Banks without Parachutes — Competitive Effects of Government Bail-out Policies†

Hendrik Hakenes* and Isabel Schnabel**

Max Planck Institute for Research on Collective Goods, Bonn

Abstract: The explicit or implicit protection of banks through government bail-out policies is a universal phenomenon. We analyze the competitive effects of such policies in two models with different degrees of transparency in the banking sector. Our main result is that the bail-out policy unambiguously leads to higher risk-taking at those banks that do not enjoy a bail-out guarantee. The reason is that the prospect of a bail-out induces the protected bank to expand, thereby intensifying competition in the deposit market and depressing other banks' margins. In contrast, the effects on the protected bank's risk taking and on welfare depend on the transparency of the banking sector.

Keywords: Government bail-out, banking competition, transparency, "too big to fail", financial stability.

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*Address: Group Martin Hellwig, MPI for Research on Collective Goods, Kurt-Schumacher-Str. 10, D-53113 Bonn, Germany, hakenes@coll.mpg.de.

**Address: Group Martin Hellwig, MPI for Research on Collective Goods, Kurt-Schumacher-Str. 10, D-53113 Bonn, Germany, schnabel@coll.mpg.de.

1 Introduction

In most countries, part of the banking sector is protected through implicit or explicit government guarantees. Some of these guarantees, such as deposit insurance, affect all banks more or less in the same way; others privilege a subset of banks, such as public banks or large banks that are "too big to fail". Such asymmetric bail-out policies are the subject of our study.

Political and academic discussions have focused on the detrimental effects of such guarantees on the risk-taking behavior of the protected banks. In contrast, the reactions of the *remaining* banks in the banking system have not been dealt with in the literature. We close this gap by analyzing the competitive effects of government bail-out policies on those banks that do *not* enjoy a public guarantee. An understanding of other banks' reactions is indispensable for the judgment of overall welfare effects of public bail-out policies.

The relevance of such competitive effects can be illustrated with an example from Japan. Since the 1990s, Japanese private banks' profitability has been compromised by thin interest margins. These have been attributed to the competition from government financial institutions as well as from (mostly large) banks receiving disguised subsidies.[1] In particular, private banks face strong competition from Japan's postal savings system, the biggest deposit taker in the world, which benefits from an explicit government guarantee and tax exemptions and is subject to limited prudential supervision. The extent of welfare losses arising from this type of "unfair competition" (Fukao, p. 25, 2003b) depends essentially on how smaller private banks adjust their risk-taking in reaction to shrinking profitability due to the subsidization of public and larger banks.

The relationship between banks' profit margins and their risk-taking is one of the central themes in the literature on competition and stability in the banking sector. The basic idea is that competition tends to reduce rents in the banking sector. In reaction, banks increase their asset risk because of the well-known risk-shifting problem described by Jensen and Meckling (1976). If one applies this idea to a setting with public bail-out guarantees to a subset of banks, one can imagine that there will be a similar risk-shifting problem at those banks that are not expected to be bailed out. This effect will be the driving force in our model.

Our starting point is a paper by Allen and Gale (2000, chapter 8.3) who analyze the tradeoff between competition and stability in a static agency model.[2] Because of its clarity and simplicity, the model is well suited to capture the effect of the size of rents on banks' risk-taking behavior. Like Allen and Gale, we model competition on the liabilities side of banks' balance sheets in a Cournot fashion.

[1] See Fukao (2003a,b) and Kashyap (2002) for an extensive overview of these problems. See also the diagnosis in the Annual Report of the Bank for International Settlements (2002, pp. 133).

[2] See also Allen and Gale (2004).

We then modify that model by introducing an asymmetric government bail-out policy where some bank is bailed out with a higher probability than the others. In contrast to Allen and Gale (2000, chapter 8.3) who assume full deposit insurance for all banks, depositors care about the risk of banks' assets and demand default premia to be compensated for expected losses from bank insolvencies.

Moreover, we consider two time structures with different patterns of information revelation: In the first model, banks are *opaque* in the sense that depositors cannot observe risk before setting deposit rates. Hence, default premia are set before deposit volumes and risk choices are determined.[3] In the second model, we reverse the timing. Depositors can observe their bank's risk choice and the level of deposits before setting default premia. We call banks *transparent* in this case.

Our main result is that the government bail-out policy unambiguously leads to higher risk-taking at banks that do not enjoy a government guarantee. The reason is that the subsidization induces the protected bank to expand its deposit volume, no matter whether banks are opaque or transparent. Since deposit volumes are strategic substitutes in our model, the other banks react by decreasing their deposit volumes. However, the overall effect on aggregate deposits is positive so that there is an increase in the deposit rate, depressing smaller banks' margins and inducing them to take higher risks.

Another important result concerns the *protected* bank's risk-taking. Contrary to conventional wisdom, the effect of the guarantee on the protected bank appears to depend on the transparency in the banking sector. In the model with opaque banks, the protected bank may have lower incentives to take risks, because the subsidy increases the bank's rents.[4] With transparent banks, risk-taking unambiguously increases in the bail-out probability. Here, the argument is similar to the literature discussing excessive risk-taking in the context of unfairly priced deposit insurance.

The overall effect on welfare is ambiguous. With opaque banks, a government bail-out policy ex ante increases welfare for many parameter constellations, while welfare is generally reduced in the model with transparent banks. Hence welfare effects are much more complicated than suggested by public discussions of government bail-out policies and depend, among other things, on the information structure in the banking sector.

The paper proceeds as follows: Section 2 contains a brief review of the related literature. In section 3, we derive the competitive effects of an asymmetric bail-out policy for the cases of opaque and transparent banks. In both models, we first analyze the monopoly case and then the oligopoly situation with banks having different bail-out probabilities. Welfare implications are discussed for each model. Section 4 summarizes our major findings and discusses some extensions to our model.

[3] This is the time structure chosen by Allen and Gale (2000, chapter 8.3).

[4] This effect is comparable to that of Keeley (1990) and Allen and Gale (2000, chapter 8).

2 Related literature

Our paper is related to two strands of literature: first, to the extensive literature on competition and stability in the banking sector, and second, to the literature on the effects of public bail-out guarantees.

The work by Keeley (1990) was the first of a large number of papers to establish the trade-off between competition and stability in the banking sector. In a simple model, Keeley shows that the reduction of rents through competition exacerbates the risk-shifting problem at banks caused by limited liability and/or unfairly priced deposit insurance.[5] Hence, the creation of "charter value" (i. e., the present discounted value of future rents) through restrictions on competition can induce banks to refrain from overly risky behavior if the expected loss of the charter value is larger than the gains from increased risk-taking.

The work by Keeley has been extended in a number of ways, with differing conclusions about the existence of the presumed tradeoff. Allen and Gale (2000, chapter 8.3) generalize Keeley's results in a static agency model, confirming the negative relationship between competition and stability.[6] While the tradeoff appears to be robust to the introduction of product differentiation,[7] it typically breaks down in the presence of competition in loan markets (and not just deposit markets).[8] Dynamic models yield contradictory results.[9]

Similar to the theoretical literature, the empirical literature yields ambiguous results as to the trade-off between competition and stability. Keeley (1990) presents some evidence for the view that the surge in bank failures in the 1980s in the United States may be explained by the disappearance of monopoly rents in banking due to financial deregulation. Similarly, the accumulation of systemic banking crises in developed and developing countries in the past two decades has been attributed to financial liberalization, which has also been shown to be accompanied by declining charter values in banking (see Demirguc-Kunt and Detragiache, 1999).[10] In contrast, a recent cross-country study by Beck, Demirguc-Kunt, and Levine (2003) shows that systemic banking crises are less likely in countries with more concentrated banking

[5]This literature review is restricted to the papers most closely related to ours. For more detailed surveys on the relationship between competition and stability in banking, see Canoy, van Dijk, Lemmen, de Mooij, and Weigand (2001) and Carletti and Hartmann (2003). Allen and Gale (2004) provide a useful overview of what type of models tend to yield what type of results as to the sign of the relationship between competition and stability.

[6]This framework will be used as the basis of our analysis.

[7]See Matutes and Vives (2000) and Cordella and Yeyati (2002).

[8]See Koskela and Stenbacka (2000), Caminal and Matutes (2002), and Boyd and De Nicolò (2003).

[9]See Hellman, Murdoch, and Stiglitz (2000), Perotti and Suarez (2002), and Allen and Gale (2000, chapter 8.4).

[10]In a similar vein, Demirguc-Kunt, Laeven, and Levine (2004) find that banks' interest margins are higher in countries with tighter restrictions on competition in banking.

sectors, but more likely in countries with tighter restrictions on entry and banking activities. These findings are inconsistent with the "charter value hypothesis", according to which crises should be less likely in the latter case as well.

The second strand of literature related to our paper concerns the effects of public bail-out guarantees. With respect to public banks, the literature is scarce. The most important empirical findings are that government ownership of banks is pervasive all over the world and that it tends to be associated with poorly operating financial systems and slower growth performance.[11]

In contrast, there exists a fairly large number of papers on the so-called "*too-big-to-fail*" (hereafter TBTF) problem. Large banks may be subject to an incentive problem because the public authorities cannot credibly commit to not supporting these banks in case of impending failure. The effects on risk-taking are similar to the ones discussed in the deposit insurance literature. In fact, a TBTF policy can be described as a complete insurance of all deposits and other liabilities at zero costs. Since Merton (1977), it is well-known that unfair deposit insurance entails a risk-shifting problem, similar to the problem arising from limited liability.[12] Hence one may expect that a more concentrated banking sector with TBTF banks entails higher risk-taking at the largest banks, and thus higher fragility.[13] Since a higher concentration implies less competition, this result is just the opposite of what would be predicted by the "charter value literature" described above. Our paper aims at resolving this apparent contradiction.

The TBTF problem also seems to be an empirically relevant phenomenon. Boyd and Gertler (1994) document a TBTF problem at the largest commercial banks in the United States in the 1980s. Schnabel (2003, 2004) describes a similar phenomenon at the so-called "great banks" in Germany at the time of the Great Depression. The episode studied most intensively is the near-failure of Continental Illinois in 1984 and the consequent public announcement by regulators that the 11 largest US banks were too big to be allowed fail. In an event study, O'Hara and Shaw (1990) find significant positive abnormal returns for the TBTF bank after the announcement, which is consistent with the existence of a positive subsidy to TBTF banks. Studies using bond market data tend to confirm the existence of conjectural government guarantees.[14]

Another strand of the empirical literature looks at the question whether the prospect

[11]See Barth, Caprio, and Levine (1999) and Porta, de Silanes, and Shleifer (2002). For an analysis of public banks in Germany, see Sinn (1999).

