

Underlying Inflation – A Common Trends Approach

by

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Abstract

Quah and Vahey (1995) discussed the dichotomy between increases in the consumer price index and equilibrium models embodying nominal price increases, and outlined an identification scheme for defining a core inflation component. The identification scheme is based on imposing the long-run restriction of a vertical Phillips curve, and defining core inflation as the component of observable inflation that does not have a long-run effect on output. We use their identification scheme, but implement it in a common trends framework, which has some computational advantages as well as being easier to generalise to allow for cointegration. The method is applied to computing core inflation for Canada, Germany, Italy, Japan, Sweden, the UK and the US.

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1 Introduction

A number of countries have begun adopting inflation-targets as goals for monetary policy. This has rekindled the debate on how inflation should be measured. Quah & Vahey (1995) (hereafter QV) argue that there is a conceptual mismatch between current methods for calculating inflation and economic theory, which goes deeper than just measurement error. Although price indexes, such as the Consumer Price Index (CPI), measure the costs of particular goods and services in the economy, the economic notion of inflation refers to sustained increases in the price level. While economic theory does not suggest a particular functional form for inflation, it does suggest certain characteristics for general increases in the price level. Any model embodying the vertical long-run aggregate supply (LRAS) curve must predict that nominal (aggregate demand) disturbances have no long-run effect on output, though they may have short-run transient effects. This property implies a restriction on the co-movements of output and observed inflation, however measured. Increases in CPI do not obey any such restrictions (except by chance). The objective of this paper, building on the work of QV, is to compute a measure that has this property.

It is widely recognised that increases in price indexes can at times be misleading. In the UK, for example, mortgage home repayments are part of the retail price index (RPI), but this component is often removed (RPI-X) before calculating inflation because otherwise a tightening of monetary policy may bring about an increase in inflation. Changes in tax rates are also problematic. If a tax-raise affects a good in the CPI basket, there will be an increase in the price

level, but it does not stem from an increase in demand and moreover does not signify rising inflation-pressures.

There are several methods available to address this problem and smooth inflation. One particularly simple approach has been to attach a zero weight to “undesirable” components of the price index. For example, removing housing costs (RPI-X), or tax-increases. The rationale for disregarding components of the price index that are significantly affected by such changes is that they may no longer be representative for the general price movements. Although it is undesirable that the index should be sensitive to such (exogenous) factors, removing a “distorting” component of the index may involve losing valuable information: some of the increase in the price of the good may contribute to an increase in the aggregate price level.

Other approaches to this measurement problem rely more on statistical methods. These typically amount to smoothing observed increases in the index in some way, such as constructing a moving average; or assume some functional form, such as autoregressive-integrated-moving average (ARIMA). Such models can be put into state-space form and smoothed with Kalman filter techniques.

The problem with these measures is that they assume certain features of inflation that do not have a strong backing in economic theory. In particular, economic theory does not suggest a specific functional form for inflation - at least not one that is uncontroversial.

Another statistical approach has been suggested by Bryan and Cecchetti (1993). They calculate core inflation by trimming the tails of the distribution of price shocks, thereby removing outliers and high frequency noise.

However, there would seem to be no economic rationale for choosing this method to smooth observed inflation. And, moreover, cutting off the tails of the distribution may again cause loss of information - as not all outliers are high frequency noise.

The paper by QV outlines a method for extracting a core inflation component (a term used interchangeably with underlying inflation) that does not involve losing (cross-sectional) information of price movements from attaching a zero weight to some sub-component of the index, by arbitrarily cutting off the tails of the distribution, or by assuming some ad-hoc functional form. Their starting point is to translate the economic implication of the vertical long-run (LR) Phillips curve into a restriction imposed on data. The restriction defines core inflation as that component of increases in the price index that has no LR effect on output; the remaining term is disregarded as transient noise. Thus, core inflation is by construction consistent with the vertical LRAS.

The purpose of this paper is to compute core inflation for Canada, Germany, Italy, Japan, Sweden, the UK, and the US. Garter and Wehinger (1997) have also computed core inflation for several OECD countries with the QV method. In this paper the same identification scheme is used but is implemented in a common trends (CT) framework instead. Using the results in Warne (1993), this is an equivalent way to impose the QV identifying restrictions, but which may be easier to interpret and implement. Furthermore, extensions to larger systems with cointegration can easily be made.

We will not be discussing a specific model that has the vertical LR Phillips curve, but one such model is given in Fisher (1977). The paper is

outlined as follows. Section two introduces the CT model; section three is the main part of the paper which sets out the econometric identification of core inflation in a model with industrial production and CPI; the fourth section discusses some issues connected to the LR identification scheme, while section five discusses the results of estimation. Section six concludes.

2 The Common Trends Model

The identification scheme we use is that of QV, imposing restrictions on long run multipliers in vector autoregressions (VAR:s). The scheme is similar to that of Blanchard & Quah (1989), Shapiro and Watson (1988). This follows the VAR tradition, employing impulse response analysis and variance decompositions, but departs from it by not relying on recursive orderings to identify shocks. It is well known that in Cholesky-based identification the ordering of the variables matter, and, in particular, variables appearing higher up in the ordering have contemporaneous effects on those below, but not the other way around.

