Information variables for monetary policy in a small structural model of the Euro area.

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Comments Welcome

Abstract

This paper estimates a small New-Keynesian model with imperfect information and optimal discretionary policy using data for the Euro area. The estimated model is used to: (1) Compare the values of key structural parameters and monetary policy targets with those commonly used in calibrations. (2) Assess the imperfect information problem and the usefulness of monetary aggregates and unit labor costs as information variables for monetary policy.

The estimates show that forward-looking behavior plays an important role in the dynamics of both inflation and output. The estimated weights for the objectives of monetary policy indicate that a considerable importance is attributed to interest-rate smoothing, greater than what is typically adopted in this literature. As to indicator variables it is shown that the information content of a monetary indicator based on M3 is rather limited. A more useful role emerges for an indicator based on unit labor costs, which contains information that helps reducing the volatility of the output gap.

JEL Classification Numbers: E5  
Key Words: monetary policy, Kalman filter, inflation, output gap.

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1. Introduction

Dynamic stochastic models of the “new keynesian” variety developed by Rotemberg and Woodford (1997), Woodford (1999) and Clarida, Gali and Gertler (1999) have acquired a solid position in the analysis of monetary policy. Such models have proved useful, e.g. to analyze the properties of various interest rate rules (Jensen, 2002), to quantify the welfare effects of simple versus optimal policy (Dennis and Söderström, 2002) and of imperfect information (Ehrmann and Smets, 2001). Several central banks employ variants of these models to inform the policy analysis.

This paper estimates the structural parameters of such a model using data for the Euro area, under the assumptions of imperfect information and optimal discretionary policy. We think that our exercise adds two useful elements to existing analyses.

First, in comparison to calibrations in which parameters are taken from microeconomic studies or from the match with selected moments of the data, the maximum likelihood estimation exploits all the information contained in the time series. Since a model’s quantitative predictions hinge upon the value of some key parameters it is important to let the data “speak” about these magnitudes and to quantify the uncertainty which surrounds them. For instance, Dennis and Söderström (2002) show that the welfare gains delivered by commitment in comparison to discretion vary significantly, from almost nil to very large, depending on the degree of forward-looking behavior in the inflation equation and on the weight attached to the interest rate stabilization objective by the monetary authority. Estimating the value of these structural parameters is therefore important to assess the quantitative importance of commitment gains. In particular, the estimation of the monetary authorities’ objectives, obtained under the assumption of optimal discretionary policy, distinguishes this paper from previous pioneering estimation exercises, e.g. Ireland (2001) for the United States or Smets and Wouters (2002) for the Euro area, in which a “simple” instrument rule (i.e. restricted to depend on a few key variables) is used to describe monetary policy.\(^1\)

Second, the paper explicitly accounts for imperfect information. This issue is central in the new keynesian model because one of its key variables, “potential

\(^1\)Our endeavour is similar to Dennis (2002), who estimates the policy preferences of the US Federal Reserve. One remaining difference is our consideration of imperfect information aspects.
output” (i.e. the flexible price level of output), is not observable. This aspect adds on top of the fact that information about several other variables of interest, such as contemporaneous GDP or inflation, is usually available only with lags and subject to statistical revisions. Svensson and Woodford (2000) show how the imperfect information problem can be rigorously dealt with by solving a filtering and a control problem. The implementation of their method requires the specification of the economy’s structural parameters and the information structure. Previous studies, e.g. Ehrmann and Smets (2001) and Coenen, Levin and Wieland (2002), deal with this specification problem by separately estimating the information structure and the structural parameters (estimated and/or calibrated). This separation is in principle problematic because, with imperfect information, the equilibrium motion of all the variables in the system depends on both the structural parameters and the information structure. An advantage of the maximum likelihood estimation pursued here is that it allows this issue to be dealt with in a consistent way, by jointly determining the economy’s structural parameters and the noisiness of each indicator.\footnote{The joint presence of an optimization and a filtering problem is an important difference with respect to Ireland (2001), who estimates a small structural model for the US assuming perfect information and an exogenous policy rule.}

The main results are the following. First, forward-looking behavior is important for both output and inflation, even though it plays a greater role in the dynamics of the latter. This finding is consistent with previous studies, e.g. Gali and Gertler (1999), who reject an either fully-backward or fully-forward specification. Second, the estimates for the monetary authority’s objectives show that the weight attached to inflation is almost twice that of the interest-smoothing target and about four-times that of the output-gap. Several previous papers also suggest utilizing a non-zero weight on interest-smoothing in order to fit the persistence of short term rates. This weight, however, is usually chosen to be smaller than that of the output gap. The relatively large weight on interest-smoothing suggested by our estimates implies that the welfare gains delivered by commitment are smaller than in typical parametrizations. This happens because it makes the discretionary monetary response to shocks gradual, replicating an essential feature of commitment policy (see Woodford, 1999).

As to the imperfect information problem, monetary aggregates and a mea-
sure of unit labor costs are considered as indicator variables. We analyze the information content of indicators and assess their welfare effect. It is shown that the information and welfare effect of indicator variables ought to be distinct. In an economy with forward-looking agents the additional information provided by the indicators does not necessarily increase the policy maker’s welfare. This happens because better information about current state variables may cause some forward-looking variables to be more responsive to new information, increasing their volatility. The estimates reveal that monetary aggregates contain little information about the state variables of interest for the conduct of stabilization policy. Under the estimated model parameterization, moreover, this information turns out to decrease welfare, albeit by a tiny amount, because it leads to a greater variability of the policy targets. The unit labor cost indicator, instead, contains information that helps reducing the forecast error about the output gap, a key non-observable variable in the new-keynesian model. This, in turn, increases the policy maker’s welfare because, by allowing for a better identification of the potential output shocks, leads to a smaller variability of inflation and output.

