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May 2015

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Johan Gars* and Conny Olovsson†

Sveriges Riksbank Working Paper Series
No. 299

May 2015

Abstract

We set up an endogenous growth model in which the efficiency of both capital and fossil energy can be improved, whereas the efficiency of one alternative energy source is limited. With capital and energy as complements, there exist two steady states: one stagnant where energy is fully derived from the alternative energy source, and one with balanced growth where energy is fully sourced from fossil fuel. Heterogeneity in initial TFP levels can generate the Great Divergence. The demand for fossil fuel in technologically advanced countries drives up its price and makes fossil fuel too costly in less advanced countries that choose the alternative and stagnant energy input.

Keywords: Growth, Malthusian stagnation, Industrial Revolution, Great Divergence, Technological progress

JEL: O11, O14, O33, O41, O50

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1 Introduction

Economic growth is a relatively recent phenomenon. The growth rate of income per capita between year 1 and the beginning of the 16th century was basically zero. Roughly 200 years ago however, Western Europe and the Western offshoots (i.e., United States, Canada, Australia and New Zealand) began to emerge from this so called Malthusian trap and embarked on a transition path to sustained economic growth.[1] This process began in England around 1760, and is often referred to as the Industrial Revolution.

The transition to sustained growth has not, however, been universal. Many countries around the world have either remained stagnant or experienced only limited growth throughout history. Since some countries have reached a state of sustained growth whereas others have remained stagnant, there are large and growing differences in income per capita and output across countries today.[2] This is often referred to as the Great Divergence.

Even though several important contributions have helped to significantly increase our understanding of the Industrial Revolution and the Great Divergence, these phenomena are still far from fully understood.[3] The question of exactly what factors were crucial for the transition from stagnation to sustained growth remains at least partly unanswered. The same is true for the question of why the transition to sustained growth has not been universal. In a world where ideas can rapidly flow between countries, new ideas and machines that increase production in some countries should also be able to do so in other countries. Since the welfare effects from economic growth are so large, the question of exactly which factors can explain the sudden takeoff from stagnation to growth in some countries and the persistent stagnation in others is sometimes viewed as the most important one within the field of social science.[4]

This paper analyzes the potential importance of one specific factor in simulta-

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[1] Galor and Weil (2000) and Hansen and Prescott (2002).

[2] Countries at the top of the distribution are roughly 30 to 50 times richer than those at the bottom.

[3] Important contributions on the transition from Malthusian stagnation to sustained economic growth include Lucas (1988), Galor and Weil (2000), Jones (2001) Hansen and Prescott (2002). The Great Divergence is analyzed in Basu and Weil (1998), Acemoglu and Zilibotti (2001), Aghion, Howitt and Mayer-Foulkes (2005) and Acemoglu, Aghion and Zilibotti (2006).

[4] For instance, Acemoglu (2012, p.7) writes that "understanding how some countries can be so rich while others are so poor is one of the most important, perhaps the most important, challenges featuring social science". Galor (2005) argues that "the discovery of a unified theory of economic growth that could account for the intricate process of development in the last thousands of years is one of the most significant research challenges facing researchers in the field of growth and development."

neously accounting for the pre-industrial period of stagnation, the post-industrial period of balanced growth, as well as the Great Divergence: accessibility and/or affordability of fossil energy. Without making any statement about causality, the focus on fossil energy is motivated by the simple observation that income and economic growth tend to be high in regions and time periods in which a relatively large share of the total energy supply is derived from fossil energy.

This observation is consistent with the Industrial Revolution. Prior to the Industrial Revolution, the standard of living was roughly constant and economic growth was limited and temporary.[5] The sources of energy in all pre-industrial civilizations were human and animal labor, water, wind and biomass fuels (such as wood, crop residue and dried dung). As shown in Figure 1, the takeoff into sustained growth then occurs at the beginning of the 19th century. This is also the time period during which these energy inputs were gradually replaced by fossil energy. Only a hundred years later, several European countries were almost completely energized by coal.[6]

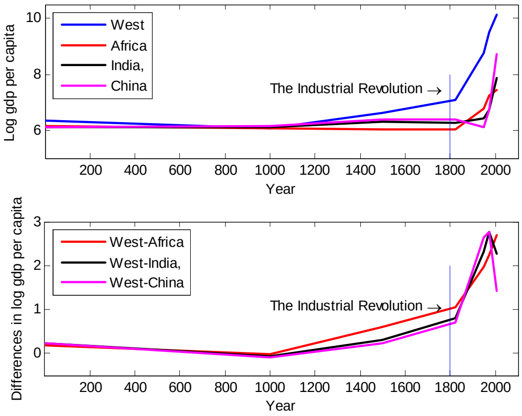

Figure 1: Top graph: the log of GDP/capita over the last 2000 years. Bottom graph: differences in log(GDP/capita) between the Western world and other regions. Source: Maddison (2008).

