Incentives for Information Production in Markets where Prices Affect Real Investment

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Abstract

A fundamental role of financial markets is to gather information on firms’ investment opportunities, and so help guide investment decisions. We analyze the incentives of speculators to produce information and the implications for firms’ investments and real values. Speculators’ incentives to produce information about an investment opportunity increase when they think the investment is more likely to be undertaken. As a result, there will be more information when ex-ante profitability is higher. This creates an amplification mechanism, whereby small changes in fundamentals can cause large shifts in investment and firm value. In some cases, a small decline in fundamentals can lead to a market breakdown where information production stops and there is a collapse in firm investment and value.
1 Introduction

“In certain circumstances, financial markets can affect the so-called fundamentals which they are supposed to reflect.”

George Soros

Financial markets play a vital role in the economy. Even when no capital issuance is directly involved – i.e., in secondary financial markets – market prices indirectly guide many decisions in the real economy. For example, Baker, Stein, and Wurgler (2003), Luo (2005), and Chen, Goldstein, and Jiang (2007) provide evidence that market prices affect firms’ investments via managerial learning and/or the firm’s access to new capital. As a result, there is a feedback effect, whereby financial markets not only reflect the cash flows generated by traded assets but also affect them.

Despite this feedback, traditional analysis of secondary financial markets – e.g., Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980), and the extensive market microstructure literature following Kyle (1985) and Glosten and Milgrom (1985) – limits attention to assets whose cash flows are unknown, but exogenous.¹ Only a handful of recent papers, reviewed below, allow for the presence of feedback. Several of these papers have shown that incorporating feedback into models of financial markets leads to new insights about the price formation process. Given the feedback effect, this then has implications for the real economy.

In this paper we analyze a model of feedback focusing on the incentives of financial-market speculators to produce information when they know that market prices affect firms’ investments. The analysis generates a new insight on the interaction between financial markets and the real economy. We show that due to the feedback effect and the endogeneity of information production, financial markets amplify small shocks in fundamentals into large changes in real investments and firm values. The amplification is sometimes very large, as a small decrease in fundamentals may

¹Note that general equilibrium REE models such as Radner (1981) do allow for resource allocation to be endogenous, but do not explicitly model the interdependence between a firm’s security prices and its investment policy. Also related is q-theory, the literature on investments and asset prices initiated by Tobin (1969). Despite the link between asset prices and investments in this literature, q-theory does not analyze a causal relation from the financial markets to real investment.
lead to a discontinuous drop in firms’ investments and values.

In the model, a firm has to decide whether to undertake an investment opportunity. A mass of competitive speculators in the financial market choose whether to incur a cost to acquire information about the profitability of the investment, which they can then use to trade in the firm’s security. Some of the information produced by speculators gets reflected in the price of the stock. The firm then uses this information in its decision whether to undertake the investment. The informativeness of the security price for the firm’s decision depends on how many speculators choose to produce information. This is determined in equilibrium by a break-even condition, such that the marginal speculator’s benefit from information acquisition does not exceed his cost of information acquisition.

The amplification result in this framework is based on two effects. First, speculators have stronger incentives to produce information about firms’ investment opportunities when these opportunities are ex-ante more profitable. When ex-ante profitability decreases, the firm is less likely to invest. Then, the value of the security becomes less sensitive to the information about the investment opportunity, and so this information loses its speculative value. Hence, speculators are discouraged from producing the information. Second, the information produced in the financial market has a positive effect on firm value because it enables managers (or providers of capital), who infer some of this information from market prices, to make more efficient investment decisions. Combining these two effects we see that a decrease in fundamentals leads to less information production, amplifying the decrease in firm value beyond the direct effect of the decrease in fundamentals.

As ex-ante fundamentals deteriorate further, the amplification mechanism is strengthened by the emergence of strategic complementarities among speculators. Strategic complementarities in our model result from the feedback effect. When ex-ante fundamentals are weak, the firm will only consider making the investment if the amount of information coming from the market is sufficiently large (and positive). Hence, in this region of the fundamentals, speculators’ profits increase when more other speculators produce information, as this increases the chance that the firm will make the investment. Indeed, as ex-ante fundamentals get weaker and strategic complementarities emerge, a small deterioration in fundamentals can lead to a discontinuous collapse in information production,
with associated large drops in investment activity and firm valuations.

In Section 5, we analyze an extension where the firm can choose between three levels of capital. Depending on the parameters of the model, this may lend itself to several interpretations. For example, one can think of a firm with an asset in place that may either sell its existing asset, buy another asset, or keep the status quo, depending on whether the news it receives from the market is bad, good, or in between, respectively. The amplification result goes through in the extended framework: a decrease in fundamentals makes the firm more likely to divest and less likely to invest, reducing the incentive for information production and amplifying the decrease in firm value. There is a sense in which amplification is now even stronger, as a decrease in profitability of one unit of capital decreases information production, which harms the efficiency of the decision on both units.

Aside from the amplification result, our model generates several new empirical implications on the properties of information production in financial markets. These predictions can be tested using market microstructure measures for the amount of information in the market. The first prediction, following the discussion above, is that there will be more information production about investment opportunities that have strong fundamentals. This implies more information production in good times than in bad times and more information production about good firms than about bad firms. The second prediction is about the effect of uncertainty on the amount of information. While traditional models always predict a positive link – more uncertainty encourages more information production by making information more valuable – our model says that the link might be negative when fundamentals are good. This is because in this region, more uncertainty implies lower probability that the investment will be undertaken, and this reduces the incentive to produce information. The third prediction is about the effect of real options on information production. In our model, adding a real option to expand increases information production in the financial market, while adding a real option to abandon decreases it.

The amplification of small changes in fundamentals into large changes in firm value and investment is the subject of a large literature on fluctuations over the business cycle (see for example, Bernanke and Gertler (1989), Greenwald and Stiglitz (1993), Kiyotaki and Moore (1997), and Suarez and Sussman (1997)). This literature typically links these fluctuations to capital market
imperfections, which limit firms’ access to capital in downturns of the business cycle. One explanation in the literature (the balance-sheet channel) is that during a bust a firm’s collateral is less valuable and therefore it has more restricted access to external finance. An alternative explanation (the credit channel) is that banks’ own balance sheets and hence their ability to lend is reduced in recessions. We identify a different mechanism for sharply reduced investment levels. In our setting, the ability to identify good investment projects is weakened during a recession because markets’ incentives to produce information are reduced. The model proposed in this paper is the first that links changes in the economic outlook to endogenous changes in jointly-determined information production and investment behavior. We think this link is fundamental since information frictions are at the core of financial markets and investment behavior.

Our paper emphasizes the role of information produced by financial markets in firms’ investment decisions. The justification for the usefulness of information in financial markets for firms’ investment decisions is that markets gather information from many different participants, who are too numerous to communicate with the firm outside the trading process (see Subrahmanyan and Titman (1999)). This idea goes back to Hayek (1945), who argues that markets provide an efficient mechanism for information production and aggregation. The ability of financial markets to produce information that accurately predicts future events has also been demonstrated empirically. For example, the literature on prediction markets shows that markets provide better forecasts than polls and other devices (see Wolfers and Zitzewitz (2004)). Roll (1984) shows that private information of citrus futures traders regarding weather conditions gets impounded into citrus futures’ prices, so that prices improve even public predictions of the weather. By focusing only on information produced in financial markets, we do not deny the importance of alternative information producers such as banks or large shareholders, which have been extensively analyzed in the corporate finance literature (see Allen and Gale (2000), for a review). Rather, we try to extend the analysis to a class of information providers which has been somewhat neglected in the corporate finance literature.

A small number of theory papers study market equilibrium in the presence of a feedback effect from asset prices to the real economy. They include Fishman and Hagerty (1992), Leland (1992), Khanna, Sleza, and Bradley (1994), Boot and Thakor (1997), Dow and Gorton (1997),
Subrahmanyam and Titman (1999), Fulghieri and Lukin (2001), Dow and Rahi (2003), Foucault and Gehrig (2008), Goldstein and Guembel (2008), Bond, Goldstein, and Prescott (2009), and Hennessy (2009). \(^2\) Several of these papers use endogenous information production as a tool for analyzing specific questions. In Khanna, Slezak and Bradley (1994), it is used to analyze the effect of insider trading on the incentives for an outsider to acquire and trade on information that may potentially be useful for the firm. In other papers, it is used to compare bank and market financing (Boot and Thakor (1997), Dow and Gorton (1997)) or debt and equity (Fulghieri and Lukin (2001), Hennessy (2009)). In contrast to these papers, our paper studies more general implications of the feedback effect in the presence of endogenous information production. In particular, our main result is the amplification of small changes in fundamentals into large (sometimes discontinuous) changes in investments and firm values. This result is new in the feedback literature.

The remainder of the paper is organized as follows. In Section 2, we describe the basic model of feedback. Section 3 derives the equilibrium outcomes. In Section 4, we analyze the effect of expected profitability and uncertainty on information production, and demonstrate the amplification result. In Section 5, we extend the model to have three investment levels. Section 6 concludes. All proofs are relegated to the appendix.

2 A Model of Feedback

2.1 Modelling assumptions

There is a firm with an investment opportunity, requiring an amount \( I \). The investment decision is taken to maximize firm value (there is no shareholder/manager agency problem). The final payoff of the investment \( R_\omega \) depends on equally likely states of the world \( \omega \in \{l,h\} \). Assume that the investment is worth undertaking when the state of the world is \( h \) but not when it is \( l \), so \( R_h > I > R_l \). Define

\[^2\]The IPO literature has also used the assumption that stock-market participants have information about some aspects of the firm, which is not available to the firm’s managers. See, for example, Rock (1986), Benveniste and Spindt (1989), Benveniste and Wilhelm (1990), and, Biais, Bossaerts, and Rochet (2002).
as a convenient measure of the *ex-ante* profitability of the investment. Since ex-ante the states are equally likely to occur, when \( \gamma > \frac{1}{2} \) the ex-ante net present value (NPV) is positive, whereas when \( \gamma < \frac{1}{2} \), it is negative.

