# Natural Rate Measures in an Estimated DSGE Model of the U.S. Economy

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# Background

- In 2007 the Research and Statistics Division at the FRB moved into production an estimated DSGE model for use in addressing quotidian and *ad hoc* policy questions.
  - The model, called the FRB/EDO (Estimated Dynamic Optimization) model, serves as a complement to FRB/US.
- Quotidian questions currently addressed by the model include the generation of an alternative model forecast.
  - Going forward other uses of the model are anticipated.
- This paper uses an earlier but similar version of FRB/EDO to consider the usefulness for policy purposes of natural rate measures derived from an estimated DSGE model.

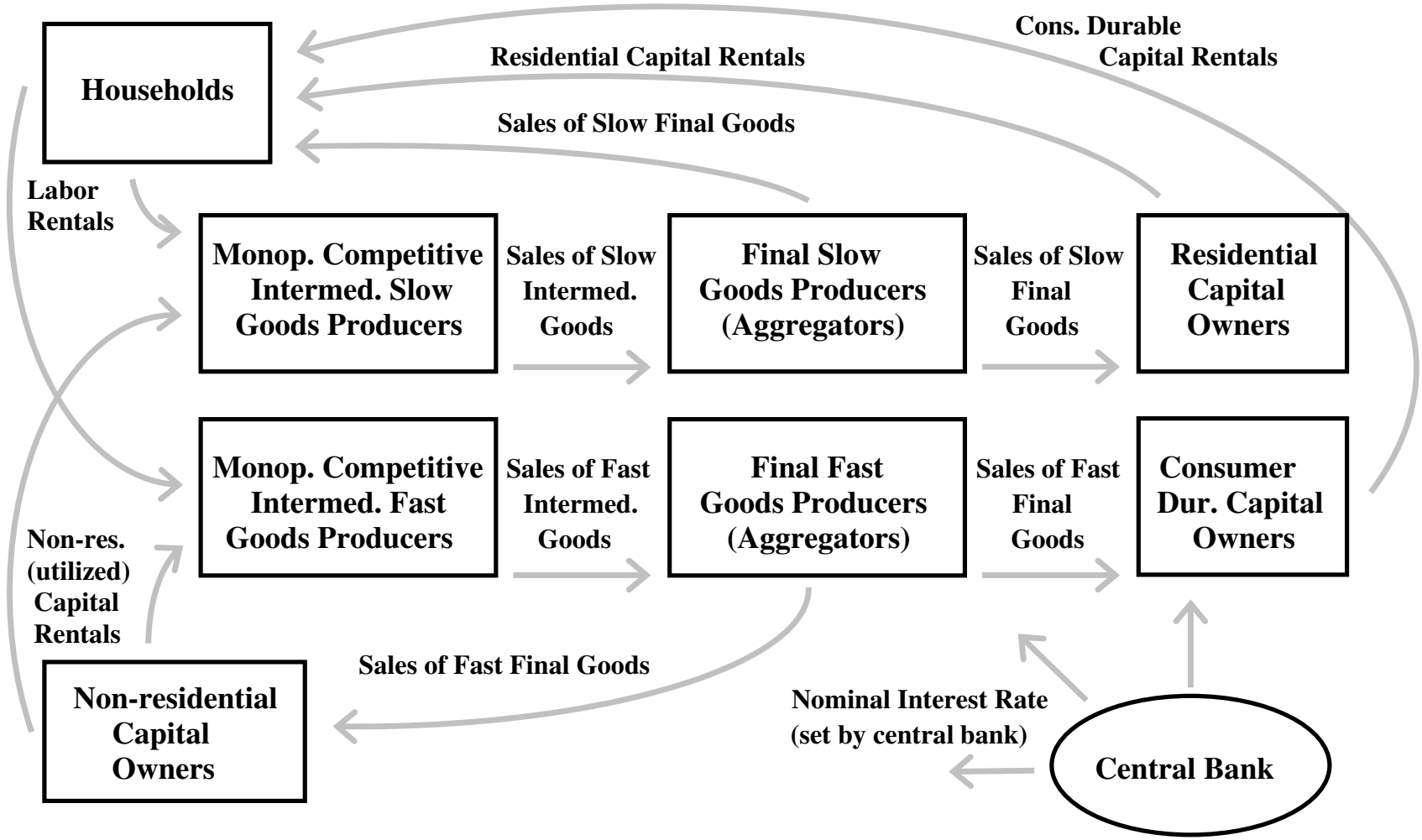
# Outline of the presentation

- Describe the version of FRB/EDO model used in this paper.
  - Emphasize the features of the model that differ from the Christiano-Eichenbaum-Evans/Smets-Wouters models.
- Discuss briefly the estimation of the model.
- Turn to the analysis of the model's natural rate measures; specifically,
  - The output gap; and,
  - The natural real rate of interest.
- Sum up.

# Model overview

- Our model is more disaggregated than most closed-economy models.
- The model has two production sectors:
  - A slow-growing goods producing sector, $X^{slw}$; and,
  - A fast-growing goods producing sector, $X^{fst}$.
- The model has multiple expenditure aggregates:
  - Expenditures on consumer nondurable goods and non-housing services, $E^{cnn}$;
  - Expenditures on consumer durable goods, $E^{cd}$;
  - Expenditures on residential capital, $E^{r}$;
  - Expenditures on non-residential capital, $E^{nr}$; and,
  - Public, net-foreign demand, $E^{exog}$.

