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Mikael Carlsson and Andreas Westermark

APRIL 2007

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Sveriges Riksbank Working Paper Series

No. 206

Revised November 2007

Abstract

We develop a New Keynesian model with staggered price and wage setting where downward nominal wage rigidity (DNWR) arises endogenously through the wage bargaining institutions. It is shown that the optimal (discretionary) monetary policy response to changing economic conditions then becomes asymmetric. Interestingly, in our baseline model we find that the welfare loss is actually slightly smaller in an economy with DNWR. This is due to that DNWR is not an additional constraint on the monetary policy problem. Instead, it is a constraint that changes the choice set and opens up for potential welfare gains due to lower wage variability. Another finding is that the Taylor rule provides a fairly good approximation of optimal policy under DNWR. In contrast, this result does not hold in the unconstrained case. In fact, under the Taylor rule, agents would clearly prefer an economy with DNWR before an unconstrained economy ex ante.

Keywords: Monetary Policy; Wage Bargaining; Downward Nominal Wage Rigidity.

JEL classification: E52, E58, J41.

[†]Research Department, Sveriges Riksbank, SE-103 37, Stockholm, Sweden. e-mail: mikael.carlsson@riksbank.se.

[‡]Department of Economics, Uppsala University, P.O. Box 513, SE-751 20 Uppsala, Sweden. e-mail: andreas.westermark@nek.uu.se.

Introduction

A robust empirical finding is that money wages do not fall during an economic downturn, at least not to any significant degree. A large number of studies report substantial downward nominal wage rigidity in the U.S. as well as in Europe and Japan.[1] Overall, the evidence points towards a sharp asymmetry in the distribution of nominal wage changes around zero. That is, money wages rise but they seldom fall. This may not have any noticeable real effects in periods with sufficiently high inflation rates to allow for a reduction of real wages in response to adverse shocks without reducing nominal wages. However, inflation rates have come down in many countries in recent decade(s) and periods of very low inflation rates are no longer out of the picture. Recent examples are Japan, Sweden and Switzerland which have all experienced prolonged episodes with average CPI-inflation rates below one percent (see below). Still, downward nominal wage rigidity may not be a concern for real outcomes, if it is not a feature of low inflation environments, as conjectured by e.g. Gordon (1996). However, the empirical evidence shows that the downward rigidity of nominal wages persists even in low inflation environments (see Agell and Lundborg, 2003, Fehr and Goette, 2005, and Kuroda and Yamamoto 2003a, 2003b). This, in turn, opens up for potentially important real effects of downward nominal wage rigidity in the current era of low inflation rates.

The purpose of this paper is to study the implications for monetary policy in situations where declining nominal wages are not a viable margin for adjustment to adverse economic conditions. To this end, we develop a New Keynesian DSGE model that can endogenously account for downward nominal wage rigidity. More specifically, this is achieved by introducing wage bargaining between firms and unions as is done in Carlsson and Westermark (2006a), but modified in line with Holden (1994). Then, downward nominal wage rigidity arises as a rational outcome.

In the model, price and wage setting are staggered. The main difference with our approach, relative to standard New Keynesian DSGE models including an explicit labor market (see Erceg, Henderson and Levin, 2000) is that we model wages as being determined in bargaining between firms and unions (households).[2] We follow Carlsson and Westermark (2006a), and assume that the household is attached to a firm.

Wage bargaining is opened with a fixed probability each period, akin to Calvo (1983). Moreover,

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[1] The empirical evidence ranges from studies using data from personnel files presented in Altonji and Devereux (2000), Baker, Gibbs, and Holmstrom (1994), Fehr and Goette (2005), and Wilson (1999), survey/register data in Altonji and Devereux (2000), Akerlof, Dickens, and Perry (1996), Dickens, Goette, Groshen, Holden, Messina, Schweitzer, Turunen, and Ward (2006), Fehr and Goette (2005), Holden and Wulfsberg (2007), Kuroda and Yamamoto (2003a, 2003b) to interviews or surveys with wage setters like Agell and Lundborg (2003), and Bewley (1999), just to mention a few.

[2] For this purpose, we must modify the simplifying assumption of Erceg, Henderson, and Levin (2000), that all households work at all firms. Otherwise, each individual household works an infinitesimal amount at each firm, implying that the effect of the individual household's wage on firm surplus is zero. Thus, in the standard setup, there is no surplus to be negotiated over, hence rendering bargaining irrelevant.

bargaining is non-cooperative as in the Rubinstein-Ståhl model, with the addition that if there is disagreement but no party is willing to call a conflict, work takes place according to the old contract. As argued by Holden (1994), this is in line with the labor market institutions in the U.S. and most western European countries. Moreover, as in Holden (1994), there are costs associated with conflicts in addition to costs stemming from impatience, such as disruptions in business relationships, startup costs and deteriorating management-employee relationships. These costs sometimes render threats of conflict non-credible, leading to agreement on the same wage as in the old contract. Since it is reasonable to assume that these costs are much larger for firms than for workers, workers can credibly threaten firms with conflict, whereas firms cannot. Since workers only use the threat to bid up wages, downward nominal wage rigidity will result.

Given our setup, a non-linear restriction on wage inflation due to downward nominal wage rigidity arises endogenously. Then, given the constraints from private sector behavior, the central bank solves for optimal (discretionary) monetary policy.[3]

The optimal response to changing economic conditions is asymmetric, and not only in the wage inflation dimension. Interestingly, the welfare loss is actually slightly smaller in an economy with downward nominal wage rigidities in our baseline case. The reason is that downward nominal rigidity is not an additional constraint on the problem. Instead, it is a constraint that changes the choice set and opens up for potential welfare gains. Another finding is that the Taylor rule estimated by Rudebusch (2002), provides a fairly good approximation of optimal discretionary policy in terms of welfare under downward nominal wage rigidity. Experimenting with using the original Taylor (1993) parameters for the Taylor rule indicates that the exact specification of the Taylor rule actually plays a minor role for this property. In contrast, neither of these results seem to hold in the unconstrained case. A corollary is that, under the Taylor rule, agents would clearly prefer an economy with downward nominal wage rigidities to an unconstrained economy ex ante. That is, downward nominal wage rigidity actually helps stabilizing the economy in the wage inflation dimension, whereas it does not induce much more variation in inflation and the output gap.

In sections 1 and 2, we outline the model and discuss the equilibrium, respectively. In section 3, we characterize the policy problem facing the central bank. Section 4 discusses optimal policy paths for endogenous variables as well as the welfare implications of downward nominal wage rigidity under optimal policy. Moreover, we also discuss the outcome of using a simple instrument (Taylor) rule instead of the optimal policy. Finally, section 5 concludes.

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[3]We focus on the discretionary policy case, since this is closest to the actual practice of central banks.

1 The Economic Environment

The model outlined below is in many respects similar to that in Erceg, Henderson, and Levin (2000). Goods are produced by monopolistically competitive producers using capital and labor. Producers set prices in staggered contracts as in Calvo (1983). There are also some important differences, however. In contrast to Erceg, Henderson, and Levin (2000), we follow Carlsson and Westermark (2006a), and assume that a household is attached to each firm.[4,5] Thus, firms do not perceive workers as atomistic. In each period, bargaining over wages takes place with a fixed probability. Accordingly, wages are staggered as in Calvo (1983), but, in contrast to Erceg, Henderson, and Levin (2000), they are determined in bargaining between the household/union and the firm. Households derive utility from consumption, real balances and leisure, earning income by working at firms and from capital holdings. Below, we present the model in more detail and derive key relationships (for a full derivation, see Appendix C and the Technical Appendix to Carlsson and Westermark, 2006a).

1.1 Firms and Price Setting

Since households will be identical, except for leisure choices, it simplifies the analysis to abstract away from the households' optimal choices for individual goods. Thus, we follow Erceg, Henderson, and Levin (2000) and assume a competitive sector selling a composite final good, which is combined from intermediate goods to the same proportions as those that households would choose. The composite good is

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $\sigma > 1$ and $Y_t(f)$ is the intermediate good produced by firm f. The price P_t of one unit of the composite good is set equal to the marginal cost

$$P_t = \left[\int_0^1 P_t(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}}. \tag{2}$$

By standard arguments, the demand function for the intermediate good f, is

$$Y_t(f) = \left(\frac{P_t(f)}{P_t} \right)^{-\sigma} Y_t. \tag{3}$$

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[4] Several households could be attached to a firm, if these negotiate together.
[5] There is no reallocation of workers among firms. This is obviously a simplifying assumption, but it enables us to describe the model in terms of very simple relationships.

The production of firm f in period t, $Y_t(f)$, is given by the following constant returns technology

$$Y_t(f) = A_t K_t(f)^\gamma L_t(f)^{1-\gamma}, \tag{4}$$

where A_t is the technology level common to all firms and $K_t(f)$ and $L_t(f)$ denote the firms' capital and labor input in period t, respectively. Since firms have the right to manage, $K_t(f)$ and $L_t(f)$ are optimally chosen, taking the rental cost of capital and the bargained wage $W_t(f)$ as given. Moreover, as in Erceg, Henderson, and Levin (2000), the aggregate capital stock is fixed at $\bar{K}$. Standard cost-minimization arguments then imply that the marginal cost in production is given by

$$MC_t(f) = \frac{W_t(f)}{MPL_t(f)}, \tag{5}$$

where $MPL_t(f)$ is the firm's marginal product of labor.[6]

1.1.1 Prices

The firm is allowed to change prices in a given period with probability $1 - \alpha$ and renegotiate wages with probability $1 - \alpha_w$. In addition, any firm that is allowed to change wages is also allowed to change prices, but not vice versa. Thus, the probability of a firm's price remaining unchanged is $\alpha_w \alpha$. The latter assumption greatly simplifies our problem; in particular, it eliminates any intertemporal interdependence between current and future price decisions via its effect on wage contracts for a given firm. Besides convenience, this assumption is in line with the micro-evidence on price-setting behavior presented in Altissimo, Ehrmann, and Smets (2006), where price and wage changes are to a large extent synchronized in time (see especially their figure 4.4). Here, we assume that wage changes induce price changes, since assuming the reverse would imply that the duration of wage contracts could never be longer than the duration of prices, which seems implausible in face of the empirical evidence, see section 3.1. Furthermore, since intertemporal interdependencies are eliminated, this allows us to describe the goods market equilibrium by a similar type of forward looking new Keynesian Phillips curve as in Erceg, Henderson, and Levin (2000) (see equation (21)).

The producers choose prices to maximize

$$\max_{p_t(f)} E_t \sum_{k=0}^\infty (\alpha_w \alpha)^k \Psi_{t,t+k} \left[(1+\tau) P_t(f) Y_{t+k}(f) - TC(W_{t+k}(f), Y_{t+k}(f)) \right] \tag{6}$$

$$\text{s. t. } Y_{t+k}(f) = \left(\frac{P_t(f)}{P_{t+k}} \right)^{-\sigma} Y_{t+k},$$

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[6]In contrast to Erceg, Henderson, and Levin (2000), the marginal cost is generally not equal among firms, since firms face different wages out of steady state.

where $TC\left(W_{t+k}\left(f\right), y_{t+k}\left(f\right)\right)$ denotes the cost function, Ψ_{t+k} is the households' valuation of nominal profits in period $t+k$ when in period t and τ is a tax/subsidy on output. The term inside the square brackets is just firm profits in period $t+k$, given that prices were last reset in period t. The first-order condition is

$$E_t \sum_{k=0}^{\infty} \left(\alpha_w \alpha\right)^k \Psi_{t,t+k} \left[\frac{\sigma-1}{\sigma} \left(1+\tau\right) P_t\left(f\right) - MC_{t+k}\left(f\right)\right] Y_{t+k}\left(f\right) = 0. \tag{7}$$

The subsidy τ is determined so as to set $\frac{\sigma-1}{\sigma}\left(1+\tau\right) = 1$; that is, we assume that fiscal policy is used to alleviate distortions due to monopoly price setting.[7]

1.2   Households

The economy is populated by a continuum of households, also indexed on the unit interval, which each supplies labor to a single firm. This setup can alternatively be interpreted as a unionized economy with firm-specific unions. In such a framework, each household can be considered as the representative union member.

The expected life time utility of the household working at firm f in period t is given by

$$E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[u\left(C_s\left(f\right)\right) + l\left(\frac{M_s\left(f\right)}{P_s}\right) - v\left(L_s\left(f\right)\right) \right] \right\}, \tag{8}$$

where period s utility is additively separable in three arguments, final goods consumption $C_s(f)$, real money balances $\frac{M_s(f)}{P_s}$, where $M_s\left(f\right)$ denotes money holdings, and the disutility of working $L_s\left(f\right)$.[8] Finally, $\beta \in (0,1)$ is the household's discount factor.

The budget constraint of the household is

$$\frac{\delta_{t+1,t} B_t\left(f\right)}{P_t} + \frac{M_t\left(f\right)}{P_t} + C_t\left(f\right) = \frac{M_{t-1}\left(f\right) + B_{t-1}\left(f\right)}{P_t} + \left(1 + \tau_w\right) \frac{W_t\left(f\right) L_t\left(f\right)}{P_t} + \frac{\Gamma_t}{P_t} + \frac{T_t}{P_t}. \tag{9}$$

The term $\delta_{t+1,t}$ represents the price vector of assets that pays one unit of currency in a particular state of nature in the subsequent period, while the corresponding elements in $B_t\left(f\right)$ represent the quantity of such claims bought by the household. Moreover, $B_{t-1}\left(f\right)$ is the realization of such claims bought in the previous period. Also, $W_t\left(f\right)$ denotes the household's nominal wage and τ_w is the tax/subsidy on labor income. Each household owns an equal share of all firms and the aggregate capital stock. Then,

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[7]Thus, we abstract from any Barro-Gordon type of credibility problems (see Barro and Gordon, 1983a, and Barro and Gordon, 1983b).

[8]In the Technical Appendix, we also introduce a consumption shock and a labor-supply shock as in Erceg, Henderson, and Levin (2000). However, introducing these shocks does not yield any additional insights here. In fact, it can easily be shown that under optimal policy, all disturbances in the model (introduced as in Erceg, Henderson and Levin, 2000) can be reduced to a single disturbance term (being a linear combination of all these shocks).

Γ_t is the household's aliquot share of profits and rental income. Finally, T_t denotes nominal lump-sum transfers from the government. As in Erceg, Henderson, and Levin (2000), we assume that there exist complete contingent claims markets (except for leisure) and equal initial wealth across households. Then, households are homogeneous with respect to consumption and money holdings, i.e., we have $C_t(f) = C_t$, and $M_t(f) = M_t$ for all t.

1.3 Wage Setting

When a firm/household pair is drawn to renegotiate the wage, bargaining takes place in a setup similar to the model by Holden (1994) and is here introduced in a New Keynesian framework following Carlsson and Westermark (2006a). There are two key features of the bargaining model in Holden (1994). First, there are costs of invoking a conflict, which are different from the standard costs in bargaining due to impatience. Instead, they are caused by e.g., disrupting business relationships, startup costs and deteriorating management-employee relationships (see Holden, 1994). Second, there is an old contract in place at the firm and if no conflict is called and no new contract is signed, the workers work according to the old contract. As pointed out by Holden (1994), this is a common feature of many western European countries as well as of the U.S.

The union and the firm only have incentives to call for a conflict when the negotiated contract gives a higher payoff than the old contract. As soon as a conflict is called, payoffs are determined in a standard Rubinstein-Ståhl bargaining game and the conflict costs are paid out of the parties' respective pockets. However, the costs of conflict imply that it is sometimes not credible to threaten with a conflict in equilibrium. Specifically, if the difference between the old contract and the Rubinstein-Ståhl solution is small relative to the conflict cost, a party cannot credibly threaten with a conflict and force the new contract into place. Then, no new agreement is struck and work continues according to the old contract, resulting in nominal rigidity. If the difference is sufficiently large, however, then conflict is a credible threat. Note, though, that there will be no conflicts in equilibrium, since it is optimal to immediately agree on the Rubinstein-Ståhl solution, rather than waiting and enduring a conflict.[9]

To derive only downward nominal rigidity, asymmetries in conflict costs are required. Specifically, if the costs are large for the firm and negligible for the union, the firm can never credibly threaten with a conflict (at least not close to the steady state), whereas the union can always do so when the Rubinstein-Ståhl solution is larger than the old contract. In reality, conflict costs for the workers are probably not zero, but small. Then, wages would be adjusted only if the Rubinstein-Ståhl solution exceeded some threshold value $\bar{\omega} > W_{t-1}$ (instead of $\bar{\omega} = W_{t-1}$). For simplicity, we restrict the

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[9]That agreement is immediate follows from e.g. Rubinstein (1982).

attention to the case when conflict costs are zero for workers.[10]

Note that downward nominal rigidity implies that there is a potential relationship between wage negotiations today and in the future. This interdependence comes from two sources. First, the wage contract is a state variable in future negotiations and second, the wage set today affects prices set in the future which, in turn, may affect future wage negotiations. The first interdependence is eliminated by using the steady state distribution in the log linearization of the model (see Appendix C for details and the caveat in section 2 for a further discussion). The second interdependence is eliminated by the assumption that prices can be changed whenever wages are allowed to change. Then, given these two steps, each wage negotiation can be analyzed separately, as in the standard Calvo setup, thereby leading to a very simple and tractable framework. Note also that there will be no intertemporal interdependence in price setting decisions for a given firm either. To see this, note that since prices can be adjusted in any direction, the current price is not a state variable in future price setting. Any interdependence in price setting over time must thus come via wage negotiations, but such interdependence is ruled out by the assumption that prices change whenever wages change.

Unions

The union at a firm represents all workers at the firm and maximizes the welfare of all members. Defining per-period utility (in the cash-less limiting case), for a given contract wage, as

$$\Upsilon_{t,t+k}(f) = u\left(C_{t+k}\right) - v\left(L_{t,t+k}\left(f\right)\right), \tag{10}$$

where $L_{t,t+k}\left(f\right)$ denotes labor demand in period $t+k$ when prices were last reset in period t. Moreover, let

$$\zeta_{t+k}\left(d_{t+k}(f)\right) = \alpha_w + (1 - \alpha_w)\, F_{t+k}\left(d_{t+k}(f)\right), \tag{11}$$

denote the probability that firm f's wages are unchanged in period $t + k$. The term $F_{t+k}\left(d_{t+k}(f)\right)$ is then firm f's probability that the wage is not adjusted conditional on renegotiation taking place, which is a function of

$$d_{t+k}(f) = \frac{W_{t+k}^o\left(f\right)}{W_t\left(f\right)}, \tag{12}$$

where $W_t\left(f\right)$ is the current contract and $W_{t+k}^o\left(f\right)$ denotes the unconstrained optimal wage in period $t + k$ for firm f, defined as the wage upon which parties would agree in period $t + k$ if all conflict costs were temporarily removed in period $t + k$. Then, let U_{t+k}^u denote union utility when the wage is

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[10] A full explanation of downward nominal wage rigidity is likely to include several mechanisms that may be complementary. Studies like Bewley (1999), and others point towards psychological mechanisms involving fairness considerations and managers' concern over workplace morale. Moreover, the workers' yardstick for fairness seems to be what happens to nominal rather than real wages. However, here we focus on the fully rational explanation proposed by Holden (1994).

renegotiated in period $t + k$. Union utility in period t, U_t^u, is then a probability weighted discounted sum of future per-period payoffs, i.e.

$$
\begin{aligned}
U_t^u &= E_t \sum_{k=0}^{\infty} (\alpha_w \alpha \beta)^k \, \Upsilon_{t,t+k}(f) + E_t \sum_{k=1}^{\infty} \prod_{i=0}^{k-1} \zeta_{t+i} \left(d_{t+i}(f)\right) \qquad (13) \\
&\times \left[\left(\zeta_{t+k} \left(d_{t+k}(f)\right) - \alpha_w \alpha\right) \beta^k \sum_{j=0}^{\infty} (\alpha_w \alpha \beta)^j \, \Upsilon_{t+k,t+k+j}(f) + \left(1 - \zeta_{t+k} \left(d_{t+k}(f)\right)\right) \beta^k U_{t+k}^u \right].
\end{aligned}
$$

To see the intuition behind the summations in (13), note that the first summation in (13) corresponds to the case when prices are never changed in the future, whereas the second summation corresponds to outcomes that include future price changes. To understand the second summation in (13), first note that the terms inside the squared bracket are multiplied by the probability of the wage not having been changed up to period $k - 1$ (i.e. $\prod_{i=0}^{k-1} \zeta_{t+i} \left(d_{t+i}(f)\right)$). Then, within a period, $t + k$, prices can change in two ways. First, the price can change without the wage changing, which happens with probability $(\zeta_{t+k} \left(d_{t+k}(f)\right) - \alpha_w \alpha)$.[11] Then, this probability is the weight for the utility associated with a reset price in period $t + k$.[12] The second way in which prices are changed in period $t + k$ is if the wage changes, which happens with probability $(1 - \zeta_{t+k} \left(d_{t+k}(f)\right))$. Then, this probability is the weight for the utility associated with resetting the wage (and price) in period $t + k$. Note that U_{t+k}^u is in itself independent of the (unconstrained) wage bargained over today. Finally, for confirmation, we note that the sum of probabilities inside the squared bracket at period $t + k$ equals the probability of prices being changed within period $t + k$ (i.e., $(1 - \alpha_w \alpha)$).