The firm’s shares give a proportional entitlement to the final payoff, but no other securities have payoffs that are contingent on \( \omega \). Markets are thus incomplete and spanning is endogenous to the firm’s production decision. An example of such a situation would be the gains from synergies in a hypothetical merger, where the actual gains will not be observed unless the merger takes place.

The shares are traded in a market similar to the one in Kyle (1985). There are three types of traders: noise traders, speculators, and a market maker. Speculators are atomistic, risk neutral, and indexed by \( i \in [0, \infty) \). Each speculator can learn \( \omega \) at cost \( c > 0 \), and can then trade \( x_i \), where \( x_i \in [-1, 1] \). That is, there are frictions (such as limited wealth) that constrain trade size to a maximum of 1. Denote by \( \alpha \) the measure of speculators that become informed about \( \omega \). Noise trade \( n \) is normally distributed with 0 mean and variance \( \sigma^2 \). Total order flow \( X \) is:

\[
X = n + \int_0^\alpha x_i \, di.
\]  

The total order flow is submitted to a risk neutral market maker who observes \( X \), but not its components. He then sets the price equal to the expected value of the firm conditional on the order flow.\(^4\)

The key ingredient in our model is the feedback from the price to the firm’s investment. The firm’s manager does not observe the state of the world \( \omega \). He observes the share price and uses this information to update his belief about \( \omega \) and consequently about the NPV of the investment. He then invests if and only if the NPV is positive. Of course, this feedback effect is taken into account by the market maker when setting the price.

\(^3\)Uninformed speculators in our model optimally choose not to trade, because they would incur a loss in expectation from trading.

\(^4\)As in Kyle (1985), this can be justified as a result of a perfectly-competitive market-making industry.
2.1.1 Discussion

Before describing the model in more detail, we discuss two elements of our model.

**Learning from the market:** Our model is based on the assumption that the firm learns from the market. For simplicity, we do not allow the firm to produce information in-house. We should clarify that our results do not require that there is no in-house information production. The important element is that there are some types of information that the market has an advantage in producing.

The main reason for which markets may have an advantage in information production was put forward by Hayek (1945). He emphasized that markets are useful for aggregating many small pieces of dispersed information. The firm may have difficulty in replicating this internally, or find it prohibitively expensive. For example, the number of people who may potentially come across a piece of information about the firm during their daily activities may be very large (this includes customers, suppliers, etc.). Another possibility is that many stock market traders collect small pieces of information about many different securities. When they decide to spend their time analyzing the markets (incurring the opportunity cost of information production), they do not know in advance exactly which security they will learn about. Hence, it would not be worthwhile for a single company to employ all the traders full time to produce information about itself, since much of the information produced would relate to other companies. For simplicity we do not explicitly model this, but rather take the usefulness of market information as given.

Another rationale for external production of information is that people inside the firm may be unable to produce objective information about pet projects because of corporate culture or individual psychology. For example, if firms tend to accumulate employees who fit closely with established corporate practices, as argued in Carrillo and Gromb (2007), such employees may not have a comparative advantage in producing criticism of the firm. Also, contracts to produce information within the firm may be beset with incentive problems. Employees may be subject to contracting difficulties, such as a limited liability constraint whereby penalties are impossible. Dow and Gorton (1997) compared information production via the financial market and internal information production with limited liability constraints. They showed that internal production
has noise which (depending on the parameter values) may make internal information production suboptimal.

Tirole (2006, section 8.1.3) discusses similar issues in connection with monitoring (via information production) by outsiders who buy shares rather than agents with permanent relationships with the firm (such as employees and long term private owners). He mentions collusion between monitors and management, the difficulty of foreseeing in advance who will be the best monitor, and limited funding of potential monitors, as potential problems with in-house monitoring.

**Length of time period:** We interpret the length of the time period in our model to be in the range from a week to a year. For example, over a couple of weeks a firm might announce an acquisition of another firm, the share price might react negatively, and the firm might cancel the acquisition. Over a couple of months a firm might announce a new product, the share price might react favorably, and the firm might respond by planning to build new manufacturing facilities. Over a year, a firm might announce a new strategy, the change might be poorly received by the stock market, and the firm might react by reviewing the new strategy or firing the CEO.

In reality, stocks trade more frequently than this so the length of the period is clearly a modeling simplification. This simplification is assumed in many other papers. The seminal paper by Grossman and Stiglitz (1976), for example, considers a time period to be the length of time between successive crops on a farm. Other papers where stock prices feed back into investment decisions employ a single trading period, and hence, the period must implicitly be assumed to be long (e.g. Fishman and Hagerty (1992), Dow and Gorton (1997), Boot and Thakor (1997), Subrahmanyam and Titman (1999)).

### 2.2 Trading decisions and investment policy

We now continue to discuss and analyze the model. From risk neutrality and because each speculator is too small to have a price impact, we know that if speculators acquire information, they will trade the maximum size possible\(^5\). So when the state is \(\omega = h\) all informed speculators submit buy orders of size 1, and total order flow is \(X = n + \alpha\). Conversely when \(\omega = l\) informed traders

\(^5\)Except when they expect the price to exactly equal the value, an outcome which does not arise in our model.
submit sell orders, and total order flow is $X = n - \alpha$.

Observing the order flow, the market maker updates his beliefs. We define $\theta(X) \equiv \Pr(\omega = h|X)$ as his updated probability that the state is high, given the observed order flow $X$ and a belief about the measure of informed speculators $\alpha$. By Bayes’ rule,

$$
\theta(X) = \frac{\varphi(X - \alpha)}{\varphi(X - \alpha) + \varphi(X + \alpha)},
$$

where $\varphi(n)$ is the density function of the normal distribution with mean 0 and variance $\sigma^2$. The investment is worth undertaking if and only if the NPV based on the updated probability $\theta(X)$ is positive:

$$(R_h - I) \theta(X) + (R_l - I) (1 - \theta(X)) > 0.$$  

Since the normal distribution satisfies the monotone likelihood ratio property, we know that $\theta(X)$ is strictly increasing in $X$ as long as $\alpha > 0$. Thus, there is a cut-off value $\overline{X}$ such that the project has a positive NPV if and only if $X > \overline{X}$. The threshold $\overline{X}$ is defined by

$$(R_h - I) \theta(\overline{X}) + (R_l - I) (1 - \theta(\overline{X})) = 0.  \tag{4}$$

Using the properties of the normal distribution,

$$
\overline{X} = \frac{\sigma^2}{2\alpha} \ln \frac{1 - \gamma}{\gamma}.  \tag{5}
$$

In a first step, we will solve for the equilibrium in the continuation game after all speculators have chosen whether or not to acquire information. As mentioned before, their choice of trades is trivial and we therefore do not carry around an explicit description of its determination as part of the equilibrium. An equilibrium of the continuation game then consists of the following: (a) A ‘fair’ price set by the market maker conditional on observed order flow, given a belief about $\alpha$ and the firm’s investment policy, (b) an investment policy by the firm that maximizes its expected value conditional on the observed stock price, and beliefs about $\alpha$ and the pricing rule. Moreover, the beliefs must be correct in equilibrium.
Lemma 1 For a given $\alpha$, the continuation game has an equilibrium where the market maker uses the pricing rule:

$$P(X) = \begin{cases} 
R_h \theta(X) + R_t (1 - \theta(X)) - I & \text{if } X > \overline{X}, \\
0 & \text{if } X \leq \overline{X}
\end{cases}$$

and the firm invests if and only if

$$P(X) > 0.$$  

(6)

(7)

An important feature of the equilibrium given by (6) and (7) is that the investment is undertaken if and only if it ought to be undertaken based on the information revealed to the market maker by the order flow. The market maker observes the order flow and sets a price that guides the firm to make the right decision. There are also equilibria without this feature. For example, when $\gamma < \frac{1}{2}$, there is a "coordination failure" equilibrium where the market maker sets $P(X) = 0$ for any $X$ and the firm optimally never invests. In this case, the ex-ante NPV is negative, and so, given that the price reveals no new information, it is optimal for the firm not to invest. Similarly, even when $\gamma > \frac{1}{2}$ there could be equilibria, where there is some $X' > \overline{X}$ and a pricing rule where the price is 0 below $X'$. This can be an equilibrium if observing a price of 0 (given this pricing rule) provides sufficiently bad news that the firm chooses not to invest. Such equilibria, as will be clear later in the paper, are Pareto inferior in the sense of reducing the value of the firm while leaving the other agents unaffected. Moreover, the equilibrium described in Lemma 1 is the only equilibrium that survives the slight modification of the model in which the firm can observe order flow directly. Given that prices are set under the implicit assumption that there are many competing market makers it would be plausible to assume that order flow is public information and thus also observed by the firm.\footnote{In models with feedback, it is quite natural for several different order flows to correspond to the same price in equilibrium, in addition to the coordination issues that arise in this model. For example, Dow and Rahi (2003) use a normally distributed functional form in which a signal of $x$ and a signal of $-x$ have the same implications for firm value but opposite implications for desired investment; they assume conditioning on volume as well as price to maintain tractability.} Because of these properties, we will focus on the equilibrium in Lemma 1 for the continuation game.
2.3 Trading profits

After characterizing the equilibrium in the continuation game, we now turn to the equilibrium amount of information production. For this, we need to calculate the expected trading profits of an informed speculator, as a function of how many other speculators become informed. Denote by \( \pi (\alpha) \) the expected profits and \( \bar{X} (\alpha) \) as the investment threshold (see (5)) in a candidate equilibrium in which a measure \( \alpha \) of speculators become informed.