The paper is organized as follows. The next section specifies a dynamic stochastic monetary policy model that incorporates an imperfect information problem, based on Ehrmann and Smets (2001). The solution of this model, following Svensson and Woodford (2000), maps the structural parameters into a vector autoregression. Section 3 discusses how to estimate the model parameters using the Kalman filter following a methodology proposed by Sargent (1989) and Ireland (2001). The data, the estimation results, and some properties of the estimated model are also presented. Section 4 utilizes the estimated model to analyze the effects of imperfect information and to quantify the welfare effects of indicator variables. A quantification of the welfare gains delivered by commitment policy concludes.
2. The model

We model policy by assuming that the central banks aims at minimizing the intertemporal loss function

\[ \Lambda_t = E[ \sum_{\tau=0}^{\infty} \beta^\tau L_{t+\tau} \mid I_t ] \] (2.1)

where \( \beta \in (0,1) \) is the intertemporal discount factor and period losses are given by

\[ L_t \equiv \left[ (\pi_t)^2 + \lambda(y_t - \bar{y}_t)^2 + \nu(i_t - i_{t-1})^2 \right]. \]

where \( \pi_t, y_t, \bar{y}_t \) and \( i_t \) denote, respectively, inflation, output, potential output and the nominal interest rate.

Our benchmark model, taken from Ehrmann and Smets (2001), consists of the following structural equations:

\[ y_t = \delta y_{t-1} + (1 - \delta) y_{t+1\mid t} - \theta \left( i_t - \pi_{t+1\mid t} \right) + u_{y,t} \] (2.2)
\[ \pi_t = \alpha \pi_{t-1} + (1 - \alpha) \pi_{t+1\mid t} + \kappa \left( y_t - \bar{y}_t \right) + u_{\pi,t} \] (2.3)
\[ \bar{y}_t = \rho \bar{y}_{t-1} + u_{\bar{y},t} \] (2.4)
\[ m_t = \gamma_1 m_{t-1} + \gamma_2 m_{t+1\mid t} + \gamma_y y_t - \gamma_i i_t + u_{m,t} \] (2.5)

where \( m_t \) is real money. There are four structural iid innovations in the model with covariance matrix \( \Sigma_u^2 \): a preference shock \( u_{p,t} \), a cost-push shock \( u_{c,t} \), a potential output shock \( u_{\pi,t} \) and a money demand shock \( u_{m,t} \).

One reason for choosing this model is its relative simplicity, which allows for a clear interpretation of the transmission mechanism of structural shocks. Moreover, the specification encompasses purely forward looking models, as the ones used, for example, by Rotemberg and Woodford (1997) and Clarida, Galí and Gertler (1999), and more backward looking models as the one described in Rudebusch (2002).

The presence of lagged values in the output, inflation and real money equations has been shown to be important to fit the dynamics of the data. Smets and Wouters (2002) and Christiano, Eichenbaum and Evans (2001) show that lagged
terms in the output and inflation equations arise in the presence of, respectively, habits-in-consumption and Calvo-pricing firms with indexation to last period inflation. Similarly, lagged and future real money in the money demand equation are introduced by assuming costly adjustment for money holdings. The shocks can also be given a microfoundation. The cost-push shock $u_{c,t}$ that appears in the inflation equation emerges with a time-varying mark-up in the goods market (e.g. Smets and Wouters (2002)), while the shock $u_{p,t}$ is obtained by introducing a random disturbance to the utility function of the representative households. The money demand shock can be justified as a shock to the real balances component of the utility function.

Information about the variables in the economy is obtained from the following vector of measurables:

\[
\begin{align*}
y_t^o &= y_{t-1} + v_{y,t} & \quad (2.6a) \\
\pi_t^o &= \pi_t + v_{\pi,t} & \quad (2.6b) \\
m_t^o &= m_t + v_{m,t} & \quad (2.6c) \\
x_t^o &= x_{t-1} + v_{x,t} & \quad (2.6d)
\end{align*}
\]

where $y_t^o$ is the indicator output variable, given by a noisy observation on the previous period output level. This assumption models the fact that information on output $y_t$ in a given quarter is not contemporaneously available and that, moreover, output observations are subject to revisions, which justifies the existence of noisy measurement ($v_{y,t}$). The indicators $\pi_t^o$ and $m_t^o$ posit that inflation and real money balances are observed contemporaneously, possibly with noise. Although no direct role for money exists in this model, as it does not affect any of the payoff relevant variables or their transmission mechanism, the monetary indicator may contain useful information on current output through the money demand equation (2.5), which may help reducing the imperfect information problem.

The last indicator, $x_t^o$, is a noisy measure of the previous period real unit labor cost. Rotemberg and Woodford (1997) show, among others, that such costs are proportional to the output gap:

\[
x_t = \mu(y_t - \bar{y}_t)
\]
The measurement errors in the vector $v$ are assumed to be iid with covariance matrix $\Sigma^2_v$.

### 2.1. The Economy under a Discretionary Equilibrium

We focus on the discretionary (i.e. Markov perfect) equilibrium, whereby the strategy of both the policy maker and the agents are constrained to be functions of the predetermined (natural) state variables alone (i.e. history-dependent strategies are ruled out).³

To solve the above model it is convenient to rewrite the system in the state-space form following a Svensson and Woodford (2000), defining the vector $X'_t \equiv \begin{bmatrix} y_{t-1} & \pi_{t-1} & m_{t-1} & \mathbf{g}_t & u_{p,t} & u_{c,t} & u_{m,t} & i_{t-1} & \mathbf{g}_{t-1} \end{bmatrix}$ of predetermined state variables and the vector $x'_t \equiv \begin{bmatrix} y_t & \pi_t & m_t \end{bmatrix}$ of non-predetermined (forward looking) variables (see Appendix B).