We impose the LR restriction in a CT model. The advantage with this approach is that the interpretation of the restrictions is transparent. Moreover, the formulation of CT model in terms of permanent and transient shocks is a particularly appealing way to deal with both innovations to the trend and transient fluctuations around the trend. The CT model was originally formulated in seminal papers by Stock & Watson (1988 a and b), and King, Plosser, Stock & Watson (1991).

Although this formulation is equivalent to the scheme in QV, it may offer some advantages when computing core inflation. Apart from ease of

interpretation, a closed form solution to the transformation matrix implementing the restrictions has been developed by Warne (1993).

The starting point for the analysis is a slightly simplified version of the CT model in Warne (1993). Consider a $n \times 1$ vector x_t , where the only permissible type of explosive behaviour is unit-roots. This process has a CT representation given by

$$x_t = x_0 + \pi r_t + \Phi(L)\varphi_t, \quad (1)$$

where x_0 is an unknown vector of constants, L is the lag operator with the property that $L^j x_t = x_{t-j}$, φ_t is a white noise disturbance with expectation zero and variance normalised to the identity matrix, and $\Phi(z)$ is a $n \times n$ matrix polynomial. Note that φ_t can have *both* permanent and transient effects: $\Phi(z)$ gives the transient fluctuations; and the growth component is given by πr_t , where

$$r_t = \mu + r_{t-1} + \varphi_t, \quad (2)$$

is a n dimensional random walk with drift μ .

3 Identifying Core Inflation: bivariate case

In this section we will use the results for the CT model in Warne (1993) to identify the core inflation component introduced by QV. The model can be written

$$\begin{cases} x_t = x_0 + \begin{pmatrix} \pi_{11} & 0 \\ \pi_{21} & \pi_{22} \end{pmatrix} r_t + \Phi(L)\varphi_t \\ r_t = \mu + r_{t-1} + \varphi_t, \end{cases} \quad (3)$$

where

$$x_t = \begin{pmatrix} y_t \\ p_t \end{pmatrix}, \quad \varphi_t = \begin{pmatrix} \varphi_{y,t} \\ \varphi_{p,t} \end{pmatrix}, \quad (4)$$

Here y_t and p_t are output and prices in logarithms respectively; $\varphi_{y,t}$ is the (real) output shock, and $\varphi_{p,t}$ is the (nominal) price shock, assumed to be uncorrelated for all leads and lags.

The most important feature in (3) is the zero-element in the π matrix: it imposes the restriction that price shocks should have no LR effect on output but allows the first shock $\varphi_{y,t}$ to have permanent effects on both output and prices.

How can this setup be used to find core inflation? Denote the representation we want to find as

$$\Delta x_t = \delta + D(L)\varphi_t, \quad (5)$$

where $\delta = (\delta_y \quad \delta_p)'$ is a vector of constants, $D(z) = \sum_{k=0}^{\infty} D_k z^k$, and

$$D_k = \begin{pmatrix} d_{11}^{(k)} & d_{12}^{(k)} \\ d_{21}^{(k)} & d_{22}^{(k)} \end{pmatrix}. \quad (6)$$

Analogously, the individual entries in $D(z)$ are defined as $d_{ij}(z) = \sum_{k=0}^{\infty} d_{ij}^{(k)} z^k$.

If $d_{12}(1) = 0$, the LR restriction is satisfied, and $D(1) = \pi$. The intuition for this is clear: the loading matrix of the CT gives the LR impact of the shocks.

With this notation we can give the QV¹ definition of core inflation, as

$$\hat{q}_t \equiv \delta_p + d_{22}(L)\varphi_{p,t}. \quad (7)$$

It is the component of changes in prices that does not have an LR effect on output, consistent with the vertical LRAS.

How can we compute (7)? First, we estimate the VAR

¹ Note that they use a different ordering of the shocks, but this is only a matter of convenience as the shocks are defined not by their (arbitrary) ordering, but by the statistical properties we attribute to them.

$$B(L)\Delta x = \theta + \varepsilon, \quad (8)$$

where $B(z) = I_n - \sum_{k=1}^p B_k z^k$, $\varepsilon \sim \text{iid}(0, \Sigma)$, and Σ is a positive definite matrix. The system is stationary and invertible if the zeros of $|B(z)|$ are inside the unit circle, in which case a unique Wold decomposition exists, obtained by inverting the VAR.

