Firm Dynamics and Aggregate Volatility with Endogenously Segmented Credit Markets

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Abstract

Recent empirical studies show that credit market segmentation changes in response to interest rate shocks: while large, bond-financed firms issue debt counter-cyclically, small, bank-dependent firms do not. We provide a unified quantitative account of these facts within a stochastic dynamic general equilibrium model of firm dynamics, where the size and distribution of firms and the segmentation of credit markets are jointly determined. Firms finance investment through optimal long-term imperfectly enforceable contracts with either banks or bond-holders. We develop a novel search-theoretic approach to this dynamic security design problem, which emphasizes the imperfect substitutability between bank and bond debt: although banks effectively relax firms' borrowing constraints, their lending ability is limited by the occasionally binding capital adequacy constraints imposed by prudential regulation. We derive a recursive characterization of the resulting dynamic trade-off between the relative tightness of borrowers' and lenders' constraints and show that bank and bond financing coexist in equilibrium. Through plausibly calibrated numerical solutions we study the characteristics of the endogenous distribution of firms over different sources of financing. At the micro level, we find that (i) small firms rely exclusively on bank debt, and (ii) firms shift from bank to bond debt over their life-cycle. This segmentation is consistent with stylized facts. At the macro level, adverse interest rate shocks lower the expected value of bank financing as capital adequacy constraints bind. In response, only the marginal, relatively larger firms optimally switch into bond financing. The resulting (constrained-efficient) fluctuations in credit market segmentation are shown to be a key determinant of aggregate investment and output volatility.

1 Introduction

Does the macroeconomic transmission of aggregate shocks depend on the characteristics of the financial sector? This paper articulates an affirmative answer to this classical question. We develop a general equilibrium model where the size and distribution of firms and the structure of financial markets are jointly determined. Firms have access to heterogeneous sources of financing. In particular, they issue different types of securities, i.e. private (bank) and public (bond) debt, which are imperfectly substitutable due to two factors: (i) managers have different incentives to default on their contracts with different financiers, which endogenously constrains their ability to sign contracts with either banks or bond holders; (ii) banks have to satisfy capital adequacy requirements imposed by prudential regulation, which endogenously constrains their ability to supply capital. As a result, at every point of time one observes a non-trivial distribution of firms across different sources of financing. Aggregate shocks generate fluctuations in security issues as banks' capital adequacy constraints occasionally bind. Some firms optimally substitute between bank loans and bonds while others are endogenously constrained in their ability to do so. This results in (constrained-efficient) fluctuations in the aggregate composition of financing which are shown to be consistent with a number of empirically documented regularities of debt structure.

To formalize the link between firm size and source of financing, we embed a dynamic multiple security design problem along the lines of the classical search-theoretic approach of Lucas and Prescott (1974) and Hopenhayn (1992)[1] into a general equilibrium model of firm growth with limited commitment. Firms' decision to issue different types of securities, i.e. private (bank) versus public (bond) debt, is modeled explicitly as a dynamic choice between long-term imperfectly enforceable contracting regimes. To model the trade-off between bank and bond financing we build on recent advances in the microeconomics of banking (see Freixas and Rochet (1996) for a detailed survey). In particular, bonds are an imperfect substitute for bank debt, since, while enjoying higher protection of their cash flow rights than bond holders, banks face occasionally binding capital adequacy constraints imposed by prudential regulation.

Under imperfect enforceability of financial contracts, the relatively higher (constrained-) ef-

[1]Gomes, Greenwood and Rebelo (2001) and Alvarez and Veracierto (2001) have constructed models of equilibrium unemployment of this class.

ficiency of banks creates a role for bank lending in fostering firm growth since access to bank financing effectively relaxes firms' borrowing constraints. Key to the existence of a non-trivial trade-off between bank and market financing is the fact that banks are subject to prudential regulation of the type introduced by the Basel Accord. In particular, they face occasionally binding capital adequacy constraints which determine an endogenous spread between bank lending and bonds. In the model, borrowing constraints are typically tighter for small firms which, consequently, are those that benefit most from bank financing and are willing to pay the spread. Hence, consistent with the empirical evidence (Cantillo and Wright (2000), Faulkender (2003), Petersen and Rajan (1994)), the model predicts that small firms borrow from financial intermediaries, while large firms borrow from public markets. An important corollary is that the size distribution of firms is a key determinant of the aggregate composition of financing. Moreover, corporate governance institutions that limit (foster) banks' protection, as observed in Anglo-Saxon (Continental Europe) systems, results in a market (bank)-based financial system. This result is a straightforward implication of the trade-off between bank and bonds: as banks are less capable of preventing managerial misconduct, ex-ante the benefits of bank lending are reduced, which induces more firms to switch to the cheaper bond financing.

Since enforcement problems prevent firms from easily substituting between bank and market debt, firms that do not have access to public debt markets are exposed to the risk that banks may face binding capital adequacy constraints. Plausibly calibrated numerical solutions reveal that due to this fact, capital adequacy requirements have sizable consequences for aggregate welfare even if firms are allowed to substitute bond for bank financing. The model is simulated to gain further insight into the quantitative implications of this fact and, in particular, to explore whether the equilibrium co-existence of bank and market lending can help us to rationalize documented key features of macroeconomic transmission of aggregate shocks, both at the micro and macro level. Two sources of aggregate uncertainty are considered: (i) technology shocks which change managers' outside options and affect managerial incentives to default; (ii) interest rate shocks, which change the endogenous spread between bank and market financing and affect banks' ability to provide capital.

We find that the arrival of a new technology induces higher aggregate volatility in market-based than in bank-based financial systems. To build intuition for this result, it is useful to consider that in the model banks are more (constrained-) efficient providers of finances than markets. The arrival of a new highly productive technology increases the value of managers' outside options. Both banks and bond holders will have to advance more capital to managers to prevent them from defaulting. Crucially, given the increase in managers' outside options, banks will have to advance relatively less capital than bond holders due to their higher efficiency. This, together with the fact that some firms are constrained in their ability to substitute bond for bank financing, will result in lower volatility the higher the proportion of firms financed by banks. This result is interesting from a substantive perspective since it lends formal support to a commonly heard argument about the role of corporate governance institutions in funneling the investment boom of the 90's in the US. Moreover, the finding that the aggregate composition of financing and not the overall amount of financing is a key determinant of aggregate volatility sharply contrasts with the previous literature which has looked entirely at the effect of shocks on the overall amount of debt (Cooley, Marimon, and Quadrini (2001); Bernanke, Gertler, and Gilchrist (1998)). A more articulated "corporate governance view" of the role of the financial sector in the transmission of aggregate shocks naturally emerges.

Finally, and perhaps most importantly, the model is consistent with key regularities of the macroeconomic transmission of interest rate shocks and implies that interest rate fluctuations induce asymmetric, ample, and persistent responses of output. The banking sector and the imperfect substitutability of bank and bond financing are key to these results. At the micro level, consistent with recent evidence (Gertler and Gilchrist (1994), Perez-Quiros and Timmermann (1998), Kashyap, Stein, and Wilcox (1993)), firms switch out of bank loans into commercial paper when bank spreads increase. Moreover, the increase in bank spreads arises endogenously as a result of interest rate shocks since when interest rates increase more banks face binding capital adequacy constraints. While marginal firms, that is, relatively large and unconstrained firms, can opt out of bank financing into bond financing, smaller firms, who are constrained in their ability to substitute between sources of financing, bear the consequences of the tightening most. At the macro level, the model predicts that an important response to a negative shock to the interest rate is a surge in commercial paper issuance. Moreover, the inability of small constrained firms to move from one debt market to others (namely from the private debt markets to the public debt markets), amplifies the impact of interest rate shocks on aggregate investment and output. It also opens up a set of intriguing implications for the cyclical properties of prudential regulation.

At the methodological level, the contribution of the paper is twofold: first, techniques developed in the recursive contracting literature (Marcet and Marimon (1999), Spear and Srivastava (1987)) are extended to study dynamic contracting with double-sided limited commitment that arises when banks are subject to capital adequacy constraints. The paper shows that the homogeneity property of the value function can be usefully employed to circumvent the curse of dimensionality which would result from applying standard methods. Moreover, the dynamic contracting approach is extended along a second critical dimension to encompass the case of multiple security issue. Both these extensions enable the study of the macroeconomic implications of firms' heterogeneity across sources of financing.

Prudential regulation is only one example of a broader set of factors that make outside capital more costly for banks than for markets. Not only are intermediation costs large (in rich countries such as the US the spread between average borrowing and average lending rates is about 5 percent), but also large amounts of resources are used in financial intermediation (in the United States the total product of the financial sector, i.e. the resources used, are approximately 9 percent of GNP). This paper considers both exogenous and endogenous specifications of these costs. Regardless of the specific nature of intermediation costs, the model robustly predicts that smaller firms are willing to pay them since they benefit most from the amelioration of incentives, while larger firms can avoid this cost and raise more capital.

Related literature In the model bank debt has an incentive advantage over bonds since banks enjoy higher protection of their cash flow rights. This special role of banks as "delegated monitors" (e.g. Diamond (1991)) is well recognized in the microeconomic literature on banking (for a survey see Freixas and Rochet (1996), Bhattacharya and Thakor (1993)) and has been documented in the empirical literature (James (1987)). More in general, Hoshi, Kashyap and Scharfstein (1990a, 1990b), Petersen and Rajan (1994), and Berger and Udell (1995) confirm the importance of lending relationships in relaxing credit constraints.

By identifying the type of firms that have access to the public bond market, this paper can address several issues eluded by the previous literature which has mostly focused on why some types of lenders (active monitors such as banks) developed to cater to certain types of firms

(Diamond (1991)). Thus far, however, there has been little work on firms' choice of the source of capital. At the micro level the model provides an empirically realistic (see Cantillo and Wright (2000)) account of firms' choice of the source of capital, hence contributing to understanding how firms and lenders are matched. Further, in our model the source of firms' debt, i.e. whether they have access to public debt markets, has a strong influence on their capital choices. This enable us to explore, to our knowledge for the first time, the macroeconomic implications of frictions that limit firms' access to capital markets.

There is an extensive literature focusing on the importance of financial factors on the investment behavior of firms (see, for example, Hubbard (1998)) and on the problem of debt renegotiation (see, for example, Hart and Moore (1998)). Important contributions embed financial frictions in a general equilibrium framework (Bernanke, Gertler and Gilchrist (1998), Carlstrom and Fuerst (1997), DenHaan, Ramey and Watson (1998), Kiyotaki and Moore (1997), Smith and Wang (1999)). However, in most of these attempts, firms heterogeneity is either exogenous or it does not play an important role. By contrast, firm heterogeneity both over capital sizes and types of financing is central to the present work.

A recent literature employs dynamic contracting methods to study the aggregate implications of limited contract enforceability (Cooley, Marimon and Quadrini (2001); Castro, Clementi, MacDonald (2003); see Albuquerque and Hopenhayn (2001) and DeMarzo and Fishman (2002, 2003) for microfoundations). Our work builds on this literature and embeds a dynamic security design problem into an equilibrium model of firm dynamics with limited enforcement and aggregate uncertainty. By formalizing the choice of the source of financing we are able to identify a novel transmission mechanism of aggregate shocks which relies on cyclical changes in the aggregate composition of financing. Monge (1999) studies the transmission of interest rate shocks through the cross-section of active firms. He does not allow for different sources of financing besides bank lending and stresses fluctuations in entry and exit which we rule out by assuming exogenous entry and exit rates.

Holmstrom and Tirole (1997), Repullo and Suarez (1998), Bolton and Freixas (2000, 2001), Besanko and Kanatas (1993), Boot and Thakor (1997) characterize the equilibrium co-existence of bank lending and market financing through securities issues. This literature does not entertain a full fledged dynamic general equilibrium analysis of the type pursued here. Moreover, the present model complements these studies as it considers the effect of the aggregate state of the economy on the co-existence of bank lending and market financing.

A final set of related papers investigates the consequences of capital adequacy requirements both at the micro and the macro level (Kim and Santomero (1988), Furlong and Keeley (1989), Gennotte and Pyle (1991), Rochet (1992) and Besanko and Kanatas (1996), Hellman, Murdock and Stiglitz (1998, 2000), Van den Heuvel (2001)).

Outline of the paper The paper is organized as follows: the first section details a set of stylized facts of the determinants of firms' sources of capital. The second presents the setup of the model. The third defines and characterizes the optimal choice of the financing mode. The fourth defines recursive competitive equilibrium and applies Brower-Schauder-Tychonoff fixed-point theorem to prove the existence of a competitive equilibrium both with and without aggregate shocks. The fifth section describes the cross-sectional implications of the model and both the steady state and cyclical properties of aggregate quantities. Impulse response functions to aggregate technology and interest rate shocks under alternative endogenously determined financial systems are presented. The final section summarizes and concludes.

2 Stylized facts

This section details a set of stylized facts of firms' sources of capital. In particular, it reviews some salient cross-sectional, business cycle, and cross-country features of firms' choice of financing mode that have been documented in the empirical literature.

2.1 Micro level facts:

Size and age patterns:

- The composition of bank finance and direct finance varies across firms: bond financing is found predominantly in mature and relatively safe firms whereas bank finance and equity are the main source of funding for start-up firms and risky ventures (Petersen and Rajan, 1994, 1995; Zarutskie, 2003). In general, the bigger the firm, the bigger the share of securities in its financial structure. This intuition is the basis for the empirical literature which has examined firms choice of lender. Smaller firms are most likely to borrow from financial intermediaries (Cantillo and Wright, 2000, Faulkender, 2003, Petersen and Rajan, 1994). Larger firms are more likely to borrow from arms length capital markets. Firms with a debt rating are appreciably larger. Whether we examine the book value of assets, the market value of assets, or sales, firms with a debt rating are about 300 percent larger (difference in natural logs) than firms without a debt rating (Faulkender and Petersen, 2003).

- The average bank relationship lasts between 7 and 30 years (Ongena and Smith, 1997). Older firms are more likely to have access to the public bond markets. The variable with the largest economic impact is the size of the firm. Raising the market value of the firm's assets from the 25th percentile to 75th percentile, raises the probability of having a bond rating by 26 percentage points (from 3 to 29%). The transformation of the Japanese economy in the eighties provides further support to this facts: the lending and financing markets became more competitive. The greater competition shifted the source of financing for Japanese firms away from the banks and toward the public debt markets (Hoshi, Kashyap and Scharfstein, 1990). The strongest firms financially were the first to reduce their bank borrowing and instead issue public debt.

