Liquidity, Risk-Taking, and the Lender of Last Resort

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Abstract

This paper studies the strategic interaction between a bank whose deposits are randomly withdrawn, and a lender of last resort (LLR) that bases its intervention decision on supervisory information on the quality of the bank's assets. The bank, which is subject to a capital requirement, chooses the liquidity buffer that it wants to hold and the risk of its loan portfolio. The equilibrium choice of risk is shown to be decreasing in the capital requirement, and increasing in the interest rate charged by the LLR. Moreover, the risk chosen without penalty rates is the same as in the absence of a LLR. Thus, in contrast with the general view, the existence of a LLR does not increase the bank's incentives to take risk, while penalty rates do.

Keywords: central bank, lender of last resort, penalty rates, moral hazard, bank supervision, capital requirements, deposit insurance.

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1 Introduction

From their inception, central banks have assumed as one of their key responsibilities the potential provision of liquidity to banks unable to find it elsewhere. The classical doctrine on the lender of last resort (LLR) was put forward by Bagehot (1873, pp. 96-97): "Nothing, therefore, can be more certain than that the Bank of England... must in time of panic do what all other similar banks must do... And for this purpose there are two rules: First, that these loans should only be made at a very high rate of interest... Secondly, that at this rate these advances should be made on all good banking securities." The contemporary literature on the LLR has disagreed on whether the aim of "staying the panic" may be achieved by open market operations (see, for example, Goodfriend and King, 1988, or Kaufman, 1991), or it requires lending to individual banks (see, for example, Flannery, 1996, or Goodhart, 1999).[1] However, both sides seem to agree on the proposition that such lending creates a moral hazard problem. As argued by Solow (1982): "The existence of a credible LLR must reduce the private cost of risk taking. It can hardly be doubted that, in consequence, more risk will be taken."

The purpose of this paper is show that this proposition is not generally true. Specifically, we model the strategic interaction between a bank and a LLR. The bank is funded with insured deposits and equity capital, is subject to a minimum capital requirement, and can invest in two assets: a safe and perfectly liquid asset, and a risky and illiquid asset, whose risk is privately chosen by the bank. Deposits are randomly withdrawn. If the withdrawal is larger than the funds invested in the safe asset (the liquidity buffer), the bank will be forced into liquidation unless it can secure emergency lending from the LLR. In this setting, we show that when the LLR does not charge penalty rates, the bank chooses the same level of risk and a smaller liquidity buffer than in the absence of a LLR. Moreover, the equilibrium choice of risk is increasing in the penalty rate.

[1] All these references (and more) are usefully collected in Goodhart and Illing (2002).

To explain the intuition for these results, consider a setup in which a risk-neutral bank raises a unit of deposits at an interest rate normalized to zero, and invests λ in a safe asset with a zero interest rate,[2] and $1-\lambda$ in a risky asset which yields a gross return $R(p)$ with probability p, and zero otherwise. Moreover, suppose that p is chosen by the bank at the time of investment, and that the success return $R(p)$ is decreasing in p, so riskier investments yield a higher success return.

Without deposit withdrawals, with probability p the bank gets the return λ of its investment in the safe asset, plus the return $(1-\lambda)R(p)$ of its investment in the risky asset, minus the amount due to the depositors, that is $\lambda + (1-\lambda)R(p) - 1 = (1-\lambda)[R(p)-1]$, and with probability $1-p$ the bank fails. Under limited liability, the bank maximizes $p(1-\lambda)[R(p)-1]$, which gives $p^* = \arg\max p[R(p)-1]$.

With random deposit withdrawals that do not depend on the bank's choice of risk, and without a LLR, the probability π that the bank has sufficient liquidity to cover the withdrawal only depends on the bank's liquidity buffer λ. Hence, the bank would maximize $p\pi(\lambda)(1-\lambda)[R(p)-1]$, which gives $p^* = \arg\max p[R(p)-1]$. Clearly, if there were a LLR whose decision did not depend on the bank's choice of risk, the function $\pi(\lambda)$ would be different, but the same p^* would obtain. However, although it may argued that insured depositors have no incentive to gather information about the quality of the bank's assets, LLR's decisions usually rely on supervisory information on asset quality.

Specifically, suppose that the LLR observes a signal on the ex-post realization of the return of the risky asset. Then, conditional on the realization of the high return state, there will be a probability distribution of signals, which implies a new function $\pi(\lambda\ |\text{high})$ that describes the conditional probability that either the withdrawal is smaller than λ or the LLR supports the bank. The bank now maximizes $p\pi(\lambda\ |\text{high})(1-\lambda)[R(p)-1]$, and we get again $p^* = \arg\max p[R(p)-1]$. Hence, we conclude that the existence of an informed LLR does not increase the bank's incen-

[2] The deposit rate should be understood as including all the relevant operating costs, so to a first approximation it is reasonable to assume that it is equal to the safe interest rate.

tives to take risk. There is, however, an effect on the liquidity buffer λ, which will be lower in the presence of a LLR.[3]

As for the result on penalty rates, the intuition is that they increase the expected interest payments in the high return state and, consequently, push the bank towards choosing higher risk and higher return strategies (i.e. a lower p).

It is important to note that the result that the LLR does not have an effect on risk-taking does not depend on its objective function, or on the quality of the supervisory information, since this only affects the functional form of $\pi(\lambda \mid \text{high})$, which factors out of the maximization problem.

In this paper we propose a specific objective function for the LLR. Following Repullo (2000) and Kahn and Santos (2001), we adopt a political economy perspective according to which government agencies have objectives that need not correspond with the maximization of social welfare. In particular, we assume that the LLR cares about (1) the revenues and costs associated with its lending activity, and (2) whether the bank fails. This may be justified by relating the payoff of the officials in charge of LLR decisions with the surpluses or deficits of the agency, as well as with the possible reputation costs associated with a bank failure.

The fact that the objective function of the LLR does not correspond with the maximization of social welfare would be irrelevant if the supervisory information were verifiable, because then the intervention rule could be specified ex-ante, possibly in order to implement the socially optimal decision. However, the information coming from bank examinations is likely to contain many subjective elements that are difficult to describe ex-ante, so it seems reasonable to assume that it is nonverifiable. In this case, the decision will have to be delegated to a LLR, which will simply compare its conditional expected payoff of supporting and not supporting the bank.

Unlike Repullo (2000) and Kahn and Santos (2001), which analyze a normative issue, namely what is the optimal allocation of LLR responsibilities between a central

[3] Interestingly, this may or may not be efficient, depending on whether the safe asset is dominated by the risky asset in expected return.

bank and a deposit insurer, this paper focuses on positive issues. Specifically, we characterize the equilibrium choice of capital, liquidity, and risk in the context of a game between a bank and a LLR that is separated from the deposit insurer, and can therefore be identified with the central bank.

To facilitate the presentation, the analysis starts with a basic model in which the bank is fully funded with deposits and can only invest in the risky asset. Then the model is extended to the case where the bank can invest in the safe asset and raise equity capital, and is subject to a minimum capital requirement.[4] We characterize the Nash equilibrium of the game between the bank and the LLR, where the former chooses the level of risk (and in the general model its capital and liquidity buffer) and the latter its contingent lending policy.

The LLR's equilibrium strategy is straightforward: the LLR will support the bank if and only if the liquidity shortfall is smaller than or equal to a critical value that is decreasing in the ex-post (i.e. conditional on the supervisory signal) probability of bank failure.