This yields

$$\Delta x_t = \delta + C(L)\varepsilon_t, \quad (9)$$

where $\delta = B(1)^{-1}\theta$, $C(L) = B(L)^{-1}$, and $C(z) = I_n + \sum_{k=1}^{\infty} C_k z^k$. How can we use this to obtain the representation in (5)? The problem is to find a transformation matrix Γ such that

$$\begin{cases} \varphi_t = \Gamma \varepsilon_t \sim \text{iid}(0, I_n) \\ D(1) = C(1)\Gamma^{-1}, \end{cases} \quad (10)$$

where $D(1) = \pi$. The first equation contains three restrictions on Γ , one orthogonality and two normalisation restrictions; the second equation provides a fourth, on the LR multiplier.

Now, using (10)

$$\begin{aligned} \Delta x_t &= \delta + \sum_{k=0}^{\infty} C_k \varepsilon_{t-k}, \\ &= \delta + \sum_{k=0}^{\infty} C_k \Gamma^{-1} \Gamma \varepsilon_{t-k} \\ &= \delta + D(L)\varphi_t, \end{aligned} \quad (11)$$

where $D(z) = D_0 + \sum_{k=1}^{\infty} D_k z^k$, $D_k = C_k \Gamma^{-1}$ for $k > 0$, and $D_0 = \Gamma^{-1}$.

In the appendix, it is shown how C_k can be computed from the VAR coefficients, and how to avoid the ‘‘inversion’’ of the VAR above by instead using simulation to compute core inflation. The remaining unknown matrix is Γ , and this turns out to be the explicit connection to the CT parameters. It is shown in Warne (1993) that

$$\Gamma = (\pi' \pi)^{-1} \pi' C(1), \quad (12)$$

and that π is the solution to

$$\pi \pi' = C(1) \Sigma C(1)'. \quad (13)$$

We will simply use the Cholesky square root. Note that Γ will not be lower triangular, so that contemporaneous effects of price shocks on output are not ruled out. Indeed, the identification scheme does not take a stance on the issue of how fast (nominal) price shocks become output neutral.

4 Discussion of the Identification Scheme

4.1 The LR Effect of Shocks to Core Inflation

From our estimated parameters we can compute a different measure, not discussed in QV, but which is perhaps also of interest to policy-makers, given by

$$\tilde{q}_t \equiv \delta_p + d_{22}(1) \phi_{p,t}, \quad (14)$$

where $d_{22}(1) = \pi_{22}$. This measures the permanent effect of a (nominal) price shock occurring at time t . The two measure are related to each other. If we let

$$d_{ij}^*(s) = -\sum_{k=s+1}^{\infty} d_{ij}^{(k)}, \text{ then } d_{ij}(z) = d_{ij}(1) + (1-z) \sum_{s=0}^{\infty} d_{ij}^*(s).$$

The permanent effect of a shock can be of interest to central banks, especially if there is an inflation target. It might be used as one indication of whether or not a change in monetary policy is needed to avoid large deviations from the target. In practice, the uncertainty associated with both the shock and the estimated parameters may be large, and such inferences should be treated with caution.

4.2 Problems with the LR Identification Scheme

The VAR approach has been criticised in a number of papers, see for example Cooley & Leroy (1985) and Hansen & Sargent (1991). In particular, the zero

restrictions on some contemporaneous shocks imposed by the Cholesky scheme is contentious, since they are rarely justifiable from economic models. The LR-identification scheme does not suffer from this problematic feature.

The LR-identification scheme is a more flexible approach, allowing all shocks in a (bivariate) system to have immediate effect. However, Faust & Leeper (1995) criticise the LR identification scheme. Essentially they argue that there is no information in a finite sample to make restrictions based on an infinite sum believable. In particular, under standard assumptions yielding consistent estimates, any confidence interval of $C(1)$ that is finite with probability one also has confidence level zero (Sims (1972), Faust (1994)). The simplest solution to this problem is to assume a maximum order for the lag-length. Faust & Leeper (1995) also discuss other ways to determine whether or not the LR restriction gives reliable results, such as testing for trend stationarity.

QV take different approach. They impose the restriction in the medium to LR, and find that their results do not change. This robustness may, of course, be confined to their particular data set or the question posed. This potential problem is ignored here. More research is needed to assess when LR restrictions are reliable.

Finally, Lippi & Reichlin (1993) raise another issue in a comment to Blanchard & Quah (1989). See also the reply by Blanchard & Quah (1993) and Quah (1993). The classical Wold theorem holds that all stationary processes have a unique representation as the sum of an infinite moving average process consisting of white noise, a representation which might be labelled “fundamental”. The roots of the moving average polynomial we obtain after

inverting the VAR are all outside the unit circle by virtue of stationarity. The problem is that there are other processes, non-fundamental ones, in which not all roots of the MA polynomial are outside the unit circle that gives us the same correlation structure. In this paper, the problem of non-fundamental processes will be assumed away.

5 Results

We use industrial production as a measure of output, since it is available on a monthly basis; consumer price indexes are used as prices with the exception of the UK, where the retail price index (RPI) is used. The sources of the data series and other details are given in appendix B.

The data is transformed into first differences in logarithms; Dickey-Fuller tests (not displayed) indicate that Δy_t and Δp_t can be treated as stationary. We choose twelve lags in the VAR for each country: shorter lag-lengths than nine (not reported) tended to give different results, while longer lag lengths (up to eighteen) did not alter the results.