[12]Empirical evidence for the adverse effects of deposit insurance on banking stability has been presented by Demirguc-Kunt and Detragiache (2002).

[13]Erlenmaier and Gersbach (1999) show that, in the absence of risk-shifting problems, a TBTF policy welfare-dominates random bail-out schemes, whereas the random scheme leads to higher stability.

[14]See Flannery and Sorescu (1996) and Morgan and Stiroh (2002).

of becoming TBTF is a motivation for bank mergers.[15] While Benston, Hunter, and Wall (1995) reject this hypothesis for the years 1981 to 1986, the evidence for the 1990s in Kane (2000) and Penas and Unal (2001) is consistent with the hypothesis.

One striking gap in the literature concerns the impact of a TBTF policy on smaller banks. O'Hara and Shaw (1990) find negative effects on banks not included in the list of banks deemed to be "too big to fail" and attribute this finding to the self-financing character of the deposit insurance system. Apart from this finding, we were not able to find any theoretical or empirical paper dealing with the banks that are "too small to be saved". Our paper contributes to closing this gap.

3 The model

The setup of our model is similar to the one in Allen and Gale (2000, chapter 8.3). We consider an economy with n chartered banks, indexed by $i = 1, \ldots, n$. Bank i collects deposits d_i and invests them in risky projects. Projects yield per invested unit a return y_i with probability $p(y_i)$, otherwise they return zero. The success probability is a decreasing and concave function of the target return, i. e., $p'(y_i) < 0$, and $p''(y_i) \leq 0$. Each bank can choose the "risk level" of its investment by fixing y_i. The aggregate amount of deposits in the economy is $D = \sum_{i=1}^{n} d_i$. Depositors demand an expected return $R(D)$, with $R'(D) > 0$ and $R''(D) \geq 0$. Banks and depositors are assumed to be risk neutral.

So far the model is identical to Allen and Gale (2000, chapter 8.3). However, Allen and Gale assume that deposits are *fully* insured, so that depositors do not need to care about default probabilities. In contrast, we assume that bank i is bailed out by the government with probability $\beta_i \in [0; 1]$ in the case of failure.[16] The government can commit itself to this exogenous bail-out probability.[17] Given this, depositors are repaid with probability $p(y_i) + \beta_i\,(1 - p(y_i))$; with probability $(1 - \beta_i)\,(1 - p(y_i))$, they receive nothing. In order to obtain an expected return of $R(D)$, they demand a nominal return of $\rho_i\,R(D)$; the "default premium" ρ_i will depend on β_i and y_i.

The expected profit of bank i consists of three factors, the probability of success $p(y)$, the deposit volume d, and some "margin" given by the difference between y and the nominal repayment $\rho\,R(D)$. It hence is a function of four endogenous variables, namely its risk level y_i, its default premium ρ_i, its deposit volume d_i, and the competitors' deposit volume, $D_{-i} = \sum_{k \neq i} d_k = D - d_i$,

$$\Pi_i(y_i, \rho_i, d_i, D_{-i}) = p(y_i)\,[y_i\,d_i - \rho_i\,R(d_i + D_{-i})\,d_i] = p(y_i)\,d_i\,[y_i - \rho_i\,R(D)]. \quad (1)$$

[15] This argument is similar to the dynamic arguments in the literature on competition and stability. It was first stated by Hunter and Wall (1989) and Boyd and Graham (1991).

[16] Depositors in our model should be thought of as investors who are not (fully) insured through a deposit insurance scheme. Because of the risk neutrality of depositors, β_i can also be interpreted as the fraction of deposits that the government refunds in the case of bank failure.

[17] A relaxation of the exogeneity assumption is discussed in section 4.

Figure 1: Time structure when banks are opaque

- For each bank i, the government announces a bail-out probability β_i.
- Depositors (anticipating d_i and y_i) set a default premium ρ_i.
- Banks choose d_i (anticipating y_i), $R(D)$ is determined in the deposit market.
- Banks choose y_i and invest.
- Projects mature and return y_i with probability $p(y_i)$. Banks pay $\rho_i\, R(D)$ to their depositors if possible. Otherwise, the government pays $\rho_i\, R(D)$ with probability β_i.

Within this setting, we define two games characterized by the degrees of *transparency* in the banking sector, modeled through different time patterns of actions and information revelation. In each game, we first discuss the monopoly case with $n = 1$. This yields insights into the banks' incentives to take risks and expand volume, abstracting from competitive effects. These insights will be useful in the subsequent analysis of the oligopoly case.

3.1 Opaque banks

In the first model, depositors set the default premia ρ *before* banks choose their deposit volumes d and their target returns y (see figure 1). This means that depositors cannot exert any *market discipline* because they cannot react to the actual risk-taking of banks, which is revealed only after depositors have set the default premia. Therefore, we call banks *opaque* in this case. The model structure is equivalent to a commitment problem, in which banks cannot commit to a particular risk level. If there was a possibility for commitment or if the risk taking of banks was contractible, depositors could discipline banks by demanding default premia rising with risk taking. The given time structure generates a moral hazard problem between depositors and banks, known as risk-shifting or asset substitution.[18]

[18] To give the market for deposits a micro foundation, assume that we have an auctioneer who first asks each infinitesimal depositor j to report her individual supply function $r^j(D)$ for deposits, and the default premium ρ_i^j she would demand from each bank i. We assume that depositors are homogenous and name identical functions $r^j(D)$ and values ρ_i^j. The auctioneer aggregates the supply, $R(D) = \int r^j(D)\, dj$, and communicates supply and default premia to the banks. Now each bank i chooses a volume d_i and communicates it to the auctioneer. Then the auctioneer determines the aggregate deposit volume $D = \sum_i d_i$ and the risk-free market rate $R(D)$, and he fixes the nominal deposit rate $\rho_i\, R(D)$ for each bank.

3.1.1 Monopoly

To abstract from competitive effects, we first look at the case with only one bank ($n = 1$), so that $D = d_1$. For readability, we omit all indices. As usual, we analyze the problem backwards. First, we determine y for given β, d and ρ,

$$\frac{\partial \Pi}{\partial y} = d\left[p(y) + [y - \rho\, R(d)]\, p'(y)\right] = 0. \tag{2}$$

If a solution to (2) exists, it is unique given our assumptions on $p(y)$.[19] Thus the maximization yields an implicit function $y(d, \rho)$, from which we can derive

$$y_d = \frac{\partial y}{\partial d} = \underbrace{\frac{\rho\, \overset{-}{p'(y)}\, \overset{+}{R'(d)}}{2\, p'(y) + [y - \rho\, R(d)]\, p''(y)}}_{-} > 0.$$

The intuition for the positive relationship between risk and deposit volume is straightforward. If d rises, $\rho\, R(d)$ goes up, and the bank compensates the shrinking margin by increasing y. Analogously,

$$y_\rho = \frac{\partial y}{\partial \rho} = \underbrace{\frac{\overset{-}{p'(y)}\, \overset{+}{R(d)}}{2\, p'(y) + [y - \rho\, R(d)]\, p''(y)}}_{-} > 0.$$

An increase in ρ reduces the margin, which is compensated for by raising the risk level y.[20] We now turn to the determination of the deposit volume. Banks choose d given the default premium ρ and anticipating y. Incorporating the dependence of y on d and ρ into the profit function (1), the bank's expected profits are given by

$$\Pi = p(y(d, \rho))\, d\, [y(d, \rho) - \rho\, R(d)]. \tag{3}$$

The first-order condition to this maximization problem yields an implicit relation between d and ρ, which we denote by $d(\rho)$. To keep our proofs tractable we assume from now on that $R(d)$ and $p(y)$ are linear functions.[21] We can then derive the following lemma.[22]

Lemma 1 (Optimal d for given ρ) *The optimal deposit volume decreases in the default premium, i. e., $d'(\rho) < 0$.*

[19] Below we will see cases where a solution does not exist for all β.

[20] Notice that both results are driven by the same mechanism, namely a decrease in the margin that translates into an increase in risk. This mechanism is central to the "charter value literature".

[21] This assumption is stronger than actually required. What we need is that $R(d)$ and $p(y)$ do not bend too much in the neighborhood of the equilibrium, so that they can be approximated reasonably well by a first-order Taylor-approximation.

[22] All proofs can be found in the appendix.

As before an increasing default premium reduces the margin $y - \rho\, R(d)$. The bank can react by either increasing y or decreasing d. Lemma 1 implies that d decreases in any case, at least if $R(d)$ and $p(y)$ do not bend too much. This rules out the possibility that there is a strong increase in y, accompanied by a weak increase in d.

Finally, we turn to the determination of ρ. Anticipating y and d, depositors set a fair default premium ρ, so that they obtain an expected return of $R(d)$, yielding

$$\rho = \frac{1}{p(y) + \beta\,(1 - p(y))}. \tag{4}$$

The default premium ρ does not directly depend on d. However, there is an indirect dependence through y. The resulting function $\rho(d)$ is characterized by lemma 2.

Lemma 2 (Fair ρ for given d) *The fair default premium increases in the deposit volume, i. e., $\rho'(d) > 0$.*

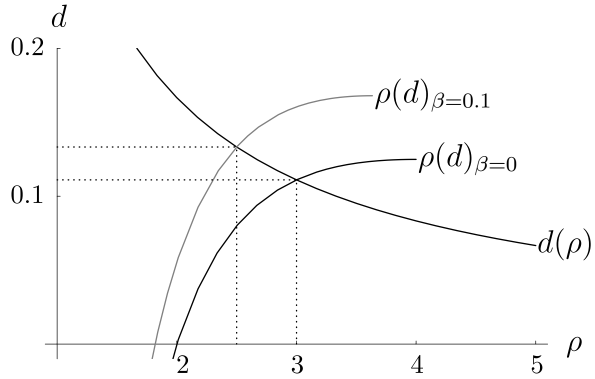
The intuition for this result is as follows: An increase in d reduces the margin $y - \rho\, R(d)$. The bank compensates for this reduction by increasing risk, inducing the depositors to demand a higher default premium. The increase in ρ provokes further risk shifting, leading to a multiplier effect reinforcing the initial effect. Hence, the overall effect on the default premium is positive.

In equilibrium, the default premium must be fair, given the anticipated deposit volume, and the deposit volume has to be optimal, given the default premium. Therefore, we can determine the equilibrium by looking at the intersection of the two curves $d(\rho)$ and $\rho(d)$. Figure 2 displays such equilibria for two different bail-out policies β, taking into account the results from lemmata 1 and 2. The following proposition characterizes the effects of different bail-out policies on the equilibrium.