Information is described by the set $J_t \equiv \{Z_\tau, \Omega; \tau = t, t-1, \ldots, 0\}$ i.e. all agents in the model are supposed to know the model parameters

$$\Omega \equiv [\alpha, \beta, \delta, \gamma_1, \gamma_2, \gamma_y, \gamma_i, \lambda, \nu, \kappa, \theta, \rho, \Sigma^2_u, \Sigma^2_v]$$

and the history of the four observable variables (2.6), stacked in the vector $Z'_t \equiv [y'_t, \pi'_t, m'_t, x'_t]$, up to and including period $t$.

We use the algorithms of Gerali and Lippi (2003) to solve for the optimal Markov perfect policy ($i_t = FX_{t[t]}$) and to compute the equilibrium representation of the model, i.e. the law of motion of the state variables ($X_t$), forward-looking ($x_t$) variables and the optimal prediction for $X_t$ computed by the Kalman filter:

$$X_{t+1} = HX_t + JX_{t|t} + C_au_{t+1}$$

$$x_t = GX_{t|t} + G^1(X_t - X_{t|t})$$

$$X_{t|t} = X_{t|t-1} + K[L(X_t - X_{t|t-1}) + v_t]$$

where the matrices $F, H, J, C_u, G, G^1, L$ and $K$ depend on the primitive parameters in $\Omega$ (see Svensson and Woodford, 2000).

³Alternatively, the model could be solved for the optimal Ramsey policy, under the assumption that the central bank can commit (see section 4.3).
The linear quadratic structure of this problem and the certainty equivalence principle imply that the optimal interest rate rule in this model, \( i_t = FX_{it} \), is a linear function of the estimate of the states which does not depend on the uncertainty in the system. Of course uncertainty affects the way in which an innovation in the observables is mapped into an updated estimate of the state variables, which occurs through the Kalman gain matrix: \( K \).

3. Bringing the model to the data

The evolution of the whole economic system (2.8) can be expressed in a compact notation using the following vector autoregression representation:

\[
Q_{t+1} = \hat{A}Q_t + \hat{G}w_{1,t+1}
\]

where

\[
Q_{t+1} = \begin{bmatrix} X_{t+1} \\ X_{t+1|t} \end{bmatrix}, \quad \hat{A} = \begin{bmatrix} H + JKL & J(I - KL) \\ (H + J)KL & (H + J)(I - KL) \end{bmatrix}, \quad \hat{G} = \begin{bmatrix} C_u & JK \\ 0 & (H + J)K \end{bmatrix}
\]

\[
w_{1,t+1} = \begin{bmatrix} u_{t+1} \\ v_t \end{bmatrix}
\]

The endogenous variables are linked to the states \( Q_t \) by:

\[
\begin{bmatrix} Z_t \\ i_t \\ x_{t|t} \end{bmatrix} = \begin{bmatrix} L + MKL & M(I - KL) \\ FKL & F(I - KL) \\ GKL & G(I - KL) \end{bmatrix} \begin{bmatrix} X_t \\ X_{t|t-1} \end{bmatrix} + \begin{bmatrix} MK + I \\ FK \\ GK \end{bmatrix} \begin{bmatrix} v_t \\ w_{2,t} \end{bmatrix}
\]

The data used in the estimation are given by the 3-month interest rate, taken to be a noisy measure of the monetary policy control variable and the four observables of the theoretical model, which are taken as noisy measures of the true (lagged) output, inflation, money and the (lagged) output gap (hence \( d'_t = [Z'_t \ i_t] \)). From the first and second row in (3.10):

\[
d_t = \hat{L}Q_t + w_{2,t}
\]

\[
w_{2,t} = \hat{M}v_t + e_t
\]
where the matrix $\hat{L}$ and $\hat{M}$ are
\[
\hat{L} \equiv \begin{bmatrix} L + MKL & M(I - KL) \\ FKL & F(I - KL) \end{bmatrix} \quad \text{and} \quad \hat{M} \equiv \begin{bmatrix} MK + I \\ FK \end{bmatrix}
\]
and the vector $e_t \equiv [0 \ 0 \ 0 \ 0 \ e_{i,t}]'$ is a vector of measurement errors in the data. Since we already have measurement errors in the theoretical model (the vector $v$), the measurement errors in $e$ associated to the $Z$ variables are assumed to be identically zero to avoid redundancy. Instead, the introduction of a measurement error for the interest rate is needed to avoid a stochastic singularity problem, as the theoretical model predicts that the interest rate is a linear function of the state variables. By introducing the measurement error $e_{i,t}$ we create a wedge between the optimal rate predicted by the model and the actual rate recorded in the data which makes estimation possible. The standard deviation of the measurement error $e_{i,t}$ can be interpreted as a measure of the distance between actual policy and the optimal one prescribed by the model.

Equations (3.9) and (3.11) represent a state space system to which a Kalman filter can be applied to estimate the structural model parameters, $\Omega$. The basic insight rests on the fact that the solution of the theoretical model maps the structural parameters $\Omega$ into the matrices $\hat{A}$, $\hat{G}$, $\hat{L}$, $\hat{M}$ and $\Sigma_u^2$ and $\Sigma_v^2$ which fully characterize the system dynamics (3.9) and (3.11). Given this system, the Kalman filter provides a convenient method to compute the likelihood function associated to a vector of observations on $d_t$. The estimation problem thus consists in finding the vector of parameters $\Omega$ that maximizes the likelihood function. The idea, originally due to Sargent (1989), McGrattan (1994) and Ireland (2001), is illustrated in more detail in Appendix C.

### 3.1. Data and estimation results

The data used in the estimation are the Euro area counterparts of the variables in the vector $Z_t$ and $i_t$: output, which is measured by real GDP, the inflation rate, measured by the quarterly changes in the GDP deflator, real money, measured by the stock of M3 divided by the GDP deflator, the (lagged) output gap indicator, measured by (lagged) real unit labor costs and the nominal short-term interest rate. These data, which run from 1981:1 to 2002:3, contain a subsample during
which the Euro area was not formally established (until the beginning of 1999). Euro area data for this subsample were reconstructed aggregating national data (See Appendix A).