The observation also applies to the cross section, i.e., countries that have not embarked on any transition to sustained growth have also not chosen to substitute manual labor and biofuels for fossil energy. This is shown in the first part of the

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[5] Maddison's estimates show some slow growth between year 1 and the beginning of the 19th century. However, many historians disagree with these estimates and instead estimate that growth was very limited before the 18th century. In any case, the economic growth before the Industrial Revolution progressed, at best, at a slow pace and it was not sustained.

[6] Smil, 2004.

paper where we use data from the International Energy Agency on the supply and consumption of coal, oil, natural gas, nuclear power, biofuels and waste for 134 countries over the time period 1960-2012 to derive two stylized facts. First, rich and poor countries use different energy inputs. Specifically, countries with higher income derive a larger share of their energy from fossil energy, whereas poor countries, to a larger extent, produce with other energy sources. Second, countries that derive a large share of their energy from fossil fuel are also growing at faster rates than countries that mainly use other energy inputs. Even though the fossil share and the growth rate are highly correlated, it is much harder - if not impossible - to empirically make inferences about potential causality. The reason is that both variables are endogenous and are likely determined simultaneously.

To analyze the relationship between energy use and economic growth, we instead set up an endogenous growth model with two types of countries: those that extract and sell fossil fuel, and those that buy and use fossil fuel as an input to produce final goods. The energy needed for final-goods production can also be sourced from an alternative source, which we refer to as the pre-industrial energy input. We then solve the model analytically and show that it can account for the pre-industrial period of stagnation, the post-industrial period with balanced growth, the Great Divergence, as well as for the two stylized facts.

Two assumptions are crucial for these results. First, we assume that capital and energy are complements in the production process. Second, while the physical supply of the energy inputs cannot be increased, we assume that the energy efficiency of fossil energy can be increased by R&D investments. However, there is a definite limit to how much the energy efficiency of the pre-industrial energy input can be increased. With this model, we derive three steady states.

First, abstracting from heterogeneity, we show that if the stock of fossil energy in efficiency units is zero, there exists a stagnant steady state where final output is exclusively produced with the pre-industrial energy input. Intuitively, with capital and energy being complements and with limited possibilities to increase the energy efficiency, there are decreasing returns to improving the capital efficiency. The investment made to improve the capital-efficiency will then eventually approach zero and growth will stop. This steady state is broadly consistent with the period before the Industrial Revolution.

Second, if some parameter restrictions are satisfied, there exists a balanced growth path where growth is constant and output is exclusively produced with fossil energy. Positive investments are then continuously made to improve the efficiency of both capital and fossil energy. This steady state is broadly consistent with the experience in the Western world in the period after the Industrial Revolution.

Third, introducing heterogeneity in the distribution of initial technology levels, we show that the model can produce the Great Divergence. Specifically, with sufficiently large differences in initial technology levels, the model features an equilibrium where a technologically advanced country experience sustained growth and endogenously only produce with fossil energy, whereas a technologically less advanced country instead endogenously chooses the pre-industrial energy input and does not grow. The growth enhancing technology is then dominated by the stagnant technology from the perspective of individual firms in the less developed country. Consequently, the countries are diverging over time.

The main intuition for the Great Divergence comes from the fact that the demand for fossil energy in technologically advanced countries drives up the price and make fossil energy too expensive in the less advanced country. Since fossil energy is not used in the less developed country, there are no R&D investments to make this technology locally profitable. Hence, if technology transfers require at least some R&D investments in the receiving country, and the initial difference in technology levels is sufficiently large, the less developed country can become stagnant.[7]

All three steady states are consistent with the stylized facts, i.e., countries with relatively higher fossil shares have higher income and growth rates than countries with lower fossil shares. Also, out of steady state, the model predicts that countries with higher TFP levels and capital stocks will produce with higher shares of fossil energy than countries with lower levels.