- Firms with relationships to single banks tend to use less bank debt in proportion to total debt as their market to book ratio increases (Houston and James, 1995).

- Most of the debt of public firms is public debt. Despite the large aggregate size of the market, however, public debt is a relatively rare source of capital for most firms.

- As an extension of Albuquerque and Hopenhayn (2001) and Cooley, Marimon, and Quadrini (2001), the model also replicates documented size and age patterns of firm life-cycle, i.e. the fact that mean growth rates of firms (and their variance) decrease with their initial size and firm size increases with age.

Bank relationships have the greatest effect on the provision of credit as opposed to the price at which firms are able to borrow (Petersen and Rajan, 1994). While the evidence that relationships are valuable in the sense they lower a firms borrowing costs is weak, controlling for characteristics of the firm, the effect of building a relation with a lender has a very large effect on these small firms access to capital. Firms with longer relationships are less credit constrained. In fact, the coefficient on the length of the lending relationship is even larger than the coefficient on the age of the firm, although they are close in size. Thus firms which are building relationships find their credit constraints are shrinking more than twice as fast as those that are not. Firms whose lenders are more informed are also less capital constrained. Firms which purchase other financial services from their lender are significantly less constrained when compared to those firms which do not. As firms increase the fraction of their debt they borrow from a lender who provides them with financial services, they pay less of their trade credit late. This effect becomes stronger as the firm purchases additional informational services from their lender. Finally, credit availability for firms in more geographically concentrated banking markets is significantly higher. A firm in the most concentrated area reduces late payments by almost five percentage points when compared to a firm in the most competitive area. Concentration of the local financial market, however, has a small and statistically insignificant effect on the price of credit.

The source of capital affects capital structure (Faulkender and Petersen, 2003). In particular, firms which have a debt rating are clearly different from firms which do not (e.g. Titman, and Wessels, 1988, Barclay and Smith, 1995b, Graham, 1996, Graham, Lemmon, and Schallheim, 1998, Hovakimain, Opler, and Titman, 2001). Faulkender and Petersen (2003) find that even among firms which have access to public capital markets, i.e. that are publicly traded, and thus facing extensive disclosure requirements, not all firms are be able to choose the source of their debt capital. Firms which have access to the public debt markets (defined as having a debt rating) have leverage ratios which are more than fifty percent higher than firms which do not have access (28.4 versus 17.9 percent). For firms of equal size, a debt rating increases the firms debt by 59 percent.

2.2 Macro level facts:

Capital markets are segmented. Typically the bond market co-exists with financial intermediaries (e.g. Allen and Gale (2001), Demirguc-Kunt and Levine (2001)).

Aggregate shocks change the composition of financing at the firm level:

- Firms that exhibit low degrees of financial constraints issue debt counter-cyclically and issue equity pro-cyclically. Meanwhile, firms that exhibit higher degrees of financial constraints do not exhibit these pronounced debt issue patterns. Specifically, Choe, Masulis and Nanda (1993) document that seasoned primary equity issues are pro-cyclical and debt issues are counter-cyclical. Gertler and Gilchrist (1994) document that net short term debt issues are flatter over the business cycle for small firms. After correcting for time-variation in firm characteristics, Korajczyk and Levy (2002) document that: (1) macroeconomic conditions account for a substantial portion of the observed counter-cyclical leverage patterns for firms that they classify as financially unconstrained, (2) their constrained sample does not exhibit counter-cyclical leverage patterns.
- hot issue markets, i.e. the volume of new equity issues displays a markedly cyclical pattern (Jaffee and Ibbotson, 1975; Bayless and Chaplinsky (1996)).

Shocks to the banking sector change the aggregate composition of financing (Kashyap and Stein, 1994):

- Small firms or firms with less access to financial markets are more sensitive to changes in interest rates than their larger or less constrained counterparts (Gertler and Gilchrist, 1994; Perez-Quiros and Timmermann, 1998). Lamont, Polk, and Saa-Requejo (1998) confirm these findings: small firms have stock returns that are more pro-cyclical and more correlated with monetary policy, interest rates, and innovations in future economic activity. Nevertheless, their constrained firms are never significantly more sensitive to macroeconomic conditions than unconstrained firms. Finally, Slovin, Sushka, and Poloncheck (1993) shocks to banks' capital, which are independent of firms demand for capital, affect firms financing.

- An important response to a monetary tightening is a surge in commercial paper issuance. Firms switch out of bank loans into commercial paper (or other forms of securitized financing) when bank spreads increase (Kashyap, Stein, and Wilcox, 1993). Gertler and Gilchrist (1994) and Oliner and Rudebusch (1995) find that the main factor behind this surge in commercial paper issuance is inventory build-up by large firms financed by commercial paper issuance. Small firms do not rely on commercial paper issues at all. Moreover, these firms bear the brunt of monetary tightening: "the main effect of a monetary contraction is to shift financing of all types from small firms to large firms. This shift produces a decline in the aggregate bank-loan share because large firms rely less heavily on bank debt than do small firms." (Oliner and Rudebusch, 1995)

2.3 Cross-country facts:

The proportion of investment financed externally by firms cannot be explained by the substantial differences we observe in the legal systems and financial institutions across countries (Beck, Demirgüç-Kunt and Maksimovic, 2002). Firms in less developed systems substitute alternative forms of external financing for those used more prevalently in developed countries. Hence, financial and legal institutions significantly affect the type of external financing that firms obtain and they affect different sources of finance differently. See Levine (2002) for a contrasting perspective.

Bank financing is relatively larger in Continental Europe and Japan while in the US firms rely more on bond and equity financing (Allen and Gale, 2001; Domowitz, Glen, and Madhavan, 2001). The US and UK display a lower average size of entrants than other countries (OECD) and a higher average size of firms. Finally, the US and UK have displayed higher aggregate volatilities in the past two decades than Japan and countries in Continental Europe. Using average values over the past two decades, Figure 1 shows that both within the financially developed and underdeveloped world aggregate volatility has been markedly different across countries with bank-based and market-based financial systems[2].

[2] The GDP growth correlation for both groups is about 0.62. The index is taken from Demirguc-Kunt and Levine (2001).

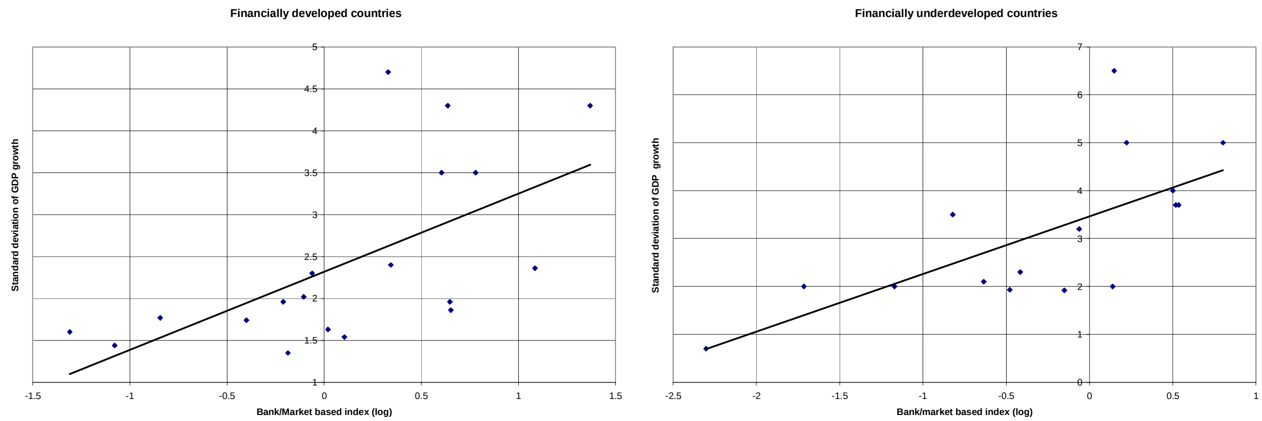

Figure 1: Financial systems and aggregate volatility

Figure 1 also makes apparent that countries with more developed financial systems are more market-based (Domowitz, Glen and Madhavan, 2001).

3 Setup

Time is discrete and the horizon is infinite. There are three types of risk-neutral agents: worker-investors, owner-managers, and banks. Consumer-worker/savers are infinitely lived and have mass m. By contrast, owner-managers and banks face a positive death probability equal to α. A fraction e of the newborn agents are owner-managers, in that they operate their own productive project, or firm, and produce output. The remaining fraction, $1-e$, are banks, in that they rent funds from the bond market and lend them to the productive units. In each period, owner-managers can choose to finance their investment by borrowing either from a bank or directly from the bond market. There is a single homogeneous final consumption good produced. This section starts by describing the technology side of the model, i.e. the characteristics of the productive projects available to owner-managers, and then turns to preferences and funding options.

3.1 Technology

Owner-managers operate a technology or investment project that, when successful, generates a positive stream of net profits or dividends, $d_t \geq 0$. A technology or investment project, at a point in time, indexed by $z \in \{z_L, z_H\}$, where z is a project-specific level of productivity (vintage), which for new entrants characterizes the state of the current technology. An agent's productivity is perfectly observable by all other agents. We introduce aggregate uncertainty by assuming that there is a large number of projects with low productivity but only a limited number N with high productivity. Given the total mass of owner-managers, e, the probability of finding a high productivity project is $p_t = \min\{N_t/e, 1\}$. The arrival of a new technology creates better investment opportunities by increasing the number of high productivity projects available, that is N_t. It is assumed that this variable follows a stationary stochastic process $\Gamma(N_{t+1}, N_t)$ to be specified later. The crucial point is that expansions are driven by the arrival of more productive projects rather than the improvement of existing ones. This resembles a model with vintage capital.

Technology z, if operated using l units of labor services and k units of the capital input, produces output $zF(k,l)$. The function F is increasing, continuous and strictly concave with respect to k, $F(\cdot, 0) = F(0, \cdot) = 0$. A project requires an initial fixed set-up investment I_0, which is sunk. If a technology is not operated, the technology is lost. The input of capital is chosen one period in advance, before observing the shock. Capital is project-specific and it cannot be reallocated to a different project once invested, so that the liquidation value of capital is zero. Existing capital depreciates at the rate δ.

The period discounted expected net profits are

$$\pi_z(k_t, l_{t+1}, w_{t+1}) = \beta\{\alpha k_t + (1-\alpha)[zF(k_t, l_{t+1}) - w_{t+1}l_{t+1} + (1-\delta)k_t]\} - k_t$$

It is convenient at this stage to assume that the production function has the Leontief property, i.e. the fact that it implies a constant capital-labor ratio, ξ, to rewrite the period profits in terms of capital only as

$$\pi_z(k_t, w(\mathbf{s}_{t+1})) = \beta\left\{\alpha k_t + (1-\alpha)\left[zF(k_t) - w_{t+1}\frac{k_t}{\xi} + (1-\delta)k_t\right]\right\} - k_t \tag{1}$$

It is worthy stressing that we assume that investment is reversible and suppress the distinction between new production and undepreciated capital. Importantly, investment decisions are made prior to the observation of the productivity level and it is not possible to re-allocate capital between agents once the productivity shock has been realized. That is, production is risky.

3.2 Households

Worker-investors are infinitely lived and have identical preferences represented by the function

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left(c_t - \varphi\left(l_t\right)\right)$$

where β is the subjective discount factor and c_t and l_t denote, respectively, household consumption and hours worked, as a fraction of the total time endowment.

Households own one unit of every date labor. Their income is derived from renting labor at the competitive wage, w_t, and lending the capital they own, a_t, at the competitive interest rate, r_t. Consequently, each period they face the following budget constraint

$$c_t + a_{t+1} = w_t l_t + \left(1 + r_t\right) a_t$$

Households' choices are summarized by the following conditions

$$\begin{aligned} \varphi'\left(l_t\right) &= w_t \\ r_t &= \frac{1}{\beta} \end{aligned}$$

We consider the implications of the model both under a constant deterministic r and under an r_t which follows a stochastic process, $\Gamma\left(r_{t+1}, r_t\right)$ to be specified later.

3.3 Financial contracts and continuing participation

New owner-managers do not have assets to finance a new project. To start a new project, they sign long-term financial contracts with either banks or bond holders in exchange for a stream of transfers, d_s. The choice between bonds and bank financing is kept at its essentials by not allowing firms to issue bonds and make loans simultaneously.

The main distinguishing features of these financial instruments are the following:

Bond financing: the owner-manager can issue bonds by signing a long-term contract with bond holders. The contract specifies a repayment schedule to bond holders, $\{\tau(\mathbf{s}_t)\}_{t=0}^{\infty}$, a stream of transfers to the owner-manager, $\{d(\mathbf{s}_t)\}_{t=0}^{\infty}$, and a stream of capital advancements, $\{k(\mathbf{s}_t)\}_{t=0}^{\infty}$. Moreover, they are not perfectly enforceable, since the owner-manager will keep his promises only as far as the following continuing participation constraint is satisfied

$$E_{s+1} \sum_{j=s+1}^{\infty} \beta^{j-s-1} d_j \geq D_z(k_s, \mathbf{s}_{s+1}) \tag{2}$$

where $D_z(\mathbf{s}, k) = \sigma z F(k, l) + V_0(\mathbf{s}) - \kappa$ is the total repudiation value for the owner-manager and κ is the fixed component of such repudiation cost. We assume that, if a contract with bond holders is repudiated, the owner-manager is able to appropriate (and consume) an amount of the cash flow revenue generated by the production process in the proportion σ of output, $zF(k, l)$. Notice that we allow for $\sigma > 1$. In addition to appropriating this cash flow, he can also start a new investment project. Repudiation, however, entails a fixed cost, κ, for the owner-manager, which can be interpreted as legal penalties that reduces his utility upon default. $V_0(\mathbf{s})$ denotes the value of a new investment project (new contract) for the entrepreneur, where s are the aggregate states of the economy as will be specified later in the paper. Finally, we assume that diverted capital cannot be invested in the new project so as to eliminate the potential complication of the dependence of the initial value of the contract on capital.