The bank's equilibrium strategy is, however, more difficult to characterize. The reason is that its objective function is likely to be convex in the capital decision, which leads to a corner solution where the bank's capital is equal to the minimum required by regulation. In this case the equilibrium level of risk only depends on the capital requirement, with higher capital increasing the bank's shareholders losses in case of default and reducing their incentives for risk-taking. We complete the analysis by deriving numerically, for a simple parameterization of the model, the bank's equilibrium liquidity. We show that in equilibrium the bank chooses the same level of risk and a lower liquidity buffer than in the absence of a LLR.

Three extensions are then discussed. First, we derive the results on penalty rates. Second, we consider the effects of introducing a higher discount rate for the LLR, which yields a forbearance result: in equilibrium, the bank is more likely to receive

[4] In line with Basel capital regulation, we assume that investment in the safe asset does not carry a capital charge.

support from the LLR. Finally, we look at the case where the LLR shares a fraction of the deposit insurance payouts (which includes, in the limit, the case where the LLR is the deposit insurer). In this case, the LLR's decision becomes more sensitive to the supervisory information.

It is important to stress that the key result on the zero effect on risk-taking of having a LLR crucially depends on the specification of the order of moves, in particular the fact that the bank cannot modify the level of risk after the receiving the support of the LLR (or cannot borrow from the LLR to undertake new investments). But in such situation, the LLR is likely to carefully monitor the bank, preventing it from engaging in any significant risk-shifting, so this seems a reasonable assumption.

Although the literature on the LLR is huge (see Freixas el al., 2000, for a recent survey), somewhat surprisingly there has been little formal modeling of the issues discussed in this paper. Most of the relevant papers invoke general results on the link between any form of insurance and moral hazard. Moreover, liquidity support is not always distinguished from capital support, which clearly has bad incentive effects whenever it translates into rescuing the shareholders of a failed bank. This paper restricts attention to liquidity support based on supervisory information on the quality of the bank's assets, and shows that under fairly general conditions this support does not encourage risk-taking.

The paper is organized as follows. Section 2 presents the basic model of the game between the bank and the LLR. Section 3 introduces equity capital and a minimum capital requirement, and allows the bank to invest in a safe asset, characterizing the equilibrium with and without a LLR, and discussing its comparative statics properties. Section 4 analyzes the effects of charging penalty rates as well as changing the objective function of the LLR to allow for higher discounting of future payouts and sharing deposit insurance payouts. Section 5 offers some concluding remarks.

2 The Basic Model

Consider a model with three dates ($t = 0, 1, 2$) and two risk-neutral agents: a *bank* and a *lender of last resort* (LLR). At date 0 the bank raises 1 unit of deposits at an interest rate which is normalized to zero, and invests these funds in an asset that yields a random *return* R at date 2. Because of asymmetric information, there does not exist a market for this asset at date 1. However, the bank can be liquidated at this date, in which case the *liquidation value* of the asset is $L \in (0, 1)$. Deposits are fully insured and can be withdrawn at either $t = 1$ or $t = 2$. To simplify the presentation, deposit insurance premia will be set equal to zero.

At date 1 a fraction $v \in [0, 1]$ of the deposits are withdrawn. Since the bank's asset is illiquid, if $v > 0$ the bank faces a liquidity problem which can only be solved by borrowing from the LLR. If no such funding is provided, the bank is liquidated at $t = 1$. Otherwise, the bank stays open until the terminal date 2. The liquidity shock v is observable, and we initially suppose that the LLR only charges the (risk-unadjusted) market interest rate which is assumed to be zero.

In order to decide whether to provide this emergency funding, the LLR supervises the bank, which yields a *signal* $s \in \{s_0, s_1\}$ on the return of the bank's risky asset. Signal s is assumed to be nonverifiable, so the LLR's decision rule cannot be specified ex ante, but will be chosen ex post by the LLR in order to maximize an objective function that will be explained below.

If the bank is liquidated at $t = 1$ or fails at $t = 2$, there is a *cost of bank failure* c which comprises the administrative costs of closing the bank and paying back depositors and the negative externalities associated with the failure (contagion to other banks, breakup of lending relationships, distortions in the monetary transmission mechanism, etc.).

The probability distribution of the final return R is given by

$$R = \begin{cases} 0, & \text{if } \theta = \theta_0 \\ R(p), & \text{if } \theta = \theta_1 \end{cases}$$

where $p \in [0,1]$ is a parameter chosen by the bank at date 0, and $\theta \in \{\theta_0, \theta_1\}$ is the realization of a *state of nature.*[5]

We introduce the following assumptions:

Assumption 1 $\Pr(\theta_1) = p$ *and* $\Pr(s_0 \mid \theta_0) = \Pr(s_1 \mid \theta_1) = q \in [\frac{1}{2}, 1]$.

Assumption 2 $R(p)$ *is decreasing and concave, with* $R(1) = 1$ *and* $1 + R'(1) < 0$.

Assumption 1 states that the parameter p chosen by the bank at date 0 is the probability of the high return state θ_1, so $1-p$ measures the riskiness of the bank's portfolio. Assumption 1 also introduces a new parameter q that describes the quality of the supervisory information.[6] By Bayes' law, it is immediate to show that

$$\Pr(\theta_1 \mid s_0) = \frac{p(1-q)}{p(1-q)+(1-p)q}, \tag{1}$$

and

$$\Pr(\theta_1 \mid s_1) = \frac{pq}{pq+(1-p)(1-q)}. \tag{2}$$

Hence when $q = \frac{1}{2}$ we have $\Pr(\theta_1 \mid s_0) = \Pr(\theta_1 \mid s_1) = p$, so the supervisory signal is uninformative, while when $q = 1$ we have $\Pr(\theta_1 \mid s_0) = 0$ and $\Pr(\theta_1 \mid s_1) = 1$, so the signal completely reveals the state of nature. Since $\Pr(\theta_1 \mid s_0) < p < \Pr(\theta_1 \mid s_1)$ for $q > \frac{1}{2}$, signals s_0 and s_1 will be called the bad and the good signal, respectively.

Assumptions 1 and 2 imply that the expected final return of the risky asset, $E(R) = pR(p)$, reaches a maximum at $\widehat{p} \in (0,1)$ which is characterized by the first order condition

$$R(\widehat{p}) + \widehat{p}R'(\widehat{p}) = 0. \tag{3}$$

To see this, notice that the first derivative of $pR(p)$ with respect to p equals $R(0) > 0$ for $p = 0$ and $1 + R'(1) < 0$ for $p = 1$, and the second derivative satisfies $2R'(p) + pR''(p) < 0$. Thus, increases in p below (above) $\widehat{p}$ increase (decrease) the expected

[5] The liquidation value L could also depend on the state of nature, with $0 < L(\theta_0) < L(\theta_1) < 1$, but this would not change the results.

[6] More generally, we could have $\Pr(s_0 \mid \theta_0) = q_0 \in [\frac{1}{2}, 1]$ and $\Pr(s_1 \mid \theta_1) = q_1 \in [\frac{1}{2}, 1]$, with $q_0 \neq q_1$, but this would not change the results.

final return of the risky asset. Moreover, we have $\widehat{p}R(\widehat{p}) > R(1) = 1$. Assumption 2 is borrowed from Allen and Gale (2000, Chapter 8), and allows to analyze in a continuous manner the risk-shifting effects of different institutional settings.