Firms

Let real per-period profits in period $t + k$, when the price was last rewritten in period t, be denoted as

$$
\phi_{t,t+k} \left(W_t \left(f\right)\right) = (1 + \tau) \frac{P_t^o \left(f\right)}{P_{t+k}} Y_{t+k} \left(f\right) - tc \left(\frac{W_t \left(f\right)}{P_{t+k}}, Y_{t+k} \left(f\right)\right), \qquad (14)
$$

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[11] To understand this probability, note that we have the outcome that the price but not the wage changes in two cases: First, if the firm is drawn for a price change but not for a wage change (which happens with probability $(1 - \alpha)\alpha_w$) and second, if the firm is drawn for wage bargaining but downward nominal wage rigidity prevents a wage change (which happens with probability $(1 - \alpha_w)F_{t+k} \left(d_{t+k}(f)\right)$).

[12] Although the utility from a reset price in period $t + k$ is formulated as if the price would never again change in the future, it is straightforward to show that the summations here keep track of outcomes where the price is changed more than once.

where tc denotes real total cost. Firm payoff U_t^f is then

$$U_t^f = E_t \sum_{k=0}^{\infty} (\alpha_w \alpha)^k \psi_{t,t+k} \phi_{t,t+k} (W_t(f)) \tag{15}$$

$$+ E_t \sum_{k=1}^{\infty} \left(\prod_{i=0}^{k-1} \zeta_{t+i}(d_{t+i}(f)) \right) \left(\zeta_{t+k}(d_{t+k}(f)) - \alpha_w \alpha \right) \sum_{j=0}^{\infty} (\alpha_w \alpha)^j \psi_{t+k,t+k+j} \phi_{t+k,t+k+j} (W_t(f))$$

$$+ E_t \sum_{k=1}^{\infty} \left(\prod_{i=0}^{k-1} \zeta_{t+i}(d_{t+i}(f)) \right) \left(1 - \zeta_{t+k}(d_{t+k}(f)) \right) \psi_{t,t+k} U_{t+k}^f,$$

where the term $\psi_{t,t+k}$ denotes how the households (which own an aliquot share of each firm) value real profits in period $t + k$ when in period t. The intuition behind the sums in (15) is analogous to that of the sums in (13) discussed above.

Bargaining

Since the Rubinstein-Ståhl solution can be found by solving the Nash Bargaining problem, we can solve for the unconstrained wage from

$$\max_{W_t(f)} (U_t^u - U_o)^\varphi \left(U_t^f \right)^{1-\varphi}, \tag{16}$$

where φ is the household's relative bargaining power and U_o its threat point. The threat point is the payoff when there is disagreement (i.e., strike or lockout). The payoff of the firm when there is a disagreement is assumed to be zero. Households are assumed to receive a share of steady-state (after tax) income and not spend any time working. This interpretation of threat points is in line with a standard Rubinstein-Ståhl bargaining model with discounting and no risk of breakdown as presented in Binmore, Rubinstein, and Wolinsky (1986) (see also Mortensen, 2005, for an application of this bargaining setup). A constant U_o leads to a very convenient and simple analysis; more complicated models of threat points, e.g. based on workers having the opportunity to search for another job, could also be introduced in this model. However, as argued by Hall and Milgrom (2005), the threat points should not be sensitive to factors like unemployment or the average wage in the economy, since delay is the relevant threat as opposed to permanently terminating the relationship between the firm and the workers. For example, United Auto Workers permanently walking away from GM is never on the table during wage negotiations, as pointed out by Hall and Milgrom (2005). The first-order condition to problem (16) is

$$\varphi U_t^f \frac{\partial U_t^u}{\partial W(f)} + (1 - \varphi)(U_t^u - U_o) \frac{\partial U_t^f}{\partial W(f)} = 0. \tag{17}$$

Then, if $W_t^o(f)$ is the solution to the above problem, which is equal across all firms that are allowed

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to renegotiate, the resulting wage for firm f with the old contract $W_{t-1}(f)$ is

$$\max\{W_t^o(f), W_{t-1}(f)\}. \tag{18}$$

Thus, in the case that the unconstrained optimal wage is lower than the present wage contract, the old wage contract prevails due to the conflict cost structure outlined above.

As in price setting, we eliminate the distortions that result from bargaining. Since there are two instruments that can be used to achieve this, i.e., τ_w and U_o, one of them is redundant. Here, we use the method in Carlsson and Westermark (2006a), relying on adjusting τ and U_o to achieve efficiency.

1.3.1 Wage Evolution

Taking into account that firms cannot substitute across workers, the average wage is determined by

$$
\begin{aligned}
W_t &= \alpha_w \int_0^1 W_{t-1}(f)\,df + (1-\alpha_w)\int_{W_{t-1}(f)>W_t^o(f)} W_{t-1}(f)\,df \\
&\quad + (1-\alpha_w)\int_{W_{t-1}(f)\leq W_t^o(f)} W_t^o(f)\,df,
\end{aligned} \tag{19}
$$

where the second term of (19) is due to downward nominal wage rigidity.

1.4 Steady State

As discussed above, downward nominal wage rigidity is not likely to have any noticeable real effects in periods with high inflation rates. However, inflation rates have come down in most countries in recent decades and prolonged periods of very low inflation rates are no longer uncommon. In figure 1, we plot the CPI-inflation rate (fourth quarter-to-quarter) for Japan, Sweden and Switzerland and put a shade on low inflation periods, identified as quarters where the five-point moving average of CPI inflation is below 1 percent. As can be seen in figure 1, lengthy periods where the maneuvering space for adjusting real wages without reducing nominal wages is seriously limited is very much a real world possibility.

To set ideas and capture the main mechanisms at work, we focus on a zero steady state inflation regime in this paper. It is possible to allow for a (small) positive steady state inflation rate. However, in order to retain tractability, we then need to index wages and prices that cannot be changed.[13] But indexation implies that welfare is independent of the steady state inflation rate. To see this, note that indexation implies that the downward nominal rigidity will be centered around the positive steady

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[13]Indexation is needed since it is otherwise impossible to eliminate expectations of variables for more than one period ahead in the first-order conditions for wage and price setting. Thus, in the absence of indexation, it is necessary to keep track of infinite sums.
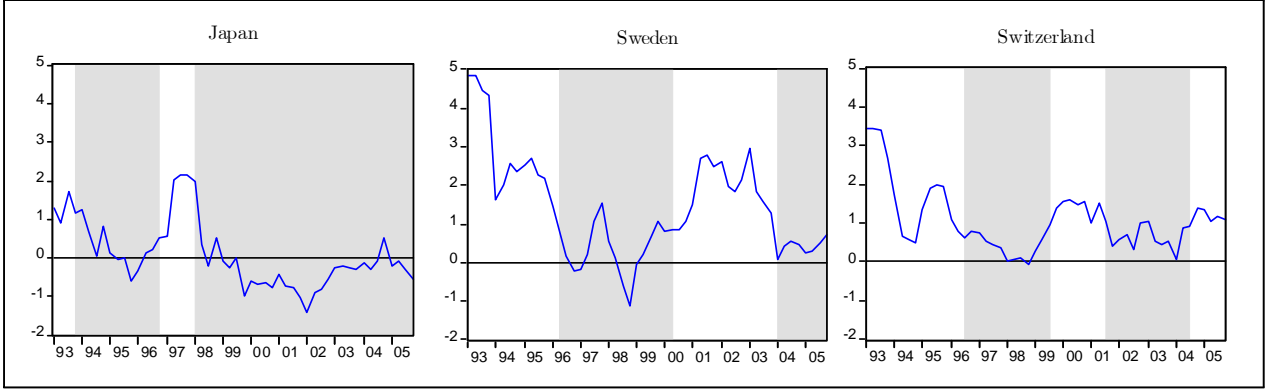
11

Figure 1: CPI-inflation rate in percentage units (fourth quarter-to-quarter). Shaded Regions indicate periods with average inflation (five point moving average) below 1 percent.

state inflation rate instead of zero. Or, in other words, wages cannot grow slower than the steady state inflation rate. This then gives rise to a an identical problem where downward wage rigidity binds just as often as in the zero steady state case; hence the focus on a zero steady state regime here.

In the zero-inflation non-stochastic steady state, A_t is equal to its steady-state value, $\bar{A}$. Moreover, all firms produce the same (constant) amount of output, i.e. $\overline{Y}(f) = \overline{Y}$, using the same (constant) quantity of labor and all households supply the same amount of labor, i.e. $\overline{L}(f) = \overline{L}$. Moreover, we will have that $\overline{C} = \overline{Y}$ and that $B = 0$. M and P are constant.

To find the steady state of the model, we use the production function (4) together with the efficiency condition $\overline{MPL} = \overline{MRS}$ (which holds due to having eliminated distortions as in Carlsson and Westermark (2006a)) to solve for $\overline{L}$ and, in turn, $\overline{Y}$ and $\overline{C}$.

2   Equilibrium

First, let the superscript * denote variables in the flexible price and wage equilibrium, to which we refer below to as the natural equilibrium, and a hat above a small letter variable denotes log-deviations from the steady-state level of the variable. Linearizing around the steady state then gives the following system of equations, where the parameters are given in Appendix B,

$$\hat{x}_t = E_t\left(\hat{x}_{t+1} - \frac{1}{\rho_C}\left(\hat{i}_t - \hat{\pi}_{t+1} - \widehat{r}_t^*\right)\right), \tag{20}$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + (1-\gamma)\left(\hat{\pi}_t^\omega - \beta E_t \hat{\pi}_{t+1}^\omega\right) + \Pi\left(\hat{w}_t - \hat{w}_t^*\right) + \frac{\gamma}{1-\gamma}\Pi\hat{x}_t, \tag{21}$$

$$\hat{\pi}_t^\omega = \max\left\{0, \frac{1+\alpha_w}{2\alpha_w}\beta E_t \hat{\pi}_{t+1}^\omega - \Omega_w\left(\hat{w}_t - \hat{w}_t^*\right) - \Omega_x \hat{x}_t\right\}, \tag{22}$$

$$\hat{w}_t = \hat{w}_{t-1} + \hat{\pi}_t^\omega - \hat{\pi}_t. \tag{23}$$

For clarity, all parameters are defined to be positive.

Equation (20) is a standard goods-demand (Euler) equation which relates the output gap $\hat{x}_t$, i.e. the log-deviation between output and the natural output level, to the expected future output gap and the expected real interest rate gap $(\widehat{i}_t - \hat{\pi}_{t+1} - \widehat{r}_t^*)$, where $\widehat{i}_t$ denotes the log-deviation of the nominal interest rate from steady state and $\widehat{r}_t^*$ is the log-deviation of the natural real interest rate from its steady state.[14] This relation is derived taking standard steps and using the households' first-order condition with respect to consumption, i.e., the consumption Euler equation.

The price-setting (Phillips) curve, equation (21), is derived using the firms' first-order condition (7), (see Carlsson and Westermark, 2006b, for details) and is similar in shape to the price-setting curve derived by Erceg, Henderson, and Levin (2000), with the exception that current and expected future wage inflation also enter the expression. Thus, price setting is affected by the real wage gap, i.e., the log deviation between the real wage and the natural real wage $(\hat{w}_t - \hat{w}_t^*)$, the output gap $\hat{x}_t$, future inflation $E_t\hat{\pi}_{t+1}$ and current and future wage inflation $\hat{\pi}_t^\omega$, $E_t\hat{\pi}_{t+1}^\omega$. As can be seen from Carlsson and Westermark (2006b), the relevant real marginal cost measure driving inflation depends on the real wage gap in firms that actually change prices (and, naturally, capital prices and productivity). However, since we are interested in a price-setting relationship expressed in terms of the economywide real wage gap, we need to adjust for the fact that interdependence in price and wage setting implies that the economywide real wage gap and the real wage gap in firms that actually change prices are different in our model.[15] This motivates the "correction term" $(1 - \gamma)\left(\hat{\pi}_t^\omega - \beta E_t\hat{\pi}_{t+1}^\omega\right)$. Thus, in expression (21), the real wage change in firms that change prices has been decomposed into the aggregate real wage change $\hat{w}_t$ and wage inflation terms $\hat{\pi}_t^\omega$, $E_t\hat{\pi}_{t+1}^\omega$.

Equation (22) describes the wage setting behavior (see Appendix C and Carlsson and Westermark, 2006b, for details). From section (1.3) above, we know that wages are set according to (18). This implies that wage inflation is non-negative and set according to the last term in the max operator of (22) when positive. Hence, the max operator captures the restriction from wage setting in (22). For positive wage inflation rates, wage inflation increases with higher expected wage inflation. The coefficient in front of $E_t\hat{\pi}_{t+1}^\omega$, i.e. $\frac{1+\alpha_w}{2\alpha_w}\beta$, is the probability adjusted discount rate from the wage negotiations $\frac{1+\alpha_w}{2}\beta$ (where $\frac{1+\alpha_w}{2}$ is the (unconditional) steady-state probability that wages remain unchanged in the next period) multiplied by $\frac{1}{\alpha_w}$, which governs how relative wages today (conditional on $\hat{\pi}_t^\omega > 0$) feed into wage-inflationary pressure. Moreover, as in Erceg, Henderson, and Levin (2000), wage inflation is influenced by the real wage gap and the output gap. Since the parameters associated

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[14] The nominal interest rate I_t is defined as the rate of return on an asset that pays one unit of currency under every state of nature at time $t + 1$.

[15] Specifically, since all firms that are allowed to change wages are also allowed to change prices, the share of wage-changing firms among the firms that change prices differs from the economywide average.

with these variables are determined by the bargaining problem, the size (and even the sign) of them depend on e.g. the relative bargaining strength. See Carlsson and Westermark (2006a) for a detailed discussion on wage setting in the unconstrained case.[16]

A caveat is in place here since the linearized wage-setting curve (22) is derived using the steady state wage distribution. In general, since the last period's wage is a state variable in today's wage setting problem, the aggregate wage outcome today will depend on the history of wage changes in the economy, described by the wage distribution. However, starting from an initial distribution where all firm/union pairs have the same wage, this will not be a problem when downward nominal wage rigidity binds, since no one will reduce the wage anyway, although for periods beyond the first when wage inflation is positive, the wage distribution potentially affects the aggregate wage inflation outcome. We take this approach since it allows us to retain analytical tractability of the problem. Moreover, as discussed above, this simplification should not lead us too far astray.

Finally, the evolution for the real wage (23) follows from the definition of the aggregate real wage and states that today's real wage is equal to yesterday's real wage plus the difference between the rates of wage and price change $(\hat{\pi}_t^\omega - \hat{\pi}_t)$.

As a comparison to the results from the economy with downward nominal wage rigidity, it is useful to look at an economy where wages can adjust symmetrically. As shown in Carlsson and Westermark (2006a), the unconstrained economy is described by (20), (21), (23) and replacing (22) with

$$\hat{\pi}_t^\omega = \beta E_t \hat{\pi}_{t+1}^\omega - \Omega_w^{uc} \left(\hat{w}_t - \hat{w}_t^* \right) - \Omega_x^{uc} \hat{x}_t \tag{24}$$

where, once more, the parameter definitions are given in Appendix B.

3   The Monetary Policy Problem

The central bank is assumed to maximize social welfare. Here, we focus on the discretionary policy case. Although studying optimal policy is in essence a normative enterprise, given that no central bank formally commits to a policy rule it is natural to focus on the discretionary case. Following the main part of the monetary policy literature, we focus on the limiting cashless economy (see e.g. Woodford (2003) for a discussion) with the social welfare function

$$E_t \sum_{t=0}^{\infty} \beta^t \left(u\left(C_t \right) - \int_0^1 v\left(L_t\left(f \right) \right) df \right). \tag{25}$$

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[16]See also Carlsson and Westermark (2006a), for a detailed comparison between the unconstrained version of (22), i.e. equation (24) below, and the wage setting curve resulting from the Erceg, Henderson, and Levin (2000) model.

Following Rotemberg and Woodford (1997), Erceg, Henderson, and Levin (2000), and others, we take a second-order approximation to (25) around the steady state. This yields a standard expression for the welfare gap (see Appendix C.5 for a detailed derivation, also c.f. Erceg, Henderson, and Levin (2000)), i.e., the discounted sum of log-deviations of welfare from the natural (flexible price and wage welfare level)

$$E_t \sum_{t=0}^{\infty} \beta^t \left(\theta_x \left(\hat{x}_t \right)^2 + \theta_\pi \left(\hat{\pi}_t \right)^2 + \theta_{\pi^\omega} \left(\hat{\pi}_t^\omega \right)^2 \right), \tag{26}$$

where we have omitted higher order terms and terms independent of policy. As usual, $\theta_x < 0$, $\theta_\pi < 0$ and $\theta_{\pi^\omega} < 0$ (see Appendix B for definitions). The first term captures the welfare loss (relative to the flexible price and wage equilibrium) from output gap fluctuations stemming from the fact that $\widehat{mpl}$ will differ from $\widehat{mrs}$ whenever $\hat{x}_t \neq 0$. However, even if $\hat{x}_t = 0$, there will be welfare losses due to nominal rigidities. The reason is that nominal rigidities imply a non-degenerate distribution of prices and wages. A non-degenerate distribution of prices and wages implies a non-degenerate distribution of output across firms and working hours across households. This leads to welfare losses due to a decreasing marginal product of labor and an increasing marginal disutility of labor.

Note that welfare only depends on variables $\hat{x}_t$, $\hat{\pi}_t$ and $\hat{\pi}_t^\omega$ which, in turn, can solely be determined from equations (21) to (23). To find the optimal rule under discretion, the central bank then solves the following problem

$$V \left(\hat{w}_{t-1}, \hat{w}_t^* \right) = \max_{\{\hat{x}_t, \hat{\pi}_t, \hat{\pi}_t^\omega, \hat{w}_t\}} \theta_x \left(\hat{x}_t \right)^2 + \theta_\pi \left(\hat{\pi}_t \right)^2 + \theta_{\pi^\omega} \left(\hat{\pi}_t^\omega \right)^2 + \beta E_t V \left(\hat{w}_t, \hat{w}_{t+1}^* \right), \tag{27}$$

subject to equations (21) to (23), disregarding that expectations can be influenced by policy.