To derive the function \( \pi (\alpha) \) note that when \( \omega = h \) (which happens with probability \( \frac{1}{2} \)), informed speculators buy the security and so the total order flow \( X \) is equal to \( n + \alpha \). Thus, the investment is undertaken if and only if the order from noise traders \( \nu \) is above \( \bar{X} (\alpha) - \alpha \). In this range, for each realization of \( n \), informed speculators make a profit equal to the difference between the true value of the security \( \chi = \nu\omega - \alpha \) and the price \( P(X) \). On the other hand, if the noise traders’ order is low \( (n \leq \bar{X} (\alpha) - \alpha) \), the investment is not undertaken, and the speculator ends up buying an asset with liquidation value 0 at price 0, so he makes no profit. Similarly, when \( \omega = l \), informed speculators sell the security and make a profit of \( P(X) - (R_l - I) \) whenever \( n > \bar{X} (\alpha) + \alpha \). In sum, the expected trading profits can be written as:

\[
\pi (\alpha) = \frac{1}{2} \int_{\bar{X}(\alpha) - \alpha}^{\infty} \varphi (n) \left( R_h - I - P(n + \alpha) \right) dn + \frac{1}{2} \int_{\bar{X}(\alpha) + \alpha}^{\infty} \varphi (n) \left( P(n - \alpha) - R_l + I \right) dn.
\]

Using the price function (6), we can rewrite the expected profit of a speculator after observing a positive signal (the first integral in (8)) as:

\[
\int_{\bar{X}(\alpha) - \alpha}^{\infty} \varphi (n) \left( R_h - I - P(n + \alpha) \right) dn = (R_h - R_l) \int_{\bar{X}(\alpha) - \alpha}^{\infty} \varphi (n) \frac{\varphi (n + 2\alpha)}{\varphi (n) + \varphi (n + 2\alpha)} dn = (R_h - R_l) \int_{\bar{X}(\alpha)}^{\infty} \frac{\varphi (x - \alpha) \varphi (x + \alpha)}{\varphi (x - \alpha) + \varphi (x + \alpha)} dx,
\]

where the second equation follows from the change of variable \( x = n + \alpha \). Similarly (and using a change of variable \( x = n - \alpha \)) we can rewrite the expected profit of a speculator after observing a
negative signal (the second integral in (8)) as:

\[ \int_{X(\alpha)-\alpha}^{\infty} \varphi(n) (P(n-\alpha) - R_l + I) \, dn \]

\[ = (R_h - R_l) \int_{X(\alpha)}^{\infty} \frac{\varphi(x-\alpha) \varphi(x+\alpha)}{\varphi(x-\alpha)+\varphi(x+\alpha)} \, dx. \quad (10) \]

Then, profits are:

\[ \pi(\alpha) = (R_h - R_l) \int_{X(\alpha)}^{\infty} \frac{\varphi(x-\alpha) \varphi(x+\alpha)}{\varphi(x-\alpha)+\varphi(x+\alpha)} \, dx. \quad (11) \]

Defining

\[ H(x) = \frac{\varphi(x-\alpha) \varphi(x+\alpha)}{\varphi(x-\alpha)+\varphi(x+\alpha)} \quad (12) \]

we can write expected profits as:

\[ \pi(\alpha) = (R_h - R_l) \int_{X(\alpha)}^{\infty} H(x) \, dx. \quad (13) \]

### 2.4 Information acquisition in equilibrium

Speculators decide whether to acquire information by comparing its cost \( c \) to the profit \( \pi(\alpha) \). Using \( \alpha \) to denote the equilibrium level of \( \alpha \), an equilibrium with information production (\( \alpha > 0 \)) is obtained when, given that a measure \( \alpha \) of speculators choose to produce information, a speculator who acquires information breaks even in expectation:

\[ \pi(\alpha) = c. \quad (14) \]

Alternatively there may be a corner solution for \( \alpha \): an equilibrium with no information production (\( \alpha = 0 \)) is obtained when, given that none of the speculators produce information, the cost of producing information is greater than the expected trading profit:

\[ \pi(0) \leq c. \quad (15) \]

### 3 Characterization of Equilibrium Outcomes

We now solve for the equilibrium in information acquisition decisions, taking as given the equilibrium of the trading game from Lemma 1. The characterization of equilibrium outcomes is different
depending on whether the investment project has an ex-ante positive or negative NPV. We first analyze the case where the investment project has a positive NPV ex ante.

### 3.1 Ex-ante positive NPV investment

This case occurs when $I < \frac{1}{2}(R_h + R_l)$, i.e., $\gamma > \frac{1}{2}$. In the absence of information about the underlying state $\omega$, the firm will choose to invest. Proposition 1 characterizes the equilibrium outcomes for this case.

**Proposition 1** When the investment has an ex-ante positive NPV ($\gamma > \frac{1}{2}$), there exists a unique equilibrium. For $c < \frac{R_h - R_l}{2}$, a positive measure of speculators become informed ($\bar{\alpha} > 0$), and for $c \geq \frac{R_h - R_l}{2}$, no information is produced ($\bar{\alpha} = 0$).

According to Proposition 1, no information is produced if the cost of information production is too high, whereas a positive measure of speculators choose to become informed if information is not too costly.

### 3.2 Ex-ante negative NPV investment

Consider now the case where $\gamma \leq \frac{1}{2}$. Proposition 2 characterizes the equilibrium outcomes for this case.

**Proposition 2** When the investment has an ex-ante negative NPV ($\gamma \leq \frac{1}{2}$):

(i) For any $c > 0$, there exists an equilibrium with no information production ($\bar{\alpha} = 0$).

(ii) For $c \leq \max_{\alpha \in \mathbb{R}^+} \pi(\alpha)$, there also exist equilibria where a positive measure of speculators become informed ($\bar{\alpha} > 0$).

(iii) For $c > \max_{\alpha \in \mathbb{R}^+} \pi(\alpha)$, the equilibrium with $\bar{\alpha} = 0$ is unique.

Unlike the positive NPV case, there may now be multiple equilibria. There is always an equilibrium with no production of information. If the cost of information production is high, this is the only equilibrium, whereas if it is not too high, there are additional equilibria with positive measures of informed speculators.
Figure 1: The figure shows trading profits as a function of $\alpha$ for the case of $\gamma = 0.55$ (dotted line) and for the case of $\gamma = 0.45$ (solid line). The other parameters are set at $\sigma = 1$ and $R_h - R_l = 1$. The figure also shows two different costs of information production $c'$ and $c''$. $\alpha_1$ and $\alpha_2$ are two equilibrium values of the measure of informed traders when $\gamma < \frac{1}{2}$ and the cost of information production is $c'$.

3.3 Discussion

Figure 1 depicts the expected trading profits as a function of the measure of speculators who choose to acquire information. The solid curve is for the case where the investment has a negative NPV ex-ante (here, $\gamma = 0.45$), while the dotted curve is for the case where the investment has a positive NPV ex-ante (here, $\gamma = 0.55$). We can see that when the ex-ante NPV is negative, the profit function is hump-shaped, whereas when the ex-ante NPV is positive, the profit function is monotonically decreasing. This illustrates why the negative NPV case may have multiple equilibria, while the positive NPV case has a unique equilibrium.
To understand the differences between the two cases, it is useful to isolate the different underlying economic effects that a change in the number of informed traders has on each trader’s profits. First, there is the standard effect in models of informed trading with exogenous investment (e.g., Grossman and Stiglitz (1980)). As more speculators become informed, the equilibrium price becomes closer to the value of the stock, and profits are reduced. This causes a downward slope in the profit function. We call this the competitive effect. It generates strategic substitutability in agents’ decisions to produce information. Going back to the expression for profits in (13), this effect is captured by the fact that $H(x)$ is decreasing in $\alpha$.

Second, there is the effect caused by the endogeneity of the firm’s investment decision, captured by the effect of $\alpha$ on $\mathbb{X}(\alpha)$. The direction of this effect depends on whether the project has a positive or a negative NPV ex-ante. In case of positive ex-ante NPV, the firm invests in the absence of information. Additional information then leads the firm not to invest some of the time, so that the overall likelihood of investment falls when more information becomes available ($\mathbb{X}(\alpha)$ increases in $\alpha$). This reinforces the competitive effect because as more speculators produce information, the investment is undertaken less often and the value of the stock becomes less sensitive to the information. Trading profits therefore decrease. This is why in the positive NPV case, the profit function is downward sloping and the equilibrium is unique.

In case of a negative ex-ante NPV, the firm’s default position without any information is not to invest. As speculators produce information, the firm may sometimes learn positive news and invest ($\mathbb{X}(\alpha)$ decreases in $\alpha$). An increase in the number of informed speculators therefore increases the likelihood of investment taking place, which renders firm value more sensitive to an individual speculator’s private information. There is an informational leverage effect, where information becomes more valuable as more agents produce it. Information production exhibits strategic complementarity. The interaction between the competitive effect and the informational leverage effect causes the profit function to be non-monotone.

As a result of the non-monotonicity, we have multiple equilibria in the case of an ex-ante negative NPV. First, there always exists an equilibrium in which no information is produced. This happens

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7 We thank Rohit Rahi for suggesting this terminology.
8 Boot and Thakor’s (1997) model exhibits a similar non-monotonicity, although they do not explore this feature.
for the following reason. When nobody produces information, the firm does not invest. Then, it
does not pay for an individual to become informed, since the firm’s securities never gain exposure
to the information that the speculator collected. Second, when the cost of information production
is not too high, there are equilibria with a positive amount of information. From Figure 1, we can
see that for \( c < \max_{\alpha \in \mathbb{R}^+} \pi (\alpha) \), the profit function will intersect twice with the cost of producing
information, generating two equilibria with positive amount of information. For example, if \( c = c' \),
we can see in the figure that one equilibrium (\( \alpha_1 \)) is obtained at the upward sloping part of the
profit function, representing less information than the other equilibrium (\( \alpha_2 \)) that is obtained at
the downward sloping part of the profit function. Essentially, there is a coordination problem in
information production among multiple speculators. A coordination failure may obtain if they
coordinate on producing no information or if the amount of information is \( \alpha_1 \) rather than \( \alpha_2 \).\(^9\)
In case of multiple equilibria, the most informative equilibrium Pareto dominates the others when
we consider the value of the firm, the market maker, and the speculators. This is because, as
will become clear later, the value of the firm increases in the amount of information, while the
speculators and the market maker always make a profit of 0.

From now on, in case of multiple equilibria we will focus on the most informative equilibrium.
The notation \( \hat{\alpha} \) will refer to this equilibrium only.

4 Information Production and Amplification

4.1 Profitability and information production

To analyze the effect of expected profitability on information production, we study the effect of \( \gamma \) (as
defined in (1)) on the equilibrium amount of information \( \hat{\alpha} \). In varying \( \gamma \), we wish to consider only
the effect of the profitability of the investment without changing anything else in the factors that
determine \( \hat{\alpha} \). Inspecting the profit function in (13), which is the main determinant of equilibrium,
we can see that this amounts either to changing \( I \) or to changing \( R_b \) and \( R_t \) by the same amount.