From the point of view of the initial date 0, the deposit withdrawal v is a continuous random variable with support $[0, 1]$ and cumulative distribution function $F(v)$.[7] Since deposits are fully insured, it is natural to assume that the withdrawal v is independent of the state of nature θ. Also, v is assumed to be independent of the supervisory signal s.

The bank and the LLR play a sequential game in which the bank chooses at date 0 the riskiness of its portfolio p, and if $v > 0$, the LLR decides at date 1 whether to support the bank based on two pieces of information: the size of the shock v, and the supervisory signal s. Importantly, the LLR does not observe the bank's choice of p, so we have a game of complete but imperfect information.

In this game, the LLR is assumed to care about the expected value of its final wealth net of a share α of the cost c incurred in the event of bank failure. As noted above, this objective function may be justified by relating the payoff of the officials in charge of LLR's decisions with the income generated or lost thought its lending activity and the cost associated with a bank failure. To simplify the presentation, we will assume that $\alpha = 1$, so the LLR fully internalizes the cost of bank failure.[8]

Consider a situation in which $v > 0$, and let s be the signal observed by the LLR. The payoff of the LLR if it supports the bank is computed as follows. With probability $\Pr(\theta_1 \mid s)$ the bank will be solvent at date 2 and the LLR will recover its loan v, while with probability $\Pr(\theta_0 \mid s)$ the bank will fail and the LLR will lose v and incur the cost c, so the LLR's expected payoff is $-(v+c)\Pr(\theta_0 \mid s)$. On the other hand, if the LLR does not support the bank, it will be liquidated at date 1, and the

[7] The distribution function $F(v)$ could have a mass point at $v = 0$, in which case $F(0) > 0$ would be the probability that the bank does not suffer a liquidity shock at date 1.

[8] Clearly, this assumption does not affect the characterization of the equilibrium of the game, since it is equivalent to a change in the cost c. Interestingly, Repullo (2000) assumes $\alpha < 1$, while Kahn and Santos (2001) assume $\alpha > 1$.

LLR's payoff will be $-c$. Hence the LLR will support the bank if

$$-(v+c)\Pr(\theta_0 \mid s) \geq -c.$$

Using the fact that $\Pr(\theta_1 \mid s) = 1 - \Pr(\theta_0 \mid s)$ this condition simplifies to

$$v \leq \frac{c\Pr(\theta_1 \mid s)}{\Pr(\theta_0 \mid s)}.$$

Substituting (1) and (2) into this expression, it follows that when the LLR observes the bad signal s_0 it will support the bank if

$$v \leq v_0 \equiv \frac{cp(1-q)}{(1-p)q}, \tag{4}$$

and when the LLR observes the good signal s_1 it will support the bank if

$$v \leq v_1 \equiv \frac{cpq}{(1-p)(1-q)}. \tag{5}$$

The critical values v_0 and v_1 defined in (4) and (5) satisfy

$$v_1 = \left(\frac{q}{1-q}\right)^2 v_0,$$

which implies $v_1 > v_0$ whenever $q > \frac{1}{2}$. Hence if the signal is informative, the LLR is more likely to provide support to the bank when it observes the good signal s_1 than when it observes the bad signal s_0. Moreover, the critical value v_0 is decreasing in the quality q of the signal, with $\lim_{q\to 1} v_0 = 0$, and the critical value v_1 is increasing in q, with $\lim_{q\to 1} v_1 = \infty$.[9] Thus, when the signal is perfectly informative, the bank will never be supported if the signal is bad, and will always be supported if it is good.

The critical values v_0 and v_1 are increasing in the cost of bank failure c because when this cost is high the LLR has a stronger incentive to lend to the bank in order to save c when the good state θ_1 obtains. They are also increasing in $p = \Pr(\theta_1)$, because when this probability is high the LLR is more likely to recover its loan v and save the cost c.

[9] The fact that v_1 may be greater than 1 is not a problem, because the support of v is $[0,1]$ which implies $\Pr(v > v_1) = 1 - F(v_1) = 0$. Thus, in this case the bank would be supported with probability one.

By limited liability, the bank gets a zero payoff if it is liquidated at date 1 or fails at date 2, and gets $R(p) - 1$ if it succeeds at date 2. This event happens when the good state of nature θ_1 obtains and either the LLR observes the bad signal s_0 and the liquidity shock satisfies $v \leq v_0$, or it observes the good signal s_1 and the liquidity shock satisfies $v \leq v_1$. By Assumption 1 and the independence of v we have

$$\Pr(\theta = \theta_1, s = s_0, \text{ and } v \leq v_0) = \Pr(\theta_1) \Pr(s_0 \mid \theta_1) \Pr(v \leq v_0) = p(1-q)F(v_0),$$

$$\Pr(\theta = \theta_1, s = s_1, \text{ and } v \leq v_1) = \Pr(\theta_1) \Pr(s_1 \mid \theta_1) \Pr(v \leq v_1) = pqF(v_1).$$

Hence, the bank's objective function is

$$U_B = p[(1-q)F(v_0) + qF(v_1)][R(p) - 1]. \tag{6}$$

A *Nash equilibrium* of the game between the bank and the LLR is a choice of risk p^* by the bank, and a choice of maximum liquidity support by the LLR contingent on the bad and the good signal, v_0^* and v_1^*, such that p^* maximizes

$$p[(1-q)F(v_0^*) + qF(v_1^*)][R(p) - 1],$$

and

$$v_0^* = \frac{cp^*(1-q)}{(1-p^*)q} \quad \text{and} \quad v_1^* = \frac{cp^*q}{(1-p^*)(1-q)}.$$

In this definition it is important to realize that since the LLR does not observe the bank's choice of risk, the critical values v_0^* and v_1^* only depend on the equilibrium p^*. This in turn implies that the term $[(1-q)F(v_0^*) + qF(v_1^*)]$ in the bank's objective function does not depend on p, so the bank's problem reduces to maximize $p[R(p) - 1]$.[10] The first-order condition that characterizes the equilibrium choice of risk p^* is

$$R(p^*) + p^*R'(p^*) = 1. \tag{7}$$

Since $R(p) + pR'(p)$ is decreasing by Assumption 2, conditions (3) and (7) imply that p^* is strictly below $\widehat{p}$, so the bank will be choosing too much risk. This is just the standard risk-shifting effect that follows from debt financing under limited liability.

[10] In a sequential game of complete information, the characterization of equilibrium would be more complicated, since the critical values v_0^* and v_1^* would depend on the bank's choice of p at date 0.

It should be noted that the bank's choice of p changes the probability distribution of the signals, increasing $\Pr(s_1) = 1 - q + p(2q - 1)$ and decreasing $\Pr(s_0) = 1 - \Pr(s_1)$ (as long as $q > \frac{1}{2}$). However, by Assumption 1, p does not affect the distribution of the signals conditional on the good state of nature θ_1, which implies that the bank's probability of getting $R(p) - 1$ is linear in $\Pr(\theta_1) = p$.

To sum up, we have set up a model of a bank and a LLR in which the former chooses the riskiness of its portfolio, and the latter chooses whether to lend to the bank to cover random deposit withdrawals. The LLR's decision depends on a signal on quality of the bank's portfolio. Importantly, the signal is not about the bank's choice of action but about the consequences of that action.[11] This assumption is justified by the fact that at the time of making its decision the LLR only cares about the ex-post quality of the portfolio that guarantees its lending. Thus, contingent on the realization of the good state of nature, the bank's action does no have any effect on the LLR's signal. For this reason, the bank's equilibrium choice of risk is independent of the distribution of the liquidity shocks and the other parameters that determine the LLR's decision like the quality of the supervisory information or the cost of bank failure.

