Liquidity Risk and Contagion*

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Abstract

This paper explores liquidity risk in a system of interconnected financial institutions when these institutions are subject to regulatory solvency constraints. When the market's demand for illiquid assets is less than perfectly elastic, sales by distressed institutions depress the market price of such assets. Marking to market of the asset book can induce a further round of endogenously generated sales of assets, depressing prices further and inducing further sales. Contagious failures can result from small shocks. We investigate the theoretical basis for contagious failures and quantify them through simulation exercises. Liquidity requirements on institutions can be as effective as capital requirements in forestalling contagious failures.

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1 Introduction

Prudential regulations in the form of liquidity or capital requirements are designed to enhance the resilience to shocks of financial systems by requiring institutions to maintain prudent levels of liquidity and capital under a broad range of market conditions. However, at times of market turbulence the remedial actions prescribed by these regulations may have perverse effects on systemic stability. Forced sales of assets may feed back on market volatility and produce a downward spiral in asset prices, which in turn may affect adversely other financial institutions. This paper investigates these issues. In particular, it looks at the consequences of combining liquidity risk with externally imposed regulatory solvency requirements, when mark-to-market accounting of firms' assets are also in place.

We construct a model that incorporates two channels of contagion - direct balance sheet interlinkages among financial institutions and contagion via changes in asset prices. The former has been studied extensively, but the latter has received only scant attention. Our aim is to redress the balance. Changes in asset prices may interact with externally imposed solvency requirements or the internal risk controls of financial institutions to generate amplified endogenous responses that are disproportionately large relative to any initial shock. An initial shock that reduces the market value of a firm's balance sheet will elicit the disposal of assets or of trading positions. If the market's demand is less than perfectly elastic, such disposals will result in a short run change in market prices. When assets are marked to market at the new prices, the externally imposed solvency constraints, or the internally imposed risk controls may dictate further disposals. In turn, such disposals will have a further impact on market prices. In this way, the combination of mark-to-market accounting and solvency constraints have the potential to induce an endogenous response that far outweighs the initial shock.

Importantly, asset price contagion by itself cannot be used to argue against prudential regulations and transparency, for two reasons. First, we model only the *ex-post* stability effects of capital requirements and mark to market for given portfolio choices and not the positive *ex-ante* effects on incentives. For example, capital requirements and mark-to-market rules may deter financial institutions from taking excessive risks *ex-ante*. Second, even if we modelled these effects explicitly, the level of optimised liquid assets and capital held by financial institutions would still be suboptimal from the point of view of minimising systemic risk as individual institutions do not internalise the externality of network membership.

Regulators are familiar with the potentially destabilizing effect of solvency constraints in distressed markets. To take a recent instance, in the days following the September 11th attacks on New York and Washington financial markets around the world were buffeted by unprecedented turbulence. In response to the short term disruption, the authorities suspended various solvency tests applied to large financial institutions such as life insurance firms. In the U.K., for instance, the usual 'resilience test' applied to life insurance companies in which the firm has to demonstrate solvency in the face of a further 25% market decline was suspended for several weeks. Also, following the decline in European stock markets in the summer of 2002, the Financial Services Authority — the U.K. regulator — diluted the resilience test so as to preempt the destabilizing forced sales of stocks by the major market players.[1]

The LTCM crisis of 1998 can also be seen as an instance where credit

[1]FSA Guidance Note 4 (2002), "Resilience test for insurers". See also FSA Press Release, June 28th 2002, no FSA/PN/071/2002, "FSA introduces new element to life insurers' resilience tests".

interconnections and asset prices acted in concert as the main channel propagating widespread market distress (see BIS (1999), IMF (1998), Furfine (1999), Morris and Shin (1999)). Furfine, for instance, cites the arguments used by the Federal Reserve to justify intervention during the LTCM crisis in 1998. On one side, the Fed wanted to contain the disruption that the liquidation of LTCM would impose on the markets, where LTCM was a significant player, in order to avoid the spillover to other market participants without direct credit relationships with LTCM. On the other, the Fed was concerned of knock-on effects that the liquidation of LTCM would imply on banks with high direct exposures to LTCM.

There has been a substantial body of work that has examined balance sheet interlinkages as a possible source of contagious failures of financial institutions. Most papers calibrate the models using actual cross-exposures in real banking systems (or an approximation of them) and simulate the effects of a shock to the system resulting from the failure of one or more institutions. Sheldon and Maurer (1998) study the Swiss banking system. Upper and Worms (2002) consider the German system. Furfine (1999) analyses interlinkages in the US Federal Funds market. Wells (2002) focuses on the UK banks. Elsinger et al (2002) consider an application to the Austrian banking system, and provide a stochastic extension of the framework (using the concept of value at risk). Cifuentes (2002) uses the same framework to analyse the link between banking concentration and systemic risk.

The main focus of these papers is on finding estimates of interbank credit exposures. Once this is determined, systemic robustness is assessed by simulating the effects on the system of the failure of one bank at a time. Importantly, solvency is assessed based on fixed prices that do not change through time. Such an assumption would be appropriate if the assets of the institutions do not undergo any changes in price, or if solvency is assessed based on historical prices. Invariably, a consistent finding of these papers is that systemic contagion is never significant in practice, even in the presence of large shocks. In the absence of price effects, this is hardly surprising, as interbank loans and deposits represent only a limited fraction of banks' balance sheets. Conventional wisdom is also that collateralisation may have mitigated this risks further.

Our paper suggests that systemic risk in these networks may be larger than thought, even in the presence of collateralisation. The reason is that the risk that materialises is not a credit risk but a market risk. This is a new dimension to systemic contagion illustrated by recent events. The value of any collateral backing a credit exposure is clearly subject to this risk, and hence not immune to systemic risk through this channel.

For commercial banks whose assets consist mainly of corporate or retail loans, the use of backward-looking prices in assessing solvency may be a reasonable approach, although even such banks would also hold some financial assets on their trading book that would be marked to market. For financial firms that hold mainly marketable assets - such as insurance companies, hedge funds or investment banks - the assumption of fixed prices would be highly unrealistic. Even for commercial banks, whose assets are currently accounted for on an accruals basis, our analysis can be seen as a hypothetical thought-experiment on the consequences of the introduction of the mark-to-market accounting of assets.

Our paper can be seen in the light of the recent theoretical literature on banking and financial crises that has emphasised the limited capacity of the financial markets to absorb sales of assets (see Allen and Gale (2002), Gorton and Huang (2003) and Schnabel and Shin (2002)), where the price repercussions of asset sales have important adverse welfare consequences. Similarly, the ineffecient liquidation of long assets in Diamond and Rajan (2000) has an analogous effect. The shortage of aggregate liquidity that such liquidations bring about can generate contagious failures in the banking system.

One important conclusion of our paper is that prudential regulation (in the form of minimum capital requirement ratios or other solvency constraints) when combined with mark-to-market rules can sometimes generate undesirable spillover effects. Marking to market enhances transparency but it may introduce a potential channel of contagion and may become an important source of systemic risk.[2] Liquidity requirements can mitigate contagion, and can play a similar role to capital buffers in curtailing systemic failure.

The paper is organised as follow. Section 2 illustrates the framework. Section 3 presents the algorithm. Section 4 discusses the main results. Section 5 concludes.