Since we do not model the search process for new fossil-fuel discoveries, there is no deterministic transition from stagnation to sustained growth. Transitions are outside the scope of research in this paper. The model therefore has nothing to say about why the Industrial Revolution began in England as opposed to, say, in China. These are all important issues and they can potentially be addressed in an extended version of the model, but they are left for future research.

The model in this paper builds on several previous contributions. The presence of two different energy inputs bears some resemblance with Hansen and Prescott (2002), the endogenous multi-country growth model builds on Howitt (2000), and the features of directed technical change build on Acemoglu (2002, 2003). Finally,

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[7] This result reflects those in Basu and Weil (1998) and Acemoglu and Zilibotti (2001) in that the technologies that are developed at the frontier are not appropriate for less developed country. In our setting, the frontier focuses, among other things, on developing the efficiency of fossil energy, which is an energy source that the less developed country does not use.

the assumption that technology transfers require R&D investments can also be found in Aghion, Howitt and Mayer-Foulkes (2005).

In focusing specifically on the role of fossil energy, we abstract from other important aspects such as human capital accumulation, institutions and demographic change. These issues have all been analyzed in great detail elsewhere. Lucas (1988), Becker, Murphy and Tamura (1994) and Galor and Weil (2000) argue that human capital is important for explaining the transition from preindustrial stagnation to modern growth. Other papers, such as Jones (2001) and Acemoglu, Johnson and Robinson (2005) stress the importance of institutions for economic growth. Finally, our theory is silent about issues related to demographic change.[8]

The argument in this paper is not that fossil energy in any way would be *sufficient* for economic growth. Instead, the argument is that with capital and energy as complements, a *necessary* condition for long-run growth is that the inputs of energy and capital in efficiency units can both be continuously increased.[9] For economic growth to actually materialize, other factors such as sound institutions must also be in place. In the model, all institutions are assumed to be well functioning.

This paper is structured as follows. Section 2 derives two stylized facts about income, growth and energy use, Section 3 sets up the model, Section 4 presents the results, and Section 5 concludes.

2 Stylized facts about income, economic growth and the choice of energy

In this section, we employ data from the International Energy Agency to document some relations between GDP/capita, economic growth and the choice of different energy inputs. The data contains annual information on the supply and consumption of coal, oil, natural gas, nuclear power, biofuels and waste. All energy inputs are denoted in units of kilo tonne of oil equivalents (ktoe). For the 34 OECD countries, the data covers the period 1960 to 2012 and for the more

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[8] Specifically, we cannot explain why population growth rates tend to be increasing in standards of living in early stages of development and decreasing at later stages. See Goodfriend and McDermott (1995), Galor and Weil (2000) and Jones (2001).

[9] We focus on fossil energy but the necessary increase in the energy supply in efficiency units could also come from other sources, such as geothermal energy on Iceland or from nuclear power as in Sweden. Alternative energy sources are discussed in detail in Section 3.1.

than 100 non-OECD countries, the data covers the period 1971 to 2012. Income is denoted in 2005 U.S. dollars and is PPP adjusted.

Richer countries can trivially be expected to consume larger amounts of fossil fuel since they may consume larger quantities of all goods and inputs. It is, however, not obvious that the *composition* of energy should differ across countries. One of the main variables of interest is thus the share of energy that is coming from fossil fuel, which we refer to as the fossil share. The relation between the logarithm of GDP/capita and the fossil share is plotted in Figure 2.

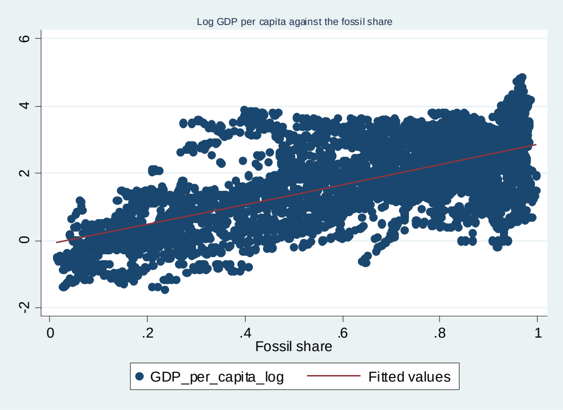

Figure 2: Log gdp per capita in PPP against the share of energy that is coming from fossil energy. Annual data for the period 1960-2013.