Proposition 1 (Effects of bail-out policy in monopoly) *In an opaque monopolistic banking system, an increase in the bail-out probability induces depositors to demand a lower default premium, $\partial\rho/\partial\beta < 0$. The bank reacts by choosing a higher deposit volume, $\partial d/\partial\beta > 0$. It chooses a higher risk level ($\partial y/\partial\beta > 0$) if and only if the supply of deposits is inelastic.*

Figure 2 illustrates the first part of this proposition. An increase in β implies that depositors are compensated with a higher probability (or to a higher degree) in the case of bank failure, which reduces the fair default premium for a given d; hence, the function $\rho(d)$ is shifted to the left. In contrast, $d(\rho)$ does not depend on β for a given ρ. From the graph, it is clear that in reaction to an increasing β the bank always expands and that the default premium falls. The effect on risk taking is not obvious, because the effects of d and ρ on y go into opposite directions. Proposition 1 states that the overall effect on risk taking depends on the elasticity of the supply

Figure 2: Default premium and deposit volume for varying β in the monopoly



This example is based on the functions $p(y) = 1-y$ and $R(d) = d$. For comparability, the same functions are used throughout the paper.

of deposits. If the supply of deposits is elastic, the inverse function $R(d)$ is inelastic. Therefore, the expansion of the bank has only a small effect on the deposit rate $R(d)$. As a result, the effect working through ρ dominates the effect working through d, meaning that risk taking actually decreases. If, on the other hand, the supply of deposits is inelastic, $R(d)$ is elastic, and an expansion of d leads to a large increase in the deposit rate $R(d)$. In this case, there is a clear tendency for banks to increase risk. This ambiguous result contradicts the conventional wisdom according to which a higher bail-out probability *always* leads to an increase in risk taking.[23]

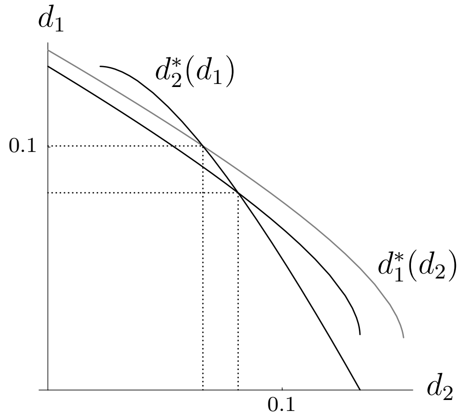
3.1.2 Oligopoly

Assume now that n banks have been chartered instead of just one. We are interested in how the market as a whole reacts when the government changes the bail-out policy for one bank. Without loss of generality, assume that the government raises the bail-out probability β_1 of bank 1.

Assume for the moment that the deposit volume D_{-1} of competitor banks is given. Then proposition 1 implies that, just as in the monopoly case, the increase in β_1 leads to a fall in the default premium ρ_1. This induces bank 1 to increase its volume d_1. The question then is how bank 1's behavior affects the remaining banking sector. In our model, banks interact only in the deposit market, namely through the deposit rate $R(D)$. In equilibrium, the deposit volume of each bank must be an optimal

[23]In the example used in the figures, the elasticity of the deposit supply is equal to 1, so there is no effect on risk taking in the monopoly case. The effects on risk taking observed in the oligopoly case can hence be attributed to competitive effects alone.

Figure 3: Reaction functions in the deposit market for varying β_1



The functions used for the graph are the same as in the monopoly. Here and in the following graphs we consider an oligopoly with two banks. Black lines stand for $\beta_1 = \beta_2 = 0.1$, the gray line for $\beta_1 = 0.13$. The equilibria are indicated by dotted lines. Notice that the increase in d_1 is larger than the decrease in d_2 so that aggregate deposits increase.

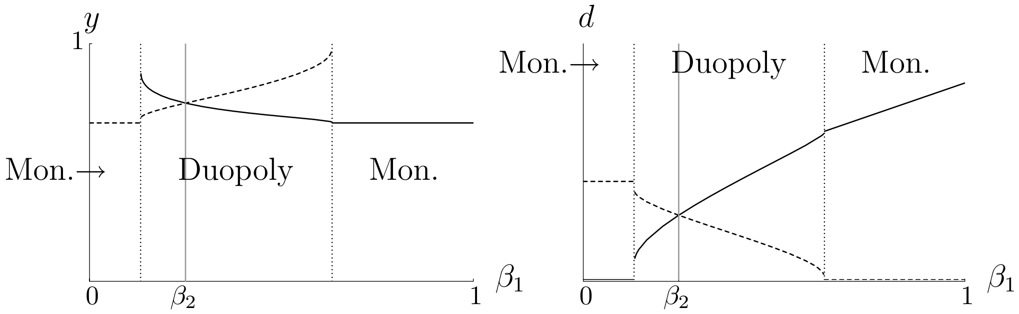
reaction to the volume choices of all competitors. Lemma 3 summarizes the strategic interactions in the deposit market.

Lemma 3 (Strategic interactions in the deposit market) *The reaction function of any bank i, $d_i(d_j)$, is a strictly decreasing function. Starting from an equilibrium with $d_1 > 0$ and $d_j > 0$, an outward shift of bank 1's reaction function leads to an increase in bank 1's and a decrease in bank j's deposit volume. Since the former effect dominates the latter, aggregate deposits D increase.*

The first part of lemma 3 implies that deposit volumes are strategic substitutes in our model. Figure 3 plots the reaction functions for a numerical example. From proposition 1, we know that the reaction function of bank 1 shifts outward as β_1 increases, while the reaction functions of the competitors remain unchanged. The second part of lemma 3 implies that an increase in β_1 leads to an expansion of deposits at the subsidized bank and a contraction of deposits at the remaining banks. Finally, the overall effect is an expansion of the aggregate deposit volume. This last point is crucial: It means that the market rate $R(D)$ increases, implying higher risk taking by the competitor banks due to shrinking margins. The following proposition sums up actions and reactions of bank 1 and its competitors.

Proposition 2 (Competitive effects of bail-out policy) *In an opaque banking system, an increase in the bail-out probability β_1 leads to*

Figure 4: Effects of an asymmetric bail-out policy



Solid lines stand for bank 1, dashed lines for a competitor bank 2. In this example, $\beta_2 = 0.25$, so for $\beta_1 = 0.25$, both banks are symmetric (gray vertical line).

1. *an expansion of deposits at bank 1 and a contraction of deposits at its competitor banks* $j \neq 1$, $\partial d_1/\partial\beta_1 > 0$ *and* $\partial d_j/\partial\beta_1 < 0$*;*
2. *an increase or decrease in risk at bank 1, depending on the elasticity of the deposit supply;* [24] *in either case, the default premium falls,* $\partial\rho_1/\partial\beta_1 < 0$*;*
3. *an increase in risk at the competitor banks* j*, accompanied by higher default premia,* $\partial y_j/\partial\beta_1 > 0$ *and* $\partial\rho_j/\partial\beta_1 > 0$*.*

Proposition 2 is illustrated in figure 4 for a numerical example with two banks. As the bail-out probability β_1 rises, bank 1 raises its deposit volume. At the same time, bank 2 is crowded out. Due to a lower nominal deposit rate $\rho_1\, R(D)$, bank 1 reduces its riskiness. For bank 2, the nominal rate rises, leading to higher risk taking. If bail-out policies become too asymmetric, the less protected bank's incentives to take risk become overwhelming, inducing depositors to demand ever higher default premia, which in turn fuel risk taking, so that the process reaches no new equilibrium. The less protected bank closes. If there are only two banks initially, the other bank is left with a monopoly, as described in section 3.1.1. At that point, deposits at the remaining bank jump up and risk taking drops.

[24] The risk taking of bank 1 decreases if the supply of deposits is elastic ($\varepsilon > 1$) or not too inelastic (if $\overline{\varepsilon} < \varepsilon \leq 1$ for some $\overline{\varepsilon}$). Then there is an intermediate region ($\underline{\varepsilon} \leq \varepsilon \leq \overline{\varepsilon}$ for some $\underline{\varepsilon} < \overline{\varepsilon}$) where risk taking may increase of decrease. If the deposit supply is very inelastic (if $\varepsilon < \underline{\varepsilon}$), bank 1 increases risk.

3.1.3 Welfare analysis

We now turn to the welfare effects of public bail-out policies. Aggregate welfare is given by

$$W = \sum_{i=1}^{n} d_i \, p(y_i) \, y_i - \int_0^D R(\mathcal{D}) \, \mathrm{d}\mathcal{D}. \tag{5}$$

The first term denotes the projects' expected returns, summed up over all banks. The second term gives the depositors' opportunity costs, defined by the integral over the returns of opportunity investments. Banks' expected repayments to depositors are welfare-neutral, so they do not appear in (5). Similarly, the bail-out payments from the government are welfare-neutral in our model.[25]

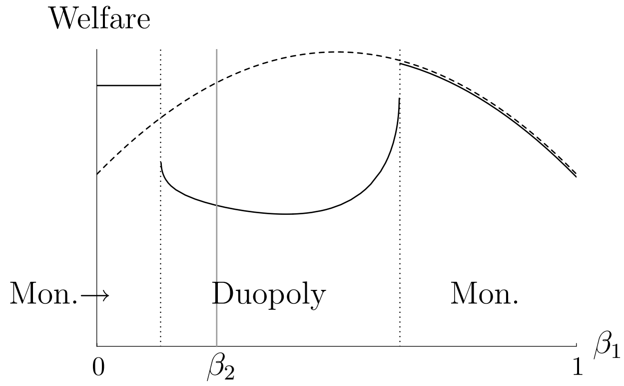
Welfare is affected by three factors: risk taking, the level of deposits, and the entrance or exit of banks. Since risk taking is excessive in this model, an increase in risk taking always decreases welfare (and vice versa). An expansion of deposits of bank i increases welfare as long as expected returns $y_i \, p(y_i)$ outweigh opportunity costs $R(D)$. In our model, banks can grow excessively when they receive a subsidy via the protection. In that case, an expansion of deposits reduces welfare. Finally, an asymmetric bail-out policy may crowd out the least protected banks completely. In that case, risk taking of all other banks drops down, aggregate deposits drop down, and welfare may jump up or down. The direction of the welfare jump depends on the relative impact of deposit contraction and risk reduction.

Given the multitude of effects going into different directions, there is no clear prediction about welfare effects of bail-out policies. Already in the monopoly case, the welfare effects are ambiguous (see the dashed curve in figure 5). As stated in proposition 1, an increase in β_1 leads to increased risk taking of bank 1 if and only if the deposit supply is inelastic. This tends to decrease welfare. A welfare increase due to mitigated risk shifting occurs only if the deposit supply is elastic. In the example of figure 5, risk taking is constant at $y = 2/3$ because the elasticity is just 1. The welfare increase observed for small β is entirely due to the expansion of deposits. For large β, welfare declines due to excessive deposit growth: For $\beta = 1$, $R(d) = 1/3$ whereas $y \, p(y) = 2/9$, so a marginal unit of deposits leads to a welfare loss of $1/9$. Because the bank still grows for large β, welfare bends downwards.