2 Framework

There are n interlinked financial institutions (for simplicity, we can think of these as being banks). The liability of bank i to bank j is denoted by L_{ij}. The total liability of bank i is then the sum

$$\bar{x}_i \equiv \sum_j L_{ij}$$

[2] It seems intuitive to conjecture that when players are faced with illiquid markets, they would try to insure against liquidity black holes by holding more liquid assets. In other words, the argument in Jackson, Perraudin and Saporta (2002) should apply also to liquidity - ie that market discpline would push banks to hold more liquid assets. That said, each individual bank will have no incentive to internalise any network membership externalities, resulting in suboptimal level of liquidity.

Denote by x_i the market value of bank i's interbank liabilities. This can be different from the notional value because the debtor may be unable to repay these liabilities in full. Interbank claims are of equal seniority, so that if the market value falls short of the notional liability, then the bank's payments are proportional to the notional liability. Let $\pi_{ij} = L_{ij}/\bar{x}_i$. Then, the payment by i to j is given by

$$x_i \pi_{ij} \tag{1}$$

while the total payment received by bank i from all other banks is

$$\sum_j x_j \pi_{ji}$$

Bank i's endowment of the illiquid asset is given by e_i. The price of the illiquid asset is denoted by p. In addition, bank i has holdings of the liquid asset given by c_i. Thus, the *net worth* or *equity value* of bank i is

$$pe_i + c_i + \sum_j x_j \pi_{ji} - x_i$$

Limited liability of the bank implies that its equity value is non-negative. Priority of debt over equity implies that equity value is strictly positive only when $x_i = \bar{x}_i$ (i.e. bank i's payment is equal to its notional obligation). Thus, the vector of payments $x = (x_1, x_2, \cdots, x_n)$ is such that for each i,

$$x_i = \min \left\{ \bar{x}_i,\ w_i(p) + \sum_j x_j \pi_{ji} \right\} \tag{2}$$

where $w_i(p) = pe_i + c_i$ is the marked-to-market value of the liquid and illiquid assets of bank i. More succinctly, we can write (2) in vector form as

$$x = \bar{x} \wedge \left(w(p) + \Pi^T x \right) \tag{3}$$

where $w(p) = (w_1(p), \cdots, w_n(p))$, Π^T is the transpose of the exposure matrix Π, and $\wedge$ is the pointwise minimum operator. Thus, a clearing vector x that satisfies (3) is a fixed point of the mapping

$$H(x) \equiv \bar{x} \wedge \left(w(p) + \Pi^T x\right)$$

$H(.)$ is an increasing function on the lattice $\mathbb{R}^n_+$ (with infimum defined by the operator $\wedge$), and where $H(0) \geq 0$ and $H(\bar{x}) \leq \bar{x}$. Hence, by Tarski's fixed point theorem, there is at least one fixed point of $H(.)$, and hence at least one clearing vector x. Eisenberg and Noe (2001) have proved that under mild regularity conditions, there is a unique fixed point of such a function. A sufficient condition for the existence of a unique fixed point is that, first, the system is *connected* in the sense that the banks cannot be partitioned into two or more unconnected sub-systems, and that there is at least one bank that has positive equity value in the system. By drawing on the results of Eisenberg and Noe, we can proceed as follows. For any fixed value of p, the net worth of each bank is determined fully. Hence, by appealing to the result of Eisenberg and Noe (2001), we have the following lemma.

Lemma 1 *Suppose the banking system is connected, and that at price p, there is at least one bank that has positive equity value. Then, there is a unique clearing vector x such that*

$$x = \bar{x} \wedge \left(w(p) + \Pi^T x\right)$$

Let us write $x(p)$ to be the unique clearing vector when the price of the illiquid asset is given by p. Then each payment x_{ij} is determined by the pro rata rule (1). Hence, this lemma allows us to write each x_{ij} as a function of p. We will use this feature in what follows.

2.1 Capital Adequacy Ratio

Assets held by the bank attract a regulatory minimum capital ratio, which stipulates that the ratio of the bank's equity value to the mark to market value of its assets must be above some pre-specified ratio r^*. When a bank finds itself violating this constraint, it must sell some of its assets so as to reduce the size of its balance sheet. Denote by t_i the units of the liquid asset sold by bank i, and denote by s_i the units of the illiquid asset sold by bank i. The liquid asset has constant price of 1. The illiquid asset has price p, which is determined in equilibrium.

The capital adequacy constraint puts a lower bound on the capital asset ratio of the bank. The constraint is given by

$$\frac{pe_i + c_i + \sum_j x_j \pi_{ji} - x_i}{p\left(e_i - s_i\right) + \left(c_i - t_i\right) + \sum_j x_j \pi_{ji}} \geq r^* \tag{4}$$

The numerator is the equity value of the bank where the interbank claims and liabilities are calculated in terms of the expected payments. The denominator is the marked-to-market value of its assets after the sale of s_i units of the illiquid asset and sale t_i of the liquid asset. The underlying assumption is that the assets are sold for cash, and that cash does not attract a capital requirement. Thus, if the bank sells s_i units of the illiquid asset, then it has ps_i in cash (assuming for simplicity that it starts with zero cash), and holds $p\left(e_i - s_i\right)$ worth of the illiquid asset. Hence, we have the sum of these (given by pe_i) on the numerator, while we have only the mark to market value of the illiquid asset (given by $p\left(e_i - s_i\right)$) on the denominator. Similar remarks apply to the liquid asset. Thus, by selling its assets for cash, the bank can reduce the size of its balance sheet and hence reduce the denominator, making the capital asset ratio larger.

We make two assumptions. First, the bank cannot short sell the assets.

Thus,

$$s_i \in [0, e_i] \quad \text{and} \quad t_i \in [0, c_i]$$

Second, we assume that the bank sells all its liquid assets before it starts selling its illiquid assets. Thus, $s_i > 0$ only if $t_i = c_i$. Any value maximizing bank will follow this rule, and hence this assumption is not a strong one.

2.2 Equilibrium

An equilibrium is the triple (x, s, p) consisting of a vector of payments x, vector of sales of illiquid asset s, and the price p of the illiquid asset such that:

1. For all banks i, $x_i = \min \left\{ \bar{x}_i,\ pe_i + c_i + \sum_j x_j \pi_{ji} \right\}$
2. For all banks i, s_i is the smallest sale that ensures that the capital adequacy condition is satisfied. If there is no value of $s_i \in [0, e_i]$ for which the capital adequacy condition is satisfied, then $s_i = e_i$.
3. There is a downward sloping inverse demand function $d^{-1}(.)$ such that $p = d^{-1}\left(\sum_i s_i\right)$.

The first clause is reiterating the limited liability of equity holders, and the priority and equal seniority of the debt holders. The second clause says that either the bank is liquidated altogether, or its sales of illiquid assets (possibly zero) reduces its assets sufficiently to comply with the capital adequacy ratio. Finally, the third clause states that the price of the illiquid asset is determined by the intersection of a downward sloping demand curve and the vertical supply curve given by aggregate sales.