The graph shows a strong positive correlation between log GDP/capita and the fossil share, implying that rich and poor countries are producing with different energy inputs. Specifically, higher income countries derive a relatively larger share of their energy demand from fossil energy than do lower income countries. Even though it is not possible to say anything meaningful here about causality between the two variables, the data suggests that the production process transforms alongside with increasing income.

Figure 3 plots average annual growth rates for countries with different average annual fossil shares over the period 1960-2013. The figure shows that also growth rates and fossil shares are highly positively correlated. Hence, countries that derive a large share of their energy from fossil fuel are also growing at higher rates than countries that mainly use other energy inputs. In particular, the countries with the lowest fossil shares also experience the lowest growth rates.

Again, both growth and the fossil share are endogenous variables. It would therefore be much harder, if not impossible, to empirically try to make inferences about any potential causality. Indeed, increasing the fossil share in one specific country requires investments in capital and new machines, and these investments are determined simultaneously with the rate of growth in that country. Both variables are, therefore, likely determined by some underlying, potentially unobservable variable.[10]

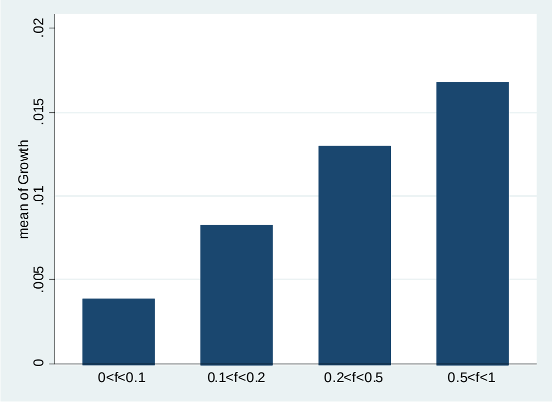

Figure 3: Average annual growth rates for countries with different average annual fossil shares (f) for the period 1960-2013.

Because of these endogeneity and identification problems, we instead analyze the relations between income, growth and energy use with a theoretical model. A successful model can provide intuition and identify the underlying unobservable variables. These insights may then be useful in future empirical studies. Note however, that most existing growth models would have nothing to say about these relations for the simple reason that they typically abstract completely from energy as an input, let alone, several energy inputs. Any model that attempts to understand the growth facts outlined in the Introduction and in this section should model the choice of the energy source, income and growth as endogenous variables. In the next section, we set up an economic model with these properties.

3 The model

In this section, we specify a multi-country-endogenous-growth model. The aim is to set up a model that can simultaneously account for periods of stagnation, periods of balanced growth, the Great Divergence, as well as for the stylized facts that are laid out in Section 2. The motivation for a model with endogenous growth

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[10] Using the generalized-method-of-moments estimator as suggested by Arellano and Bond (1991) does not help much because the fossil share is highly auto correlated.

is the finding in the empirical literature that total-factor-productivity (TFP) differences are important for understanding income differences around the world.[11] Based on this fact, the ambition is to set up a model that can endogenously generate differences in income and TFP between the richest set of countries and the poorest set of countries that are of the same magnitudes as in the data.

The model features two types of countries: those that extract and sell fossil fuel, and those that buy and use fossil fuel as an input to produce final goods. In this setting, the final-good-producing countries use capital, labor, and two perfectly substitutable energy inputs in production. One energy input is thought of as a pre-industrial source, such as wood, and the other is fossil fuel. Two assumptions are crucial in the model and they are discussed in the following section.

3.1 Two assumptions

First, for some of our results, we will assume that the elasticity of substitution between capital and energy is less than one. This makes the supply of energy more important for long-run growth. Even though almost all past macroeconomic research abstract from energy as an input, recent research show convincingly that capital/labor and energy are, in fact, strong complements. Hassler, Krusell and Olovsson (2012) estimate the elasticity of substitution between capital/labor and energy to be close to zero.