The wage inflation restriction (22) can be replaced by

$$\hat{\pi}_t^\omega \geq \frac{1 + \alpha_w}{2\alpha_w} \beta E_t \hat{\pi}_{t+1}^\omega - \Omega_x \hat{x}_t - \Omega_w \left(\hat{w}_t - \hat{w}_t^* \right) - \hat{\pi}_t^\omega, \tag{28}$$

$$\hat{\pi}_t^\omega \geq 0. \tag{29}$$

Note that the problem with the original max constraint (23) and the problem with inequality constraints (28) and (29) need not be equivalent. It is obviously true that a solution $(\hat{x}_t, \hat{\pi}_t, \hat{\pi}_t^\omega, \hat{w}_t)$ to the problem with the original max constraint also satisfies the two inequality constraints. However, it is possible that there is a solution $(\hat{x}_t, \hat{\pi}_t, \hat{\pi}_t^\omega, \hat{w}_t)$ to the problem with inequality constraints, so that none of the inequality constraints is binding, thus leading to a violation of the original max constraint. However, this is ruled out by the following Lemma.

Lemma 1 *At least one of the inequality constraints (28) and (29) must be binding.*

Proof: See Appendix D. ∎

Thus, this possibility is ruled out by the above Lemma, thereby implying that the problems are equivalent. The intuition for the result is the following. Since the constraints (28) and (29) both put lower bounds on $\hat{\pi}_t^\omega$ and, as can be seen from expression (26), welfare is decreasing in $\hat{\pi}_t^\omega$, the central bank sets $\hat{\pi}_t^\omega$ as low as possible, implying that one of the inequality constraints (28) and (29) must bind.

From the above, it follows that the central banks' problem (27) gives rise to two systems depending on whether the inequality constraint binds. These systems, in turn, consist of the case specific first-order conditions for optimal policy and restrictions from private sector behavior (see Appendix D for details).

3.1 Numerical Solution and Calibration

To solve the model, we find the paths for $\hat{x}_t$, $\hat{\pi}_t$, $\hat{\pi}_t^\omega$ and $\hat{w}_t$ that maximize welfare, as suggested by Woodford (2003).[17,18] As in Erceg, Henderson, and Levin (2000), we look at the effects of a technology shock, which is assumed to follow an AR(1). It is straightforward to show that there is a positive linear relationship between $\hat{w}_t^*$ and $\hat{A}_t$.[19] Then, if technology follows an AR(1) process, $\hat{w}_t^*$ also follows an AR(1) process. We can thus model $\hat{w}_t^*$ as

$$\hat{w}_t^* = \eta \hat{w}_{t-1}^* + \varepsilon_t, \tag{30}$$

where ε_t is an (scaled) i.i.d. (technology) shock with standard deviation σ_ϵ.

For our numerical exercises, we follow Erceg, Henderson, and Levin (2000), and assume that

$$u(C_t) = \frac{1}{1-\chi_C}\left(C_t - \bar{Q}\right)^{1-\chi_C}, \tag{31}$$

and that

$$v(L_t) = -\frac{1}{1-\chi_L}\left(1 - L_t - \bar{Z}\right)^{1-\chi_n}. \tag{32}$$

Here, we introduce $\bar{Q}$ and $\bar{Z}$ in order to facilitate the comparison with Erceg, Henderson, and Levin

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[17]We solve the problem in a different way than Erceg, Henderson, and Levin (2000), where an interest rate rule is postulated and the parameters are chosen to maximize welfare.

[18]To solve for the optimal instrument rule, the paths can be used together with the Euler equation and suitable criteria for the shape of the rule; see Woodford (2003), for a discussion.

[19]It is possible to allow for other shocks. In the Technical Appendix of Carlsson and Westermark (2006a), we also introduce a consumption shock and a labor-supply shock as in Erceg, Henderson, and Levin (2000). However, introducing these shocks does not yield any additional insights here. In fact, it can easily be shown that under optimal policy, all disturbances in the model (introduced as in Erceg, Henderson and Levin, 2000) can be reduced to a single disturbance term (being a linear combination of all these shocks).

(2000), by mimicking the preferences and the steady state of their model.[20] The calibration of the deep parameters, presented in Table 1, also follows Erceg, Henderson, and Levin (2000), when possible (thus, e.g., we do not follow Erceg, Henderson and Levin, 2000, when calibrating α and α_w, since they have a different interpretation in our model).

Table 1: Baseline Calibration of the Model

Deep Parameters	Baseline values	Derived Parameters	Baseline values Constrained (Unconstrained)
d_p	2	Π	0.505
d_w	6	Ω_x (Ω_x^{uc})	0.005 (0.002)
β	0.99	Ω_w (Ω_w^{uc})	0.110 (0.044)
γ	0.30	θ_x	-0.962
χ_C	1.5	θ_π	-1.043
χ_n	1.5	θ_{π^ω} $(\theta_{\pi^\omega}^{uc})$	-2.676 (-7.458)
σ	4	α (α^{uc})	0.750 (0.600)
η	0.95	α_w (α_w^{uc})	0.667 (0.833)
σ_ϵ	0.0067		
φ	0.5		

Moreover, to find the steady state of the model, we also follow Erceg, Henderson, and Levin (2000) and set: $\overline{Q} = 0.3163, \overline{Z} = 0.03, \overline{K} = 30\overline{Q}$ and $\overline{A} = 4.0266$. Then, using the scheme outlined in section (1.4) we obtain $\overline{L} = 0.27$. Thus, $\overline{L}$ and $\overline{Z}$ stand for about one quarter of the households' time endowment. Further, $\overline{Y} = \overline{C} = 3.1627$, giving rise to a steady state capital-output ratio of about three. Moreover, to achieve symmetric Nash bargaining (equally shared surplus), we set the bargaining power of the union φ to 0.5.

Here, we treat price and wage contract durations as deep parameters. The probabilities of price and wage adjustment are then derived from price and wage contract durations. This is due to the fact that when comparing economies with and without downward nominal wage rigidity, we can either keep price and wage resetting probabilities fixed or price and wage contract durations fixed. We find it natural to compare economies with the same contract durations. Letting d_p and d_w denote the duration of price and wage contracts, respectively, we have $d_p = d_p^{uc} = 1/(1-\alpha_w\alpha)$ and $d_w = 1/(1-(1+\alpha_w)/2)$ and $d_w^{uc} = 1/(1-\alpha_w)$ with and without downward nominal wage rigidity. Starting with wage contract duration, Taylor (1999), summarizes the evidence and argues that overall, the evidence points toward a wage contract duration of about one year. However, Cecchetti (1987), found that average duration increases in periods with low inflation, which is what we want to capture here. In fact, during the 1950s and 1960s when inflation was low in the U.S., the wage contract duration was about two years for the large union sector. In the baseline calibration, we set the duration to six quarters, which

is higher than suggested by Taylor (1999), but still being conservative relative to the low inflation estimate from Cecchetti (1987).

For price contract duration, the micro evidence presented by Bils and Klenow (2004), suggests a price duration of about five months, whereas the micro evidence presented by Nakamura and Steinsson (2007), and the survey evidence in Blinder, Canetti, Lebow, and Rudd (1998), suggest about eight months. In our baseline calibration, we set the price duration to two quarters which is in the range given by the above studies.

Then, using the steady state solution together with the definitions for the derived parameters (presented in appendix A) yields the values presented in Table 1.

It is interesting to see the high coefficient for wage inflation variance in the loss function (θ_{π^ω}). Starting with the constrained case, we see that the coefficient on wage inflation variance (θ_{π^ω}) is about three times larger than the coefficient on the variance in the output gap (θ_x) and the coefficient on the variance in inflation (θ_π). Thus, variation in wage inflation is associated with considerable welfare losses. Moreover, when relaxing the downward nominal wage rigidity constraint, the coefficient on wage inflation almost triples in size relative to the constrained economy. The reason is that since wages never fall in the constrained economy there will be a cap on the relative wage misalignment that a given wage inflation gives rise to.

To obtain a ball-park estimate of σ_ϵ, we make use of a the Taylor (1993) rule estimated by Rudebusch (2002) (see expression (33) below) as an approximation of actual monetary policy and impose it on the unconstrained version of the model. Then, we set σ_ϵ to match the standard deviation of quarterly inflation in the model with the actual standard deviation of the U.S. quarterly CPI inflation (1987:Q4-1999:Q4).[21] This results in a standard deviation of the innovation to the $\hat{w}_t^*$ process of 0.0067 $(= \sigma_\epsilon)$.

Numerically, we solve the model by iterating on the policy functions and updating the value function given the new policy functions. In the procedure, we also take into account how expectations in the constraints are affected by this (see (121) in Appendix D).

A standard reference for algorithms with occasionally binding constraints is Christiano and Fisher (2000). Unfortunately, we cannot use this algorithm since our model is slightly different. Specifically, our problem includes expectations of the control variables in the constraints. The way in which we take care of this problem is to use that the control variables are functions of the state variables (i.e., the policy functions) and rewrite the constraint set in terms of state variables only.[22] This is related

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[21] We focus on inflation for the calibration, since this is the only variable we can directly observe without resorting to some filtering technique.

[22] Note that policy functions are potentially nonlinear, since there is a non-linear constraint to the problem (i.e. constraint (22)).

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to the method used in e.g., Soderlind (1999). However, instead of using the policy functions from the previous iteration as is done in Soderlind (1999), we use the current policy functions, as in the algorithm used in e.g., Krusell, Quadrini, and Rios-Rull (1996). Note that the algorithm in Krusell, Quadrini, and Rios-Rull (1996) can be considered as analyzing a one-period deviation from a proposed policy. Iteration finishes when there are no gains from deviating from the proposed policy. The full algorithm is outlined in appendix A.

4   Optimal Policy V.S. Simple Rules

First, we solve the model for the calibration described above, both in the case with and without downward nominal wage rigidity under optimal discretionary monetary policy. In figure 2, we plot the impulse responses to a one standard deviation negative shock to the natural real wage $\hat{w}_t^*$.
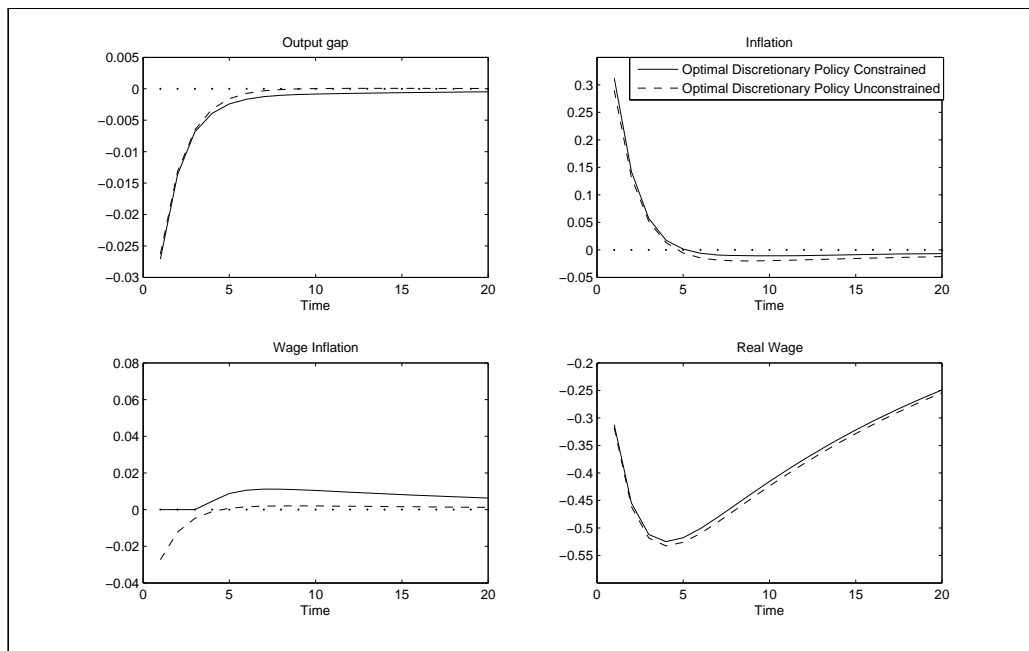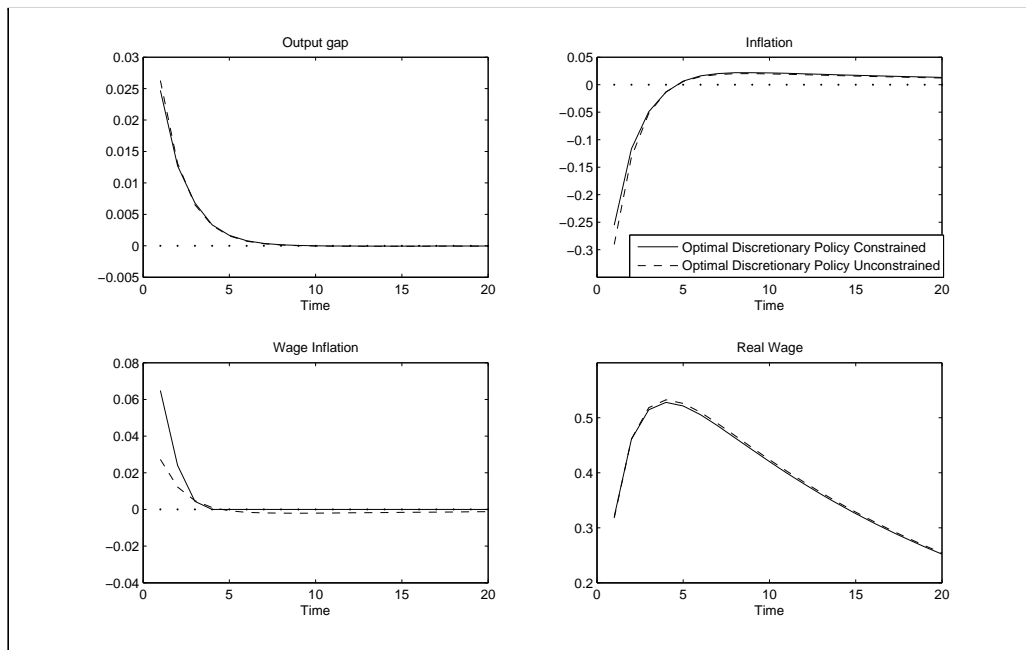

Figure 2: Impulse responses to a one standard deviation negative shock in the natural real wage. Scale corresponds to percentage units.

Starting with the unconstrained case, the negative shock drives down the natural real wage implying that the actual real wage is higher than the natural real wage, thus initially causing a positive real-wage gap. The real wage can be adjusted by changing inflation and wage inflation. Holding the inflation rate above the wage inflation rate decreases the real wage. However, since it is costly to stabilize the real wage gap, in terms of the implied variation in inflation, wage inflation and the output gap, it is optimal not to fully compensate for the shock. For the same reason, optimality requires that inflation and wage inflation should be kept (approximately) at opposite sides of zero (the path of $\hat{x}_t$ must also

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be considered). Thus, the optimal initial inflation response is positive, whereas the wage inflation response is negative. But the difference between them is not sufficiently large to immediately fully close the real-wage gap.

Given the AR(1) structure of the shock, the natural real wage increases towards the steady value of zero after the initial negative shock. So at some point, the central bank needs to start increasing the real wage in order to continue to stabilize the economy. This is also what we see after approximately four quarters. For this purpose, the relationship between inflation and wage inflation needs to be reversed, which also happens at this point in time.

In the constrained case, wage inflation cannot be used to initially lower the real wage. Instead, optimal policy prescribes a stronger initial inflation reaction relative to the unconstrained case, and also allow for a larger output gap in order to stabilize the real-wage gap. However, the initial optimal inflation response is not sufficient to stabilize the real wage gap to the same extent as in the unconstrained case. This is reflected by the initial real wage response in the constrained case lying above the real wage response in the unconstrained case.

In figure 3, we plot the resulting impulse responses to a one standard deviation positive shock to $\hat{w}_t^*$. For the unconstrained case, the impulse responses are a mirror image through the horizontal
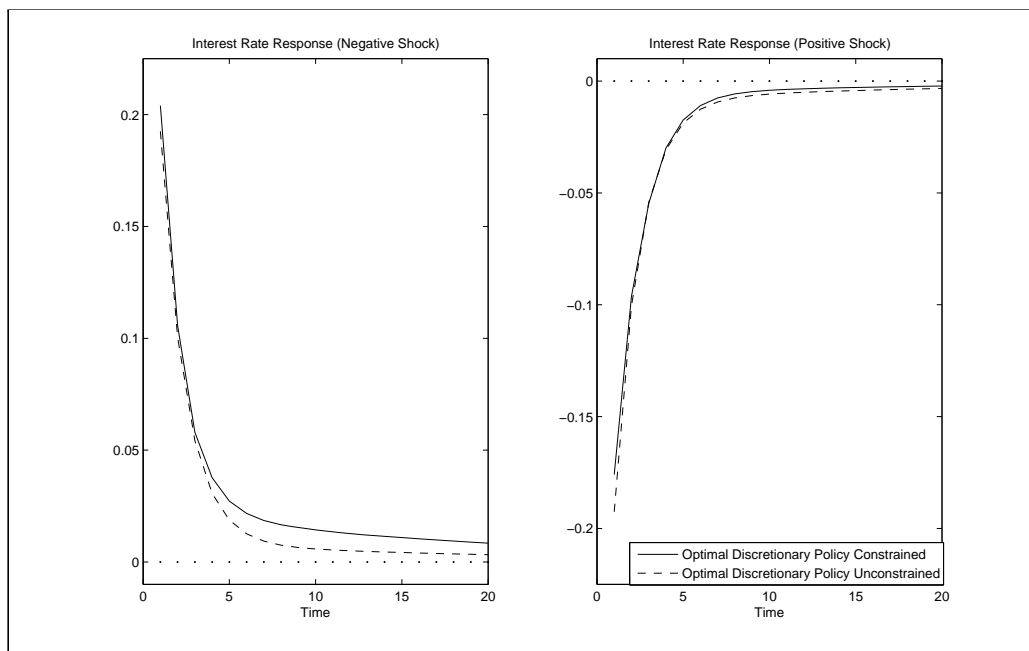

Figure 3: Impulse responses to a one standard deviation positive shock in the natural real wage. The scale corresponds to percentage units.

axis of the negative case. We now see that the initial optimal inflation and output gap responses are smaller than in the unconstrained case. However, the initial wage inflation response is larger than

in the unconstrained case. Note that downward nominal wage rigidity does not only affect private sector behavior but also the parameters in the loss function in making the parameter for wage inflation smaller. Below we will decompose the effects on welfare from the change in the loss function from the effects stemming from changes in private sector behavior. However, any asymmetry in the impulse response paths across positive and negative shocks must stem from private sector behavior and not from the (symmetric) loss function.

In figure 4, we plot the optimal interest rate path (in terms of deviations from steady state) for the unconstrained and constrained case, respectively. As can be seen in figure 4, the optimal interest

Figure 4: Optimal interest-rate responses to a one standard deviation positive and negative shock in the natural real wage. The scale corresponds to percentage units.

rate response is asymmetric in the constrained case. Especially, the interest rate response is larger in the constrained case for negative shocks and smaller for positive shocks relative to the unconstrained case.