By re-arranging the capital adequacy condition (4) together with the condition that s_i is positive only if $t_i = c_i$, we can write the sale s_i as a function of p, where $s_i = 0$ if the capital adequacy condition can be met by sales of the liquid asset or from no sales of assets, but otherwise is given by

$$s_i = \min\left\{ e_i,\ \frac{x_i - (1 - r^*)\left(\sum_j x_j \pi_{ji} + p e_i\right) - c_i}{r^* p} \right\}$$

The interbank payments x_{ij} are all functions of p. Thus, s_i itself is a function of p, and we write $s_i(p)$ the sales by bank i are a function of the price p. Let

$$s(p) = \sum_i s_i(p)$$

be the aggregate sale of the illiquid asset given price p. Since each $s_i(.)$ is decreasing in p, the aggregate sale function $s(p)$ is decreasing in p.

2.3 Equilibrium Price

The inverse demand curve for the illiquid asset is assumed to be

$$p = e^{-\alpha\left(\sum_i s_i\right)} \tag{5}$$

where $\alpha > 0$ is a positive constant. The maximum price is $p = 1$, which occurs when sales are zero. We impose two regularity conditions on the demand and sales functions. First, we require that the banking system does not spiral down into zero net worth when all the illiquid assets are sold. When the entire endowment of illiquid assets in the system are sold, there is at least one bank that has positive equity value.[3] Let $\underline{p}$ be the price of the illiquid asset when the entire endowment of the illiquid asset is sold. That is $\underline{p} = d^{-1}\left(\sum_i e_i\right)$. Our first regularity condition is

$$s\left(\underline{p}\right) < d\left(\underline{p}\right) \tag{6}$$

[3] This condition is only needed to show the existence of an interior solution, and will be removed when we run our empirical simulations later.

Our second regularity condition is at the opposite end of the price spectrum. We require that when the price of the illiquid asset is at its highest, given by $p = 1$, no bank is forced to sell any of its illiquid assets. In other words, $s(1) = 0$. From (5), we have $d(1) = 0$. Together, we have

$$s(1) = d(1) \tag{7}$$

An equilibrium price of the illiquid asset is a price p for which

$$s(p) = d(p)$$

From (7), we have at least one equilibrium price, given by $p = 1$. This is the status quo price where the banking system has not suffered any adverse shock. However, an equilibrium price lower than 1 is possible provided that the $s(p)$ curve lies above the $d(p)$ curve for some ranges of price (see figure 1).

The price adjustment process can be depicted as a step adjustment process in the arc below the $s(p)$ curve, but above the $d(p)$ curve. The process starts with a downward shock to the price of the illiquid asset. At the lower price p_0, the forced sales of the banks puts quantity $s(p_0)$ on the market. However, this pushes the price further down to $p_1 = d^{-1}(s(p_0))$. This elicits further sales, implying total supply of $s(p_1)$. Given this increased supply, the price falls further to $p_2 = d^{-1}(s(p_1))$, and so on. The price falls until we get to the nearest intersection point where the $d(p)$ curve and $s(p)$ curve cross.

Equivalently, we may define the function $\Phi : [\underline{p}, 1] \rightarrow [\underline{p}, 1]$ as

$$\Phi(p) = d^{-1}(s(p))$$

and an equilibrium price is a fixed point of the mapping $\Phi(.)$. The function $\Phi(.)$ has the following interpretation. For any given price p, the value $\Phi(p)$

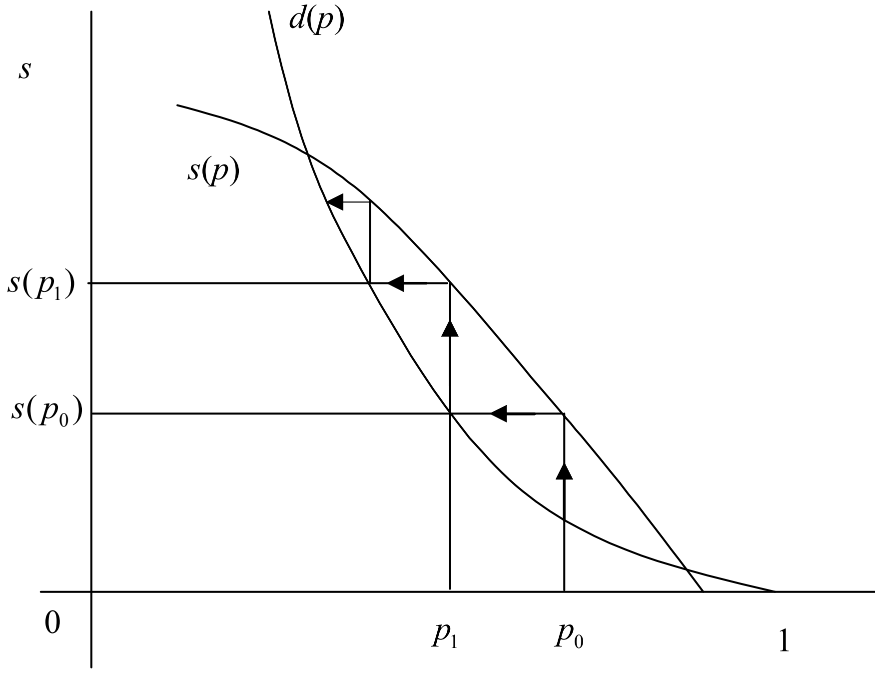

Figure 1:

is the market-clearing price of the illiquid asset that results when the price of the illiquid asset on the banks' balance sheets are evaluated at price p. Thus, when $\Phi(p) < p$, we have the precondition for a downward spiral in the illiquid asset's price. The price that results from the sales is lower than the price at which the balance sheets are evaluated. We can summarize our results as follows.

Proposition 2 *If $\Phi(p) \geq p$ for all p, there is a unique equilibrium in which $p = 1$. In this case, the value of the banking system declines only by the size of the initial shock.*

Proposition 3 *If $\Phi(p) < p$ for some values of p, then there is an equilibrium in which p is strictly below 1, and in which there are sales of the illiquid asset. In this case, the banking system will reach this equilibrium by the step adjustment process provided that the initial shock is big enough.*

The first proposition is immediate. Thus, when the $\Phi(p)$ curve lies above the 45-degree line, there is no endogenous fall in the asset value of the banking system. The only effect of the initial shock is to reduce the banking sector's value by the amount of the initial shock. The second proposition follows from the continuity of the $\Phi(.)$ mapping, which inherits its continuity from the continuity of $d(p)$ and $s(p)$. In this case, there is an amplification effect that arises from the endogenous responses generated by the forced sales.

3 Simulations

We now illustrate the effects of illiquidity as given in Proposition (3) by means of several examples. The basic structure of the model is the same as that outlined in the previous section. But to make the example more realistic we include deposits as an additional liability in banks' balance sheets. We use these to explore the implications on systemic robustness of changes in a wide set of systemic and policy parameters.

3.1 The algorithm

To identify the equilibrium of the model, we devise an iterative procedure whose structure is designed to obtain the equilibrium to the adjustment procedure defined in the previous section. The algorithm can be described as follows.

Given the level of the minimum capital ratio r^*, the algorithm checks that the equity ratio of each bank (r_k) satisfies condition (4). Failure to comply with this requirement triggers a resizing of the bank's balance sheet and possibly the liquidation of the bank.

There are two possible cases.