Stationarity of the time series is achieved by means of the Hodrick-Prescott filter with the only exception of the inflation rate for which we used deviations from an annual rate of 2 per cent and the nominal interest rate for which we used deviations from an annual rate of 4.1 per cent. A figure of the detrended data is reported below.

Figure 1: Detrended data

The likelihood function is constructed using the Kalman filter and is maximized with respect to the structural parameters of the model presented in Section 2. The discount factor in the loss function, $\beta$, is set to 0.9949 implying a steady state real interest rate of two per cent. The value was calibrated as the average three-month real interest rate between 1998 and 2002 a period in which annual inflation rate was fluctuating around its steady state. Moreover, the parameter $\mu$ linking output gap to unit-labor cost (equation 2.7) did not appear to be pinned

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4 The steady state interest rate is computed dividing the steady state inflation rate by the discount factor.
down very precisely by the data independently of the value of $\kappa$ (see equation 2.3). A unit value was therefore chosen for $\mu$ in the estimation, which somewhat amounts to a normalization on the value of $\kappa$. Estimation of the model parameters with alternative values of $\mu$ do not significantly alter the quantitative conclusions of this paper.

The estimated parameters are reported in Table 1. All parameters are statistically significant at conventional 5 per cent confidence level.

To the best of our knowledge no previous empirical analysis has attempted the estimation of the weights of the target variables for the euro area (output gap, inflation and interest rate changes). The range of the calibrated values used in the literature on optimal monetary policy is wide. The coefficient of the output gap ranges between 0 and 1 while a smaller coefficient, between 0 and 0.5, is usually chosen for the weight on interest rate changes (e.g. Ehrmann and Smets (2001) and Dennis and Söderström (2002)). These parameters are crucial in quantifying, for example, the gains from commitment, as Dennis and Söderström (2002) have shown and in shaping the dynamics of output, inflation and the nominal interest rate.

Our estimates of the loss function weights indicate that the monetary authority is more concerned with fluctuations in the inflation rate and the interest rate than the output gap. These values differ substantially from the ones used in the literature: for example in the benchmark calibration in Ehrmann and Smets (2001) the weights are set to 1 and 0.1 for, respectively, the output gap and the changes in the interest rate. Our estimates indicate a much smaller weight for the output gap (0.25) and a greater weight for the interest rate term (0.5).

The estimates suggest that output exhibits a large degree of backwardness (high $\delta$) and a low sensitivity to changes in the expected real interest rate (small $\theta$). The first result contrasts with the estimates in Andres et al. (2001) and Smets and Wouters (2002) who suggest a larger degree of forwardness in the output equation. With respect to the interest rate elasticity our value is smaller than the estimate in Andres et al. (2001) and Smets and Wouters (2002).

The likelihood function is maximized using the algorithm \texttt{csminwel.m} written by C. Sims. This routine is robust to discontinuities in the objective function although being a gradient-based method.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.2457</td>
<td>0.0091</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.4952</td>
<td>0.0059</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.7293</td>
<td>0.0969</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1045</td>
<td>0.0034</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.5244</td>
<td>0.0028</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.0032</td>
<td>0.0001</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.7663</td>
<td>0.0041</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.7965</td>
<td>0.0196</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>0.0527</td>
<td>0.0001</td>
</tr>
<tr>
<td>$\gamma_y$</td>
<td>0.0616</td>
<td>0.0009</td>
</tr>
<tr>
<td>$\gamma_i$</td>
<td>0.0094</td>
<td>0.0005</td>
</tr>
<tr>
<td>$\sigma_{u,p}$</td>
<td>0.0056</td>
<td>0.0001</td>
</tr>
<tr>
<td>$\sigma_{u,c}$</td>
<td>0.0013</td>
<td>0.0001</td>
</tr>
<tr>
<td>$\sigma_{u,\gamma}$</td>
<td>0.0098</td>
<td>0.0004</td>
</tr>
<tr>
<td>$\sigma_{u,m}$</td>
<td>0.0038</td>
<td>0.0002</td>
</tr>
<tr>
<td>$\sigma_{v,y}$</td>
<td>0.0196</td>
<td>0.0002</td>
</tr>
<tr>
<td>$\sigma_{v,\pi}$</td>
<td>0.0012</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\sigma_{v,m}$</td>
<td>0.0006</td>
<td>0.0001</td>
</tr>
<tr>
<td>$\sigma_{v,\pi}$</td>
<td>0.0086</td>
<td>0.0002</td>
</tr>
<tr>
<td>$\sigma_{e,i}$</td>
<td>0.0106</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
With respect to the parameters of the New Phillips curve equation we find a large degree of forwardness in inflation ($\alpha = 0.52$), as in Andres et al. (2001) and Smets and Wouters (2002). The estimated value of the elasticity of inflation to the output gap ($\kappa = 0.003$), suggests a very flat supply curve and is close to the value in Smets and Wouters (2002). There is no consensus in the literature on the value of the slope of the new Phillips curve. The estimates range from a minimum value of 0.015 for the U.S. (Galí and Gertler (1999)) to a maximum of 0.39 (Orphanides and Wieland (2000)). With respect to the euro area Smets and Wouters (2002) estimate a slope of 0.007, which is pretty close to our estimate. Calibrated values for this parameter for the euro area range between 0.03 and 0.08 (Casares (2001, 2002)).

The estimated money demand equation suggests a large degree of backwardness (large $\gamma_1$) and a small interest rate elasticity. The long-run elasticity to output and the interest rate are equal to, respectively, 0.38 and -0.06.

The estimates of the standard deviation of the structural shocks are small, with innovations in potential output being the most volatile. The latter result is in line with the empirical findings of Ireland (2001) for the United States and of Smets and Wouters (2002) for the Euro area. The measurement errors in the observables are also small. The variable which is measured with the highest precision is real money (the standard deviation of the measurement error is 0.06 per cent) while the variable which is measured with the largest errors is output (2.0 per cent). The standard deviation of the measurement error in the interest rate, $\sigma_{ei}$, is equal to 1 per cent. The implications of these findings are discussed in Section 4.

