MONETARY NON-NEUTRALITY FROM HETEROGENEOUS BELIEFS

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ABSTRACT. In a model where uncertainty about future monetary and fiscal policy has no effect on real allocations under the assumption of a single common probability distribution for policy among all agents, heterogeneity in agent's beliefs about policy does have real effects. Agents in effect bet with each other about future policy by taking leveraged positions in asset markets. For given differences in beliefs as measured by odds ratios, the effect of policy uncertainty on leverage increases as the difference across agents in expected inflation shrinks. The phenomena that arise in this model suggest a route by which beliefs abaout monetary policy before the 2008 crash might have contributed to the increasing leverage before the crash. They also suggest additional reasons why transparency and clear communication by central banks and fiscal authorities are important.

This paper originated in my trying to understand what kind of model would support the view that a period in which monetary policy held nominal interest rates low could generate a bubble-like asset price boom and subsequent crash. Though one often sees claims that a policy of low interest rates in the US laid the foundation for the housing price boom and crash in 2007-9, there are few if any general equilibrium models that support this claim. Monetary policy controls nominal, not real, interest rates. In models without market imperfections or irrationality, the central bank's ability to control nominal rates gives it no influence at all on real rates. Once

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frictions are introduced that give the central bank some, probably temporary, influence on real rates, the nature of those frictions should become central to discussion of the effects of interest rate policy. A model that simply treats the central bank as able to control real rates directly can't be taken seriously.

But it is possible, in a model with no market imperfections and no irrationality, for *uncertainty* about monetary policy to have real effects that look like the housing price boom: highly leveraged investment in a real asset, followed by large-scale wealth redistribution as uncertainty is resolved. The model depends critically on the uncertainty being accompanied by heterogeneity across the population in beliefs about likely outcomes. Without the heterogeneity, the model makes monetary policy and uncertainty about it completely neutral, having no influence on real allocations.

The idea that heterogeneity of beliefs can have major effects on asset markets is far from new. Harrison and Kreps (1978) presented a model in which belief heterogeneity, combined with short sales constaints, affected asset prices. Brunnermeier and Julliard (2008) displayed a model in which belief heterogeneity (with one type of agent being treated as "naive") could have substantial effects on the housing market. Scheinkman and Xiong (2004) survey much of this literature. This paper's contribution is to examine the effects of uncertainty and belief differences about monetary policy, when monetary policy is otherwise completely neutral.

The first section below lays out this model and its results. The next section deconstructs the result. It shows that the model is formally similar to one in which one simply introduces a "horse race" form of extraneous uncertainty and allows betting on it. Bets on monetary policy, though, are most directly implemented via leveraged investment and shorting, which put stresses on the financial system quite different from those generated by pure horse-race type bets. But betting based on heterogeneous beliefs raises some knotty questions for welfare evaluation of policy that are similar, whether the betting is on horses or central bank policy. The final section of the paper contrasts horse races and monetary policy uncertainty, arguing that it is much clearer that monetary policy transparency is a good thing than that horse racing should be outlawed.

I. THE MODEL

There are two periods, two assets — nominal bonds and a real asset with a known rate of return — two representative agents with differing beliefs about future policy, and a government that redeems the stock of bonds in the second period by lump-sum taxation of all agents in equal amounts. Agents have constant relative risk aversion utility. Agents at the initial date $t = 1$ are each endowed with nominal bonds in the amount b_0 and one unit of the real consumption-investment good. Asset markets are competitive, and borrowing, lending, and short-selling of the real asset are permitted.

We index the two agents as a, b. Policy sets the level of lump sum taxation in the second period at one of two levels, τ_f or τ_m, with $\tau_f < \tau_m$. The government fixes a gross interest rate R on bonds in the first period. Agent i places probability π_i on the event that the taxation level will be τ_m. Formally, agent i solves

$$\max_{c_i, c'_{if}, c'_{im}, k_i, b_i} \frac{c_i^{1-\sigma}}{1-\sigma} + \pi_i \frac{c_{i2m}^{1-\sigma}}{1-\sigma} + (1-\pi_i)\frac{c_{i2f}^{1-\sigma}}{1-\sigma} \quad \text{subject to} \tag{1}$$

