

Central Bank Lending and Money Market Discipline*

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PRELIMINARY

Abstract

This paper provides a theory for the joint existence of lending on a private money market and public lending by a central bank. When banks' investment and balance sheet are private information, the bargaining solution on the money market may eliminate excessive risk taking, at the cost of terminating some projects that would have been continued under full information. Central bank lending solves this inefficiency when it extends collateralized loans at a penalty rate. We solve for the optimal collateral haircut and penalty rate. We show that the joint existence of the money market and central bank lending improves upon a money market alone, or central bank lending alone.

1 Introduction

Implementation of monetary policy relies crucially on the link between the interest rate set by a central bank and other interest rates throughout the economy. Central

*The views expressed in this paper do not necessarily reflect those of the European Central Bank, the Eurosystem, the Federal Reserve Bank of Philadelphia, or the Federal Reserve System.

banks typically aim at steering a short-term (overnight) money market rate, which in turn affects medium- and long-term rates.

The degree to which a central bank successfully controls the target money market rate is, however, different over time and across countries. For example, while the volatility of the overnight rate was very high in the United Kingdom (UK) over the past five years (average standard deviation of daily changes in rates was about 10 basis points), Canada experienced a much lower volatility, with standard deviation being less than half that of the UK. Moreover, while money markets function rather smoothly in normal times, their functioning became severely impaired during the recent financial crisis. The volatility of rates increased substantially in many countries and short-term rates “decoupled” from the longer-term rates.

High volatility of the target rate may pose a problem as it jeopardizes the credibility of the central bank in achieving its goals, complicates its communication as well as its accountability towards the public.¹

Recently, new tools to control the target rate have been introduced. Central banks provide a lending and a deposit facility, where banks can satisfy their liquidity needs or earn interest on their liquidity surplus. These facilities effectively provide the bounds for the equilibrium rate on the money market. The difference between the lending and deposit facility rates defines a corridor for the money market rate and every central bank offering such facilities sets a positive corridor. If the goal is to control the money market rate, however, why don't central banks set the corridor to zero? In other words, why do central banks enable the money market to exist?

We start from the presumption that market discipline plays an important role in the unsecured money market, as argued by Rochet and Tirole (1996). We view market discipline as an *ex ante* provision of incentives to banks to conduct business

¹For instance, one of the explicitly stated objectives of the Bank of England's monetary policy implementation framework is that “overnight market interest rates be in line with the official Bank Rate, ..., with very limited day-to-day or intraday volatility in market interest rates...”. For more details, see “The Framework for the Bank of England's Operations in the Sterling Money Markets,” 2006.

in a safe and sound manner. To model market discipline, we use a framework with bargaining under asymmetric information: borrowing banks know more about the risk of their investments than lending banks do. We assume that borrowers can choose between safe-liquid and risky-illiquid projects. Each project needs to be refinanced or otherwise is liquidated at a cost. If a bank does not have enough funds for refinancing – akin to a liquidity shock – it can obtain refinancing by contacting a lender on the money market.

If the borrower undertakes a risky project and it fails, the lender is not repaid. As a borrower is protected by limited liability, he may take on more risk than what's socially optimal. The bilateral interaction between a borrower and its lender in the over-the-counter money market ensures that the borrower does not take any more risk than is socially desirable. We show that under the bargaining solution, the lender controls the risk taking behavior of the borrower by playing on two margins: either by always refinancing a small amount thus reducing the size of the initial investment, or by refinancing randomly thus rationing the borrower. Such market discipline is successful in eliminating excessive risk taking, yet it is socially suboptimal: in equilibrium, some safe projects will be liquidated.

This gives a role for a central bank lending facility where borrowers can obtain refinancing. Such facility can improve welfare as long as it does not distort the borrowers' incentives. We show that necessary conditions are that the lending facility offers loans against collateral and that the lending rate is strictly above the money market rate. Both elements imply that borrowers face a penalty when accessing the lending facility. If this is not the case, borrowers may take on more risk than desirable and obtain refinancing at the lending facility instead of the money market. The collateral requirement is costly implying that the central bank is unable to achieve the first-best welfare level. Still, a lending facility can improve welfare by preventing the liquidation of the safe project. Moreover, the central bank can offer a policy that ensures that the scale of the initial investment in the safe project is optimal. Therefore, the cost of operating the lending facility only comes from the cost of

pledging collateral. When this cost is small, the central bank solution will be close to the first-best.

We derive the optimal penalty rate and collateral haircut. We show that the penalty rate is given by the return on the safe liquid investment *less* the collateral cost. A lower rate would encourage risk taking, while a higher rate would discourage refinancing.

The notion that unsecured lending on the money market provides incentives for lenders to monitor their borrowers is highlighted in Rochet and Tirole (1996). They consider a model of interbank monitoring in which a lender can, at a cost, reduce a borrower's private benefits from misbehaving. They show that when the cost is sufficiently small, monitoring by lenders reduces the moral hazard problem of the borrowing bank and allows it to invest more than without monitoring. In our model, to ensure that borrowers' investments are safe, lenders may reduce the refinancing amount or refuse to refinance some borrowers. Collateralized lending by a central bank at a penalty rate arises as a means to refinance borrowers rejected in the money market.²

The effects of asymmetric information on the functioning of debt markets were first examined by Stiglitz and Weiss (1981). In their analysis, a lender with market power prefers to ration credit rather than to raise the interest rate to clear away the excess demand since he fears that a higher interest rate is only attractive to riskier borrowers.³ Since in our model, a borrower's type is endogenous, rationing acts as an incentive device that ensures that borrowers only undertake safe investments *ex ante*.

²In a different context, Calomiris and Kahn (1991) show that demandable debt provides incentives for depositors to monitor bankers who have a comparative advantage in making investment decisions but who may abscond with funds entrusted to them.

³See also Arnold and Riley (2008). Armendariz and Gollier (2000) show that formation of peer groups is a way to reduce interest rates and remove credit rationing arising due to adverse selection. Freixas and Holthausen (2005) consider asymmetric information across countries and show that the integration of interbank markets may fail when cross-border information about banks is less precise than home-country information.

Similarly, the role of collateral as a means to screen borrowers of *fixed* types is subject of a large literature (see, for example, Bester, 1985, 1987; Besanko and Thakor, 1987). In our approach, anticipation of refinancing on the money market influences the borrower's choice of his type, and the only signal a lender gets is the size of the loan a borrower requests. Collateral plays a role when borrowing from a central bank as a way of preserving money market discipline.

The remainder of the paper is organized as follows. In Section 2, we describe the model setup. In Section 3, we analyze the problem of the social planner. In Section 4, we analyze the decentralized solution on the money market, focusing on the case in which the risk of banks' investments is private information. In Section 5, we introduce a lending facility of a central bank and derive the optimal lending rate and collateral haircut. In Section 6, we consider extensions of the benchmark model. In Section 7, we offer a discussion of our results. All proofs are in the Appendix.

2 The Model

The economy lasts three periods $t = 0, 1, 2$. It is populated by two risk neutral banks. At $t = 0$, both banks are endowed with an amount K of initial funds. The opportunity cost of using these funds is their gross return $R_m = 1$ at $t = 2$.

At $t = 0$, a randomly selected bank, that we will call the borrower, receives a new investment opportunity: he can invest in a safe or a risky project. The initial investment has to be rolled over in period 1 for the investment to bear fruit in period 2. That is, the initial investment needs refinancing. Refinancing requirements are proportional to the size of the initial investment. If k has been invested in either project, then the refinancing needs equal ρk , where $0 < \rho \leq 1$ can be small. To consider borrowing on the money market, we assume that the borrower is unable to refinance his project on his own, even a fraction of the necessary amount.⁴

⁴Refinancing using own funds for a fraction of the amount does not modify the gist of the argument. In Section 6, we extend our results to the case where banks can save funds they don't

The projects' returns are as follows. If k is invested in the safe project and it is refinanced, then it yields $g(k)$ for sure. We assume that $g(\cdot)$ is increasing and strictly concave and that $g(0) = 0$ and $g'(0) = \infty$. If the project is not refinanced, it can be liquidated to yield k .⁵ Therefore the safe project is riskless and liquid in that (1) it yields a sure return if it is refinanced and (2) it is pure storage otherwise. Alternatively, if k is invested in the risky project and it is refinanced, it yields θk with probability p and 0 otherwise. If the project is not refinanced, it yields 0 at $t = 2$.⁶ We assume that the borrower cannot invest in both projects simultaneously. In this sense, the borrower is choosing an overall riskiness of his investment portfolio.

At $t = 0$, the other bank, that we will call the lender, does not receive any investment opportunity. Therefore, it can use its initial funds to refinance the borrower's project at $t = 1$. Crucially, the choice of the borrower between safe and risky investments is not observable by the lender.

3 Efficient Allocation

We denote by k_s the amount invested in the safe projects and by k_r the amount invested in the risky one. Suppose the borrower's choice is observable. Then a planner who allocates resources across projects solves the following problem, where μ is the probability to invest in the safe project

$$\begin{aligned} \max_{k_s, k_r, \mu} & \mu [g(k_s) - (1 + \rho)k_s] + (1 - \mu) [p\theta k_r - (1 + \rho)k_r] \\ \text{s.t.} & k_r \leq K \text{ and } k_s \leq K \end{aligned}$$

Let k^* be the solution to $g'(k^*) = 1 + \rho$. We assume that $K > k^*$ so that the efficient investment level in the safe project is feasible. Therefore, if the planner invests in

invest and use their savings to refinance at $t = 1$.

⁵Once we consider a deposit facility of a central bank, k could be deposited and yield $R_d k$ instead.

⁶This is for simplicity only. Our results would be qualitatively unchanged if we instead assumed that the recovery value of the risky project which is not refinanced is positive, as long as there is a non-trivial tradeoff between liquidity and return on the two projects for a range of parameters.

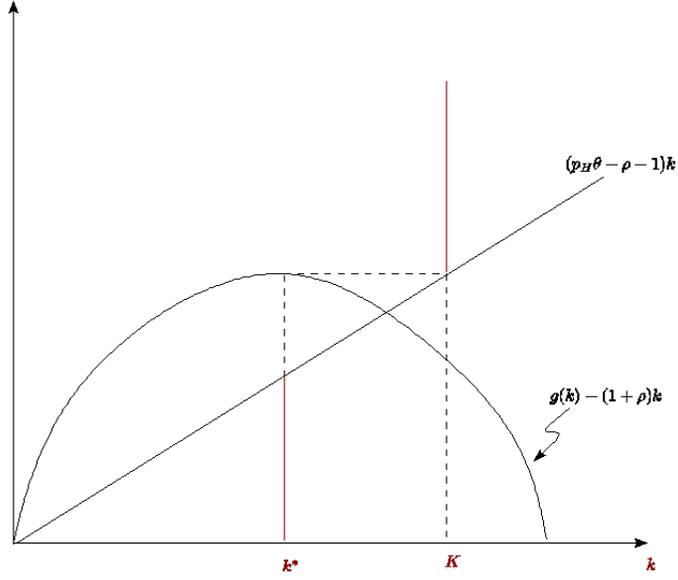


Figure 1: Optimal Allocation

the safe project, he invests k^* . This defines the planner's solution k_s^* . Also, since the return of investing in the risky project is linear, the planner either invests everything or nothing in it. Hence, given he invests in the risky project, he invests $k_r^* = K$ if and only if $p\theta \geq 1 + \rho$ and nothing otherwise.

To determine which project is used, the planner compares the return under the above optimal investment strategy. Therefore, the planner invests k^* in the safe project if and only if

$$g(k^*) - (1 + \rho)k^* \geq [p\theta - (1 + \rho)]K$$

and the planner invests K in the risky project otherwise. The red line in Figure 1 traces out the optimal level of investment as the success probability p varies, where p_H is defined by the above equation holding with equality. Below the line $(p_H\theta - \rho - 1)k$, it is optimal to invest k^* , and above the line it is optimal to invest K .

The planner's decision internalizes all refinancing cost. However, the borrower may not. This leads to a possible wedge between the planner's solution and the decentralization through a money market. We study this next.

4 Equilibrium

4.1 No Private Information

To gain some intuition, consider first the case where the choice of project is observable. In this case, the borrower can offer the lender the interest rate that makes him indifferent between lending the required amount and not lending at all. The break-even rate of interest when the safe project is used is just $R = 1$, since the loan is always repaid. When the borrower invests k_s in the safe project his payoff is $g(k_s) - (1 + R\rho)k_s$.