As is the case for previous studies, the model forecasting performance within sample is rather modest. About half of the cyclical variability in inflation and real balances is captured, but much less is achieved for output (5 per cent) and the interest rate (13 per cent). However the model performance with respect to an unconstrained VAR is reasonable: the ratio between the likelihood of our structural VAR and the likelihood of the corresponding unconstrained VAR is 0.83. This seems to suggest that a great portion of the cyclical volatility of these variables is not easy to fit.

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6The ratio between the standard deviation of the forecast errors in a given variable and the standard deviation of that same variable is usually quite high, equal to 0.55 for inflation, 0.95 for output, 0.54 for real money, 0.78 for the real CLUP and 0.87 for the interest rate.
3.2. Robustness of the estimates

[TBW]

3.3. Analysis of the model

The estimated model is characterized by the optimal monetary policy rule \( i_t = F X_{t|t} \), the coefficients of which are reported in the Table 2. The standard error are computed by means of Monte Carlo methods. The optimal rule reacts strongly to the cost-push shock which has important effects on inflation (also see Figure 5 below). The weight on lagged inflation is also large. The coefficient on potential output is negative and significant: an increase in potential output forces the central bank to accommodate the shock to stabilize inflation and the output gap. The coefficients on lagged real money and the money demand shock are zero: these two variables have no direct effect on the target variables. Therefore it is optimal for the central bank not to react to them.

![Figure 2](image)

Figure 2 reports the time series for the interest rate that is implied by the optimal rule, together with 95 per cent confidence bands (dashed lines) and the realized 3-month interest rate (solid line) over the estimation period. It shows that the optimal rate implied by the theoretical model tracks the actual interest rate on average. The latter, however, appears to be less volatile: the interest rate
Table 2. The optimal policy function

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficient</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{t-1</td>
<td>t}$</td>
<td>0.49</td>
</tr>
<tr>
<td>$\pi_{t-1</td>
<td>t}$</td>
<td>0.77</td>
</tr>
<tr>
<td>$m_{t-1</td>
<td>t}$</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{y}_{t</td>
<td>t}$</td>
<td>-0.27</td>
</tr>
<tr>
<td>$u_{p,t</td>
<td>t}$</td>
<td>0.67</td>
</tr>
<tr>
<td>$u_{c,t</td>
<td>t}$</td>
<td>1.39</td>
</tr>
<tr>
<td>$u_{m,t</td>
<td>t}$</td>
<td>-</td>
</tr>
<tr>
<td>$i_{t-1}$</td>
<td>0.61</td>
<td>0.01</td>
</tr>
<tr>
<td>$\bar{y}_{t-1</td>
<td>t}$</td>
<td>-</td>
</tr>
</tbody>
</table>

was significantly higher than the optimal one in the 1993-99 period and below it in the 2001-02 period.

In order to have a more intuitive interpretation of the optimal monetary policy rule reported in Table 2, we estimated a Taylor-type of rule over the data generated by a simulation of the estimated model under the optimal discretionary policy. This rule constrains the interest rate to be a linear function of the contemporaneous estimate of the output gap and inflation: $i_t = \phi_x x_{t|t} + \phi_\pi \pi_{t|t}$. The ordinary least square regression explains about 80 per cent of the variability of the optimal rule. The estimated coefficients are 0.6 on the output gap and 1.6 on inflation. These values are remarkably close to those originally proposed by Taylor for the U.S. (0.5 and 1.5 for, respectively, the output gap and inflation).

The qualitative behavior of the estimated model can be described by means of impulse responses to the structural shocks. An innovation in potential output (Figure 3) can be interpreted as a positive productivity shock which determines a decrease in real marginal costs, and hence inflation, and an increase in output. The central bank reduces the interest rate in order to increase output and stabilize the output gap. The initial decrease in the output gap reduces inflation. Real money increases as a consequence of the reduction in the interest rate and the increase in output.
A positive demand shock (Figure 4), which in standard sticky price models is interpreted as a preference shock, increases output and the output gap and, through the Phillips curve, inflation. The central bank increases the interest rate to stabilize the target variables output gap and inflation. Real money increases reflecting mainly the increase in output which is partially compensated by the increase in the interest rate.

A positive cost-push shock (Figure 5) increases inflation on impact. The reaction of the monetary authority is to increase strongly the interest rate which
reduces output and the output gap. As a result real money decreases.

It is important to underline that in response to all the shocks, the central bank reacts gradually because changes in the interest rate are costly.

4. The Role of Information

This section discusses the estimates of the measurement errors that are present in the model ($\Sigma^2_v$) and then proceeds to assess the role of these errors in affecting macroeconomic performance and the policy makers welfare. The estimates reported above provide quantitative information on the extent of the imperfect information problem in the model. Table 1 shows that the estimated measurement errors pertaining to output, inflation, money and the output gap are significant. The two largest errors pertain to output and the output gap, with standard deviation of respectively around 1.9 and 0.9 percent. This finding is not surprising given that these two variables are the ones for which no contemporaneous information is available. Much smaller measurement errors are computed for inflation and the monetary indicator. This is consistent with the empirical observation that information about these variables is available at higher frequency and that they are subject to much smaller statistical revisions.