Bank debt: the owner-manager can take a loan by signing a long-term contract with a bank. The contract specifies a repayment schedule to the bank, $\{\hat{\tau}(\mathbf{s}_t)\}_{t=0}^{\infty}$, a stream of transfers to the owner-manager, $\left\{\hat{d}(\mathbf{s}_t)\right\}_{t=0}^{\infty}$, and a stream of capital advancements, $\left\{\hat{k}(\mathbf{s}_t)\right\}_{t=0}^{\infty}$. Banks enjoy a relatively higher legal protection than generic bond holders. Within the present context, we operationalize this assumption by positing a proportional difference, Λ, between the values of defaulting under bank, $\hat{D}_z(\mathbf{s}, k)$, and market, $D_z(\mathbf{s}, k)$, finance, i.e.

$$\hat{D}_z(\mathbf{s}, k) = (1 - \Lambda)(\sigma z F(k, l) + V_0(\mathbf{s}) - \kappa) < D_z(\mathbf{s}, k) \tag{3}$$

where Λ can be interpreted as a proportional credit the bank carries over to the next contract. Bank lending entails a cost and in what follows we start by studying the case of an exogenous cost of bank capital and then we turn to consider fully endogenous costs that arise due to the presence of prudential regulations and the consequent need for the bank to satisfy capital adequacy constraints.

3.4 Timing

A newborn owner-manager searches for a high productivity project and a financier. Given the available project characterized by the productivity parameter z he will choose to sign a long-term contract with either a bank or a bond holder. The contract provides the funds for the initial set-up investment I_0 and the initial variable capital k_t. At the beginning of the next period then, conditional on survival, the owner-manager observes N_{t+1} and decides whether to repudiate the contract and search for a new investment project. In case of repudiation the old project is permanently lost. If instead the contract is not repudiated, the firm hires labor and production takes place. The revenue from production, net of the labor cost, are used to make payments to the owner-manager, to the financier and to finance the new capital k_{t+1}.

4 The choice of financing regimes

This section employs dynamic programming techniques to characterize the dynamic security design problem of the owner-manager. We adapt the methods developed in Marcet and Marimon (1997), to derive a recursive characterization. In particular, we use the Lagrange multipliers associated with the continuing participation constraint of the owner-manager as state variables.

4.1 Exogenous cost of bank capital

Banks' capital is more costly than markets' (Gomes, 2002). This does not appear to be an altogether unrealistic assumption in the light of a number of plausible considerations. For example, banks are likely to face liquidity and information dilution costs (Myers and Majluf (1984)) in their financing: when a bank decides to raise additional capital, the market tends to undervalue the issue for the better banks. But since it is the better banks that drive the decision whether to raise capital, the overall effect on all banks' capital issues is to reduce the amount of capital raised relative to the full information optimum. Thus, because of information asymmetries about the fundamental value of the bank, a cost of capital can be rationalized, and by implication, a cost of bank lending.

Accordingly, the period discounted expected net profits under bank financing become

$$\hat{\pi}_z\left(k_t, w\left(\mathbf{s}_{t+1}\right)\right) = \beta\left\{\alpha k_t + (1-\alpha)\left[zF\left(k_t\right) - w_{t+1}\frac{k_t}{\xi} + (1-\delta)\,k_t\right]\right\} - (1+\lambda)\,k_t \tag{4}$$

4.1.1 The optimal choice between contracts

The choice of the owner-manager between optimally contracting with bond holders or a bank can be formulated as the maximization of the expected discounted payments to the owner-manager subject to the no-default constraints and the participation constraint for the financiers:

$$\begin{aligned}
V_z^I\left(\mathbf{s}_t\right) &= \max_{\{d_s,k_s,I_s\}_{s=t}^{\infty}} E_t\sum_{s=t}^{\infty}\beta^{s-t}\left\{I_s d_s + \left(I_{s-1}-I_s\right)V_z^B\left(\mathbf{s}_s\right)\right\} \quad (5)\\
\text{s.t. } E_{s+1}\sum_{j=s+1}^{\infty}\beta^{j-s-1}d_j &\geq D_z\left(k_s, \mathbf{s}_{s+1}\right), \text{ for } s\geq t\\
E_s\sum_{j=s}^{\infty}\beta^{j-s}\left[\pi_z\left(k_s, w\left(\mathbf{s}_{s+1}\right)\right) - d_s\right] &\geq I_0\\
d_s &\geq 0
\end{aligned}$$

$$\begin{aligned}
V_z^B\left(\mathbf{s}_t\right) &= \max_{\{d_s,k_s,\hat{I}_s\}_{s=t}^{\infty}} E_t\sum_{s=t}^{\infty}\beta^{s-t}\left\{\hat{I}_s d_s + \left(\hat{I}_{s-1}-\hat{I}_s\right)V_z^I\left(\mathbf{s}_s\right)\right\} \quad (6)\\
\text{s.t. } E_{s+1}\sum_{j=s+1}^{\infty}\beta^{j-s-1}d_j &\geq \hat{D}_z\left(k_s, \mathbf{s}_{s+1}\right), \text{ for } s\geq t\\
E_s\sum_{j=s}^{\infty}\beta^{j-s}\left[\hat{\pi}_z\left(k_s, w\left(\mathbf{s}_{s+1}\right)\right) - d_s\right] &\geq I_0\\
d_s &\geq 0
\end{aligned}$$

where D_z, $\hat{D}_z$, π_z, and $\hat{\pi}_z$ are defined respectively in (2), (3), (1), and (4). The appendix shows that this problem can be equivalently written in recursive form as follows:

$$\begin{aligned} W_z^I(\mathbf{s},\mu) &= \min_{\mu'} \max_{d,k} \pi_z(k,\mathbf{s}') - (1-\mu)\, d - \beta E\left(\mu(\mathbf{s}') - \mu\right) D_z(k,\mathbf{s}') \\ &\quad + \beta E \max\left[W_z^B(\mathbf{s}',\mu(\mathbf{s}')), W_z^I(\mathbf{s}',\mu(\mathbf{s}'))\right] \\ \text{s.t. } d &\geq 0, \qquad \mu(\mathbf{s}') \geq \mu \\ \mathbf{s}' &\sim M(\mathbf{s}) \end{aligned} \tag{7}$$

$$\begin{aligned} W_z^B(\mathbf{s},\mu) &= \min_{\mu'} \max_{d,k} \hat{\pi}_z(k,\mathbf{s}') - (1-\mu)\, d - \beta E\left(\mu(\mathbf{s}') - \mu\right) \hat{D}_z(k,\mathbf{s}') \\ &\quad + \beta E \max\left[W_z^B(\mathbf{s}',\mu(\mathbf{s}')), W_z^I(\mathbf{s}',\mu(\mathbf{s}'))\right] \\ \text{s.t. } d &\geq 0, \qquad \mu(\mathbf{s}') \geq \mu, \\ \mathbf{s}' &\sim M(\mathbf{s}) \end{aligned} \tag{8}$$

and M is the distribution function for the next period aggregate states, given the current states. The aggregate state is $\mathbf{s} = (Z, \Omega)$, where Ω denotes the distribution (measure) of the fractions of firms which are bank and market financed over the variables z and μ, and Z denotes the number of new investment projects with high productivity.

To gain further insight into the characteristics of such distribution, denote the contract policies under market financing, i.e. the decision rules under (17), are $d = d^I(\mu, z; \mathbf{s})$, $k = k^I(\mu, z; \mathbf{s})$, $\mu' = \mu^I(\mu, z; \mathbf{s})$, while under bank financing, , i.e. the decision rules under (18) $d = d^B(\mu, z; \mathbf{s})$, $k = k^B(\mu, z; \mathbf{s})$, $\mu' = \mu^B(\mu, z; \mathbf{s})$. An owner-manager will choose market financing in the current period if $W_z^I(\mathbf{s},\mu) \geq W_z^B(\mathbf{s},\mu)$. Otherwise, he will choose bank financing. Let $\Phi(\mu, z; \mathbf{s})$ denote the decision rule governing whether an individual chooses market or bank financing. It is defined as

$$\Phi(\mu, z; \mathbf{s}) = \begin{cases} 1 & \text{if} \quad W_z^I(\mathbf{s},\mu) \geq W_z^B(\mathbf{s},\mu) \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

Clearly, the owner-manager's choice between financing modes is based on the comparison of the expected present discounted value of each financing regime.

Finally, an owner-manager in state $(\mu, z; \mathbf{s})$ will accumulate capital according to

$$\mu' = \mu(\mu, z; \mathbf{s}) = \Phi(\mu, z; \mathbf{s})\, \mu^I(\mu, z; \mathbf{s}) + (1 - \Phi(\mu, z; \mathbf{s}))\, \mu^B(\mu, z; \mathbf{s})$$

The Appendix contains a proof of the following

Proposition 1 *The functions $W_z^I(\mathbf{s},\mu)$ and $W_z^B(\mathbf{s},\mu)$ exist and are monotone in μ.*

Key to the argument is the observation that the operator $\left(W^{I,j+1}, W^{B,j+1}\right) = T\left(W^{I,j}, W^{B,j}\right)$ satisfies Blackwell's sufficient conditions (Stokey, Lucas, and Prescott, 1989).

Characterization Given the choice rule $\Phi(\mu, z; \mathbf{s})$, the optimal policies are given by

$$\frac{\partial \cdot}{\partial d} : \quad \mu \leq 1$$

$$\frac{\partial \cdot}{\partial \mu'} : \quad \Phi(\mu, z; \mathbf{s})\, D_z(k, \mathbf{s}') + (1 - \Phi(\mu, z; \mathbf{s}))\, \hat{D}_z(k, \mathbf{s}') \leq \frac{\partial W_z(\mathbf{s}',\mu(\mathbf{s}'))}{\partial \mu(\mathbf{s}')} \qquad (= \text{if } \mu(\mathbf{s}') > \mu)$$

$$\frac{\partial \cdot}{\partial k} : \quad \begin{array}{c} \Phi(\mu, z; \mathbf{s})\, E\left[\frac{\partial \pi_z(k,w(\mathbf{s}'))}{\partial k} - \beta(\mu(\mathbf{s}') - \mu)\frac{\partial D_z(k,\mathbf{s}')}{\partial k}\right] \\ +(1 - \Phi(\mu, z; \mathbf{s}))E\left[\frac{\partial \hat{\pi}_z(k,w(\mathbf{s}'))}{\partial k} - \beta(\mu(\mathbf{s}') - \mu)\frac{\partial \hat{D}_z(k,\mathbf{s}')}{\partial k}\right] = 0 \end{array}$$

and the envelope is

$$\frac{\partial \cdot}{\partial \mu} : \quad \frac{\partial W_z(\mathbf{s},\mu)}{\partial \mu} = d + \left\{ \begin{array}{cc} \begin{array}{c} \Phi(\mu, z; \mathbf{s})\, \beta E D_z(k, \mathbf{s}') \\ +(1 - \Phi(\mu, z; \mathbf{s}))\, \beta E \hat{D}_z(k, \mathbf{s}') \end{array} & if \quad \mu(\mathbf{s}') > \mu \\ \beta E \frac{\partial W_z(\mathbf{s}',\mu)}{\partial \mu} & if \quad \mu(\mathbf{s}') = \mu \end{array} \right.$$

Non trivial life-cycle of financial contracts The Appendix contains a proof of the following

Proposition 2 *The exists a generic set of parameters values for which the function $\Phi(\mu, z; \mathbf{s})$ is nondegenerate. In particular, there exists a $\tilde{\mu}$ such that:*

1. *for all $\mu \geq \tilde{\mu}$, $W_z^I(\mathbf{s},\mu) \geq W_z^B(\mathbf{s},\mu)$;*

2. for all $\mu < \tilde{\mu}$, $W_z^B(\mathbf{s},\mu) > W_z^I(\mathbf{s},\mu)$.

The existence of a switching threshold depends crucially on the properties of the (endogenous) opportunity cost of forgone bank financing. In particular, it is the very fact that financing constraints are more binding on smaller firms to create scope for a decreasing opportunity cost of bank financing over a firm's life-cycle.

This proposition is crucial to identify who which firms borrow from banks and which from markets. In particular, even under an exogenous cost of bank capital, the model is consistent with the stylized fact that smaller firms are most likely to borrow from financial intermediaries (Cantillo and Wright, 2000, Faulkender, 2003, Petersen and Rajan, 1994).

Proposition 3 *Ceteris paribus, firms have on average higher investment and lower growth under bank financing.*

Value of a new firm The analysis of the previous section takes the relative weight assigned to the entrepreneur in a new contract signed at time t, χ_t, as given. In equilibrium this weight is endogenous and is important to close the model since it determines the value of starting a new contract and, in turn, the repudiation values $\hat{D}_z(k,\mathbf{s})$ and $D_z(k,\mathbf{s})$.

In fact, the value of searching for an investment project is

$$
\begin{aligned}
V_0^I(\mathbf{s}) &= pV_{z_H}^I\left(\mathbf{s},\lambda_{z_H}^I(\mathbf{s})\right) + (1-p)\,V_{z_L}^I\left(\mathbf{s},\lambda_{z_L}^I(\mathbf{s})\right) \\
V_0^B(\mathbf{s}) &= pV_{z_H}^B\left(\mathbf{s},\lambda_{z_H}^B(\mathbf{s})\right) + (1-p)\,V_{z_L}^B\left(\mathbf{s},\lambda_{z_L}^B(\mathbf{s})\right)
\end{aligned}
$$

where $V_z^I\left(\mathbf{s},\lambda_z^I(\mathbf{s})\right)$ and $V_z^B\left(\mathbf{s},\lambda_z^B(\mathbf{s})\right)$ are the end-of-period values of the market and bank financing contracts for the entrepreneur as defined in (5) and (6)and $\lambda_z^I(\mathbf{s})$ and $\lambda_z^B(\mathbf{s})$ are the Lagrange multipliers associated with the break-even condition for the market and the bank respectively.

4.2 Endogenous cost of bank capital

Each period, given the initial Lagrange multiplier, μ, and the initial state, s, the contract specifies a triple (d,τ,k),where the payments to investors are denoted by τ , the payments to the entrepreneur are denoted by d, and the next period capital input is denoted by k, and a pair of future lagrange multipliers $(\mu(\mathbf{s}'), \lambda(\mathbf{s}'))$ respectively for the entrepreneur and for the intermediary. In turn, future lagrange multipliers will dictate future investment and repayments. The aggregate state, $\mathbf{s} = (Z, \Phi)$, is given by the distribution (measure) of firms over the variables z and μ, Φ, and by the number of new investment projects with high productivity, Z. The existence of capital adequacy requirements complicates the analysis since it creates the need for an additional state variable, λ, with respect to the one-sided commitment case previously studied in the literature (Albuquerque and Hopenhayn (2001), Cooley, Marimon and Quadrini (2001)). This inevitably creates a curse of dimensionality which makes the problem potentially cumbersome to compute. We show how to circumvent this problem by using the homogeneity properties of the Lagrangian. We show that the original contract can be equivalently reformulated using "relative" lagrange multipliers.