\(^9\)Note that the equilibrium point \( \alpha_1 \) is not stable in the sense that a slight perturbation of the amount of in-
formed speculators around the equilibrium point would induce all speculators to change their decisions away from
the equilibrium.
An increase in $\gamma$ can be interpreted as a decrease in $I$ or as an increase in $R_h$ and $R_I$ while holding $(R_h - R_I)$ constant.

Proposition 3 establishes the effect of $\gamma$ on $\hat{\alpha}$. Trivially, when the equilibrium is at the corner with $c > \pi(0)$, small changes in the model parameters will not affect the amount of information production. For the comparative statics presented in the following proposition we therefore focus on the case where $\hat{\alpha} > 0$.

**Proposition 3** Suppose that the equilibrium amount of information $\hat{\alpha}$ is strictly positive:

The amount of information production $\hat{\alpha}$ increases in the ex-ante profitability of the project $\gamma$.

The intuition for this result is as follows. As economic fundamentals deteriorate and expected profitability decreases, for each level of information production, the firm invests less frequently. Then, speculators’ expected trading profits are reduced because the value of the firm is less exposed to the information about the profitability of the investment. As a result, in equilibrium, fewer speculators choose to pay the cost of information, and the equilibrium amount of information decreases.

It is interesting to point out that while the equilibrium amount of information produced by speculators increases monotonically in $\gamma$, the value to the firm of that information does not. There will thus be a wedge between the amount of information actually produced in equilibrium and the importance of information for the efficiency of investment decisions. We can calculate the value of information to the firm, which we denote $S(\gamma)$, by taking the difference between expected firm value with and without information about $\omega$. If the firm knows $\omega$ before the investment decision is taken, ex-ante expected firm value is $\frac{1}{2} (R_h - I)$. If the firm invests without knowing anything about $\omega$ expected firm value is zero in case $\gamma < \frac{1}{2}$, and $\frac{1}{2} (R_h + R_I) - I$ in case $\gamma \geq \frac{1}{2}$. So

$$ S(\gamma) = \begin{cases} \frac{\gamma R_h - R_I}{2} & \text{if } \gamma < \frac{1}{2} \\ (1 - \gamma) \frac{R_h - R_I}{2} & \text{if } \gamma \geq \frac{1}{2} \end{cases} $$

Information is most valuable to the firm when $\gamma = \frac{1}{2}$. At this point the cost of taking the wrong investment decision when uninformed is highest. Hence, the value added from learning $\omega$ is also highest here. As $\gamma$ gets closer to 0 or 1, the expected error made under the optimal uninformed
investment policy is relatively low. For example, when \( \gamma \) is close to 1, the loss from investing in the bad state \( \omega = l \) is quite small and information is less valuable. Whether markets produce too little or too much information therefore depends on the properties of the project. Loosely speaking, when \( \gamma \) is high, too much information will be produced, while too little will be produced when \( \gamma \) is in some intermediate range.\(^{10}\)

Proposition 3's characterization of the equilibrium amount of information production has two empirical implications. First, it implies that the amount of information produced will vary over time in response to aggregate fluctuations in investment prospects. When prospects are poor, firms are more likely to cancel investments and this reduces available trading profits and information production incentives. This hypothesis can be tested by analyzing changes in market microstructure measures of informed trading across stages of the business cycle. Alternatively, one can look at analyst activity as a measure of information production and analyze what happens to measures capturing analyst activity over the business cycle. Consistent with our hypothesis, anecdotal evidence seems to suggest that financial firms’ employment policies are highly cyclical, much more so than employment policies of other firms. Moreover, Eisfeldt and Rampini (2006) provide empirical evidence that is consistent with our analysis that information production is strongly procyclical. They find that there is much less capital re-allocation among firms during recessions than in booms.

Second, the result suggests that information production should vary cross-sectionally with firms’ investment prospects. Taking analyst activity as a proxy for information production in financial markets, there is some support for this hypothesis in the empirical literature. McNichols and O’Brien (1997) investigate analysts’ decisions to initiate or drop coverage of specific stocks. They find that analysts bias their coverage towards those firms about which they have more favorable expectations. Building on these findings, Sun (2003) shows that the initiation or dropping decision itself predicts future firm performance. Das, Guoh and Zhang (2006) find that among newly listed firms, analysts selectively cover those firms for which they have more positive expectations. Firms that receive more coverage perform better afterwards.

\(^{10}\) This recalls Hirshleifer’s (1971) seminal insight that the social and private values of information may diverge. The case where \( \gamma \) approaches 1 is Hirshleifer’s “foreknowledge” where information is produced about something that will happen anyway.
Veldkamp (2005, 2006) suggests some different explanations of cyclicality in information production. In Veldkamp (2005), when the economy is in a good state, many projects are undertaken and this automatically generates many signals about the projects. The result is that the economy responds quickly to new information (and the opposite in a bad state). In Veldkamp (2006), there are economies of scale in information production, so in a boom, the amount of information is much greater than in a bust. The mechanism in our model, on the other hand, stresses the effect of the likelihood of investment on the decision to produce information about the investment, and how the information-acquisition decision is mediated by the stock price.

4.2 Amplification

We now turn to analyze the effect of ex-ante investment profitability on the expected value of the firm. Our main result is that endogenous information production amplifies the impact that a change in fundamentals has on the firm’s expected value.

Let \( V(\tilde{\alpha}, \gamma) \) be the expected value of the firm as a function of the equilibrium amount of information produced \( \tilde{\alpha} \) and the investment profitability \( \gamma \). We showed above that the firm invests whenever noise trading is above a certain threshold. In the good state \( (\omega = h) \), this threshold is \( \mathcal{X}(\tilde{\alpha}, \gamma) - \tilde{\alpha} \), where \( \mathcal{X}(\tilde{\alpha}, \gamma) = \frac{\sigma^2}{2\eta} \ln \frac{1-\gamma}{\gamma} \) (see (5)), while in the bad state \( (\omega = l) \), the firm invests whenever \( n \) is above \( \mathcal{X}(\tilde{\alpha}, \gamma) + \tilde{\alpha} \). Hence, the expected value of the firm is:

\[
V(\tilde{\alpha}, \gamma) = \frac{1}{2} \int_{\mathcal{X}(\tilde{\alpha}, \gamma) - \tilde{\alpha}}^{\infty} \varphi(n) (R_h - I) \, dn + \frac{1}{2} \int_{\mathcal{X}(\tilde{\alpha}, \gamma) + \tilde{\alpha}}^{\infty} \varphi(n) (R_l - I) \, dn. \tag{17}
\]

After performing changes of variables in both integrals, this can be written as:

\[
V(\tilde{\alpha}, \gamma) = \frac{1}{2} \int_{\mathcal{X}(\tilde{\alpha}, \gamma)}^{\infty} [(R_h - I) \varphi(x - \tilde{\alpha}) + (R_l - I) \varphi(x + \tilde{\alpha})] \, dx \tag{18}
\]

\[
= \frac{1}{2} \int_{\mathcal{X}(\tilde{\alpha}, \gamma)}^{\infty} [\gamma \varphi(x - \tilde{\alpha}) - (1 - \gamma) \varphi(x + \tilde{\alpha})] \, dx.
\]

We can now calculate how firm value changes in response to a change in the project’s funda-
mentals, measured by project profitability $\gamma$.\footnote{Note that $\hat{\alpha}$ and therefore $V(\hat{\alpha}, \gamma)$ may be discontinuous in $\gamma$ so the following derivative is only defined on the continuous and differentiable segments of $V(\hat{\alpha}, \gamma)$. A discontinuity occurs when $\pi'(\hat{\alpha}) = 0$ and therefore the equilibrium amount of informed speculators drops from a strictly positive number to zero.}

$$\frac{dV(\hat{\alpha}, \gamma)}{d\gamma} = \frac{\partial V}{\partial \gamma} + \frac{\partial V}{\partial \hat{\alpha}} \cdot \frac{\partial \hat{\alpha}}{\partial \gamma}. \tag{19}$$

The first term in (19) captures the direct effect that a change in the project’s characteristic has on firm value. The second term captures the indirect effect through the information channel. As the following proposition shows, the information channel amplifies the direct effect of fundamentals on firm value.

**Proposition 4** Suppose that the equilibrium amount of information $\hat{\alpha}$ is strictly positive:

The endogenous response of information production to a change in project profitability amplifies the direct effect that the change in profitability has on firm value:

$$\frac{dV(\hat{\alpha}, \gamma)}{d\gamma} = \frac{\partial V}{\partial \gamma} + \frac{\partial V}{\partial \hat{\alpha}} \cdot \frac{\partial \hat{\alpha}}{\partial \gamma} > \frac{\partial V}{\partial \gamma} > 0.$$

Intuitively, the direct effect, $\frac{\partial V}{\partial \gamma}$, is positive given that an increase in ex-ante profitability directly increases the value of the firm. This is amplified by the indirect effect through the information channel, as $\frac{\partial V}{\partial \hat{\alpha}} \cdot \frac{\partial \hat{\alpha}}{\partial \gamma}$ is also positive. The mechanism behind the indirect effect reflects two forces. First, as we know from Proposition 3, the amount of information produced in the market increases when ex-ante profitability improves ($\frac{\partial \hat{\alpha}}{\partial \gamma} > 0$). Second, the presence of more information enables the firm to make a more efficient investment decision – i.e., invest more often in the good state and less often in the bad state (see (18)) – and this increases the value of the firm ($\frac{\partial V}{\partial \hat{\alpha}} > 0$).