We do not display the estimation results on the VAR:s, but the resulting $C(1)$, Σ , π and Γ matrices along with the estimation periods are given in Appendix D. The longest available sample is used for all countries except Germany, for which we have excluded data after unification. For all countries, the permanent effect of output shocks on prices is negative ($\pi_{21} < 0$); the immediate impact of price shocks on output is positive ($\Gamma_{12}^{-1} > 0$). The impulse responses of the shocks $\varphi_{y,t}$ and $\varphi_{p,t}$ on Δy_t and Δp_t are given in appendix C. For all countries, the period needed for price shocks to inflation to die out is rather

long, more than three years for all countries. Price shocks, by contrast, become output neutral much faster, typically in less than 15 months.

Inflation and core inflation are plotted in figures 1 and 2. The headline inflation series is computed as (one-hundred times) the twelve month change in the logarithm of the price level². The core inflation component that we have identified is in terms of one-month changes; to obtain a comparable series of twelve month changes, we simply sum twelve one-month changes which is given

$$\text{by } \hat{\pi}_t^y = \sum_{i=0}^{11} \hat{q}_{t-i}.$$

Common to all countries is the high inflation in the mid 70s, after the oil shocks. But the identification scheme does not yield response of core inflation to such peaks. Canada, Sweden, the UK and the US all have core inflation series that are unresponsive to short-run fluctuations in inflation, but which show the trend of inflation. By contrast, Germany, Italy, and Japan have core inflation series that track the short-run movements of inflation much closer.

² The official Central Statistics Office (SCB) in Sweden does not compute Swedish inflation by simple percentage increases: a long-term link, calculated in December, is used as a correction term.

Figure 1

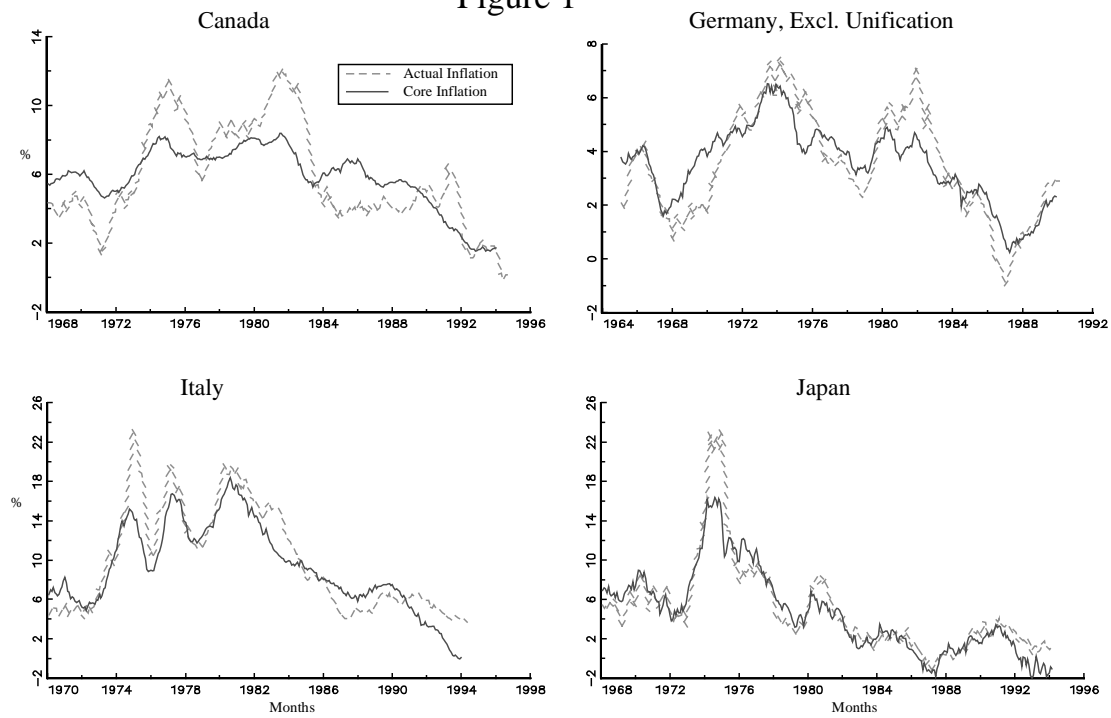
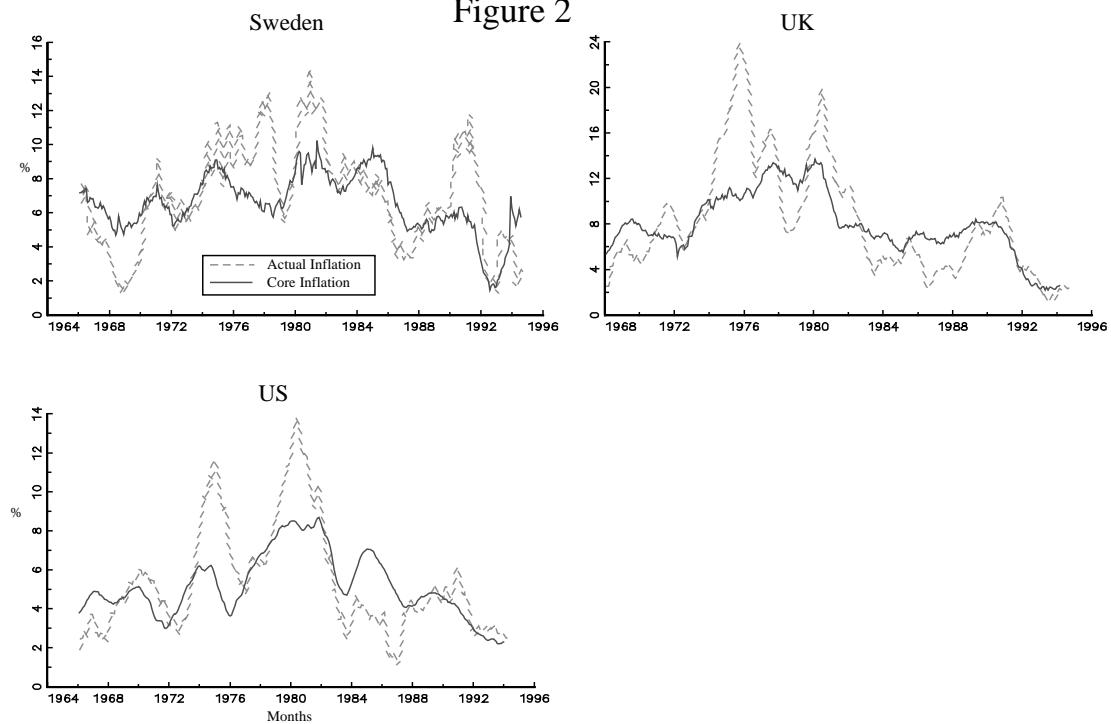