$$c_i + k_i + \frac{b_i}{p} = 1 + \frac{b_0}{p} \tag{2}$$

$$c_{i2j} = R\frac{b_i}{p'_j} + Ak_i - \tau_j\,, \quad j = m, f\,. \tag{3}$$

Here p is the price level at time 1, p'_j the price level at time 2 with policy choice j. A is the gross rate of return on the real asset and R the gross nominal rate of return on the nominal debt. The government issues no debt and imposes no taxes at time $t = 1$, so

$$b_a + b_b = 2b_0\,. \tag{4}$$

At time 2, the price level is determined by the amount of real resources generated by taxes that are available to redeem the debt:

$$\frac{Rb_0}{p'_j} = \tau_j\,, \quad j = m, f\,. \tag{5}$$

The first-order conditions for the agent's problem are

$$\partial c_i : \qquad c_i^{-\sigma} = \lambda_i \tag{6}$$

$$\partial c_{i2m} : \qquad \pi_i c_{i2m}^{-\sigma} = \mu_{im} \tag{7}$$

$$\partial c_{i2f} : \qquad (1 - \pi_i) c_{i2f}^{-\sigma} = \mu_{if} \tag{8}$$

$$\partial k_i : \qquad \lambda_i = A(\mu_{im} + \mu_{if}) \tag{9}$$

$$\partial b_i : \qquad \frac{\lambda_i}{p} = R\left(\frac{\mu_{im}}{p'_m} + \frac{\mu_{if}}{p'_f}\right)\,. \tag{10}$$

If there were no debt in the model, and thus no uncertainty and no heterogeneity of beliefs, we would have a solution with (dropping i and j subscripts because agents are identical and there is no uncertainty)

$$c_2 = Ak = A(1 - c_1)\,. \tag{11}$$

When debt is present, but there is no uncertainty, this is still the solution. In this case debt and k appear equivalent to the agents, and both must pay the same real return, so

$$\frac{Rp}{p'} = A \tag{12}$$

$$\frac{Rb_0}{\tau} = p'\,, \tag{13}$$

so p and p' are uniquely determined, but changes in τ or b_0 affect only prices, not the real variables c_1, c_2 and k. That is, inflation policy is neutral.

Even if uncertainty is present, so long as there is no heterogeneity in beliefs (i.e., in π_i values), the real allocation remains the same. In this case agents do not see bonds and capital as equivalent. The former is risky, because of inflation uncertainty;

the latter is not. But they choose to make $b_i = b_0$, which results in the uncertainty about the period-2 value of bonds exactly mirroring the uncertainty about period-2 taxes. That is, the government budget constraint (5) in this case eliminates uncertainty about second period consumption. In order for agents to find this pattern of asset holding optimal, it must satisfy the debt first-order condition (10) for both agents. But since both agents have the same beliefs and the same first and second period consumption, (10) has the same form for both agents. That condition gives π an influence on p (not p'_m or p'_f, because they are determined by (5)), but π has no effect on first or second period consumption or on k. In this equilibrium, each agent recognizes that it is optimal to hold the original endowment of debt, b_0, because this perfectly hedges uncertainty about second-period taxes.

But with heterogeneity in beliefs, real allocations are affected by policy uncertainty. The reason is apparent from looking at the b first-order condition (10), with its Lagrange multipliers substituted out:

$$\frac{1}{pc_i^\sigma} = R\left(\frac{\pi_i}{c_{i2m}^\sigma p'_m} + \frac{1-\pi_i}{c_{i2f}^\sigma p'_f}\right). \tag{14}$$

If π's differ across agents, while their c choices are all the same, this equation can't hold. The differing π_i values must be offset on the right-hand side of (14) by differing c_{i2j} values. The agent who sees low inflation (i.e. τ_m) as more likely will see bonds as a better investment. He will therefore buy bonds from the other agent, financing the purchases by lower real investment k. Since there are no constraints on lending or on short selling, the agent seeing bonds as a better investment may end up shorting k, and the other agent may not only sell some of her bond endowment, but sell all of it and short bonds — i.e. engage in nominal borrowing. Since the two agents have different views of the expected return on the bond asset, the agent expecting less inflation will shift his portfolio toward bonds until it becomes, according to his subjective probabilities, so risky that further increases in expected yield by shifting toward bonds are matched by the disutility from the additional risk. The other agent also makes her portfolio riskier. She undoes the hedge against tax uncertainty provided by holding the original endowment b_0, because selling bonds or borrowing to invest in k offers a favorable expected return.

