')]^{-1} E_\theta(y_{t-1}y_t'), \\ \Sigma_u^*(\theta) &= E_\theta(y_t y_t') - E_\theta(y_t y_{t-1}')[E_\theta(y_{t-1}y_{t-1}')]^{-1} E_\theta(y_{t-1}y_t'). \end{aligned}$$

Product moments, e.g. $E_\theta(y_t y_t')$, may be computed from the state-space representation of the log-linearized DSGE model.

PRIOR

'Mental algorithm' for constructing the prior:

1. Simulate artificial time series data from the DSGE model
2. Assume the artificial time series follow a VAR and compute the posterior distribution of the VAR coefficients on this data
3. Use this posterior as a prior for the actual data.

$$p(\Phi, \Sigma_u, \theta) = p(\Phi, \Sigma_u|\theta)p(\theta)$$

The prior on (Φ, Σ_u) conditional on θ centers Φ over $\Phi^*(\theta)$ and Σ_u over $\Sigma_u^*(\theta)$.

PRIOR

A single hyperparameter, $\lambda \geq 0$, determines the extent to which prior probability mass is distributed around the DSGE-VAR.

- $\lambda \to 0$: the prior becomes infinitely spread out around this subspace. DGP $\to$ unrestricted VAR.
- $\lambda \to \infty$: the prior assigns zero probability mass outside the subspace defined by the restriction functions. DGP $\to$ DSGE-VAR. (Prop. 1).

COMMENTS AND QUESTIONS

Is $\theta = \theta$? By embedding the DSGE model in a VAR, the posterior distribution of θ becomes dependent on λ. Relation to direct (non-embedded) Bayesian analysis of θ?

What is the message in Section 3.3.2 'Learning about the DSGE model parameters' and Proposition 2?

How sensitive are the posterior inferences on θ to changes in the lag length of the VAR? Why no lag length shrinkage?

How does ex post λ ($\hat{\lambda}$) relate to the Bayes factor of the DSGE-restricted VAR vs unrestricted VAR?

COMMENTS AND QUESTIONS

Suggestion: Compute the posterior distribution over the models:

- 1) VAR with DSGE-prior
- 2) DSGE-VAR
- 3) DSGE
- 4) VAR with Minnesota prior

Alt. suggestion: Simulate data from DSGE model. Compute $\hat{\lambda}$. Forecast with:

- 1) DSGE
- 2) DSGE-VAR
- 3) VAR with DSGE-prior ($\hat{\lambda}$)

Repeat.

COMMENTS AND QUESTIONS

Suggestion: Plot $\hat{\lambda}$ against forecast horizon.

Equation (19) is not correct. Equation (20) makes a strong independence assumption. Equation (21) is OK as a definition.

Direct state-space modelling without VAR approximation - what is the challenge?

Impulse responses are identified from the initial response of the time series to the structural shocks in the DSGE model.
One controversial choice (identifying restrictions) replaced by another (choice of DSGE model)?

Non-stationary and possibly cointegrated data. How shall we proceed?

$$\Delta y_t = \alpha\beta' y_{t-1} + \epsilon_t$$

$$p(\Phi, \Sigma_u, \theta) = p(\alpha, \Sigma_u|\beta, \theta)p(\beta|\theta)p(\theta).$$

Extending the framework beyond DSGE models?