1. If $r_k \geq r^*$, then the bank satisfies the capital adequacy ratio and no action is required.

2. If $r_k < r^*$, then the bank violates the capital adequacy ratio and needs to liquidate assets.

In the second case, depending on the size of its equity capital, the bank can resize its balance sheet, scaling down the size of its assets to a new level consistent with the actual level of equity capital available. Alternatively, if this is not possible the bank is liquidated.[4] We assume that liquidation

[4] In principle, the bank could also raise equity capital in the markets. However we rule out this option on the grounds that at times of stress raising equity may be expensive, may take time, and may be impossible in some cases, if capital markets are shut.

occurs if equity capital is insufficient to support more assets than the outstanding claims in the interbank market. In other words, the threshold level of equity capital for technical solvency is given by:

$$r^* \sum_{i=1}^{n} L_{ij}$$

For a bank that violates the capital adequacy ratio, the resizing routine is activated. This entails a reduction of the size of the bank's balance sheet until the bank's assets can be supported by the given equity size. Assets are liquidated according to their degree of liquidity. First, banks liquidate their liquid asset and then they move to the illiquid asset. When the bank becomes insolvent, the liquidation routine is activated. All the bank's (liquid and illiquid) assets are liquidated and used to settle liabilities, according to the principles set out in the previous section (priority of debt claims, proportionality, limited liability). In particular, when default occurs, the defaulting bank pays all claimants in proportion of the size of their nominal claims on the bank's assets. This implies that the loss is distributed proportionally among all bank's creditors. Because of this, interbank assets and liabilities cannot be netted, as netting would effectively give priority to some claimants over others, implying that the loss is not evenly spread among holders of interbank assets.

Different liquidation rules apply to the three categories of bank assets when they have to be used to settle liabilities. Liquid assets are sold for cash at the notional value. Illiquid assets have to be liquidated at market price. The proceeds from the sale can then be used to settle liabilities. Interbank assets are not cashed but are redirected, or redistributed, at face value proportionally among the holders of the bank's liabilities who essentially take over the credit line given by the defaulting or resizing bank and become the new creditor of the contract. This liquidation rule of interbank assets reflects the fact that interbank loans normally cannot be expected to be recalled early in the event of default of the lender. In other words, default of the lender cannot trigger early repayment of a loan.

This variety of liquidation rules acknowledges the fact that different asset types carry different risks. In particular, liquid assets are always exchanged at face value and bear no risk. Illiquid assets are exposed to market and liquidity risks as their final value is determined endogenously as a market price, which in turn depends on the equilibrium of demand and supply. Interbank assets are exposed to credit (or counterpart) risk, as their value ultimately depends on whether the borrower is able or not to repay the loan in full, and on the recovery rate in the event of default.

Clearly, because different asset types imply exposure to a range of different risks, the actual asset composition of a bank's portfolio has a direct bearing on the bank's intrinsic creditworthiness, on its capacity to withstand shocks and on its susceptibility to contagion. Banks with significant holdings of liquid assets as a proportion of total assets are generally more resilient to shocks and less susceptible to contagion, as they are overall less exposed to fluctuations of the price of the illiquid asset and face lower credit risk. Additionally, if these banks default or have to resize, they create less externalities on the rest of the system, as they can settle their liabilities through the liquid asset, whose price is fixed. Thus, they would sell smaller amounts of the illiquid asset in the market, and would create less systemic contagion through movements of asset prices.

Importantly, when choosing their portfolio allocation banks do not internalise the positive externalities that holding more liquidity has on the stability of the system. Therefore, the privately determined liquidity will be sub-optimal. We do not model explicitly the banks' individual choices of liquidity (and capital). However, because banks do not internalise the externalities of network membership, the introduction of an *ex-ante* portfolio allocation to the problem would not necessarily guarantee that liquidity in equilibrium coincides with the level that minimises systemic risk. As a consequence, liquidity and capital requirements would need to be externally imposed. Moreover, they should be set in relation to a bank's contribution to systemic risk, and not on the basis of the bank's idiosyncratic risk.

One distinguishing feature of the algorithm is that for defaulting and oversized banks the algorithm keeps track of the quantity of the illiquid asset dropped in the market. In other words, payments in liquidations are kept under separate accounts according to their origin, in order to allow re-pricing of the illiquid asset when the market price changes. Combining this information with the given demand function of the illiquid asset allows calculating the new equilibrium price of the illiquid asset. Mark-to-market rules imply that all banks have to re-price their stock holding of illiquid assets in their balance sheet at the new (lower) given market price, which in turn may mean that banks that were previously safe, may now become illiquid or insolvent.

Formally, the algorithm determines in each round the set of banks that are oversized or insolvent and calculates the quantity of the illiquid asset that these nodes need to drop in the market. Given this quantity, it then determines the new price of the illiquid asset using the specified demand function. Then, the illiquid asset is re-priced by all nodes in the system, according to mark-to-market requirements. Finally the algorithm checks that all banks are solvent under the new price. If there is at least one that is not, the algorithm is iterated again until an equilibrium is found where all banks satisfy the solvency condition.

3.2 Parameterisation

For a first set of basic results we model a highly stylised banking system. We keep a number of background parameters constant and explore the effects on systemic stability of a number of state and policy parameters.

The main background parameters are:

- **Initial banks' balance sheet:** we assume that banks are homogenous, i.e. that they all have the same initial balance sheet, which takes the form:

Liquid and illiquid assets	70	Equity	7
Interbank assets	30	Deposits	63
		Interbank liabilities	30
Total assets	100	Net worth and liabilities	100

- **Initial interbank claims:** the initial overall size of the inter-bank market is fixed and constant across all simulations; additionally banks have zero net interbank exposures (i.e. gross interbank liabilities and assets match for each bank);

- **Regulatory capital requirement:** banks' equity must be at least 7% of total assets; if equity falls below this threshold then banks would need to scale down their balance sheet and eventually liquidate;

- **Initial number of banks:** We set this equal to 10;

- **Demand function of the illiquid asset:** this takes an exponential form and is given by equation (5), and we set values for the parameter α, as described below.

These assumptions identify a 'neutral' banking system, where all agents are alike and where results do not depend on the size of banks' balance sheets. Moreover, results can be interpreted regardless of considerations of market concentration. Clearly, calibrating a realistic financial system with differentially sized firms would be an obvious extention of this research.

We explore how systemic stability is affected by the following parameters:

- **Capital buffer:** defined as the margin of a bank's equity above regulatory capital. A high capital buffer allows a bank to withstand larger shocks before it is pushed below the threshold for regulatory solvency and forced to resize or liquidate. Thus a bank with high capital buffer is more resilient to shocks and generates less systemic risk through asset price movements and links in the interbank market.

- **Liquidity ratio:** defined as the initial proportion of liquid and illiquid assets in banks' balance sheets. Intuitively, systemic risk is lower in more liquid banking systems as banks can resize their balance sheets without creating large movements in the price of the illiquid asset, for any given price elasticity of the demand curve.

- **Banking interlinkages:** defines the structure of banks' interconnections through the interbank market and is given by the number and combinations of interbank links, for a given size of the interbank market. In particular, given the size of interbank loans and deposits, we fix the number of possible counterparts and then randomly simulate all the possible combinations that can be created for that given number of counterparts.