Next, we turn to welfare analysis. Note first that it need not be the case that the model with the downward nominal wage rigidity constraint necessarily leads to lower welfare relative to the unconstrained case. The reason is that this is not just an additional constraint on the problem, i.e., a constraint that makes the feasible set smaller. Instead, it is a constraint that changes the choice set.[23] Also, another factor is that the parameter on wage inflation variance in the welfare function changes

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[23]To see this, consider the relationship between, say $\hat{\pi}^{\omega}$ and $\hat{x}$ in equation (22), treating other variables as constants. Then, in the unconstrained case, the relationship is linear with slope Ω_x. In contrast, in the constrained case, the relationship is piecewise linear with zero slope below some critical value of $\hat{x}$ and with slope Ω_x above this value.

when imposing downward nominal wage rigidity. Thus, overall, the welfare effect from downward nominal wage rigidity is ambiguous a priori.

To compute welfare, we construct sequences of shocks for 1000 periods and use these to find paths for the variables $\hat{x}_t$, $\hat{\pi}_t$, $\hat{\pi}_t^\omega$ and $\hat{w}_t$. Then, welfare is computed from these paths using the welfare criterion (26), ignoring the periods $t > 1000$. This is repeated 1000 times to generate an approximation of the expectation. Finally, to express the welfare loss as a fraction of steady state consumption, we scale the welfare difference (26) by $1/\left(u_C\left(\bar{C}, \bar{Q}\right)\bar{C}\right)$.

In the unconstrained case, we find a welfare difference relative to the natural (flexible price and wage) welfare level of 0.186 percentage units of steady state consumption. Interestingly, the result for the constrained case is almost identical with a difference of 0.181. Thus, downward nominal wage rigidity does not necessarily lead to large welfare losses, as is often considered. In fact, in our model we find a (very) modest welfare gain. Now, since introducing downward nominal wage rigidity does not only affect the behavior of the private sector, but also the parameter in the loss function on variation in wage inflation (c.f. Table 1 above), it is interesting to try to isolate the effects. To this end, we both solve for the unconstrained optimal policy, as well as calculate the welfare loss using the parameters for the loss function from the constrained case. The resulting welfare difference is 0.178, which is very close to the original results for the unconstrained case (0.186). Thus, the similarity between the welfare outcomes in the unconstrained and constrained cases is not driven by the increase in the loss-function parameter on wage inflation variance.

The intuition for the small welfare effects is that downward nominal wage rigidity is not all that harmful since it may help keep down wage dispersion (see below). This, in turn, can be exploited by the central bank when designing optimal monetary policy.

A Simple Rule

Next, we turn to analyzing the effects on welfare from relying on a simple instrument rule instead of the optimal policy rule. To this end, we impose a Taylor (1993) rule, i.e.

$$\widehat{i}_t = 1.24\hat{\pi}_t + 0.33\hat{x}_t, \tag{33}$$

where the parameters for (33) are calibrated to match the estimates in Rudebusch (2002). Note that this rule does not take any asymmetry into account when setting the interest rate, although the economy will react asymmetrically to positive and negative shocks, due to private sector behavior.[24]

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[24]To implement the Taylor rule, we replace the central bank's first-order condition in systems (135) and (139) in Appendix C with the sticky price Euler equation (20), where we have used the corresponding flexible-price Euler equation to eliminate the real natural interest rate and the Taylor rule to eliminate the nominal interest rate. For the system

In figure 5, we plot the resulting impulse responses to a one standard deviation negative shock to $\hat{w}_t^*$ when the nominal interest rate is governed by a Taylor rule as well as under optimal policy.
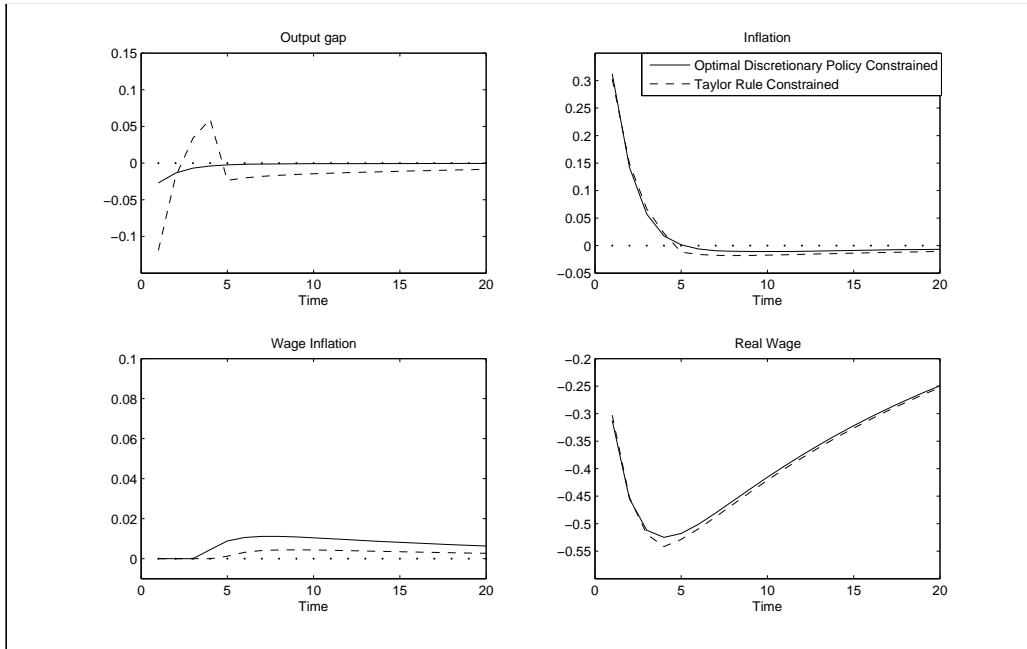

Figure 5: Impulse responses to a one standard deviation negative shock to $\hat{w}_t^*$ when the nominal interest rate is governed by a Taylor rule as well as under the optimal policy. Scale corresponds to percentage units.

Note that the impulse responses are not smooth as under optimal policy. The reason for this is that the Taylor rule is not optimally chosen and has the same functional form for both the case when downward nominal wage rigidity binds and when it does not. Interestingly, the Taylor rule responses for inflation are in fact fairly similar to the optimal responses. However, for wage inflation, the Taylor rule undershoots slightly when the constraint stops binding, all in all leading to substantial excess volatility in the output gap. In figure 6, we plot the resulting impulse responses to a one standard deviation positive shock to $\hat{w}_t^*$. Once more, the Taylor rule responses for inflation are similar to the optimal responses. However, the initial wage inflation response now overshoots. Once more, the volatility in the output gap is substantially larger than in the optimal responses. [25]

Next we turn to welfare. Note that it is not a trivial result that optimal discretionary policy outperforms the Taylor rule, since the Taylor rule is, in fact, a commitment rule and hence, could perform better than the optimal discretionary rule. However, we do find that optimal discretionary

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under the Taylor rule, there is no need to iterate on the value function. Instead, we can directly solve the system for the policy functions (i.e. we only do step 1 in the numerical algorithm outlined in Appendix A). Then, we can simulate the model and evaluate welfare as done above.

[25] For brevity, we do not plot interest rate responses, since they are mainly a reflection of differences in the output gap responses.
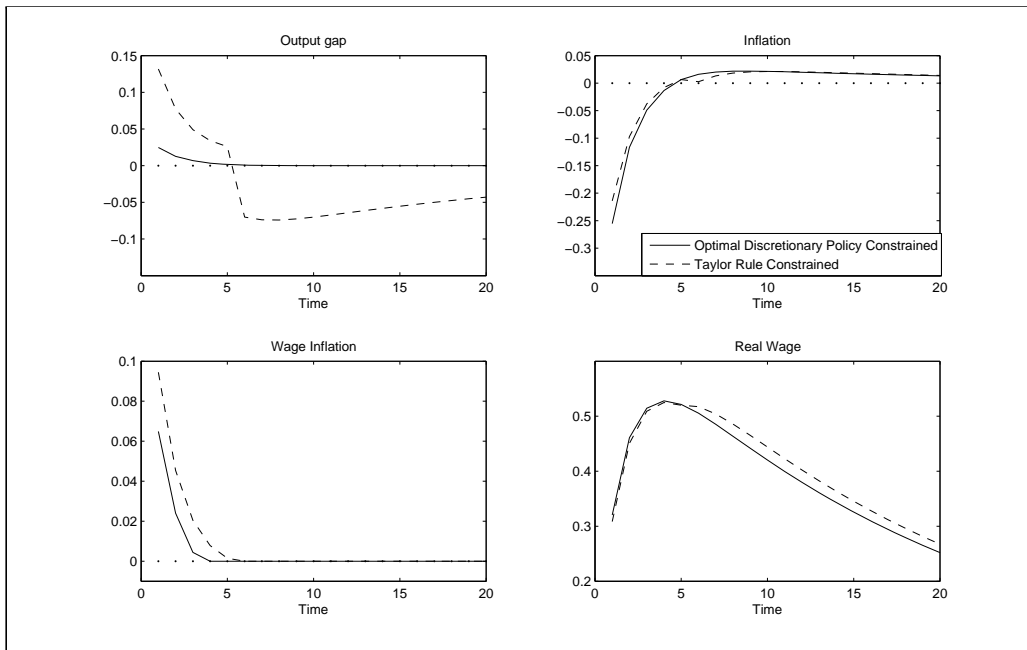
Figure 6: Impulse responses to a one standard deviation positive shock to $\hat{w}_t^*$ when the nominal interest rate is governed by a Taylor rule as well as under the optimal policy. Scale corresponds to percentage units.

policy performs better than the policy prescribed by the Taylor rule. In terms of steady state consumption, the additional loss is about 0.07 percentage units of steady state consumption. But, all in all, the Taylor rule seems to be a fairly good approximation of optimal discretionary monetary policy in the presence of downward nominal wage rigidity.

Finally, we look at the impulse responses for the unconstrained economy versus an economy with downward nominal wage rigidity under the Taylor rule. In figure 7, we plot the resulting impulse responses to a one standard deviation negative shock to $\hat{w}_t^*$. The key result here is that downward nominal wage rigidity actually helps stabilize the economy in the wage inflation dimension, whereas it does not induce much more variation in inflation and the output gap. A fairly similar result appears from figure 8, where we plot the resulting impulse responses to a one standard deviation negative shock to $\hat{w}_t^*$. Most notably, the wage inflation variability is not substantially larger in the downward rigid case. This is also the dimensions about which the households care most (c.f. table 1).

When looking at welfare differences, we find that welfare loss is quite a bit lower in the economy with downward nominal wage rigidity relative to the unconstrained economy when monetary policy is governed by the Taylor rule. In terms of steady state consumption, the welfare gain is about 0.14 percentage units of steady state consumption. Thus, an agent would in this case prefer an economy with downward nominal wage rigidity over an economy with downward nominal wage flexibility ex ante.
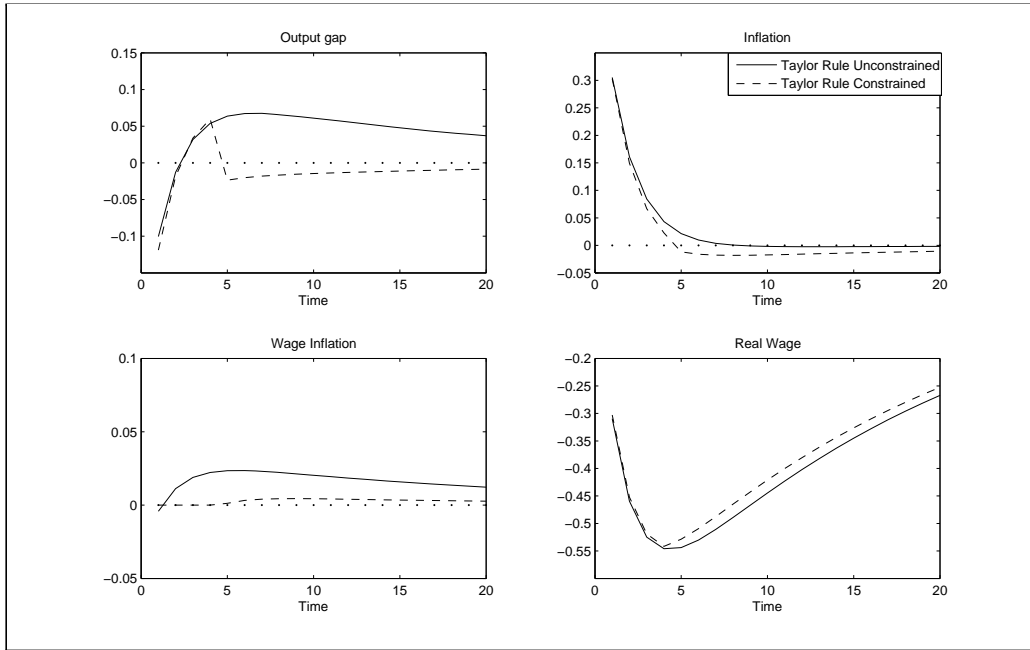
24

Figure 7: Impulse responses to a one standard deviation negative shock to $\hat{w}_t^*$ when the nominal interest rate is governed by a Taylor rule. Scale corresponds to percentage units.
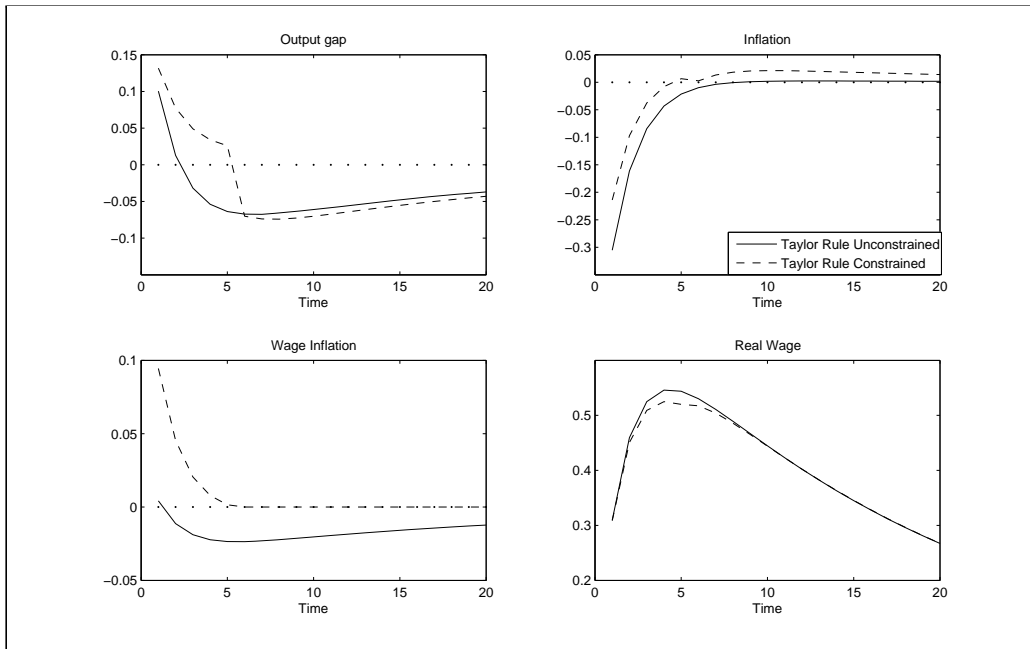

Figure 8: Impulse responses to a one standard deviation positive shock to $\hat{w}_t^*$ when the nominal interest rate is governed by a Taylor rule. Scale corresponds to percentag units.

Evaluating the unconstrained economy with the loss function of the constrained economy suggests a welfare gain from downward nominal wage rigidity of about 0.08 percent of steady state consumption. Thus, once more, the above result is not just driven by the increase in the loss-function parameter on wage inflation, when relaxing the constraint while keeping the duration of price and wage contracts constant. Instead, private sector behavior plays an important part in explaining the results.

Wage-Inflation Distributions and Welfare

To see the consequences for welfare, we illustrate the wage-inflation distributions in four histograms; see figure 9. First, if we vary policy, i.e., compare the Taylor rule and optimal policy, the distribution
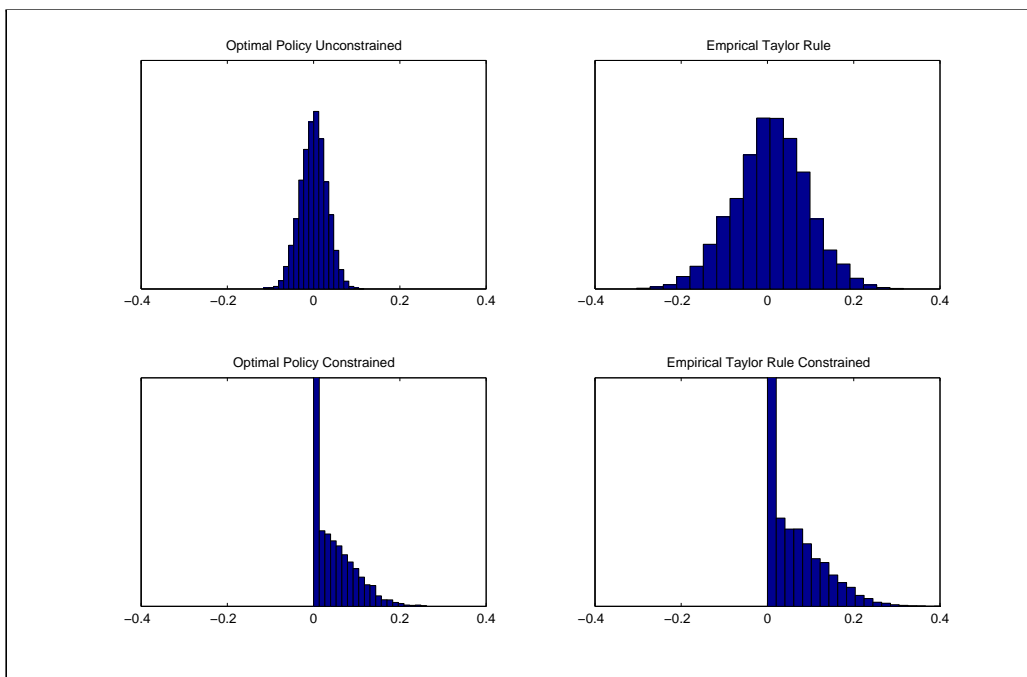
Figure 9: Histograms of simulated wage-inflation paths.

is more dispersed under the Taylor rule. This seems to hold irrespectively whether there are downward nominal wage rigidities or not. If we fix the policy regime, on the other hand, the distribution seems to have lower variability with downward nominal wage rigidities than without.

4.1 Robustness

A key parameter for welfare evaluation is the standard deviation of the innovation to $\hat{w}_t^*$, σ_ϵ. We now consider increasing σ_ϵ by 50 percent. The results of this experiment are presented in Table 2. We see that this does not change the qualitative conclusions from our baseline simulation, in terms of welfare rankings. But increasing the shock size does increase the welfare loss in all four cases.

Table 2: Robustness - Shock Size

	Baseline ($\sigma_\epsilon = 0.0067$)	Big Shocks ($\sigma_\epsilon = 0.0101$)
	Welfare Differences Relative to Flex Price	
Unc. Opt.	-0.1856	-0.4176
Con. Opt.	-0.1809	-0.4125
Unc. Taylor	-0.3872	-0.8723
Con. Taylor	-0.2508	-0.6118

The values are expressed in terms of percentage units of steady state consumption.