The model is difficult to solve analytically, but simple enough that numerical experiments with it provide a good understanding of how it works. First, we show results in which $\sigma = 1$, i.e. log utility, and $\pi_a = .6 = 1 - \pi_b$. With log utility, agents always choose to consume in the first period half the present value of their wealth, and they see this wealth as 1, the initial endowment. They do not see the initial real debt b_0/p as wealth, because it is perfectly offset by the present value of taxes, regardless of the agent's beliefs about the probabilities of high or low taxes; the tax obligation in the second period exactly matches the real value of b_0/p'_j, regardless of whether $j = m$ or $j = f$.

It may be initially puzzling that agents' perceptions of their own wealth are unaffected by the presence of differing beliefs about p'_j. After all, the agent who thinks high inflation likely will see the availability of other agents whose beliefs lead them to lend at low rates as a boon. Certainly expected lifetime utility is higher when the economy contains other agents perceived as having "mistaken" beliefs. But in this model, without borrowing or short sales constraints, this higher utility does not translate into higher initial wealth. The reason is that betting against other agents is undertaken to the point where *at the margin* the increased risk fully offsets the expected yield. In other words, evaluated at shadow prices, the availability of the bet has zero value.

The tax levels and second-period price levels in this batch of solutions vary, as do the agents' portfolio allocations between capital and bonds. But the second-period consumption outcomes are invariant across solutions. There is a bet available in this economy, on which of two values the price level will take in the second period. It does not matter to the real allocation what the price levels are in the two possible

TABLE 1. Solutions with τ_f and τ_m varying

τ_f	1.0	1.07	1.05	1.095	1.00	1.00	1.00
τ_m	1.1	1.10	1.10	1.100	1.05	1.03	1.01
k_a	−1.600	−6.733	−3.800	−14.067	−3.600	−6.267	−19.600
k_b	2.600	7.733	4.800	15.067	4.600	7.267	20.600
b_a	3.200	8.333	5.400	15.667	5.400	8.333	23.000
b_b	−1.200	−6.333	−3.400	−13.667	−3.400	−6.333	−21.000
p	1.048	1.014	1.023	1.007	1.073	1.084	1.095
p'_f	1.100	1.028	1.048	1.014	1.100	1.100	1.100
p'_m	1.000	1.000	1.000	1.000	1.048	1.067	1.089

TABLE 2. Parameters and variables constant across solutions that vary τ's

R	1.1
σ	1.0
b_0	1.0
A	1.1
π_a	0.6
π_b	0.4
c_a	0.50
c_b	0.50
c_{a2f}	0.44
c_{a2m}	0.66
c_{b2m}	0.44
c_{b2f}	0.66

states — all that matters is that there are two states, and the two agents put different probabilities on the two states.

The amount of leverage and short-selling required to implement the bet with this set of available securities varies tremendously with the values of the second-period taxes and prices in the two states, however. The amount of leverage and short selling *increases* as the variance of the second-period price level decreases. For analyzing the effects of betting based on disagreement about probabilities, the appropriate measure of differences in beliefs is not a difference in expected values, but a measure of the difference in probability measures, for example total variation distance. Total variation distance for a finite measure like this is the sum of the absolute difference in probabilities across the points in the space, for this first solution batch .4 in all cases. This may help to explain why people discussing the run-up to the 2008 crash suggested that low interest rates drove market participants to increase leverage, seeking high yields. If there were investors betting on beliefs about future inflation or interest rates, with disagreements in probability distributions stable while the range of likely values for future inflation and interest rates shrank, increased leverage was a likely outcome.

It might not seem plausible that the distributions agents have for monetary policy quantities would simply scale down by a common factor (which would keep their total variation distance unchanged). One might think that everyone expecting inflation to be likely to remain in a narrow range would mean that people's beliefs would overlap more (total variation distance among beliefs would be smaller). But some people's beliefs about the interest rate and the price level may be "naive", so that their decisions suggest they believe that interest rates or the price level will not change. This is the kind of belief whose implications are explored, e.g. in Brunnermeier and Julliard (2008). If these agents interact with others who believe in greater uncertainty about interest rates and prices, increased leverage with reduced variance of future policy seems quite possible.