The lender being risk neutral, the break-even rate of interest when investing in the risky project is $R = 1/p \geq 1$. When the borrower invests k_r in the risky project his payoff is $-k_r + p(\theta k_r - R\rho k_r)$.

Replacing the break even rates in the borrower's payoffs, it is easy to see that his problem yields the efficient allocation. We now turn to the case with private information, when the choice of project is not observable.

4.2 Private Information

We will model the refinancing stage as an over-the-counter market where the borrower directly bargains with the lender over the terms of the loan. A loan is a pair (k, R) , where k is the amount of investment to be refinanced and R is the gross interest rate. As in Rocheteau and Li (2010), we assume that the borrower makes a take-it-or-leave-it offer to the lender. Because the borrower is informed, the offer will reveal some information to the lender and will affect his beliefs about the borrower's initial choice. Therefore there is a multitude of equilibrium of this game that are supported by the appropriate out-of-equilibrium beliefs.

To solve the game, we use the method proposed by In and Wright (2008) and used by Rocheteau and Li (2010). They ingeniously suggest to reverse the sequence of moves, so that the borrower first posts a contract and only then chooses his investment. Since the resulting payoffs of this modified game are identical to the ones of

the original game, the solution to this modified game will also be a solution to the original game.

To make things interesting, we make the following assumption:

Assumption 1 $p(\theta - \rho) > 1$.

In this case, the borrower may want to invest in the risky project whenever the lender expects him to use the safe project. Indeed, if the lender believes that the borrower uses the safe project, he is willing to accept the break-even rate $R = 1$. However, under the assumption above, this may instead induce the borrower to invest in the risky project.

We first show that, in equilibrium, a borrower would not want to post a refinancing contract that would specify a higher amount than what he actually needs.

Proposition 1 *If the borrower posts the contract (k, R) , then the lender knows the initial investment was of size k .*

That is, by posting a contract (k, R) , the lender knows that the borrower invested k and not less. A borrower may have the incentive to post a higher amount than what he actually needs, if the lender then believes that he invested in the safe project. However, this also implies an additional cost of refinancing. We can show that either the payoff of investing in any project is strictly increasing in the investment size, *given* the posted refinancing amount is below a threshold κ (in this case the investment equals the posted refinancing amount), or the borrower would not want to invest in the safe project (since it displays decreasing return to scale), *given* the refinancing amount is above κ . To summarize, the lender can figure out the amount of the initial investment because a borrower who posts a refinancing need above κ necessarily invested in the risky project, and a borrower who posts a refinancing need below κ necessarily invested as much in either the safe or risky project.

We now describe three regimes, Regime I, II and III, and we show in the Appendix that these define four types of equilibrium, as a function of the rate of success p of

the risky project. Regime I and III are the obvious candidates: When p is very large, the risky project succeeds with high probability and the lender expects the borrower to invest in it. Therefore the borrower posts the contract $(K, 1/p)$ and the lender refinances for sure. This is Regime I. On the other end of the spectrum, when p is very small, the risky project's expected return is low and the lender expects the borrower to invest in the safe project instead. Therefore, the borrower posts a contract $(k^*, 1)$ and the lender refinances for sure. This is Regime III. Finally, for intermediate values of p , the borrower's payoff of investing in the risky project when he is refinanced for sure at the rate $R = 1$ dominates his payoff of investing k^* in the safe project. Still, his payoff of investing in the risky project at the break-even rate $R = 1/p$ is lower than investing k^* in the safe project. For these intermediate values, the borrower posting $(k^*, 1)$ cannot signal credibly that he invested in the safe project. To credibly signal that he invested in the safe project, the borrower will either reduce its initial investment (loan size limits), or will accept to be rationed in the sense that his project may not be refinanced. This is Regime II. We describe each regime in detail below, starting from Regime I, moving to Regime III and finishing with Regime II.

4.2.1 Regime I - Risky Project

To describe the parameter space delimiting Regime I, it is first useful to define the success rate p_H that solves

$$(p_H\theta - \rho - 1)K = g(k^*) - (1 + \rho)k^*. \quad (1)$$

In words, given the lender knows the borrower invested in the risky project and accepts to refinance at the rate $R = 1/p_H$, the probability p_H is the success rate of the risky project so that it yields at least as much as the safe project, as shown in Figure 2. Therefore, whenever $p > p_H$, investing K in the risky project dominates any investment in the safe one. As a consequence, whenever $p > p_H$, the lender expects the borrower to invest in the risky project and he accepts any offer $(K, 1/p)$.

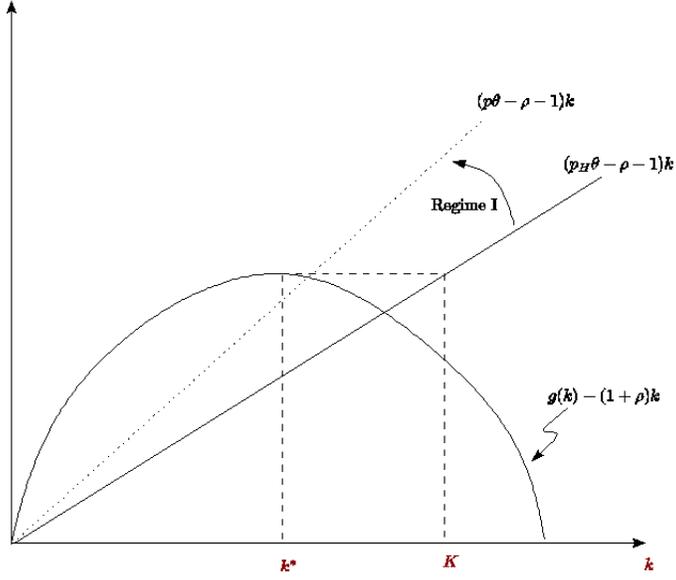


Figure 2: Regime I

4.2.2 Regime III - Safe Project

To describe the parameter space delimiting Regime III, it is first useful to define the success rate p_L that solves

$$(p_L \theta - p_L \rho - 1) k^* = g(k^*) - (1 + \rho) k^* \quad (2)$$

In words, at p_L , given the lender accepts the offer $(k^*, 1)$, the borrower is indifferent between investing k^* in the risky and the safe projects, as shown in Figure 3.

So, for all $p < p_L$, a lender who faces the offer $(k^*, 1)$ knows that the borrower invested in the safe project. This does not mean however that the borrower will invest in the safe project, as it may still be optimal to invest K in the risky project. So, we have to distinguish two cases. First, suppose $p_H < p_L$. Then for all $p < p_H$, the borrower posts $(k^*, 1)$, he invests k^* in the safe project and the lender refinances for sure. Second, suppose $p_L < p_H$. Then for all $p < p_L$, the borrower posts $(k^*, 1)$, invests in the safe project and the lender refinances for sure. For all $p > p_H$, we are in Regime I above.

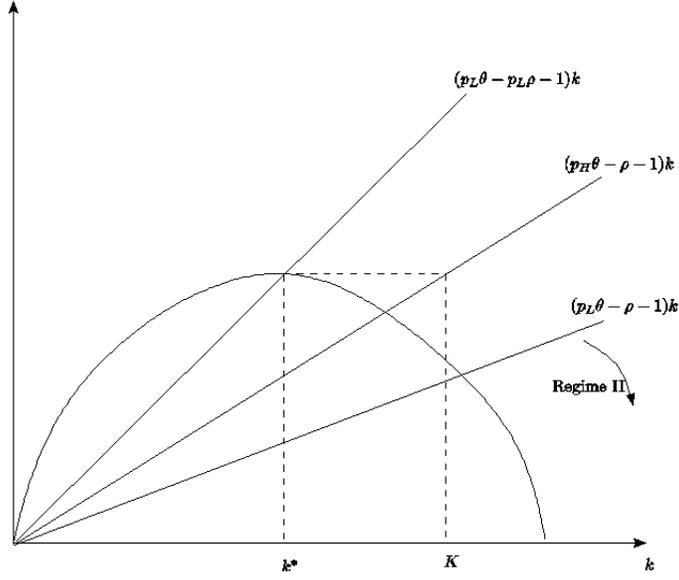


Figure 3: Regime II

4.2.3 Regime II - Safe Project, Rationing

Regime II exists whenever there is a p in the interval (p_L, p_H) .⁷ Given the definition of p_L and p_H , for any $p \in (p_L, p_H)$, we have from (2)

$$(p(\theta - \rho) - 1)k^* > g(k^*) - (1 + \rho)k^* \quad (3)$$

This inequality implies that a borrower posting the contract $(k^*, 1)$ would rather invest in the *risky* project than in the safe one. Figure 4 shows the difference between the two payoffs in red. Also, we obtain from (1)

$$(p\theta - \rho - 1)K < g(k^*) - (1 + \rho)k^*, \quad (4)$$

so that a borrower would rather invest in the safe project, if the lender knows his initial investment's choice and refinances the risky project at the break-even rate $R = 1/p$.

In Regime II, the borrower faces the following conundrum: he would prefer to invest in the safe project if the lender knew this information, but he cannot credibly

⁷In the Appendix, we derive the conditions under which this is the case. Here, it suffices to say that $K = k^*$ and $\theta k^* > g(k^*)$ is sufficient for $p_H > p_L$.

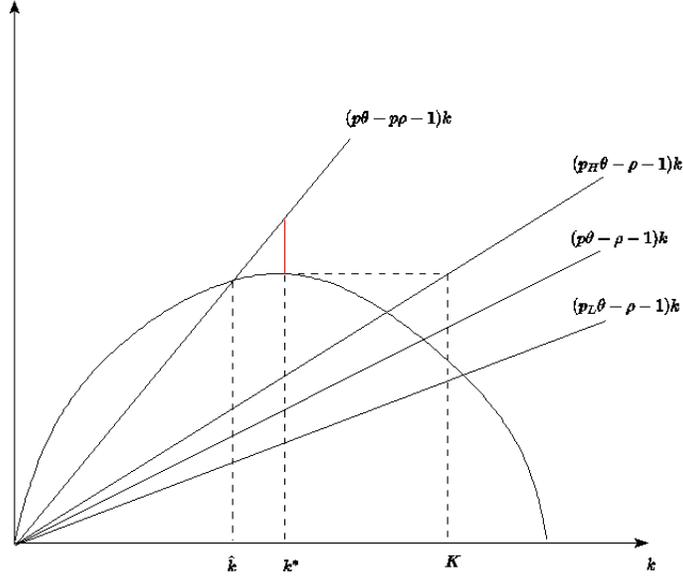


Figure 4: Incentive to Invest in the Risky Technology.

commit to do so if he knows the lender accepts the contract $(k^*, 1)$. In the Appendix, we show that the borrower can use two disciplining devices: First, the borrower can reduce his initial investment in the safe project to a level $\hat{k} < k^*$ such that

$$(p(\theta - \rho) - 1)\hat{k} = g(\hat{k}) - (1 + \rho)\hat{k} \quad (5)$$

With the contract $(\hat{k}, 1)$, the borrower is indifferent between investing in the risky and safe projects and so the lender can safely expect him to invest in the safe project and will refinance for sure.

Second, the borrower can offer the contract $(k^*, 1)$ but he will be rationed by the lender, i.e. the lender will not refinance the project for sure. The rationing intensity will depend on the incentives to invest in the risky project: the higher the payoff from cheating is, the more rationing will take place. Let π denote the probability of refinancing, so that $1 - \pi$ is rationing. Then, there is $\pi^* < 1$ such that expecting refinancing with probability π^* , the borrower is indifferent between investing in either projects. For all $\pi \leq \pi^*$, the borrower prefers investing k^* in the safe project, as

liquidating the risky project is very costly. Probability π^* is defined as⁸

$$\pi^* = \frac{k^*}{p(\theta - \rho)k^* - g(k^*) + (1 + \rho)k^*} \quad (6)$$

However, probability of refinancing π cannot be too low: for instance if π is close to zero, the borrower will certainly prefer to invest \hat{k} in the safe project, post the contract $(\hat{k}, 1)$ and be refinanced for sure. This defines a threshold rationing intensity $\hat{\pi}$ below which the borrower prefers to invest \hat{k} in the safe project, and above which the borrower prefers to invest k^* and be refinanced only with probability $\pi \in (\hat{\pi}, \pi^*)$. Probability $\hat{\pi}$ is set so that

$$\hat{\pi} = \frac{g(\hat{k}) - (1 + \rho)\hat{k}}{g(k^*) - (1 + \rho)k^*} \quad (7)$$

In words, the borrower is indifferent between the contract $(\hat{k}, 1)$ where the lender refinances for sure and the contract $(k^*, 1)$ where the lender refinances with probability $\hat{\pi}$. Finally notice that there is a continuum of rationing equilibria, but they are dominated by the equilibrium where lenders refinance with probability π^* .