3 The General Model

We now introduce in our basic model two features of banking in the real world that are relevant to the problem under discussion. First, on the asset side of the bank's balance sheet, we suppose that, apart from the risky asset, the bank can invest in a safe and perfectly liquid asset that can be used as a buffer against liquidity shocks. Second, on the liability side, we suppose that the bank can be funded with both deposits and equity capital, and that the bank is subject to a minimum capital requirement. The LLR observes both the bank's equity capital and its investment in the liquid asset. We characterize the equilibrium of the new game between the bank and the LLR,

[11] See Prat (2003) for a detailed discussion about these two types of signals in the context of a principal-agent model.

compare it with that of a model without a LLR, and examine its comparative statics properties.

Specifically, suppose that at date 0 the bank raises k equity capital and $1-k$ deposits, and invests λ in the safe asset and $1-\lambda$ in the risky asset, so the size of the bank's balance sheet is normalized to 1. Bank capital has to satisfy the constraint $k \geq \kappa(1-\lambda)$, where $\kappa \in (0,1)$. Thus, the capital requirement depends on the (observable) bank's investment in the risky asset, but not on the (unobservable) bank's choice of risk.

We assume that the return of the safe asset is equal to the deposit rate, which has been normalized to zero. On the other hand, bank capital is provided by a special class of agents, called bankers, who require an expected rate of return $\delta \geq 0$ on their investment. A strictly positive value of δ captures either the scarcity of bankers' wealth or, perhaps more realistically, the existence of a premium for the agency and asymmetric information problems faced by the bank shareholders.[12]

3.1 Characterization of equilibrium

At date 1 a fraction $v \in [0,1]$ of the deposits are withdrawn. Since the bank has $1-k$ deposits, then $v(1-k)$ deposits are withdrawn at this date. There are two cases to consider. First, if $v(1-k) \leq \lambda$ the bank can repay the depositors by selling the required amount of the safe asset, so it keeps $\lambda - v(1-k)$ invested in the safe asset. In this case the bank shareholders' payoff in the good state of nature θ_1 equals the return of its investment in the safe asset, $\lambda - v(1-k)$, plus the return of its investment in the risky asset, $(1-\lambda)R(p)$, minus the amount paid to the remaining depositors, $(1-v)(1-k)$, that is

$$\lambda - v(1-k) + (1-\lambda)R(p) - (1-v)(1-k) = (1-\lambda)[R(p)-1] + k.$$

[12] See Holmström and Tirole (1997) and Diamond and Rajan (2000) for explicit models of why δ might be positive. The same assumption is made by Bolton and Freixas (2000), Hellmann, Murdock and Stiglitz (2000), and Repullo and Suarez (2003), among others.

Second, if $v(1-k) > \lambda$ the bank needs to borrow $v(1-k)-\lambda$ from the LLR in order to avoid liquidation. If such funding obtained, the bank shareholders' payoff in the good state of nature θ_1 equals the return of its investment in the risky asset, $(1-\lambda)R(p)$, minus the amount paid to the remaining depositors, $(1-v)(1-k)$, minus the amount paid to the LLR, $v(1-k)-\lambda$, that is

$$(1-\lambda)R(p)-(1-v)(1-k)-[v(1-k)-\lambda]=(1-\lambda)[R(p)-1]+k.$$

In both cases, if the bad state of nature θ_0 obtains the bank's net worth is $\lambda-(1-k)$, which will be negative as long as the bank's investment in the liquid asset, λ, does not exceed its deposits, $1-k$, which will generally obtain in equilibrium.[13] Hence, by limited liability, the bank shareholders' payoff in the bad state will be zero. Obviously, their payoff will also be zero when $v(1-k) > \lambda$ and the LLR does not support the bank.

The decision of the LLR in the case when $v(1-k) > \lambda$ is characterized as follows. If the LLR observes signal s and decides to support the bank, with probability $\Pr(\theta_1 \mid s)$ the bank will be solvent at date 2 and the LLR will recover its loan $v(1-k)-\lambda$, while with probability $\Pr(\theta_0 \mid s)$ the bank will fail and the LLR will lose $v(1-k)-\lambda$ and incur the cost c. If, on the other hand, the LLR does not support the bank, it will be liquidated at date 1, and the LLR's payoff will be $-c$. Hence the LLR will support the bank if

$$-[v(1-k)-\lambda+c]\Pr(\theta_0 \mid s) \geq -c.$$

As before, substituting (1) and (2) into this expression, it follows that when the LLR observes the bad signal s_0 it will support the bank if $v(1-k)-\lambda \leq v_0$, that is if

$$v \leq \frac{v_0+\lambda}{1-k}, \tag{8}$$

and when it observes the good signal s_1 it will support the bank if $v(1-k)-\lambda \leq v_1$, that is if

$$v \leq \frac{v_1+\lambda}{1-k}, \tag{9}$$

[13] In particular, we will show that under plausible conditions the capital requirement will be binding, so $k=\kappa(1-\lambda)$, which implies $\lambda-(1-k)=-(1-\kappa)(1-\lambda)<0$.

where the critical values v_0 and v_1 are given by (4) and (5), respectively. Thus, the probability that the bank will be supported by the LLR is increasing in its investment in the safe asset λ and its equity capital k. This is explained by the role of the safe asset as a buffer against liquidity shocks, and the fact that the higher the bank capital the lower its deposits and hence the size of the liquidity shocks.

The bank's objective function is to maximize the expected value of the payoff of its shareholders net of the opportunity cost of their initial infusion of capital. The latter is simply $(1+\delta)k$. To compute the former, notice that bank shareholders get a zero payoff if the bank is liquidated at date 1 or fails at date 2, and get $(1-\lambda)[R(p)-1]+k$ if it succeeds at date 2. This event happens when the good state of nature θ_1 obtains and either the LLR observes the bad signal s_0 and the liquidity shock v satisfies (8), or it observes the good signal s_1 and the liquidity shock v satisfies (9). As before, we have

$$\Pr\left(\theta=\theta_1, s=s_0, \text{ and } v \leq \frac{v_0+\lambda}{1-k}\right) = p(1-q)F\left(\frac{v_0+\lambda}{1-k}\right),$$
$$\Pr\left(\theta=\theta_1, s=s_1, \text{ and } v \leq \frac{v_1+\lambda}{1-k}\right) = pqF\left(\frac{v_1+\lambda}{1-k}\right).$$

Hence, the bank's objective function in the general model is

$$U_B = p\left[(1-q)F\left(\frac{v_0+\lambda}{1-k}\right)+qF\left(\frac{v_1+\lambda}{1-k}\right)\right][(1-\lambda)[R(p)-1]+k]-(1+\delta)k. \quad (10)$$

Obviously, this coincides with the objective function (6) in the previous section when $\lambda=0$ and $k=0$.