- **Price elasticity:** defined by the parameter α in equation (5). A low value of α implies an elastic demand for the illiquid asset, so that price changes will be smaller for a given amount of the illiquid asset sold in the market. In most simulations we assume a value of α such that p falls by 50% if all the illiquid assets held by banks are dropped in the market, or that there is a floor for p equal to 0.5.[5]

For convenience, we treat the capital buffer and the liquidity ratio as 'policy' parameters. The interlinkages structure, the price elasticity, and the size of the shock are 'state' parameters. Results presented here assume an idiosyncratic shock.

4 Results

Results are presented in terms of the final system equilibrium, when nodes do not further adjust their balance sheets and equilibrium prices are used to evaluate assets. However, the transition from the initial state to the final equilibrium contains additional information. We will use this in one specific example.

The shock that we simulate is the failure of one institution, which occurs with a certain initial loss given default (LGD).[6] The initial LGD is the excess of nominal liabilities over the value of the assets of the failed bank. Expressed as a fraction of total initial assets, it indicates the percentage loss that creditors suffer if assets are recovered at their liquidation value. Clearly

[5] In coding the algorithm we replace the first regularity condition for equilibrium (that at least one bank survives with positive equity when the entire endowment of the illiquid asset is sold in the market - Section 2.3) with a floor on the price of the illiquid asset. When the regularity condition is removed, it is possible, under certain circumstances, that all the insitutions in the financial system may go bankrupt. In this case, we assume by definition that the equilibrium price of the illiquid asset is given by the floor value.

[6] Alternatively, the initial shock could have been a shock to the price of the illiquid asset. This would have affected all banks in the same way. In this case, the shock takes the form of a shift to the left of the supply function $s(.)$ (Figure 1)

in our model the initial LGD does not necessarily coincide with the final losses suffered by banks. If the price elasticity of the illiquid asset is below infinity, total losses in equilibrium may be higher than the initial simulated shock as falls in the price of the illiquid asset imply destruction of equity value. This is an important distinction from Eisenberg and Noe, where the initial loss is simply reallocated among the nodes, there is no destruction of system value, and the final loss is always equal to the size of the simulated shock.

We present the result in terms of the total number of banks failed as a consequence of the initial shock, using heatmap charts. Thus contagion is measured by the number of banks that fail after the first bank is shocked. The figures presented assume that the size of the shock, the price elasticity and the size of the capital buffer are fixed. The number of credit counterparts and liquidity vary. For the latter, we vary liquid assets as fraction of total non-interbank assets.

4.1 System Resilience when $LGD = 0$

In the first set of simulations we consider a bank that has to be liquidated because it has exactly zero equity ($LGD = 0$). Therefore any contagion will stem from price effects. Figure 2 shows these results.

[Figure 2 here]

Panel A reports the limiting case of infinite demand elasticity ($\alpha = 0$). This is equivalent to the case in Eisenberg and Noe, and can be thought of as a case of historical cost accounting. It shows that contagion never occurs, as the price of the illiquid asset is constantly equal to one. However, when p reacts ($\alpha > 0$), contagion may occur if liquidity is low (Panel B). The liquidation of a failed bank implies the selling of assets in the market, which triggers a fall in the price of the illiquid asset. In turn, this generates two effects. Banks with direct exposures to the failed bank will be unable to recover the full amount of their loans. In addition, all banks will suffer a loss from the fall in the market price of the illiquid asset, if mark-to-market rules are in place. These losses imply that eventually banks may have to adjust their balance sheets in order to comply with capital adequacy requirements. These additional sales of assets cause further falls in prices, which in turn may feed back on banks' resilience. A vicious circle may be unleashed. Since all banks are identical, results tend to concentrate in the corners: either all banks fail or none of them does. The vicious circle does not necessarily end up in the collapse of the whole system. Under certain circumstances, the algorithm may converge to a solution where banks remain solvent after losing some capital.

The cases where banks remain solvent are those where liquidity is high. This is because banks can adjust their balance sheets by selling liquid assets, which can be sold at the notional value. Therefore, in this case the pressure on banks' balance sheets arising from the falls in the price of the illiquid asset is lower. Importantly, banks' liquidity is lower in the final equilibrium. This case shows that asset prices may be a powerful channel of systemic contagion. It also shows that liquidity holding can help to avoid contagion.

Panel C illustrates the relation between number of interlinkages and systemic resilience. Two basic results can be highlighted. First, the panel includes a case where nodes are not linked via reciprocal loans and deposits (autarky). Nodes in this system may be thought of as insurance companies or mutual funds, which are only exposed to asset price contagion.

Our simulations show that an autarkic system can be more resilient than an interlinked system. The explanation is straightforward. In autarky, there is no channel for the transmission of credit losses among financial institutions. The losses are borne entirely by the customers of these financial institutions. Other institutions in the system are affected only via price effects. By contrast, when there are credit linkages among nodes, credit losses are transmitted to other nodes in addition to the aggregate price effect. Therefore, autarky may become more stable than a system of interconnected banks.[7]

A second result is that when there are credit relationships, the system tends to be more resilient to shocks when the number of counterparts is higher. This is shown in Panel C. Intuitively, a given credit loss is spread across a higher number of agents and thus each faces a loss that is proportionally smaller. In this case, lower capital buffers may be enough to withstand the loss. This result is straightforward, and goes in line with Allen and Gale (2000) in that systems that are more interconnected are also safer. However, when we move to a case of LGD greater than zero, this result can be different.

4.2 System Stability and Interlinkages

As explained in the previous section, more diversified interbank credit structures may lead to safer systems. If a given credit loss is absorbed by more agents, the amount that each of them has to face is smaller and therefore it is more likely that agents can bear the loss without further failures. However, this result may not hold when asset prices are an additional channel of contagion.

This can be illustrated with a simple example. Consider the case of an insolvent bank, which has liabilities towards one other bank only. Suppose that losses imply that the creditor bank also fails. The failure of the second

[7] Intuitively, autarky would be safest also in a world à la Allen and Gale (2000), (ie without price contagion).

bank implies additional sales of illiquid assets, with the consequential price impact. Notice that this impact is limited by the amount of the illiquid asset held by the failed bank. Consider now the case of several creditor banks. Suppose that, given that the loss is spread among more creditors, none of them fails in the first round. But in order to adjust their balance sheets, they have to sell illiquid assets. Notice that the amount sold in the market now is not limited by the balance sheet of one bank, but by the sum of the balance sheets of all the banks that are exposed to the first default. It is possible therefore, that the fall in the price of the illiquid asset may be higher in the case of more interconnections. If the price fall is larger, adjustments to comply with capital requirements by other banks will also be higher. This implies that the endogenous process of price reduction that is being unleashed can be of wider magnitude in the case of a higher number of counterparts.

[Figure 3 here]

A case where this happens is shown in Figures 3 to 5. These figures describe the case of a 30% LGD, $\alpha = 0.5$ and initial asset-to-capital ratio of 8%. In Figure 3 we look at the transition between the initial and final steady state by showing the number of insolvencies at each round by different total number of counterparts. In the case with one credit counterpart there is only one failure in the first round. The cases with two to four counterparts show the situation just described. In the initial rounds there are no failures, given that initially no bank receives a big shock. But the process generated in the market of the illiquid asset ends up in a higher number of defaults in later rounds.