Regime II is inefficient, as either borrowers limit their initial investment below the efficient level or they are rationed at the refinancing stage. Rationing creates the scope for a central bank that would lend to a borrower that was denied refinancing on the money market. We study this next.

5 Central Bank Lending Facility

In Regime II, the borrower that posts the contract $(k^*, 1)$ does not always obtain refinancing. This is inefficient as the (safe) project has to be liquidated. In this case, it only returns k^* , while it would return $g(k^*) - \rho k^* > k^*$ otherwise. Therefore, there is scope for a central bank that offers lending to a borrower in case he is denied refinancing on the money market. The issue is the refinancing terms: if they are too generous, the borrower may prefer to invest in the risky project and finance himself

⁸Notice that $\pi^* < 1$ follows from (3).

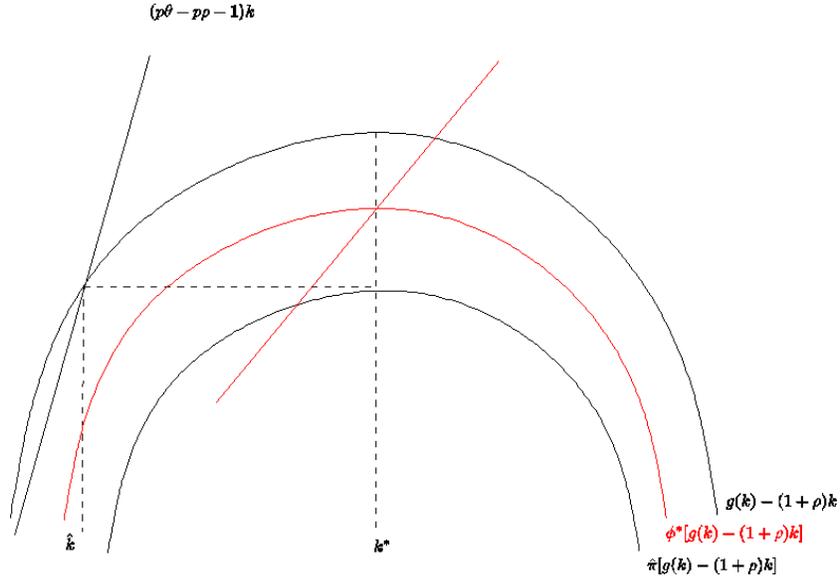


Figure 5: Loan Size Limit and Rationing.

at the lending facility. To the contrary, if they are too strict, the borrower will not use the lending facility and will inefficiently liquidate his investment.

5.1 Uncollateralized Lending Facility

We now assume that the central bank offers a lending facility where the borrower can always obtain refinancing at a gross rate R_l . In addition, we assume that the central bank offers a deposit facility at a gross interest rate $R_d \geq 1$. So, the borrower has to offer an interest rate $R \geq R_d$. For simplicity, we assume that the central bank knows all the parameters in this economy (later we will assume that the CB does not know the realization of p). Aside from these two additions, the model remains the same.

If borrowers undertake the safe project with an initial investment of k , they will be willing to refinance at the lending facility only if R_l satisfies the following condition

$$g(k) - R_l \rho k \geq k \quad (8)$$

since the borrower could always liquidate its project for a value k .

The presence of a lending facility modifies the borrower's payoff of investing in

either project, as well as the size of the investment. In particular, his payoff of investing k in the safe project and having the contract (k, R) being accepted with probability π on the money market is

$$g(k) - k - [\pi R + (1 - \pi) R_l] \rho k \quad (9)$$

Similarly, the borrower's payoff from investing in the risky project is

$$p\theta k - k - p[\pi R + (1 - \pi) R_l] \rho k \quad (10)$$

The size of the investment in the safe project is then k_l such that

$$k_l = \arg \max -k + g(k) - \pi R \rho k - (1 - \pi) R_l \rho k \quad (11)$$

The optimal lending rate in Regime I is any $R_l \geq 1/p$ for $p > p_H$, and the optimal lending rate in Regime III is any $R_l \geq 1$ for $p < p_L$. To see this, first notice that the money market achieves the efficient allocation in Regime I and III. Therefore, the optimal lending facility rate should not distort the money market equilibrium. This is achieved by setting the above rates. Indeed, facing these lending facility rates, the borrower prefers to refinance his investment in the money market since he knows he gets refinanced for sure at a lower rate. In this case, it follows from (9) and (10) that the borrower's payoff is unaffected by the lending facility. So the analysis conducted above still applies.

We now have the following result:

Proposition 2 *Central bank's uncollateralized lending is welfare decreasing.*

In Regime II, $p \in (p_L, p_H)$, the existence of a simple lending facility (sure refinancing with no collateral requirement) eliminates the rationing equilibrium: Indeed, the risky project becomes very attractive since it is always refinanced, independently of what happens in the money market, and it is only repaid when it succeeds, while the safe project has to always be repaid. So, offering uncollateralized lending is welfare decreasing because the only equilibrium is the one where the investment in the safe

project is \hat{k} , but, absent such a facility, we showed that the rationing regime can dominate it.

In the next section, we consider collateralized lending by the central bank.

5.2 Collateralized Lending Facility

We now assume that any borrower can always obtain refinancing at the central bank at a gross rate R_l , as long as they provide proper collateral. We denote the required collateral by C , and the cost to pledge collateral by $\gamma(C)$, which is increasing and convex. Collateral is an illiquid asset that matures too late to be used for refinancing.⁹ The cost makes it unappealing to pledge collateral and explains why borrowers may seek refinancing on the uncollateralized money market. In this sense, the uncollateralized money market is here to save on collateral. Again, for simplicity, we assume that the central bank has no uncertainty regarding the parameters (it knows as much as the other players).

If the borrower refinanced his project at the central bank and the project fails, the central bank seizes the collateral. Otherwise, the central bank hands it over back to the borrower. Since the borrower only gets the collateral back with probability p if he invests in the risky project, this makes the risky investment more costly. Therefore, collateral gives the borrower the incentives to invest in the safe project.

In the Appendix, we show several properties of the equilibrium with collateralized central bank lending. 1) The higher the central bank's haircut, the lower the refinancing cost, R_l is decreasing in haircut. This is to insure that borrowers still want to refinance their safe project in spite of the collateral cost. 2) The higher the haircut, the higher the probability to be refinanced in the money market. Therefore, the optimal haircut policy will trade off refinancing probability in the money market and collateral cost. 3) The optimal haircut policy is such that borrowers are indifferent between investing \hat{k} with sure refinancing and investing k^* with random refinancing. However, the welfare is higher since no safe project is ever liquidated. Still welfare

⁹See Kocherlakota (2001) for collateral in a version of Holmstrom and Tirole (1998).

does not reach its first best level as collateral is costly to pledge. We now give more details regarding the derivation of these results.

If borrowers undertake the safe project with an initial investment of k , they will be willing to refinance at the lending facility only if R_l satisfies the following condition

$$g(k) - R_l \rho k - \gamma(C) \geq k \quad (12)$$

since the borrower could always liquidate its project with a value k and collateral costs $\gamma(C)$. Access to the lending facility modifies the borrower's payoff of investing in either project, as well as the size of the investment. In particular, his payoff of investing k in the safe project and having the contract (k, R) being accepted with probability π on the money market is

$$g(k) - k - [\pi R + (1 - \pi) R_l] \rho k - (1 - \pi) \gamma(C) \quad (13)$$

Similarly, the borrower's payoff from investing in the risky project is

$$p\theta k - k - p[\pi R + (1 - \pi) R_l] \rho k - (1 - \pi) \gamma(C) - (1 - \pi)(1 - p)C \quad (14)$$

where the last term is the loss of collateral when the risky project fails. The size of the investment in the safe project is then k_l such that

$$k_l = \arg \max -k + g(k) - \pi R \rho k - (1 - \pi) R_l \rho k - (1 - \pi) \gamma(C) \quad (15)$$

The optimal lending rate in Regime I is any $R_l \geq 1/p$ for $p > p_H$, and the optimal lending rate in Regime III is any $R_l \geq 1$ for $p < p_L$. To see this, first notice that the money market achieves the efficient allocation in Regime I and III. Therefore, the optimal lending facility rate should not distort the money market equilibrium. This is achieved by setting the above rates. Indeed, facing these lending facility rates, the borrower prefers to refinance his investment in the money market since he knows he gets refinanced for sure at a lower rate. In this case, it follows from (9) and (10) that the borrower's payoff is unaffected by the lending facility. So the analysis conducted above still applies. We now have the following result:

Proposition 3 *Central bank's collateralized lending is welfare improving.*

By choosing the proper collateral haircut and lending rate, the central bank can guarantee that borrowers prefer the rationing regime II(b) while investing k^* in the safe technology. When the central bank requires haircut h when it lends k , the cost incurred by borrowers is $\gamma(h)$. This affects the incentives to invest k^* in the safe project. For example, by setting $R_l(k) = 1$ for all k and haircut $h(k)$, the initial investment size will be $k < k^*$.¹⁰ However, the lending rate below guarantees that borrowers are willing to refinance, *and* that they invest k^* in the safe technology,

$$R_l(k) = \frac{g(k) - k - \gamma(h)}{\rho k} \quad (16)$$

Indeed, this rate leaves them indifferent between refinancing or not at the central bank, and therefore leaves them with the same payoff as if they did not refinance. It therefore, does not affect their incentive to invest k^* in the safe project.

Lenders will believe that borrowers invested in the safe project only if the haircut – defined as $h = C - R_l \rho k$ – is high enough, or

$$h(k) \geq \frac{p\theta k - g(k)}{1 - p} \equiv \bar{h}(k) \quad (17)$$

where $\bar{h}(k)$ denotes the minimum collateral haircut necessary. It is such that the expected loss of collateral $(1 - p)\bar{h}(k)$ equals the difference in expected return. For any higher haircut, the expected loss does not justify the higher expected return of investing in the risky project. Given the haircut policy, borrowers only invest in the safe project if the probability of being refinanced in the money market is lower than π_l^* , where

$$\pi_l^* = \frac{h(k) - \bar{h}(k)}{\rho k + h(k)} \quad (18)$$

For all $\pi > \pi_l^*$, borrowers get refinanced too often and will opt to invest in the risky project. For all $\pi \leq \pi_l^*$, the penalty from using the lending facility is high enough that the borrower will not want to invest in the risky project, running the risk of losing

¹⁰See the Appendix for the details.

their collateral. Obviously, the higher the haircut, the higher can be the probability of being refinanced in the money market. As before, π cannot be too low, and below the threshold $\hat{\pi}$ as defined in (7), the borrower prefers to invest \hat{k} in the safe project and being refinanced for sure. Above $\hat{\pi}$, the borrower prefers to invest k^* and be refinanced only with probability $\pi \in (\hat{\pi}, \pi^*)$.

To summarize the effects of the two instruments in the hand of the central bank: The size of the haircut affects the *incentives* to undertake the safe project, while the lending rate affects the *scale* of the initial investment.

We can now solve for the optimal policy. In the Appendix, we show that the central bank maximizes the social surplus of investing k^* in the safe project, or

$$\begin{aligned} & \max_h g(k^*) - (1 + \rho)k^* - (1 - \pi)\gamma(h) \\ \text{s.t. } & \hat{\pi}_l \leq \pi \leq \pi_l^*(h) = \frac{h(k^*) - \bar{h}(k^*)}{\rho k^* + h(k^*)} \end{aligned}$$

Indeed, the central bank only has a role in Regime II(b). In this regime, borrowers invest k^* in the safe technology and always obtain refinancing, either on the money market with probability $\pi(h)$, or at the lending facility with probability $1 - \pi(h)$. The surplus from investing in the safe project is simply $g(k^*) - (1 + \rho)k^*$. However, borrowers incur the collateral cost $\gamma(h)$ whenever they use the lending facility, and this lowers the surplus. The constraint insures that the market equilibrium is indeed in Regime II(b).