The Kalman filter provides a convenient way to assess the consequences of these measurement errors for the information that the policy maker is able to
Table 3. Information and forecast error about fundamental shocks

<table>
<thead>
<tr>
<th>Indicator:</th>
<th>With Measurement Error</th>
<th>Without Meas. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ULC and Money</td>
<td>ULC</td>
</tr>
<tr>
<td></td>
<td>{1}</td>
<td>{2}</td>
</tr>
<tr>
<td>$Var [u_{p,t} - u_{p,t</td>
<td>t}]$</td>
<td>0.316</td>
</tr>
<tr>
<td>$Var [u_{c,t} - u_{c,t</td>
<td>t}]$</td>
<td>0.008</td>
</tr>
<tr>
<td>$Var [y_t - y_{t</td>
<td>t}]$</td>
<td>1.381</td>
</tr>
<tr>
<td>$Var [u_{m,t} - u_{m,t</td>
<td>t}]$</td>
<td>0.007</td>
</tr>
</tbody>
</table>

extract about the fundamental shocks that hit the economy and, consequently, the true value of the state variables at each point in time. Column {1} of Table 3 reports the (unconditional) variance of the contemporaneous forecast errors about the fundamental shocks that the agents in the model face when information is processed optimally (using the Kalman filter) and both the monetary and the unit labor cost indicators are used. It appears that the largest forecast errors pertain to the innovations in potential output. This is partly due to the relatively large size of the innovations hitting this variable (see Table 1) and partly to the relative noisiness of the unit labor cost indicator that is used to form a forecast of this variable.

The other columns in this table analyze how the forecast errors change as we vary the information available to agents. When the monetary indicator is taken out of the vector of observables $Z_t$, the forecast errors concerning the money demand innovation obviously increase (they almost double, see column {2}) but the forecast errors about the innovations in output (the preference shock) and potential output increase only by a tiny amount. This finding suggests that the M3 monetary aggregate contains relatively little information about the current and potential output, while it contains information on the innovation in the demand for real balances. This result is in stark contrast with the experiment reported in column {3}, in which the unit labor cost indicator is dropped from the information set of the policy maker. It appears that the forecast errors about potential output are almost doubled, while the forecast errors in the other variables are essentially unchanged.

Column {4} reports, as a benchmark of comparison, the variance of the forecast errors that are produced by the model if there is no measurement error on
the vector of observables (i.e. when $\Sigma_v^2 = 0$). It shows that even with perfect measurement an incomplete information problem persists about actual and potential output given the assumption that information on this variables is available only with a lag. This benchmark shows that when the monetary indicator is used the forecast errors on output are as small as they would be if there was no measurement error on the lagged output indicator. Forecast errors about potential output instead remain above this benchmark even when the unit labor cost indicator is used (columns \{1\} and \{2\}).

4.1. Effects of information on outcomes and welfare

The forecast errors discussed above influence the unconditional variances of the main variables in the model. Table 4 reports the variance of the three goal variables (output gap, inflation and interest rate changes) together with the unconditional value of expected losses. The four columns of Table 4 report the values obtained under four alternative information assumptions. As before, the spirit of the exercise is to use the estimated model to analyze how economic performance (volatilities, welfare) changes in each of these scenarios.

The results for the benchmark case in which both the monetary and the unit labor cost indicator used appear in column \{1\}. Let us compare the volatility of the targets in this case with the one which is obtained when no monetary indicator is available and only the unit labor cost indicator is used (column \{2\}). As Table 3 showed, this change in the information set causes forecast errors about innovations in current and potential output to increase by a tiny amount. This (small) worsening in the information about the fundamental shocks causes monetary policy to be less active (smaller variability of interest rate changes) and the output gap volatility to be smaller. No effect is detected on the volatility of inflation. Smaller variances in two of the three goal variables lead to a moderate decrease in the losses enjoyed by the policy maker. Hence, less information about output innovations turns out to be good for welfare as it results in smaller volatility of target variables (this point is elaborated in the next subsection). The improvement however is quantitatively very limited (losses decrease by 0.2 per cent).

Quantitatively more noticeable consequences emerge when the output gap indicator is taken away from the agent’s information set (column \{3\}). In this case,
Table 4. Targets volatility and the value of losses

<table>
<thead>
<tr>
<th>Indicators:</th>
<th>With Measurement Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UL and Money</td>
</tr>
<tr>
<td>( Var [y_t - y_t^-] )</td>
<td></td>
</tr>
<tr>
<td>( Var [\pi_t] )</td>
<td>0.413</td>
</tr>
<tr>
<td>( Var [i_t - i_{t-1}] )</td>
<td>0.175</td>
</tr>
<tr>
<td>( \Lambda_t )</td>
<td>237.2</td>
</tr>
<tr>
<td>% change in ( \Lambda_t ) w.r. to {1}</td>
<td>-</td>
</tr>
</tbody>
</table>

The greater noise surrounding the potential output indicator leads to a significant reduction in monetary policy activism (as indicated by the smaller volatility of interest rate changes) and to a significantly greater output gap volatility. Due to the certainty equivalence feature of this problem, policy effects stemming from imperfect information arise entirely from the way uncertainty influences the estimates of the states (i.e. through the matrix \( K \) in the updating equation (2.8c)), since the vector \( F \) of the optimal control rule \( (i_t = FX_{it}) \) does not depend on the uncertainty (see Svensson and Woodford, 2000). As shown in the bottom line of the table, these changes increase the losses of the policy maker in comparison to the case in which both indicators are available by approximately 3 percent. This finding indicates that the unit labor cost indicator is useful as it allows the policy maker to implement a welfare superior stabilization policy.

4.2. Forward-looking behavior and the welfare effects of information

It was mentioned above that the welfare effects of information depend on the degree of forward-looking behavior of the agents. This subsection illustrates this point by studying the effects of “more precise information” in two different models: the one estimated above and a calibrated “backward” model in which the forward-looking terms are removed from the inflation, output and money equations. None of the other parameters is changed. By the wording “more precise information” we mean that the covariance matrix \( \Sigma^2 \) is identically zero, i.e. a context in which

\[\text{This is done by setting } \alpha = .999, \delta = .999, \gamma_1 = .999, \gamma_2 = .001.\]
Table 5. The value of information in forward versus backward models

<table>
<thead>
<tr>
<th></th>
<th>Our Model</th>
<th>Backward-model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Var[y_t - \bar{y}_t]$</td>
<td>{1} 3.0 3.2</td>
<td>{B1} 7.0 6.6</td>
</tr>
<tr>
<td>$Var[\pi_t]$</td>
<td>0.4 0.6</td>
<td>1.9 1.9</td>
</tr>
<tr>
<td>$Var[i_t - i_{t-1}]$</td>
<td>0.18 0.25</td>
<td>0.32 0.31</td>
</tr>
<tr>
<td>$\Lambda_t$</td>
<td>237.2 293.3</td>
<td>533.7 516.5</td>
</tr>
<tr>
<td>% change w.r. to Bench.</td>
<td>- 23.7</td>
<td>- 3.2</td>
</tr>
</tbody>
</table>

the measurement error is completely eliminated.  