# Model diagram



# Why do we disaggregate production?

- U.S. NIPA data are *not* consistent with a one-sector growth model.
  - There are sizable differences in long-run real growth rates across NIPA categories with large trends in relative prices.
  - But nominal growth rates are broadly similar across NIPA categories.

| | Avg. Real Growth | Avg. Rel. Price Change | Avg. Nom. Growth |
|---|---|---|---|
| Cons. NDS $E^{cnn}$ | $3\frac{1}{4}$ percent | n.a. | $6\frac{1}{4}$ percent |
| Cons. durables, $E^{cd}$ | $6\frac{3}{4}$ percent | -3 percent | $6\frac{1}{2}$ percent |
| Res. investment, $E^{r}$ | $3\frac{3}{4}$ percent | $\frac{1}{2}$ percent | $7\frac{1}{2}$ percent |
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# Long-run properties of a two-sector growth model

- A two-sector growth model with

  $X_t^{slw} = (K_t^{slw})^{\alpha}(Z_t^{ntrl} L_t^{slw})^{1-\alpha}$ and $X_t^{fst} = (K_t^{fst})^{\alpha}(Z_t^{ntrl} Z_t^{fst} L_t^{fst})^{1-\alpha}$

  where $\ln Z_t^s = \ln Z_{t-1}^s + \ln \Gamma_*^{z,s} + \gamma_t^{z,s}$ for $s = ntrl, fst$ implies that:

  - There are differential real growth rates across sectors.
  - There are differential inflation rates across sectors.
  - The same nominal growth rate across sectors.

| Sector | Real Gr. $(\Gamma_*^{x,s})$ | Price Ch. $(\Pi_*^{p,s})$ | Nom. Gr. $(\Gamma_*^{x,s}\Pi_*^{p,s})$ |
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# Why do we further disaggregate expenditures?

- Different expenditure categories have different cyclical properties.
  - Consumer durables and residential investment tend to lead business cycles; non-residential *fixed* investment tends to lag.
  - Consumer durables and residential investment may be more responsive to monetary policy.
- Policy-makers are concerned with movements in detailed aggregates.
- Growth accounting requires that the (latent) capital stock series used in the model's production functions correspond to conventional definitions of productive capital.

# Model features

- Nominal rigidities:
  - Sticky prices and sticky wages, where both lagged inflation and steady-state inflation determine adjustment costs.
- Real rigidities:
  - Habit formation in consumption;
  - Adjustment costs to investment;
  - Adjustment costs to cross-sectoral factor movements; and,
  - Variable utilization of non-residential capital.

# Household preferences

- Utility is defined over leisure and the following goods:
  - Consumer non-durable goods and non-housing services, $E_t^{cnn}$;
  - Consumer durable capital stock, $K_t^{cd}$; and,
  - Residential capital stock, $K_t^r$.
- Within-period utility is:

$$\varsigma^{cnn}\Xi_t^{cnn}\ln\left(E_t^{cnn}(i)-h^{cnn}E_{t-1}^{cnn}(i)\right)+\varsigma^{cd}\Xi_t^{cd}\ln\left(K_t^{cd}(i)-h^{cd}K_{t-1}^{cd}(i)\right)$$
$$+\varsigma^{r}\Xi_t^{r}\ln\left(K_t^{r}(i)-h^{r}K_{t-1}^{r}(i)\right)\ ]-\varsigma^{l}\Xi_t^{l}\frac{\left(L_t^{slw}(i)+L_t^{fst}(i)\right)^{1+\nu}}{1+\nu}.$$

- $\Xi_t^{cnn}$, $\Xi_t^{cd}$, $\Xi_t^{r}$, and $\Xi_t^{l}$, are persistent preference shocks.

# Capital evolution processes

- The economy possesses multiple capital stocks:
  - Non-residential capital, $K_t^{nr}$;
  - Residential capital stock, $K_t^r$; and,
  - Consumer durable capital stock, $K_t^{cd}$.
- Investment incurs adjustment costs; *e.g.*, res. capital evolves:

$$K_{t+1}^r(k) = (1-\delta^r)K_t^r(k) + A_t^r E_t^r(k) - \frac{100\cdot\chi^r}{2}\left(\frac{E_t^r(k) - E_{t-1}^r(k)\Gamma_t^{x,slw}}{K_t^r}\right)^2 K_t^r$$

- $A_t^r$ is a transitory, but persistent, capex. "efficiency" shock.
- Non-residential and consumer durables capital evolve similarly.

# Nominal rigidities

- Intermediate goods producing firms and households face convex adjustment costs in altering prices and nominal wages.
- Adjustment costs depend on lagged and steady-state inflation.
- The log-linearized price Phillips curve in sector $s$ is:

$$\pi_t^{p,s} = \frac{\eta^{p,s}}{1+\beta\eta^{p,s}} \cdot \pi_{t-1}^{p,s} + \frac{\beta}{1+\beta\eta^{p,s}} \cdot E_t\pi_{t+1}^{p,s} + \frac{1}{1+\beta\eta^{p,s}} \cdot \frac{\left(\Theta_*^{x,s}-1\right)}{100\cdot\chi^p(\Pi_*^{p,s})^2} \left(mc_t^s - \frac{1}{\Theta_*^{x,s}-1} \cdot \theta_t^{x,s}\right).$$

- The log-linearized wage Phillips curve in sector $s$ is:

$$\pi_t^{w,s} = \frac{\eta^{w,s}}{1+\beta\eta^{w,s}} \cdot \pi_{t-1}^{w,s} + \frac{\beta}{1+\beta\eta^{w,s}} \cdot E_t\pi_{t+1}^{w,s} + \frac{1}{1+\beta\eta^{w,s}} \cdot \frac{\left(\Theta_*^{l}-1\right)}{100\cdot\chi^w(\Pi_*^{w,s})^2} \left(mrs_t - w_t^s - \frac{1}{\Theta_*^{l}-1} \cdot \theta_t^{l}\right).$$

# Monetary policy

- The log-linearized monetary policy rule is:

$$r_t = \phi^r \cdot r_{t-1} + (1 - \phi^r) \cdot \bar{r}_t + \epsilon_t^r.$$

- The target nominal interest rate, $\bar{r}_t$, is:

$$\bar{r}_t = \phi^{gr,gdp} . gr_t^{gdp} + \phi^{\Delta gr,gdp} . \Delta gr_t^{gdp} + \phi^{\pi,gdp} . \pi_t^{p,gdp} + \phi^{\Delta \pi,gdp} . \Delta \pi_t^{p,gdp}$$

  where $gr_t^{gdp}$ = GDP growth and $\pi_t^{p,gdp}$ = GDP price inflation.