In the baseline calibration, the wage-contract duration is six quarters. Since this may seem to be on the high side (even though it is in line with the evidence of wage-contract durations in low-inflation environments), we also see to what extent our results are robust to varying the wage-contract duration. In table 3, we present the results from decreasing the wage duration by a quarter. As can be seen in the table, decreasing the duration of the wage contracts leads to slightly smaller welfare

Table 3: Robustness - Contract Durations

Contract Durations	$d_p = 2, d_w = 6$	$d_p = 2, d_w = 5$
	Welfare Differences Relative to Flex Price	
Unc. Opt.	-0.1856	-0.1759
Con. Opt.	-0.1809	-0.1761
Unc. Taylor	-0.3872	-0.3767
Con. Taylor	-0.2508	-0.2436

The values are expressed in terms of percentage units of steady state consumption.

losses in all cases. Also, note that the welfare ranking between the optimal policy in the constrained and unconstrained case shifts to a tiny advantage for the unconstrained case. However, the key point that downward nominal wage rigidity does not imply large welfare losses is robust.

Finally, we have also experimented to see how sensitive the results are to the exact values of the parameters in the Taylor rule. In table 4, we present the result from using the original values from Taylor (1993) (i.e., $\beta_{\hat{\pi}} = 1.5$ and $\beta_{\hat{x}} = 0.5$). When using the Taylor (1993) parameter values for

Table 4: Robustness - Taylor Rule Parameters

Taylor Rule	$\beta_{\hat{\pi}} = 1.24, \beta_{\hat{x}} = 0.33$	$\beta_{\hat{\pi}} = 1.5, \beta_{\hat{x}} = 0.5$
	Welfare Differences Relative to Flex Price	
Unc. Taylor	-0.3872	-0.3224
Con. Taylor	-0.2508	-0.2604

The values are expressed in terms of percentage units of steady state consumption.

the Taylor rule, the welfare loss decreases without downward nominal wage rigidity, while the welfare loss almost remains unchanged when downward nominal wage rigidities are present. Thus, the results indicate that downward nominal wage rigidity makes the welfare outcome less sensitive to the exact calibration of the Taylor rule.

Overall, the key points from the previous section seem to be robust.

5 Concluding remarks

In this paper, we study the implications for optimal monetary policy when declining nominal wages do not constitute a viable margin for adjustment to adverse economic conditions. To this end, a New Keynesian model is developed that can endogenously account for downward nominal wage rigidity. This is achieved by introducing wage bargaining between firms and unions in the model as in Holden (1994). Under asymmetric conflict costs, downward nominal wage rigidity arises as a rational endogenous outcome.

Focusing on optimal discretionary monetary policy, we show that when money wages cannot fall, the optimal policy response to changing economic conditions becomes asymmetric. More specifically, inflation and the output gap respond more when the downward nominal wage rigidity constraint binds.

Interestingly, in our baseline case the welfare loss is actually slightly smaller in an economy with downward nominal wage rigidities. The reason is that downward nominal rigidity is not an additional constraint on the problem. Instead, it is a constraint that changes the choice set and opens up for potential welfare gains. Another effect of downward nominal wage rigidity is that the loss function parameter for wage inflation variation is changed, although this latter effect seems to play a small role in explaining the welfare effects of downward nominal wage rigidity (at least under optimal policy). We also find that the Taylor rule estimated by Rudebusch (2002), provides a fairly good approximation of optimal discretionary policy in terms of welfare under downward nominal wage rigidity. Experimenting with using the original Taylor (1993), parameters for the Taylor rule indicates that the exact specification of the Taylor rule actually plays a minor role for this property. In contrast, the Taylor rule does not provide such a good approximation of optimal policy in the unconstrained case. A corollary then is that, under the Taylor rule, agents would clearly prefer an economy with downward nominal wage rigidities rather than an unconstrained economy ex ante. That is, since downward nominal wage rigidity actually helps stabilizing the economy in the wage inflation dimension and hence, reduces wage variability, but does not induce much more variation in inflation and the output gap.

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Appendix

A   Numerical Algorithm

The main outline of the algorithm follows algorithm 12.2 in Judd (1998). We first define $N \subset R^2$ nodes over the state space.

Step 0. Guess policy functions U_i^0, i.e., parameter values in second-order complete polynomials

$$\hat{\pi}^0 \left(\hat{w}_{t-1}, \hat{w}_t^* \right),$$
$$\hat{\pi}^{\omega 0} \left(\hat{w}_{t-1}, \hat{w}_t^* \right), \tag{34}$$
$$\hat{x}^0 \left(\hat{w}_{t-1}, \hat{w}_t^* \right),$$

and value function

$$V^0 \left(\hat{w}_{t-1}, \hat{w}_t^* \right). \tag{35}$$

Then proceed to step 2.

Step 1. Consider systems (135) and (139) derived by using the first three first-order conditions in (130) to eliminate the Lagrange multipliers in the last first-order condition, together with the three constraints (21) to (23) (see the discussion in section 4 and Appendix C) We find the new guess for the policy functions U_i^{l+1} by solving for these from systems (135) and (139), respectively, with a collocation method. While solving for policy functions, we take into account that the policy functions affect the expectations.

Step 2. Compute current period utility P_i^{l+1} for $i = 1, \ldots, N$, given policy function guesses U_i^{l+1}.[26]

Step 3. Update the value function $V^{l+1} \left(\hat{w}_{t-1}, \hat{w}_t^* \right)$ using

$$V^{l+1} = \left(I - \beta Q^{U^{l+1}} \right)^{-1} P^{l+1}, \tag{36}$$

where $Q^{U^{l+1}}$ is the transition matrix defined by the new guess for the policy functions for inflation and wage inflation and the flow equation for real wages.

Step 4. If $\left\| V^{l+1} - V^l \right\| < \varepsilon$ stop. Otherwise, go to step 1.

B   Parameter Definitions

The parameters in the equilibrium relations are defined as

---

[26] In terms of Judd (1998) p. 416, compute $\pi \left(y_i, U_i^{l+1} \right)$ where U_i^{l+1} consists of $\hat{\pi}_t$, $\hat{\pi}_t^\omega$ and $\hat{x}_t$ and $y_i = \left(\hat{w}_{t-1}, \hat{w}_t^* \right)_i$, which gives $P_i^{l+1} = \pi \left(y_i, U_i^{l+1} \right)$.

$$\Omega_x = \Pi_1 \frac{\Phi_x}{\Phi_d}, \quad \Omega_x^{uc} = \Pi_1^{uc} \frac{\Phi_x}{\Phi_d},$$
$$\Omega_w = \Pi_1 \frac{\Phi_w}{\Phi_d}, \quad \Omega_w^{uc} = \Pi_1^{uc} \frac{\Phi_w}{\Phi_d}, \tag{37}$$

where

$$\Pi_1 = \left(1 - \frac{1+\alpha_w}{2}\beta\right) \frac{1-\alpha_w}{\alpha_w}, \quad \Pi_1^{uc} = (1 - \alpha_w\beta) \frac{1-\alpha_w}{\alpha_w},$$
$$\Pi = \frac{1-\alpha_w\alpha}{\alpha_w\alpha} (1 - \alpha_w\alpha\beta), \tag{38}$$

and Φ_x, Φ_w and Φ_d are defined as (see the Appendix C below for details)

$$\Phi_d = \varepsilon_L \frac{1}{\sigma-1} \left(\varphi\left(2 + (1+\rho_L)\varepsilon_L\right) - (1+\varepsilon_L)\right) \bar{v}_L\bar{L} - \varphi \frac{\bar{v}_L\bar{L}}{1-\gamma}(1-\gamma)^2 \tag{39}$$

$$\Phi_x = \left(-\varphi\frac{\varepsilon_L}{\sigma-1}\left(\rho_C - \rho_L\frac{1-\sigma\gamma}{1-\gamma}\right) - (1-\varphi)\sigma\gamma\right)\bar{v}_L\bar{L} + \varphi\frac{1}{\sigma-1}\frac{\bar{v}_L\bar{L}}{1-\gamma}(1-\sigma\gamma-\rho_C(1-\gamma))$$

$$\Phi_w = -\left(-\varphi\frac{\varepsilon_L}{\sigma-1}(1-\sigma\rho_L) + (1-\varphi)(1-\gamma)\sigma\right)\bar{v}_L\bar{L} + \varphi\frac{1}{\sigma-1}\frac{\bar{v}_L\bar{L}}{1-\gamma}(1+\varepsilon_L)$$

where

$$\varepsilon_L = -(\gamma + \sigma(1-\gamma)). \tag{40}$$

Let

$$\rho_C = -\frac{\bar{u}_{CC}\bar{C}}{\bar{u}_C} = \chi_C\frac{\bar{C}}{\bar{C}-\bar{Q}} \text{ and } \rho_L = -\frac{\bar{v}_{LL}\bar{L}}{\bar{v}_L} = -\chi_n\frac{\bar{L}}{1-\bar{L}-\bar{Z}}. \tag{41}$$

The loss function parameters in (26) are then defined as

$$\theta_x = \frac{\bar{C}}{2}\bar{u}_C\left(-\rho_C + \rho_L\frac{1}{1-\gamma} - \frac{\gamma}{1-\gamma}\right),$$
$$\theta_\pi = -\frac{\bar{v}_L\bar{L}}{2}\frac{\sigma(\gamma(1-\sigma)+1+\gamma)}{1-\gamma}\frac{1}{\Pi}, \tag{42}$$
$$\theta_{\pi^\omega} = -\frac{\bar{v}_L\bar{L}}{2}\left(\frac{-\gamma\varepsilon_L + (1-\gamma)\sigma(1+\gamma(1-\sigma))}{\Pi_1} - (1-\gamma)\frac{\sigma(\gamma(1-\sigma)+1+\gamma)}{\Pi}\right).$$

and for the unconstrained case we need to replace θ_{π^ω} with $\theta_{\pi^\omega}^{uc}$ defined as

$$\theta_{\pi^\omega}^{uc} = -\frac{\bar{v}_L\bar{L}}{2}\left(\frac{-\gamma\varepsilon_L + (1-\gamma)\sigma(1+\gamma(1-\sigma))}{\Pi_1^{uc}} - (1-\gamma)\frac{\sigma(\gamma(1-\sigma)+1+\gamma)}{\Pi}\right). \tag{43}$$

C   Some Derivations

Most of the derivations in the paper are similar to those in Carlsson and Westermark (2006b). Here, we present the derivations that must be modified due to downward nominal wage rigidity. Thus, the changes concern the sections where wage setting is a part. Hence, wage setting itself must be modified, as well as part of the steady state analysis and the welfare computations.

C.1   Wages

As in Carlsson and Westermark (2006b), we use gradient notation (∇) to indicate derivatives. We can write the derivative of U_u^t as

$$
\begin{aligned}
\nabla_W U_u^t \;=\; & E_t \sum_{k=0}^{\infty} (\alpha_w \alpha \beta)^k \nabla_W \Upsilon_{t,t+k} + E_t \sum_{k=1}^{\infty} \prod_{i=0}^{k-1} \zeta_{t+i}(d_{t+i}) \left(\zeta_{t+k}(d_{t+k}) - \alpha_w \alpha \right) \beta^k \\
& \times \sum_{j=0}^{\infty} (\alpha_w \alpha \beta)^j \nabla_W \Upsilon_{t+k,t+k+j} + E_t \sum_{k=1}^{\infty} \sum_{i=1}^{k-1} \left(\prod_{\substack{m=0 \\ m \neq i}}^{k-1} \zeta_{t+m}(d_{t+m}) \right) (1 - \alpha_w) \nabla_W \zeta_{t+i}(d_{t+i}) \\
& \times \left(\left(\zeta_{t+k}(d_{t+k}) - \alpha_w \alpha \right) \beta^k \sum_{j=0}^{\infty} (\alpha_w \alpha \beta)^j \Upsilon_{t+k,t+k+j} + \left(1 - \zeta_{t+k}(d_{t+k}) \right) \beta^k U_u^{t+k} \right) \quad (44) \\
& + E_t \sum_{k=1}^{\infty} \left(\prod_{i=0}^{k-1} \zeta_{t+i}(d_{t+i}) \right) \nabla_W \zeta_{t+k}(d_{t+k}) \beta^k \left(\sum_{j=0}^{\infty} (\alpha_w \alpha \beta)^j \Upsilon_{t+k,t+k+j} - U_u^{t+k} \right)
\end{aligned}
$$

and the derivative of U_f^t as

$$
\begin{aligned}
\nabla_W U_f^t \;=\; & E_t \sum_{k=0}^{\infty} (\alpha_w \alpha)^k \psi_{t,t+k} \nabla_W \phi_{t,t+k} + E_t \sum_{k=1}^{\infty} \prod_{i=0}^{k-1} \zeta_{t+i}(d_{t+i}) \left(\zeta_{t+k}(d_{t+k}) - \alpha_w \alpha \right) \\
& \times \sum_{j=0}^{\infty} (\alpha_w \alpha)^j \psi_{t,t+k+j} \nabla_W \phi_{t+k,t+k+j} + E_t \sum_{k=1}^{\infty} \sum_{i=1}^{k-1} \left(\prod_{\substack{m=0 \\ m \neq i}}^{k-1} \zeta_{t+m}(d_{t+m}) \right) \nabla_W \zeta_{t+i}(d_{t+i}) \quad (45) \\
& \times \left(\left(\zeta_{t+k}(d_{t+k}) - \alpha_w \alpha \right) \sum_{j=0}^{\infty} (\alpha_w \alpha)^j \psi_{t,t+k+j} \phi_{t+k,t+k+j}(W(f)) + \left(1 - \zeta_{t+k}(d_{t+k}) \right) \psi_{t,t+k} U_{f,t+k} \right) \\
& + E_t \sum_{k=1}^{\infty} \left(\prod_{i=0}^{k-1} \zeta_{t+i}(d_{t+i}) \right) \nabla_W \zeta_{t+k}(d_{t+k}) \left(\sum_{j=0}^{\infty} (\alpha_w \alpha)^j \psi_{t,t+k+j} \phi_{t+k,t+k+j} - \psi_{t,t+k} U_{f,t+k} \right) .
\end{aligned}
$$

Note that, evaluating at the steady state distribution, we have

$$\nabla_W U_f^t = E_t \sum_{k=0}^{\infty} (\alpha_w \alpha)^k \psi_{t,t+k} \nabla_W \phi_{t,t+k} \tag{46}$$

$$+ E_t \sum_{k=1}^{\infty} \prod_{i=0}^{k-1} \zeta_{t+i} (d_{t+i}) (\zeta_{t+k} (d_{t+k}) - \alpha_w \alpha) \sum_{j=0}^{\infty} (\alpha_w \alpha)^j \psi_{t,t+k+j} \nabla_W \phi_{t+k,t+k+j}$$

with $\nabla_W \phi_{t+k,t+k+j} < 0$ and hence, $\nabla_W U_f^t < 0$.

Then, using the first-order condition (17) in steady state, we have

$$\nabla_W U_u^t = -\frac{1-\varphi}{\varphi} \frac{U_{u,t} - U_o}{U_{f,t}} \nabla_W U_f^t > 0 \tag{47}$$

and hence

$$\frac{\partial W(f)}{\partial \varphi} = \underbrace{\frac{\frac{1-\varphi}{\varphi} + 1}{SOC}}_{(-)} \underbrace{(U_{u,t} - U_o)}_{(+)} \underbrace{\nabla_W U_f^t}_{(-)} > 0. \tag{48}$$

The ideal wage for the workers is when $\varphi = 1$ and the ideal wage for the firm is when $\varphi = 0$. Thus, the desired wage of the workers is larger than the ideal wage for the firm. Thus, for small shocks around the the steady state, the union would never want to reduce the wage.

C.2   Steady state

We now turn to the (non-stochastic) steady state of the model.[27] Note that the steady state of the real variables is the same in the flexible price model and the sticky price model. In the steady state, R, C, $Y(f)$ and B are constant. Moreover, $B = 0$. M and P also grow at the rate $\bar{\pi}$, i.e., we have $\frac{P_{t+1}}{P_t} = \bar{\pi}$ and $\bar{I} = \bar{R}\bar{\pi}$.

Now, let us analyze the Nash Bargaining solution in steady state. Since $F_t(d_t) = 0$ for all $d_t \geq 1$ and $F_t(d_t) = 1$ for all $d_t < 1$, the first-order condition (17) is well defined for any $W(f) \neq P_t \bar{w}$, given that all other variables are at their steady state values[28], and we have

$$\Xi(W(f)) = \varphi \bar{U}_f(W(f)) \nabla_W \bar{U}_u(W(f)) + (1-\varphi) (\bar{U}_u(W(f)) - \bar{U}_o) \nabla_W \bar{U}_f(W(f)), \tag{49}$$

where $\bar{U}_u(W(f))$ etc. indicates that all variables except $W(f)$ are at steady state levels, noting that the steady state value of $\psi_{t,t+k}$ is $\bar{\psi}_k = \beta^k$. When $W(f) > P_t \bar{w}$, we have $d_{t+k} < 1$ and $F_{t+1}(d_{t+k}) = 1$

---

[27] That is, a situation where the disturbance A_t is equal to its mean value at all dates.
[28] If all other variables are at their steady state values $d_t = 1 \iff W(f) = P_t \bar{w}$.

and hence, $\zeta_{t+i}(d_{t+k}) = 1$

$$\bar{U}_u(W(f)) = \frac{1}{1-\beta}\bar{\Upsilon}(W(f)), \tag{50}$$

$$\bar{U}_f(W(f)) = \frac{1}{1-\beta}\bar{\phi}(W(f)).$$

Letting

$$\bar{U}_o = \frac{1}{1-\beta}\bar{\Upsilon}_o \tag{51}$$

and

$$\Xi^D(W(f)) = \varphi\bar{\phi}(W(f))\nabla_W\bar{\Upsilon}(W(f)) + (1-\varphi)\left(\bar{\Upsilon}(W(f)) - \bar{\Upsilon}_o\right)\nabla_W\bar{\phi}(W(f)). \tag{52}$$

When $W(f) > P_t\bar{w}$, we have

$$\Xi(W(f)) = \frac{\varphi}{1-\beta}\bar{\phi}(W(f))\frac{1}{1-\beta}\nabla_W\bar{\Upsilon}(W(f)) + \frac{1-\varphi}{1-\beta}\left(\bar{\Upsilon}(W(f)) - \bar{\Upsilon}_o\right)\nabla_W\bar{\phi}(W(f)) \tag{53}$$

and hence

$$\Xi(W(f)) = \left(\frac{1}{1-\beta}\right)^2\Xi^D(W(f)). \tag{54}$$

When $W(f) < P_t\bar{w}$, we have

$$\Xi(W(f)) = \left(\frac{1}{1-\alpha_w\beta}\right)^2\Xi^D(W(f)) +$$

$$+ \frac{\beta(1-\alpha_w)}{(1-\alpha_w\beta)^2}\left[\varphi\bar{U}_f(P_t\bar{w})\nabla_W\bar{\Upsilon}(W(f)) + (1-\varphi)\left(\bar{U}_u(P_t\bar{w}) - \bar{U}_o\right)\nabla_W\bar{\phi}(W(f))\right].$$

As $P_t\bar{w} \to W(f)$, the term on the second row converges to $\vartheta\Xi^D(P_t\bar{w})$ where $\vartheta > 0$, since $\bar{U}_f(P_t\bar{w})$ and $\bar{U}_u(P_t\bar{w})$ converges to $\bar{U}_f(W(f))$ and $\bar{U}_u(W(f))$, respectively. Hence, we must have

$$\Xi^D(P_t\bar{w}) = 0$$

in steady state. Since per-period utility and profits are concave and continuously differentiable in wages, there is a unique wage such that this holds.