Table 5 reports the outcomes under the different scenarios. Columns \{1\} and \{4\} report, respectively, the outcomes under the benchmark estimated model and under a counterfactual scenario in which the measurement error is set to zero. It appears that more information leads to a greater variance of the output-gap, inflation and the interest-adjustment. As shown in the bottom line of Table 5, this raises losses of about 23 per cent in comparison to the benchmark case. Columns \{B1\} and \{B2\} repeat the same exercise for backward-looking model. It appears that more information (column \{B2\}) reduces the variance of the output gap, while leaving the variance of the other targets basically unchanged, reducing the policy maker’s losses by about 3 percent.

[...]To be completed...

4.3. The welfare gains of commitment

Svensson (1997), Clarida et al. (1999) and Woodford (1999) highlighted that the forward-looking elements in the new keynesian model give rise to a form of time inconsistency.\(^9\) This is also known as the problem of the “stabilization bias”. In a model with forward-looking agents the optimal prescription is that shocks should be stabilized gradually, i.e. that the optimal policy is displays a form of history dependence. This allows policy to stabilize expectations and to smooth stabiliza-

---

\(^8\)This assumption does not coincide with full-information since the number of states is greater than the number of observables and therefore a non-degenerate filtering problem remains.

\(^9\)This is an instance of the more general time-inconsistency problem brought out by Kydland and Prescott (1977).
Table 6. The welfare effects of commitment

<table>
<thead>
<tr>
<th></th>
<th>Our Model</th>
<th>Model with:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(λ = 0.246 and ν = 0.495)</td>
<td>λ = 0.5 and ν = 0.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Discretion</td>
<td>Commitment</td>
<td>Discretion</td>
</tr>
<tr>
<td></td>
<td>(λ = 0.246 and ν = 0.495)</td>
<td>λ = 0.5 and ν = 0.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>{1}</td>
<td>{5}</td>
<td>{W_d}</td>
</tr>
<tr>
<td>( Var [y_t - \bar{y}_t] )</td>
<td>3.0</td>
<td>3.2</td>
<td>2.3</td>
</tr>
<tr>
<td>( Var [\pi_t] )</td>
<td>0.41</td>
<td>0.25</td>
<td>0.66</td>
</tr>
<tr>
<td>( Var [i_t - i_{t-1}] )</td>
<td>0.18</td>
<td>0.20</td>
<td>1.13</td>
</tr>
<tr>
<td>( \Lambda_t )</td>
<td>237.2</td>
<td>222.0</td>
<td>365.6</td>
</tr>
<tr>
<td>% change w.r. to discr.</td>
<td>-</td>
<td>-6.4</td>
<td>-</td>
</tr>
</tbody>
</table>

One interpretation of the commitment solution is that, after a shock hits the economy, the policy maker announces a path of current and future policy responses to this shock and sticks to it afterwards. Unfortunately, this plan cannot be implemented under a markov-perfect equilibrium in which policy cannot be made contingent on past “promises”. Therefore, under discretion, output stabilization is excessive and inflation is too volatile in comparison to the optimal commitment benchmark. The estimated model allows us to quantify the welfare gains of commitment.

Columns \{1\} and \{5\} of Table 6 report the outcomes of the target variables which originate, respectively, under discretion and commitment. It appears that under commitment the output gap is more volatile and inflation more stable. This reduces losses \( \Lambda \) (equation 2.1) of about six per cent. Following Jensen (2002), this reduction can alternatively be expressed as a permanent deviation of inflation from target: by this metric, the welfare loss of discretion correspond to a permanent deviation of inflation (measured on an annual basis) from target of about 1.1 percentage points in the estimated model.

Columns \{W_d\} and \{W_c\} of Table 6 compare the outcomes of discretion and commitment for a model in which the preference weights of the monetary authority

\[ \text{Var} [y_t - \bar{y}_t] \]

\[ \text{Var} [\pi_t] \]

\[ \text{Var} [i_t - i_{t-1}] \]

\[ \Lambda_t \]

% change w.r. to discr.
are closer to the numerical values usually adopted in the literature, i.e. a relatively small weight is chosen for interest smoothing ($\nu = 0.1$) and a relatively large one is given to the output gap ($\lambda = 0.5$). The bottom line of the Table shows that under this parametrization the gains from commitment are greater than under our estimated model: losses are reduced by about 10 per cent. The equivalent permanent deviation of (annual) inflation from target is 1.7 percentage points.

Note how, despite the fact that output-gap volatility is greater under commitment, the welfare gains of commitment are higher under the parametrization in which this target is more important. One likely explanation of this result is that a more marked "preference" for a interest-rate smoothing (high $\nu$) makes discretionary policy more gradualist. This replicates a feature of commitment policy which, as shown above, reduces the gains from commitment.

[...To be completed...]
A. Appendix: Data source

All data are quarterly and seasonally adjusted. The source for the real GDP, the GDP deflator and unit labour costs is EUROSTAT for the period running from 1991:1 to 2002:3. The data for the period from 1981:1 to 1990:4 are constructed recursively using the starting value (1991:1) of the EUROSTAT series and the growth rates of the corresponding series from the Area Wide Model (AWM) database constructed by Fagan, Henry and Mestre (2001). The source for the stock of nominal money M3 is the European Central Bank. The source for the short-term nominal interest rate is the AWM database for the period running from 1981:1 to 1998:4. For the period up to 2002:3 the interest rate is taken to be the three-month Euribor rate. The source is the European Central Bank.