- GDP growth and GDP price inflation are consistent with NIPA definitions; that is, both are Tornquist indexes.

# Model equations

- Utility maximization by households (and cost minimization by capital owners) implies:
  - Non-durables & non-housing services consumption demand, consumer durables demand, residential capital goods demand.
  - Labor supply (i.e., wage Phillips curves).
- Cost minimization by firms (and capital owners) implies:
  - Economy-wide labor demand.
  - Non-residential capital goods demand.
- Profit-maximization by firms implies:
  - Aggregate supply for goods (i.e., price Phillips curves).
- The monetary authority follows an interest rate feedback rule.
- Public and foreign demand evolve exogenously.

# Estimation overview (1)

The model is estimated with Bayesian techniques using 11 data series:

| | |
|---|---|
| Real variables | • GDP<br>• NDS (ex. housing) consumption<br>• Durables consumption<br>• Residential investment<br>• Non-residential investment<br>• Hours in the NFB sector |
| Wages and prices | • Real comp. per hour in the NFB sector<br>• GDP deflator<br>• NDS (ex. housing) consumption deflator<br>• Non-residential investment goods deflator |
| Monetary policy | • Federal funds rate |

All other (latent) model variables, including the models exogenous processes, are estimated as part of the Kalman-filter routine.

The estimation period 1984q3 to 2004q4.

# Estimation overview (2)

The model assumes 14 exogenous structural shocks

| | |
|---|---|
| Technology shocks | • Sector-neutral TFP |
| | • Sector-specific TFP |
| Mark-up shocks | • Slow sector prices over m.c. |
| | • Fast sector prices over m.c. |
| | • Wages over the m.r.s. |
| Preference shocks | • Labor supply |
| | • Durables consumption |
| | • Housing services consumption |
| | • NDS (ex. housing) consumption |
| Capex. efficiency shocks | • Non-residential investment |
| | • Residential investment |
| | • Consumer durables |
| Demand shock | • Exog. public, net-foreign demand |
| Monetary policy shock | • Federal funds rate |

We also add measurement error for all of the observed series except the nominal interest rate and aggregate hours.