As in Carlsson and Westermark (2006a), we have

$$
\begin{aligned}
\bar{\Upsilon} &= \bar{u} - \bar{v}, \\
\bar{\Upsilon}_o &= u\left(\bar{C} - b\bar{w}\bar{L}, \bar{Q}\right) - v\left(0, \bar{Z}\right), \\
\bar{\phi} &= \tau\bar{y}, \\
\frac{\partial\bar{\Upsilon}}{\partial W\left(f\right)} &= \bar{u}_C\left(1 + \tau_w\right)\left(1 - \left(\gamma + \sigma\left(1 - \gamma\right)\right)\right)\frac{\bar{w}\bar{L}}{W\left(f\right)} + \bar{v}_L\left(\gamma + \sigma\left(1 - \gamma\right)\right)\frac{\bar{L}}{W\left(f\right)}, \\
\frac{\partial\bar{\phi}}{\partial W\left(f\right)} &= -\left(1 - \gamma\right)\frac{\overline{tc}}{W\left(f\right)},
\end{aligned} \tag{55}
$$

and $\Xi^D\left(W\left(f\right)\right) = 0$ can be written as

$$
\varphi\tau\bar{y}\left[\bar{u}_C\left(1 + \tau_w\right)\left(1 - \sigma\right)\bar{w}\left(1 - \gamma\right) + \bar{v}_L\left(\gamma + \sigma\left(1 - \gamma\right)\right)\right]\bar{L} - \left(1 - \varphi\right)\left(\bar{u} - \bar{v} - \bar{\Upsilon}_o\right)\left(1 - \gamma\right)\bar{y} = 0. \tag{56}
$$

Potentially, the parties can settle on wages where $\Xi^D\left(W\left(f\right)\right) \leq 0$. However, in the case when prices are flexible, there is a unique wage agreement on the wage where $\Xi^D\left(W\left(f\right)\right) = 0$. We focus on the same point here.

C.2.1   Efficiency

Since there are two distortions and three instruments, one instrument is redundant. Here we use the same method as in Carlsson and Westermark (2006a), relying on adjusting τ and $\bar{\Upsilon}_o$, i.e., the per period version of $\bar{U}_o$, to achieve efficiency.

C.3   Optimal Wages and the Wage Setting "Phillips" Curve

Here, it is important to distinguish the period when the wage contract was last rewritten, the period when the price was last changed and the current period. Therefore, we use the following notation $x_{t+k,t+k+j}^t$ to denote the value of variable x in period $t + k + j$, when the wage contract was last renegotiated in t and the price was last changed in $t + k$.

C.4   Case 1 Wages are adjusted

In this section, we derive the wage setting "Phillips" curve. Assume that the downward nominal wage rigidity constraint does not bind. Log-linearizing the first-order condition (17) gives

$$
\begin{aligned}
0 &= \varphi\nabla_W\bar{U}_u\widehat{\nabla_W U_u^t} + \left(1 - \varphi\right)\left(\frac{1}{\bar{U}_f}\nabla_W\bar{U}_f\bar{U}_u\hat{U}_u^t - \frac{\bar{U}_u - \bar{U}_o}{\left(\bar{U}_f\right)^2}\nabla_W\bar{U}_f\bar{U}_f\hat{U}_f^t\right) \\
&\quad + \left(1 - \varphi\right)\frac{\bar{U}_u - \bar{U}_o}{\bar{U}_f}\nabla_W\bar{U}_f\widehat{\nabla_W U_f^t}.
\end{aligned} \tag{57}
$$

The four terms in the above expressions are[29]

$$
\begin{aligned}
\bar{U}_u \hat{U}_u^t &= E_t \sum_{k=0}^{\infty} (\alpha_w \alpha \beta)^k \, \hat{\Upsilon}_{t,t+k}^t \\
&+ E_t \sum_{k=1}^{\infty} \left(\frac{1+\alpha_w}{2} \right)^{k-1} \left(\alpha_w (1-\alpha) + \frac{1-\alpha_w}{2} \right) \beta^k \sum_{j=0}^{\infty} (\alpha_w \alpha \beta)^j \, \hat{\Upsilon}_{t+k,t+k+j}^t,
\end{aligned}
\tag{58}
$$

$$
\begin{aligned}
\bar{U}_f \hat{U}_f^t &= E_t \sum_{k=0}^{\infty} (\alpha_w \alpha)^k \, \bar{\psi}_k \bar{\phi} \hat{\phi}_{t,t+k}^t \left(W(f) \right) \\
&+ E_t \sum_{k=1}^{\infty} \left(\frac{1+\alpha_w}{2} \right)^{k-1} \left(\alpha_w (1-\alpha) + \frac{1-\alpha_w}{2} \right) \sum_{j=0}^{\infty} (\alpha_w \alpha)^j \, \bar{\psi}_{k+j} \bar{\phi} \hat{\phi}_{t+k,t+k+j}^t \left(W(f) \right),
\end{aligned}
\tag{59}
$$

$$
\begin{aligned}
\nabla_W \bar{U}_u \widehat{\nabla_W U}_u^t &= E_t \sum_{k=0}^{\infty} (\alpha_w \alpha \beta)^k \, \overline{\nabla_W \Upsilon} \widehat{\nabla_W \Upsilon}_{t,t+k}^t \\
&+ E_t \sum_{k=1}^{\infty} \left(\frac{1+\alpha_w}{2} \right)^{k-1} \left(\alpha_w (1-\alpha) + \frac{1-\alpha_w}{2} \right) \beta^k \sum_{j=0}^{\infty} (\alpha_w \alpha \beta)^j \, \overline{\nabla_W \Upsilon} \widehat{\nabla_W \Upsilon}_{t+k,t+k+j}^t,
\end{aligned}
\tag{60}
$$

and

$$
\begin{aligned}
\nabla_W \bar{U}_f \left(-\widehat{\nabla_W U_f^t} \right) &= E_t \sum_{k=0}^{\infty} (\alpha_w \alpha)^k \, \bar{\psi}_k \overline{\nabla_W \phi} \left(-\widehat{\nabla_W \phi}_{t,t+k}^t \right) + \\
&+ E_t \sum_{k=1}^{\infty} \left(\frac{1+\alpha_w}{2} \right)^{k-1} \left(\alpha_w (1-\alpha) + \frac{1-\alpha_w}{2} \right) \\
&\times \sum_{j=0}^{\infty} (\alpha_w \alpha)^j \, \bar{\psi}_{k+j} \overline{\nabla_W \phi} \left(-\widehat{\nabla_W \phi}_{t+k,t+k+j}^t \right).
\end{aligned}
\tag{61}
$$

Leading the first-order condition for wages one period, multiplying with $\alpha_w \beta$ and taking the expectation at t gives

$$
\begin{aligned}
0 &= \varphi \nabla_W \bar{U}_u \left(\widehat{\nabla_W U_u^t} - \frac{1+\alpha_w}{2} \beta E_t \widehat{\nabla_W U_u^{t+1}} \right) + (1-\varphi) \frac{1}{\bar{U}_f} \nabla_W \bar{U}_f \bar{U}_u \left(\hat{U}_u^t - \frac{1+\alpha_w}{2} \beta E_t \hat{U}_u^{t+1} \right) \\
&+ (1-\varphi) \left(-\frac{\bar{U}_u - \bar{U}_o}{(\bar{U}_f)^2} \nabla_W \bar{U}_f \bar{U}_f \left(\hat{U}_f^t - \frac{1+\alpha_w}{2} \beta E_t \hat{U}_f^{t+1} \right) \right) \\
&+ (1-\varphi) \frac{\bar{U}_u - \bar{U}_o}{\bar{U}_f} \nabla_W \bar{U}_f \left(-\widehat{\nabla_W U_f^t} - \frac{1+\alpha_w}{2} \beta E_t \left(-\widehat{\nabla_W U_f^{t+1}} \right) \right).
\end{aligned}
\tag{62}
$$

---

[29]The last summation in the expressions contains all future price changes during the present wage contract.

We define $\hat{n}^t(f)$ as the log-linearized relative wage $\left(n_t(f) = \frac{W(f)}{W_t}\right)$. Let

$$\Delta \hat{n}^t(f) = \frac{1}{1 - \frac{1+\alpha_w}{2}\beta}\left(\hat{n}^t(f) - \frac{1+\alpha_w}{2}\beta\left(E_t\hat{n}^{t+1}(f) + E_t\hat{\pi}^\omega_{t+1}\right)\right) \qquad (63)$$

and, from Carlsson and Westermark (2006b)

$$\hat{n}^t = \int \hat{n}^t(f)\,df = \frac{\alpha_w}{1-\alpha_w}\hat{\pi}^\omega_t. \qquad (64)$$

Moreover, we need to distinguish the period where the wage contract was last rewritten for the terms $\hat{\phi}_{t+k,t+k+j}$ and $\hat{\Upsilon}_{t+k,t+k+j}$ as well as for the corresponding derivatives. As for firm payoff and union utility, we indicate the wage contract period with superscripts, i.e., we use the notation $\hat{\phi}^t_{t+k,t+k+j}$ and $\hat{\Upsilon}^t_{t+k,t+k+j}$.

We eliminate the two distortions in the economy stemming from monopoly power in the intermediate goods market and from union bargaining power in the labor market as in Carlsson and Westermark (2006a). Following the method in Carlsson and Westermark (2006b), we then have

$$\Phi_d\Delta\hat{n}^t + \Phi_x\hat{x}_t + \Phi_w(\hat{w}_t - \hat{w}^*_t) = 0, \qquad (65)$$

where

$$
\begin{aligned}
\Phi_d &= \left(\varepsilon_L\frac{1}{\sigma-1}\left(\varphi\left(2 + (1+\rho_L)\varepsilon_L\right) - (1+\varepsilon_L)\right) - \varphi(1-\gamma)\right)\bar{v}_L\bar{L},\\
\Phi_x &= \left(\left(-\varphi\frac{\varepsilon_L}{\sigma-1}\left(\rho_C - \rho_L\frac{1-\sigma\gamma}{1-\gamma}\right) - (1-\varphi)\sigma\gamma\right) + \varphi\frac{1-\sigma\gamma-\rho_C(1-\gamma)}{(\sigma-1)(1-\gamma)}\right)\bar{v}_L\bar{L}, \quad (66)\\
\Phi_w &= \left(-\left(-\varphi\frac{\varepsilon_L}{\sigma-1}(1-\sigma\rho_L) + (1-\varphi)(1-\gamma)\sigma\right) + \varphi\frac{1+\varepsilon_L}{(\sigma-1)(1-\gamma)}\right)\bar{v}_L\bar{L}.
\end{aligned}
$$

Using (66), when dividing expression (65) with Φ_d and dividing through with $1+\varepsilon_L$, we get

$$
\begin{aligned}
\frac{\Phi_x}{\Phi_d} &= \frac{\varphi\left(\frac{1-\sigma\gamma}{1-\gamma}\frac{1+\rho_L\varepsilon_L}{1+\varepsilon_L} - \rho_C\right) + (1-\varphi)\sigma\frac{\gamma}{1-\gamma}}{\varphi\left(\frac{\varepsilon_L}{1+\varepsilon_L}(1+\rho_L\varepsilon_L)+1\right) - (1-\varphi)\varepsilon_L}, \qquad (67)\\
\frac{\Phi_w}{\Phi_d} &= \frac{\varphi\left(\frac{\varepsilon_L}{1+\varepsilon_L}(1-\sigma\rho_L) + \frac{1}{1-\gamma}\right) + (1-\varphi)\sigma}{\varphi\left(\frac{\varepsilon_L}{1+\varepsilon_L}(1+\rho_L\varepsilon_L)+1\right) - (1-\varphi)\varepsilon_L}.
\end{aligned}
$$

Using (63) and (64)

$$\int \Delta\hat{n}^t(f)\,df = \frac{1}{1 - \frac{1+\alpha_w}{2}\beta}\left(\frac{\alpha_w}{1-\alpha_w}\hat{\pi}^\omega_t - \frac{1}{1-\alpha_w}\frac{1+\alpha_w}{2}\beta E_t\hat{\pi}^\omega_{t+1}\right), \qquad (68)$$

39

and using (38), the labor market Phillips curve is

$$\hat{\pi}_t^\omega = \frac{1 + \alpha_w}{2\alpha_w} \beta E_t \hat{\pi}_{t+1}^\omega - \Pi_1 \left(\frac{\Phi_x}{\Phi_d} \hat{x}_t + \frac{\Phi_w}{\Phi_d} \left(\hat{w}_t - \hat{w}_t^* \right) \right). \tag{69}$$

C.4.1   Case 2. Wages are not adjusted

With negative shocks, we get

$$\hat{n}^t = \hat{\pi}_t^\omega = 0. \tag{70}$$

C.5   Welfare

First, consider the relationship between relative prices and wages. This can be derived from the price setting relationship; see Carlsson and Westermark (2006b). As in the Technical Appendix, we consider the relative price $\hat{q}_t(f)$ and the relative wage in period t (where prices were last rewritten in some period $s < t$). By similar arguments as in the Technical Appendix of Carlsson and Westermark (2006a), we then have the following relationship between relative prices and wages

$$\hat{q}_t(f) = (1 - \gamma) \hat{n}_t(f) + K'. \tag{71}$$

Since prices and wages are not fully flexible, variances are persistent. We want to find the variance today as a function of previous variances and inflation. For this purpose, let us express $var_f(\log P_t(f))$ and $var_f(\log W_t(f))$ in terms of squared inflation and wage inflation. Let $\bar{P}_t = E_f \log P_t(f)$. We have

$$var_f(\log P_t(f)) = E_f \left(\log P_t(f) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2 - \left(\Delta \bar{P}_t \right)^2 \tag{72}$$

where

$$\Delta \bar{P}_t = \bar{P}_t - \log \bar{\pi} - \bar{P}_{t-1}. \tag{73}$$

Let us rewrite $\Delta \bar{P}_t$ in terms of inflation. Since $\log P_t = E_f \log P_t(f) = \bar{P}_t$, we have [30]

$$\Delta \bar{P}_t = \log P_t - \log \bar{\pi} - \log P_{t-1} = \log \pi_t - \log \bar{\pi} = \hat{\pi}_t. \tag{74}$$

---

[30] This follows from using a first-order Taylor approximation of the price level $P_t^{1-\sigma} = \int_0^1 P_t(f)^{1-\sigma} df$

$$\bar{P} + (1 - \sigma) \bar{P}^{-\sigma} \left(\bar{P} \hat{P}_t - \bar{P} \right) = \int_0^1 \left(\bar{P} + (1 - \sigma) \bar{P}^{-\sigma} \left(\bar{P} \hat{P}_t(f) - \bar{P} \right) \right) df.$$

Since $\bar{P} = \bar{P}(f)$, we have

$$\hat{P}_t = \int_0^1 \hat{P}_t(f) df$$

or, by the definition of $\hat{P}_t$ and $\hat{P}_t(f)$, we have (74) (they are constructed as log-deviations around the same mean).

Similarly, we have [31]

$$\Delta \bar{W}_t = \log W_t - \log \bar{\pi}^\omega - \log W_{t-1} = \hat{\pi}_t^w. \tag{75}$$

We can write $var_f(\log P_t(f))$ as

$$
\begin{aligned}
& E_f \left(\log P_t(f) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2 - \left(\Delta \bar{P}_t \right)^2 \\
= \ & \alpha_w \alpha E_f \left(\log \bar{\pi} P_{t-1}(f) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2 + (1-\alpha) \alpha_w E_f \left(\log P_t^o(W_t(f)) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2 \\
& + (1-\alpha_w) \int_{W_{t-1}(f) > W_t^o(f)} E_f \left(\log P_t^o(W_t(f)) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2 dF(d_{t,t}) \\
& + (1-\alpha_w) \int_{W_{t-1}(f) \leq W_t^o(f)} E_f \left(\log P_t^o(W_t^o(f)) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2 dF(d_{t,t}) - \left(\Delta \bar{P}_t \right)^2,
\end{aligned}
\tag{76}
$$

recalling that superscript o indicates an optimally chosen price and wage. When evaluating at the steady state, using that when wages are changed, they are the same for all firms, i.e., $W_t^o(f) = W_t^o$ for all f, and hence optimal prices are the same, we get [32]

$$
\begin{aligned}
& E_f \left(\log P_t(f) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2 - \left(\Delta \bar{P}_t \right)^2 \\
= \ & \alpha_w \alpha E_f \left(\log \bar{\pi} P_{t-1}(f) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2 \\
& + (1-\alpha) \alpha_w E_f \left(\log P_t^o(W_t(f)) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2 \\
& + (1-\alpha_w) \left(\log P_t^o(W_t^o) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2 - \left(\Delta \bar{P}_t \right)^2
\end{aligned}
\tag{77}
$$

when $\log W_t^o - \log \bar{\pi}^w - \bar{W}_{t-1} \geq 0$ and

$$
\begin{aligned}
& E_f \left(\log P_t(f) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2 - \left(\Delta \bar{P}_t \right)^2 \\
= \ & \alpha \alpha_w E_f \left(\log \bar{\pi} P_{t-1}(f) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2 \\
& + ((1-\alpha)\alpha_w + (1-\alpha_w)) E_f \left(\log P_t^o(W_t(f)) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2 - \left(\Delta \bar{P}_t \right)^2
\end{aligned}
\tag{78}
$$

otherwise, i.e., when $\log W_t^o - \log \bar{\pi}^w - \bar{W}_{t-1} < 0$.

Let us express $\Delta \bar{P}_t$ in terms of $E_f \left(\log P_t^o(W_t(f)) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2$ and $\Delta \bar{W}_t$. First, suppose that

---

[31] This follows from using a first-order Taylor approximation of the price level $W_t = \int_0^1 W_t(f)\, df$;

$$\hat{W}_t = \int_0^1 \hat{W}_t(f)\, df,$$

or, by the definition of $\hat{W}_t$ and $\hat{W}_t(f)$ (they are constructed as log-deviations around the same mean);

$$\log W_t = E_f \log W_t(f) = \bar{W}_t.$$

[32] A caveat is in place since our approach disregards any effects stemming from that the wage distribution may be non-degenerate. This dramatically reduces the complexity of the problem, c.f. the discussion of the wage distribution in section 2.

$\log W_t^o - \log \bar{\pi}^w - \bar{W}_{t-1} \geq 0$. We have

$$\Delta \bar{P}_t = \bar{P}_t - \log \bar{\pi} - \bar{P}_{t-1} = (1-\alpha) \, \alpha_w \left(E_f \log P_t^o \left(W_t \left(f \right) \right) - \log \bar{\pi} - \bar{P}_{t-1} \right) \tag{79}$$
$$+ (1-\alpha_w) \left(E_f \log P_t^o \left(W_t^o \right) - \log \bar{\pi} - \bar{P}_{t-1} \right).$$

Note that, from the firms' optimal capital and labor choice and the optimal pricing decision, we can write the optimal price as $P_t^o = \varrho \left(W \left(f \right) \right)^{1-\gamma}$ where ϱ only depend on aggregate variables. We thus can write

$$\log P_t^o \left(W_t^o \right) = \log P_t^o \left(\bar{\pi}^\omega W_{t-1} \left(f \right) \right) + (1-\gamma) \left(\log W_t^o - \log \bar{\pi}^\omega W_{t-1} \left(f \right) \right) \tag{80}$$

Using that we have $W_t \left(f \right) = \bar{\pi}^\omega W_{t-1} \left(f \right)$ for firms that do not change prices and since $E_f \left(\log \bar{\pi}^\omega W_{t-1} \left(f \right) \right) = \log \bar{\pi}^\omega + \bar{W}_{t-1}$, we have

$$\Delta \bar{P}_t - (1-\gamma) \, \Delta \bar{W}_t = ((1-\alpha) \, \alpha_w + (1-\alpha_w)) \left(E_f \log P_t^o \left(W_t \left(f \right) \right) - \log \bar{\pi} - \bar{P}_{t-1} \right) \tag{81}$$

and

$$\Delta \bar{W}_t = (1-\alpha_w) \left(\log W_t^o - \log \bar{\pi}^\omega - \bar{W}_{t-1} \right) \tag{82}$$

when $\log W_t^o - \log \bar{\pi}^w - \bar{W}_{t-1} \geq 0$.