Figure 2



For Germany, there is only one peak when core and headline inflation differ much, about 3%, occurring in 1982. For Italy and Japan there is also only one peak when the two series differ much, but it occurs during the oil-crisis.

Swedish core inflation shows a large rise towards the end of the sample, in 1993. With the hindsight of more data, the rise in headline inflation did not materialise, and inflation was kept below three percent. This was not the only measure that showed inflationary pressures at the time: both expected future interest rates from forward curves and two independent surveys indicated expected inflation to be rising, see Svensson (1995).

This points to the potential problem of structural breaks. In January 1993, Sveriges Riksbank announced that from 1995 it would have an inflation-target of 2 percent with a tolerance interval of ± 1 percentage points as goal for monetary policy. This is effectively a change in the central bank's historical reaction function, which may change some "deep" parameters in the economy, making inference from VAR:s (and other standard models) problematic. All countries have structural breaks in one form or another; whether or not they are important is likely to depend on the question posed: for an application such as this, changes in targeting practice by the central bank should matter, but more data in the new regime is needed for evaluation.

As a robustness check, we have estimated VAR:s for all countries without the first one-hundred observations. Apart from minor deviations, the resulting core inflation series were very close to the full sample ones, except for Japan. This is an encouraging result, especially so for an identification scheme based on LR restrictions. For the UK, however, core inflation differs from that in QV even if we use their shorter sample; Possible explanations for this are that they use seasonal dummies in the VAR and they treat the price level as $I(2)$ so that CPI must be twice differenced to be stationary. Although it is sometimes not possible

to reject the unit root specification for inflation – such as QV – we would argue that this is only due to power problems with such tests. Essentially we “know” that inflation is stationary – even in countries with hyperinflation: eventually inflation becomes the most acute economic problem, starting a political process aimed at stabilising inflation.

Finally, let us briefly examine the measure in (14). It can be obtained immediately from the estimated π matrices in appendix D. For example, consider the US where $\pi_{22} = 1.44$. This indicates that a 1% price shock in a given month will have a total impact of 1.44% on the price level after the adjustment process is completed. Confidence intervals of impulse responses and of the total response can be computed using the results in Warne (1993).

6 Concluding Remarks

In this paper we have computed core inflation for Canada, Germany, Italy, Japan, Sweden, the UK, and the US in a CT framework using the identification scheme in QV. Other schemes of smoothing observed inflation, such as a moving average filter, have no particular economic interpretation; by contrast, the QV measure is based on the LR neutrality of nominal shocks, a property manifest in all macroeconomic models with a vertical LR Phillips curve. Using this restriction, we can identify a series of core inflation that is theory based, yet permits rich short-run response patterns. In particular, there is no need to impose a zero restriction on contemporaneous shocks, as is the case with Cholesky based identification schemes.