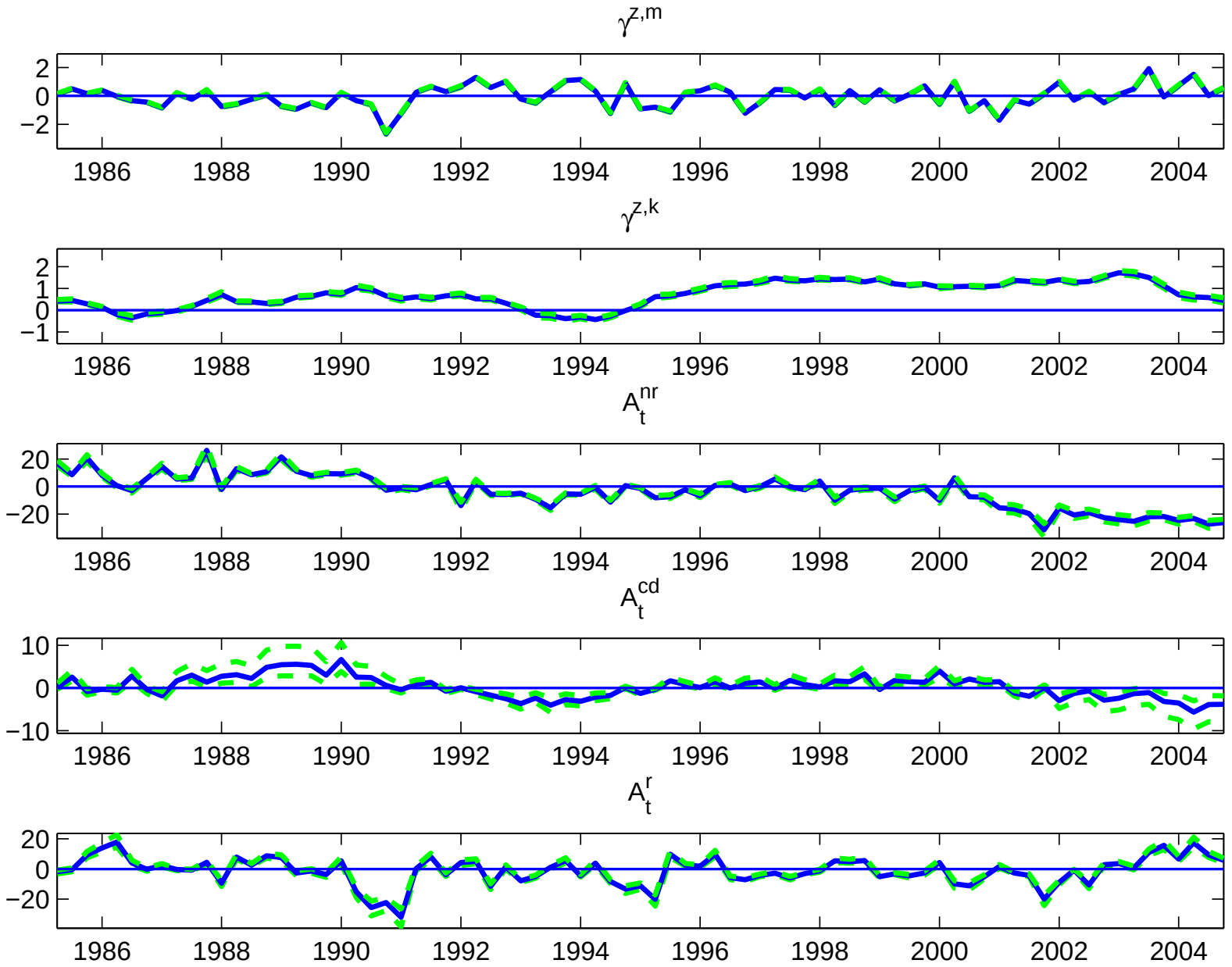
# Key estimation results

- Nominal rigidities:
  - Nominal rigidities are important; prices appear less sensitive to their driving process than wages.
  - In both the wage and price Phillips curves, the weight on the lead of inflation is larger than on the lag.
  - The weight on lagged inflation is larger for wages than prices.
- Real rigidities:
  - Habit formation for NDS (ex. housing) consumption is large.
  - Habit formation for the service flow from consumer durables and residential capital is lower but the data is not informative.
  - Adjustment costs are important for explaining the dynamics of residential and non-residential investment spending.
  - Adjustment costs in the movement of labor between the two production sectors are important for aggregate comovement.
- Most of the structural shocks follow persistent processes.

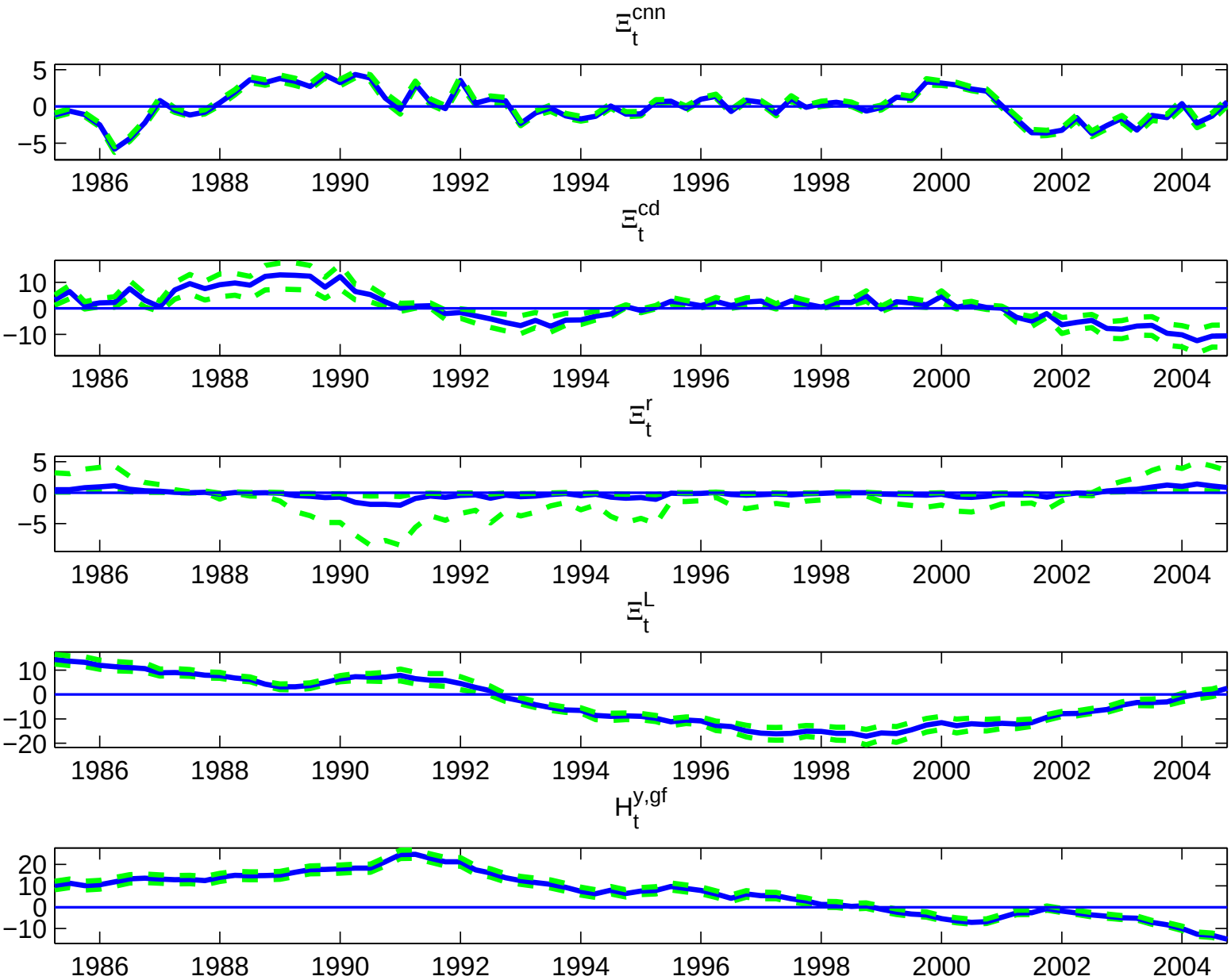
# Deriving the DSGE model's natural rate measures

- Natural-rate variables are those implied by a flexible wage and price version of the model.
  - This model eliminates sticky wages and prices ($\chi^w = \chi^p = 0$) and mark-up and monetary policy shocks ($\theta_t^w = \theta_t^{p,s} = \epsilon_r^r = 0$) but is otherwise identical to the estimated model.
- Natural-rate variables are generated using the flexible wage and price model and all other structural shocks of the estimated model:
  - That is, technology, capex. efficiency, preference, and public, net-foreign demand shocks.