TABLE 3. Solutions with σ varying

σ	.5	1	2	10
k_a	-40.716	-19.600	-9.398	-1.470
k_b	41.784	20.600	10.372	2.426
b_a	46.149	23.000	11.819	3.132
b_b	-44.149	-21.000	-9.819	-1.132
p	1.095	1.095	1.095	1.095
p'_f	1.100	1.100	1.100	1.100
p'_m	1.089	1.089	1.089	1.089
c_a	0.466	0.500	0.513	0.523
c_b	0.466	0.500	0.513	0.523
c_{a2f}	0.361	0.440	0.481	0.515
c_{a2m}	0.814	0.660	0.590	0.537
c_{b2m}	0.361	0.440	0.481	0.515
c_{b2f}	0.813	0.660	0.590	0.537

In these first solutions, first period consumption, and hence aggregate savings and investment, are unaffected by belief heterogeneity. Therefore aggregate second-period consumption is also unaffected. When utility is not logarithmic, though, belief heterogeneity does affect real aggregates. Except for σ, the results in Table 3 all have parameters at their values in Table 2. τ_f and τ_m are at the values 1 and 1.01 that generated the last column of Table 1 (so the second column of Table 3 matches the last column of 1).

First-period consumption is positively related to σ, so aggregate investment is negatively related to σ. High risk aversion makes agents who see that through betting they have a high expected second period consumption want to raise their first-period consumption. With low risk aversion, leverage becomes extreme and the agent investing in k does so so enthusiastically that aggregate k is larger than it would be without betting. However these effects are much more modest than the effects of betting on the distribution of asset portfolios and second-period consumption across agents.

For many real assets, single-family houses in particular, shorting the asset is difficult or impossible. We can solve this model with short sales of the real asset forbidden. Table 4 shows solutions with the short sales constraint on k in the first two columns, and a third column with the same parameters as the second but with short sales allowed. The short sales constraint reduces the amount of betting and thus makes second-period consumption vary less across agents and states. Here the values of τ_f and τ_m do matter for the real allocation. The short sales constraint holds betting farther from the unconstrained equilibrium when τ_f and τ_m (and hence p'_f and p'_m) are close to each other.

II. AN ECONOMY WITH HORSE RACING AND NO GOVERNMENT

With no government debt, no taxes and no price level, we intoduce a second-period wager. Either Firefly or Magus will win a horse-race in the second period. Agent a believes Magus will win with probability .6, agent b believes Firefly will win with probability .6. The amount that agent i bets is w_i and the equilibrium odds are q, so that if a's favored horse Magus wins, he collects $w_a q$ from agent b, while if agent b's favorite, Firefly, wins, she collects w_b from agent a. In equilibrium $w_a = w_b$. Now the budget constraints are

$$c_i + k_i = 1\,, \quad i \in \{a, b\} \tag{15}$$

$$c_{a2m} = Ak_a + w_a q \tag{16}$$

$$c_{a2f} = Ak_a - w_a \tag{17}$$

$$c_{b2m} = Ak_b - w_b q \tag{18}$$

$$c_{b2f} = Ak_f + w_b \tag{19}$$

TABLE 4. Effect of a short sale constraint on k

τ_f	1	1	1
τ_m	1.1	1.01	1.01
k_a	0	0	−9.398
k_b	.974	.976	10.372
b_a	1.512	1.534	11.819
b_b	.488	.466	−9.819
p	1.053	1.096	1.095
p'_f	1.100	1.100	1.100
p'_m	1.000	1.089	1.089
c_a	.513	.512	0.513
c_b	.512	.512	0.513
c_{a2f}	.512	.534	0.481
c_{a2m}	.563	.540	0.590
c_{b2m}	.508	.534	0.481
c_{b2f}	.559	.539	0.590

Note: the last column is taken from Table 3, where there is no short sale constraint. All parameters are as in Table 2, except σ=2.