Extensions with more nominal variables, such as money aggregates, may allow us to disentangle the contributions of different components to core inflation, and allow an assessment of the robustness of the results. Future research may also shed light on the effects of changes in regime for the transmission mechanism. In particular, what are the effects of the recent adoption of inflation targets in Canada, Finland, New Zealand, Sweden, UK, and for the future European Central Bank?

Finally, core inflation may be seen as a useful complement to other sources and methods, especially when known changes, e.g. in taxes, make adjustments to the official inflation figure desirable.

Appendix A

In this appendix we show how to avoid the inversion of the VAR discussed in the main part of the paper; these methods are not new, but are applied in a particular way. Instead of inversion, we will approximate core inflation using simulation.

The VAR estimated was

$$B(L)\Delta x_t = \theta + \varepsilon_t, \quad (\text{A.1})$$

where $B(z) = I_n - \sum_{k=1}^p B_k z^k$ and $\varepsilon \sim \text{iid}(0, \Sigma)$. Put this into companion form VAR according to

$$Y_t = J' \theta + B Y_{t-1} + J' \varepsilon_t, \quad (\text{A.2})$$

where

$$Y_t = \begin{bmatrix} \Delta x_t \\ \Delta x_{t-1} \\ \vdots \\ \Delta x_{t-p+1} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 & B_2 & \cdots & B_p \\ I_n & 0 & \cdots & 0 \\ \vdots & \ddots & & \vdots \\ 0 & \cdots & I_n & 0 \end{bmatrix}, \quad (\text{A.3})$$

and the $n \times np$ matrix $J = [I_n \ 0 \ \cdots \ 0]$. The Wold moving average representation is obtained by inverting (A.2), and is given by

$$Y = \sum_{k=0}^{\infty} B^k J' \theta + \sum_{k=0}^{\infty} B^k J' \varepsilon_{-k}. \quad (\text{A.4})$$

Pre-multiply (A.4) by J , and we obtain

$$y_t = \sum_{k=0}^{\infty} J B^k J' \theta + \sum_{k=0}^{\infty} J B^k J' \varepsilon_{t-k}. \quad (\text{A.5})$$

Note that this gives explicit expressions for the unknown terms in (9)

based on the estimated VAR coefficients, namely $\delta = J(I_{np} - B)^{-1} J' \theta$ and

$C_k = J B^k J'$ Next, using $\varphi_t = \Gamma \varepsilon_t$ in (A.5), we find

$$\begin{aligned}
y_t &= \delta + \sum_{k=0}^{\infty} C_k \Gamma^{-1} \Gamma \varepsilon_{t-k} \\
&= \delta + \sum_{k=0}^{\infty} D_k \varphi_{t-k}.
\end{aligned} \tag{A.6}$$

Thus, $D_k = C_k \Gamma^{-1}$ for $k > 0$, and $D_0 = \Gamma^{-1}$.

Finally, we can now show a convenient method to obtain core inflation defined in (7). Let $N = J' \Gamma^{-1}$, and let the $np \times T$ matrix $\tilde{Y} = [\tilde{Y}_1 \cdots \tilde{Y}_T]$, obtained by simulating

$$\tilde{Y}_t = J' \theta + B \tilde{Y}_{t-1} + N_1 \varphi_{p,t}, \tag{A.7}$$

where N_i is the i :th column of N . As starting value for the simulation we choose

$\tilde{Y}_0 = \mathbb{E}[Y_t] = (I_{np} - B)^{-1} J' \theta$. Core inflation can now be extracted as the second row

of \tilde{Y} , or more formally $q_t^1 = e_2' J \tilde{Y}_t$, where e_i is the i :th column of an identity

matrix.

Appendix B. Data definitions

All data used is on a monthly basis. The main sources are Sveriges Riksbank (RB), or the Bank of Sweden; OECD's Main Economic Indicators (MEI); and IMF's International Financial Statistics (IFS).

| | Source | Name in Source | Start | Stop |
|----------------|--------|----------------|-------|-------|
| Canada | | | | |
| CPI | RB | VEBA CA01 | 60.01 | 94.09 |
| IP | IFS | 15666..CZF | 57.01 | 93.12 |
| Germany | | | | |
| CPI | RB | VEBA DE01 | 48.06 | 94.05 |
| IP | IFS | 13466..CZF | 57.01 | 94.02 |
| Italy | | | | |
| CPI | MEI | ITA 475000 9H | 60.01 | 94.04 |
| IP | IFS | 13666..CZF | 57.01 | 93.12 |
| Japan | | | | |
| CPI | MEI | JPN 475000 9H | 60.01 | 94.04 |
| IP | IFS | 15866..CZF | 57.01 | 94.02 |
| Sweden | | | | |
| CPI | RB | VEBA SE01 | 55.01 | 94.10 |
| IP | IFS | 14466..CZF | 57.01 | 94.02 |
| UK | | | | |
| RPI | RB | VEBA GB01 | 61.01 | 94.09 |
| IP | IFS | 11266..CZF | 57.01 | 94.03 |
| USA | | | | |
| CPI | RB | VEBA US01 | 13.01 | 94.05 |
| IP | IFS | 11166..CZF | 57.01 | 93.12 |