The oligopoly case is exemplified by the solid curve with two discontinuities (see figure 5). The mechanisms affecting welfare are the same as above. However, in the case of an oligopoly, the endogenous variables of different banks influence each other. If the β of different banks are very asymmetric, the less protected bank does not enter the market. In the example with just two banks, we are back in the monopoly case with a high margin and little risk taking (compare figure 4); welfare jumps up

[25] In reality, distortions from taxation to finance bail-outs can be substantial. In that case, (5) overestimates welfare in the presence of bail-outs, especially if bail-out probabilities are large.

Figure 5: Aggregate welfare in the charter value model



In this example, $\beta_2 = 0.25$. For $\beta_1 = 0.25$, both banks are symmetric (gray vertical line). The dashed curve denotes welfare for the monopoly case, whereas the solid line refers to an oligopoly with two banks.

in that case. As before, welfare decreases for very large β_1 due to excessive deposit growth. In the intermediate region, there are several countervailing effects. With increasing β_1, bank 1 grows and reduces its risk taking, whereas bank 2 shrinks and increases risk. For relatively small β_1, the welfare decreasing effects dominate, while the opposite is true for larger β_1. Overall it appears that some (but not too much) asymmetry between banks enhances welfare.

One should be careful in drawing policy implications from this analysis. The analysis restricts itself to the comparison of different bail-out policies. However, there may be alternative policies (e. g., the regulation of the number of banks, deposit rate floors, direct subsidies for deposits) that may increase welfare with less detrimental side-effects on risk shifting.

3.2 Transparent banks

In our second model the time structure is reversed (see figure 6), so that depositors observe banks' risk choices before setting default premia. Here depositors can (and do) exert *market discipline*. We call banks *transparent* in this case. In making their risk choices, banks take into account that they are punished for excessive risk taking. Again the model could alternatively be phrased in terms of a commitment problem. If banks could commit to a certain risk level or if risk taking were contractible, default premia would depend directly on the level of risk taking, exerting discipline on banks.

This disciplining effect can be seen most clearly in the extreme case where $\beta = 0$, i. e., the bank is never bailed out. In this case, depositors demand the fair premium

Figure 6: Time structure when banks are transparent

- For each bank i, the government announces a bail-out probability β_i
- Banks choose y_i (anticipating d_i and ρ_i)
- Banks choose d_i, $R(D)$ is determined in the deposit market
- y_i is revealed, depositors set a default premium ρ_i, banks invest
- Projects mature and return y_i with probability $p(y_i)$. Banks pay $\rho_i\, R(D)$ to their depositors if possible. Otherwise, the government pays $\rho_i\, R(D)$ with probability β_i

$\rho = 1/p(y)$, and expected profits of the bank are given by $\Pi = d\,(y\,p(y) - R(D))$. Then the bank's optimal y is equal to the first-best solution because the bank must itself bear the entire costs from excessive risk taking. If $\beta > 0$, there is again a risk-shifting problem because the costs of the implicit government guarantee are not borne by the bank.

3.2.1 Monopoly

Again we start with the monopoly case to abstract from competitive effects. First, we determine the fair default premium ρ, given the level of deposits d and the bank's risk choice y. The expression for ρ looks exactly as in (4),

$$\rho = \frac{1}{p(y) + \beta\,(1 - p(y))}. \tag{6}$$

ρ increases in y and does not depend directly on d. Furthermore, it decreases in β. In the extreme case with $\beta = 1$, there is no default premium, independent of the chosen risk level, i. e., $\rho = 1$. We can directly incorporate the fair default premium into the expected profit function (1),

$$\Pi = p(y)\,d\left(y - \frac{R(d)}{p(y) + \beta\,(1 - p(y))}\right). \tag{7}$$

Now there are only two endogenous variables left, y and d. First, we determine the optimal level of deposits d for a given β and y,

$$\frac{\partial \Pi}{\partial d} = p(y)[y - \rho(y)\,[R(\overset{+}{d}) + d\,R'(\overset{+}{d})]] \overset{!}{=} 0. \tag{8}$$

An increase in d has two effects: First, it increases profits at a given margin; second, it decreases the margin due to an increase in the deposit rate $R(d)$. At the optimum, these two effects balance. If a solution to (8) exists, it is unique given our assumptions on $R(d)$. Thus the maximization yields an implicit function $d(y)$, from which we can derive

$$d_y = \frac{1 - \overset{+}{\rho'(y)}\,[R(d) + \overset{+}{d\,R'(d)}]}{\underset{+}{\rho(y)}\,[\underset{+}{2\,R'(d) + d\,R''(d)}]}. \tag{9}$$

One can distinguish two effects: First, an increase in y directly increases the margin, inducing the bank to expand its deposit volume. Second, there is an indirect effect working through ρ. A rise in y pushes up the default premium, and the corresponding decrease in the margin leads to a countervailing effect on d. The following lemma states that the direct effect dominates the indirect effect. Again we assume that $p(y)$ and $R(d)$ are linear functions to keep the proofs manageable. Additionally, we assume that β is small.[26]

Lemma 4 (Optimal d for given y) *In the neighborhood of the optimal y, $d_y > 0$.*

The intuition for this result is as follows: For any β, the bank will choose the highest deposit volume when its expected return from the project is maximal. For $\beta = 0$, the bank's risk choice coincides with the first-best level of risk. Hence, the function $d(y)$ will have a maximum at the first-best y, so that $d_y = 0$ at this point; the direct and the indirect effect on d just cancel at this point. For $\beta > 0$ and y close to the first-best, the direct effect on d is stronger than the indirect effect since the increase in ρ does not reflect the full increase in risk. Hence, the volume-maximizing y will be larger than the first-best, and the function $d(y)$ will be strictly increasing at the first-best.[27]

In the final step, we determine the bank's optimal risk choice by taking the derivative of the profit function with respect to y. From this we get the implicit relation

$$p'(y)\,d\,[y - \rho(y)\,R(d)] + p(y)\,d\,[1 - \rho'(y)\,R(d)] = 0. \tag{10}$$

An increase in y has two effects: it decreases profits through the success probability; and it increases profits through a rising margin. Again, these two effects just balance at the optimum. This leads to the following relationship between y and d,

$$y_d = \frac{\partial y}{\partial d} = \frac{\overset{+}{R'(d)}\,[\overset{-}{p'(y)}\,\rho(y) + p(y)\,\overset{+}{\rho'(y)}]}{p''(y)\,[y - \rho(y)\,R(d)] + 2\,p'(y)\,[1 - \rho'(y)\,R(d)] - p(y)\,\rho''(y)\,R(d)}. \tag{11}$$

The denominator of this equation is negative if we assume that a solution to the optimization problem exists. In the numerator, we can distinguish two countervailing effects: First, a rise in d mitigates the effect of y working through the success

[26] All results are proven in a neighborhood of $\beta = 0$ (which does not necessarily preclude large β). Numerical calculations suggest that all of our results hold for $\beta \in [0;\, 1]$.

[27] Figure A2 in the appendix shows the function $d(y)$ for different choices of β.

probability, inducing the bank to raise y. Second, an increasing d also mitigates the effect of y on the margin, reducing the bank's incentive to raise y. The following lemma states that the first effect always dominates the second.

Lemma 5 (Optimal y for given d) *If $\beta = 0$ then $y_d = 0$, and if $\beta > 0$ then $y_d \geq 0$.*

Intuitively, for $\beta = 0$ any increase in y is accompanied by a "fair" increase in ρ. The bank therefore always opts for the first-best risk level, which is constant. Hence the two effects add up to zero, and $y_d = 0$. For $\beta > 0$, the increase in ρ is less than fair, and the bank has an incentive to increase y above the first-best level even if part of the increase in returns is eaten up by the resulting rise in the default premium $\rho(y)$.

The equilibria are given by the intersections of the curves $d(y)$ and $y(d)$. We are interested in the reactions of the optimal y and d to a change in β.

First, we examine how the curve $d(y)$ moves when β rises. At a given risk level y, an increase in β leads to a decrease in ρ. This induces the bank to expand its deposit volume d. Hence, the curve $d(y)$ shifts upwards. The movement of $y(d)$ is somewhat more complicated: On the one hand, an increase in β lowers ρ and thus reinforces the effect working through the success probability. This induces the bank to reduce y. On the other hand, an increase in β lowers the influence of y on the default premium ρ, i. e., $\rho'(y)$ falls. Therefore, raising risk is less costly for the bank, giving the bank an incentive to raise y. Hence, the movement of the curve is not monotonous.[28] One can show, however, that the second effect dominates the first one for small β. The following proposition characterizes the effect of an increase in the bail-out probability on the equilibrium choices of y and d.

Proposition 3 (Effects of bail-out policy in monopoly) *In a transparent monopolistic banking system, an increase in the bail-out probability induces the bank to raise its deposit volume d and choose a higher risk y. The default premium ρ decreases if $R(d)$ is small enough.*

The monopolistic bank reacts to the subsidization by the government by expanding its deposit volume and increasing risk because part of the potential losses can be shifted to the government. With respect to ρ, there are two countervailing effects: the rise in β and the rise in y. Proposition 3 states that the effect of the rising β dominates for small $R(d)$ and small β.

[28] This can also be seen in the example in figure A2.

3.2.2 Oligopoly

Now we turn to the oligopoly case with n banks. Banks first announce y_i simultaneously; then they simultaneously take in deposits d_i.[29] Analytically, we first have to solve the system $(\partial\Pi_i/\partial d_i = 0)_i$, and then in a second step the system $(\partial\Pi_i/\partial y_i = 0)_i$. As in section 3.1.2, assume without loss of generality that the government raises β_1.

The chain of reactions is almost identical to the one in section 3.1.2. There is a direct effect on bank 1 as described in proposition 3. Thereby, the rise in β_1 leads to an increase in bank 1's risk and deposit volume, taking the deposit volumes of competitors as given. As before, the behavior of bank 1 spills over to the other banks through the deposit market. Lemma 6 describes how competitor banks $j \neq 1$ react to the increase in d_1, neglecting for the moment further reactions of bank 1.

Lemma 6 (Reactions of competitors) *An increase in D_{-j} leads to*

1. *a decrease in bank j's deposit volume, i. e., $\partial d_j/\partial D_{-j} < 0$, and*
2. *an increase in bank j's risk if $\beta_j > 0$, i. e., $\partial y_j/\partial D_{-j} > 0$. If $\beta_j = 0$, y_j remains constant.*

The effects of an increase in D_{-j} (or d_1 in our case) work through a rising deposit rate $R(D)$. As in the model with opaque banks, this induces bank j to raise its risk level y_j and to lower its deposit volume d_j. As before, deposit volumes are strategic substitutes. In fact, banks' reactions functions look like the ones in figure 3 with slightly different slopes.[30] Therefore, the overall effect on total deposits will again be positive. As a result, the market rate rises, and competitors' risk levels increase accordingly. Proposition 4 summarizes the reactions of bank 1 and its competitors.