Clearly, the objective function is maximized whenever $\pi = \pi_l^*(h)$, and the convexity of the cost function implies that the objective function is decreasing in haircuts. This implies that the central bank's optimal haircut policy is the minimum haircut level so as to be in Regime II(b). That is, the CB requires h such that $\pi_l^* = \hat{\pi}_l$. Using (7) and (18), we obtain that the optimal haircut $h(k^*)$ is

$$h(k^*) = \frac{G(\hat{k})\rho k^* + G(k^*)\bar{h}(k^*)}{G(k^*) - G(\hat{k})} \quad (19)$$

where $G(k) = g(k) - (1 + \rho)k$. Finally, the central bank offers a lending facility with haircut (19) whenever the optimal collateral costs are low enough. In particular,

the net return of investing k^* in the safe asset should cover the collateral costs, $G(k^*) > \gamma(h)$. Notice that (16) and (19) fully define the optimal policy of the central bank.

6 Extensions

6.1 Savings

In this section, we consider the case in which both a borrowing and a lending bank can store funds between any of the periods. Banks with an investment opportunity can thus save whatever they do not invest at $t = 0$ and use such savings in order to self-finance in period 1.

Consider the problem of a bank which receives an investment opportunity. If its initial endowment is K and the bank chooses to invest $k \leq K$, it saves $K - k$. To make the problem interesting, we assume that $K - k < \rho k$, so that the bank cannot refinance fully using its own funds. Alternatively, ρ could be random and if ρ is large, the bank can't refinance on its own. Therefore, the bank will seek funds on the interbank market, and if it borrows ℓ on the interbank market at a gross interest rate $R \geq 1$, its payoff of investing k in the safe project is

$$g(k) + K - k - (\rho k - \ell) - R\ell$$

where $\rho k - \ell$ is the amount of refinancing using own funds, where $\rho k - \ell \leq K - k$. So the payoff of investing in the safe project is

$$g(k) + K - (1 + \rho)k - (R - 1)\ell$$

Similarly, the payoff of investing k in the risky project is

$$\begin{aligned} & p[\theta k - R\ell] + K - k - (\rho k - \ell) \\ &= p\theta k + K - (1 + \rho)k - (pR - 1)\ell \end{aligned}$$

The analysis is analogous to the case without savings. There are three regimes that depend on p . If $p \geq p_H$, the bank finds it optimal to invest K in the risky

project, not save, and get refinancing ρK at $R = 1/p$. Note that since there is no savings, the solution is the same as before. In particular, p_H is such that the bank prefers to invest in the risky project for any $p \geq p_H$ rather than invest k^* in the safe project and pay $R = 1$ when he refinances. The solution for p_H is

$$p_H \theta K - (1 + \rho) K = g(k^*) - (1 + \rho) k^*$$

so that p_H is as before.

If $p < p_L$, the bank prefers to invest k^* in the safe project and get refinanced at a rate $R = 1$ rather than investing k^* in the risky project and getting refinanced at $R = 1$. Then p_L solves

$$g(k^*) + K - (1 + \rho) k^* = p_L \theta k^* + K - (1 + \rho) k^* - (p_L - 1) \ell$$

where ℓ is given by $\rho k - \ell = K - k$ (this is the amount of borrowing that minimizes money market refinancing and therefore hurts a risky borrower most since he has to use all of his savings). Therefore p_L solves

$$p_L = \frac{g(k^*) - (1 + \rho) k^* + K}{\theta k^* - (1 + \rho) k^* + K}$$

Notice that when $K = k^*$, we obtain the same expression for p_L as before. However, with savings, i.e. $K > k^*$, p_L is higher than with no savings, as investing savings in the risky project is less attractive compared to the safe project and therefore requires a higher probability of success for indifference to hold. As a consequence, the safe project is preferred for a wider range of p than with no savings.

Regime II exists whenever $p \in (p_L, p_H)$. This set is non-empty whenever $p_L < p_H$, and for K close enough to k^* , the inequality will always hold for θ relatively large. In Regime II, at k^* the bank prefer to invest in the risky project when he knows he is refinanced at $R = 1$, or

$$g(k^*) + K - (1 + \rho) k^* < p \theta k^* + K - (1 + \rho) k^* - (p - 1) \ell$$

which, using $\rho k^* - \ell = K - k^*$, simplifies to

$$g(k^*) - (1 + \rho) k^* + K < p [\theta k^* - (1 + \rho) k^* + K]$$

As before, we have two cases. First, the bank could invest \hat{k} in the safe project and get refinanced at $R = 1$ for sure. The bank's payoff is

$$g(\hat{k}) + K - (1 + \rho) \hat{k}$$

and \hat{k} is then defined by

$$g(\hat{k}) - (1 + \rho) \hat{k} + K = p[\theta \hat{k} - (1 + \rho) \hat{k} + K]$$

The second case is when the bank chooses to invest k^* in the safe project and get refinanced with probability $\pi < 1$ at an interest rate $R = 1$. Then π is given by

$$\begin{aligned} & \pi [g(k^*) + K - k^* - (\rho k^* - \ell) - \ell] + (1 - \pi) (K - k^* + k^*) \\ = & \pi [p(\theta k^* - \ell) + K - k^* - (\rho k^* - \ell)] + (1 - \pi) (K - k^*) \end{aligned}$$

On the left-hand side is the payoff of the borrower when he chooses the safe project: When he is refinanced, he uses his savings, but if he is not refinanced, he just liquidates his project and gets his initial investment k^* back. On the right-hand side is his payoff when he invests in the risky project: If the project is not refinanced, he loses the initial investment, but carries over his savings to period 2. Using the expression for ℓ , this simplifies to

$$\pi = \frac{k^*}{p(\theta - \rho)k^* - G(k^*) - (1 - p)(K - k^*)}$$

Notice that if there is no savings ($K = k^*$), then we obtain the same expression for π as before. However, with savings, the probability of getting refinanced on the money market is higher than without, as the safe project is more attractive. In particular, a risky borrower loses his savings with probability p , and lenders know that. Hence, the ability of borrowers to save and use their savings to refinance increases the probability of getting refinancing in the money market.

The random refinancing is preferred to investing \hat{k} and being refinanced for sure whenever $\pi \geq \hat{\pi}$, where $\hat{\pi}$ solves

$$G(\hat{k}) + K = \hat{\pi} [G(k^*) + K] + (1 - \hat{\pi}) K$$

which simplifies to

$$\hat{\pi} = \frac{G(\hat{k})}{G(k^*)}$$

which is the same as before. This means that, with savings, the range of refinancing probability that is consistent with the safe investment at k^* is wider. The rest of the analysis follows the same steps as in the benchmark model.

6.2 Refinancing with multiple lenders and the CB

We now assume that the size of the initial investment is known. However, the quality of the investment remains unknown. When there is no central bank, the analysis is similar to the one in as in the benchmark model. However, when there is a central bank, the borrowing bank could ask for a money market loan and then get additional funds from the central bank.

We first consider the case in which the CB offers unsecured loans and we show that the only equilibrium is one in which the borrowing bank invests $\hat{k} < k^*$. Then we consider the case in which the CB lends against collateral and charges a haircut. We show that the best equilibrium is one in which the CB charges no interest rate penalty, but imposes a haircut and borrowing banks always refinance at the CB. However, this is dominated by the equilibrium with random refinancing.

6.2.1 Unsecured

In regime II, we know that

$$p[\theta k^* - \ell] + (K - k^*) - (\rho k^* - \ell) > g(k^*) + K - k^* - (\rho k^* - \ell) - \ell$$

or, equivalently,

$$p[\theta k^* - \ell] > g(k^*) - \ell$$

where

$$\ell = (1 + \rho) k^* - K$$

Replacing ℓ in the inequality above, we get

$$p\theta k^* + (1 - p) [(1 + \rho) k^* + K] > g(k^*) \quad (20)$$

Now, suppose the bank only borrows ℓ^b in the money market knowing that it will have to borrow L at the CB (alternatively, suppose the bank borrows ℓ^b from one bank at $R = 1$ and L from another at a higher interest $R^l > 1$). Then, the bank only invest in the safe technology if

$$p[\theta k^* - \ell^b - R^l L] \leq g(k^*) - \ell^b - R^l L$$

where

$$\ell^b = (1 + \rho) k^* - L - K$$

Replacing this value in the inequality above, we obtain

$$p\theta k^* + (1 - p) ((1 + \rho) k^* - K) + (1 - p) (R^l - 1) L \leq g(k^*)$$

which is impossible since $R^l \geq 1$ and (20) has to hold. Hence, there is no pair (R^l, L) such that the safe investment is undertaken at its optimal scale, even when the borrowing bank can refinance by getting smaller unsecured loans from multiple lenders (including the CB).

6.2.2 Secured

We now consider a case in which the bank can use the unsecured money market to refinance partially its project and then obtain the remaining funds from the CB through a secured loan.

Suppose the bank only borrows ℓ^b in the money market knowing that it will have to borrow L at the CB an interest R^l against collateral with a haircut h . The cost of pledging collateral with a haircut h is $\gamma(h)$.

The CB seeks to implement the efficient level of investment k^* in the safe project. It does *not* distort the incentives to invest k^* if and only if for $\alpha \in (0, 1)$,

$$(R^l(k) - 1) L(k) + \gamma(h(k)) = \alpha [g(k) - (1 + \rho) k] \quad (21)$$

Indeed, in such a case, the borrower's payoff is

$$\begin{aligned} & g(k) - (1 + \rho)k + K - (R^l(k) - 1)L(k) - \gamma(h(k)) \\ &= (1 - \alpha)[g(k) - (1 + \rho)k] + K \end{aligned}$$

which is maximized at k^* . This implies that the interest rate from (21) is

$$R^l(k) = 1 + \frac{\alpha G(k^*) - \gamma(h)}{L}$$

and the borrower's payoff is

$$(1 - \alpha)G(k^*) + K$$

Then the borrower invests in the safe technology if

$$\begin{aligned} & p[\theta k^* - \ell^b - R^l L] - (1 - p)C + (K - k^*) - (\rho k^* - \ell^b - L) - \gamma(h) \\ &< (1 - \alpha)G(k^*) + K \end{aligned}$$

On the left-hand side is the expected payoff of investing in the risky project, given that an amount ℓ^b is borrowed in the unsecured money market at $R = 1$ and an amount L is borrowed from the CB at R^l against collateral C with a haircut h . If the risky project fails, the borrower loses his collateral, incurring cost $-C$. On the right-hand side is the expected payoff of investing in the safe project. The inequality simplifies to

$$p\theta k^* - (1 - p)C - (1 + \rho)k^* + (1 - p)\ell^b + (1 - pR^l)L - \gamma(h) < (1 - \alpha)G(k^*)$$

where $C = R^l L + h$ and

$$\ell^b = (1 + \rho)k^* - L - K$$

Replacing the expressions for C and ℓ^b we obtain

$$p[\theta k^* - (1 + \rho)k^*] - (1 - p)[h + K + L] < G(k^*)$$

and

$$h > \frac{p\theta k^* - g(k^*)}{1 - p} + (1 + \rho)k^* - L - K \equiv \bar{h}(L)$$

The right-hand side defines the minimum haircut \bar{h} necessary for the borrower to choose the safe project. Notice that if there is no savings so that $k^* = K$ and the borrower refinances entirely at the CB, the minimum haircut is what we found in the benchmark model. The CB's problem is to choose L and h to maximize the borrowers's payoff given he chooses the safe project and operates it at the optimal scale k^* . This adds the following constraints on the CB. First, getting refinancing at the CB has to yield a higher payoff than just liquidating the project, or

$$g(k^*) + K - k^* - (\rho k^* - \ell^b - L) - \ell^b - R^l L - \gamma(h) \geq K$$

which simplifies to

$$G(k^*) \geq \gamma(h) + (R^l - 1)L \quad (22)$$

Second, investing in the safe project k^* and refinancing at the CB at a penalty should yield a higher payoff than investing \hat{k} and being fully refinanced in the unsecured market, or

$$g(k^*) + K - k^* - (\rho k^* - \ell^b - L) - \ell^b - R^l L - \gamma(h) \geq g(\hat{k}) + K - (1 + \rho)\hat{k}$$

which simplifies to

$$G(k^*) - G(\hat{k}) \geq \gamma(h) + (R^l - 1)L \quad (23)$$

Notice that (23) implies (22), so the planner is only constrained by (23). Also, combining (21) with (23) we obtain

$$G(k^*) - G(\hat{k}) \geq \alpha G(k^*)$$

so that

$$1 - \alpha \geq \frac{G(\hat{k})}{G(k^*)}$$

Hence, the CB solves

$$\begin{aligned} \max \quad & g(k^*) + K - (1 + \rho)k^* - (R^l - 1)L - \gamma(h) + T \\ \text{s.t.} \quad & 1 - \alpha \geq \frac{G(\hat{k})}{G(k^*)} \\ & h > \frac{p\theta k^* - g(k^*)}{1 - p} + (1 + \rho)k^* - L - K \\ & L \leq \rho k^* \end{aligned}$$

where $T = (R^l - 1)L = \alpha G(k^*) - \gamma(h)$ is the transfer of the CB revenue back to the borrower. Since the CB's objective function is decreasing in both L and h , it will set $h = \bar{h}(L)$, so that its problem becomes

$$\begin{aligned} \max \quad & G(k^*) + K - \gamma(\bar{h}(L)) \\ \text{s.t.} \quad & G(k^*) - [(R^l - 1)L + \gamma(\bar{h}(L))] \geq G(\hat{k}) \\ \bar{h}(L) = & \frac{p\theta k^* - g(k^*)}{1 - p} + (1 + \rho)k^* - L - K \end{aligned}$$

The CB wants to set L as high as possible since the minimum haircut is decreasing in L . The reason is that collateral is costly for the borrowers that undertook the risky project. Therefore asking for a lot of collateral penalizes only them and allows for a reduction in the haircut which is imposed on all banks (not only those who invested in the risky project). The highest L is achieved when $R^l = 1$ (from the first constraint). Then the borrower refinances his entire project at the CB, i.e. $L^* = \rho k^*$, there is no refinancing in the money market, and the CB asks for

$$\bar{h}(L^*) = \frac{p\theta k^* - g(k^*)}{1 - p} + k^* - K$$

This is inefficient and borrowers prefer the random refinancing equilibrium.