Appendix C

Figure C.1

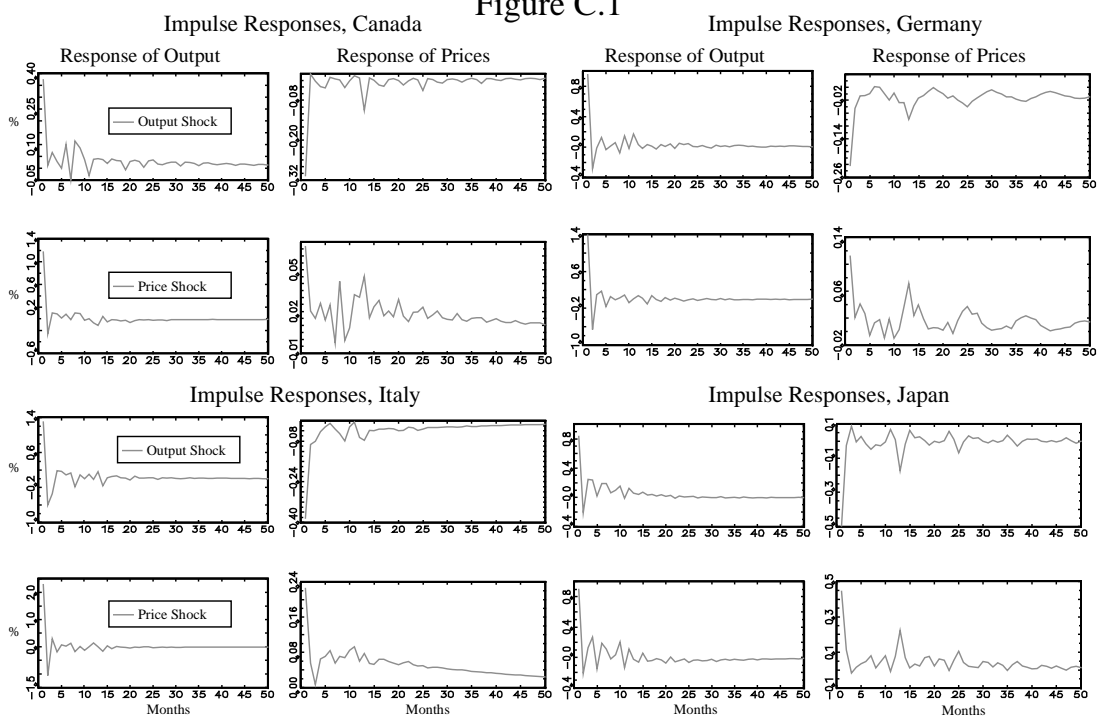
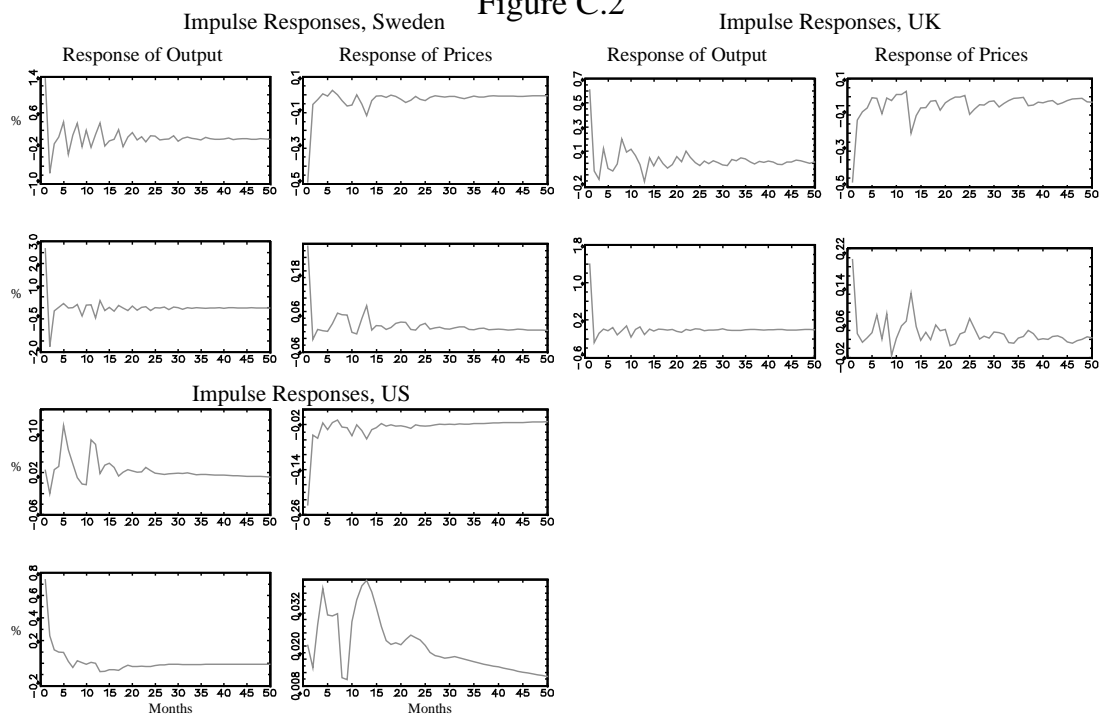