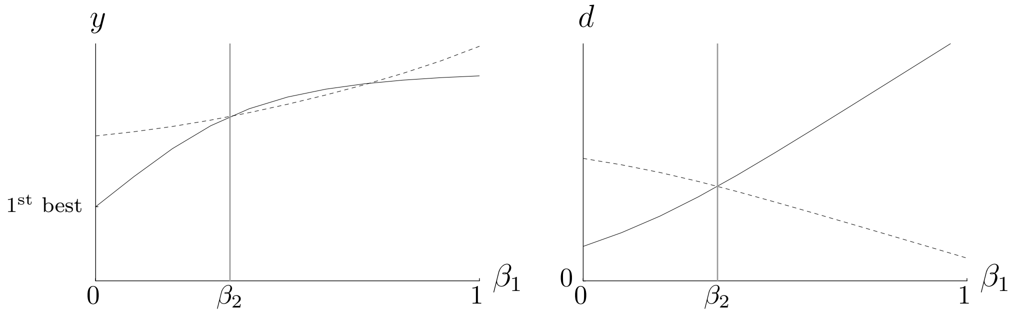
Proposition 4 (Competitive effects of bail-out policy) *In a transparent banking system, an increase in the bail-out probability β_1 leads to*

1. *an expansion of deposits at bank 1 and a contraction of deposits at its competitor banks $j \neq 1$, i. e., $\partial d_1/\partial \beta_1 > 0$ and $\partial d_j/\partial \beta_1 < 0$;*

[29] Note that the other banks' y_i are revealed only after banks have decided on their deposit volumes. As an alternative time structure, one may assume that banks can observe their mutual risk choices when fixing deposit volumes. In this case, banks will tend to choose higher risk levels for strategic reasons: Since a bank's optimal deposit volume rises in its exposure to risk, competitor banks interpret increased risk taking as a commitment to choose a high volume afterwards in the deposit market. Numerical calculations suggest that our main results remain valid in this case.

[30] In the special case of $R''(d) = 0$, reaction functions are linear.

Figure 7: Effects of an asymmetric bail-out policy



Solid lines stand for bank 1, dashed lines for competitor bank 2. In this example, $\beta_2 = 0.35$, so for $\beta_1 = 0.35$, both banks are identical.

2. *an increase in risk at bank 1, but a decrease in the default premium for* $R(D)$ *small enough, i. e.,* $\partial y_1/\partial\beta_1 > 0$ *and* $\partial\rho_1/\partial\beta_1 < 0$*;*
3. *an increase in risk at the competitor banks* j*, accompanied by higher default premia, i. e.,* $\partial y_j/\partial\beta_1 > 0$ *and* $\partial\rho_j/\partial\beta_1 > 0$*.*

The proposition is illustrated in figure 7 for the case of two banks. As the bail-out probability β_1 rises, bank 1 expands, while bank j is crowded out. Since the increase in d_1 is larger than the decrease in d_j, the aggregate deposit volume increases, leading to a higher market rate $R(D)$. As a result, the risk-shifting problem at bank j is exacerbated. In the case of transparent banks, y_1 also increases unambiguously. Interestingly, the effect of competition on risk taking may be even stronger than the direct effect: For very large β_1, y_2 exceeds y_2 in the numerical example.

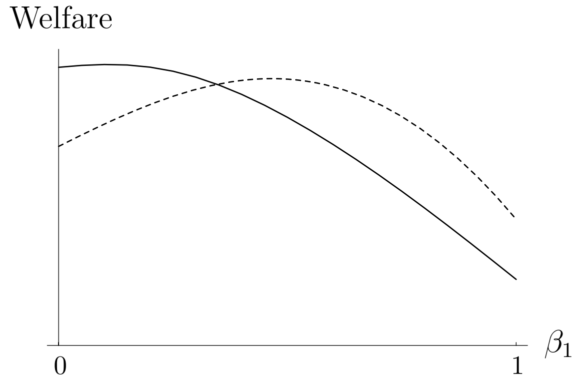
The most important result is that the competitive effects of the bail-out policy on the remaining banking sector are independent of the time and information structure of the model: In both of our models, the subsidized bank expands, causing a rise in the market rate, which aggravates the risk-shifting problems at competitor banks.

3.2.3 Welfare analysis

The welfare measure in the transparent banking system is the same as in the opaque system (see equation (5)). The mechanisms at work are also very similar. The increase in risk taking at *all* banks clearly decreases welfare in this model. The effect of an expansion of deposits is again ambiguous because deposit growth can be excessive. This introduces some ambiguity as to the overall welfare effects.

First, consider the monopoly case. For $\beta = 0$, the level of risk is first best, but the deposit volume is suboptimally low. An increase in β_1 increases risk taking and hence

Figure 8: Aggregate welfare in the charter value model



In this example, $\beta_2 = 0.35$. The dashed curve denotes welfare for the monopoly case, whereas the solid curve refers to an oligopoly with two banks.

reduces welfare, but it also induces the monopolist bank to expand, which increases welfare as long as $y\,p(y)$ exceeds $R(D)$. Either of the effects may dominate. For large β_1, deposit growth is likely to be excessive, implying an unambiguous decrease in welfare. The dashed curve in figure 8 shows an example where welfare increases for small β, but decreases for large β.

In the oligopoly, welfare decreases in most cases. Only if the number of banks is low, the welfare gain due to the deposit expansion of bank 1 may outweigh the welfare losses due to excessive risk taking, at least for small β. The larger the number of banks, the closer is the banking system to being competitive. If $\beta_i = 0$ for all i and i is large, the level of aggregate deposits is close to the efficient level and expected returns of the projects are nearly equal to refinancing rates, $y_i\,p(y_i) \approx R(D)$. Therefore, little is to be gained from a deposit expansion. Even for the duopoly case, the scope for welfare gains may be limited as is apparent from the solid curve in figure 8, which rises only slightly for small β_1. In many cases, welfare will be decreasing even for small β in the oligopoly.

4 Conclusion

We started from the question of how government bail-out policies affect competition in the banking sector. While the existing literature has focused on the effects of bail-out policies on the bank that enjoys the public guarantee, our main interest is in the competitive effects of such policies on the remaining banking sector.

We have presented two models, differing only with respect to their time and information structures. In the first model with *opaque* banks, risk taking is unobservable by depositors, so that there is no market discipline. Therefore, a bank's risk choice does not affect its refinancing costs directly. In the second model with *transparent* banks, investments are perfectly observable, and depositors exert market discipline. As a consequence, deposit rates react promptly to a bank's risk choice.

Our main contribution is to show that an increase in the bail-out probability of one bank unambiguously leads to an *increase* in the risk taking of the *competitor banks*. At the same time, competitor banks are crowded out. In contrast, the effect on the protected bank's risk taking depends, among other things, on the degree of transparency in the banking system. If banks are opaque, the protected bank *may* take less risk, while it always assumes more risk in a transparent environment. This qualifies the existing literature that suggests that an increase in the bail-out probability always leads to higher risk taking at the protected bank. As a direct consequence, the welfare effects of raising the bail-out probability are ambiguous. Welfare may increase or decrease, depending on the transparency of the banking system, the degree of competition within the system, the degree of protection, and the asymmetry of banks.

The competitive effects are particularly strong in the opaque setting. The reason is a multiplier effect whereby the original effect from risk taking on the default premium is reinforced through the feedback from the default premium to risk taking. The observation that protected banks do not take higher risk would simply indicate that the banking system is rather opaque, implying that competitor banks are especially exposed to increased risk taking – this would be anything but reassuring.

There is a number of interesting extensions to our paper. One of the most important issues is the assumed exogeneity of the bail-out probability. In the case of public banks, one can reasonably argue that the bail-out probability is exogenous. In contrast, the bail-out probability should depend on the size of banks in the case of a "too-big-to-fail" policy. By allowing the bail-out probability to depend on size, we would get an additional strategic effect. Since high bail-out probabilities are beneficial for banks, a strategic tendency towards increased volume would develop. This raises the deposit rate, exacerbating the risk-shifting problem.

Another extension concerns the chosen market structure. In our model, the only market where banks interact is the market for deposits, and this interaction is modelled in a Cournot fashion. One may change the market structure of our model in two ways. First, competition in the market for deposits may be modelled as price competition with product differentiation or transportation costs.[31] We believe that our central result remains valid, so that competitor banks are still pushed towards higher risk taking. Suppose, for example, that banks are located on a Salop circle and that the bail-out probability of one bank increases. The direct effect is a decrease in the default premium of the protected bank, leading to cheaper refinance

[31] This has been done by Matutes and Vives (2000) and Cordella and Yeyati (2002), however without considering government bail-outs.

opportunities. In reaction, the protected bank will expand. Neighboring banks find themselves threatened by the competition from the protected bank and may react by increasing their deposit rates to regain some "territory" and move away from the protected banks. This effect spills over to the neighbors of the neighbors and so forth. In equilibrium, all competitor banks have lost some territory and have increased their deposit rates, accompanied by higher risk taking.

Second, competition may take place in the market for loans, and not only in the deposit market. In this case, it is less clear that our main result is still valid. In fact, Boyd and De Nicolò (2003) have shown that the main effect driving the "charter-value" literature – and our model – disappears, once one introduces competition in deposit and loan markets simultaneously. However, there are a number of different ways how to introduce competition in the loan market. We conjecture that the results would depend on the exact model specification.

As a normative implication of our model, governments should refrain from bail-out policies, especially in transparent banking markets. The overall welfare effects of such policies are highly ambiguous, and the effects on the competitor banks are always detrimental. Only the subsidized bank may profit, at the cost of an increased instability of the remaining banking sector. Regulatory initiatives towards greater transparency should be accompanied by a "zero bail-out policy", if such a policy is at all credible.[32] Market transparency and government intervention are substitutes, they should never prevail at the same time.

A Appendix

A.1 Proofs for opaque banks

A.1.1 Monopoly

Proof of lemma 1: In the monopoly, $D = d$. $R(d)$ and $p(y)$ are assumed to be linear, so we can write

$$p(y) = p_0 - p_1\, y, \tag{A1}$$
$$R(d) = R_0 + R_1\, d. \tag{A2}$$

[32] Our results are independent of whether the government does in fact bail out some banks, or whether markets only expect the government to do so. Since the negative impact arises from the expectations of market participants, the government should try to build up a reputation of being committed to a "zero bail-out policy".