A *Nash equilibrium* of the game between the bank and the LLR is a choice of liquidity λ^*, capital $k^* \geq \kappa(1-\lambda^*)$, and risk p^* by the bank, and a choice of maximum liquidity support by the LLR contingent on the bad and the good signal, v_0^* and v_1^*, such that (λ^*, k^*, p^*) maximizes

$$p\left[(1-q)F\left(\frac{v_0^*+\lambda}{1-k}\right)+qF\left(\frac{v_1^*+\lambda}{1-k}\right)\right][(1-\lambda)[R(p)-1]+k]-(1+\delta)k, \quad (11)$$

and

$$v_0^* = \frac{cp^*(1-q)}{(1-p^*)q} \quad \text{and} \quad v_1^* = \frac{cp^*q}{(1-p^*)(1-q)}. \quad (12)$$

As in the basic model, it is important to note that since the LLR does not observe the bank's choice of risk, the critical values v_0^* and v_1^* only depend on the equilibrium p^*. This in turn implies that the bank's choice of risk is characterized by the first-order condition

$$(1-\lambda^*)[R(p^*)-1]+k^*+p^*(1-\lambda^*)R'(p^*)=0,$$

which simplifies to

$$R(p^*)+p^*R'(p^*)=1-\frac{k^*}{1-\lambda^*} \tag{13}$$

Comparing this expression with (3) and (7), and taking into account that $R(p)+pR'(p)$ is decreasing by Assumption 2, it follows that the bank's equilibrium choice of risk p^* is going to be closer to $\widehat{p}$ than in the model without the capital requirement. This is just the standard *capital-at-risk effect*: higher capital implies higher losses for the banks' shareholders in case of default, and hence lower incentives for risk-taking.[14] Moreover, this effect is stronger when the bank's investment in the risky asset, $1-\lambda^*$, is small, because then the expected payoff from investing in this asset is proportionally smaller.

It should be noticed that if the bank's equilibrium choice of capital k^* is at the corner $\kappa(1-\lambda^*)$, then the first-order condition (13) further simplifies to

$$R(p^*)+p^*R'(p^*)=1-\kappa. \tag{14}$$

In this case, the equilibrium p^* only depends on the capital requirement κ. Moreover, Assumption 2 implies that $R(p)+pR'(p)$ is decreasing, which gives $dp^*/d\kappa>0$. Hence, the higher the capital requirement the lower the risk chosen by the bank.[15]

In general, it is difficult to prove that this case will obtain, since the properties of the bank's objective function (10) depend on the shape of the distribution function of the liquidity shock $F(v)$.[16] One can show that U_B is convex in k if $F(v)$ is convex,

[14] See Hellmann, Murdock and Stiglitz (2000) and Repullo (2003) for a recent discussion of this effect.

[15] Notice that for $\kappa=1$, that is a 100% capital requirement, (3) and (14) imply $p^*=\widehat{p}$.

[16] However, finding that k is at the minimum required by regulation is standard in both static and dynamic models of banking (see, for example, Repullo and Suarez, 2003, and Repullo, 2003).

which implies that the equilibrium level of capital must be either $k^* = \kappa(1-\lambda^*)$ or $k^* = 1$. Since the second corner where the bank takes no deposits is generally suboptimal, we get the desired result.

However, the assumption that $F(v)$ is convex (or, equivalently, that the density function $F'(v)$ is increasing) implies that big liquidity shocks are more likely than smaller shocks, which is not very appealing. For this reason, in what follows we will work with a specific concave parameterization of $F(v)$ for which the bank's problem has a corner solution in k. In particular, assume that

$$F(v) = v^\eta,$$

where $\eta \in (0,1)$.[17] In this case, one can show that $\partial^2 U_B/\partial k^2 > 0$, so we have $k^* = \kappa(1-\lambda^*)$.

This result implies that the equilibrium of the game between the bank and the LLR is easy to characterize. The risk p^* chosen by the bank is the unique solution of the first-order condition (14). This in turn determines the critical values v_0^* and v_1^* that characterize the behavior of the LLR. Substituting $p = p^*$ and $F(v) = v^\eta$ into the bank's objective function (11), we then find the value of λ^* by maximizing

$$p^*\left[(1-q)F\left(\frac{v_0^*+\lambda}{1-k}\right)+qF\left(\frac{v_1^*+\lambda}{1-k}\right)\right][(1-\lambda)[R(p^*)-1]+k]-(1+\delta)k, \quad (15)$$

subject to $k = \kappa(1-\lambda)$. Finally, we get $k^* = \kappa(1-\lambda^*)$.

Going analytically beyond this point is however complicated, because although the bank's objective function (15) is concave in λ, this is in general no longer the case once we substitute the constraint $k = \kappa(1-\lambda)$ into (15).[18] For this reason, our results on equilibrium liquidity and capital will be will be derived from numerical simulations for a simple parameterization of the model.

[17] Notice that this is a simple special case of a beta distribution.

[18] Recall that (15) is convex in k.

3.2 Equilibrium without a LLR

We now compare the equilibrium behavior of the bank when there is a LLR with its behaviour when there is no LLR. The objective function of the bank in a such model is a special case of (10) when we set $v_0^* = v_1^* = 0$ (i.e. no last resort lending), which gives

$$U_B = pF\left(\frac{\lambda}{1-k}\right)[(1-\lambda)[R(p)-1]+k]-(1+\delta)k.$$

Thus, bank shareholders get $(1-\lambda)[R(p)-1]+k$ in the good state of nature θ_1, which obtains with probability p, but only if they have sufficient liquidity to cover the deposit withdrawals at date 1, that is if $v(1-k) \leq \lambda$, an event that happens with probability $F(\lambda/(1-k))$.

From here we can follow our previous steps to conclude that, if the bank's capital k is at the corner $\kappa(1-\lambda^*)$, then its choice of risk p will be characterized by (14), so we get exactly the same p^* as in the model with the LLR. In other words, contrary to what has been taken for granted in the banking literature, our model predicts that *the existence of a LLR does not have any effect on the bank's incentives to take risk.*

Computation of the effects of having a LLR on liquidity and capital decisions requires to specify the functional forms of the return of the risky asset in the good state of nature, $R(p)$, and the cumulative distribution function of the liquidity shock, $F(v)$, as well as the parameter values of the capital requirement κ, the cost of capital δ, the informativeness of the supervisory signal q, and the cost of bank failure c. Since our focus is on the qualitative results, we will not calibrate the model to obtain plausible numerical results, but instead choose simple functional forms and round parameter values.

Specifically, the functional forms $R(p) = 3 - 2p^2$ and $F(v) = v^\eta$, with $\eta = 0.25$, are going to be maintained in all our simulations,[19] and our baseline parameter values are $\kappa = \delta = c = 0.10$, and $q = 0.60$.

[19] Clearly, $R(p) = 3 - 2p^2$ is decreasing and concave, with $R(1) = 1$ and $1 + R'(1) = -3 < 0$, so Assumption 2 is satisfied.

The corresponding equilibria with and without a LLR are shown in Table 1. As noted above, the level of risk p^* chosen by the bank is the same in both models, and may be obtained by substituting $R(p) = 3 - 2p^2$ into (14), which gives $p^* = \sqrt{(2+\kappa)/6} = 0.59$. As expected, the liquidity buffer λ^* is much larger in the absence of a LLR.[20] Given that $k^* = \kappa(1 - \lambda^*)$, this in turn implies a lower level of capital.

Table 1

Equilibrium with and without a LLR

	With LLR	Without LLR
λ^*	0.11	0.23
k^*	0.09	0.08
p^*	0.59	0.59
v_0^*	0.10	–
v_1^*	0.22	–

When the deposit withdrawal $v(1-k^*)$ is below the bank's liquidity λ^*, an event that happens with probability 0.58 in the model with the LLR, and with probability 0.71 in the model without it, the bank will be able to repay the depositors by selling the required amount of the safe asset. Moreover, in the first model, when $v(1-k^*)$ is greater than λ^* the LLR will provide liquidity up to $v_0^* = 0.10$ when it observes the bad signal s_0, and up to $v_1^* = 0.22$ when it observes the good signal s_1.