Figure C.2



Appendix D

These tables gives the estimation period and some of parameters; the VAR coefficients are available from the author on request. The displayed matrices $C(1)$ and Σ can be used to derive the remaining two, using $\Gamma = (\pi' \pi)^{-1} \pi' C(1)$ and a Cholesky decomposition of $\pi \pi' = C(1) \Sigma C(1)'$; small discrepancies from the displayed values is due to rounding errors. Finally, it can be verified that $\text{Var}[\varphi_i] = \Gamma \Sigma \Gamma' \approx I_2$.

Table D.1

Canada

60/1-93/12, 408 observations

$$C(1) = \begin{pmatrix} 0.39 & -6.89 \\ 0.81 & 7.88 \end{pmatrix}, \Sigma = \begin{pmatrix} 1.51 & -0.04 \\ -0.04 & 0.09 \end{pmatrix}, \pi = \begin{pmatrix} 2.26 & 0 \\ -2.12 & 1.48 \end{pmatrix}, \Gamma = \begin{pmatrix} 0.17 & -3.04 \\ 0.79 & 0.97 \end{pmatrix}$$

Germany

Period: 60/1-89/12, 360 observations

$$C(1) = \begin{pmatrix} 0.47 & -5.77 \\ 0.36 & 5.42 \end{pmatrix}, \Sigma = \begin{pmatrix} 2.83 & -0.06 \\ -0.06 & 0.06 \end{pmatrix}, \pi = \begin{pmatrix} 1.74 & 0 \\ -0.87 & 1.11 \end{pmatrix}, \Gamma = \begin{pmatrix} 0.26 & -3.33 \\ 0.54 & 2.30 \end{pmatrix}$$

Italy

Period: 62/1-93/12, 384 observations

$$C(1) = \begin{pmatrix} 0.33 & -3.35 \\ 0.65 & 8.57 \end{pmatrix}, \Sigma = \begin{pmatrix} 6.87 & 0.02 \\ 0.02 & 0.20 \end{pmatrix}, \pi = \begin{pmatrix} 1.71 & 0 \\ -2.44 & 3.41 \end{pmatrix}, \Gamma = \begin{pmatrix} 0.19 & -1.96 \\ 0.33 & 1.11 \end{pmatrix}$$

Japan

Period: 61/01-94/02, 398 observations

$$C(1) = \begin{pmatrix} 1.19 & -2.42 \\ 1.27 & 2.97 \end{pmatrix}, \Sigma = \begin{pmatrix} 1.50 & -0.01 \\ -0.01 & 0.44 \end{pmatrix}, \pi = \begin{pmatrix} 2.20 & 0 \\ -0.42 & 2.45 \end{pmatrix}, \Gamma = \begin{pmatrix} 0.54 & -1.10 \\ 0.61 & 1.02 \end{pmatrix}$$

Table D.2**Sweden**

Period: 61/01-94/07, 403 observations

$$C(1) = \begin{pmatrix} 0.32 & -3.42 \\ 0.07 & 3.19 \end{pmatrix}, \Sigma = \begin{pmatrix} 9.02 & 0.01 \\ 0.01 & 0.30 \end{pmatrix}, \pi = \begin{pmatrix} 2.11 & 0 \\ -1.46 & 1.01 \end{pmatrix}, \Gamma = \begin{pmatrix} 0.15 & -1.62 \\ 0.30 & 0.83 \end{pmatrix}$$

UK

Period: 61/01-94/02, 398 observations

$$C(1) = \begin{pmatrix} 0.40 & -2.85 \\ 0.77 & 7.72 \end{pmatrix}, \Sigma = \begin{pmatrix} 2.33 & -0.01 \\ -0.01 & 0.26 \end{pmatrix}, \pi = \begin{pmatrix} 1.58 & 0 \\ -3.16 & 2.60 \end{pmatrix}, \Gamma = \begin{pmatrix} 0.25 & -1.80 \\ 0.61 & 0.78 \end{pmatrix}$$

US

Period: 59/01-93/12, 420 observations

$$C(1) = \begin{pmatrix} 0.19 & -6.89 \\ 1.72 & 8.16 \end{pmatrix}, \Sigma = \begin{pmatrix} 0.55 & 0.01 \\ 0.01 & 0.06 \end{pmatrix}, \pi = \begin{pmatrix} 1.62 & 0 \\ -1.87 & 1.44 \end{pmatrix}, \Gamma = \begin{pmatrix} 0.12 & -4.26 \\ 1.35 & 0.14 \end{pmatrix}$$

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