When $\log W_t^o - \log \bar{\pi}^w - \bar{W}_{t-1} < 0$, we have

$$\Delta \bar{P}_t = \bar{P}_t - \log \bar{\pi} - \bar{P}_{t-1} = ((1-\alpha) \, \alpha_w + (1-\alpha_w)) \left(E_f \log P_t^o \left(W_t \left(f \right) \right) - \log \bar{\pi} - \bar{P}_{t-1} \right), \tag{83}$$

and $\Delta \bar{W}_t = 0$. Note that we always have

$$E_f \left(\log P_t^o \left(W_t \left(f \right) \right) - \log \bar{\pi} - \bar{P} \bar{p}_{t-1} \right)^2$$
$$= E_f \left(\log P_t^o \left(W_t \left(f \right) \right) - E_f \log P_t^o \left(W_t \left(f \right) \right) + E_f \log P_t^o \left(W_t \left(f \right) \right) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2 \tag{84}$$
$$= var_f \log P_t^o \left(W_t \left(f \right) \right) + \left(E_f \log P_t^o \left(W_t \left(f \right) \right) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2.$$

Using (81) and (84) when $\log W_t^o - \log \bar{\pi}^w - \bar{W}_{t-1} \geq 0$, we have

$$E_f \left(\log P_t^o \left(W_t \left(f \right) \right) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2 = var_f \log P_t^o \left(W_t \left(f \right) \right) + \left(E_f \log P_t^o \left(W_t \left(f \right) \right) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2$$
$$= var_f \log P_t^o \left(W_t \left(f \right) \right) + \frac{\left(\Delta \bar{P}_t - (1-\gamma) \, \Delta \bar{W}_t \right)^2}{((1-\alpha) \, \alpha_w + (1-\alpha_w))^2}. \tag{85}$$

and, using (83) and (84)

$$E_f \left(\log P_t^o \left(f \right) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2 = var_f \log P_t^o \left(W_t \left(f \right) \right) + \frac{1}{\left(\left(1 - \alpha \right) \alpha_w + \left(1 - \alpha_w \right) \right)^2} \left(\Delta \bar{P}_t \right)^2 \qquad (86)$$

otherwise.

Furthermore, by a similar method as in the Technical Appendix of Carlsson and Westermark (2006b), using the fact that $\log P_t^o \left(W_t^o \right)$ is the same for all firms that change wages and the log-linearization of $\log P_t^o \left(W_t^o \right)$ i.e., (80) we can write

$$\left(\log P_t^o \left(W_t^o \right) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2 \qquad (87)$$

$$= \quad E_f \left(\log P_t^o \left(\bar{\pi}^\omega W_{t-1} \left(f \right) \right) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2 + E_f \left(\left(1 - \gamma \right) \left(\log W_t^o - \log \bar{\pi}^\omega W_{t-1} \left(f \right) \right) \right)^2$$

$$+ 2 \left(1 - \gamma \right) E_f \left(\log P_t^o \left(\bar{\pi}^\omega W_{t-1} \left(f \right) \right) - \log \bar{\pi} - \bar{P}_{t-1} \right) \left(\log W_t^o - \log \bar{\pi}^\omega W_{t-1} \left(f \right) \right)$$

where, using (84)

$$E_f \left(\log P_t^o \left(\bar{\pi}^\omega W_{t-1} \left(f \right) \right) - \log \bar{\pi} - \bar{P}_{t-1} \right)^2 \qquad (88)$$

$$= \quad var_f \log P_t^o \left(W_t \left(f \right) \right) + \frac{1}{\left(\left(1 - \alpha \right) \alpha_w + \left(1 - \alpha_w \right) \right)^2} \left(\Delta \bar{P}_t - \left(1 - \gamma \right) \Delta \bar{W}_t \right)^2,$$

using (75);

$$E_f \left(\left(1 - \gamma \right) \left(\log W_t^o - \log \bar{\pi}^\omega W_{t-1} \left(f \right) \right) \right)^2 = \left(1 - \gamma \right)^2 \left(\frac{1}{\left(1 - \alpha_w \right)^2} \left(\Delta \bar{W}_t \right)^2 + \left(\bar{\pi}^\omega \right)^2 var_f \log W_{t-1} \left(f \right) \right) \qquad (89)$$

and, using (75), (81) and (84) that

$$E_f \left(\log P_t^o \left(\bar{\pi}^\omega W_{t-1} \left(f \right) \right) - \log \bar{\pi} - \bar{P}_{t-1} \right) \left(\log W_t^o - \log \bar{\pi}^\omega W_{t-1} \left(f \right) \right) \qquad (90)$$

$$= \quad \frac{1}{\left(1 - \alpha \right) \alpha_w + \left(1 - \alpha_w \right)} \left(\Delta \bar{P}_t - \left(1 - \gamma \right) \Delta \bar{W}_t \right) \frac{1}{1 - \alpha_w} \Delta \bar{W}_t - \left(\bar{\pi}^\omega \right)^2 \left(1 - \gamma \right) var_f \log W_{t-1} \left(f \right).$$

From (71) we have $var_f \log P_t^o \left(W_t \left(f \right) \right) = \left(1 - \gamma \right)^2 var_f \log W_t \left(f \right)$. Then, using (88), (90) and (89) in (87), (77) and (85), we have that

$$var_f \left(\log P_t \left(f \right) \right) = \alpha_w \alpha var_f \left(\log P_{t-1} \left(f \right) \right) + \left(1 - \alpha \right) \alpha_w \left(1 - \gamma \right)^2 var_f \log W_t \left(f \right) \qquad (91)$$

$$+ \frac{\alpha_w}{\left(1 - \alpha \right) \alpha_w + \left(1 - \alpha_w \right)} \left(\alpha \left(\Delta \bar{P}_t \right)^2 + \frac{1 - \alpha}{1 - \alpha_w} \left(1 - \gamma \right)^2 \left(\Delta \bar{W}_t \right)^2 \right)$$

when $\log W_t^o(f) - \log \bar{\pi}^w - \bar{W}_{t-1} \geq 0$ and, from (78) and (86);

$$
\begin{aligned}
var_f\left(\log P_t\left(f\right)\right) \quad = \quad & \alpha_w \alpha var_f\left(\log P_{t-1}\left(f\right)\right) + \left(\left(1-\alpha\right)\alpha_w + 1 - \alpha_w\right)\left(1-\gamma\right)^2 var_f \log W_t\left(f\right) \quad (92) \\
& + \frac{\alpha_w \alpha}{\left(1-\alpha\right)\alpha_w + \left(1-\alpha_w\right)}\left(\Delta \bar{P}_t\right)^2,
\end{aligned}
$$

otherwise.

For wages, using a similar method as in (72), we can write when $\Delta \bar{W}_t > 0$

$$
var_f\left(\log W_t\left(f\right)\right) = E_f\left(\log W_t\left(f\right) - \log \bar{\pi}^w - \bar{W}_{t-1}\right)^2 - \left(\Delta \bar{W}_t\right)^2 \qquad (93)
$$

and

$$
var_f\left(\log W_t\left(f\right)\right) = var_f\left(\log W_{t-1}\left(f\right)\right), \qquad (94)
$$

otherwise. When $\Delta \bar{W}_t > 0$, we have

$$
\begin{aligned}
& E_f\left(\log W_t\left(f\right) - \log \bar{\pi}^w - \bar{W}_{t-1}\right)^2 - \left(\Delta \bar{W}_t\right)^2 \\
= \quad & \alpha_w E_f\left(\log \bar{\pi}^w W_{t-1}\left(f\right) - \log \bar{\pi}^w - \bar{W}_{t-1}\right)^2 \qquad (95) \\
& + \left(1-\alpha_w\right)\left(\log W_t^o\left(f\right) - \log \bar{\pi}^w - \bar{W}_{t-1}\right)^2 - \left(\Delta \bar{W}_t\right)^2.
\end{aligned}
$$

Using (75), we have

$$
var_f\left(\log W_t\left(f\right)\right) = \alpha_w var_f\left(\log W_{t-1}\left(f\right)\right) + \frac{\alpha_w}{1-\alpha_w}\left(\Delta \bar{W}_t\right)^2. \qquad (96)
$$

Using $\Delta \bar{W}_t = \hat{\pi}_t^w$ gives

$$
var_f\left(\log W_t\left(f\right)\right) = \alpha_w var_f\left(\log W_{t-1}\left(f\right)\right) + \frac{\alpha_w}{1-\alpha_w}\left(\hat{\pi}_t^w\right)^2 + o\left(\|\xi\|^3\right) \qquad (97)
$$

when $\log W_t^{opt}(f) - \log \bar{\pi}^w - \bar{W}_{t-1} \geq 0$ and

$$
var_f\left(\log W_t\left(f\right)\right) = var_f\left(\log W_{t-1}\left(f\right)\right) + o\left(\|\xi\|^3\right), \qquad (98)
$$

otherwise.

C.5.1 Loss Function

We focus on the limiting cashless economy. The social welfare function is then

$$E_0 \sum_{t=0}^{\infty} \beta^t SW_t \tag{99}$$

where

$$SW_t = u\left(C_t, Q_t\right) - \int_0^1 v\left(L_t\left(f\right), Z_t\right) df. \tag{100}$$

As in the Technical Appendix of Carlsson and Westermark (2006a), the total welfare difference is

$$
\begin{aligned}
E_0 \sum_{t=0}^{\infty} \beta^t \left(SW_t - SW_t^*\right) &= -E_0 \frac{\bar{v}_L \bar{L}}{2} \sum_{t=0}^{\infty} \beta^t \left(\sigma var_f \hat{P}_t\left(f\right) + \left(\gamma^2 + 2\sigma\gamma\left(1-\gamma\right)\right) var_f \hat{W}_t\left(f\right)\right) \\
&\quad + E_0 \frac{\Lambda^* \bar{C}}{2} \sum_{t=0}^{\infty} \beta^t \left(\hat{Y}_t - \hat{Y}_t^*\right)^2 + tip + o\left(\|\xi\|^3\right).
\end{aligned} \tag{101}
$$

In this expression, the price and wage variances are different than in Carlsson and Westermark (2006b), due to the downward nominal wage rigidities, since these are affected by wage setting. Given the path $\hat{\pi}^w$, let $T_t\left(\hat{\pi}^w, r\right)$ denote the number of times wage inflation is positive between t and s

$$T_t\left(\hat{\pi}^w, s\right) = \left|\left\{r : \hat{\pi}_r^w > \bar{\pi}^w, t \geq r \geq s\right\}\right|. \tag{102}$$

Repeatedly substituting (97) and (98) into themselves (forwardly) using (74), starting at 0 gives

$$var_f\left(\log W_t\left(f\right)\right) = \left(\alpha_w^{T_t\left(\hat{\pi}^w, 0\right)}\right) var_f\left(\log W_{-1}\left(f\right)\right) + \frac{\alpha_w}{1-\alpha_w} \sum_{s=0}^{t} \alpha_w^{T_t\left(\hat{\pi}^w, s\right)-1}\left(\hat{\pi}_s^w\right)^2 + o\left(\|\xi\|^3\right). \tag{103}$$

Multiplying by β^t on both sides, using that $var_f\left(\log W_{-1}\left(f\right)\right)$ is independent of policy and summing from 0 to infinity gives

$$\sum_{t=0}^{\infty} \beta^t var_f\left(\log W_t\left(f\right)\right) = \sum_{t=0}^{\infty} \beta^t \frac{\alpha_w}{1-\alpha_w} \sum_{s=0}^{t} \alpha_w^{T_t\left(\hat{\pi}^w, s\right)-1}\left(\hat{\pi}_s^w\right)^2 + tip + o\left(\|\xi\|^3\right). \tag{104}$$

Taking the expectation at $t = 0$ and using the fact that the expression is evaluated at the steady state and hence that the probability that the constraint binds is $\frac{1}{2}$ gives

$$E_0 \sum_{t=0}^{\infty} \beta^t var_f\left(\log W_t\left(f\right)\right) = \frac{\alpha_w}{1-\alpha_w} \frac{1}{1-\beta\frac{1+\alpha_w}{2}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{\pi}_t^w\right)^2 + tip + o\left(\|\xi\|^3\right). \tag{105}$$

Now consider prices again and expressions (91) and (92). These can be rewritten as

$$var_f\left(\log P_t\left(f\right)\right) = \alpha_w\alpha var_f\left(\log P_{t-1}\left(f\right)\right) + \left(1-\alpha\right)\alpha_w\left(1-\gamma\right)^2 var_f\log W_{t-1}\left(f\right) \tag{106}$$
$$+\frac{\alpha_w\alpha}{\left(1-\alpha\right)\alpha_w+\left(1-\alpha_w\right)}\left(\hat{\pi}_t\right)^2 + \frac{\left(1-\alpha\right)\alpha_w}{\left(1-\alpha_w\right)\left(\left(1-\alpha\right)\alpha_w+\left(1-\alpha_w\right)\right)}\left(1-\gamma\right)^2\left(\hat{\pi}_t^w\right)^2$$

when $\log W_t^o\left(f\right)-\log\bar{\pi}^w-\bar{W}_{t-1}\geq 0$ and, from (86) and (78)

$$var_f\left(\log P_t\left(f\right)\right) = \alpha_w\alpha var_f\left(\log P_{t-1}\left(f\right)\right) + \left(\left(1-\alpha\right)\alpha_w+1-\alpha_w\right)\left(1-\gamma\right)^2 var_f\log W_{t-1}\left(f\right)$$
$$+\frac{\alpha_w\alpha}{\left(1-\alpha\right)\alpha_w+\left(1-\alpha_w\right)}\left(\hat{\pi}_t\right)^2, \tag{107}$$

otherwise.

Repeatedly substituting this expression into itself (forwardly), starting at 0, taking expectations at time 0 and evaluating at the steady state gives, using that the previous variance is passed through with $\frac{1}{2}\left(1-\alpha\right)\alpha_w+\frac{1}{2}\left(\left(1-\alpha\right)\alpha_w+\left(1-\alpha_w\right)\right)$

$$E_0 var_f\left(\log P_t\left(f\right)\right) = E_0\sum_{s=0}^{t-1}\left(\alpha_w\alpha\right)^{t-s}\left(\left(\left(1-\alpha\right)\alpha_w+\frac{1-\alpha_w}{2}\right)\left(1-\gamma\right)^2\right)var_f\left(\log W_s\left(f\right)\right)$$
$$+E_0\sum_{s=0}^{t}\left(\alpha_w\alpha\right)^{t-s}\left(1-\gamma\right)^2\frac{\left(1-\alpha\right)\alpha_w}{\left(1-\alpha_w\right)\left(\left(1-\alpha\right)\alpha_w+\left(1-\alpha_w\right)\right)}\left(\hat{\pi}_s^w\right)^2 \tag{108}$$
$$+E_0\sum_{s=0}^{t}\left(\alpha_w\alpha\right)^{t-s}\frac{\alpha_w\alpha}{\left(1-\alpha\right)\alpha_w+\left(1-\alpha_w\right)}\left(\hat{\pi}_s\right)^2 + tip + o\left(\|\xi\|^3\right).$$

Multiplying by β^t on both sides, using that $var_f\left(\log W_{-1}\left(f\right)\right)$ is independent of policy and summing from 0 to infinity gives

$$E_0\sum_{t=0}^{\infty}\beta^t var_f\left(\log P_t\left(f\right)\right) = \beta\frac{\left(\left(1-\alpha\right)\alpha_w+\frac{1-\alpha_w}{2}\right)\left(1-\gamma\right)^2}{1-\beta\alpha_w\alpha}E_0\sum_{t=0}^{\infty}\beta^t var_f\left(\log Ww_t\left(f\right)\right)$$
$$+\frac{\left(1-\gamma\right)^2\left(1-\alpha\right)\alpha_w}{\left(1-\beta\alpha_w\alpha\right)\left(1-\alpha_w\right)\left(\left(1-\alpha\right)\alpha_w+\left(1-\alpha_w\right)\right)}E_0\sum_{t=0}^{\infty}\beta^t\left(\hat{\pi}_t^w\right)^2 \tag{109}$$
$$+\frac{\alpha_w\alpha}{\left(1-\beta\alpha_w\alpha\right)\left(\left(1-\alpha\right)\alpha_w+\left(1-\alpha_w\right)\right)}E_0\sum_{t=0}^{\infty}\beta^t\left(\hat{\pi}_t\right)^2 + tip + o\left(\|\xi\|^3\right).$$

Using (109) and (105) in (101) gives

$$E_0\sum_{t=0}^{\infty}\beta^t\left(SW_t-SW_t^*\right) = E_0\sum_{t=0}^{\infty}\beta^t\mathcal{L}_t + tip + o\left(\|\xi\|^3\right) \tag{110}$$

where

$$\mathcal{L}_t = \theta_x\left(\hat{x}_t\right)^2 + \theta_\pi\left(\hat{\pi}_t\right)^2 + \theta_{\pi^\omega}\left(\hat{\pi}_t^\omega\right)^2, \tag{111}$$

and using similar methods as in Carlsson and Westermark (2006b), and expression (38) we get

$$
\begin{aligned}
\theta_x &= \frac{\bar{C}}{2}\bar{u}_C\left(-\rho_C + \rho_L\frac{1}{1-\gamma} - \frac{\gamma}{1-\gamma}\right), \\
\theta_\pi &= -\frac{\bar{v}_L\bar{L}}{2}\frac{\sigma\left(\gamma\left(1-\sigma\right)+1+\gamma\right)}{1-\gamma}\frac{1}{\Pi}, \\
\theta_{\pi^\omega} &= -\frac{\bar{v}_L\bar{L}}{2}\left(\frac{-\gamma\varepsilon_L + \left(1-\gamma\right)\sigma\left(1+\gamma\left(1-\sigma\right)\right)}{\Pi_1} - \left(1-\gamma\right)\frac{\sigma\left(\gamma\left(1-\sigma\right)+1+\gamma\right)}{\Pi}\right).
\end{aligned}
\tag{112}
$$

Note that $\theta_x < 0$, $\theta_\pi < 0$ and $\theta_{\pi^\omega} < 0$.

D   Optimal Discretionary Policy

To find the optimal rule under discretion, the central bank solves the following problem

$$
V\left(\hat{w}_{t-1},\hat{w}_t^*\right) = \max_{\{\hat{x}_t,\hat{\pi}_t,\hat{\pi}_t^\omega,\hat{w}_t\}} \theta_x\left(\hat{x}_t\right)^2 + \theta_\pi\left(\hat{\pi}_t\right)^2 + \theta_{\pi^\omega}\left(\hat{\pi}_t^\omega\right)^2 + \beta E_t V\left(\hat{w}_t,\hat{w}_{t+1}^*\right) + tip + o\left(\|\xi\|^3\right) \tag{113}
$$

subject to

$$
\hat{x}_t = E_t\left(\hat{x}_{t+1} - \frac{1}{\rho_C}\left(\hat{i}_t - \hat{\pi}_{t+1} - \widehat{r}_t^*\right)\right), \tag{114}
$$

$$
\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \left(1-\gamma\right)\left(\hat{\pi}_t^\omega - \beta E_t\hat{\pi}_{t+1}^\omega\right) + \Pi\left(\hat{w}_t - \hat{w}_t^*\right) + \frac{\gamma}{1-\gamma}\Pi\hat{x}_t, \tag{115}
$$

$$
\hat{\pi}_t^\omega = \max\left\{0, \frac{1+\alpha_w}{2\alpha_w}\beta E_t\hat{\pi}_{t+1}^\omega - \Omega_w\left(\hat{w}_t - \hat{w}_t^*\right) - \Omega_x\hat{x}_t\right\}, \tag{116}
$$

$$
\hat{w}_t = \hat{w}_{t-1} + \hat{\pi}_t^\omega - \hat{\pi}_t. \tag{117}
$$

Note that the max restriction can be replaced by

$$
\hat{\pi}_t^\omega \geq \frac{1+\alpha_w}{2\alpha_w}\beta E_t\hat{\pi}_{t+1}^\omega - \Omega_w\left(\hat{w}_t - \hat{w}_t^*\right) - \Omega_x\hat{x}_t, \tag{118}
$$

$$
\hat{\pi}_t^\omega \geq 0. \tag{119}
$$

The reason for the restriction $\hat{\pi}_t^\omega \geq \frac{1+\alpha_w}{2\alpha_w}\beta E_t\hat{\pi}_{t+1}^\omega - \Omega_x\hat{x}_t - \Omega_w\left(\hat{w}_t - \hat{w}_t^*\right)$ is the following. First, if the desired wage change is positive, it is set according to the first-order condition, implying that the condition holds with equality. Second, if the desired wage change is negative, downward rigidity kicks in so that wages are higher than those prescribed by the first-order condition.