[20] Gonzalez-Eiras (2003) provides some interesting empirical evidence on this result. He shows how the contingent credit line agreement signed by the Central Bank of Argentina with a group of international banks in December 1996 enhanced its ability to act as LLR, and led to a significant decrease in the liquidity holdings of domestic Argentinian banks.

The probability that the bank will get a positive payoff in the model with a LLR is

$$p^* \left[(1-q) F\left(\frac{v_0^* + \lambda^*}{1-k^*} \right) + qF\left(\frac{v_1^* + \lambda^*}{1-k^*} \right) \right] = 0.44,$$

while the corresponding probability in the model without a LLR is

$$p^* F\left(\frac{\lambda^*}{1-k^*} \right) = 0.42.$$

Since in the first model the bank is investing a higher proportion of its portfolio in the risky asset, it is not surprising that its equilibrium expected payoff is significantly higher with a LLR (0.45 against 0.37).

3.3 Comparative statics

We next analyze the effect on the equilibrium of the game between the bank and the LLR of changes in the capital requirement κ, the cost of capital δ, the informativeness of the supervisory signal q, and the cost of bank failure c. The results summarized in Table 2 are derived by computing the equilibrium corresponding to deviations in κ, δ, q, and c from the baseline case.

Table 2

Equilibrium effects of changes in the capital requirement κ, the cost of capital δ, the informativeness of the signal q, and the cost of bank failure c

x	$\frac{dp^*}{dx}$	$\frac{d\lambda^*}{dx}$	$\frac{dk^*}{dx}$	$\frac{dv_0^*}{dx}$	$\frac{dv_1^*}{dx}$
κ	$+$	$+$	$+$	$+$	$+$
δ	0	$+$	$-$	0	0
q	0	$-/0$	$+/0$	$-$	$+$
c	0	$-/0$	$+/0$	$+$	$+$

As noted above, an increase in the capital requirement κ leads to an increase in p^*, which by (12) increases the maximum liquidity support, v_0^* and v_1^*, provided by the LLR contingent on the bad and the good signal. The effect on λ^* is also positive, so higher capital requirements lead to higher investment in liquidity. Two reasons explain this result. First, the higher capital requirement makes investment in the risky asset relatively less attractive for bank shareholders. Second, the higher p^* reduces the success payoff of the risky asset, $R(p^*)$, and makes it relatively less attractive than the safe asset. On the other hand, the higher liquidity support offered by the LLR reduces the bank's incentives to invest in the safe asset, but the numerical results show that this effect is dominated by the other two.

With regard to the other comparative statics results, note first that as shown analytically, the value of p^* chosen by the bank only depends on the capital requirement κ, so the effect of the other three parameters is zero.

Since p^* does not depend on δ, the cost of capital does not have any effect on the maximum liquidity support, v_0^* and v_1^*, provided by the LLR contingent on the bad and the good signal. The cost of capital δ has a positive effect on equilibrium liquidity λ^*, because when capital is more expensive investing in the safe asset, which does not carry a capital charge, is relatively more attractive. Since $k^* = \kappa(1 - \lambda^*)$, this in turn explains why an increase in δ has a negative effect on k^*.

As noted in Section 2, the critical value v_0^* is decreasing in the quality q of the supervisory signal, while the critical value v_1^* is increasing in q, so with better information the bank is less (more) likely to be supported by the LLR when the signal is bad (good). The sign of the derivative of λ^* with respect to q is negative, which means that the positive effect of having less support when the signal is bad is dominated by the negative effect of having more support when the signal is good. Since $k^* = \kappa(1 - \lambda^*)$, this in turn explains why an increase in q has a positive effect on k^*. However, when q is sufficiently large we may get to the corner $\lambda^* = 0$ and $k^* = \kappa$, where the derivatives become zero.

As also noted in Section 2, the critical values v_0^* and v_1^* are increasing in the cost of bank failure c, because when this cost is high the LLR has a stronger incentive to lend to the bank in order to save c when the good state obtains. This clearly explains why the bank has less incentive to invest in liquidity, so λ^* will be lower and $k^* = \kappa(1 - \lambda^*)$ will be higher. In fact, when c is sufficiently large we may get to the corner $\lambda^* = 0$ and $k^* = \kappa$, where the derivatives become zero. If we consider that the cost of failure increases more than proportionately with the size of the bank's balance sheet (which we have normalized to 1), c will be higher for large banks, which implies a "too big to fail" result: large banks are more likely to be supported by the LLR, and consequently they will hold smaller liquidity buffers.

4 Extensions

4.1 Penalty rates

The classical doctrine on the LLR put forward by Bagehot (1873) required "these loans should only be made at a very high rate of interest." We now examine how the results in Section 3 are modified when the LLR charges a penalty rate $r > 0$. Importantly, we assume that the rate r is exogenously given ex ante, and not chosen by the LLR ex post.

To characterize the equilibrium of the new game between the bank and the LLR, suppose that $v(1-k)$ deposits are withdrawn at date 1. If $v(1-k) \leq \lambda$, the bank can repay the depositors by selling the required amount of the safe asset, so there is no change with respect to our previous analysis. If, on the other hand, $v(1-k) > \lambda$, the bank needs to borrow $v(1-k)-\lambda$ from the LLR. If such funding obtained, the bank's payoff in the good state of nature θ_1 equals the return of its investment in the risky asset, $(1-\lambda)R(p)$, minus the amount paid to the remaining depositors, $(1-v)(1-k)$, minus the amount paid to the LLR, $(1+r)[v(1-k)-\lambda]$, that is

$$(1-\lambda)R(p)-(1-v)(1-k)-(1+r)[v(1-k)-\lambda] = (1-\lambda)[R(p)-1]+k-r[v(1-k)-\lambda].$$

The last term in this expression is the interest payments to the LLR, which is positive whenever $r > 0$.

The decision of the LLR in the case when $v(1-k) > \lambda$ is now characterized as follows. If the LLR observes signal s and decides to support the bank, with probability $\Pr(\theta_1 \mid s)$ the bank will be solvent at date 2 and the LLR will recover its loan $v(1-k) - \lambda$ and net $r[v(1-k) - \lambda]$ in interest payments, while probability $\Pr(\theta_0 \mid s)$ the bank will fail and the LLR will lose $v(1-k)-\lambda$ and incur the bankruptcy cost c, so the LLR's expected payoff is

$$r[v(1-k) - \lambda] \Pr(\theta_1 \mid s) - [v(1-k) - \lambda + c] \Pr(\theta_0 \mid s).$$

On the other hand, if the LLR does not support the bank, it will be liquidated at date 1, and the LLR's payoff will be $-c$. Hence the LLR will support the bank if

$$r[v(1-k) - \lambda] \Pr(\theta_1 \mid s) - [v(1-k) - \lambda + c] \Pr(\theta_0 \mid s) \geq -c.$$

Substituting (1) and (2) into this expression, it follows that when the LLR observes the bad signal s_0 it will support the bank if $v(1-k) - \lambda \leq v_0$, that is if

$$v \leq \frac{v_0 + \lambda}{1-k},$$

where

$$v_0 \equiv \frac{c \Pr(\theta_1 \mid s_0)}{\Pr(\theta_0 \mid s_0) - r \Pr(\theta_1 \mid s_0)} = \frac{cp(1-q)}{(1-p)q - rp(1-q)},$$

and when the LLR observes the good signal s_1 it will support the bank if $v(1-k)-\lambda \leq v_1$, that is if

$$v \leq \frac{v_1 + \lambda}{1-k},$$

where

$$v_1 \equiv \frac{c \Pr(\theta_1 \mid s_1)}{\Pr(\theta_0 \mid s_1) - r \Pr(\theta_1 \mid s_1)} = \frac{cpq}{(1-p)(1-q) - rpq}.$$

As before, it is easy to check that $v_1 > v_0$ whenever $q > \frac{1}{2}$. Also, notice that both v_0 and v_1 are increasing in the penalty rate r, so the LLR will be softer with the bank, providing emergency funding for a larger range of liquidity shocks.