Given the stylized nature of our model, it is obviously difficult to assess the quantitative significance of the amplification effect identified above. This raises the question whether amplification is merely a theoretical possibility or something that can potentially help explain significant changes in firm value and investment. The next proposition shows that the amplification effect can be unboundedly large. As such it is obvious that there are parameter values for which the amplification effect will be quantitatively very significant.
Proposition 5 Suppose that the cost of information production $c < \max_{\alpha \in \mathbb{R}^+} \pi (\alpha)$ for some profitability $\gamma < \frac{1}{2}$, so that there exists an equilibrium with positive amount of information: $\tilde{\alpha} > 0$. Then there exists a $\gamma^*$ such that the limit from above

$$\lim_{\gamma^* \to \gamma^+} \frac{\partial V}{\partial \tilde{\alpha}} \cdot \frac{\partial \tilde{\alpha}}{\partial \gamma} = \infty.$$ 

Intuitively, the information channel has the most drastic impact on firm value near the point where a small reduction in the project’s fundamental $\gamma$ drives out all informed trade. This happens near the point where the profit function reaches its maximum (Figure 1). At this point, a small decrease in profitability causes informed trading to drop discontinuously from a strictly positive amount to zero (i.e., $\frac{\partial \tilde{\alpha}}{\partial \gamma} \to \infty$). The reason for this result is the presence of strategic complementarities in information production when the ex-ante NPV of the investment is negative. In this case, the firm only invests if there is enough information in the price. Hence, a speculator finds it worthwhile to produce information when sufficiently many other speculators do so. A small decrease in fundamentals can then lead informed speculation to dry up completely in a market breakdown: traders stop producing information and abandon the market. Since the market in our model has an important real effect in guiding resource allocation, this has a substantial negative effect on the firm’s real investment and value. Empirically, this would be manifested in excess volatility in the sense of volatility that cannot be attributed to changes in fundamentals.

Other models have used strategic complementarities in information production to explain excess volatility. The source of complementarities is different in these models. In Veldkamp (2006), complementarities result from fixed costs in the production of information. In Froot, Scharfstein and Stein (1992), they stem from speculators’ short-termism. In our model, the introduction of feedback from the financial market to real investments is sufficient to generate complementarities. There are two distinguishing features in our model relative to the other papers. First, the involvement of the real sector in our model implies that complementarities have an effect on firm value and investment, and not just on prices like in the other papers. Second, complementarities arise in our framework only when fundamentals are low – in particular, when the NPV of the investment is ex-ante negative – and so the extreme amplification and excess volatility arise only then. Hence, while the other models do not say when excess volatility is more likely to arise, ours says it is more likely to arise
when fundamentals are relatively weak.\footnote{Ozdenoren and Yuan (2008) study a model of feedback, but with no learning or information production, i.e., they assume that prices affect firm value for exogenous reasons. This generates strategic complementarities in speculators' trading, since when many speculators sell the value of the firm is expected to drop, leading others to sell as well.}

Another interesting point is that a loss of information has a particularly strong impact on firm value when $\gamma$ is close to $\frac{1}{2}$ (see (16). In this case, $\frac{dV}{d\alpha}$ tends to be large. In particular, if $\gamma$ is slightly below $\frac{1}{2}$, the firm may be quite valuable when high quality information is available: a little bit of positive news from the stock price will mean that the firm invests and is thus valuable ex ante. If the information flow dries up, it is no longer worthwhile for the firm to invest, so its value drops sharply. While in our model we consider $\gamma$ to be exogenous, one could think of a richer setting where the level of $\gamma$ for the marginal firm is determined endogenously. In such a setting, it is quite likely that the marginal firm will have $\gamma$ that is close to $\frac{1}{2}$. This is because projects with very high profitability have already been exploited, while those with very low profitability are not viable enough. Then, in this setting, the amplification effect is indeed expected to be quite large for the marginal investment.

The amplification result is demonstrated in Figure 2. Here, we let $\gamma$ decrease from 0.6 to 0.4, which could be thought of as a worsening of project profitability, maybe in response to a business cycle downturn. The dotted line shows the effect of $\gamma$ on firm value when the amount of information remains constant at its equilibrium level $\hat{\alpha}$ for $\gamma = 0.6$. The solid line shows the effect of $\gamma$ with endogenous information production. Consistent with Proposition 4, the solid line is steeper: the change in firm value resulting from a change in $\gamma$ is amplified by the endogenous production of information. Moreover, Figure 2 illustrates Proposition 5 and shows that there is a point at which a small change in $\gamma$ causes a collapse in firm value due to the breakdown of informed trading.

We can also use our model to think about the amount of investment in equilibrium. It has been well documented that (private) investment fluctuates strongly over the business cycle. Although it only makes up around 20% of GDP on average, investment fluctuations account for more than 90% of GDP variability (Barro, 1997). It has therefore been a key challenge of Real-Business-Cycle theory to understand how small shocks to fundamentals can lead to large fluctuations in investment. It is thus worthwhile investigating the quantitative effect that a small technology shock (a change
Figure 2: The figure shows the value of the firm as a function of $\gamma$ for the case where $\alpha$ is exogenous (dotted line) and for the case where $\alpha$ is endogenous (solid line). In the first case $\alpha$ is kept constant at $\tilde{\alpha} (0.6)$, while in the second case, $\alpha$ varies as a function $\gamma$ according to $\tilde{\alpha} (\gamma)$. The other parameters are set at $\sigma = 1$, $R_h - R_l = 1$, and $c = 0.45$. 
Figure 3: The figure shows the frequency of investment as a function of $\gamma$ for the case where $\alpha$ is exogenous (dotted line) and for the case where $\alpha$ is endogenous (solid line). In the first case $\alpha$ is fixed at $\hat{\alpha}(0.6)$, while in the second case, $\alpha$ varies with $\gamma$ according to $\hat{\alpha}(\gamma)$. The other parameters are set at $\sigma = 1$, $R_h - R_l = 1$, and $c = 0.45$.

in $\gamma$) has in our model on the ex ante probability (the ‘frequency’) of investment. We can think of this as translating into aggregate corporate investment when there are many firms with a common $\gamma$, but idiosyncratic projects.

The results are provided in Figure 3. Again, the dotted line shows the effect of $\gamma$ when the amount of information remains constant, while the solid line shows the effect of $\gamma$ with endogenous information production. We can see that, as a result of a decrease in expected profitability $\gamma$, there is a sharp drop in the frequency of investment only when information production is endogenous. When the production of information is not affected by profitability, the decrease in investment as a result of a decrease in profitability is minor.

Finally, a robustness question that comes up with regard to our amplification result is whether it
depends on the assumption that the manager cannot produce any information on his own. Presumably, anticipating the decrease in information production in the market, the manager will produce more information when fundamentals are bad, weakening the amplification effect. Speculating outside our model, the amplification effect should remain significant as long as there are some types of information that the market has substantial advantage in producing. (We provided motivation for this premise in Section 2.1.1.) As long as this holds, then market information is not easily replaceable, and thus the logic behind the amplification result remains intact. Proving this formally requires a richer model than the one presented here, as the informational advantage of the market has to become explicit, e.g., by assuming that the market aggregates many heterogeneous signals, and the firm can produce only several such signals.

4.3 The effect of uncertainty

Another parameter of interest is the level of uncertainty about the fundamental. In market microstructure models (e.g., Kyle, 1985) an increase in cash flow volatility typically increases the value of private information and trading profits. We now explore whether this is the case in the presence of the feedback mechanism.

Since cash flows are endogenous in our model, we conduct comparative statics on uncertainty as captured by \((R_h - R_t)\), which has been kept fixed in the analysis so far. Going back to the informed traders' profit function in (13), we can see that \((R_h - R_t)\) has two effects on profits (and thus on the equilibrium amount of information). The first effect is direct, and the second one is via the effect of \(\gamma\) on \(\alpha\) (see (5)). Using (1), we can write \(\gamma\) as:

\[
\gamma = \frac{1}{2} + \frac{R_h + R_t}{R_h - R_t} - I
\]

\[
= \frac{1}{2} + \frac{\nu}{R_h - R_t},
\]

where \(\nu \equiv \left(\frac{R_h + R_t}{2} - I\right)\) is the ex-ante NPV of the investment. Since we are interested here only in the effect of uncertainty, we will study the effect of changes in \((R_h - R_t)\) while keeping the expected profitability, measured by the ex-ante NPV \(\nu\), constant. Proposition 6 establishes the effect of \((R_h - R_t)\) on \(\alpha\). As before, our comparative statics focus on the case \(\alpha > 0\).
Proposition 6  Consider an increase in uncertainty $R_h - R_l$ such that the expected NPV $\nu$ remains constant.

(i) When the ex-ante NPV is positive ($\gamma > \frac{1}{2}$) and the equilibrium amount of information $\hat{\alpha}$ is greater than 0, then $\hat{\alpha}$ may decrease or increase in $R_h - R_l$.

(ii) When the ex-ante NPV is negative ($\gamma \leq \frac{1}{2}$) and the equilibrium amount of information $\hat{\alpha}$ is greater than 0, then $\hat{\alpha}$ increases in $R_h - R_l$.

This result follows from the two effects that the uncertainty ($R_h - R_l$) has on the trading profits. The direct effect is standard in financial market models: when uncertainty increases, private information is more valuable, and thus trading profits increase. Mathematically, this is captured by the fact that $(R_h - R_l)$ multiplies the expression for trading profits in (13). The second effect is indirect. Changes in uncertainty affect the threshold order flow above which the investment is undertaken and thus the expected frequency of investing. The direction in which this affects trading profits depends on whether the investment has a positive or a negative NPV ex-ante.

In case of a positive ex-ante NPV ($\gamma > \frac{1}{2}$), an increase in uncertainty (keeping the ex-ante NPV $\nu$ constant) makes it less likely that the investment will be undertaken, and thus decreases trading profits. To see this, substitute $\gamma$, given from (20) into (5). This yields

$$X = \frac{\sigma^2}{2\alpha} \ln \left[ \frac{1}{2} \frac{\frac{\nu}{\nu - \frac{\nu}{R_h - R_l}}} {\frac{\nu}{R_h - R_l}} \right].$$

When the ex-ante NPV $\nu$ is positive ($\gamma > \frac{1}{2}$), the threshold order flow $X(\alpha)$ above which the firm invests is negative, and so the firm invests more than half the time. However, this effect of the ex-ante NPV is weaker when uncertainty is higher. Thus, high uncertainty implies less frequent investment when the ex-ante NPV is positive, and this lowers trading profits.