Second, while the physical supply of the energy inputs cannot be increased, we will assume that it is possible (by R&D investments) to continuously increase the energy efficiency of fossil energy. There is, however, a definite limit to how much the energy efficiency of the pre-industrial energy input can be increased. In fact, it is assumed that this limit has been reached, so that the energy-efficiency of the pre-industrial energy input cannot be increased at all. This assumption incorporates the argument that has been put forward by historians such as Smil (1994), Pomeranz (2000) and Wrigley (2010), i.e., that accessibility and/or affordability of fossil fuel can relax the constraints imposed by pre-industrial energy sources.

Putting this argument into context, before the Industrial Revolution, people were, for all practical purposes, limited to what they and their domesticated animals could ingest in the way of chemical energy in their food. This chemical energy was then transformed into heat and mechanical energy through human and animal muscle power. It was, in effect, a solar energy regime where plants

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[11] Klenow and Rodrígues-Clare (1997), Hall and Jones (1999), Caselli (2005) and Hsieh and Klenow (2010).

turned less than one percent of incoming solar energy into chemical energy via photosynthesis. A tiny proportion of those plants would then be used as food.

The kinetic energy generated by water and wind power added to the human energy supply. Several important technological improvements took place during this long period, but the process was slow and the improvements were limited.[12] Some of the major problems with water and wind power are that they only exist in selected and often remote locations, and that they are non-portable, non-storable and often seasonally unreliable. As a result, water and wind could only be used for a few specific chores, such as sailing and milling grain. Consequently, they only slightly added to the total energy use.

Wood is another energy source that has been continuously used throughout history. Wood is, however, heavy and bulky in relation to its heat content and this makes it difficult and expensive to transport and store. For the same reason, wood is not suitable as fuel for transportation. Wood has therefore, historically, almost exclusively added to the quantity of heat energy and not to the mechanical energy. For that, there was no substitute for human and animal muscle power.

Hence, as long as mechanical energy was coming from pre-industrial energy sources, the maximum attainable level of productivity was bound to be low. Even with important technological innovations such as the collar harness, the horseshoe and mechanical devices such as pulleys, traditional farming (the main part of final output during this period) could only produce limited improvements in average harvests. In fact, according to Smil (1994) no agriculture that only uses pre-industrial energy sources has consistently been able to produce enough resources to eliminate malnutrition, regardless of the historical period, the environmental setting, or the mode of cropping and intensification. In addition, Wrigley (2010) argues that it would have been physically impossible to produce iron and steel on the scale needed for the modern world if the heat energy needed to smelt and process the iron and steel had come from wood and charcoal.

In contrast, fossil fuel is easily stored and transported from one place to another; it is an abundant and reliable energy source that works independent of the weather and it can be used for almost any purpose.[13] As a result, coal driven steam engines — unlike watermills and windmills — could be put anywhere, even on ships and locomotives. In this way, and also unlike water and wind power, the exploitation of fossil energy could increase productivity in a vast range of activi-

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[12] It took, for instance, nearly 800 years to increase the capacity of waterwheels by an order of magnitude (Smil, 1994).

[13] The major disadvantage is that the burning of fossil fuel causes pollution and contributes to global warming.

ties. It is in this sense that fossil energy could relax the constraints imposed by the pre-industrial energy sources. Furthermore, Hassler, Krusell and Olovsson (2012) provide support for the assumption that the energy efficiency of fossil energy can be increased by showing that this energy efficiency has been increasing steadily in the U.S. over the period 1949-2009.[14]

Note, however, that the limitations of the the pre-industrial sources may not fully apply to the developed countries today. Water and wind (as well as other sources such as geothermal energy and nuclear power) can now produce electricity, and there is a national power grid in place to deliver this electricity to firms and households. However, these limits are potentially important when studying the period before and after the Industrial Revolution because the age of electricity (in the Western world) began only at the end of the 1900th century. They may also impose constraints in developing countries today. In fact, the rural electrification rate in Africa is only 25 percent, and it is below 15 percent in Sub-Saharan Africa. The limitations of the pre-industrial energy sources can therefore impose the same constraint in these regions today as they once did in pre-industrial Europe.[15]

3.2 Consumers

We now turn to the details of the model. Households in all countries value consumption streams by the discounted value of the utility stream they provide. Preferences are

$$U = \sum_{t=0}^{\infty} \beta^t \log(C_t), \tag{1}$$

where $\beta < 1$ is the discount factor.