4.2.1 Prudential regulation and capital adequacy requirements

Intermediaries must satisfy capital adequacy requirements of the following form

$$k_s \leq \frac{1}{1-\gamma} E_s \sum_{j=s}^{\infty} \beta^{j-s} \tau_j \tag{10}$$

where τ_j indicates the transfer received by the intermediary. According to this constraint, banks can only raise capital up to a given constant proportion, $\frac{1}{1-\gamma}$, of their wealth. Such proportion is typically specified by the prudential regulator. For example, the BIS capital adequacy rules set $1-\gamma = 0.08$. Finally, it is straightforward to observe that (10) can be equivalently rewritten as a limited commitment constraint as follows:

$$E_s \sum_{j=s}^{\infty} \beta^{j-s} \tau_j \geq (1-\gamma) k_s$$

In this interpretation, capital adequacy requirements, by limit the ability of banks to commit to capital advancements beyond a fixed proportion of their wealth, define an endogenous lower bound on the value of the contract for the bank.

Capital adequacy requirements can also be interpreted as collateral requirements on bank capital: at every point of time the amount of capital banks can raise is limited by the sum of their wealth and the value of their collateralizable assets, γk. Accordingly $k_s \leq E_{s+1} \sum_{j=s+1}^{\infty} \beta^{j-s-1} \tau_j + \gamma k_s$, which immediately yields (10) .

4.2.2 The contract with banks only

The contract between banks and the entrepreneur can be formulated as follows:

$$
\begin{aligned}
V_z^B(\mathbf{s}_t) &= \max_{\{d_s,\tau_s,k_s\}_{t=0}^{\infty}} E_t \sum_{s=t}^{\infty} \beta^{s-t} (\chi_t d_s + \zeta_t \tau_s) \qquad (11) \\
\text{s.t. } E_s \sum_{j=s}^{\infty} \beta^{j-s} [\hat{\pi}_z (k_s, w(\mathbf{s}_{s+1})) - d_s] &\geq I_0 \\
E_{s+1} \sum_{j=s+1}^{\infty} \beta^{j-s-1} d_j &\geq \hat{D}_z (k_s, \mathbf{s}_{s+1}) \\
E_s \sum_{j=s}^{\infty} \beta^{j-s} \tau_j &\geq (1-\gamma) k_s
\end{aligned}
$$

and $d_s \geq 0$, where $\hat{\pi}_z (k, w(\mathbf{s}'))$ is defined in (4) .

The appendix shows that using the homogeneity of the Lagrangian in the multiplier of the bank, the contract can be written in the following "intensive" recursive form using "relative" multipliers, $\mu_\lambda = \frac{\mu}{\lambda}$:

$$
\begin{aligned}
W_z^B(\mathbf{s},\mu_\lambda) &= \min_{\mu_\lambda',\gamma_\lambda} \max_{d,k} \hat{\pi}_z (k, \mathbf{s}') - (1-\mu_\lambda) d - \beta E (\mu_\lambda(\mathbf{s}') - \mu_\lambda) \hat{D}_z (k, \mathbf{s}') \\
&\quad - E((\gamma_\lambda - 1)(1-\gamma) k) + \beta \gamma_\lambda E W_z^B (\mathbf{s}',\mu_\lambda(\mathbf{s}')) \\
\text{s.t. } d &\geq 0, \qquad \mu_\lambda(\mathbf{s}') \geq \mu_\lambda, \qquad \gamma_\lambda \geq 1 \\
\mathbf{s}' &\sim H(\mathbf{s})
\end{aligned}
$$

Characterization of the contract The first order conditions characterize the dynamic features of a firm induced by an optimal financial contract. They are given by:

$$\begin{aligned}
\frac{\partial\cdot}{\partial d} &: \mu_\lambda \leq 1 \quad (= if\ d > 0) \\
\frac{\partial\cdot}{\partial k} &: E\left[\frac{\partial\hat{\pi}_z\left(k, w\left(\mathbf{s}'\right)\right)}{\partial k} - \beta\left(\mu_\lambda\left(\mathbf{s}'\right) - \mu_\lambda\right)\frac{\partial\hat{D}_z\left(k, \mathbf{s}'\right)}{\partial k} - \left(\gamma_\lambda - 1\right)\left(1-\gamma\right)\right] = 0 \\
\frac{\partial\cdot}{\partial\mu_\lambda'} &: \hat{D}_z\left(k, \mathbf{s}'\right) \leq \gamma_\lambda\frac{\partial W_z^B\left(\mathbf{s}', \mu_\lambda\left(\mathbf{s}'\right)\right)}{\partial\mu_\lambda\left(\mathbf{s}'\right)} \quad (= if\ \mu_\lambda\left(\mathbf{s}'\right) > \mu_\lambda) \\
\frac{\partial\cdot}{\partial\gamma_\lambda} &: \left(1-\gamma\right)k \leq \beta W_z^B\left(\mathbf{s}', \mu_\lambda\left(\mathbf{s}'\right)\right) \quad (= if\ \gamma_\lambda > 1)
\end{aligned}$$

and the envelope is

$$\frac{\partial\cdot}{\partial\mu_\lambda} : \frac{\partial W_z^B\left(\mathbf{s}, \mu_\lambda\right)}{\partial\mu_\lambda} = d + \begin{cases} \beta E\hat{D}_z\left(k, \mathbf{s}'\right) & if \quad \mu_\lambda\left(\mathbf{s}'\right) > \mu_\lambda \\ \beta E\frac{\partial W_z^B\left(\mathbf{s}', \mu_\lambda\right)}{\partial\mu_\lambda} & if \quad \mu_\lambda\left(\mathbf{s}'\right) = \mu_\lambda \end{cases}$$

Patterns of firms' growth

Proposition 4 *When contracts are fully enforceable and banks capital adequacy requirements are not binding, firms operate at the optimal scale* $\overline{k}_z\left(\mathbf{s}\right)$ *that solves* $E\frac{\partial\tilde{\pi}_z\left(k, w\left(\mathbf{s}'\right)\right)}{\partial k} = 1$. *With limited enforceability, binding capital adequacy requirements imply that firms never reach the optimal scale.*

Notice that in the case of limited enforceability and no bank capital adequacy requirements, firms are initially small and grow on average until they reach the optimal scale (Cooley, Marimon and Quadrini (2001), Albuquerque and Hopenhayn (2001)).

Dividend policy and evolution of entrepreneur's value

Proposition 5 *With limited enforceability, binding capital adequacy requirements determine the dividend policy of the firm.*

Value of a new firm The analysis of the previous section takes the relative weight assigned to the entrepreneur in a new contract signed at time t, χ_t, as given. In equilibrium this weight is endogenous and is important to close the model since it determines the value of starting a new contract and, in turn, the repudiation value $\hat{D}_z(k, \mathbf{s})$.

In fact, the value of searching for an investment project is

$$V_0^B(\mathbf{s}) = pV_{z_H}^B\left(\mathbf{s}, \chi_{z_H}(\mathbf{s})\right) + (1-p)\, V_{z_L}^B\left(\mathbf{s}, \chi_{z_L}(\mathbf{s})\right)$$

where $V_z^B(\mathbf{s}, \chi_z(\mathbf{s}))$ is the end-of-period value of the contract for the entrepreneur and $\chi_z(\mathbf{s})$ is determined by the break-even condition for the bank.

4.2.3 The optimal choice between contracting with banks and markets

The contract can be written in the following recursive form

$$\begin{aligned}
W_z^I(\mathbf{s},\mu_\lambda) &= \min_{\mu'} \max_{d,k} \pi_z(k,\mathbf{s}') - (1-\mu_\lambda)\, d - \beta E\left(\mu_\lambda(\mathbf{s}') - \mu_\lambda\right) D_z(k,\mathbf{s}') \\
&\quad + \beta E \max\left[W_z^I(\mathbf{s}',\mu_\lambda(\mathbf{s}')), W_z^B(\mathbf{s}',\mu_\lambda(\mathbf{s}'))\right] \\
\text{s.t. } d &\geq 0, \qquad \mu_\lambda(\mathbf{s}') \geq \mu_\lambda \\
\mathbf{s}' &\sim M(\mathbf{s})
\end{aligned}$$

$$\begin{aligned}
W_z^B(\mathbf{s},\mu_\lambda) &= \min_{\mu'_\lambda,\gamma_\lambda} \max_{d,k} \hat{\pi}_z(k,\mathbf{s}') - (1-\mu_\lambda)\, d - \beta E\left(\mu_\lambda(\mathbf{s}') - \mu_\lambda\right) \hat{D}_z(k,\mathbf{s}') \\
&\quad - E\left((\gamma_\lambda - 1)(1-\gamma)\, k\right) + \beta\gamma_\lambda E \max\left[W_z^I(\mathbf{s}',\mu_\lambda(\mathbf{s}')), W_z^B(\mathbf{s}',\mu_\lambda(\mathbf{s}'))\right] \\
\text{s.t. } d &\geq 0, \qquad \mu_\lambda(\mathbf{s}') \geq \mu_\lambda, \qquad \gamma_\lambda \geq 1 \\
\mathbf{s}' &\sim M(\mathbf{s})
\end{aligned}$$

and M is the distribution function for the next period aggregate states, given the current states. The aggregate state is $s = (Z, \Omega)$, where Ω denotes the distribution (measure) of the fractions of firms which are bank and market financed over the variables z and μ_λ, and Z denotes the number of new investment projects with high productivity.

Characterization of the contract An owner-manager will choose market financing in the current period if $W_z^I(\mathbf{s},\mu_\lambda) \geq W_z^B(\mathbf{s},\mu_\lambda)$. Otherwise, he will choose bank financing. Let

$\Phi(\mu_\lambda, z; \mathbf{s})$ denote the decision rule governing whether an individual chooses market or bank financing. It is defined as

$$\Phi(\mu_\lambda, z; \mathbf{s}) = \begin{cases} 1 & \text{if} \quad W_z^I(\mathbf{s}, \mu_\lambda) \geq W_z^B(\mathbf{s}, \mu_\lambda) \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

Clearly, the owner-manager's choice between financing modes is based on the comparison of the expected present discounted value of each financing regime.

Finally, an owner-manager in state $(\mu_\lambda, z; \mathbf{s})$ will accumulate capital according to

$$\mu'_\lambda = \mu(\mu_\lambda, z; \mathbf{s}) = \Phi(\mu_\lambda, z; \mathbf{s})\, \mu_\lambda^I(\mu_\lambda, z; \mathbf{s}) + (1 - \Phi(\mu_\lambda, z; \mathbf{s}))\, \mu_\lambda^B(\mu_\lambda, z; \mathbf{s})$$

Hence, the optimal policies are given by

$$\begin{aligned}
\frac{\partial \cdot}{\partial d} &: \quad \mu_\lambda \leq 1 \quad (= if \;\; d > 0) \\
\frac{\partial \cdot}{\partial k} &: \quad \begin{array}{c} \Phi(\mu_\lambda, z; \mathbf{s})\, E\left[\frac{\partial \pi_z(k, w(\mathbf{s}'))}{\partial k} - \beta(\mu_\lambda(\mathbf{s}') - \mu_\lambda)\frac{\partial D_z(k, \mathbf{s}')}{\partial k}\right] \\ + (1 - \Phi(\mu_\lambda, z; \mathbf{s}))\, E\left[\frac{\partial \hat{\pi}_z(k, w(\mathbf{s}'))}{\partial k} - \beta(\mu_\lambda(\mathbf{s}') - \mu_\lambda)\frac{\partial \hat{D}_z(k, \mathbf{s}')}{\partial k} - (\gamma_\lambda - 1)(1 - \gamma)\right] = 0 \end{array}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \cdot}{\partial \mu'_\lambda} &: \quad \Phi(\mu_\lambda, z; \mathbf{s})\, D_z(k, \mathbf{s}') + (1 - \Phi(\mu_\lambda, z; \mathbf{s}))\, \hat{D}_z(k, \mathbf{s}') \leq \gamma_\lambda \frac{\partial W_z(\mathbf{s}', \mu_\lambda(\mathbf{s}'))}{\partial \mu_\lambda(\mathbf{s}')} \quad (= if \;\; \mu_\lambda(\mathbf{s}') > \mu_\lambda) \\
\frac{\partial \cdot}{\partial \gamma_\lambda} &: \quad (1 - \Phi(\mu_\lambda, z; \mathbf{s}))\left[(1 - \gamma)k - \beta W_z(\mathbf{s}', \mu_\lambda(\mathbf{s}'))\right] \leq 0 \quad (= if \;\; \gamma_\lambda > 1)
\end{aligned}$$

and the envelope is

$$\frac{\partial \cdot}{\partial \mu_\lambda} : \frac{\partial W_z(\mathbf{s}, \mu_\lambda)}{\partial \mu_\lambda} = d + \begin{cases} \begin{array}{c} \Phi(\mu_\lambda, z; \mathbf{s})\, \beta E D_z(k, \mathbf{s}') \\ + (1 - \Phi(\mu_\lambda, z; \mathbf{s}))\, \beta E \hat{D}_z(k, \mathbf{s}') \end{array} & if \quad \mu_\lambda(\mathbf{s}') > \mu_\lambda \\ \beta E \frac{\partial W_z(\mathbf{s}', \mu_\lambda)}{\partial \mu_\lambda} & if \quad \mu_\lambda(\mathbf{s}') = \mu_\lambda \end{cases}$$

It is straightforward to observe that this version of the model inherits the main properties discussed in the previous sections. In particular, it is still possible to prove the following

Proposition 6 *The functions $W_z^I(\mathbf{s}, \mu)$ and $W_z^B(\mathbf{s}, \mu)$ exist and are monotone in μ.*

Key to the argument is the observation that the operator $\left(W^{I,j+1}, W^{B,j+1}\right) = T\left(W^{I,j}, W^{B,j}\right)$ satisfies Blackwell's sufficient conditions (Stokey, Lucas, and Prescott, 1989).