The opposite is true when the ex-ante NPV $\nu$ is negative ($\gamma \leq \frac{1}{2}$): investment occurs less than half the time, and high uncertainty increases this frequency and raises profits. As a result, the overall effect of uncertainty on trading profits when projects are ex-ante profitable is ambiguous, while when they are ex-ante negative it is positive.
5 Extension to Three Investment Levels

Our basic model has either an investment of fixed size or no investment. We now explore robustness of the main results for a richer set of investment choices, where the firm can expand its investment or shrink it to a level above zero. Assume the investment can be of three different sizes: \( K \in \{0, 1, 2\} \). The first unit of investment costs \( I_1 \), while the second unit costs \( I_2 \). As before the payoff on any unit of the investment is \( R_\omega \), depending on the realization of equally likely states of the world \( \omega \in \{l, h\} \). We assume that \( R_h > I_2 > I_1 > R_l \), since this is the only configuration of parameters where the firm effectively decides among three investment levels.\(^{13}\) Following the basic model, we can now describe the firm’s projects by two profitability parameters:

\[
\gamma_1 = \frac{R_h - I_1}{R_h - R_l} > \gamma_2 = \frac{R_h - I_2}{R_h - R_l}.
\]

If the ex-post probability that \( \omega = h \) is above \( 1 - \gamma_1 \) then investment in the first unit is worthwhile, while if it is above \( 1 - \gamma_2 \) investment in the second unit is also worthwhile. Define the optimal investment level in the absence of information by \( K^* \), which takes the following values:

\[
K^* = \begin{cases} 
0 & \text{if } 0 < \gamma_2 < \gamma_1 \leq \frac{1}{2} \\
1 & \text{if } 0 < \gamma_2 \leq \frac{1}{2} < \gamma_1 < 1 \\
2 & \text{if } \frac{1}{2} < \gamma_2 < \gamma_1 < 1 
\end{cases}
\]

By changing the values of \( \gamma_1 \) and \( \gamma_2 \) we can see how the model captures different investment scenarios:

(i) Consider the case where \( 0 < \gamma_2 < \frac{1}{2} < \gamma_1 < 1 \). This corresponds to a situation where, ex-ante, the first unit is a positive-NPV investment, and the second unit is a negative-NPV investment. We will take as a benchmark the situation where the firm has no information. At this benchmark the firm has one unit: \( K^* = 1 \). If the firm receives good news, it may invest more, and scale up to \( K = 2 \). If the firm receives bad news, it may divest to \( K = 0 \). Thus, this captures a situation where the firm may choose to invest more or to divest its existing capital (or neither). We can describe this as assets in place with an option to expand and an option to abandon.

\(^{13}\)First, both \( R_l < I_1 < R_h \) and \( R_l < I_2 < R_h \), so that there will be a meaningful decision about each unit. Second, \( I_1 < I_2 \), as otherwise the firm either invests in two units or does not invest at all.
(ii) Alternatively, consider the case where \( \frac{1}{2} < \gamma_2 < \gamma_1 < 1 \). Here, both units are ex-ante positive NPV investments, so the benchmark is \( K^* = 2 \). Receiving bad news, the firm may divest to \( K = 1 \), while receiving even worse news, the firm may shut down completely and set \( K = 0 \). We can describe this as assets in place with an option to contract and an option to abandon.

(iii) Finally, consider the case where \( 0 < \gamma_2 < \gamma_1 < \frac{1}{2} \). This corresponds to a scenario where both units are ex-ante negative NPV investments, so the benchmark is \( K^* = 0 \). Receiving good news, the firm may invest in one unit (\( K = 1 \)), and receiving even better news, it may invest in two units (\( K = 2 \)). We can describe this as a firm with no assets in place, with an option on a possible project and a further option to scale up the project.

Apart from the change in the firm’s investment opportunities, we leave the basic model unchanged. It thus follows that when the true state is \( \omega = h \) all \( \alpha \) informed speculators optimally choose to submit buy orders and total order flow is then \( X = n + \alpha \). Conversely, when \( \omega = l \) informed traders sell and total order flow is \( X = n - \alpha \). As before, define \( \theta(X) \) as the probability that \( \omega = h \) given the order flow \( X \). This is given in (3) and is increasing in \( X \). Then, we have two cut-off values of \( X \): \( X_1 < X_2 \). Investment in the first unit is worthwhile if \( X \) is above \( X_1 \) and investment in the second unit is worthwhile if \( X \) is above \( X_2 \). These cut-off values can be expressed as:

\[
X_1 = \frac{\sigma^2 \ln 1 - \gamma_1}{2\alpha} \\
X_2 = \frac{\sigma^2 \ln 1 - \gamma_2}{2\alpha}.
\]  

(24)

As before, we focus on the equilibrium where, given \( \alpha \), the investment is undertaken if and only if it ought to be undertaken based on the information revealed to the market maker by the order flow. The price is given by:

\[
P(X) = \begin{cases} 
2(\theta(X)R_h + (1 - \theta(X)) R_l) - I_1 - I_2 & \text{if } X > X_2 \\
\theta(X)R_h + (1 - \theta(X)) R_l - I_1 & \text{if } X_1 < X \leq X_2 \\
0 & \text{if } X \leq X_1.
\end{cases}
\]

(25)

Note that \( P(X) \) is invertible for \( X > X_1 \) and the firm therefore optimally conditions its investment
policy on $P$ in the following way:

$$K(P) = \begin{cases} 
2 & \text{if } P > P(X_2) \\
1 & \text{if } 0 < P \leq P(X_2) \\
0 & \text{if } P = 0.
\end{cases} \quad (26)$$

When the price is 0, the firm knows that the NPV of investing in the first unit is negative, and does not invest at all. When the price is above 0, the firm optimally invests in the first unit. When the price is above $P(X_2)$ the firm optimally invests in both units.

Following steps similar to those in the basic model, we can write the expected profit from speculation as:

$$\pi(\alpha) = (R_h - R_t) \left( \int_{X_1(\alpha)}^{\infty} H(x)dx + \int_{X_2(\alpha)}^{\infty} H(x)dx \right). \quad (27)$$

Following the same line of argument as that in the proof of Proposition 1, it can be shown that $\lim_{\alpha \to \infty} \pi(\alpha) = 0$. Thus, for $c \leq \max_{\alpha \in \mathbb{R}^+} \pi(\alpha)$ there is an equilibrium with information production ($\widehat{\alpha} > 0$) while for $c > \max_{\alpha \in \mathbb{R}^+} \pi(\alpha)$ there is only an equilibrium with no information production. As before, in case of multiple equilibria, we consider the one with the largest measure of informed traders. This is denoted $\bar{\alpha}$.

### 5.1 Amplification

Our amplification result holds in this extended framework. There are now two parameters capturing profitability: $\gamma_1 = \frac{R_h - I_1}{R_h - R_t}$ and $\gamma_2 = \frac{R_h - I_2}{R_h - R_t}$. Following an analysis similar to that in the proof of Proposition 3, we can see that trading profits are increasing in $\gamma_1$ and $\gamma_2$, and as a result, the equilibrium amount of information production $\widehat{\alpha}$ is increasing in both $\gamma_1$ and $\gamma_2$. This does not depend on the level of $K^*$ and thus on the types of real options that the firm has. Essentially, a decrease in NPV of any unit implies that the firm is less likely to invest (or more likely to divest) in that unit. This reduces the incentive to produce information. Then, following a similar analysis to that in the proof of Proposition 4, we can see that the endogenous response of information production amplifies the effect that changes in $\gamma_1$ and $\gamma_2$ have on firm value.

Formally, Let $V(\alpha, \gamma_1, \gamma_2)$ be the expected value of the firm as a function of the equilibrium
amount of information produced $\hat{\alpha}$ and the investment profitability parameters $\gamma_1$ and $\gamma_2$:  

$$
V(\hat{\alpha}, \gamma_1, \gamma_2) = \frac{1}{2} (R_h - R_l) \left[ \int_{X_1(\hat{\alpha}, \gamma_1)}^{\infty} [\gamma_1 \varphi(x - \hat{\alpha}) - (1 - \gamma_1) \varphi(x + \hat{\alpha})] \, dx + \int_{X_2(\hat{\alpha}, \gamma_2)}^{\infty} [\gamma_2 \varphi(x - \hat{\alpha}) - (1 - \gamma_2) \varphi(x + \hat{\alpha})] \, dx \right],
$$

(28)

where as we saw above, $X_i(\alpha, \gamma_i) = \frac{\sigma^2}{2\alpha} \ln \frac{1 - \gamma_i}{\gamma_i}$. As before, the amplification effect implies that:

$$
\frac{dV(\hat{\alpha}, \gamma_1, \gamma_2)}{d\gamma_i} = \frac{\partial V}{\partial \gamma_i} + \frac{\partial V}{\partial \hat{\alpha}} \cdot \frac{\partial \hat{\alpha}}{\partial \gamma_i} > 0, \quad i = 1, 2.
$$

(29)

Moreover, just like in Proposition 5, the amplification effect is unbounded when $\pi(\hat{\alpha})$ approaches $\max_{\alpha \in \mathbb{R}^+} \pi(\alpha)$. As before, for this to happen, the profit function has to be hump-shaped, which is now always the case when $\gamma_1 < \frac{1}{2}$ and may be the case when $\gamma_2 < \frac{1}{2} < \gamma_1$ (depending on parameter values).

Interestingly, there is a sense in which the amplification effect is stronger in the extended model (where the decision is between 0, 1, or 2 investment units) than in the basic model (where the decision is between 0 or 1 investment units). In the extended model, an improvement in the profitability of one of the two units will increase the incentive for information production. This will then increase the efficiency of the investment decision in this unit, but will also have a positive spillover effect on the efficiency of the investment decision in the other unit, leading to a larger effect on firm value.