We solve by using the Lagrange method. The Lagrangian is

$$
\begin{aligned}
L = \ & \theta_x \left(\hat{x}_t \right)^2 + \theta_\pi \left(\hat{\pi}_t \right)^2 + \theta_{\pi^\omega} \left(\hat{\pi}_t^\omega \right)^2 + \beta E_t V \left(\hat{w}_t, \hat{w}_{t+1}^* \right) \\
& - \lambda_t^\pi \left(\beta E_t \hat{\pi}_{t+1} + \left(1 - \gamma \right) \left(\hat{\pi}_t^\omega - \beta E_t \hat{\pi}_{t+1}^\omega \right) + \Pi \left(\hat{w}_t - \hat{w}_t^* \right) + \frac{\gamma}{1 - \gamma} \Pi \hat{x}_t - \hat{\pi}_t \right) \\
& - \lambda_t^w \left(\hat{w}_{t-1} + \hat{\pi}_t^\omega - \hat{\pi}_t - \hat{w}_t \right) \\
& - \mu_t^{\pi^\omega} \left(\frac{1 + \alpha_w}{2 \alpha_w} \beta E_t \hat{\pi}_{t+1}^\omega - \Omega_x \hat{x}_t - \Omega_w \left(\hat{w}_t - \hat{w}_t^* \right) - \hat{\pi}_t^\omega \right) \\
& - \mu_t^0 \left(-\hat{\pi}_t^\omega \right) .
\end{aligned}
\tag{120}
$$

The first-order conditions are

$$
\begin{aligned}
0 = \ & 2\theta_x \hat{x}_t - \lambda_t^\pi \frac{\gamma}{1 - \gamma} \Pi + \mu_t^{\pi^\omega} \Omega_x, \\
0 = \ & 2\theta_\pi \hat{\pi}_t + \lambda_t^\pi + \lambda_t^w, \\
0 = \ & 2\theta_{\pi^\omega} \hat{\pi}_t^\omega - \lambda_t^\pi \left(1 - \gamma \right) - \lambda_t^w + \mu_t^{\pi^\omega} + \mu_t^0, \\
0 = \ & \beta E_t V_1 \left(\hat{w}_t, \hat{w}_{t+1}^* \right) - \lambda_t^\pi \left(\beta E_t \frac{\partial \hat{\pi}_{t+1}}{\partial \hat{w}_t} - \left(1 - \gamma \right) \beta E_t \frac{\partial \hat{\pi}_{t+1}^\omega}{\partial \hat{w}_t} + \Pi \right) \\
& + \lambda_t^w - \mu_t^{\pi^\omega} \left(\frac{1 + \alpha_w}{2 \alpha_w} \beta E_t \frac{\partial \hat{\pi}_{t+1}^\omega}{\partial \hat{w}_t} - \Omega_w \right) .
\end{aligned}
\tag{121}
$$

Note that we restrict the attention to Markov-perfect equilibria, i.e., we do not consider any equilibria with reputational effects that could arise from complex non-Markov behavior. However, we need to take into account that the real wage is an endogenous state variable. Therefore, expected inflation and expected wage inflation will depend on lagged real wages in equilibrium. An implication of this is that when designing monetary policy, even in the absence of a commitment mechanism, the central bank should take into account how changes in the real wage today affect private sector expectations.

We also have the complementary slackness conditions

$$
\begin{aligned}
\mu_t^{\pi^\omega} \left(\frac{1 + \alpha_w}{2 \alpha_w} \beta E_t \hat{\pi}_{t+1}^\omega - \Omega_x \hat{x}_t - \Omega_w \left(\hat{w}_t - \hat{w}_t^* \right) - \hat{\pi}_t^\omega \right) &= 0, \\
\mu_t^0 \left(-\hat{\pi}_t^\omega \right) &= 0.
\end{aligned}
\tag{122}
$$

Note that the original constraint (116) and the inequality constraints (118) and (119) need not be equivalent. It is obviously true that a $\hat{\pi}_t^\omega$ satisfying the original constraint satisfies the two inequality constraints. However, it is possible that there is a solution to the problem with inequality constraints so that none of the inequality constraints is binding, leading to a violation of the original constraints. The following Lemma shows that at least one of the inequality constraints must be binding, ruling out this possibility.

Lemma 1. At least one of the inequality constraints (118) and (119) must be binding.

Proof: We prove this by contradiction. Suppose that

$$\hat{\pi}_t^\omega > \frac{1 + \alpha_w}{2\alpha_w} \beta E_t \hat{\pi}_{t+1}^\omega - \Omega_x \hat{x}_t - \Omega_w (\hat{w}_t - \hat{w}_t^*),$$

(123)

$$\hat{\pi}_t^\omega > 0.$$

Then $\mu_t^{\pi^\omega} = \mu_t^0 = 0$ and the first-order conditions simplifies to

$$0 = 2\theta_x \hat{x}_t - \lambda_t^\pi \frac{\gamma}{1-\gamma} \Pi,$$

$$0 = 2\theta_\pi \hat{\pi}_t + \lambda_t^\pi + \lambda_t^w,$$

(124)

$$0 = 2\theta_{\pi^\omega} \hat{\pi}_t^\omega - \lambda_t^\pi (1 - \gamma) - \lambda_t^w,$$

$$0 = \beta E_t V_1 (\hat{w}_t, \hat{w}_{t+1}^*) - \lambda_t^\pi \left(\beta E_t \frac{\partial \hat{\pi}_{t+1}}{\partial \hat{w}_t} - (1 - \gamma) \beta E_t \frac{\partial \hat{\pi}_{t+1}^\omega}{\partial \hat{w}_t} + \Pi \right) + \lambda_t^w.$$

Using the first three first-order conditions, we get

$$2\theta_x \hat{x}_t + (2\theta_\pi \hat{\pi}_t + 2\theta_{\pi^\omega} \hat{\pi}_t^\omega) \frac{1}{1-\gamma} \Pi = 0.$$

(125)

Suppose that we change $\hat{\pi}_t^\omega$ and keep $(\hat{w}_t - \hat{w}_t^*)$ fixed. To ensure that the two equality constraints hold, we require that, for the second constraint, $\frac{d\hat{\pi}_t}{d\hat{\pi}_t^\omega} = 1$ and, for the first constraint, $\frac{d\hat{x}_t}{d\hat{\pi}_t^\omega} = \frac{1-\gamma}{\Pi}$

$$1 = \frac{1}{1-\gamma} \Pi \frac{d\hat{x}_t}{d\hat{\pi}_t^\omega}.$$

(126)

The effect on the objective of a change in $\hat{\pi}_t^\omega$ is then

$$\left(2\theta_x (\hat{x}_t) \frac{d\hat{x}_t}{d\hat{\pi}_t^\omega} + 2\theta_\pi (\hat{\pi}_t) \frac{d\hat{\pi}_t}{d\hat{\pi}_t^\omega} + 2\theta_{\pi^\omega} (\hat{\pi}_t^\omega) \right) d\hat{\pi}_t^\omega = \left(2\theta_x (\hat{x}_t) \frac{1-\gamma}{\Pi} + 2\theta_\pi (\hat{\pi}_t) + 2\theta_{\pi^\omega} (\hat{\pi}_t^\omega) \right) d\hat{\pi}_t^\omega = 0$$

(127)

using expression (125). From the last first-order condition we have

$$\beta E_t V_1 (\hat{w}_t, \hat{w}_{t+1}^*) = -\frac{2\theta_\pi \hat{\pi}_t + 2\theta_{\pi^\omega} \hat{\pi}_t^\omega}{\gamma} \left(\beta E_t \frac{\partial \hat{\pi}_{t+1}}{\partial \hat{w}_t} - (1 - \gamma) \beta E_t \frac{\partial \hat{\pi}_{t+1}^\omega}{\partial \hat{w}_t} + \Pi \right)$$

(128)

$$-2\theta_\pi \hat{\pi}_t - \frac{2\theta_\pi \hat{\pi}_t + 2\theta_{\pi^\omega} \hat{\pi}_t^\omega}{\gamma}.$$

The effect on the right-hand side of the change in $\hat{\pi}_t^\omega$, while keeping real wages constant, is

$$\left(-(2\theta_\pi + 2\theta_{\pi^\omega}) \left(\frac{\Pi + 1}{\gamma} \right) - 2\theta_\pi \right) d\hat{\pi}_t^\omega.$$

(129)

Since the term within the parenthesis is positive and $d\hat{\pi}_t^\omega$ can be changed in any direction, this implies that the last first-order condition is violated. Since the change led to an unaffected value

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of the problem, there is another change in the variables $\hat{\pi}_t$, $\hat{\pi}_t^\omega$, $\hat{x}_t$ and $\hat{w}_t$ which leads to a strict improvement in the value of the problem, ruling out optimality of a solution where both inequality constraints holds strictly as in (123). $\blacksquare$

First, suppose $\hat{\pi}_t^\omega > 0$. Then, we get $\mu_t^0 = 0$ from the complementary slackness condition. The first-order conditions then simplify to

$$
\begin{aligned}
0 &= 2\theta_x \hat{x}_t - \lambda_t^\pi \frac{\gamma}{1-\gamma} \Pi + \mu_t^{\pi^\omega} \Omega_x, \\
0 &= 2\theta_\pi \hat{\pi}_t + \lambda_t^\pi + \lambda_t^w, \\
0 &= 2\theta_{\pi^\omega} \hat{\pi}_t^\omega - \lambda_t^\pi (1-\gamma) - \lambda_t^w + \mu_t^{\pi^\omega}, \\
0 &= \beta E_t V_1 \left(\hat{w}_t, \hat{w}_{t+1}^* \right) - \lambda_t^\pi \left(\beta E_t \frac{\partial \hat{\pi}_{t+1}}{\partial \hat{w}_t} - (1-\gamma) \beta E_t \frac{\partial \hat{\pi}_{t+1}^\omega}{\partial \hat{w}_t} + \Pi \right) \\
&\quad + \lambda_t^w - \mu_t^{\pi^\omega} \left(\frac{1+\alpha_w}{2\alpha_w} \beta E_t \frac{\partial \hat{\pi}_{t+1}^\omega}{\partial \hat{w}_t} - \Omega_w \right).
\end{aligned}
\tag{130}
$$

We also have the constraints

$$
\begin{aligned}
\hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + (1-\gamma) \left(\hat{\pi}_t^\omega - \beta E_t \hat{\pi}_{t+1}^\omega \right) + \Pi \left(\hat{w}_t - \hat{w}_t^* \right) + \frac{\gamma}{1-\gamma} \Pi \hat{x}_t, \\
\hat{w}_t &= \hat{w}_{t-1} + \hat{\pi}_t^\omega - \hat{\pi}_t, \\
\hat{\pi}_t^\omega &= \frac{1+\alpha_w}{2\alpha_w} \beta E_t \hat{\pi}_{t+1}^\omega - \Omega_x \hat{x}_t - \Omega_w \left(\hat{w}_t - \hat{w}_t^* \right).
\end{aligned}
\tag{131}
$$

Using the first three first-order conditions gives

$$
\begin{aligned}
2\theta_x \hat{x}_t - \lambda_t^\pi \frac{\gamma}{1-\gamma} \Pi + \mu_t^{\pi^\omega} \Omega_x &= 0, \\
2\theta_\pi \hat{\pi}_t + \lambda_t^\pi + \lambda_t^w &= 0, \\
2\theta_{\pi^\omega} \hat{\pi}_t^\omega - \lambda_t^\pi (1-\gamma) - \lambda_t^w + \mu_t^{\pi^\omega} &= 0.
\end{aligned}
\tag{132}
$$

We get;

$$
\begin{pmatrix} \lambda_t^\pi \\ \lambda_t^w \\ \mu_t^{\pi^\omega} \end{pmatrix} = -2 \begin{pmatrix} -\frac{\gamma}{1-\gamma}\Pi & 0 & \Omega_x \\ 1 & 1 & 0 \\ -(1-\gamma) & -1 & 1 \end{pmatrix}^{-1} \cdot \begin{pmatrix} \theta_x \hat{x}_t \\ \theta_\pi \hat{\pi}_t \\ \theta_{\pi^\omega} \hat{\pi}_t^\omega \end{pmatrix}.
\tag{133}
$$

We can write the Lagrange multipliers as functions of $\hat{x}_t$, $\hat{\pi}_t$ and $\hat{\pi}_t^\omega$ and hence, we can eliminate the Lagrange multipliers from the last first-order condition

$$
\begin{aligned}
0 &= \beta E_t V_1 \left(\hat{w}_t, \hat{w}_{t+1}^* \right) - \lambda_t^\pi \left(\hat{x}_t, \hat{\pi}_t, \hat{\pi}_t^\omega \right) \left(\beta E_t \frac{\partial \hat{\pi}_{t+1}}{\partial \hat{w}_t} - (1-\gamma) \beta E_t \frac{\partial \hat{\pi}_{t+1}^\omega}{\partial \hat{w}_t} + \Pi \right) \\
&\quad + \lambda_t^w \left(\hat{x}_t, \hat{\pi}_t, \hat{\pi}_t^\omega \right) - \mu_t^{\pi^\omega} \left(\hat{x}_t, \hat{\pi}_t, \hat{\pi}_t^\omega \right) \left(\frac{1+\alpha_w}{2\alpha_w} \beta E_t \frac{\partial \hat{\pi}_{t+1}^\omega}{\partial \hat{w}_t} - \Omega_w \right).
\end{aligned}
\tag{134}
$$

We then get the following system of equations

$$
\begin{aligned}
0 &= \beta E_t V_1 \left(\hat{w}_t, \hat{w}_{t+1}^* \right) - \lambda_t^\pi \left(\hat{x}_t, \hat{\pi}_t, \hat{\pi}_t^\omega \right) \left(\beta E_t \frac{\partial \hat{\pi}_{t+1}}{\partial \hat{w}_t} - (1-\gamma) \beta E_t \frac{\partial \hat{\pi}_{t+1}^\omega}{\partial \hat{w}_t} + \Pi \right) \\
&\quad + \lambda_t^w \left(\hat{x}_t, \hat{\pi}_t, \hat{\pi}_t^\omega \right) - \mu_t^{\pi^\omega} \left(\hat{x}_t, \hat{\pi}_t, \hat{\pi}_t^\omega \right) \left(\frac{1+\alpha_w}{2\alpha_w} \beta E_t \frac{\partial \hat{\pi}_{t+1}^\omega}{\partial \hat{w}_t} - \Omega_w \right), \\
\hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + (1-\gamma) \left(\hat{\pi}_t^\omega - \beta E_t \hat{\pi}_{t+1}^\omega \right) + \Pi \left(\hat{w}_t - \hat{w}_t^* \right) + \frac{\gamma}{1-\gamma} \Pi \hat{x}_t, \qquad (135) \\
\hat{w}_t &= \hat{w}_{t-1} + \hat{\pi}_t^\omega - \hat{\pi}_t, \\
\hat{\pi}_t^\omega &= \frac{1+\alpha_w}{2\alpha_w} \beta E_t \hat{\pi}_{t+1}^\omega - \Omega_x \hat{x}_t - \Omega_w \left(\hat{w}_t - \hat{w}_t^* \right).
\end{aligned}
$$

Second, suppose that $\hat{\pi}_t^\omega = 0$. Then, we get $\mu_t^{\pi^\omega} = 0$ from the complementary slackness condition. The first-order conditions then simplify to

$$
\begin{aligned}
0 &= 2\theta_x \hat{x}_t - \lambda_t^\pi \frac{\gamma}{1-\gamma} \Pi, \\
0 &= 2\theta_\pi \hat{\pi}_t + \lambda_t^\pi + \lambda_t^w, \qquad (136) \\
0 &= -\lambda_t^\pi (1-\gamma) - \lambda_t^w + \mu_t^0, \\
0 &= \beta E_t V_1 \left(\hat{w}_t, \hat{w}_{t+1}^* \right) - \lambda_t^\pi \left(\beta E_t \frac{\partial \hat{\pi}_{t+1}}{\partial \hat{w}_t} - (1-\gamma) \beta E_t \frac{\partial \hat{\pi}_{t+1}^\omega}{\partial \hat{w}_t} + \Pi \right) + \lambda_t^w
\end{aligned}
$$

or

$$
\begin{pmatrix} \lambda_t^\pi \\ \lambda_t^w \\ \mu_t^0 \end{pmatrix} = -2 \begin{pmatrix} -\frac{\gamma}{1-\gamma}\Pi & 0 & 0 \\ 1 & 1 & 0 \\ -(1-\gamma) & -1 & 1 \end{pmatrix}^{-1} \cdot \begin{pmatrix} \theta_x \hat{x}_t \\ \theta_\pi \hat{\pi}_t \\ 0 \end{pmatrix}. \qquad (137)
$$

Thus, we can write the Lagrange multipliers as functions of $\hat{x}_t$ and $\hat{\pi}_t$ and hence, once more, eliminate them from the last first-order condition. The last first-order condition becomes

$$
\beta E_t V_1 \left(\hat{w}_t, \hat{w}_{t+1}^* \right) - \lambda_t^\pi \left(\hat{x}_t, \hat{\pi}_t \right) \left(\beta E_t \frac{\partial \hat{\pi}_{t+1}}{\partial \hat{w}_t} - (1-\gamma) \beta E_t \frac{\partial \hat{\pi}_{t+1}^\omega}{\partial \hat{w}_t} + \Pi \right) + \lambda_t^w \left(\hat{x}_t, \hat{\pi}_t \right) = 0. \qquad (138)
$$

We then get the following system of equations

$$
\begin{aligned}
0 &= \beta E_t V_1 \left(\hat{w}_t, \hat{w}_{t+1}^* \right) - \lambda_t^\pi \left(\hat{x}_t, \hat{\pi}_t \right) \left(\beta E_t \frac{\partial \hat{\pi}_{t+1}}{\partial \hat{w}_t} - (1-\gamma) \beta E_t \frac{\partial \hat{\pi}_{t+1}^\omega}{\partial \hat{w}_t} + \Pi \right) + \lambda_t^w \left(\hat{x}_t, \hat{\pi}_t \right), \\
\hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + (1-\gamma) \left(\hat{\pi}_t^\omega - \beta E_t \hat{\pi}_{t+1}^\omega \right) + \Pi \left(\hat{w}_t - \hat{w}_t^* \right) + \frac{\gamma}{1-\gamma} \Pi \hat{x}_t, \qquad (139) \\
\hat{w}_t &= \hat{w}_{t-1} + \hat{\pi}_t^\omega - \hat{\pi}_t, \\
\hat{\pi}_t^\omega &= 0.
\end{aligned}
$$

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