6.3 Uncertainty

1) The Central Bank Ignores p

Conjecture: if the CB does not know p , i.e., it does not know the regime, it should set R_l equal to (16) and stand ready to refinance up to k^* .

7 Discussion

It is tempting to interpret each regime as different economic environments. Good times are when p is relatively high, and it is optimal that banks take some risk. Every market participants recognize that risk is taken, however the chance of failure is small enough that they are willing to refinance all projects. In this case, the central

bank lending facility is not used, as is usually the case in normal times. When p declines and crosses the threshold into Regime II, risk taking becomes an issue and market discipline eliminates it. In this case, the central bank facility is used, in proportion to the rationing intensity. Interestingly, many banks were cut-off from the euro-area money market during the crisis, at a time where it was uncertain whether banks were risky or not, and access to the lending facility increased.

One could also think of K as related to bank size: large banks have larger investment capabilities and therefore larger liquidity needs. In our model, a larger K induces more investment in the risky technology more often: i.e. given p , it can be optimal that large banks exploit their size to invest in the constant return to scale, risky project, while it is optimal for small banks to invest in the safe project. For a given change in p , our analysis suggests that relatively small banks will be cut-off from the money market first, while large bank can still obtain funding on the money market. Figure 5 illustrates this point. [add]

Also, notice that ρ could be linked to the aggregate demand for liquidity in the money market. Interestingly, as the liquidity need increases, i.e. ρ edges higher, the penalty rate of the central bank should optimally decline. This rationalizes central banks' practices during times when liquidity is scarce. The refinancing parameter ρ can be also be interpreted as a liquidity shock. Suppose that instead of knowing ρ in period 0, borrowers only know the expected value $E\rho = E(\rho)$ and invest k^* as a function of $E\rho$. In period 1, their liquidity shock ρ is realized and they have to refinance ρk^* .

In our analysis, we ruled out the case where borrowers would invest k^* , refinance up to $k < k^*$ in the money market and the rest in the standing facility. It is certainly feasible to study this case and it could yield interesting results. For instance, the lending facility is a relatively nice outside option to borrowers and its availability might induce them to invest in the risky technology. The central bank could then introduce haircuts (or higher rates) for higher amounts. We leave this for future research.

We also leave for future research the task of embedding this framework in a dynamic setting where banks would trade money and invest it into assets of different quality. This is a daunting task, as the tools to study dynamic bargaining under private information have yet to be developed.

References

- [1] Acharya, V., D. Gromb, and T. Yorulmazer (2008). “Imperfect Competition in the Interbank Market for Liquidity as a Rationale for Central Banking,” Working Paper, London Business School.
- [2] Arnold, L. and J. Riley (2008). “On the Possibility of Credit Rationing in the Stiglitz-Weiss Model,” forthcoming in the *American Economic Review*.
- [3] Armendariz, A. and C. Gollier (2000). “Peer Group Formation in an Adverse Selection Model,” *Economic Journal* 110, 632-643.
- [4] Berentsen, A and C. Monnet (2008). “Monetary Policy in a Channel System,” *Journal of Monetary Economics* 55, 1067-1080.
- [5] Besanko, D. and A. Thakor (1987). “Collateral and Rationing: Sorting Equilibrium in Monopolistic and Competitive Credit Markets,” *International Economic Review* 28, 671-689.
- [6] Bester, H. (1987). “The Role of Collateral in Credit Markets with Imperfect Information,” *European Economic Review* 31, 887-899.
- [7] Bester, H. (1985). “Screening vs. Rationing in Credit Markets with Imperfect Information,” *American Economic Review* 75, 850-855.
- [8] Bindseil, U. and J. Jablecki (2010). “The Optimal Width of the Standing Facility Corridor,” mimeo, European Central Bank.
- [9] Calomiris, C. and C. Kahn (1991). “The Role of Demandable Debt in Structuring Optimal Banking Arrangements,” *American Economic Review* 81, 497-513.
- [10] Freixas, X. and C. Holthausen (2005). “Interbank Market Integration under Asymmetric Information,” *Review of Financial Studies* 18, 459-490.

- [11] Freixas, X. and J. Jorge (2008). “The Role of Interbank Markets in Monetary Policy: A Model with Rationing,” *Journal of Money, Credit and Banking* 40, 1151-1176.
- [12] Holmstrom, B. and J. Tirole (1998). “Private and Public Supply of Liquidity,” *Journal of Political Economy* 106, 1-40.
- [13] King, T. (2008). “Discipline and Liquidity in the Interbank Market,” *Journal of Money, Credit and Banking* 40, 295-317.
- [14] Kocherlakota, N. (2001). “Risky Collateral and Deposit Insurance,” *B.E. Journal of Macroeconomics* 1.
- [15] Rochet, J. C. and J. Tirole (1996). “Interbank Lending and Systemic Risk,” *Journal of Money Credit and Banking* 28, 733-762.
- [16] Stiglitz, J. E. and A. Weiss (1981). “Credit Rationing in Markets with Imperfect Information,” *American Economic Review* 71, 393-410.

Appendix A

A.1 Condition under which $p_L < p_H$.

Given (1) and (2), we obtain the following expressions for p_H and p_L , where $G^* = g(k^*) - (1 + \rho)k^*$

$$\begin{aligned} p_H &= \frac{G^* + (1 + \rho)K}{\theta K} \\ p_L &= \frac{G^* + k^*}{(\theta - \rho)k^*} \end{aligned}$$

Therefore, $p_H - p_L$ is given by

$$p_H - p_L = \frac{G^* + (1 + \rho)K}{\theta K} - \frac{G^* + k^*}{(\theta - \rho)k^*}$$

which sign is given by

$$g^*(k^*)(\theta - \rho)k^* + (1 + \rho)(\theta - \rho)k^*(K - k^*) - (g^*(k^*) - \rho k^*)\theta K$$

If $K = k^*$, then this becomes

$$-g(k^*)\rho k^* + \rho\theta k^{*2}$$

and therefore it is enough that

$$\theta k^* > g(k^*)$$

for the $p_L < p_H$. Therefore for K close to k^* and θ relatively large, the inequality will still hold.

A.2 Initial Investment

Given a contract (k, R) , we denote by $\eta \in [0, 1]$ the lender's belief that the borrower invested in the safe project, and π denotes the borrower's belief that the lender will refinance his project.

We first show that given (k, R) , the lender can infer that the initial investment, in either project, is k . Given he posts the contract (k, R) , a borrower surely invested less

than k , since he needs to be fully refinanced in order to carry his project to fruition. Therefore, we can restrict our attention to initial investment $\tilde{k} \leq k$. The payoff of a borrower who invested $\tilde{k} \leq k$ in the safe project is, given he expects refinancing with probability π ,

$$\begin{aligned} & -\tilde{k} + \pi [g(\tilde{k}) - R\rho k] + (1 - \pi) \tilde{k} \\ = & \pi [g(\tilde{k}) - \tilde{k} - R\rho k] \end{aligned}$$

Clearly, this is a function of $g(\tilde{k}) - \tilde{k}$. We let k_1 be the solution to $g'(k_1) = 1$. Therefore, either a borrower invests $\tilde{k} = k_1$ if $k_1 < k$ or he invests $\tilde{k} = k$ in the safe project. The payoff of a borrower who invested in the risky project $\tilde{k} \leq k$ is

$$-\tilde{k} + \pi p [\theta \tilde{k} - R\rho k]$$

The borrower only invests in the risky project if π is such that $\pi p (\theta - R\rho) > 1$ which means that the return is increasing in \tilde{k} . So, if the borrower gets refinancing when he posts (k, R) , it is optimal for him to also invest $\tilde{k} = k$. Therefore, when the lender observes the contract (k, R) he can infer the size of the initial investment: It is k_1 in the safe project if $k_1 \leq k$ and it is k in the risky and safe projects in all other cases. However, given he invests in the safe project, a borrower will never invest more than k^* . Recall that k^* is defined as $g'(k^*) = 1 + \rho$, so that $k^* < k_1$. Therefore, we are always in the case where the lender can perfectly infer the initial investment perfectly.

A.3 Derivation of the Equilibrium

The payoff of the borrower who invests k in the safe project and posts a contract (k, R) that he believes is accepted with probability π is

$$-k + \pi [g(k) - R\rho k] + (1 - \pi) k$$

Under the same condition, his payoff when he invests k in the risky project is

$$-k + \pi p (\theta k - R\rho k)$$

Therefore lenders' beliefs η that the borrower invested in the safe project, given the contract (k, R) are as follows

$$\begin{aligned} \frac{(1-\pi)}{\pi}k + g(k) - R\rho k > p(\theta k - R\rho k) & \quad \text{then } \eta = 1 \\ . = . & \quad \text{then } \eta \in (0, 1) \\ . < . & \quad \text{then } \eta = 0 \end{aligned}$$

Also, the lender's payoff if he accepts the offer (k, R) to refinance is given by

$$-\rho k + \eta R\rho k + (1-\eta)pR\rho k$$

The lender has to incur his own investment costs $-\rho k$, but he gets reimbursed for sure if the borrower invested in the safe project and only when the risky project succeeds otherwise. The lender's payoff if he rejects is just 0. Therefore the borrower's beliefs about getting refinancing given the contract (k, R) are as follows

$$\begin{aligned} \eta + (1-\eta)p > \frac{1}{R} & \quad \text{then } \pi = 1 \\ . = . & \quad \text{then } \pi \in (0, 1) \\ . < . & \quad \text{then } \pi = 0 \end{aligned}$$

We define U_s^* as

$$\begin{aligned} U_s^* &= \max_{(k,R)} g(k) - (R\rho + 1)k \\ \text{s.t.} \quad & R \geq 1 \\ & g(k) - R\rho k \geq p(\theta - R\rho)k \end{aligned}$$

In words, U_s^* is the utility level that a borrower can achieve by investing in the safe project under the constraint that the he has a lower payoff from investing in the risky project under the same term R .

A Nash equilibrium of the subgame that follows an offer (k, R) is a pair (π, η) such that these sets of conditions are satisfied. Since the payoff is non-positive if the project is not refinanced, $\pi = 0$ and $k = 0$ is always an equilibrium (autarky). For

$\pi > 0$, there are six possible cases, $\pi = \eta = 1$, $(\pi, \eta) \in (0, 1)^2$, $(\pi, \eta) \in \{1\} \times (0, 1)$, $(\pi, \eta) \in (0, 1) \times \{1\}$, and $(\pi, \eta) \in (0, 1) \times \{0\}$ and $(\pi, \eta) = \{1\} \times \{0\}$. We study each in turn.