We derive the function $y(d, \rho)$ by taking the derivative of profits with respect to y and solving for y:

$$\begin{aligned}
\frac{\partial \Pi}{\partial y} &= d\,(p_0 - p_1\,y) + d\,p_1\,(y - \rho\,(R_0 + R_1\,d)) \overset{!}{=} 0, \\
y(d,\rho) &= \frac{p_0 + p_1\,(R_0 + R_1\,d)\,\rho}{2\,p_1}. \qquad \text{(A3)}
\end{aligned}$$

Plugging this function into the profit function yields

$$\Pi = \frac{d\,(p_0 - p_1\,(R_0 + R_1\,d)\,\rho)^2}{4\,p_1}. \qquad \text{(A4)}$$

Then we derive the implicit function $d(\rho)$ by taking the derivative of A4 with respect to d, yielding

$$\frac{\partial \Pi}{\partial d} = \frac{(p_0 - p_1\,(R_0 + d\,R_1)\,\rho)\,(p_0 - p_1\,(R_0 + 3\,d\,R_1)\,\rho)}{4\,p_1} \overset{!}{=} 0. \qquad \text{(A5)}$$

Making use of the implicit function theorem, we can determine $d'(\rho)$,

$$\begin{aligned}
\frac{\partial d}{\partial \rho} &= -\frac{\partial^2\Pi/\partial d\,\partial\rho}{\partial^2\Pi/\partial d^2} \\
&= -\frac{p_0\left(\square + \sqrt{\square^2 - (1-\beta)\,12\,p_1\,R_0}\right)^2}{108\,p_1\,R_1} \quad \text{where} \\
\square &= 3\,\beta + (1-\beta)\,p_0 > 0.
\end{aligned}$$

Hence, the derivative $d'(\rho)$ is negative. ∎

Proof of lemma 2: The equation that determines ρ is

$$\rho = \frac{1}{p(y) + \beta\,(1 - p(y))} \quad \Longleftrightarrow \quad \rho\,[p(y) + \beta\,(1-p(y))] - 1 = 0. \qquad \text{(A6)}$$

The default premium ρ depends only *indirectly* on the deposit volume d. To derive $\rho'(d)$, we take the derivative of equation (A6) with respect to d, considering that $y = y(d, \rho(d))$, and solve for $\rho'(d)$,

$$\begin{aligned}
0 &= \rho'(d)\,[p(y) + \beta\,(1-p(y))] + \rho\,y_\rho\,\rho'(d)\,p'(y)\,(1-\beta) + \rho\,y_d\,p'(y)\,(1-\beta) \\
&= \frac{\rho'(d)}{\rho} + \rho\,p'(y)\,(1-\beta)\,[y_\rho\,\rho'(d) + y_d],
\end{aligned}$$

$$\rho'(d) = -\frac{(1-\beta)\,\rho^2\,\overset{-}{p'(y)}\,y_d}{1 + (1-\beta)\,\rho^2\,\underset{-}{p'(y)}\,y_\rho}. \qquad \text{(A7)}$$

There are two indirect effects through which d affects ρ. First, an increase in d leads to a rising $R(d)$, which induces the bank to raise y; this reduces $p(y)$, so depositors demand a higher ρ. This effect can be seen from the numerator in equation (A7). Second, this increase in ρ induces the bank to raise y even further, leading to a further increase in ρ. This multiplier effect is embodied in the denominator of equation (A7). Both effects point into the same direction. Because the denominator is smaller than 1, the whole fraction is greater than the numerator alone. If the denominator converges towards 0, risk becomes so large that it cannot be compensated by a finite ρ. Hence in the area of well-defined ρ, we have $\rho'(d) > 0$. ■

Proof of proposition 1: In lemma 1, we have already shown that $\rho(d)$ is a strictly increasing function. Lemma 2 states that $d(\rho)$ is strictly decreasing. Now we are interested in how these curves shift in the (ρ, d)-space when β increases (see figure 2). As can be seen from equation (A5), $d(\rho)$ does not depend on β. For $\rho(d)$, we determine the direction of the shift by taking the derivative of equation (A6) with respect to β,

$$\begin{aligned} 0 &= \rho'(\beta)\,[p(y) + \beta\,(1 - p(y))] + \rho\,(1 - p(y)), \\ \frac{\partial \rho}{\partial \beta} &= -\rho\,\frac{1 - p(y)}{p(y) + \beta\,(1 - p(y))} = -\frac{1 - p(y)}{(p(y) + \beta\,(1 - p(y)))^2} < 0. \end{aligned} \tag{A8}$$

This implies a leftward shift of the function $\rho(d)$. From these results we can conclude directly that $\partial\rho/\partial\beta < 0$ and $\partial d/\partial\beta > 0$ (see again figure 2), which proves the first part of the proposition.

The effect of β on y is not immediately clear, because the increase in d leads to a higher y, whereas the decrease in ρ reduces y. To analyze the effect on y, we explicitly calculate the equilibrium from equation (A5) and equation (A6), after plugging in the expression for y from equation (A3).

There is one possibly meaningful solution (other algebraic solutions have $d < 0$, $y < y^{\text{firstbest}}$ or $\rho < 1$),

$$d = \frac{(\Box - \sqrt{\Box^2 - 12\,(1-\beta)\,p_1\,R_0})\,p_0 - 6\,p_1\,R_0}{18\,p_1\,R_1} \quad \text{and} \tag{A9}$$

$$\rho = \frac{6}{\Box - \sqrt{\Box^2 - 12\,(1-\beta)\,p_1\,R_0}}, \tag{A10}$$

where $\Box > 0$ is defined as above.

Three problems may occur: First, d may become zero if β is small. Then the moral hazard problem is so large that depositors prefer not to lend at all,

$$d \geq 0 \quad \Longleftrightarrow \quad \beta \geq \frac{p_0\,(3 - p_0) + 6\,(p_1\,R_0 + \sqrt{p_1\,R_0\,(3 - p_0 + p_1\,R_0)})}{(3 - p_0)^2}.$$

Second, if β is small, there may be no ρ that compensates investors for the risk. A higher ρ leads to higher risk shifting y, which again induces a higher ρ. For small

β, this vicious circle may have no fixed point (the term in the square root becomes negative),

$$\square^2 - 12\,(1-\beta)\,p_1\,R_0 \geq 0 \qquad \Longleftrightarrow \qquad \beta \geq \frac{p_1\,R_0}{p_0}.$$

However, if

$$R_0 \geq \frac{p_0^2}{(3+p_0)\,p_1},$$

the effect of d going to zero dominant. Third, $p(y)$ has to be smaller than one. We have

$$p(y) < 1 \qquad \Longleftrightarrow \qquad p_0 < 3 + p_1\,R_0. \tag{A11}$$

Substituting d and ρ as given in (A9) and (A10) into $y(d,\rho)$ from (A3) and taking the derivative with respect to β, we get

$$y = \frac{5\,\square - 12\,\beta + \sqrt{\square^2 - 12\,(1-\beta)\,p_1\,R_0}}{6\,p_1\,(1-\beta)},$$
$$\frac{\partial y}{\partial \beta} = \frac{2\,p_1\,R_0\,(1-\beta) - \square + \sqrt{\square^2 - 12\,(1-\beta)\,p_1\,R_0}}{2\,p_1\,(1-\beta)^2\,\sqrt{\square^2 - 12\,(1-\beta)\,p_1\,R_0}}.$$

This derivative changes signs in two cases: for $p_0 = 3 + p_1\,R_0$ and for $R_0 = 0$. Because of (A11), we know that $p_0 < 3 + p_1\,R_0$ as long as $p(y) < 1$, hence we need to look only at the case where $R_0 = 0$. For $R_0 < 0$, $\partial y/\partial\beta$ is positive, whereas for $R_0 > 0$, it is negative. We can interpret this finding in terms of elasticities. We have

$$R(D) = R_0 + R_1\,D \qquad \Longleftrightarrow \qquad D(R) = (R - R_0)/R_1,$$
$$\varepsilon = \frac{\partial D}{\partial R}\,\frac{R}{D} = \frac{1}{R_1}\,\frac{R\,R_1}{R - R_0} = \frac{R}{R - R_0}.$$

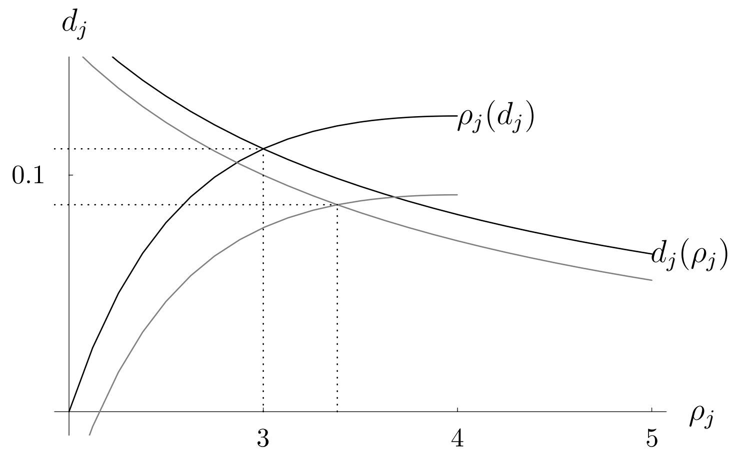
We see that the elasticity ε is smaller than one if R_0 is negative, implying that risk increases with rising β in that case (and vice versa). ■

A.1.2 Oligopoly

Proof of lemma 3: We first show that a bank's deposit volume shrinks as competitors' deposit volumes expand. We derive the function $y_j(D,\rho_j)$ just as in the monopoly case and plug it into the profit function,

$$y_j(D,\rho_j) = \frac{p_0 + p_1\,(R_0 + R_1\,D)\,\rho_j}{2\,p_1},$$
$$\Pi_j = \frac{d_j\,(p_0 - p_1\,(R_0 + R_1\,D)\,\rho_j)^2}{4\,p_1}.$$

Figure A1: Default premium and deposit volume for varying D_{-j} in the oligopoly



Black lines stand for $D_{-j} = 0$, gray lines for $D_{-j} = 1/30$.