3.3 Fossil-fuel-extracting countries

There is one representative fossil-fuel producer that operates under perfect competition. Fossil fuel is sold on a world market to the country/countries that use fossil energy as an input to produce final goods. For simplicity, the fossil-fuel producing country is assumed to derive all its income from selling fossil fuel. Fuel extraction is costless and the total resource at time t has size R_t. The budget/resource

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[14] There is also evidence showing that the fossil-energy efficiency started to improve immediately after fossil energy was first introduced. The steam engine of 1900 is, for instance, estimated to be around 30 times as powerful as that of 1800 (see McNeill, 2000).

[15] A national power grid is a classical schoolbook example of a natural monopoly, thus implying that it requires huge investments in infrastructure to have one installed.

constraint for the representative fuel-extracting country is then given by

$$C_t + p_{E,t} R_{t+1} = p_{E,t} R_t, \tag{2}$$

where $0 \leq R_{t+1} \leq R_t$ and $p_{E,t}$ is the price of fossil fuel in period t.[16] The problem for the country is to choose R_{t+1} so as to maximize (1) subject to (2). This is a standard cake-eating problem, similar to that in Dasgupta and Heal (1974) and the solution is

$$R_{t+1} = \beta R_t. \tag{3}$$

Energy producers thus choose to extract a constant fraction of the remaining stock in each period. The intuition for the result that extraction is independent of the price sequence is that income and substitution effects exactly balance with logarithmic utility. Specifically, a high price in one period implies both that extraction should increase at that time (the substitution effect), but also that extraction in all other periods should increase (the income effect). With logarithmic utility, the net effect of changes in the price path on extraction is zero. The consumption of the fossil-fuel producer equals $C_t = p_{E,t} R_t (1 - \beta)$.

3.4 Final-good-producing countries

We now specify the production side of the economy. As a starting point, a representative final-good-producing country is considered, but heterogeneous final-good-producing countries are introduced in Section 4.3. Total final-goods production in period t is given by

$$Y_t = \left[G_t^{\frac{\sigma-1}{\sigma}} + (B_t + F_t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{4}$$

where G, B and F are intermediate goods that are used in the production of final output, and σ is the elasticity of substitution between G and $(B+F)$.[17] Intermediate good G is a composite that consists of labor N and capital in the form of machines. Labor is assumed to be in fixed supply, so the total amount of G at time t is

$$G_t = N^{1-\alpha} \int_0^1 A_{i,t}^G \left(x_{i,t}^G \right)^\alpha di, \tag{5}$$

where x_i^G is the number of machines of variety $i \in [0, 1]$, and A_i^G is the productivity parameter associated with the machines of variety i. As in Aghion and

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[16] The results in the paper are robust to the alternative assumptions that the stock of fossil fuel can be increased at a cost.

[17] Note that when $\sigma = 0$, G and $(B + F)$ are perfect complements, when $\sigma = \infty$ they are perfect substitutes and when $\sigma = 1$, Y is a Cobb-Douglas function.

Howitt (1992), quality improving (or vertical) innovations replace older vintages by making the older ones obsolete.[18]

Intermediate goods B and F are perfectly substitutable energy inputs. Specifically, the technology defined by B transforms land directly into energy.[19] Land has a constant energy efficiency that cannot be increased which, without loss of generality, can be normalized to one. The amount of energy in efficiency units that is generated by B is then given by

$$B_t = \underbrace{A^B}_{=1} L_{B,t}, \tag{6}$$

where $L_{B,t}$ is the amount of land allocated to intermediate input B. Since the energy efficiency of B cannot be improved, we will occasionally refer to B as the *stagnant* (energy) input.

Production of the intermediate energy input F takes land and another set of machines as inputs. If we denote the share of land that is combined with machines by $L_{F,t}$, the total amount of energy in efficiency units that is generated by F at time t is given by

$$F_t = L_{F,t}^{1-\gamma} \int_0^1 A_{i,t}^F \left(x_{i,t}^F\right)^\gamma di, \tag{7}$$

where A_i^F is the productivity of the latest machine of variety $i \in [0,1]$.[20]

The total amount of land L, is fixed in supply, so it has to be divided between B and F., i.e., $L_{B,t} + L_{F,t} = L$. Because land also enters the production function for F, a tradeoff between the energy inputs is introduced. L_F is thus a measure of the relative importance of F. This assumption is also convenient for tractability reasons.