Given beliefs (π, η) are affected by the posted contract, the contract (k, R) proposed by the borrower solves

$$\begin{aligned} & \eta(k, R) \{-k + \pi(k, R) [g(k) - R\rho k] + (1 - \pi(k, R)) k\} \\ & + (1 - \eta(k, R)) [-k + \pi(k, R) p(\theta k - R\rho k)] \\ & \text{subject to } 0 \leq k \leq K \end{aligned}$$

CASE 1: $\pi = \eta = 1$. In this case, the definition of beliefs implies that

$$g(k) - R\rho k > p(\theta - R\rho) k$$

and

$$R > 1$$

So, given (π, η) , the contract (k, R) proposed by the borrower solves

$$\begin{aligned} & -k + g(k) - R\rho k \\ & \text{subject to } 0 \leq k \leq \min\{K, k_1\} \end{aligned}$$

The objective function is decreasing in R , so that the borrower will want to set R as low as possible such that $R = 1$, in this case, the solution is k^* if

$$g(k^*) - R\rho k^* > p(\theta k^* - R\rho k^*) \tag{A.1}$$

or this constraint binds otherwise. Therefore the solution when $\pi = \eta = 1$ achieves at least the supremum of U_s^* .

CASE 2: $(\pi, \eta) \in (0, 1)^2$. Since $\eta \in (0, 1)$, we have

$$\frac{(1 - \pi)}{\pi} k + g(k) - R\rho k = p(\theta k - R\rho k)$$

or

$$\pi = \frac{1}{1 + p(\theta - R\rho) - [g(k)/k - R\rho]} \tag{A.2}$$

Since $\pi \in (0, 1)$, this requires that

$$0 < \frac{1}{1 + p(\theta - R\rho) - [g(k)/k - R\rho]} < 1$$

Hence we need (from the < 1 inequality)

$$g(k) - R\rho k < p(\theta - R\rho)k \quad (\text{A.3})$$

so that the return on the risky project is higher when the borrower is refinanced. And from the > 0 inequality, we need

$$1 + p(\theta - R\rho) > g(k)/k - R\rho \quad (\text{A.4})$$

and since (A.3) must hold, (A.4) always holds. Replacing (A.2) in the objective function of the borrower, we get obtain the following problem

$$\begin{aligned} \max_{k, R} & -k + \frac{p(\theta - R\rho)k^2}{k + p\theta k - g(k) + R\rho(1 - p)k} \\ \text{subject to} & 0 \leq k \leq K \end{aligned}$$

The borrower's objective function is unambiguously decreasing in R . This implies that the optimal offer is $R = 1$. And since $\pi \in (0, 1)$, we have

$$\eta = \min \left\{ \frac{1 - pR}{(1 - p)R}, 1 \right\}$$

Given $R = 1$, we obtain $\eta = 1$. Therefore replacing $R = 1$, the borrower's problem becomes

$$\begin{aligned} U(k) &= -k + \frac{p(\theta - \rho)k^2}{(1 + \rho)k + p(\theta - \rho)k - g(k)} \\ \text{subject to} & 0 \leq k \leq \min\{K, k_1\} \end{aligned}$$

The objective function is decreasing in k . Taking its derivative with respect to k , we obtain, after some manipulations,

$$\frac{\partial U(k)}{\partial k} = -1 + x(k) - \frac{(1 + \rho) + p(\theta - \rho) - g'(k)}{4p(\theta - \rho)} x(k)^2$$

where

$$x(k) = \frac{2p(\theta - \rho)k}{(1 + \rho)k + p(\theta - \rho)k - g(k)}$$

Therefore the derivative is of the form $ax^2 + bx - 1$. So a zero to this equation is defined by k such that

$$x(k) = \frac{-1 \pm \sqrt{\frac{-(1+\rho)+g'(k)}{p(\theta-\rho)}}}{-\frac{(1+\rho)+p(\theta-\rho)-g'(k)}{2p(\theta-\rho)}}$$

Notice that $x(k)$ is only well defined for $g'(k) \leq 1 + \rho$. However, in this case, $a < 0$ in the quadratic function and hence, there is no $x > 0$ such that the first order condition is non-negative. Therefore, the objective function is decreasing in k . However from (A.2) the probability π is also decreasing in k , as $g(\cdot)$ is strictly concave. Therefore k will be decreased up to the point where $\pi = 1$. Therefore the supremum of the problem when $(\eta, \pi) \in (0, 1)^2$ will yield a payoff that is at most U_s^* , since it corresponds to the solution when $(\pi, \eta) = (1, 1)$ and here it cannot be achieved.

CASE 3: $(\pi, \eta) \in \{1\} \times (0, 1)$. Then we consider the case where $(\pi, \eta) \in \{1\} \times (0, 1)$. Since $\eta \in (0, 1)$, we have

$$\frac{(1-\pi)}{\pi}k + g(k) - R\rho k = p(\theta - R\rho)k$$

or

$$g(k) - R\rho k = p(\theta - R\rho)k \tag{A.5}$$

and since $\pi = 1$, we have

$$\eta + (1 - \eta)p > \frac{1}{R}$$

Given $(\pi, \eta) \in \{1\} \times (0, 1)$, the contract (k, R) proposed by the borrower solves

$$\begin{aligned} & \max_{k, R} \eta(k, R) \{-k + g(k) - R\rho k\} \\ & + (1 - \eta(k, R)) [-k + p(\theta k - R\rho k)] \\ & \text{subject to } 0 \leq k \leq K \end{aligned}$$

and using (A.5), we obtain

$$\begin{aligned} & \max_{k, R} -k + p(\theta k - R\rho k) \\ & \text{subject to } 0 \leq k \leq K \text{ and (A.5)} \end{aligned}$$

Given R , there is only one k that solves (A.5), and this level of capital is decreasing in R . Since $-k + p(\theta k - R\rho k) \geq 0$, the borrower maximizes his payoff by reducing R as much as possible, to the point where

$$\eta + (1 - \eta)p = \frac{1}{R}$$

Therefore the supremum of the borrower's objective in the case $(\pi, \eta) \in \{1\} \times (0, 1)$ is at most equal to the supremum when $(\pi, \eta) \in (0, 1)^2$, which here cannot be achieved. But we saw that the later is at most the supremum when $(\pi, \eta) = (1, 1)$, which again cannot be achieved. Therefore the supremum of the borrower's objective in the case $(\pi, \eta) \in \{1\} \times (0, 1)$ is again at most equal to U_s^* .

CASE 4: $(\pi, \eta) \in (0, 1) \times \{1\}$.

Now we consider the case where $(\pi, \eta) \in (0, 1) \times \{1\}$. Since $\eta = 1$, we have

$$\frac{(1 - \pi)}{\pi}k + g(k) - R\rho k > p(\theta - R\rho)k$$

and since $\pi \in (0, 1)$, we have

$$\eta + (1 - \eta)p = \frac{1}{R}$$

Since $\eta = 1$ in this case, it follows from the above equality that $R = 1$.

Given (π, η) , and since $\eta = 1$, and $R = 1$ the contract offered by the borrower simply solves

$$\begin{aligned} \max_k \pi(k, 1) [g(k) - (1 + \rho)k] \\ \text{subject to } 0 \leq k \leq K \end{aligned}$$

and since $R = 1$, we get that $\eta = 1$ iff

$$\pi [g(k) - (1 + \rho)k] > -k + \pi p(\theta - \rho)k \tag{A.6}$$

Let us define π^* as

$$\pi^* = \frac{k^*}{p(\theta - \rho)k^* - g(k^*) + (1 + \rho)k^*} \tag{A.7}$$

So that either $\pi > \pi^*$ and the RHS is increasing in k so that k is chosen so that this constraint binds and $k < k^*$. In this case the borrower's objective is at most the one

for the case $(\pi, \eta) \in (0, 1)^2$, which itself is at most U_s^* . When the constraint bind, the solution is \hat{k} that solves

$$g(\hat{k}) - (1 + \rho)\hat{k} = -\hat{k} + p(\theta - \rho)\hat{k} \quad (\text{A.8})$$

and $\hat{k} < k^*$.

Otherwise π is such that

$$\pi \leq \pi^* \leq 1$$

in which case the solution is k^* and $R = 1$. Since $k^* < K$, the borrower's payoff is at most $U_s(k^*) = \pi^* [g(k^*) - (1 + \rho)k^*]$.

CASE 5: $(\pi, \eta) \in (0, 1) \times \{0\}$. Since $\eta = 0$, we have

$$\frac{(1 - \pi)}{\pi}k + g(k) - R\rho k < p(\theta - R\rho)k$$

or

$$k < \pi [p(\theta - R\rho)k - g(k) + R\rho k + k]$$

Hence, we need

$$p(\theta - R\rho)k \geq g(k) - R\rho k - k$$

and so it must be that the LHS is positive, or $\theta > \rho R$. Also,

$$\pi > \frac{k}{p(\theta - R\rho)k - g(k) + R\rho k + k}$$

where the RHS is decreasing in k by concavity of the function $g(\cdot)$. Now since $\pi \in (0, 1)$, we have

$$\eta + (1 - \eta)p = \frac{1}{R}$$

and as $\eta = 0$, we obtain $R = 1/p$. In this case, the borrower's problem then becomes

$$\max_k \quad -k + \pi(k, 1/p)(p\theta - \rho)k \quad (\text{A.9})$$

subject to $0 \leq k \leq K$

Then the solution is $k = K$ iff

$$\pi > \frac{1}{p\theta - \rho}$$

and

$$\pi > \frac{1}{p\theta - \rho - g(k)/k + \rho/p + 1}$$

and $k = 0$ otherwise. The solution to (A.9) is at most as high as the solution to the borrower's problem when $\pi = 1$, the solution of which we consider next.

Case 6: $(\pi, \eta) = (1, 0)$. Since $\pi = 1$, we have that $\eta = 0$ if and only if

$$g(k) - R\rho k < p(\theta k - R\rho k)$$

and since $\eta = 0$, $\pi = 1$ if and only if $pR > 1$. In this case, the borrower's problem becomes

$$\begin{aligned} \max_{(k, R)} & -k + p[\theta - R\rho]k \\ \text{s.t.} & 0 \leq k \leq K \end{aligned}$$

Hence, the borrower sets R to the infimum $R = 1/p$ and $k = K$ if

$$p\theta \geq 1 + \rho$$

and in this case the borrower's objective value at the supremum is U_r^*

$$U_r^* = [p\theta - (1 + \rho)]K.$$

This completes the analysis of the five cases. Notice that, truly, three cases remain: 1, 4 and 6. Indeed, the solution for case 1 dominates the solution for case 2 and 3, and the solution for case 6 dominates the one for case 5. Therefore, these cases 1, 4 and 6 define three regimes, which existence depends on the parameters of the model:

(1) invest $k_s^* = \arg \max U_s^*$ in the safe project and get refinancing for sure, with a payoff U_s^* , and

(2) invest k^* in the safe project but get refinancing with probability π^* defined in (A.7) with a payoff $U_s(k^*)$

$$U_s(k^*) = \pi^* [g(k^*) - (1 + \rho)k^*]$$

(3) invest k_r^* only in the risky project and get refinancing for sure, with a payoff U_r^* , where

$$U_r^* = [p\theta - (1 + \rho)] K$$

We now derive the set of model parameters under which each regime exists.

Regime I: $p > p_H$ where p_H solves

$$(p_H\theta - \rho - 1) K = g(k^*) - (1 + \rho) k^*$$

Then the borrower invests K in the risky project and get refinanced for sure. p_H is the probability of success such that $U_r^* = U_s^*$, such that the borrower is indifferent between investing in the risky or the safe project. So, for any success probability above p_H , the borrower will choose the risky project. However, for any probability below p_H , the borrower will prefer to invest in the safe project.

Regime III: $p < p_L$ where p_L solves

$$-k^* + p_L(\theta - \rho) k^* = g(k^*) - (1 + \rho) k^* \quad (\text{A.10})$$

Then the borrower invests k^* in the safe project and get refinanced for sure. p_L is the probability of success such that the borrower get the same payoff from investing in the safe and risky project while the lender believes he invested in the safe one. For any $p < p_L$, the borrower will therefore invest in the safe project and the lender will refinance for sure. This is case 1: in this case, recall that (A.1) must hold and since $R = 1$, we obtain the definition of p_L .

Regime II: $p \in (p_L, p_H)$. This regime corresponds to case 4 above. In case 4, there are two possibilities: Either $\pi > \pi^*$ and the investment constraint binds. The solution is then at most the solution of U_s^* where $\pi = 1$ and the constraint binds. In this case the borrower invests $\hat{k} < k^*$ that satisfies (A.8) and posts $(\hat{k}, 1)$. Since $\hat{k} < k^*$, and p_L is defined by (A.10), it must be that $p > p_L$. Otherwise $\pi \leq \pi^*$ in which case the solution is k^* and $R = 1$. Since $k^* < K$, the borrower's payoff is at most $U_s(k^*) = \pi^* [g(k^*) - (1 + \rho) k^*]$.