[Figure 4 here]

Figure 4 shows the evolution of illiquid assets sold in the market in successive iterations of the simulation. In the case of one credit counterpart, only the assets of the failed bank are sold in the first round. However, as the figure shows, in the cases of two and three credit links the amount sold in the market is larger in the same round, despite the fact that no bank fails. This implies a larger fall in price, as can be seen in Figure 5.

[Figure 5 here]

The asset price channel of systemic contagion disappears when the number of interlinkages is high enough to allow banks to stand the losses without selling illiquid assets. Balance-sheet adjustments take place by selling liquid assets only. In the example, this happens when the number of counterparts is five or more.

This non-linear response to a shock with respect to the number of interconnections is one important finding of our simulation exercise. Intuitively, more interconnected systems can lead to more systemic risk also in a world without price contagion, as in Allen and Gale (2000), if shocks are large enough. However, price effects do increase the likelihood of this phenomenon significantly.[8]

4.3 Liquidity Buffer

One way to curtail systemic contagion is by requiring banks to hold more liquid assets. These assets allow banks to adjust their balance sheets without receiving adverse feedback effects from market prices. Sales of illiquid assets

[8] Our experiments suggest that to get a non-linear response in an Allen and Gale world shocks would have to be so large as to be implausible (a bank default with LGD of four times assets).

may still occur, but at a level that is below the point where the fall in prices generates systemic contagion.

[Figure 6 here]

Figure 6 shows the effects of liquidity. Moving along the horizontal axis we increase the fraction of liquid assets as a fraction of the total liquid plus illiquid assets that banks have. The figure shows that there is a threshold liquidity level beyond which no systemic contagion via asset prices occurs.[9] Additionally, for this combination of shock and price elasticity there is a clear non-linear relationship between the number of interlinkages and the liquidity threshold. The positively sloped part (from 0 to 5 interconnections) comes from the effect explained in the previous section. The negatively sloped part (from 5 to 9 links) shows that for a higher number of credit counterparts, the liquidity threshold is reduced. This implies that liquidity and interconnections can be substitutes for systemic stability for an important range of parameter values.

The substitution between liquidity and interconnections follows straightforwardly from the fact that a larger number of counterparts diminishes the size of the shock that each of them faces. The countervailing effect described in the previous section vanishes with a sufficiently large number of counterparts.

4.4 Capital Buffer

Systemic contagion can obviously also be contained by higher capital buffers. If banks have capital in excess of the amounts required by the regulator, they

[9] In the case of a single link in the interbank market, there is evidence of contagion to at least one other bank in the system, for any level of the liquidity ratio. However, this is due to direct credit exposure, and not to asset price contagion.

may not need to adjust their balance sheets when they are hit by an adverse shock, both directly through their interbank exposures, and indirectly, via asset price movements. This implies that when capital is higher than the minimum required by the regulators, the threshold liquidity levels are reduced. Figure 7 shows this result.

[Figures 7 and 8 here]

It is possible, therefore, to derive a relationship between the threshold level of liquid assets for a given level of the capital ratio. The higher the capital ratio, the lower the liquidity required to avoid systemic losses. Figure 8 shows this relation for different given levels of connectivity.

5 Conclusions

Under certain circumstances, prudential regulations can have perverse effects on the stability of a financial system. We look at the *ex-post* stability effects of capital requirements in a system of interconnected banks for given portfolio choices, when mark to market rules are in place. Because financial institutions do not internalise the externalities of network membership, banks' liquidity choices will be suboptimal. As a consequence, liquidity and capital requirements need to be imposed externally, and should be set in relation to a bank's contribution to systemic risk, rather than on the basis of the bank's idiosyncratic risk.

One message that emerges from our simulations is that for a given shock, systemic resilience and bank interconnections are non linearly related, ie under particular circumstances more interconnected systems may be riskier than less connected systems.

Another important message is that liquidity buffers play a role similar to capital buffers. In some circumstances, liquidity requirements may be more effective than capital buffers in forestalling systemic effects. When the residual demand curve is extremely inelastic (such as during periods of major financial distress when risk appetite is very low), even a large capital buffer may be insufficient to prevent contagion, since the price impact of sales into a falling market would be very high. To put it another way, even a large capital cushion may be insufficient if the stuffing in the cushion turns out to be useless. Liquidity requirements can internalize some of the externalities that are generated by the price impact of selling into a falling market.

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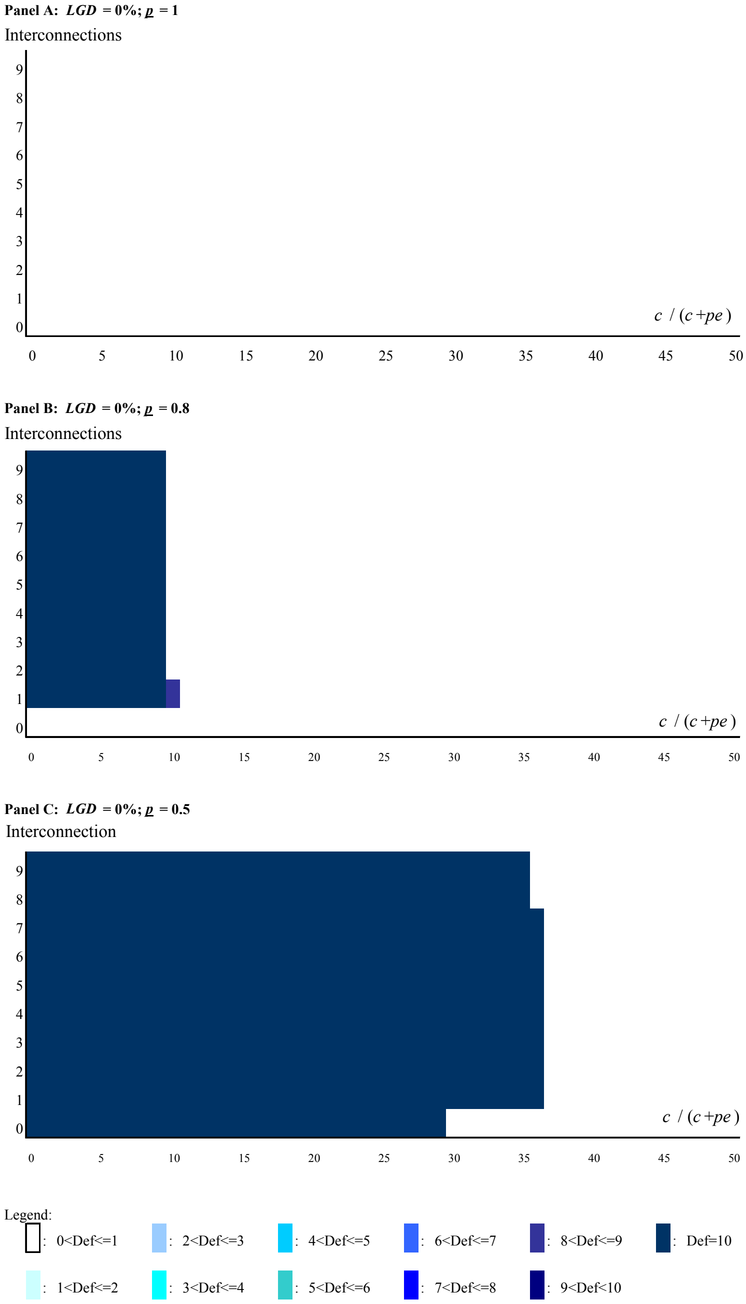