Even though some of the variables in the model are growing over time, it suffices for now to see that within each period, K, A^G, A^F and p_E are all given. We can therefore solve for the within-period-equilibrium allocation while taking these variables as given. Since all computations related to the within-period equilibrium are based on variable values from that period, we suppress time indices from our notation in sections 3.5-3.7. In addition, since labor and land are both in fixed supply, these variables are, without loss of generality, both normalized to one, i.e.,

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[18] An advantage with Schumpeterian growth models is that it is possible to eliminate the scale effect that predicts that a larger population raises the incentives to carry out R&D. Based on the result in Howitt (1999), the results in this paper should then not depend on the assumption of a fixed population size.

[19] For instance, the land produces wood that is used as fuel.

[20] To simplify notation, the integration is done over variety i for both goods G and F, but the varieties are different for the two goods.

$N = L = 1$.

3.5 Final-good and intermediate-good producers

As in most endogenous growth models, there are several lines of production: firms that produce final output, firms that produce intermediary goods and monopolies that produce the latest version of a specific machine variety. Final-good producers are price takers in all markets and they make new decisions to buy/rent all their inputs in each period. Land is owned by the consumers and it is rented out to the firms at the rental rate κ. Specifically, final-goods producers face prices p_G for G, κ for L and p_F for F. The price of the final good is normalized to one.

Similarly, the competitive producers of the intermediate good G are paid price p_G for their output and they face input prices W for N and p_i^G for x_i^G. The competitive producers of the intermediate good F are instead paid the price p_F for their output and they face input prices κ for L and p_i^F for x_i^F. The first-order conditions to these problems are laid out in Appendix A.1.

3.6 Monopolists

A successful invention of a new variety of a specific machine makes the old machine of that variety obsolete, and it entitles the innovator to a one period monopoly on producing this specific variety. The monopolies take the inverse demand function for their respective products into account when deciding how many machines to produce and sell. The monopolies are, however, price takers in all input markets.

Machines used for intermediate good G are produced with capital as the only input, whereas the machines that are used for F instead require fossil fuel as the only input. Specifically, A_i^G units of capital are needed to produce one G-machine of variety i, and A_i^F units of fossil fuel are needed to produce one F-machine of a certain variety.[21] Capital is rented from the households at the rental rate r, and is for reasons of tractability assumed to depreciate fully between any two consecutive periods. The monopoly profits from a specific machine of variety i in sector G and F are then respectively given by

$$\pi_i^G = p_G \alpha A_i^G \left(x_i^G\right)^{\alpha} - A_i^G r x_i^G \text{ and } \pi_i^F = p_F \gamma L_F^{1-\gamma} A_i^F \left(x_i^F\right)^{\gamma} - A_i^F p_E x_i^F, \tag{8}$$

where $p_G \alpha A_i^G \left(x_i^G\right)^{\alpha-1}$ is the inverse demand function for machine x_i^G, and $p_F \gamma L_F^{1-\gamma}$

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[21] The specific units that are required allow for analytical tractability, but are unlikely to be important for the results in the paper.

$A_i^F \left(x_i^F\right)^{\gamma-1}$ is the inverse demand function for machine x_i^F. Maximizing π_i^G with respect to x_i^G gives the profit-maximizing quantity

$$x^G \equiv x_i^G = \left(\frac{\alpha^2}{r} p_G\right)^{\frac{1}{1-\alpha}}, \tag{9}$$

i.e, x_i^G is independent of i. At the aggregate level, the demand for capital must equal the supply of saving, i.e., we must have $K = \int_0^1 A_i^G x_i^G di$. Furthermore, since x^G is the same for all varieties in G, it follows that

$$x^G = k \equiv \frac{K}{A^G}, \tag{10}$$

where A^G is the average productivity in sector G, i.e., $A^G = \int_0^1 A_i^G di$. Inserting (10) into (7) shows that the composite G is given by a standard Cobb-Douglas function:

$$G = A^G k^\alpha. \tag{11}$$

Similarly, maximizing π_i^F with respect to x_i^F also delivers F as a Cobb-Douglas function:[22]

$$F = \left(A^F L_F\right)^{1-\gamma} E^\gamma, \tag{12}$$

where A^F is the average productivity in sector F, i.e., $A^F = \int_0^1 A_i^F di$. Inserting (11) and (12) into (4) gives the derived production function:

$$Y = \left[\left(A^G k^\alpha\right)^{\frac{\sigma-1}{\sigma}} + \left((1-L_F) + \left(A^F L_F\right)^{1-\gamma} E^\gamma\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{13}$$