To know the borrower's choice of investment, we need to compare $U_s^*(\hat{k})$ and $U_s(k^*)$. That is, a borrower can either post the contract $(k^*, 1)$ or $(\hat{k}, 1)$. If he posts the contract k^* then with $R = 1$, the borrower gets a higher payoff from investing in the risky project and $\pi^* \in (0, 1)$, to prevent him to do so. If he posts the contract \hat{k} , then the lender knows that he invested in the safe project, as the payoff of investing \hat{k} in the risky is the same. So the lender refinances for sure.

Let us denote by $G_s(k) = g(k) - (1 + \rho)k$. First notice that in the current case 4, $U_s(k^*) = \pi^*G_s(k^*)$ where $\pi \leq \pi^*$ and $U_s^*(\hat{k}) = G_s(\hat{k})$. Since $G_s(k^*) > G_s(\hat{k})$ and $\pi \geq 0$, there exists $\hat{\pi}$ such that $\hat{\pi}G_s(k^*) = G_s(\hat{k})$ and naturally

$$\hat{\pi} = \frac{G_s(\hat{k})}{G_s(k^*)} < 1$$

Now, since $\pi \leq \pi^*$, we have two cases: **Regime II(a)** either $\hat{\pi} > \pi^*$, in which case $U_s(k^*) < U_s^*(\hat{k})$ and the borrower prefers to post $(\hat{k}, 1)$ and get refinancing for sure.

Regime II(b) Or $\hat{\pi} \leq \pi^*$, in which case $U_s(k^*) \geq U_s^*(\hat{k})$ and the borrower prefers to post $(k^*, 1)$ and get refinancing with probability π^* .

We now study the existence of both regimes. Since $\pi^* < 1$, we must have $p > p_L$. This also guarantees that $\pi^* > 0$. Also, at $p = p_L$ we have $\pi^* = 1$. Notice that p_L is such that $\hat{k} = k^*$, in which case we also have $\hat{\pi} = 1$. So, for all $p < p_L$, we are in Regime III above. For $p = p_L$, we can be in either Regime II(a) or II(b). Both π^* and $\hat{\pi}$ are decreasing in p . Therefore, either there is no $\bar{p} > p_L$ such that $\pi^* > \hat{\pi}$, or there is such a \bar{p} such that for all $p > \bar{p}$, we are in Regime II(b). The existence of Regime II(b) will depend on the concavity of the function $g(\cdot)$. If $G_s(\cdot)$ is rather flat so that $G_s(\hat{k})$ is relatively close to $G_s(k^*)$, $\hat{\pi}$ will be large and Regime II(a) will prevail. If $G_s(\hat{k})$ is very concave so that $G_s(\hat{k}) \ll G_s(k^*)$, then Regime II(b) is more likely.

Example: Consider $g(k) = k^\alpha/\alpha$ and $\alpha < 1$. Then $k^{*\alpha-1} = 1 + \rho$ and $\hat{k}^{\alpha-1} = \alpha[p\theta + (1-p)\rho]$. The expressions for $\hat{\pi}$ and π^* are then

$$\hat{\pi} = \frac{\hat{k} \hat{k}^{\alpha-1} - \alpha(1 + \rho)}{k^* k^{*\alpha-1} - \alpha(1 + \rho)}$$

and

$$\pi^* = \frac{\alpha}{\alpha p(\theta - \rho) - k^{*\alpha-1} + \alpha(1 + \rho)}$$

Therefore,

$$\hat{\pi} = \alpha \frac{\hat{k}}{k^*} \frac{p(\theta - \rho) - 1}{(1 - \alpha)(1 + \rho)}$$

and

$$\pi^* = \frac{\alpha}{\alpha p(\theta - \rho) - (1 - \alpha)(1 + \rho)}$$

Since $\hat{k} < k^*$, to check that $\hat{\pi} < \pi^*$, it is enough to verify that

$$\alpha \frac{p(\theta - \rho) - 1}{(1 - \alpha)(1 + \rho)} \leq \frac{\alpha}{\alpha p(\theta - \rho) - (1 - \alpha)(1 + \rho)}$$

Simplifying this expression we find that a sufficient condition for $\hat{\pi} < \pi^*$ is

$$p(\theta - \rho) - 1 \leq \frac{(1 - \alpha)}{\alpha} (1 + \rho)$$

which will hold if α is close to zero and the function $G(\cdot)$ is very concave.

A.3.1 $\hat{\pi}$ is decreasing in p

We compute $\partial \hat{\pi} / \partial p$. The sign is given by

$$\text{sign} [\partial \hat{\pi} / \partial p] = \text{sign} [-1 + p(\theta - \rho)] \frac{\partial \hat{k}}{\partial p} + (\theta - \rho) \hat{k}$$

From (??) we obtain

$$\begin{aligned} g'(\hat{k}) \frac{\partial \hat{k}}{\partial p} &= [p(\theta - \rho) + \rho] \frac{\partial \hat{k}}{\partial p} + (\theta - \rho) \hat{k} \\ g'(\hat{k}) \frac{\partial \hat{k}}{\partial p} &= \frac{g(\hat{k})}{\hat{k}} \frac{\partial \hat{k}}{\partial p} + (\theta - \rho) \hat{k} \\ \left[g'(\hat{k}) - \frac{g(\hat{k})}{\hat{k}} \right] \frac{\partial \hat{k}}{\partial p} &= (\theta - \rho) \hat{k} \end{aligned}$$

so that by concavity of $g(\cdot)$, we obtain $\frac{\partial \hat{k}}{\partial p} < 0$. Therefore,

$$\begin{aligned}
\text{sign} [\partial \hat{\pi} / \partial p] &= \text{sign} \left\{ [-1 + p(\theta - \rho)] \frac{\partial \hat{k}}{\partial p} + (\theta - \rho) \hat{k} \right\} \\
&= (\theta - \rho) \hat{k} \text{sign} \left\{ \frac{p(\theta - \rho) - 1}{g'(\hat{k}) - g(\hat{k})/\hat{k}} + 1 \right\} \\
&= (\theta - \rho) \hat{k} \text{sign} \left\{ 1 - \frac{p(\theta - \rho) - 1}{g(\hat{k})/\hat{k} - g'(\hat{k})} \right\} \\
&= (\theta - \rho) \hat{k} \text{sign} \left\{ \frac{g(\hat{k})/\hat{k} - g'(\hat{k}) - [p(\theta - \rho) - 1]}{g(\hat{k})/\hat{k} - g'(\hat{k})} \right\}
\end{aligned}$$

but $g(\hat{k})/\hat{k} = p(\theta - \rho) - 1$ by definition and so

$$\text{sign} [\partial \hat{\pi} / \partial p] = \text{sign} \left\{ \frac{-g'(\hat{k})}{g(\hat{k})/\hat{k} - g'(\hat{k})} \right\} < 0$$

B Central Bank Corridor

B.1 Uncollateralized Lending Facility

In Regime II, some borrowers may invest k^* but will have to liquidate their investment as they cannot get refinancing. This gives a role for a central bank facility that provides refinancing. So, we now assume that the central bank offers a lending facility where any borrower can always obtain refinancing at a gross rate R_l . In addition, we assume that the central bank offers a deposit facility where depositors get a gross interest rate $R_d \geq 1$. The interest rate offered by borrowers has to be $R \geq R_d$. For simplicity, we assume that the central bank has no uncertainty regarding the economy. Aside from these two additions, the model remains the same. However, since it modifies the outside option of the borrower, the standing facility will affect its incentive to invest in the risky project and we need to go through the derivation of the equilibrium once more.

Given a contract (k, R) is refinanced with probability π , the borrower's payoff of

investing in the safe project is

$$\begin{aligned} & -k + \pi [g(k) - R\rho k] + (1 - \pi) [g(k) - R_l\rho k] \\ = & g(k) - k - [\pi R + (1 - \pi) R_l] \rho k \end{aligned} \quad (\text{A.11})$$

where

$$g(k) - R_l\rho k \geq k \quad (\text{A.12})$$

since the borrower could always liquidate its project with a value k . The borrower's payoff from investing in the risky project under the same condition as above is

$$\begin{aligned} & -k + \pi p [\theta k - R\rho k] + (1 - \pi) p [\theta k - R_l\rho k] \\ = & p\theta k - k - p [\pi R + (1 - \pi) R_l] \rho k \end{aligned} \quad (\text{A.13})$$

We again have four regimes which now depend on R_d and R_l (where the difference in the equation follow naturally from our definition of the interest rates). Now \hat{k} is defined as

$$g(\hat{k}) - (1 + R_d\rho) \hat{k} = -\hat{k} + p(\theta - R_d\rho) \hat{k} \quad (\text{A.14})$$

In regimes I and III the borrower was getting refinanced for sure and the first best allocation is achieved. Therefore, setting $R_d = 1$ and $R_l(p) = R(p)$, the central bank does not affect the equilibrium and there is no fundamental change in the analysis. Hence, we obtain with a corridor as well,

Regime I: $p > p_H$ then the borrower invests K in the risky project and the lender always accepts the contract $(K, 1/p)$.

Regime II: $p < p_L$ then the borrower invests k^* in the safe project and the lender always accepts the contract $(k^*, 1)$.

However, the analysis in Regime IV is now altered and we go through it again.

Regime IV: $p \in (p_L, p_H)$

In our previous analysis, this corresponds to the case where $(\pi, \eta) \in (0, 1) \times \{1\}$.

Returning to the definition of the beliefs, since $\eta = 1$ we have

$$g(k) - k - [\pi R + (1 - \pi) R_l] \rho k > p\theta k - k - p [\pi R + (1 - \pi) R_l] \rho k$$

and since $\pi \in (0, 1)$, we have

$$\eta + (1 - \eta)p = \frac{R_d}{R}$$

so that with $\eta = 1$ we must have $R = R_d = 1$. Given (π, η) , the contract $(k, 1)$ proposed by the borrower solves (since $\eta = 1$)

$$\begin{aligned} & \max_k -k + \pi(k, 1)[g(k) - \rho k] + (1 - \pi(k, 1))[g(k) - R_l \rho k] \\ & \text{subject to } 0 \leq k \leq K \end{aligned}$$

Suppose the central bank sets (this is the maximum rate under which a safe project holder wants to refinance at the bank)

$$R_l(k) = \frac{g(k) - k}{\rho k} \tag{A.15}$$

Then the contract solved by the borrower solves

$$\begin{aligned} & \max_k \pi(k, 1)[g(k) - (1 + \rho)k] \\ & \text{subject to } 0 \leq k \leq K \end{aligned}$$

Absent any constraint, the solution to the borrower's problem given he believes his project will be refinance with probability π is k^* as the solution to

$$g'(k) = 1 + \rho \tag{A.16}$$

Since $R = R_d = 1$, we get that $\eta = 1$ iff

$$g(k) - k - [\pi + (1 - \pi)R_l]\rho k > p\theta k - k - p[\pi + (1 - \pi)R_l]\rho k \tag{A.17}$$

However, notice that for any $k > \hat{k}$, this inequality cannot be satisfied: (A.17) holds with equality at \hat{k} and $\pi = 1$, and is violated for all $k > \hat{k}$. Furthermore, decreasing π does not reverse the inequality since the RHS is decreasing more slowly in π than the LHS. Therefore, the solution to the borrower's problem is $\hat{k} < k^*$. The borrower then sets $R = R_d$. We conclude that the central bank following such a policy eliminates Regime II(b). Therefore by refinancing for sure albeit at a penalty rate, the central bank can actually decrease welfare.

B.2 Collateralized Lending Facility

We now assume that the central bank offers a lending facility where any borrower can always obtain refinancing at a gross rate R_l , as long as they provide proper collateral. Collateral is an illiquid asset that matures too late to be used for refinancing. We denote the required collateral by C and the cost of pledging collateral by $\gamma(C)$. We assume that $\gamma(0) = 0$, $\gamma'(C) \geq 0$ and $\gamma''(C) \geq 0$. Also, we assume the cost is small enough that the central bank will still want to operate a collateralized lending facility.