# Technology and capex efficiency shocks



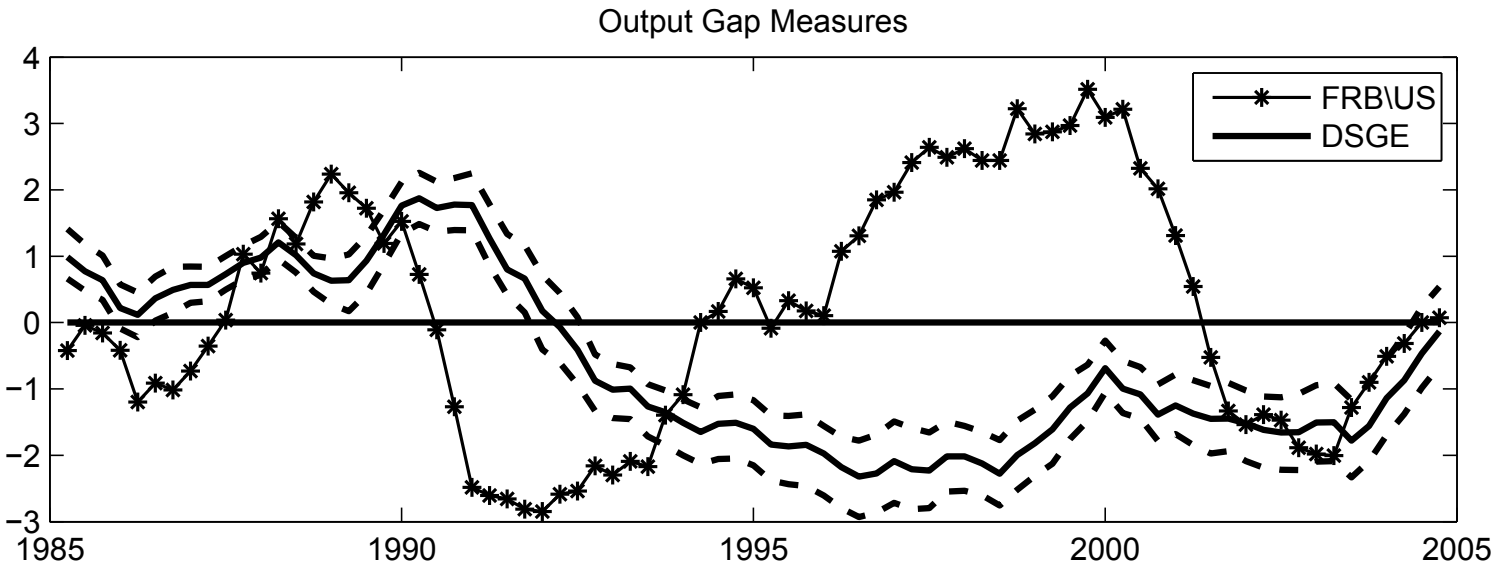
# Preference and public, net-foreign demand shocks



# Potential output in FRB/US

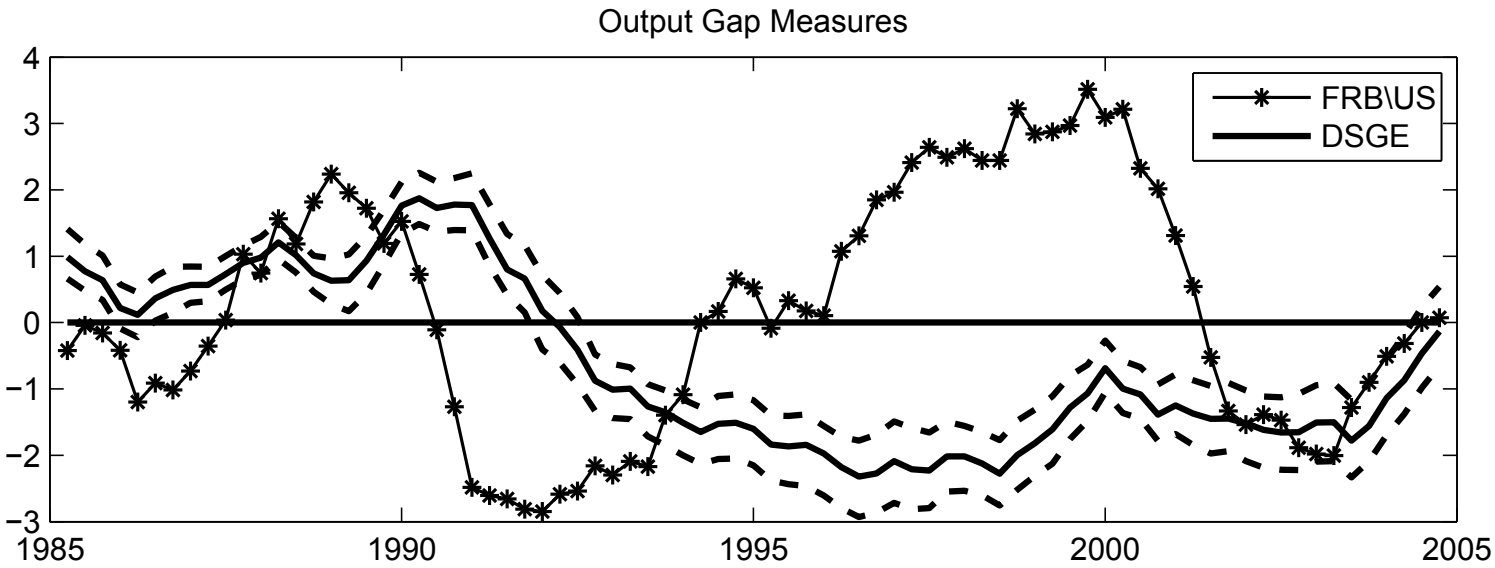
- For comparison we will consider a measure of potential output and the output gap based on the FRB/US model.
- FRB/US potential output is production-function based and smoothly evolving. In the FRB/US measure of potential:
  - The TFP series is a smoothed series for measured TFP;
  - The capital stock is the actual measured capital stock; and,
  - The labor input is a smoothed series (similar to the DSGE model's notion of steady-state).