Then we derive the equilibrium, using the same procedure as in the monopoly case,

$$d_j = \frac{(\Box_j - \sqrt{\Box_j^2 - 12\,(1-\beta_j)\,p_1\,(R_0 + D_{-j}\,R_1)})\,p_0 - 6\,p_1\,(R_0 + D_{-j}\,R_1)}{18\,p_1\,R_1} \quad \text{and}$$

$$\rho_j = \frac{6}{\Box_j - \sqrt{\Box_j^2 - 12\,(1-\beta_j)\,p_1\,(R_0 + D_{-j}\,R_1)}} \quad \text{where}$$

$$\Box_j = 3\,\beta_j + (1-\beta_j)\,p_0.$$

One can show that $d_j(\rho_j)$ is again strictly decreasing in the (ρ_j, d_j)-space, and $\rho_j(d_j)$ strictly increasing. When D_{-j} grows, both curves shift downwards, as depicted in figure A1. From the figure, it is immediately apparent that $d_j/\partial D_{-j} < 0$. Formally,

$$\frac{\partial d_j}{\partial D_{-j}} = -\frac{1}{3}\left(1 + (1-\beta_j)\,\frac{p_0}{\triangle_j}\right) < 0 \qquad \text{where} \tag{A12}$$
$$\triangle_j = \sqrt{\Box_j^2 - (1-\beta_j)\,12\,p_1\,(R_0 + D_{-j}\,R_1)},$$

which proves our first assertion.

We have shown that an increase in the deposit volume of one bank is accompanied by a decrease in deposits at the competitor banks. We now show that the aggregate effect on deposits is positive. Denote by $d_1^*, \ldots, d_n^*$ the initial equilibrium levels of deposits. When β_1 rises marginally, equilibrium levels adjust to $d_1^{**}, \ldots, d_n^{**}$. We have shown already that the direct effect of the rise of β_1 is an expansion of d_1, and that this leads to a contraction of $d_2, \ldots, d_n$. However, this contraction will never overcompensate the increase in d_1: When $d_2, \ldots, d_n$ contract so much that the original D^* is reached again, for each bank $j \neq 1$ the choice d_j^* is again optimal, and only bank 1 chooses a $d_1 > d_1^*$. Therefore, D expands.

As shown above, the contraction of $d_2, \ldots, d_n$ leads to a further expansion of d_1, which again entails a further contraction of $d_2, \ldots, d_n$. Eventually, this convergence process comes to an end. Possibly, $d_j = 0$ for some $j \in \{2, \ldots, n\}$. The contraction of $d_2, \ldots, d_n$ will never overcompensate the expansion of d_1, hence in the new equilibrium

$$d_1^{**} > d_1^*, \quad d_j^{**} < d_j^* \text{ for all } j \in \{2 \ldots, n\}, \quad \text{and} \quad D^{**} > D^*,$$

which proves our second assertion. ■

Proof of proposition 2: The direct effects of an increase in β_1 on bank 1 are as in the monopoly case: ρ_1 declines and d_1 rises. The effect on risk taking y_1 again depends on the supply elasticity of deposits. However, the elasticity of *individual* supply is *larger* than that of aggregate supply. The individual elasticity of bank 1 can be written as

$$\varepsilon_1 = \frac{\partial d_1}{\partial R(D)} \frac{R(D)}{d_1} = \frac{1}{1 + \partial D_{-1}/\partial d_1} \frac{R}{R_1\, d_1} > \frac{R}{R_1\, D} = \varepsilon = \frac{\partial D}{\partial R(D)} \frac{R(D)}{D}.$$

Hence the protected bank increases risk only if the *aggregate* supply elasticity of deposits is lower, and possibly considerably lower, than one. The rise in d_1 leads to a decrease in d_j (for $j \neq 1$), as was shown in lemma 3. Bank 1 reacts by increasing its deposit volume even further. This reinforces the direct effects on d_1 and ρ_1, and tends to reduce y_1.

We now show how the default premium ρ_j and risk y_j react to an increase in D_{-j}. Using the same procedure as above, we get (with $\triangle_j$ as in (A12))

$$\frac{\partial \rho_j}{\partial D_{-j}} = -\frac{36\, p_1\, R_1\, (1 - \beta_j)}{\triangle_j\, (\square_j + \triangle_j)^2} < 0, \tag{A13}$$

$$\frac{\partial y_j}{\partial D_{-j}} = \frac{R_1}{\triangle_j} > 0, \tag{A14}$$

where $\triangle$ is defined as above. Hence the default premium and the risk taking of banks $j \neq 1$ increase in reaction to the rise in d_1. Again these effects are reinforced by indirect effects. This completes the proof of proposition 2. ■

A.2 Proofs for transparent banks

A.2.1 Monopoly

Proof of lemma 4: Substituting $\rho(y) = 1/[p(y) + \beta\,(1 - p(y))]$, we get

$$d_y = \frac{\beta + (1 - \beta)\,[p(y) + y\, p'(y)]}{2\, R'(d) + d\, R''(d)}.$$

Now if β is small, then $p'(y) + y\,p(y) \approx 0$. Substituting this into (11) yields

$$d_y \approx \frac{\beta}{2\,R'(d) + d\,R''(d)} > 0,$$

which was to be shown. ■

Proof of lemma 5: To find out y_d, take derivatives of (7) with respect to y and d,

$$\frac{\partial \Pi}{\partial y} = d\left(p'(y)\left[y - \frac{R(d)}{p(y) + \beta\,(1-p(y))}\right] + p(y)\left[1 + \frac{(1-\beta)\,p'(y)\,R(d)}{(p(y)+\beta\,(1-p(y)))^2}\right]\right) = 0,$$

$$0 = [p(y) + y\,p'(y)]\,[p(y) + \beta\,(1-p(y))]^2 - \beta\,p'(y)\,R(d), \tag{A15}$$

$$\begin{aligned} 0 &= y_d\,[2\,p'(y) + y\,p''(y)]\,[p(y) + \beta\,(1-p(y))]^2 \\ &\quad + [p(y) + y\,p'(y)]\,2\,(1-\beta)\,y_d\,p'(y)\,[p(y) + \beta\,(1-p(y))] \\ &\quad - \beta\,\big(y_d\,p''(y)\,R(d) + p'(y)\,R'(d)\big). \end{aligned} \tag{A16}$$

If we assume that β is not too large and hence $p'(y) + y\,p(y) \approx 0$ in (A16), solving for y_d we get

$$y_d = \frac{\beta\,p(y)\,R'(d)}{\beta\,y\,\underset{-}{p''(y)}\,\underset{+}{R(d)} - [y\,\underset{-}{(2\,p'(y) + y\,p''(y))}]\,\underset{+}{[p(y) + \beta\,(1-p(y))]^2}}.$$

Again, if β is small, then $\beta\,y\,p''(y)$ becomes small and the denominator is positive. Because $R'(d) \neq 0$, the derivative y_d can vanish only if $\beta = 0$. ■

Figure A2 displays the function $y(d)$ for different choices of β.

Proof of proposition 3: We first show that c. p., volume d increases as β increases. The equation determining d implicitly is

$$\begin{aligned} 0 &= y\,(p(y) + \beta\,(1-p(y))) + R(d) + d\,R'(d) \\ 0 &= y\,(1-p(y)) + d_\beta\,(2\,R'(d) + R''(d)) \\ d_\beta &= \frac{y\,(1-p(y))}{2\,R'(d) + d\,R''(d)} > 0. \end{aligned}$$

Next, we show that y also rises as β rises. The equation determining y is (A15),

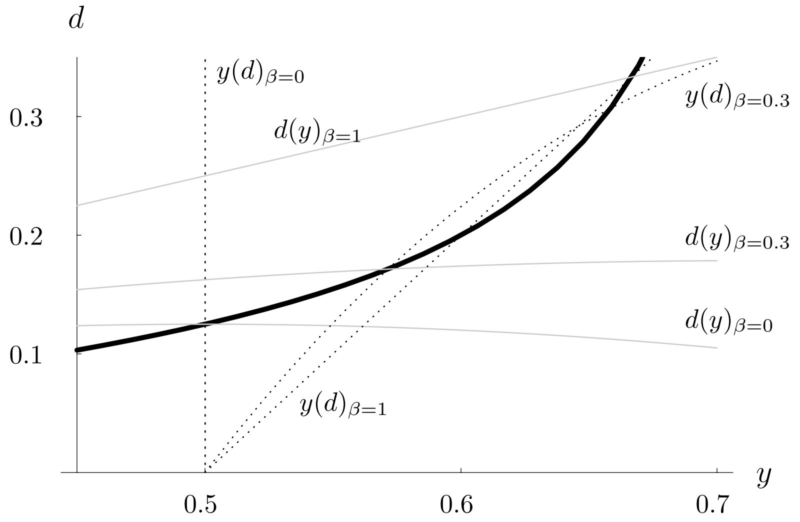
$$\begin{aligned} 0 &= [p(y) + y\,p'(y)]\,[p(y) + \beta\,(1-p(y))]^2 - \beta\,p'(y)\,R(d), \\ 0 &= y_\beta\,[2\,p'(y) + y\,p''(y)]\,[p(y) + \beta\,(1-p(y))]^2 \\ &\quad + [p(y) + y\,p'(y)]\,2\,[p(y) + \beta\,(1-p(y))]\,[1 - p(y) + y_\beta\,(1-\beta)\,p'(y)] \\ &\quad - R(d)\,p'(y). \end{aligned} \tag{A17}$$

If β is not too large, then $p'(y) + y\,p(y) \approx 0$, and (A17) simplifies to

$$0 = y_\beta\,[y^2\,p''(y) - 2\,p(y)]\,[p(y) + \beta\,(1-p(y))] - [y\,\beta\,y_\beta\,p''(y) - p(y)]\,R(d)$$

$$y_\beta = -\frac{p(y)\,R(d)}{\underset{-}{[y^2\,p''(y) - 2\,p(y)]}\,\underset{+}{[p(y) + \beta\,(1-p(y))]} - \underset{-}{y\,\beta\,p''(y)\,R(d)}}. \tag{A18}$$

Figure A2: Risk level and deposit volume for varying β in the monopoly



Dotted lines refer to $y(d)$, gray lines to $d(y)$ for $\beta = 0$, 0.3 and 1. Equilibria (i. e., intersections of curves pertaining to the same β) are marked by the solid black curve. As predicted in lemma 4, $d(y)$ rises with y near the equilibrium; it is locally constant only for $\beta = 0$. $y(d)$ rises with d, it is constant even globally if $\beta = 0$. Furthermore, the curve $d(y)$ moves up when β rises. The curve $y(d)$ moves to the right for rising β if β is small, but bends back to the left for large d. Still, the curve describing the equilibria is monotonous, i. e., $y_\beta > 0$ and $d_\beta > 0$.

Here, $\partial^2(y\,p(y))/\partial y^2 = 2\,p'(y) + y\,p''(y) < 0$, otherwise there is no solution to the first-best problem. Therefore, $-2\,p(y)/y + y\,p''(y) < 0$, and hence $y^2\,p''(y) - 2\,p(y) < 0$. Because of the assumption that β is small, the first addend in the denominator dominates the second. As a result, $y_\beta > 0$.