Market clearing requires $w_a = w_b$. We omit tables of results from this model, because they can be described very simply: They deliver real allocations, values of k_i, c_i, and c_{i2j}, that exactly match those in Tables 1 and 3 when the values of σ match. The difference is that here there is no borrowing or lending and no short-selling, just second-period betting.

III. WELFARE ANALYSIS WITH DIFFERENCES OF OPINION

An appealing characteristic of the Pareto criterion for ranking allocations is that no one in an economy should object to shifting to a Pareto-superior allocation from a current one. In this model, both agents value the opportunity to bet. Thus any policy intervention that restricts betting would be opposed, not just by some agent, but by every agent in the economy. However a planner who knew the agents utility functions and maximized some weighted sum of them would, no matter what odds the planner thought applied to the horse race or inflation policy, outlaw betting, either directly for the horse race or indirectly by forbidding asset market transactions in the inflation policy economy. The uncertainty in these two economies is entirely avoidable, without any loss of aggregate economy-wide resources.

Markus Brunnermeier and Wei Xiong, in continuing research, have suggested a welfare criterion with some appeal. They argue that it is natural to rank allocation A as better than allocation B if the sum of expected utilities across agents in the economy is lower under B than under A, according to the probability distribution of every agent in the economy. In other words, if my view is that by betting with you I gain in expected utility less than you will lose, and if you have the same view but in reverse, then the equilibrium with betting is worse than that with betting forbidden. Using this criterion would suggest policies that forbid or inhibit the direct betting in our horse-race economy or the leveraged investment betting in our model with uncertain inflation policy.

My view is that outlawing betting is not good policy. In the case of horse-race bets, where the betting is on artificially generated random events, the argument against a ban is that people enjoy betting. Of course for some it behaves like an addiction, and there is a legitimate policy issue over how to limit the damage from obsessive self-destructive gambling. But gambling is also play; it rehearses the skills needed to make decisions under uncertainty with limited information. People enjoy gambling for some of the same reasons cats enjoy pouncing on flashlight beams — it exercises skills that are important in other areas of life. Outlawing gambling is not just stamping out a spuriously attractive sin; if widely enough pursued, such a ban could lead to a less competent population. This argument, though, is an argument against banning all forms of betting and gambling that reduce welfare by Brunnermeier and Xiong's criterion. The form of the extraneous randomness underlying the betting, and side effects of the betting, may matter, and since different forms of betting are substitutes, there is plenty of room for arguing that specific types of betting should be regulated. It would be a bad idea to ban all of private poker games, dog racing, horse racing, bets on dog and cock fights, bets on athletic contests, and leveraged speculation on monetary policy. But there are legitimate arguments that the negative consequences of some of these forms of gambling are severe enought that channeling gambling into other forms would be good policy.

In the case of betting in financial markets, there is a second reason to be skeptical of policies that attempt to identify and outlaw bets that are bad by the Brunnermeier-Xiong criterion. It will generally be impossible to differentiate such transactions from contracts that arise from legitimate risk-sharing or contracts that deal with agency problems. For example, futures contracts between parties neither of whom has a natural hedging motive for entering the contract are sometimes seen as a pure "speculative" bet that might be outlawed or taxed. But in futures markets, it would not make sense to require that every contract be between a prototypical farmer with a corn crop in the field and a risk-bearing counterparty. For the market to function well, there need to be middlemen who make contracts with natural hedgers, but then lay off some of the risk they have taken on through deals with other non-farmers. Of course in deals between investors where one is laying off risk, one could take the view that the Brunnermeier-Xiong criterion for ruling out the transaction is not met. But to apply the criterion this way requires that the entire balance sheet of each participant must be assessed before deciding whether a trade between people whose business is futures trading, not farming, is "good". This seems impractical. Even if we could, say, identify trades that made each particpant's balance sheet riskier, the arguments against a general ban of gambling listed above would apply.