Figure 2: $LGD = 0$

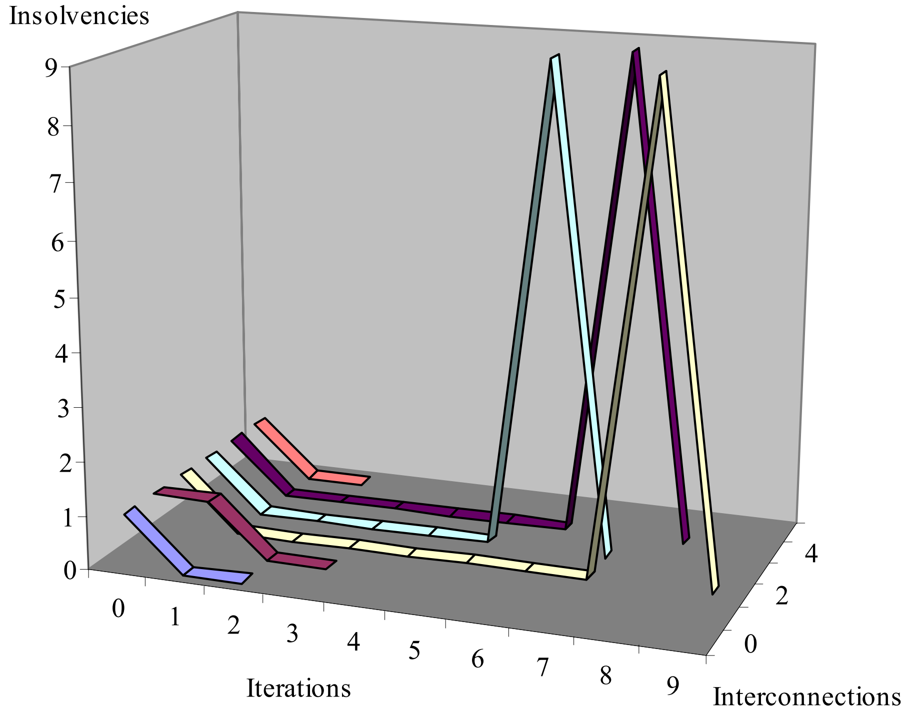

Figure 3: Insolvencies at each iteration

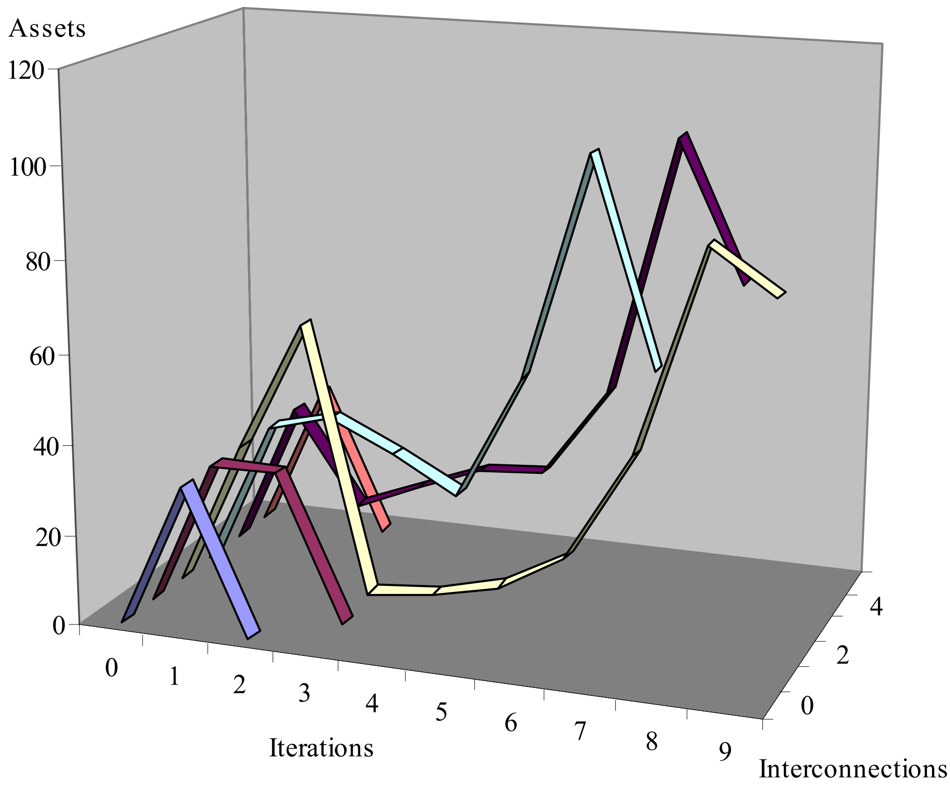

Figure 4: Assets dropped at each iteration

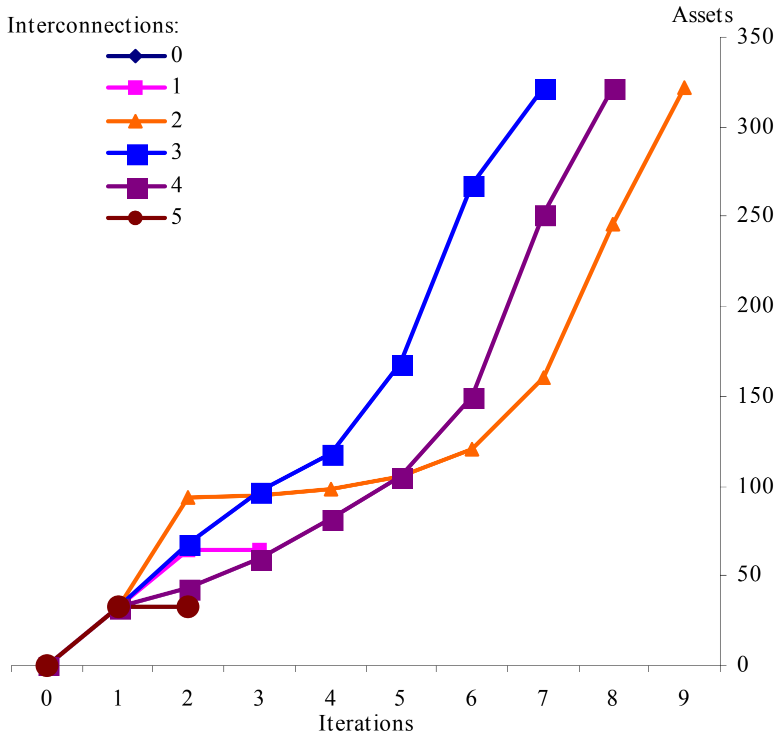

Figure 5: Total assets dropped