B. Appendix: State-space formulation of the ESM model

The model can be represented in state-space formulation:

\[
\begin{bmatrix}
X_{t+1} \\
x_{t+1|t}
\end{bmatrix} = A_1 \begin{bmatrix}
X_t \\
x_t
\end{bmatrix} + A_2 \begin{bmatrix}
X_{t|t} \\
x_{t|t}
\end{bmatrix} + B_i + C_u u_{t+1}
\]

where the vector \( X'_t \equiv [y_{t-1} \quad \pi_{t-1} \quad m_{t-1} \quad \overline{y}_t \quad u_{p,t} \quad u_{c,t} \quad u_{m,t} \quad i_{t-1} \quad \overline{y}_{t-1}] \) and \( x'_t \equiv [y_t \quad \pi_t \quad m_t] \) denote, respectively, predetermined and non-predetermined (forward looking) variables at time \( t \) and \( i_t \) is the instrument controlled by the central bank.

The observables are stacked in the vector \( Z_t \equiv [y_t^o \quad \pi_t^o \quad m_t^o \quad x_t^o] \) according to:

\[
Z_t = D_1 \begin{bmatrix}
X_t \\
x_t
\end{bmatrix} + D_2 \begin{bmatrix}
X_{t|t} \\
x_{t|t}
\end{bmatrix} + v_t
\]

and target variables are collected in the vector \( Y_t \equiv [y_t - \overline{y}_t \quad \pi_t \quad i_t - i_{t-1}] \):

\[
Y_t = C_1 \begin{bmatrix}
X_t \\
x_t
\end{bmatrix} + C_2 \begin{bmatrix}
X_{t|t} \\
x_{t|t}
\end{bmatrix} + C_i i_t
\]

Mapping the model of Section 2 into this formulation yields the following...
matrices:

\[
A_1 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]

where \( \xi \equiv (1 - \alpha)(1 - \delta) \)

\[
A_2 = [0], \quad B = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
1 \\
0 \\
\frac{\delta}{1 - \delta} \\
0 \\
\frac{\xi}{1 - \delta} \\
\end{bmatrix}, \quad C_u = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
D_1 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
\end{bmatrix}, \quad D_2 = [0]
\]

24
\[ C_1 = \begin{bmatrix}
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \quad C_2 = [0] \quad C_i = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \]

\[ W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \nu \end{bmatrix} \]

C. Appendix: Computing the likelihood function

[PRELIMINARY] In this section we describe how to compute the likelihood function for the model described in Section 2. The model, in its state-space representation, is defined by the following equations:

\[ d_t = \hat{L}Q_t + w_{2,t} \quad w_{2,t} \equiv \hat{M}v_t \]

\[ Q_{t+1} = \hat{A}Q_t + \hat{G}w_{1,t+1} \]

\[ w_{1,t+1} \equiv \begin{bmatrix} u_{t+1} \\ v_t \end{bmatrix} \]

where the first equation describe the law of motion of the unobserved states \(Q_{t+1}\) and the second equation is the observation equation linking the observed variables \(d_t\) to the states.

The vector of structural shocks \(u_t\) and the measurement errors \(v_t\) are assumed to be independent i.i.d processes with covariance matrices \(\Sigma_u^2\) and \(\Sigma_v^2\). The Kalman filter consists in a system of recursive equations that allows to forecast the unobserved state vector using the information contained in the observed variables.

The recursive system for computing the Kalman filter is given by:

\[ Q_{t+1|t} = A Q_{t|t-1} + K_t \left( d_t - d_{t|t-1} \right) \]

\[ K_t = \left( A \Sigma_{t|t-1}^2 C' + GV_3 \right) \left( C \Sigma_{t|t-1}^2 C' + V_2 \right)^{-1} \]

\[ \Sigma_{t+1|t} = \left( A \Sigma_{t|t-1}^2 A' + GV_1 G' \right) - K_t \left( A \Sigma_{t|t-1}^2 C' + GV_3 \right)' \]

where the matrix \(K_t\) is defined as the Kalman gain and \(\Sigma_{t+1|t}^2\) is the covariance matrix of the forecast of next period state vector \(Q_{t+1}\) as of time \(t\). The matrices
\( V_1, V_2 \) and \( V_3 \) are given by:

\[
V_1 = E \left( w_{1,t+1} w'_{1,t+1} \right) = \begin{bmatrix} \Sigma_u^2 & 0 \\ 0 & \Sigma_v^2 \end{bmatrix} \tag{C.1}
\]

\[
V_2 = E \left( w_{2,t} w'_{2,t} \right) = \hat{M} E \left[ v_t v'_t \right] \hat{M}' = \hat{M} \Sigma^2 \hat{M}' \tag{C.2}
\]

\[
V_3 = E \left( w_{1,t+1} w'_{2,t} \right) = \hat{M} E \begin{bmatrix} u_{t+1} \\ v_t \end{bmatrix} \begin{bmatrix} v'_{t+1} \\ v'_{t} \end{bmatrix} = \begin{bmatrix} 0 \\ \Sigma_v^2 \end{bmatrix} \tag{C.3}
\]

The prediction errors of the observed variables \( d_t \), which are used to compute the likelihood function, are given by

\[
a_t = d_t - d_{y|t-1} = d_t - \hat{L}Q_{y|t-1} \tag{C.4}
\]

and their covariance matrix by

\[
E (a_t a'_t) = C \Sigma^2_{t|t-1} C' + V_2 = \Omega_t \tag{C.5}
\]

Finally, the likelihood function is given by:

\[
\log L = - \frac{nT}{2} ln (2\pi) - \frac{1}{2} \sum_{t=1}^{T} ln |\Omega_t| - \frac{1}{2} \sum_{t=1}^{T} a'_t \Omega_t^{-1} a_t \tag{C.6}
\]
References