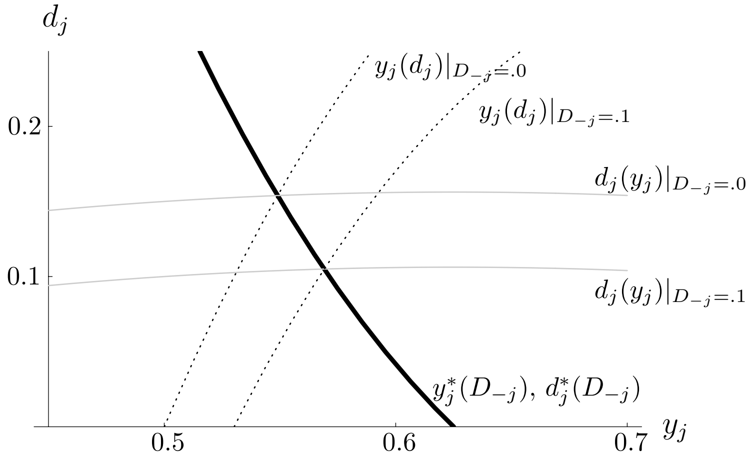
Summing up, the increase in β leads directly to a rise in y and d. Because both y_d and d_y are positive (cf. lemma 4 and 5), there is a multiplier effect into the same direction. Now look at the effect on ρ. Clearly, ρ falls if $p(y) + \beta\,(1 - p(y))$ rises. Therefore, examine

$$\begin{aligned}\frac{\partial(p(y) + \beta\,(1 - p(y)))}{\partial\beta} &= y_\beta\,p'(y) + (1 - p(y)) - \beta\,y_\beta\,p'(y)\\ &= 1 - p(y) + (1 - \beta)\,y_\beta\,p'(y).\end{aligned}$$

Now incorporate (A18) and consider $p(y) + y\,p'(y) \approx 0$, then the term becomes

$$\begin{aligned}&1 - p(y) + (1 - \beta)\,\frac{p(y)^2\,R(d)}{[y^2\,p''(y) - 2\,p(y)]\,[p(y) + \beta\,(1 - p(y))] - y\,\beta\,p''(y)\,R(d)}\\ &\overset{\beta\approx 0}{\approx} 1 - p(y) + \frac{R(d)}{y}\,\underset{-}{(y^2\,p''(y) - 2\,p(y))}.\end{aligned}$$

Figure A3: Relation between volume and risk, varying D_{-j}



This figure is the analogy to figure A1. Dashed lines show the optimal $d_j(y_j)$, gray lines show $y_j(d_j)$ for $\beta_j = 0.2$ and $D_{-j} = 0$ and 0.1. Equilibria (i. e. intersections of curves pertaining to the same d_1) are marked by the solid black curve.

This is positive whenever $R(d)$ is small enough,

$$R(d) < y\,(1 - p(y))\,(2\,p(y) - y^2\,p''(y)). \tag{A19}$$

Note that this is the condition for the case that β is small, hence that y is close to the first-best case. As β becomes larger, (A19) may be overly strict. In numerical calculations (e. g., in the example $p(y) = 1 - y$ and $R(d) = d$), ρ rises with β globally, even when (A19) does not hold. ■

A.2.2 Oligopoly

Proof of lemma 6: The proof falls into three steps, as becomes clear from figure A3. We first show that if D_{-j} rises, the curve $d_j(y_j)$ moves downwards. Second, under somewhat stricter conditions, the curve $y_j(d_j)$ moves upwards. Finally, we look at the contemporaneous reaction of the optimal y and d.

Step 1. We claim that if the competitors' deposit volume D_{-j} rises, the curve defining the optimal deposit volume d_j of bank j given the risk choice y_j moves downwards ($\partial d_j(y_j)/\partial D_{-j} < 0$).

The equation that implicitly determines the optimal d_j given y_j and D_{-j} is

$$\frac{\partial \Pi_j}{\partial d_j} = 0 = y_j\,\big(p(y_j) + \beta_j\,(1 - p(y_j))\big) + R(d_j + D_{-j}) + d_j\,R'(d_j + D_{-j}),. \tag{A20}$$

Taking the derivative with regard to D_{-j}, considering that d_j is a function of D_{-j} and then solving for $\partial d_j(y_j)/\partial D_{-j}$ yields

$$\begin{aligned} 0 &= \Big(1 + 2\,\frac{\partial d_j}{\partial D_{-j}}\Big)\,R'(D) + \Big(1 + \frac{\partial d_j}{\partial D_{-j}}\Big)\,d_j\,R''(D) \\ \frac{\partial d_i}{\partial D_{-j}} &= -\frac{R'(D) + d_j\,R''(D)}{2\,R'(D) + d_j\,R''(D)}. \end{aligned}$$

Numerator and denominator are positive, hence the derivative $\partial d_j(y_j)/\partial D_{-j}$ is negative. $\square$

Step 2. Assume that the bail-out probability of bank j is not too large ($\beta_j \approx 0$). Then we claim that if the competitors' deposit volume D_{-j} rises, the curve defining the optimal level of risk y_j of bank j given the risk choice y_j moves upwards ($\partial y_j(d_j)/\partial D_{-j} > 0$).

The equation that implicitly determines the optimal y_j given d_j and D_{-j} is

$$\frac{\partial \Pi_j}{\partial y_j} = 0 = \big(p(y_j) + y_j\,p'(y_j)\big)\,\big(p(y_j) + \beta_j\,(1 - p(y_j))\big)^2 - \beta_j\,p'(y_j)\,R(d_j + D_{-j}). \tag{A21}$$

Taking the derivative with regard to D_{-j}, considering that y_j is a function of D_{-j} and then solving for $\partial y_j(d_j)/\partial D_{-j}$ and using $p'(y_j) \approx -p(y_j)/y_j$ yields

$$\begin{aligned} \frac{\partial y_j(d_j)}{\partial D_{-j}} &= \frac{\beta_j\,p'(y_j)\,R'(D)}{[p(y_j) + \beta_j\,(1 - p(y_j))]^2\,[2\,p'(y_j) + y_j\,p''(y_j)] - \beta_j\,p''(y_j)\,R'(D)} \\ &\stackrel{\beta_j \approx 0}{\approx} \beta_j\,\frac{\overset{-}{p'(y_j)}\,\overset{+}{R'(D)}}{p(y_j)^2\,[2\,p'(y_j) + y_j\,p''(y_j)]}. \end{aligned}$$

The whole fraction is positive whenever $2\,p'(y_j) + y_j\,p''(y_j)$ is negative. Now $p'(y_j) + y_j\,p''(y_j) < 0$ for the second-order condition (global assumption), and $p'(y_j) < 0$ per assumption. Therefore, the derivative $\partial y_j(d_j)/\partial D_{-j}$ is positive. $\square$

Step 3. Finally, we must look at the equations (A20) and (A21) defining y_j and d_j simultaneously. Generally, if we have two equations $F(y, d, \epsilon = 0)$ and $\tilde{F}(y, d, \epsilon) = 0$ that implicitly define a dependence $y(\epsilon)$ and $d(\epsilon)$, then the implicit function theorem implies that

$$\begin{aligned} \frac{\partial y}{\partial \epsilon} &= \frac{F_d\,\tilde{F}_\epsilon - F_\epsilon\,\tilde{F}_d}{F_y\,\tilde{F}_d - F_d\,\tilde{F}_y} \quad \text{and} \\ \frac{\partial d}{\partial \epsilon} &= \frac{F_y\,\tilde{F}_\epsilon - F_\epsilon\,\tilde{F}_y}{F_d\,\tilde{F}_y - F_y\,\tilde{F}_d}. \end{aligned}$$

Applied to equations (A20) and (A21), we get the following,

$$\frac{\partial y_j}{\partial D_{-j}} = \frac{\beta_j R'(D)}{(1-\beta_j)\big([11\beta_j+8(1-\beta_j)p(y_j)]p(y_j)+[3\beta_j+4(1-\beta_j)\,p(y_j)]y_j p'(y_j)\big)+3\beta_j^2}$$
$$\frac{\partial d_j}{\partial D_{-j}} = -\frac{(1-\beta_j)\big([5\beta_j+4(1-\beta_j)p(y_j)]p(y_j)+[\beta_j+2(1-\beta_j)p(y_j)]y_j p'(y_j)\big)+\beta_j^2}{(1-\beta_j)\big([11\beta_j+8(1-\beta_j)p(y_j)]p(y_j)+[3\beta_j+4(1-\beta_j)p(y_j)]y_j p'(y_j)\big)+3\beta_j^2}.$$

Because β_j is small, y_j is close to the first-best, hence $p(y_j) + y_j\, p'(y_j) \approx 0$. We get

$$\frac{\partial y_j}{\partial D_{-j}} \approx \frac{\beta_j\, R'(D)}{4\,(1-\beta_j)\,[2\,\beta_j + (1-\beta_j)\,p(y_j)]\,p(y_j) + 3\,\beta_j^2} \quad \text{and}$$
$$\frac{\partial d_j}{\partial D_{-j}} \approx -\frac{2\,(1-\beta_j)\,[2\,\beta_j + (1-\beta_j)\,p(y_j)]\,p(y_j) + \beta_j^2}{4\,(1-\beta_j\,[2\,\beta_j + (1-\beta_j)\,p(y_j)]\,p(y_j) + 3\,\beta_j^2}.$$

Finally, we can also use the approximation $\beta_j \approx 0$, hence

$$\frac{\partial y_j}{\partial D_{-j}} \approx \beta_j\,\frac{R(D)}{4\,p(y_j)^2} > 0 \quad \text{and}$$
$$\frac{\partial d_j}{\partial D_{-j}} \approx -\frac{2\,p(y_j)^2}{4\,p(y_j)^2} = -\frac{1}{2} < 0. \tag{A22}$$

This was to be shown. ■

Proof of proposition 4: The proof draws heavily on proposition 3 and lemma 6. Because of (A22), competitors react to an expansion of d_1 by contracting their d_j by half the expansion of d_1. A new equilibrium emerges with increased d_1, decreased d_j (for all j) and increased D. Hence from the view of bank 1, the interest rate on the deposit market is described by a function $\tilde{R}(d_1)$ instead of $R(D)$. $\tilde{R}(d_1)$ still has a positive slope. As a result, the statements about bank 1 from proposition 4 follow directly from proposition 3 by replacing $R(d)$ with $\tilde{R}(d)$ in the monopoly case.

From the view of the competitor banks, an increase in β_1 is equivalent to a rise in the volume of one of their competitor banks. Therefore, the reaction of d_j and y_j are already described by lemma 6. The reaction of ρ_j is obvious. Because β_j remains constant, the risk level y_j rises, the default premium ρ_j must also rise. ■

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