### 5.2 Real options and trading profits

We noted above that the investment technology described in this section is a project with embedded real options. There is a natural link between real options and a model of feedback from stock prices to firm value. Without feedback, the firm is just a bundle of existing investment projects, valued at expected discounted cash flows. With feedback, the firm reacts to information conveyed by stock prices. This is like a real option, where the exercise is contingent on the firm’s own stock price movements. We can use our model to analyze the effect of real options (and hence feedback effects) on the profit from information production and hence on the amount of information production in equilibrium.
For illustration, consider the case where $0 < \gamma_2 < \frac{1}{2} < \gamma_1 < 1$. As noted above, in this case the firm can be thought of as one unit of assets in place with an option to expand and an option to abandon. If the investment project displayed no embedded options, that is, if the firm was going to keep its benchmark level of capital $K^* = 1$, trading profits would be:

$$ (R_h - R_t) \int_{-\infty}^{\infty} H(x)dx. $$

(30)

Here, speculators profit on their information about the state of the world for the unit of assets in place over the entire range of order flows $(-\infty, \infty)$. This is because this unit is never cancelled.

Now, adding the option to expand to a second unit of asset increases trading profits by $(R_h - R_t) \int_{X_2(\alpha)}^{\infty} H(x)dx$ to:

$$ (R_h - R_t) \left( \int_{-\infty}^{\infty} H(x)dx + \int_{X_2(\alpha)}^{\infty} H(x)dx \right). $$

(31)

Here, knowing that the firm may invest in another unit (which happens when order flow is above $X_2(\alpha)$), implies that information about the profitability of the project is more strongly related to the value of the security, increasing expected trading profits. In equilibrium, this leads to a higher amount of information production $\alpha$.

Finally, adding the option to abandon, reduces trading profits by $(R_h - R_t) \int_{-\infty}^{X_1(\alpha)} H(x)dx$ to:

$$ (R_h - R_t) \left( \int_{X_1(\alpha)}^{\infty} H(x)dx + \int_{X_2(\alpha)}^{\infty} H(x)dx \right). $$

(32)

Here, knowing that the firm may abandon the first unit (which happens when order flow is below $X_1(\alpha)$), implies that information about the profitability of the project becomes less related to the value of the security, reducing expected trading profits and the equilibrium amount of information production.

In summary, adding a real option to expand increases profits from informed trading and hence the equilibrium amount of information, while adding a real option to abandon decreases profits from informed trading and the equilibrium amount of information.

Our model can be linked to Hirshleifer’s (1971) distinction between two types of information: discovery and foreknowledge. Discovery means learning information that will not necessarily be
revealed otherwise, such as inventing a new technology. Foreknowledge means learning information that will in any case be revealed later on, such as learning a firm’s earnings a few days in advance. He argues that the private and social rewards to either kind of information production may diverge. In particular, there may be a private benefit from socially useless foreknowledge. In our model, foreknowledge can be thought of as information about a project’s cash flows when there is no uncertainty about whether the project will be undertaken. Discovery can be thought of as information needed to guide the decision on whether to take the project or not: if the project is not taken, information about \( \omega \) will not become available. Fixing to one unit of investment, our model predicts more information production on the former than on the latter. This is because trading profits in the foreknowledge scenario are \((R_h - R_t) \int_{-\infty}^{\infty} H(x) dx\), which are higher than in the discovery scenario: \((R_h - R_t) \int_{X_1(\alpha)}^{\infty} H(x) dx\). Essentially, speculators want to produce information on investments they know will be undertaken, but this precludes the beneficial social role of using that information for deciding whether to undertake the investment or not.

6 Conclusion

Financial markets play a central role in the economy. Even in the case of secondary markets, which have no direct effect on the real economy, market prices aggregate a lot of useful information that ends up affecting real investment decisions and consequently firm value. The model developed in this paper helps understand some of the consequences of this feedback effect from financial markets to the real economy.

Endogenizing the amount of information produced in the financial market, we show that speculators have stronger incentives to produce information about an investment opportunity when the firm is more likely to undertake the investment. This creates an amplification effect, whereby small changes in fundamentals are amplified into large changes in real investments and firm values. Importantly, our model generates a market breakdown, where a small change in fundamentals can lead information production to dry up completely, generating a collapse in firm investment and real value.

Such an amplification effect has been the focus of many papers trying to explain its sources.
To our knowledge, our paper is the first one that links such amplification to the informational role of financial markets. Information in financial markets gets produced by speculators, who are incentivized by potential speculative profits. This leads them to produce more information in good times than in bad times (or more information about good firms than about bad firms), generating the above mentioned amplification mechanism. Given that information asymmetries are among the most important frictions driving investment and financing behavior, we believe that our informational channel is an important addition to the understanding of amplification in firms’ investments and values.

Finally, our analysis can be linked to Hirshleifer’s (1971) distinction between discovery and foreknowledge. The incentive to produce information is stronger when the project is certain to be undertaken, even though the social value is higher when the project may not be undertaken and information is needed to guide the investment decision.

7 Appendix

**Proof of Lemma 1:** We first verify that the decision rule in (7) is optimal for the firm. This holds because observing a price above 0 reveals that the updated NPV of the investment is positive and thus that the investment should be undertaken, whereas a price of 0 reveals that the updated NPV is negative (or zero). Second, the pricing rule in (6) reflects the expected value of the firm given the order flow, and given the firm’s investment decision: the price is equal to 0 when the investment is not expected to be undertaken and is equal to the expected NPV when the investment is undertaken. QED.

**Proof of Proposition 1:** We first show that when $\gamma > \frac{1}{2}$, $\pi(\alpha)$ is a strictly decreasing function.
We can write
\[
\frac{d\pi (\alpha)}{d\alpha} = \frac{\partial}{\partial \alpha} \left( (R_h - R_l) \int_{\bar{X}(\alpha)}^{\infty} H(x)dx \right) \\
= (R_h - R_l) \int_{\bar{X}(\alpha)}^{\infty} \frac{\partial H(x)}{\partial \alpha} dx - (R_h - R_l) \frac{\partial \bar{X}(\alpha)}{\partial \alpha} H(\bar{X}(\alpha)) \\
= (R_h - R_l) \int_{\bar{X}(\alpha)}^{\infty} \frac{\varphi (x - \alpha) \varphi (x + \alpha) (\frac{x - \alpha}{\sigma^2} \varphi (x + \alpha) - \frac{x + \alpha}{\sigma^2} \varphi (x - \alpha))}{(\varphi (x - \alpha) + \varphi (x + \alpha))^2} dx \\
- (R_h - R_l) \frac{d\bar{X}(\alpha)}{d\alpha} H(\bar{X}(\alpha)).
\] (33)

Here, we used the definition of \(H(x)\) as given in (12) and the fact that \(\varphi' (n) = -\frac{n}{\sigma^2} \varphi (n)\). From (5) we can see that when \(\gamma > \frac{1}{2}\), we will have \(\ln \frac{1 - \gamma}{\gamma} < 0\), and so \(\frac{d\bar{X}(\alpha)}{d\alpha} > 0\). Thus, the second term in (33) is negative. To show that the first term is weakly negative, we need to show that
\[
(x - \alpha)\varphi (x + \alpha) - (x + \alpha)\varphi (x - \alpha) \leq 0
\]
i.e.
\[
x (\varphi (x + \alpha) - \varphi (x - \alpha)) \leq \alpha (\varphi (x + \alpha) + \varphi (x - \alpha)) .
\] (34)

If \(x < 0\), then, because \(\varphi\) is the density function of a normal distribution with mean 0, \(\varphi (x + \alpha) \geq \varphi (x - \alpha)\), and thus the LHS of (34) is negative while the RHS is positive, so the inequality in (34) holds. Similarly, if \(x > 0\), then \(\varphi (x + \alpha) \leq \varphi (x - \alpha)\), and again the LHS of (34) is negative while the RHS is positive, so the inequality in (34) holds. It follows that \(\frac{d\pi (\alpha)}{d\alpha} < 0\).

Next, we prove the following lemma.

**Lemma 2** \(\lim_{\alpha \to -\infty} \pi (\alpha) = 0\).

**Proof:** Since as \(\alpha \to -\infty\), \(\bar{X}(\alpha) \to 0\), for sufficient large \(\alpha\), we will have
\[
\pi (\alpha) < (R_h - R_l) \int_{-1}^{\infty} \frac{\phi (x - \alpha) \phi (x + \alpha)}{\varphi (x - \alpha) + \varphi (x + \alpha)} dx
\]
Note that \(\frac{\phi (x - \alpha) \phi (x + \alpha)}{\varphi (x - \alpha) + \varphi (x + \alpha)} < \phi (x + \alpha)\), so
\[
\pi (\alpha) < (R_h - R_l) \int_{-1}^{\infty} \phi (x + \alpha) dx = (R_h - R_l) \int_{-1}^{1} \phi (x + \alpha) dx + (R_h - R_l) \int_{1}^{\infty} \phi (x + \alpha) dx.
\]
Since \( \lim_{\alpha \to \infty} \phi(x + \alpha) = 0 \), \( \lim_{\alpha \to \infty} (R_h - R_t) \int_{-\infty}^{1} \phi(x + \alpha)dx = 0 \). For \( (R_h - R_t) \int_{-\infty}^{1} \phi(x + \alpha)dx \), note that when \( x \geq 1 \) and \( \alpha \geq 0 \), \( \phi(x + \alpha) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x+\alpha)^2}{2\sigma^2}} < \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{x^2}{2\sigma^2}} \), we have

\[
\int_{-\infty}^{1} \phi(x + \alpha)dx < \int_{-\infty}^{1} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2\sigma^2}(x+\alpha)^2} dx = \frac{1}{\sqrt{2\pi\sigma}} 2\sigma^2 e^{-\frac{x^2}{2\sigma^2}}(1+\alpha) \to 0 \text{ as } \alpha \to \infty
\]

so \( \lim_{\alpha \to \infty} (R_h - R_t) \int_{-\infty}^{1} \phi(x + \alpha)dx = 0 \). Thus \( \lim_{\alpha \to \infty} \pi(\alpha) = 0 \). QED

Thus, since \( \pi(\alpha) \) is strictly decreasing and since it approaches 0 as \( \alpha \) approaches \( \infty \), there are two possible cases: either (i) there is a unique intersection \( \pi(\alpha) = c \), or (ii) \( \pi(0) < c \). Case (i) will hold when \( c = \pi(0) = \frac{R_h - R_t}{2} \). From (15) and (14), we know that in Case (ii), the unique equilibrium is that there is no information production: \( \widehat{\alpha} = 0 \). In Case (i), there is a unique \( \widehat{\alpha} > 0 \) that satisfies \( \pi(\widehat{\alpha}) = c \). QED

**Proof of Proposition 2:** (i) We need to show that if \( \widehat{\alpha} = 0 \), the profit from producing information is 0. From (5), we can see that, for \( \gamma < \frac{1}{2} \), \( \lim_{\alpha \to 0} X(\alpha) = \infty \). Plugging this in the profit function (13), and noting that \( \lim_{\alpha \to 0} H_\alpha(x) = \frac{1}{2} \varphi(x) \), we know that \( \lim_{\alpha \to 0} \pi(\alpha) = 0 \).