Proposition 7 *The exists a generic set of parameters values for which the function* $\Phi(\mu, z; \mathbf{s})$ *is nondegenerate. In particular, there exists a* $\tilde{\mu}$ *such that:*

1. *for all* $\mu \geq \tilde{\mu}$, $W_z^I(\mathbf{s},\mu) \geq W_z^B(\mathbf{s},\mu)$;

2. *for all* $\mu < \tilde{\mu}$, $W_z^B(\mathbf{s},\mu) > W_z^I(\mathbf{s},\mu)$.

Notice that capital adequacy requirements are key to determine the switching threshold and constitute the distinguishing feature of the model in as much as the choice of financing modes depends on the non-trivial trade-off between frictions on the supply (i.e. capital adequacy requirements) and on the demand (i.e. the no-default constraint of the owner-manager) side of capital.

Value of a new firm The analysis of the previous section takes the relative weight assigned to the entrepreneur in a new contract signed at time t, χ_t, as given. In equilibrium this weight is endogenous and is important to close the model since it determines the value of starting a new contract and, in turn, the repudiation values $D_z^B(k, \mathbf{s})$ and $D_z^I(k, \mathbf{s})$.

In fact, the value of searching for an investment project is

$$\begin{aligned} V_0^I(\mathbf{s}) &= pV_{z_H}^I\left(\mathbf{s}, \lambda_{z_H}^I(\mathbf{s})\right) + (1-p)\, V_{z_L}^I\left(\mathbf{s}, \lambda_{z_L}^I(\mathbf{s})\right) \\ V_0^B(\mathbf{s}) &= pV_{z_H}^B\left(\mathbf{s}, \lambda_{z_H}^B(\mathbf{s})\right) + (1-p)\, V_{z_L}^B\left(\mathbf{s}, \lambda_{z_L}^B(\mathbf{s})\right) \end{aligned} \tag{13}$$

where $V_z^I\left(\mathbf{s}, \lambda_z^I(\mathbf{s})\right)$ and $V_z^B\left(\mathbf{s}, \lambda_z^B(\mathbf{s})\right)$ are the end-of-period values of the market and bank financing contracts for the entrepreneur as defined in (**??**) and $\lambda_z^I(\mathbf{s})$ and $\lambda_z^B(\mathbf{s})$ are the lagrange multiplier associated with the break-even condition for the market and the bank respectively.

5 General equilibrium

This section defines recursive competitive equilibrium and applies Brower-Schauder-Tychonoff fixed-point theorem to prove the existence of a competitive equilibrium both with and without aggregate shocks.

5.1 Definition of recursive competitive equilibrium

Recursive competitive equilibrium is defined by:

1. a set of value functions $W^I(\mu_\lambda, z, \mathbf{s})$, $W^B(\mu_\lambda, z, \mathbf{s})$, initial condition for a new firm $V_0^I(\mathbf{s})$, $V_0^B(\mathbf{s})$, decision rules for investment $k^I(\mu_\lambda, z, \mathbf{s})$, $k^B(\mu_\lambda, z, \mathbf{s})$, labor demand, $l^I(\mu_\lambda, z, \mathbf{s})$, $l^B(\mu_\lambda, z, \mathbf{s})$, dividend $d^I(\mu_\lambda, z, \mathbf{s})$, $d^B(\mu_\lambda, z, \mathbf{s})$, state evolution $\mu_\lambda^I(\mu_\lambda, z, \mathbf{s})$, $\mu_\lambda^B(\mu_\lambda, z, \mathbf{s})$, and financial regime, $\Phi(\mu_\lambda, z, \mathbf{s})$;
2. consumption, $c(a, \mathbf{s})$, and labor supply, $l^s(a, \mathbf{s})$, from households;
3. wage $w(s)$;
4. aggregate demand of labor from firms and aggregate supply from households;
5. aggregate investment from firms and aggregate savings from households;
6. distribution function (law of motion) $s' \sim M(\mathbf{s})$;

such that:

1. given w, M, and W^B, the decision rules $k^I(\mu_\lambda, z, \mathbf{s})$, $l^I(\mu_\lambda, z, \mathbf{s})$, $d^I(\mu_\lambda, z, \mathbf{s})$, $\mu_\lambda^I(\mu_\lambda, z, \mathbf{s})$, the value function $W^I(\mu_\lambda, z, \mathbf{s})$, and the initial condition $V_0^I(\mathbf{s})$, solve problem (17);
2. given w, M, and W^I, the decision rules $k^B(\mu_\lambda, z, \mathbf{s})$, $l^B(\mu_\lambda, z, \mathbf{s})$, $d^B(\mu_\lambda, z, \mathbf{s})$, $\mu_\lambda^B(\mu_\lambda, z, \mathbf{s})$, the value function $W^B(\mu_\lambda, z, \mathbf{s})$, and the initial condition $V_0^B(\mathbf{s})$, solve problem (18);
3. given W^I and W^B, the bank/market financing decision rule, $\Phi(\mu_\lambda, z, \mathbf{s})$, is determined by (9);
4. consumption and labor supply of households satisfy their optimality conditions;
5. the wage is the equilibrium clearing price in the labor market;
6. the capital and good markets clear, that is in equilibrium, composite output, Y, is used for consumption, C, and for investment, I;
7. the law of motion $M(s)$ is consistent with the individual decisions and the shock.

5.2 Existence of equilibrium and characterization of the financing decision - No aggregate uncertainty

To solve for equilibrium is equivalent to finding the fixed point of the following operator

$$\mathbf{V}_0^{j+1}(\mathbf{s}) = \mathbf{T}\left(\mathbf{V}_0^j\right)(\mathbf{s})$$

where $\mathbf{V}_0(\mathbf{s}) = \left(V_0^I(\mathbf{s}), V_0^B(\mathbf{s})\right)$.

Two features complicate this problem: (i) the dependence on the overall distribution of firms; (ii) the presence of the max operator on the right hand side of (17) and (18). For the case of no aggregate uncertainty $(z_H = z_L)$, the following proposition shows that both these problems can be circumvented in a rather straightforward manner and standard tools (Stokey, Lucas, and Prescott (1989)) can be employed.

We start by proving that, given an invariant distribution M, problems (17) and (18) are well defined.

Proposition 8 (Properties of V) $\mathbf{T}$ *exists and is monotone decreasing.*

Proof. *Consider the mapping* $\mathbf{V}_0^{j+1}(\mathbf{s}) = \mathbf{T}\left(\mathbf{V}_0^j\right)(\mathbf{s})$. *By applying the Theorem of the Maximum it is straightforward to see that* $\mathbf{T}$ *maps continuous functions into continuous functions. Blackwell's sufficient conditions deliver the monotonicity, which completes the proof.* ■

5.2.1 Steady state and equilibrium existence

Recall that the state of the economy is a collection of elements

$$\left(\{\mathbf{s}_t\}, \{z^i, \mu_t^i\}\right)_{i \in I, t \in T}$$

where $\{z_t^i, \mu_t^i\}$ are the states of individual firm i at date t, and $\{\mathbf{s}_t\}$ is the aggregate state of the economy, $I \equiv [0,1]$, $T \equiv [0, \infty)$. The state space is then $S \equiv P \times Z \times N$.

The individual problem is given by (13). The real function $W_z \in \Psi$ and T is an operator $T : \Psi \to \Psi$. The solution of the Bellman Equation is a fixed point w such that $Tw = w$. Given

w we obtain the optimal policy correspondence γ defined on the space F as

$$\gamma(s) = \left\{ \begin{array}{c} \mu' \geq \mu \mid \\ w(s) = \mu' d + \tau - \beta(\mu' - \mu) D_z(\mathbf{s}, k) + \beta E w(s'), \Phi' = H(\Phi) \end{array} \right\} \tag{14}$$

This optimal policy, which determines the dynamics of the individual variables, defines individual equilibrium. The solution to the individual problem provides an optimal policy γ as a function of the initial policy γ_0, say $\Xi(\gamma_0) = \gamma_{\gamma_0}$.

The aggregate state of the economy is a probability measure $\Phi_t \in P$ over individual variables $\{z_t^i, \mu_t^i\}$. The dynamics of the aggregate measure is given by the integral operator $H : P \to P$ with kernel $H(z, \mu : \Phi)$ given by

$$(H\Phi) = \int_{Z \times N} H(z, \mu; \Phi) \Phi(dz, d\mu) \tag{15}$$

An aggregate equilibrium is an individual equilibrium γ^* that generates sequences $\{z_t^i, \mu_t^i\}$ that are consistent with the law of motion of the aggregate state Φ. It can be defined as a fixed point of Ξ, that is as $\Xi(\gamma_0) = \gamma_0$, such that the kernel H provides a dynamics for Φ that is consistent with the dynamics of the individual states $\{z_t^i, \mu_t^i\}$.

Finally, an aggregate steady state is a pair (γ^*, Φ^*) such that γ^* is an aggregate solution and Φ^* is a constant.

Existence of an individual equilibrium

Theorem 9 *The operator T, given in the individual problems* (17) *and* (18), *has a unique fixed point $w \in \Psi$. The optimal policy correspondence $\gamma \in F$, defined by w as in* (14), *is a continuous function.*

Proof. *see Appendix* ■

Existence of an aggregate equilibrium

Theorem 10 *The function $\Xi : F \to F$ has a fixed point γ^* such that $\Xi(\gamma^*) = \gamma^*$.*

Proof. *see Appendix* ■

Existence of steady-state

Theorem 11 *Let $H_{\gamma_0} : P \to P$ be the operator defined in* (15) *. There exists a measure* $\Phi^* \in P$ *such that* $H_{\gamma_0}(\Phi^*) = \Phi^*$.

Proof. *see Appendix* ■

6 Quantitative implications

This section studies the aggregate implications of corporate governance institutions quantitatively. We show that institutional constraints on the monitoring ability of banks and prudential regulation are an important determinant of financial systems. Moreover, we evaluate the aggregate welfare implications of capital adequacy constraints and show that they entail sizable welfare losses since by limiting the availability of funds to firms they change the size and distribution of firms. We then present impulse responses of aggregate output and investment to both aggregate technology and interest rate shocks. We find that the model is consistent with key regularities of financial systems and the business cycle. Most notably, it implies that aggregate volatility is higher in market-based financial systems and that the banking sector plays a crucial role in the macroeconomic transmission of interest rate shocks. Finally, we present results on the aggregate implications of shocks to capital adequacy ratios. This exercise suggests that counter-cyclical prudential regulation can be an effective macroeconomic policy instrument.

6.1 Calibration

The period in the economy is one year and the intertemporal discount rate to $\beta = 0.96$. The survival probability is 0.95. The disutility from working takes the form $\varphi(l_t) = \pi l^{\nu}$. The parameter ν affects the size of the aggregate economy with and without financial frictions, which is important for the size response of output to shocks: the smaller its value (the more elastic is the supply of labor) and the larger is the response of output. However, the shape of the impulse response to shocks is not affected significantly by this parameter. In the baseline model we set $\nu = 1.1$. After fixing ν, the parameter π is chosen so that one third of available time is spent working. The mapping from π to the working time will be described below.

The production function is specified as $zF(k) = z \min\{k, \xi l\}$,where $F = zk^{\theta}$. The parameter θ is assigned the value of 0.975. We would like the steady state of the economy to have a capital-output ratio of 2.8 and a labor income share of 0.6. These indices are complicated functions of the whole distribution of firms. However, because most of the aggregate output is produced by unconstrained firms, we can choose the parameter values so that these numbers are reproduced by unconstrained firms. After normalizing the capital stock of unconstrained firms to 1, a value of 2.8 for the capital-output ratio implies $E(z) = 0.4$. This condition pins down the mean of z. The depreciation rate for the aggregate stock of capital is about 0.085.

Given the parameterization of the production sector, the model generates a stationary distribution of firms and an aggregate demand of labor. The parameters κ and σ and the initial set-up cost I_0 affect the size of new firms and the steady-state proportion of constrained firms. We set I_0 so that the set-up cost is about 20 percent the value of capital for unconstrained firms and κ so that the repudiation value is 2.5 percent the value of capital for unconstrained firms. We set σ so as to have about 40% of the firms constrained in the steady state. This implies a value of 1.2. We set Λ equal to 0.1 so that the steady state ratio of bank supplied capital to output matches the corresponding value for the US (i.e. 0.644). λ is set to 0.02 to match the observed differential between the lending and the borrowing rates.

Table 1: benchmark parametrization

Parameter	Value
Intertemporal discount rate	$\beta = .96$
Disutility from working	$\nu = 1.1$
	$\pi = .001$
Death probability	$\alpha = .05$
Production function	$\theta = .975$
Capital-labor ratio	$\xi = .003$
Depreciation rate	$\delta = .037$
Set-up cost	$I_0 = 0.2$
Cost of repudiation	$\kappa = .026$
Cost of bank capital	$\lambda = .02$
Incentive benefit of bank	$\Lambda = 0.1$
Stealing parameter	$\sigma = 1.2$

The capital-labor ratio and the utility parameter π are determined so that in the steady state equilibrium each worker spends 1/3 of available time working and unconstrained firms employ 1,000 workers. This implies $\xi = 0.0033$. The number of workers employed by unconstrained firms is not important. The results would not change if we choose a different number. Given ξ we are able to determine the steady state wage rate. Then to pin down the parameter π we consider the workers first order condition in the supply of labor, which pins down π. Finally, the mass of new firms (newborn agents with entrepreneurial skills, e) is such that the aggregate supply of labor is equal to the aggregate demand. Given these values it can be verified that the non-repudiation condition for the intermediary is satisfied. Table 1 summarizes these parametric choices.

6.2 Steady state results

In a steady state equilibrium all firms have the same productivity, i.e. there is no aggregate uncertainty and $z_L = z_H$.

The left-panel of Figure 2 plots the values of a new contract for the manager and the financiers. The value of the contract is increasing in the initial stock of capital for the manager but it is decreasing for the financiers. The assumption of competitive financial markets implies that in equilibrium the value of the contract for the financiers is zero (zero profit condition). It is worth noting that the option of issuing multiple securities induces a non-concavity in these values and, consequently, in the total value of the contract as plotted in the right-panel of Figure 2. Loosely speaking, the value functions when managers have the option of switching between financing regimes are the outer envelope of the value functions under each regime.