# The output gap in the DSGE and the FRB/US models (1)



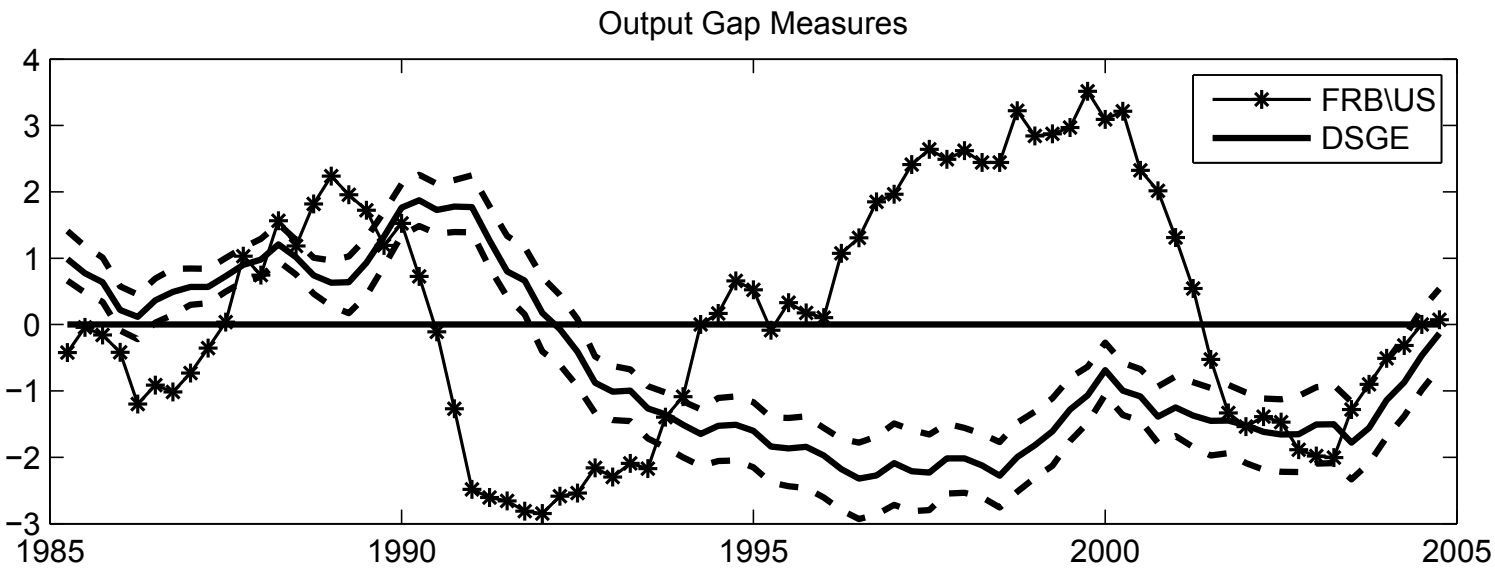

- Both output gap series show movements in actual output away from potential in the early 1990s and in 2001.
- Shocks to the preference for leisure account for much of the contour of output gap in first half of the 1990s.
- Increases in the preference for leisure and NDS consumption explain the narrowing of the output gap in 1998 and 1999.