(ii) From (13) it is clear that \( \pi(\alpha) > 0 \) for all \( \alpha > 0 \). We showed in part (i) of this proof that for \( \gamma < \frac{1}{2} \), \( \lim_{\alpha \to 0} \pi(\alpha) = 0 \) and we showed in the proof of Proposition 1 that for all \( \gamma \), \( \lim_{\alpha \to \infty} \pi(\alpha) = 0 \). Since \( \pi(\alpha) \) is a continuous function, it follows that \( \pi(\alpha) \) must have a global maximum \( \max_{\alpha \in \mathbb{R}^+} \pi(\alpha) \). Consider a cost \( c \leq \max_{\alpha \in \mathbb{R}^+} \pi(\alpha) \). It follows that there is at least one point \( \alpha > 0 \) such that \( \pi(\alpha) = c \) (note that when \( c \leq \max_{\alpha \in \mathbb{R}^+} \pi(\alpha) \), there will be at least two such points). Moreover from (14), we know that each such \( \alpha \) constitutes an equilibrium.

(iii) When \( c > \max_{\alpha \in \mathbb{R}^+} \pi(\alpha) \), there is no \( \alpha > 0 \) such that \( \pi(\alpha) = c \). Thus, following (14), there is no equilibrium with \( \widehat{\alpha} > 0 \) and the equilibrium \( \widehat{\alpha} = 0 \) identified in part (i) of this proof is unique. QED

**Proof of Proposition 3:** We start by analyzing the effect of \( \gamma \) on trading profits \( \pi(\alpha) \) for any level of information production \( \alpha \).

\[
\frac{\partial \pi(\alpha)}{\partial \gamma} = (R_h - R_t) \int_{X(\alpha)}^{\infty} \frac{\partial H(x)}{\partial \gamma} dx - (R_h - R_t) \frac{\partial X(\alpha)}{\partial \gamma} \cdot H(X(\alpha)).
\]

The first line on the RHS is 0 since \( H(x) \) does not depend on \( \gamma \) (see the definition of 12). From (5), it follows that \( \frac{\partial X(\alpha)}{\partial \gamma} < 0 \). Hence, \( \frac{\partial \pi(\alpha)}{\partial \gamma} > 0 \).
Now fix $\gamma = \gamma'$, and denote the resulting profit function $\pi_{\gamma'} (\alpha)$. Denote the resulting equilibrium quantity of information $\hat{\alpha}$; recall that this is defined as the largest point $\alpha$ at which $\pi_{\gamma'} (\alpha) = c$. Now consider a small perturbation to a smaller $\gamma = \gamma''$ and the resulting equilibrium quantity of information $\hat{\alpha}''$. Denote the resulting profit function $\pi_{\gamma''} (\alpha)$. From the analysis above we know that the decrease from $\gamma'$ to $\gamma''$ results in smaller profits for all $\alpha$: $\pi_{\gamma''} (\alpha) < \pi_{\gamma'} (\alpha)$.

We will show that a smaller $\gamma$ implies a smaller $\hat{\alpha}$: $\hat{\alpha}'' < \hat{\alpha}'$. Suppose on the contrary that $\hat{\alpha}'' \geq \hat{\alpha}'$ while $\gamma'' < \gamma'$. Since for all $\alpha$, $\pi_{\gamma''} (\alpha) < \pi_{\gamma'} (\alpha)$, and since $\pi_{\gamma''} (\hat{\alpha}'') = c$, it follows that $\pi_{\gamma'} (\hat{\alpha}'') > c$. Since $\lim_{\alpha \to \infty} \pi_{\gamma'} (\alpha) = 0$, by continuity of $\pi$ there exists a point $\alpha^*$, with $\alpha^* > \hat{\alpha}'' \geq \hat{\alpha}'$, at which $\pi_{\gamma'} (\alpha^*) = c$. This contradicts the definition of $\hat{\alpha}'$ as the largest point $\alpha$ at which $\pi_{\gamma'} (\alpha) = c$. Hence, we must have $\hat{\alpha}'' < \hat{\alpha}'$. QED

**Proof of Proposition 4:** We know from the proof of Proposition 3 that $\frac{\partial \hat{\alpha}}{\partial \gamma} > 0$. Hence, to prove the proposition, we need to show that $\frac{\partial V}{\partial \gamma} > 0$ and $\frac{\partial V}{\partial \alpha} > 0$.

Based on the expression for $V (\hat{\alpha}, \gamma)$ in (18), we know that:

$$
\frac{\partial V (\hat{\alpha}, \gamma)}{\partial \gamma} = \frac{1}{2} (R_h - R_l) \int_{X(\hat{\alpha}, \gamma)}^{\infty} [\varphi (x - \hat{\alpha}) + \varphi (x + \hat{\alpha})] dx.
- \frac{1}{2} (R_h - R_l) \frac{\partial X(\hat{\alpha}, \gamma)}{\partial \gamma} \left[ \gamma \varphi (X(\hat{\alpha}, \gamma) - \hat{\alpha}) - (1 - \gamma) \varphi (X(\hat{\alpha}, \gamma) + \hat{\alpha}) \right].
$$

Using (3) and (4), we can see that the expression in the brackets in the second line is 0. Since the expression in the first line is positive, we know that $\frac{\partial V}{\partial \gamma} > 0$.

Now we can write:

$$
\frac{\partial V (\hat{\alpha}, \gamma)}{\partial \alpha} = \frac{1}{2} (R_h - R_l) \int_{X(\hat{\alpha}, \gamma)}^{\infty} [-\gamma \varphi' (x - \hat{\alpha}) - (1 - \gamma) \varphi' (x + \hat{\alpha})] dx.
- \frac{1}{2} (R_h - R_l) \frac{\partial X(\hat{\alpha}, \gamma)}{\partial \alpha} \left[ \gamma \varphi (X(\hat{\alpha}, \gamma) - \hat{\alpha}) - (1 - \gamma) \varphi (X(\hat{\alpha}, \gamma) + \hat{\alpha}) \right].
$$

The second line is again 0. The first line can be rewritten as:

$$
\frac{1}{2} (R_h - R_l) \left[ \gamma \varphi (X(\hat{\alpha}, \gamma) - \hat{\alpha}) + (1 - \gamma) \varphi (X(\hat{\alpha}, \gamma) + \hat{\alpha}) \right] > 0.
$$

Hence, $\frac{\partial V}{\partial \alpha} > 0$. QED

**Proof of Proposition 5:** We know from the proof of Proposition 4 that

$$
\frac{\partial V}{\partial \alpha} = \frac{1}{2} (R_h - R_l) \left[ \gamma \varphi (X(\hat{\alpha}, \gamma) - \hat{\alpha}) + (1 - \gamma) \varphi (X(\hat{\alpha}, \gamma) + \hat{\alpha}) \right].
$$
This is bounded away from 0 as long as $\hat{\alpha} > 0$. Hence, we need to show that $\frac{\partial \hat{\alpha}}{\partial \gamma}$ approaches $\infty$ as $\gamma$ approaches $\gamma^*$ from above. For this, set $\gamma^*$ at that value for which $c = \max_{\alpha \in \mathbb{R}^+} \pi(\alpha)$, i.e., where the equilibrium $\hat{\alpha} = \arg \max \pi(\alpha)$. We know that such a $\gamma^*$ must exist, because there exists a $\gamma < \frac{1}{2}$ for which $\hat{\alpha} > 0$ and for $\gamma$ sufficiently close to zero $\hat{\alpha} = 0$.

By the implicit function theorem, $\frac{\partial \hat{\alpha}}{\partial \gamma} = \left( \frac{\partial \pi(\hat{\alpha})}{\partial \gamma} \right) / \left( \frac{\partial \pi(\hat{\alpha})}{\partial \alpha} \right)$. From the proof of Proposition 3, $\frac{\partial \pi(\hat{\alpha})}{\partial \gamma} > 0$ is bounded away from 0. Since $\frac{\partial \pi(\hat{\alpha})}{\partial \alpha} < 0$ approaches 0 as $\pi(\hat{\alpha})$ approaches $\max_{\alpha \in \mathbb{R}^+} \pi(\alpha)$, $\frac{\partial \hat{\alpha}}{\partial \gamma}$ approaches $\infty$. QED

**Proof of Proposition 6:** Following the proof of Proposition 3, the sign of the effect of $(R_h - R_l)$ on $\hat{\alpha}$ is the same as the sign of $\frac{\partial \pi(\hat{\alpha})}{\partial (R_h - R_l)}$. From (13):

$$\frac{\partial \pi(\hat{\alpha})}{\partial (R_h - R_l)} = \int_{X(\alpha)} H(x) dx - (R_h - R_l) \frac{\partial X(\alpha)}{\partial (R_h - R_l)} \frac{\partial \gamma}{\partial (R_h - R_l)} H(X(\alpha))$$

Using (5) and (20), we then get:

$$\frac{\partial \pi(\hat{\alpha})}{\partial (R_h - R_l)} = \int_{X(\alpha)} H(x) dx - (R_h - R_l) \left( \frac{\sigma^2}{2\alpha (1 - \gamma)} - \frac{-\gamma - (1 - \gamma)}{\gamma^2} \right) \left( -\frac{v}{(R_h - R_l)^2} \right) H(X(\alpha))$$

As a result, the overall effect of $(R_h - R_l)$ on $\pi(\hat{\alpha})$ and on $\hat{\alpha}$ is ambiguous when $v$ is positive (simulations show that it can go either way), and it is positive when $v$ is negative. QED

**References**