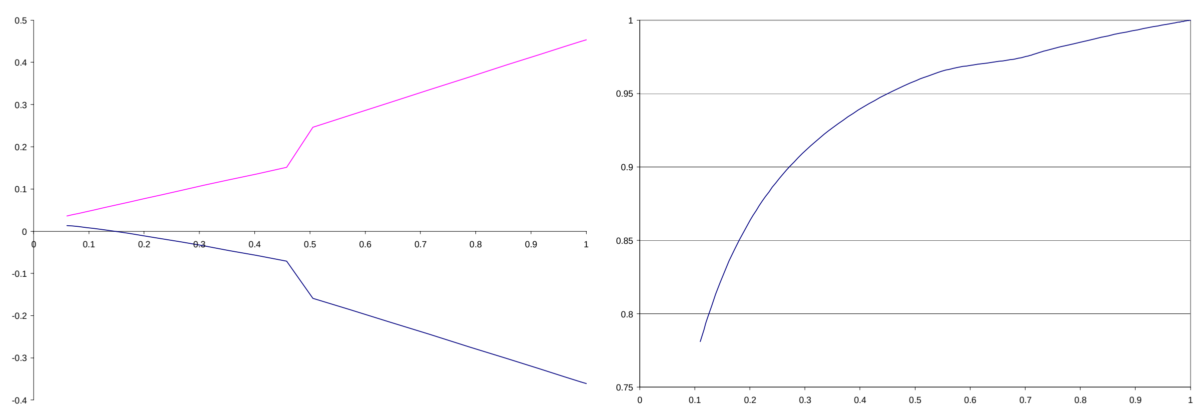

Figure 2: Contract values to manager and financiers (left-panel) and total (right-panel)

6.2.1 Corporate governance institutions and financial systems

What accounts for the cross-country differences in the importance of banks and capital market financing of investment? La Porta, et. al (1997) show that the depth of financial markets depends crucially on the degree of law enforcement. We show that institutional constraints and prudential regulation are important as well.

Table 2: Financing choice with search for $\Lambda = 0.05$

Cap. Req.	Share of capital financed by bank	Share of firms with bank
4%	15.8	37.7
8%	7.7	20.64

In fact, Table 2 reports the shares of capital and firms financed by banks under two regimes of loose and relatively tight capital adequacy requirements. In fact, higher capital adequacy requirements result in a lower proportion of projects financed by banks in equilibrium. In this sense, prudential regulations limits banks' effectiveness at preventing managerial misconduct, which results in a market-based financial system. This result is a straightforward implication of the trade-off between bank and bonds: higher capital adequacy requirement increase the spread between bank and market financing. Hence, ex-ante the benefits of bank lending are reduced, which induces more firms to switch to the cheaper bond financing.

6.2.2 The firm size distribution is less skewed in bank-based systems

Figure 3 plots the size distribution of firms in the economies with (blue lines) and without (red lines) banks.

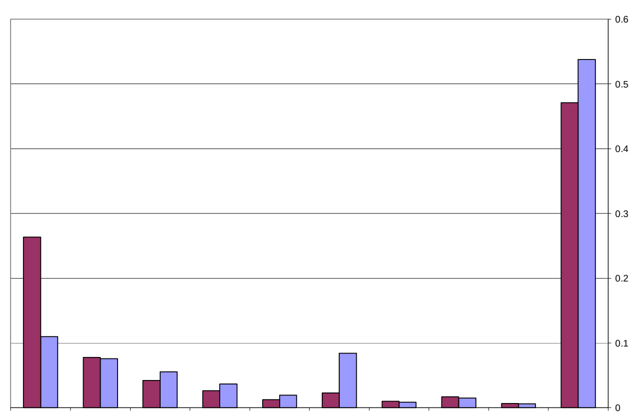

Figure 3: firm size distribution

In the economy without banks, at entrance firms are small, which motivates the large mass of firms in the smallest class. After entrance these firms grow on average and become bigger.

However, because some of them exit, the mass of firms declines as we consider larger classes, until we reach the largest class which contains unconstrained firms. Because firms do not grow once they have reached the optimal input of capital, we observe a concentration of firms in this size class.

With banks, we observe a lower concentration of firms in the smaller decile, since the option of obtaining bank financing effectively relaxes the borrowing constraint of small firms. Moreover, we observe a non-negligible fraction of firms in the intermediate deciles. These are the marginal firms, which by exercising the option of switching to bond financing are able to economize on the cost of bank financing, and hence can operate with a relatively high amount of capital. As a result, bank financing induces a shift of the firm size distribution toward the mean.

6.2.3 Welfare analysis

In this section we evaluate the welfare consequences of capital adequacy requirements. In doing so we keep the productivity of new technologies constant (no aggregate uncertainty).

Table 3: Contract with banks only

CA Req	Output loss	W. loss	W. dyn. gain	% constrained	Average Size
0%	10.65	2.43	0.79	43.5	0.68
4%	14.75	3.03	0.89	40.4	0.6
8%	22.88	4.45	0.92	36.2	0.40
10%	28.66	5.53	1.20	34.5	0.30
12%	35.04	6.75	1.67	34.0	0.22

To evaluate the welfare consequences of intermediation we conduct the following experiment. Starting from the steady state of the economy with limited enforceability and only bank debt available, we assume that all contracts become fully enforceable (including the existing ones). This unanticipated change brings the economy to a new equilibrium in which all firms with the same productivity employ the same input of capital. The transition dynamics takes only one period. The welfare gains from this transition are reported in Table 3. These gains are computed as the increase in every period consumption necessary to make all agents indifferent between staying in the economy with limited enforceability (but with the consumption increase) and in the transition to the steady state with full enforceability. The table also reports some key statistics of the steady state allocations. Different values of capital adequacy requirements are considered. In general, welfare losses are high and higher capital adequacy requirements imply higher welfare losses.

6.3 Technology shocks - a financial decelerator

Did corporate governance institutions play any role in fostering the US investment boom of the 90's? Figure 4 shows that the arrival of a new technology induces higher aggregate volatility in market-based than in bank-based financial systems. To build intuition for this result, it is useful to consider that in the model banks are more (constrained-) efficient providers of finances than markets. The arrival of a new highly productive technology increases the value of managers' outside options. Both banks and bond holders will have to advance more capital to managers to prevent them from defaulting. This relaxes the tightness of the incentive compatibility constraints and more capital can be given to these firms. Therefore, the impact of the new technology is to lessen the tightness of financial constraints (Cooley, Marimon and Quadrini (2001)).

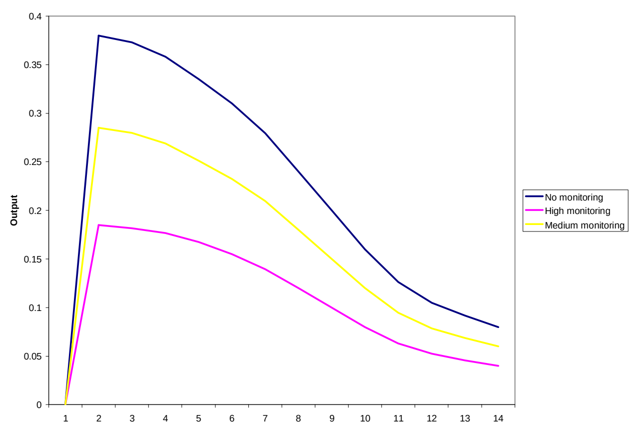

Figure 4: Impulse Response of GDP to a technology shock

The special feature of the present model is that, given the increase in managers' outside options, banks will have to advance relatively less capital than bond holders due to their higher efficiency. This follows directly from the characterization of the contract and in particular from assumption (3)

$$\hat{D}_z(\mathbf{s}, k) = (1 - \Lambda)(\sigma z F(k, l) + V_0(\mathbf{s}) - \kappa) < D_z(\mathbf{s}, k)$$

This, together with the fact that some firms are constrained in their ability to substitute bond for bank financing, will result in lower volatility the higher the proportion of firms financed by banks. This result is interesting from a substantive perspective since it lends formal support to a commonly heard argument about the role of corporate governance institutions in funneling the investment boom of the 90's in the US. Moreover, that fact that the aggregate composition of financing and not the overall amount of financing is a key determinant of aggregate volatility sharply contrasts with the previous literature which has looked entirely at the effect of shocks on aggregate debt (Cooley, Marimon, and Quadrini (2001); Bernanke, Gertler, and Gilchrist (1998)). A more articulated "corporate governance view" of the role of the financial sector in the transmission of aggregate shocks naturally emerges.

Figure 5 plots sample simulated time series of GDP and investment in an economy with bank and market financing. The time series are generated by solving the model for a time-series of iid aggregate technology shocks. Bank financing effectively completes contracts and alleviates enforcement problems, but does not eliminate them altogether. As a result, and in accord with the previous literature, investment and output display ample and serially correlated fluctuations.

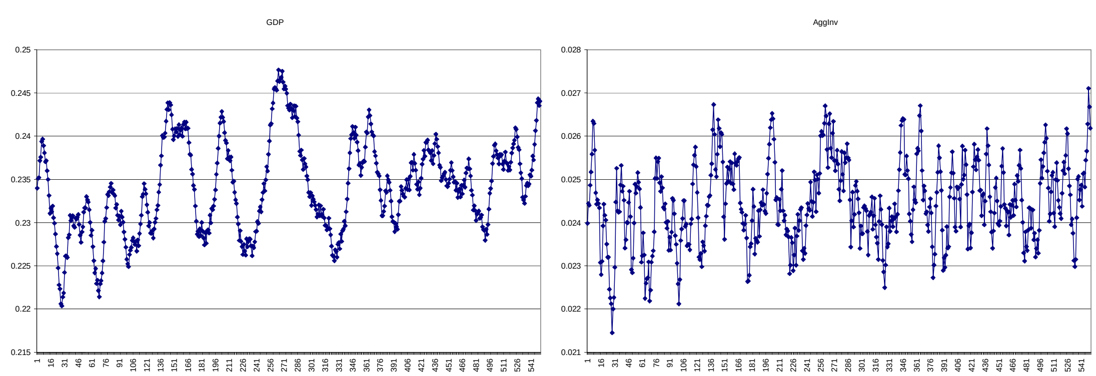

Figure 5: Simulated Time Series of GDP (left-panel) and Investment (right-panel)

6.4 Interest rate shocks

In this section we study how interest rate shocks are propagated by conducting the following experiment: starting from a steady state equilibrium in which all firms face a constant interest rate, we consider a temporary higher interest rate. The shock increases the interest rate of half percentage point. The results of this experiment is to show that imperfect substitutability of bank and market lending amplifies and delays the impact of interest rate shocks on aggregate output.

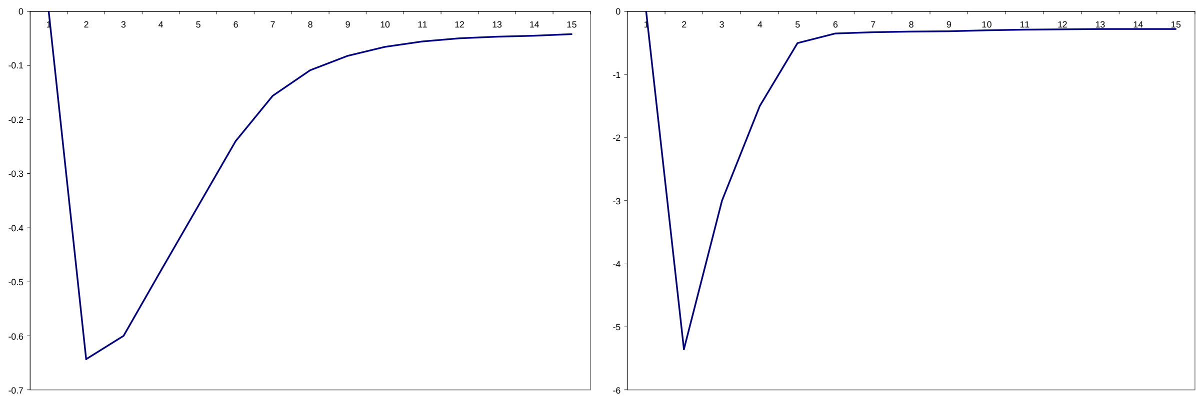

Figure 6: Impulse Response of GDP (left-panel) and Investment (right-panel) to an Interest Rate Shock

The model implies that interest rate fluctuations induce ample and persistent responses of output as displayed in Figure 6. To see this, recall the characterization of the contract for firms who are currently under bank financing and the implied expression for the capital adequacy constraint

$$\frac{1}{r}EW_z\left(\mathbf{s}',\mu_\lambda\left(\mathbf{s}'\right)\right) \geq \left(1-\gamma\right)k \tag{16}$$

A shock that increases the interest rate lowers the net worth of banks, i.e. the left hand side of (16) , and induces more firms to face binding capital adequacy constraints. While marginal firms can switch to bond financing, smaller constrained firms have to bear the cost of the tightening. In this sense, the imperfect substitutability of bank and bond financing is key to the amplification result.

As shown in Figure 7, the model is consistent with key empirical regularities of the macroeconomic transmission of interest rate shocks (Gertler and Gilchrist, 1994; Perez-Quiros and Timmermann, 1998; Kashyap, Stein, and Wilcox, 1993). In particular, at the micro level, the marginal firms switch out of bank loans into commercial paper on impact. In fact, this is due to the increase in bank spreads which arises endogenously as a result of interest rate shocks.

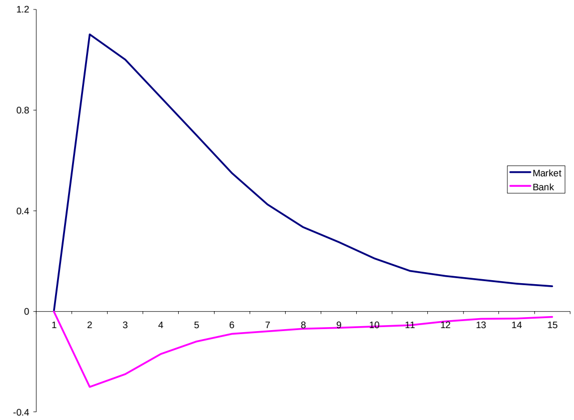

Figure 7: Impulse Response of the bond issues and bank loans to an Interest Rate Shock

While the marginal firms, that is, relatively large and unconstrained firms, can opt out of bank financing into bond financing, smaller firms, who are constrained in their ability to substitute between sources of financing, bear the consequences of the tightening most. Consequently, as displayed in the upper panel of Figure 7, the model predicts that in response to a negative shock to the interest rate there is a surge in commercial paper issuance and, as displayed in the lower panel of Figure 7, a reduction in bank credit. Moreover, the inability of small constrained firms to move from one debt market to others (namely from the private debt markets to the public debt markets), amplifies the impact of interest rate shocks on aggregate investment and output.