To compute the bank's new objective function we have to subtract from U_B in (10) the expected interest payments to the LLR. If the LLR observes the bad signal s_0, the bank borrows from the LLR when $0 < v(1-k) - \lambda \leq v_0$, that is when

$$\frac{\lambda}{1-k} < v \leq \frac{v_0 + \lambda}{1-k},$$

and the conditional expected cost of this borrowing is

$$r \left[\int_{\frac{\lambda}{1-k}}^{\frac{v_0+\lambda}{1-k}} [v(1-k) - \lambda] \, dF(v) \right] \Pr(s_0 \mid \theta_1).$$

Similarly, if the LLR observes the good signal s_1, the conditional expected cost of the bank's borrowing is

$$r \left[\int_{\frac{\lambda}{1-k}}^{\frac{v_1+\lambda}{1-k}} [v(1-k) - \lambda] \, dF(v) \right] \Pr(s_1 \mid \theta_1).$$

Hence the bank's objective function is

$$\begin{aligned} U_B &= p \left[(1-q) F\left(\frac{v_0 + \lambda}{1-k} \right) + q F\left(\frac{v_1 + \lambda}{1-k} \right) \right] [(1-\lambda)[R(p) - 1] + k] - (1+\delta)k \\ &\quad -rp \left[(1-q) \int_{\frac{\lambda}{1-k}}^{\frac{v_0+\lambda}{1-k}} [v(1-k) - \lambda] \, dF(v) + q \int_{\frac{\lambda}{1-k}}^{\frac{v_1+\lambda}{1-k}} [v(1-k) - \lambda] \, dF(v) \right]. \end{aligned}$$

Assuming that $F(v) = v^\eta$, the integrals in the previous expression can be easily solved, and we can compute for the baseline parameters the equilibrium effects of charging a penalty rate r.[21] The results are summarized in Table 3.

Table 3

Equilibrium effects of changes in the penalty rate r

x	$\frac{dp^*}{dx}$	$\frac{d\lambda^*}{dx}$	$\frac{dk^*}{dx}$	$\frac{dv_0^*}{dx}$	$\frac{dv_1^*}{dx}$
r	$-$	$-$	$+$	$+$	$+$

[21] It should be noted that this computation is complicated because now p^* depends on r, and cannot be directly solved from (14). The equilibrium is obtained by numerical iteration of the best response functions of the two players.

Thus, an increase in the penalty rate r leads to a reduction in p^*, so the bank's portfolio becomes riskier. The reason for this result is that penalty rates increase the expected interest payments in the high return state and, consequently, the bank tries to compensate this effect by choosing higher return strategies (recall that by Assumption 2 a decrease in p increases $R(p)$). The reduction in p^* would ceteris paribus lead to an decrease in both v_0^* and v_1^*, but this is more than compensated by the positive effect of the interest payments on the LLR's willingness to lend. The increase in v_0^* and v_1^* in turn explains why the bank chooses a lower liquidity buffer λ^*, so $k^* = \kappa(1 - \lambda^*)$ will be higher.

4.2 LLR's discounting of future payoffs

The LLR is a public institution that is run by officials that may have fixed terms of office. If these terms are short or they are close to finishing their terms, they may have an incentive to avoid current costs possibly at the expense of some larger future costs that would be assumed by their successors. Formally, we can incorporate this possibility into our model by introducing a discount factor $\beta < 1$ for the LLR.[22]

To analyze the effect of such discounting, consider a situation in which $v(1-k) > \lambda$, and let s be the signal observed by the LLR. The expected payoff of the LLR if it supports the bank is now

$$-\beta\,[v(1-k) - \lambda + c]\,\Pr(\theta_0 \mid s),$$

since with probability $\Pr(\theta_0 \mid s)$ the bank will fail at date 2 and the LLR will lose $v(1-k) - \lambda$ and incur the bankruptcy cost c. On the other hand, if the LLR does not support the bank, it will be liquidated at date 1, and the LLR's payoff will be $-c$. Hence the LLR will support the bank if

$$-\beta\,[v(1-k) - \lambda + c]\,\Pr(\theta_0 \mid s) \geq -c.$$

[22] This assumption is justified by Kaufman (1991) in the following terms: "The discount rate used by policy makers, who are under considerable political pressure to optimize economic performance in the short-term and whose terms of office are relatively short and not guaranteed to last until the next crisis, is likely to be overestimated."

Substituting (1) and (2) into this expression, it follows that when the LLR observes the bad signal s_0 it will support the bank if $v(1-k) - \lambda \leq v_0$, that is if

$$v \leq \frac{v_0 + \lambda}{1-k},$$

where

$$v_0 \equiv \frac{c[1 - \beta \Pr(\theta_0 \mid s_0)]}{\beta \Pr(\theta_0 \mid s_0)} = \frac{c[p(1-q) + (1-\beta)(1-p)q]}{\beta(1-p)q},$$

and when the LLR observes the good signal s_1 it will support the bank if $v(1-k)-\lambda \leq v_1$, that is if

$$v \leq \frac{v_1 + \lambda}{1-k},$$

where

$$v_1 \equiv \frac{c[1 - \beta \Pr(\theta_0 \mid s_1)]}{\beta \Pr(\theta_0 \mid s_1)} = \frac{c[pq + (1-\beta)(1-p)(1-q)]}{\beta(1-p)(1-q)}.$$

As before, it is easy to check that $v_1 > v_0$ whenever $q > \frac{1}{2}$. Also, notice that both v_0 and v_1 are decreasing in the discount factor β. This means that a LLR with $\beta < 1$ will be softer with the bank, providing funding for a larger range of liquidity shocks.

We can now compute for the baseline parameters the equilibrium effects of introducing a discount factor $\beta < 1$ for the LLR. Unlike in the model with penalty rates, the bank's choice of risk p^* is again characterized by (14), and so it only depends on the capital requirement κ. Hence the discount factor β does not have any effect on the bank's incentives to take risk. The full comparative static results are summarized in Table 4.

Table 4

Equilibrium effects of changes in the LLR's discount factor β

x	$\frac{dp^*}{dx}$	$\frac{d\lambda^*}{dx}$	$\frac{dk^*}{dx}$	$\frac{dv_0^*}{dx}$	$\frac{dv_1^*}{dx}$
β	0	$+$	$-$	$-$	$-$

As expected, an increase in the discount factor β (that is, a decrease in the corresponding discount rate) makes the LLR more willing to incur the current costs of not supporting the bank in order to save some larger future costs, so the derivative of v_0^* and v_1^* with respect to β is negative. The reduction in v_0^* and v_1^* in turn explains why the bank chooses a higher liquidity buffer λ^*, so $k^* = \kappa(1 - \lambda^*)$ will be lower.