# The output gap in the DSGE and the FRB/US models (2)



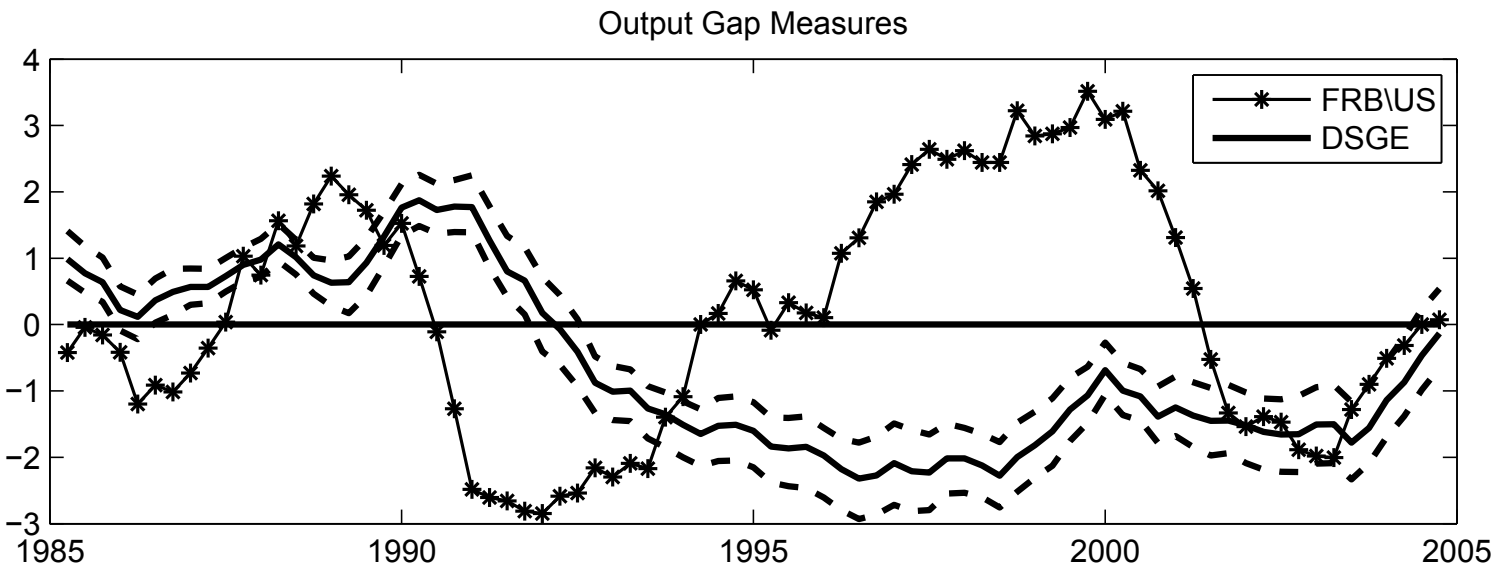

- Unfavorable non-residential capex. efficiency shocks explain the slight widening in the output gap in 2000.
- Further increases in the preference for leisure and favorable res. capex. efficiency shocks narrow the gap in 2003 and 2004.
- Investment specific technology shocks also hold output below potential in the late 1990s and narrow the gap in 2003.

# The output gap in the DSGE and the FRB/US models (3)



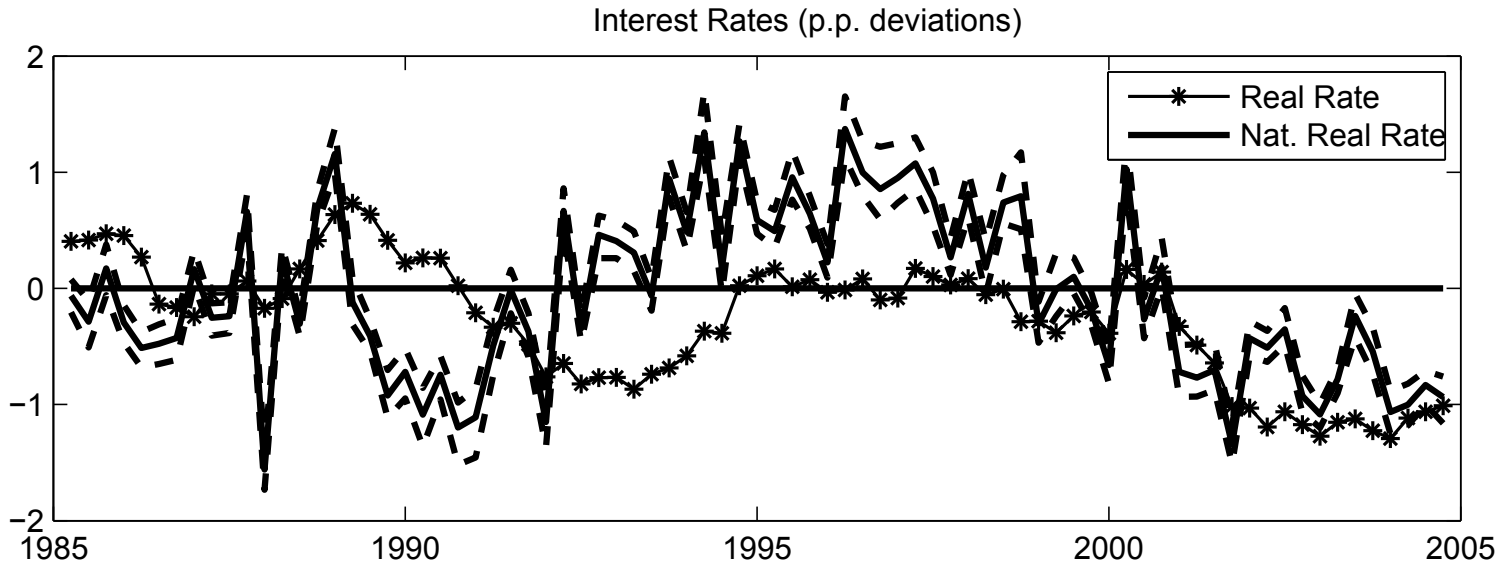

- It is useful to think about the output gap in terms of the real wage in the flexible and sticky wage/price models.
- This is the intuition of GGL's (2002) *mrs* − *mpl* gap measure.
- Note that with endogenous capital accummulation the real wage in the flexible and sticky wage/price models is related to the capital-stock gap in addition to the output gap.