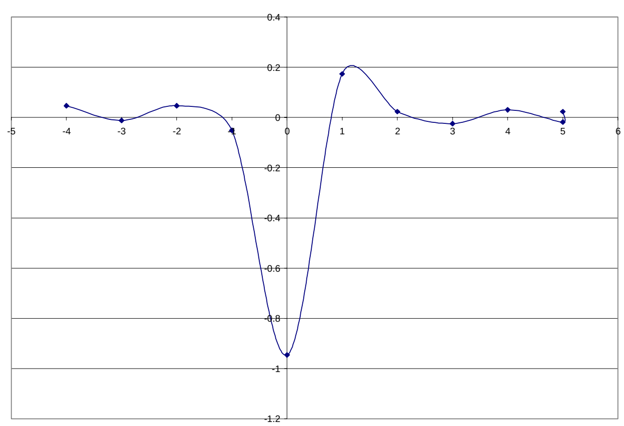

Figure 8: Correlation of GDP with the Interest Rate

Finally, as displayed in Figure 8, the pattern of correlations of interest rates and output at different lags and leads is consistent with the data.

6.5 Cyclicality of capital adequacy requirements

Should capital ratios vary with the business cycle and, if so, how? This sections details the implications of the model for the cyclical properties of prudential regulation. Starting from a steady state equilibrium in which all firms face a constant capital adequacy ratio, γ, we consider a shock that temporary increases the ratio of half percentage point. The dynamic responses of aggregate output and investment to this shock are shown in Figure 9.

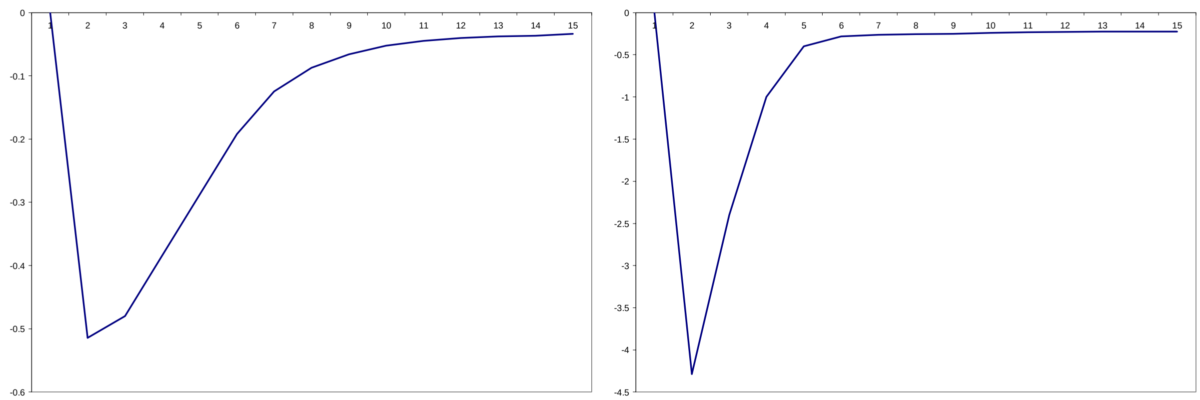

Figure 9: Impulse Response of GDP (left-panel) and Investment (right-panel) to a Capital Adequacy Shock

A straightforward implication of the impulse responses is that procyclical adequacy ratios are a potentially useful tool for aggregate policy interventions directed toward output stabilization. Nevertheless, this conclusion is tentative since an assessment of the cyclical properties of prudential regulation requires a more sophisticated representation of the banking sector which, unlike the present model, includes an explicit rationale for prudential regulation.

7 Conclusions

This paper has undertaken a quantitative evaluation of the macroeconomic implications of corporate governance institutions within a model where the size and distribution of firms and the structure of financial markets are jointly determined. We have shown that if firms adapt their financing modes to economic conditions, aggregate shocks change the aggregate composition of financing, which, in turn, is a key determinant of firm growth and aggregate volatility.

Plausibly calibrated numerical solutions reveal that capital adequacy requirements have sizable consequences for aggregate welfare. At the micro level, the model is consistent with the stylized fact that larger firms have a bigger share of securities in their financial structure (e.g. Petersen and Rajan, 1994). At the macro level, corporate governance institutions that limit (foster) banks' effectiveness at preventing managerial misconduct, as observed in Anglo-Saxon (Continental Europe) systems, results in a market (bank)-based financial system. The model is consistent with key regularities of financial systems and the business cycle. Most notably, it implies that aggregate volatility is higher in market-based financial systems and that the banking sector plays a crucial role in the macroeconomic transmission of interest rate shocks. We have also explored the question of the cyclical properties of prudential regulation. A straightforward implication of the model is that procyclical adequacy ratios are a potentially useful tool for aggregate policy interventions directed toward output stabilization. Nevertheless, more work is needed to corroborate this conclusion and to incorporate an explicit rationale for prudential regulation.

There are several dimensions along which the present work can be extended. In particular, a broader set of policy experiments is of straightforward implementation. For example, adverse shocks to the parameter Λ which measures banks' effectiveness at preventing managerial misconduct could be considered as a way to address the recent debate on the role of banks in exacerbating the economic recession in the second half of the 90's in Japan. Relaxing the assumption of risk-neutral households is perhaps the most important generalization for at least two reasons: (i) it would allow a study of the general equilibrium feedback of firms' choices on the interest rate; (ii) it would open up the possibility of exploring the asset pricing implications of cyclical changes in the aggregate composition of financing.

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8 Appendix

8.1 Proofs

The problem can be equivalently written in recursive form as follows:

$$\begin{aligned} W_z^I(\mathbf{s},\mu) &= \min_{\mu'}\max_{d,k} \pi_z(k,\mathbf{s}') - (1-\mu)d - \beta E(\mu(\mathbf{s}')-\mu) D_z(k,\mathbf{s}') \\ &\quad +\beta E \max\left[W_z^B(\mathbf{s}',\mu(\mathbf{s}')), W_z^I(\mathbf{s}',\mu(\mathbf{s}'))\right] \\ \text{s.t. } d &\geq 0, \qquad \mu(\mathbf{s}') \geq \mu \\ \mathbf{s}' &\sim M(\mathbf{s}) \end{aligned} \tag{17}$$

$$\begin{aligned} W_z^B(\mathbf{s},\mu) &= \min_{\mu'}\max_{d,k} \hat{\pi}_z(k,\mathbf{s}') - (1-\mu)d - \beta E(\mu(\mathbf{s}')-\mu) \hat{D}_z(k,\mathbf{s}') \\ &\quad +\beta E \max\left[W_z^B(\mathbf{s}',\mu(\mathbf{s}')), W_z^I(\mathbf{s}',\mu(\mathbf{s}'))\right] \\ \text{s.t. } d &\geq 0, \qquad \mu(\mathbf{s}') \geq \mu, \\ \mathbf{s}' &\sim M(\mathbf{s}) \end{aligned} \tag{18}$$

Proof. Given γ_{t+1}, the Lagrange multiplier associated with the enforcement constraint and λ_t, the Lagrange multiplier associated with the participation constraint, the Lagrangian for a currently market financed firm can be written as:

$$\mathcal{L} = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left\{ \begin{array}{c} \lambda_t \pi_z(k_s, w(\mathbf{s}_{s+1})) + (\lambda_t - 1) d_s + (I_{s-1} - 1) V_z^B(\mathbf{s}_s) \\ +\gamma_{t+1}\beta \left[\sum_{j=s+1}^{\infty} \beta^{j-s-1} d_j - D_z(k_s, \mathbf{s}_{s+1})\right] \end{array} \right\}$$

After rearranging terms, the Lagrangian can be written as:

$$\begin{aligned} \mathcal{L} &= E_t \sum_{s=t}^{\infty} \beta^{s-t} \left\{ \begin{array}{c} \lambda_t \pi_z(k_s, w(\mathbf{s}_{s+1})) + (\lambda_t - 1) d_s + (I_{s-1} - 1) V_z^B(\mathbf{s}_s) \\ -\gamma_{t+1}\beta D_z(k_s, \mathbf{s}_{s+1}) \end{array} \right\} \\ &\quad + E_t \sum_{s=t}^{\infty} \beta^{s-t} \gamma_{s+1} \sum_{j=s+1}^{\infty} \beta^{j-s} d_j \end{aligned}$$

It is straightforward to show that the following identity holds:

$$\sum_{s=t}^{\infty} \beta^{s-t} \gamma_{s+1} \sum_{j=s+1}^{\infty} \beta^{j-s} d_j = \sum_{s=t}^{\infty} \beta^{s-t} \tilde{\mu}_s d_s$$

where $\tilde{\mu}_t = 0$ and $\tilde{\mu}_{s+1} = \tilde{\mu}_s + \gamma_{s+1}$.

Using this identity to eliminate the last term in the Lagrangian we obtain:

$$\begin{aligned}\mathcal{L} &= E_t \sum_{s=t}^{\infty} \beta^{s-t} \left\{ \begin{array}{c} \lambda_t \pi_z \left(k_s, w\left(\mathbf{s}_{s+1}\right)\right) + \left(\lambda_t - 1 - \tilde{\mu}_s\right) d_s + \left(I_{s-1} - 1\right) V_z^B \left(\mathbf{s}_s\right) \\ - \left(\tilde{\mu}_{s+1} - \tilde{\mu}_s\right) \beta D_z \left(k_s, \mathbf{s}_{s+1}\right) \end{array} \right\} \\ &\quad + E_t \sum_{s=t}^{\infty} \beta^{s-t} \gamma_{s+1} \sum_{j=s+1}^{\infty} \beta^{j-s} d_j\end{aligned}$$

Dividing the Lagrangian by λ_t and defining the new state as $\mu_s = \frac{1+\tilde{\mu}_s}{\lambda_t}$, we obtain the saddle-point formulation reported in recursive form in the text.

The derivation of the saddle-point formulation for a currently bank financed firm is entirely analogous and, hence, we omit it. ■

Proof of Proposition 1. Recall the statement: The functions $W_z^I(\mathbf{s},\mu)$ and $W_z^B(\mathbf{s},\mu)$ exist and are monotone in μ.

Consider the operator $\left(W^{I,j+1}, W^{B,j+1}\right) = T\left(W^{I,j}, W^{B,j}\right)$ defined by equations (17) and (18). By applying the Theorem of the Maximum it is straightforward to see that the operator T maps $W^{I,j\prime}s$ and $W^{B,j\prime}s$ that are continuous in μ and z into $W^{I,j+1\prime}s$ and $W^{B,j+1\prime}s$ that are also continuous in μ and z. By Blackwell's sufficient conditions the operator T defines a contraction mapping in the space of continuous functions with the uniform norm. Hence, W^I and W^B exist and are continuous functions (in μ and z).

Let

$$R_z^I(\mu, \mu', \cdot) = \pi_z(k, \mathbf{s}') - (1-\mu) d - \beta E\left(\mu(\mathbf{s}') - \mu\right) D_z(k, \mathbf{s}')$$

and

$$Q_z^I(\mu', \cdot) = \beta E \max\left[W_z^B(\mathbf{s}',\mu(\mathbf{s}')), W_z^I(\mathbf{s}',\mu(\mathbf{s}'))\right]$$

Now, it needs to be shown that the operator T maps $W^{I,j\prime}s$ and $W^{B,j\prime}s$ that are nondecreasing in μ into ones that are increasing in μ. To see this, consider two levels of the lagrange multiplier μ, $\mu_1 < \mu_2$. So the question is: If $W^{I,j}$ and $W^{B,j}$ are nondecreasing in μ then will $W^{I,j+1}$ and

$W^{B,j+1}$be increasing in μ? The answer is yes since

$$\begin{aligned} W_z^{I,j+1}(\mu_1,\cdot) &= max_{\mu^0\geq\tilde{\mu}}\{R_z^I(\mu_1,\mu',\cdot)+\beta Q_z^I(\mu',\cdot)\} \\ &< max_{\mu^0\geq\tilde{\mu}}\{R_z^I(\mu_2,\mu',\cdot)+\beta Q_z^I(\mu',\cdot)\} = W_z^{I,j+1}(\mu_2,\cdot) \end{aligned}$$

A similar argument can be used to establish that $W_z^{B,j+1}(\mu_1,\cdot) < W_z^{B,j+1}(\mu_2,\cdot)$. ■

Proof of Proposition 2. Recall the statement: The exists a generic set of parameters values for which the function $\Phi(\mu,z;\mathbf{s})$ is nondegenerate. In particular, there exists a $\tilde{\mu}$ such that: for all $\mu\geq\tilde{\mu}$, $W_z^I(\mathbf{s},\mu)\geq W_z^B(\mathbf{s},\mu)$; for all $\mu<\tilde{\mu}$, $W_z^B(\mathbf{s},\mu)>W_z^I(\mathbf{s},\mu)$.

Define $J(\mu)$ as the value of the shock z at which an agent is indifferent between bank and market financing. Formally this financing threshold rule is defined by the equation

$$W^I(\mu,J(\mu),\cdot)=W^B(\mu,J(\mu),\cdot) \tag{19}$$

Since W is monotonically increasing and continuous in z, J will be a function and

$$W_z^I(\mu,\cdot)\gtreqless W_z^B(\mu,\cdot)\ as\ z\gtreqless J(\mu,\cdot)$$

To further develop intuition (in a heuristic way) make the following assumption: W^I and W^B are C^1 functions.

By the implicit function theorem it then follows that $J(\mu)$ is a C^1 function too. It transpires that (17) and (18) will have the form

$$\begin{aligned} W_z^I(\mathbf{s},\mu) &= \max_{\mu^0\geq\tilde{\mu}}\pi_z(k,\mathbf{s}')-(1-\mu)d-\beta E(\mu(\mathbf{s}')-\mu)D_z(k,\mathbf{s}') \\ &\quad +\beta E_{J(\mu^0)}\max\left[W_z^B(\mathbf{s}',\mu(\mathbf{s}')),W_z^I(\mathbf{s}',\mu(\mathbf{s}'))\right] \end{aligned} \tag{20}$$

and

$$\begin{aligned} W_z^B(\mathbf{s},\mu) &= \max_{\mu^0\geq\tilde{\mu}}\hat{\pi}_z(k,\mathbf{s}')-(1-\mu)d-\beta E(\mu(\mathbf{s}')-\mu)\hat{D}_z(k,\mathbf{s}') \\ &\quad +\beta E_{J(\mu^0)}\max\left[W_z^B(\mathbf{s}',\mu(\mathbf{s}')),W_z^I(\mathbf{s}',\mu(\mathbf{s}'))\right] \end{aligned} \tag{21}$$

Using the envelope theorem it then follows that

$$W_1^I(\mathbf{s},\mu)=(d+\beta D_z(k,\mathbf{s}'))(1+r) \tag{22}$$

and

$$W_z^B(\mathbf{s},\mu) = \left(d + \beta \hat{D}_z(k,\mathbf{s}')\right)(1+r) \tag{23}$$

■

Proof of Proposition 3. Recall the statement: Ceteris paribus, firms have on average higher investment and lower growth under bank financing.