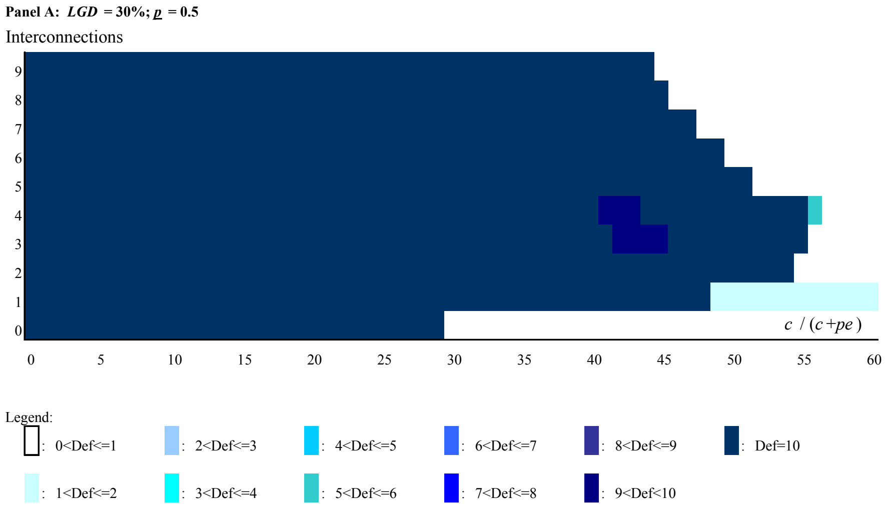

Figure 6: Liquidity ratio vs number of counterparts

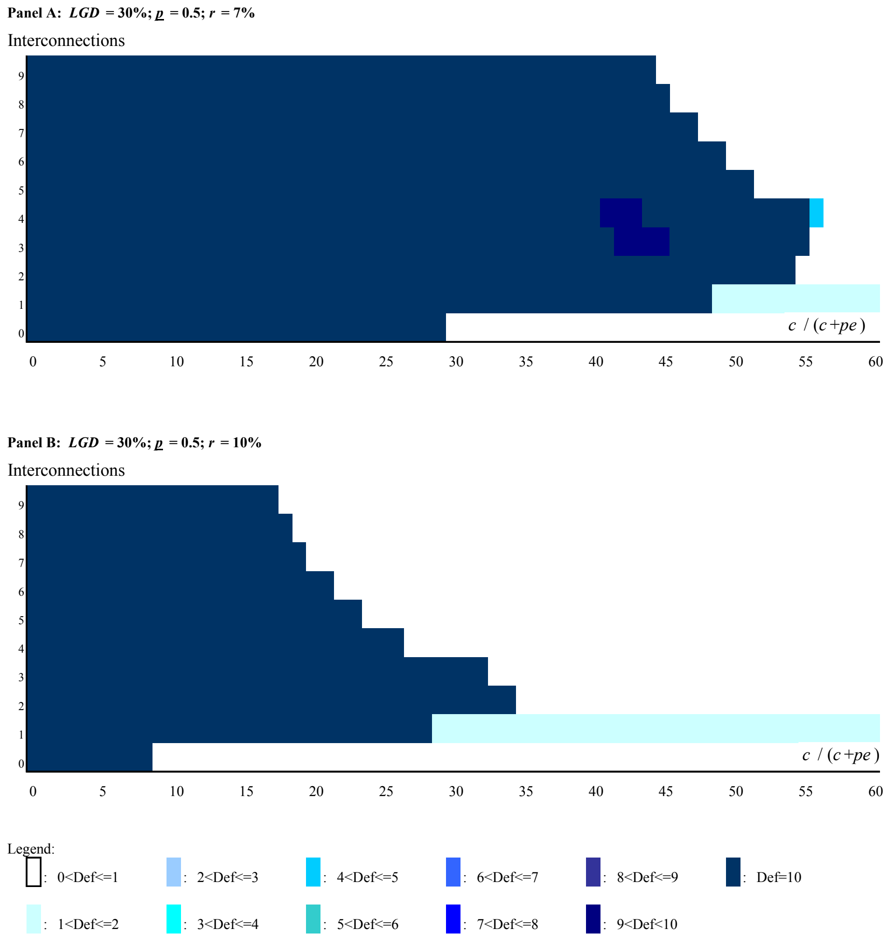

Figure 7: Capital buffers

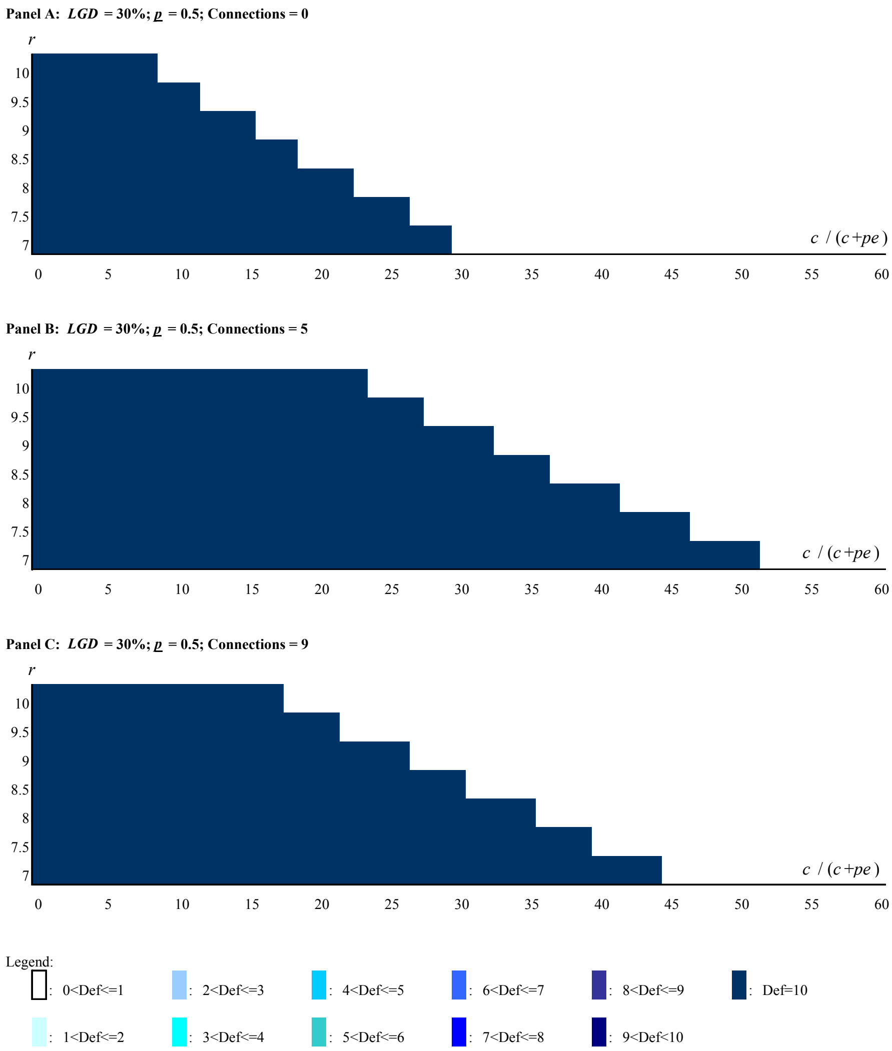

Figure 8: Liquidity vs capital buffer