# The output gap in the DSGE and the FRB/US models (4)



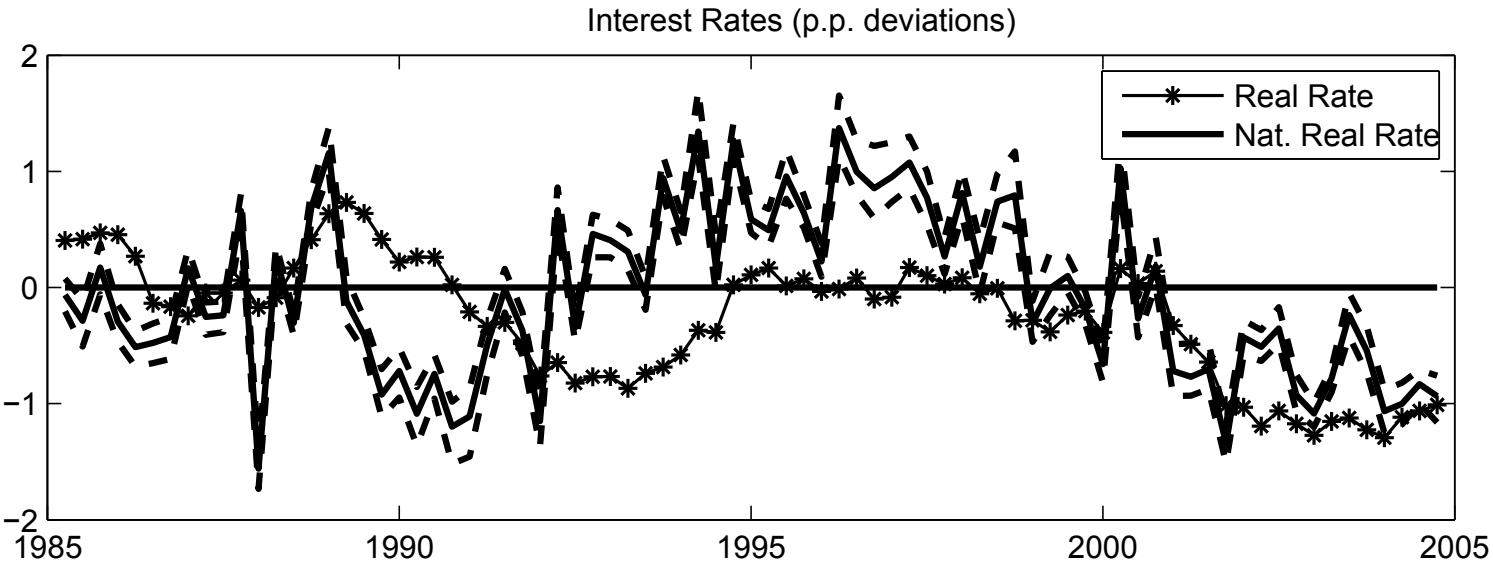

- The obvious differences between the output gaps, reflects differences in the measures' relationship with inflation.
- The DSGE output gap is a driver of inflation, so inflation has a notable bearing on the DSGE output-gap path.
- The FRB/US output gap is related to inflation only in so much as trend labor supply depends on the NAIRU.

# The natural real rate of interest (1)



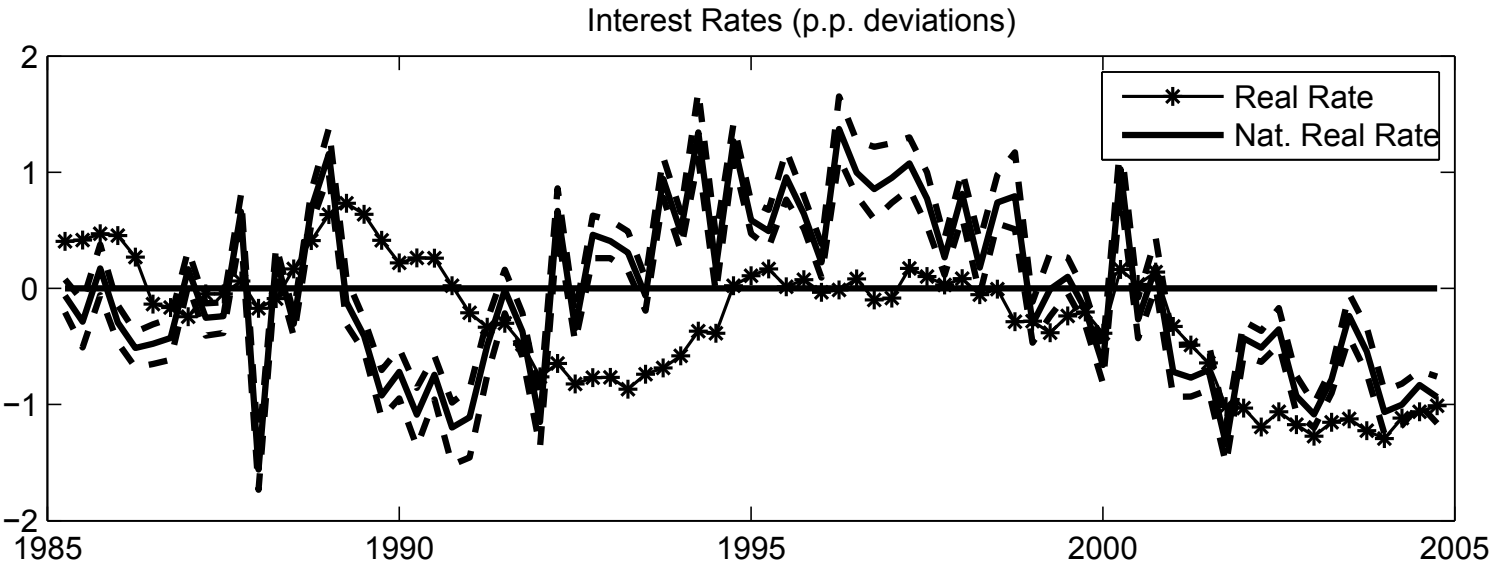

- How do actual real rates compare to the natural rate?
- Real interest rates in 1990/91 recession decreased more slowly than the decline in the natural rate and then remained below the natural rate until the end of the 1990s.
- Actual rates declined with the natural rate in 2000/01 and were a bit below the natural rate in 2003 and 2004.

# The natural real rate of interest (2)




- Why is the natural rate of interest so volatile?
- Output is supply determined in the flexible wage/price model and the natural rate of interest is what equilibrates consumption and investment expenditures with output.
- The models contains some large and "choppy" shocks; these imply big swings in spending and large interest rate changes.

# The natural real rate of interest (3)




- Also habits and capex. adjustment costs make spending less interest sensitive so even larger rate changes are needed.
- In general, most policymakers would not think of the natural rate of interest as something very volatile.
- This tends to limit the practical usefulness of DSGE model based natural real rate measures.

# Summing up

- DSGE generated natural-rate variables do differ from their counterparts derived from other more traditional models.
- Better specified models could help in making DSGE natural rate variables more consistent with conventional thinking.
  - A better specified labor market could limit the role of leisure preference shocks, which influenced the output gap quite a bit.
  - A reduced reliance on large real rigidities may be able to lessen natural real interest rate volatility.
- Nonetheless, DSGE generated natural-rate variables present a potentially useful perspective for policy conversations.