IV. Do these models have implications for policy and analysis of the 2008 crash?

Even though there is not a good general case for outlawing pure bets that generate risk for both parties, there is quite a strong case for minimizing the extent of betting on inflation policy. There are many other outlets for the desire to gamble for fun, so reducing uncertainty about inflation would not be a big loss on that score. As the model shows, betting on the price level is likely to lead to individuals taking on risky balance sheets. Within the model, these risky balance sheets are simply mechanisms to implement the same bets that could be made directly in a futures market on prices, but in reality implementing bets via leveraged investments carries substantial costs. The model assumes that there is a finite state space and contracts are completely specified for all possible contingencies. In reality, contracts are not completely specified, so that when someone goes heavily short on one asset and long on another, there is enhanced risk that contingencies could arise that require costly resolution in the courts.

If a pure futures market in the price level were available, people could bet without taking on leverage. This supports the idea that futures markets in inflation (or interest rates, or tax rates) should not be banned or restricted. But most people, particularly most people in the housing market, do not have access to futures markets. Furthermore, many of them probably do not see the process of borrowing money to buy a house, and deciding how much to borrow and how much to spend on a house, as involving a bet, so the availability of an organized futures market would probably not change their behavior.

Uncertainty about future monetary and fiscal policy arises in part because the state of the future economy is unknown, even though reaction of policy to the state is well understood. But to the extent that monetary policy is non-transparent, the uncertainty can arise simply from the opacity of the policy process. And future fiscal policy is now uncertain because of political institutions that make it difficult or impossible to reach agreement on policy principles, thus making policy very uncertain when historically rare conditions arise. This paper's model provides support for a monetary policy that minimizes uncertainty via transparency and explicit discussion of the future time path of policy under various contingencies. This is precisely what is done, to a greater or lesser extent, in the inflation policy reports of inflation-targeting central banks. It also supports the importance of central banks' paying attention to communication. Differences of opinion about probabilities of future policy action may arise from people paying limited attention to them, or from the difficulty of incorporating information from disparate sources. Good central bank communication may provide a reference point for the public's beliefs about policy that minimizes belief discrepancies.

For fiscal policy, practical prescriptions suggested by this model are more difficult to see. In this model, as in the real world, there is no sharp distinction between monetary and fiscal policy. In the model, there is a single uncertain policy action that is at once monetary and fiscal. In reality, great uncertainty about fiscal policy, as now exists in many countries, must create uncertainty about inflation, no matter how transparent and communicative is the central bank. One can imagine a new fiscal institution, perhaps with authority to adjust the level of some broad-based tax like a VAT, that is charged with maintaining long-term budget balance in much the same way that the central bank is charged with maintaining long-term price stability. Leeper (2010, section VIII) discusses possibilities for institutional reform of fiscal policy. At the current time, though, the prospects for such reform seem distant.

What about the original motivating question for this line of thinking? Does the model suggest that monetary policy before the crash is to blame for the house price boom and subsequent crash? The model does not provide much support for the notion that low nominal interest rates led to the house price boom. It is true that with short sales constraints, the model raises the first-period price of capital, and with a lower value of R the model would even generate deflation in the second period with $\tau = \tau_m$. However the increase in the price level (which in this simple model is the same as the price of the real capital good) is small. In Table 4 it looks extremely small; it can be larger when τ_f and τ_m are farther apart, but within this model it does not get larger than a few percentage points. On the other hand, that low interest rates, combined with uncertain monetary and fiscal policy, could encourage highly leveraged purchases of housing assets, with purchasers and lenders having different views about the risks they were taking on, is quite consistent with the model.

In an environment where there are traders who see themselves as having an "edge" through better understanding than other market participants of the uncertainties about future interest rates and prices, low and stable interest rates and inflation can lead to increased leverage. Most of the usual arguments that a Fed low interest rate policy dangerously increases leverage rest on treating the Fed as controlling real rates, with low rates encouraging leveraged investment in real assets. This paper's model suggests quite a different mechanism. The Fed controls only nominal rates, but by narrowing the scale of uncertainty about rates, while not eliminating differences of opinion in the market, inflation policy (or interest rate policy) can encourage increased leverage *without* substantially increasing real investment. This is not to suggest that low interest rates and low inflation are in themselves problematic. But central banks should recognize that uncertainty about whether the interest rate is going to be one percent or two percent by the end of the year can be fully as destabilizing to financial markets as uncertainty about whether it will be three percent or six percent. Expectations being "anchored", in the sense that the expected values of future inflation and interest rates are low should not make central banks complacent about the need for providing clear information about the future course of policy.

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