Thus, we conclude that the excessive discounting by the LLR of future payoffs may explain forbearance, but in line with our previous results this only translates into a lower liquidity buffer, without any effect on risk-taking.

4.3 LLR's internalization of deposit insurance payouts

We have assumed so far the LLR is separated from the deposit insurer, so the former does not take into account deposit insurance payouts in deciding whether to support the bank. We now consider a situation in which the LLR either internalizes or assumes a fraction $\gamma \in [0, 1]$ of these payouts. When $\gamma = 0$ the LLR is completely independent from the deposit insurer (e.g. a central bank with no deposit insurance role), whereas when $\gamma = 1$ the LLR also acts as deposit insurer.[23]

To analyze the effect of such relation between the LLR and the deposit insurer, consider a situation in which $v(1 - k) > \lambda$, and let s be the signal observed by the LLR. The expected payoff of the LLR if it supports the bank is now

$$-[v(1-k) - \lambda + c + \gamma(1-v)(1-k)] \Pr(\theta_0 \mid s),$$

since with probability $\Pr(\theta_0 \mid s)$ the bank will fail at date 2 and the LLR will lose $v(1 - k) - \lambda$, incur the bankruptcy cost c, and assume a fraction γ of the deposit insurance payouts which are given by $(1 - v)(1 - k)$. On the other hand, if the LLR does not support the bank, it will be liquidated at date 1, and the LLR will incur the bankruptcy cost c, and assume a fraction γ of the deposit insurance payouts which are given by $(1 - k) - \lambda - (1 - \lambda)L$, where $(1 - \lambda)L$ is the liquidation value of the

[23] Intermediate cases are also relevant. For example, until 1998 the Bank of Spain matched the contribution of the Spanish banks to the deposit insurance fund, so $\gamma = \frac{1}{2}$.

bank's risky asset. Hence the LLR will support the bank if

$$-[v(1-k)-\lambda+c+\gamma(1-v)(1-k)]\Pr(\theta_0 \mid s) \geq -[c+\gamma[(1-k)-\lambda-(1-\lambda)L]].$$

Substituting (1) and (2) into this expression, it follows that when the LLR observes the bad signal s_0 it will support the bank if $v(1-k)-\lambda \leq v_0$, that is if

$$v \leq \frac{v_0+\lambda}{1-k},$$

where

$$\begin{aligned} v_0 &\equiv \frac{[c+\gamma(1-k-\lambda)]\Pr(\theta_1 \mid s_0)-\gamma(1-\lambda)L}{(1-\gamma)\Pr(\theta_0 \mid s_0)} \\ &= \frac{[c+\gamma(1-k-\lambda)]\,p(1-q)-\gamma(1-\lambda)L[p(1-q)+(1-p)q]}{(1-\gamma)(1-p)q}, \end{aligned}$$

and when the LLR observes the good signal s_1 it will support the bank if $v(1-k)-\lambda \leq v_1$, that is if

$$v \leq \frac{v_1+\lambda}{1-k},$$

where

$$\begin{aligned} v_1 &\equiv \frac{[c+\gamma(1-k-\lambda)]\Pr(\theta_1 \mid s_1)-\gamma(1-\lambda)L}{(1-\gamma)\Pr(\theta_0 \mid s_1)} \\ &= \frac{[c+\gamma(1-k-\lambda)]\,pq-\gamma(1-\lambda)L[pq+(1-p)(1-q)]}{(1-\gamma)(1-p)(1-q)}. \end{aligned}$$

In this case, assuming that the amount of deposits is greater than or equal to the liquidation value of the bank at date 1, that is $1-k > \lambda+(1-\lambda)L$, one can check that $v_1 > v_0$ whenever $q > \frac{1}{2}$.

We can now compute for the baseline parameters the equilibrium effects of the LLR's sharing a fraction γ of the deposit insurance payouts. As in the case of discounting, the bank's choice of risk p^* is again characterized by (14), and so it only depends on the capital requirement κ. Hence, for any value of γ, the sharing does not have any effect on the bank's incentives to take risk. The full comparative static results are summarized in Table 5.

Table 5

Equilibrium effects of changes in the LLR's share of deposit insurance payouts γ

x	$\frac{dp^*}{dx}$	$\frac{d\lambda^*}{dx}$	$\frac{dk^*}{dx}$	$\frac{dv_0^*}{dx}$	$\frac{dv_1^*}{dx}$
γ	0	$-$	$+$	$-$	$+$

Thus, an increase in the share γ makes the LLR tougher when it observes the bad signal s_0 (since the critical value v_0^* is decreasing in γ), and makes it softer when it observes the good signal s_1 (since the critical value v_1^* is increasing in γ). To explain these results, note that the partial derivative of v_0 with respect to γ is negative whenever $(c+1-k-\lambda)\Pr(\theta_1 \mid s_0) < (1-\lambda)L$, which holds when $\Pr(\theta_1 \mid s_0)$ is small, that is for a sufficiently informative signal. Similarly, the partial derivative of v_1 with respect to γ is positive whenever $(c+1-k-\lambda)\Pr(\theta_1 \mid s_1) > (1-\lambda)L$, which holds when $\Pr(\theta_1 \mid s_1)$ is large, that is for a sufficiently informative signal.[24]

Finally, the sign of the derivative of λ^* with respect to γ is negative, which means that the positive effect of having less support when the signal is bad is dominated by the negative effect of having more support when the signal is good. Since $k^* = \kappa(1-\lambda^*)$, this in turn explains why an increase in γ has a positive effect on k^*.

5 Concluding Remarks

Goodhart (1999) has argued that "there are few issues so subject to myth, sometimes unhelpful myths that tend to obscure rather than illuminate real issues, as is the subject of whether a central bank should act as a LLR." The third myth in his list is that "moral hazard is everywhere and at all times a major consideration."[25] This

[24] Here we also need the assumption that the amount of deposits is greater than or equal to the liquidation value of the bank at date 1, that is $1-k > \lambda + (1-\lambda)L$.

[25] The other myths are that it is generally possible to distinguish between illiquidity and insolvency, that national central bank LLR capabilities are unrestricted whereas international bodies cannot function as LLR, and that it is possible to dispense with a LLR altogether.

paper provides a rationale for the claim that this is indeed a myth.

Although the model is special in a number of respects, we believe that the results are fairly robust. For example, full deposit insurance is not essential, because without it the deposit rate would incorporate a default premium, but this would not change the results. Also, the assumption that deposit withdrawals are purely random is not essential either, because it they were based on information (a relevant case in the absence of full deposit insurance) there would not be any change as long as they were based on signal coarser than that of the LLR. Obviously, if this were not the case, the LLR should incorporate this information in its decision, which would complicate the algebra but probably not the results.

The model also provides a rationale for a standard feature of LLR policy, namely the principle of "constructive ambiguity." This is taken to mean that LLRs do not typically spell out beforehand the procedural and practical details of their policy. One possible rationalization of this principle is based on the idea of the LLR committing to a mixed strategy (see Freixas, 1999). Our model supports a different story, suggested by Goodfriend and Lacker (1999), according to which the policy is not random from the perspective of the LLR, but it is perceived as such by outsiders that cannot observe the supervisory information on the basis of which decisions are made. Thus, the randomness lies in the supervisory information, not in the policy rule.

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