As a first pass, we abstract from the reasons why the interbank market is uncollateralized, but it is easy to see that borrowers may want to save on the collateral cost. Collateral modifies the borrower's incentives in the following way: He always gets his collateral back when he invests in the safe project. However he only gets it back with probability p , when he invests in the risky project. Therefore, given a contract (k, R) is refinanced with probability π , the borrower's payoff of investing in the safe project is unchanged and equals

$$\begin{aligned} & -k + \pi [g(k) - R\rho k] + (1 - \pi) [g(k) - R_l\rho k - \gamma(C)] \\ = & g(k) - k - [\pi R + (1 - \pi) R_l] \rho k - (1 - \pi) \gamma(C) \end{aligned} \quad (\text{A.18})$$

where

$$g(k) - R_l\rho k - \gamma(C) \geq k \quad (\text{A.19})$$

since the borrower could always liquidate its project with a value k rather than using the lending facility. However the borrower's payoff from investing in the risky project is affected by the collateral requirement in the following way,

$$\begin{aligned} & -k + \pi p [\theta k - R\rho k] + (1 - \pi) \{-C + p[\theta k - R_l\rho k + C] - \gamma(C)\} \\ = & p\theta k - k - p[\pi R + (1 - \pi) R_l] \rho k - (1 - \pi)(1 - p)C - (1 - \pi)\gamma(C) \end{aligned} \quad (\text{A.20})$$

We again have four regimes which now depend on R_d and R_l (where the difference in the equation follow naturally from our definition of the interest rates). Now \hat{k} is defined as

$$g(\hat{k}) - (1 + R_d\rho)\hat{k} = -\hat{k} + p(\theta - R_d\rho)\hat{k} \quad (\text{A.21})$$

In regimes I and III the borrower was getting refinanced for sure and the first best allocation is achieved. Therefore, setting $R_d = 1$ and $R_l(p) \geq R(p) = 1$, the central bank does not affect the equilibrium and there is no fundamental change in the analysis. Hence, we obtain with a corridor as well,

Regime I: $p > p_H$ then the borrower invests K in the risky project and the lender always accepts the contract $(K, 1/p)$.

Regime III: $p < p_L$ then the borrower invests k^* in the safe project and the lender always accepts the contract $(k^*, 1)$.

However, the analysis in Regime II is now altered and we go through it again.

Regime II: $p \in (p_L, p_H)$

In our previous analysis, this corresponds to the case where $(\pi, \eta) \in (0, 1) \times \{1\}$. Returning to the definition of the beliefs, since $\eta = 1$ we have

$$g(k) - k - [\pi R + (1 - \pi) R_l] \rho k > p \theta k - k - p [\pi R + (1 - \pi) R_l] \rho k - \nu(\pi) C$$

where $\nu(\pi) = (1 - \pi)(1 - p)$. Notice that the collateral cost cancels out. Also, since $\pi \in (0, 1)$, we have

$$\eta + (1 - \eta) p = \frac{R_d}{R}$$

so that with $\eta = 1$ we must have $R = R_d = 1$. Given (π, η) , the contract $(k, 1)$ proposed by the borrower solves (since $\eta = 1$)

$$\begin{aligned} & \max_k -k + \pi(k, 1) [g(k) - \rho k] + (1 - \pi(k, 1)) [g(k) - R_l \rho k - \gamma(C)] \\ & \text{subject to } 0 \leq k \leq K \end{aligned}$$

Suppose the central bank sets (this is the maximum rate under which a safe project holder wants to refinance at the bank)

$$R_l(k) = \frac{g(k) - k - \gamma(C)}{\rho k} \tag{A.22}$$

Then the contract solved by the borrower solves

$$\begin{aligned} U_l(\pi) &= \max_k \pi(k, 1) [g(k) - (1 + \rho) k] \\ & \text{subject to } 0 \leq k \leq K \end{aligned} \tag{A.23}$$

Absent any constraint, the solution to the borrower's problem given he believes his project will be refinanced with probability π is k^* . Since $R = R_d = 1$, we get that $\eta = 1$ iff

$$g(k) - k - [\pi + (1 - \pi) R_l] \rho k > p\theta k - k - p[\pi + (1 - \pi) R_l] \rho k - \nu(\pi) C \quad (\text{A.24})$$

As we showed in the previous section, if $C = 0$, this equation is never satisfied for $k > \hat{k}$. Also, it will not be satisfied if $C > 0$ is too small. For instance, it is not satisfied when $C = (R_l - 1) \rho k$, i.e. when the collateral just covers the interest payments. Also, notice that when π decreases, the RHS decreases more rapidly than the LHS only if $C > \rho k (R_l - 1)$. Therefore, if $C \leq (R_l - 1) \rho k$, (A.24) will not hold for any $k > \hat{k}$ even if π is lowered. Hence we need to require $C > (R_l - 1) \rho k$. On the other hand (A.24) is always satisfied for $C \rightarrow \infty$ as $\nu(\pi) > 0$. Re-arranging (A.24), we obtain that $\eta = 1$ iff

$$g(k) - p\theta k - (1 - p) \rho k R_l + (1 - p) C > \pi (1 - p) [(1 - R_l) \rho k + C]$$

Since $C > (R_l - 1) \rho k$, the RHS is positive. The LHS is also positive whenever

$$C > R_l \rho k + \frac{p\theta k - g(k)}{1 - p} \quad (\text{A.25})$$

Let $h(k)$ be the haircut on collateral for a loan of size k so that $C = R_l \rho k + h(k)$. The LHS positive means

$$h(k) > \frac{p\theta k - g(k)}{1 - p} \equiv \bar{h}(k) > 0 \quad (\text{A.26})$$

Equation (A.26) gives us the minimum collateral haircut $\bar{h}(k)$ necessary to obtain $\eta = 1$. Using (A.26) and $C = R_l \rho k + h(k)$, equation (A.24) yields:

$$\begin{aligned} \pi_l^* &= \frac{h(k)}{\rho k + h(k)} + \frac{g(k) - p\theta k}{(1 - p)(\rho k + h(k))} \\ &= \frac{h(k) - \bar{h}(k)}{\rho k + h(k)} \end{aligned} \quad (\text{A.27})$$

so that for all $\pi > \pi_l^*$, (A.24) is violated at k^* and the solution to the borrower's problem is $\hat{k} < k^*$. For all $\pi \leq \pi_l^*$, the constraint does not bind at k^* and k^* is the

solution to the borrower's problem. The borrower then sets $R = R_d = 1$. By requiring a higher haircut, the central bank can increase the probability of being refinanced in the money market.

As already noted, $\hat{k} < k^*$. Therefore the borrower can either post the contract $(\hat{k}, 1)$ and be refinanced for sure with a payoff \hat{U}_l , or the contract $(k^*, 1)$ and be refinanced with probability π_l , with a payoff $U_l^*(\pi_l)$. Notice that $U_l^*(1) > \hat{U}_l$ and that $U_l^*(0) < \hat{U}_l$. Therefore, there is $\hat{\pi}_l < 1$ such that $U_l^*(\hat{\pi}_l) = \hat{U}_l$. Then we have two cases: Either **Regime II(a)** where $\hat{\pi}_l > \pi_l^*$, in which case $\hat{U}_l > U_l^*(\pi_l^*)$ and the borrower prefer to post the contract $(\hat{k}, 1)$ and get sure refinancing. Or, **Regime II(b)** where $\hat{\pi}_l \leq \pi_l^*$, in which case $\hat{U}_l < U_l^*(\pi_l^*)$ and the borrower prefer to post the contract $(k^*, 1)$ and get random refinancing.

B.3 Central Bank Interest and Haircut Policies

The existence of both regimes II(a) and II(b) now depend on the central bank policy. In our current environment, if $\gamma(C)$ is sufficiently small, it is optimal to be in regime II(b) as the investment is the planner's solution k^* , which is refinanced for sure (randomly in the money market, but for sure at the lending facility).

Notice that while \hat{k} is not affected by the corridor policy, k_l is. Also, R_l and h affects the probability of getting refinancing in the market in Regime II(b). Therefore the central bank will want to set R_l and h to maximize the refinancing rate, and the initial investment k_l . In Regime II(b), the central bank solves

$$\begin{aligned} & \max_{R_l, h} U_l^*(\pi) + T & (A.28) \\ s.t. & \quad g(k^*) - R_l \rho k^* - \gamma(C) \geq k^* \\ & \quad \hat{\pi}_l \leq \pi \leq \pi_l^* \end{aligned}$$

where T is a transfer to all users of the facility. T equals to the interest payments the central bank received, $(1 - \pi)(R_l - 1)\rho k^*$.¹¹ We first want to show that (A.22) is an

¹¹In equilibrium the borrowers choose safe investment implying that there is no default and collateral is never seized by the central bank.

optimal interest rate schedule: First, recall from (A.12) that R_l cannot be set too large, as otherwise the facility is not used, and we need to impose $g(k) - R_l \rho k - \gamma(C) \geq k$. When this inequality binds, borrowers actually get the *same payoff* as in the case where there is no facility. However, since the project is not liquidated, the overall resources in the economy with the standing facility are higher. Second, with R_l set according to (A.22) borrowers still invest k^* in Regime II(b). Therefore, this level of interest rate achieves the constrained efficient investment level and this is the best policy that the CB can use in Regime II(b). In equilibrium the effective rate at the lending facility is therefore

$$R_l = \frac{g(k^*) - k^* - \gamma(C)}{\rho k^*}. \quad (\text{A.29})$$

Now, the CB affects the existence of Regimes II(a) and II(b) by choosing C (or h). The higher the haircut h the more likely Regime II(b) is. For simplicity, we assume that $\gamma(C) \equiv \gamma(C - R_l \rho k)$ so that γ is actually a function of haircuts. Using a slight abuse of notation, we denote it $\gamma(h)$.

Replacing in (A.28) the value for the transfer T , the expression for U_l^* from (A.23), and R_l using (A.29), the central bank's problem becomes

$$\begin{aligned} \max_h \quad & g(k^*) - (1 + \rho) k^* - (1 - \pi(h)) \gamma(h) \\ \text{s.t.} \quad & \hat{\pi}_l \leq \pi \leq \pi_l^*(h) = \frac{h(k^*) - \bar{h}(k^*)}{\rho k^* + h(k^*)} \end{aligned} \quad (\text{A.30})$$

Clearly, the objective function is maximized whenever $\pi = \pi_l^*(h)$. The derivative of the objective function then is

$$\begin{aligned} & \pi_l^{*'}(h) \gamma(h) - (1 - \pi_l^*(h)) \gamma'(h) \\ = & \frac{\rho k^* + \bar{h}(k^*)}{\rho k^* + h(k^*)} \left[\frac{\gamma(h)}{\rho k^* + h(k^*)} - \gamma'(h) \right] < 0 \end{aligned}$$

where the inequality follows from convexity as $\frac{\gamma(h)}{\rho k^* + h(k^*)} < \frac{\gamma(h)}{h(k^*)} < \gamma'(h)$. Hence, the objective function is decreasing in haircuts. So the CB will set the haircut at the minimum level so as to be in Regime II(b). That is, the CB requires h such that

$\pi_l^* = \hat{\pi}_l$. Recall that $\hat{\pi}_l$ solves

$$\hat{\pi}_l = \frac{g(\hat{k}) - (1 + \rho)\hat{k}}{g(k^*) - (1 + \rho)k^*}.$$

Using (A.27) and denoting $G(k) = g(k) - (1 + \rho)k$, we get that $h(k^*)$ should be set to

$$h(k^*) = \frac{G(\hat{k})\rho k^* + G(k^*)\bar{h}(k^*)}{G(k^*) - G(\hat{k})}. \quad (\text{A.31})$$

Finally, the central bank offers a lending facility with haircut (A.31) whenever

$$g(k^*) - (1 + \rho)k^* - \frac{\rho k^* + \bar{h}(k^*)}{\rho k^* + h(k^*)}\gamma(h) > g(\hat{k}) - (1 + \rho)\hat{k}. \quad (\text{A.32})$$

Since

$$\rho k^* + h(k^*) = \rho k^* + \frac{G(\hat{k})\rho k^* + G(k^*)\bar{h}(k^*)}{G(k^*) - G(\hat{k})} = G(k^*) \frac{\rho k^* + \bar{h}(k^*)}{G(k^*) - G(\hat{k})},$$

we have

$$\frac{\rho k^* + \bar{h}(k^*)}{\rho k^* + h(k^*)} = \frac{G(k^*) - G(\hat{k})}{G(k^*)}$$

and equation (A.32) becomes

$$G(k^*) - G(\hat{k}) > \frac{G(k^*) - G(\hat{k})}{G(k^*)}\gamma(h).$$

Hence, the lending facility is offered as long as

$$G(k^*) > \gamma(h),$$

i.e., as long as the net return of investing k^* in the safe asset covers the cost of collateral.