By applying the implicit function theorem to (19)it follows that $J(\mu)$ is increasing in μ if and only if $W_1^B(\mu, J(\mu), \cdot) > W_1^I(\mu, J(\mu), \cdot)$ since $J_1(\mu) = [W_1^B(\mu, J(\mu)) - W_1^I(\mu, J(\mu))]/W_2^I(\mu, J(\mu))$. Equations (22) and (23) imply $k^I(\mu, J(\mu)) \gtreqless k^B(\mu, J(\mu))$ as $W_1^B(\mu, J(\mu)) \gtreqless W_1^I(\mu, J(\mu))$. ■

The contract with banks only can be written in the following "intensive" recursive form:

$$\begin{aligned}
W_z^B(\mathbf{s},\mu_\lambda) &= \min_{\mu'_\lambda,\gamma_\lambda} \max_{d,k} \hat{\pi}_z(k,\mathbf{s}') - (1-\mu_\lambda)d - \beta E(\mu_\lambda(\mathbf{s}') - \mu_\lambda)\hat{D}_z(k,\mathbf{s}') \\
&\quad - E((\gamma_\lambda - 1)(1-\gamma)k) + \beta\gamma_\lambda E W_z^B(\mathbf{s}',\mu_\lambda(\mathbf{s}')) \\
\text{s.t. } d &\geq 0, \qquad \mu_\lambda(\mathbf{s}') \geq \mu_\lambda, \qquad \gamma_\lambda \geq 1 \\
\mathbf{s}' &\sim H(\mathbf{s})
\end{aligned}$$

Proof. By analogy with the proof for the case with an exogenous cost of bank capital, it is straightforward to show that the contracts with bank only can be written in recursive form as

$$\begin{aligned}
W_z^B(\mathbf{s},\mu,\lambda) &= \min_{\mu'_\lambda,\gamma_\lambda} \max_{d,k} \lambda\hat{\pi}_z(k,\mathbf{s}') - (\lambda-\mu)d - \beta E(\mu(\mathbf{s}') - \mu)\hat{D}_z(k,\mathbf{s}') \\
&\quad - E((\lambda(\mathbf{s}') - \lambda)(1-\gamma)k) + \beta E W_z^B(\mathbf{s}',\mu(\mathbf{s}'),\lambda(\mathbf{s}')) \\
\text{s.t. } d &\geq 0, \qquad \mu(\mathbf{s}') \geq \mu, \qquad \lambda(\mathbf{s}') \geq \lambda \\
\mathbf{s}' &\sim H(\mathbf{s})
\end{aligned}$$

The recursive saddle-path Lagrangian is homogenous of degree one in Lagrnage multiplier associated with the participation constriant of the bank, λ_t, i.e. $W_z^B(\mathbf{s},\mu,\lambda) = \lambda W_z^B(\mathbf{s},\mu,1)$ and $W_z^B(\mathbf{s}',\mu(\mathbf{s}'),\lambda(\mathbf{s}')) = \lambda(\mathbf{s}')W_z^B(\mathbf{s}',\mu(\mathbf{s}'),1)$, where $\gamma_\lambda = \frac{\lambda'}{\lambda}$. Substituting these identities into the Bellman equation and dividing both sides by λ allows us to rewrite it in the "intensive" form reported in the text, where $\mu_\lambda = \frac{\mu}{\lambda}$. ■

Proof of Proposition 4. Recall the statement: When contracts are fully enforceable and banks capital adequacy requirements are not binding, firms operate at the optimal scale $\overline{k}_z(\mathbf{s})$ that solves $E\frac{\partial \tilde{\pi}_z(k,w(\mathsf{s}'))}{\partial k} = 1$. With limited enforceability, binding capital adequacy requirements imply that firms never reach the optimal scale.

When capital adequacy requirements are not binding, the model reduces to the class of contracting problem studied in Cooley, Marimon and Quadrini (2001) and Albuquerque and Hopenhayn (1997). Hence, for the properties presented in the proposition see the proofs therein.

■

Proof of Proposition 5. Recall the statement: With limited enforceability, binding capital adequacy requirements determine the dividend policy of the firm.

When capital adequacy requirements are not binding, the dividend policy of the firm is indeterminate when firms are unconstrained, as any dividend sharing rule is optimal once firms reach the unconstrined status. As far as firms are constrained, they never pay dividends to entrepreneurs. In fact, it is never optimal to do so, as, due to the risk neutrality assumption, ny cash flow is optimally invested in relaxing the borrowing constraint. By contrast, when capital adequacy requirements bind, for uncostrained firms the equation $E_s \sum_{j=s}^{\infty} \beta^{j-s}\tau_j = (1-\gamma)\, k_s^*$ implicitly defines τ_j given a k_s^* defined by $E\left[\frac{\partial \pi_z(k^*,w(\mathsf{s}'))}{\partial k^*}\right] = 0$. ■

Proof of Proposition 6. Recall the statement: The functions $W_z^I(\mathbf{s},\mu)$ and $W_z^B(\mathbf{s},\mu)$ exist and are monotone in μ.

An argument analogous to the one given to prove proposition 1 can be used to establish this result. Consider the operator $\left(W^{I,j+1}, W^{B,j+1}\right) = T\left(W^{I,j}, W^{B,j}\right)$ defined by equations (17) and (18). By applying the Theorem of the Maximum it is straightforward to see that the operator T maps $W^{I,j\prime}s$ and $W^{B,j\prime}s$ that are continuous in μ and z into $W^{I,j+1\prime}s$ and $W^{B,j+1\prime}s$ that are also continuous in μ and z. By Blackwell's sufficient conditions the operator T defines a contraction mapping in the space of continuous functions with the uniform norm. Hence, W^I and W^B exist and are continuous functions (in μ and z).

Let

$$R_z^I(\mu,\mu',\cdot) = \pi_z(k,\mathbf{s}') - (1-\mu)\, d - \beta E\left(\mu(\mathbf{s}') - \mu\right) D_z(k,\mathbf{s}')$$

and

$$Q_z^I(\mu',\cdot) = \beta E \max\left[W_z^B(\mathbf{s}',\mu(\mathbf{s}')), W_z^I(\mathbf{s}',\mu(\mathbf{s}'))\right]$$

Now, it needs to be shown that the operator T maps $W^{I,j\prime}s$ and $W^{B,j\prime}s$ that are nondecreasing in μ into ones that are increasing in μ. To see this, consider two levels of the lagrange multiplier μ, $\mu_1 < \mu_2$. So the question is: If $W^{I,j}$ and $W^{B,j}$ are nondecreasing in μ then will $W^{I,j+1}$ and $W^{B,j+1}$be increasing in μ? The answer is yes since

$$\begin{aligned} W_z^{I,j+1}(\mu_1,\cdot) &= max_{\mu^0 \geq \tilde{\mu}}\{R_z^I(\mu_1,\mu',\cdot) + \beta Q_z^I(\mu',\cdot)\} \\ &< max_{\mu^0 \geq \tilde{\mu}}\{R_z^I(\mu_2,\mu',\cdot) + \beta Q_z^I(\mu',\cdot)\} = W_z^{I,j+1}(\mu_2,\cdot) \end{aligned}$$

A similar argument can be used to establish that $W_z^{B,j+1}(\mu_1,\cdot) < W_z^{B,j+1}(\mu_2,\cdot)$. ■

Proof of Proposition 7. Recall the statement: The exists a generic set of parameters values for which the function $\Phi(\mu, z; \mathbf{s})$ is nondegenerate. In particular, there exists a $\tilde{\mu}$ such that: for all $\mu \geq \tilde{\mu}$, $W_z^I(\mathbf{s},\mu) \geq W_z^B(\mathbf{s},\mu)$;for all $\mu < \tilde{\mu}$, $W_z^B(\mathbf{s},\mu) > W_z^I(\mathbf{s},\mu)$.

A straightforward application of the argument given to prove proposition 2 can be used to establish this result. ■

Recall that with no aggregate uncertainty to solve for equilibrium is equivalent to finding the fixed point of the following operator

$$\mathbf{V}_0^{j+1}(\mathbf{s}) = \mathbf{T}\left(\mathbf{V}_0^j\right)(\mathbf{s})$$

where $\mathbf{V}_0(\mathbf{s}) = \left(V_0^I(\mathbf{s}), V_0^B(\mathbf{s})\right)$.

Proof of Proposition 8. Recall the statement: **T** exists and is monotone decreasing.

Consider the mapping $\mathbf{V}_0^{j+1}(\mathbf{s}) = \mathbf{T}\left(\mathbf{V}_0^j\right)(\mathbf{s})$. By applying the Theorem of the Maximum it is straightforward to see that **T** maps continuous functions into continuous functions. Blackwell's sufficient conditions deliver the monotonicity, which completes the proof. ■

Proof of Proposition 9. Recall the statement: The operator T, given in the individual problems (17) and (18), has a unique fixed point $w \in \Psi$. The optimal policy correspondence $\gamma \in F$, defined by w as in (14), is a continuous function.

We use the Theorem of the Maximum and Banach contraction theorem. Define $S = P \times Z \times N$ and $R = P \times Z \times N \times N$. For $s \in S$, we write $\Sigma(s) = (\{\mathbf{s}\}, \{z, \mu\}, \Omega(\{\mathbf{s}\}, \{z, \mu\}))$, and for $\in R$, we write $\phi(r) = F(\{\mathbf{s}\}, \{z, \mu\}) + \beta F(\{\mathbf{s}'\}, \{z', \mu'\}, G(\mu) \mid z)$. Since G is continuous, F is continuous and Lemma 9.5 in Stokey, Lucas, and Prescott (1989) can be used to conclude that ϕ is a continuous function on a compact domain. Hence, its maximum is attained and, by the Theorem of the Maximum, $T : \Psi \to \Psi$. It is clear that T is a contraction of modulus β. Then, by the Banach Contraction Theorem, T has a unique fixed point $w \in \Psi$.

Also, by the Theorem of the Maximum, the optimal action function γ is an u.h.c. correspondence. Denote by Ψ' the subspace of concave functions of Ψ and by Ψ'' the subspace of strictly concave functions of of Ψ. Now if $f(z, \mu, \cdot) \in \Psi'$ then $Tf \in \Psi''$ given the linearity of the moment utility function and the concavity of the production function. Then by Theorem 9.8 in Stokey, Lucas, and Prescott (1989) the unique fixed point w is in Ψ''. Moreover, since F is concave the optimum is attained at a unique μ'. Hence γ is single valued and, therefore, a continuous function. ■

Proof of Proposition 10. Recall the statement: The function $\Xi : F \to F$ has a fixed point γ^*such that $\Xi(\gamma^*) = \gamma^*$.

To prove the existence of such a fixed point we apply the Schauder fixed point theorem. Note that (i) $\Xi(F)$ is an equicontinuous family, and (ii) $\Xi(F)$ is bounded, since $\|\gamma_\tau\|_\infty \leq \sup_{\mu \in N} |\mu|$, then by Arzela-Ascoli Theorem, $\overline{\Xi(\cdot)}$ is compact.

Define $A = \Xi(F)$ and $C_1 = \overline{co(A)}$. Note that C_1 is the closed convex hull containing A. In a Banach space, as our $(F, \|\cdot\|_\infty)$, if A is a compact set, then the closed convex hull containing A, $\overline{co(A)}$, is also compact. Now consider $\Xi : C_1 \to C_1$. Then $\Xi(C_1)$ is a compact map, hence the Schauder fixed point theorem applies and provides the result. ■

Proof of Proposition 11. Recall the statement: Let $H_{\gamma_0} : P \to P$ be the operator defined in (15). There exists a measure $\Phi^* \in P$ such that $H_{\gamma_0}(\Phi^*) = \Phi^*$.

The proof reduces to check the hypotheses of the Schauder-Tychonoff Theorem. The T^* topology is a locally convex topology and the space of measures P is a compact convex subset of

$$M = \{\mu : \chi = Z \times N \to R \mid \text{is finite regular Borel signed measure}\}$$

in the topology T^*. Since we have proved in proposition 8 that the operator H is a continuous operator, then the required hypotheses hold and then the operator has a fixed point μ^* such that $H_{\gamma_0}(\Phi^*) = \Phi^*$. ■

8.2 Details on the computational algorithm

The steps to solve for the steady state equilibrium are as follows:

1. We guess the wage variable w.

2. We guess the value of a new project V^0.

3. Given w and V^0 , we solve the contract on a grid of points for μ, for each z. Since μ never decreases, we use a backward procedure starting from $\mu = 1$. Grid points are joined with step-wise linear functions.

4. Using the zero profit condition for the intermediary we find the value of a new contract for the owner-manager. If this value is different from the initial guess V^0, we restart the procedure from step 2 until convergence.

5. Given the invariant distribution of firms (which can be determined without iteration given the backward structure of the solution procedure), we check the equilibrium in the labor market. If the demand of labor is different from the supply we restart the procedure from step 1 until convergence.

In computing the equilibrium with aggregate shocks, we use the following steps:

1. We solve the dynamic program of the firm by parametrizing the value of a new project $V^0(\mathbf{s})$ and, for each grid point of μ and for each z, the values $E(\mu'' - \mu)$ and EW. These values are parameterized with linear functions of the following variables:

   (a) the fraction of new projects with high productivity;

   (b) the mean value of μ for constrained firms;

   (c) the mean value of μ for unconstrained firms.

The last three variables are proxies for the distribution of firms. We guess the coefficients of the parameterized functions and, given the parameterized functions, we solve the dynamic problem of the firm and we check for aggregate labor market clearing.

2. We estimate the coefficients of the parameterized functions using the simulated data as in Krusell and Smith (1998). These estimates are used as new guesses for these functions and the procedure is restarted from the previous step until convergence.