Note that, if $\sigma < 1$, energy is essential for production but it is not necessary that energy is sourced from fossil fuel. In addition, with $L_F = 1$, the stagnant input is not used at all, thus implying that all energy is coming from fossil fuel. In this case, the derived production function is conceptually similar to the estimated function in Hassler, Krusell and Olovsson (2012). Hence, it is also potentially consistent with the post-industrial growth experience.[23]

Combining (9) and (10), we can derive an expression for the equilibrium inter-

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[22] See Appendix A.2 for an explicit derivation.

[23] The presence of two different energy inputs bears some resemblance the assumption of two different technologies as found in Hansen and Prescott's (2002). Since A^G, A^F and L_F are all endogenous in our setting, our model allows for the analysis of the conditions under which a transition from stagnation to growth can actually take place, as well as why different countries endogenously may choose different proportions of B and F.

est rate:

$$r = \alpha^2 p_G \frac{G}{K}. \tag{14}$$

Finally, equating the total demand for fossil fuel (which is given by (49) in Appendix A.2) with the supply for fossil fuel, which is given by $(1-\beta)\,R_t$, delivers the equilibrium price of fossil fuel:

$$p_E = \gamma^2 \left(\frac{A^F L_F}{(1-\beta)\,R} \right)^{1-\gamma} p_F. \tag{15}$$

At any given point in time, the fossil-fuel price depends on the level of the fossil-energy efficiency (A^F), the amount of land used in the fossil sector (L_F), and on the marginal product of the fossil technology (p_F).

3.7 Within-period equilibrium

Since energy inputs B and F are perfect substitutes, they will only both be used in production if their costs are the same, i.e., if $\kappa = p_F$. The first order condition with respect to L_F from the producers of intermediated good F can then be shown to imply that F is proportional to L_F:[24]

$$F = \frac{1}{1-\gamma} L_F. \tag{16}$$

Equating (12) and (16), and using the fact that E_t has to equal $(1-\beta)\,R_t$ gives that the amount of land that is combined with fossil fuel is given by

$$L_F = \min\left\{ \left(A^F\right)^{\frac{1-\gamma}{\gamma}} R\,(1-\beta)\,(1-\gamma)^{\frac{1}{\gamma}}, 1 \right\}. \tag{17}$$

At any given point in time, A^F and R are both given. Equation (17) therefore defines three regimes for how final output is produced at each point in time. First, if either A^F and/or R is zero, the amount of land that is combined with fossil fuel is zero. In this case, final goods are exclusively produced with the stagnant input.

Second, if $0 < \left(A^F\right)^{\frac{1-\gamma}{\gamma}} R\,(1-\beta)\,(1-\gamma)^{\frac{1}{\gamma}} < 1$, some but not all of the land is combined with fossil fuel. In this case, final output is produced using both energy inputs. Finally if $\left(A^F\right)^{\frac{1-\gamma}{\gamma}} R\,(1-\beta)\,(1-\gamma)^{\frac{1}{\gamma}} \geq 1$, the fossil input dominates the stagnant input. In this case, no land is used to produce the stagnant energy input and all energy is derived from fossil fuel. These three regimes will respectively be referred to as the *pre-industrial*, the *hybrid*, and the *fossil* regime.

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[24] See equation (46) in Appendix A.1.

Equation (17) also reveals that fossil energy is always used whenever A^F and R are both larger than zero. Intuitively, fossil fuel is then always productive and, since the sellers of fuel have zero costs of supplying it, they will accept any price larger than zero.

In a fully dynamic setting, the variables A^G, A^F, K and R are all endogenous. We are now ready to take the next step in the analysis and incorporate the evolution of these variables.

3.8 Intertemporal equilibrium: directed technical change and economic growth

To model the process for technological progress and economic growth, we build on the multi-country model in Howitt (2000). Since there are two intermediate goods whose efficiency can be improved in our setting, we also build on Acemoglu (2002, 2003) and Acemoglu et al. (2012) and expand Howitt's setting to one with directed technical change.

Specifically, there are two separate research sectors for improving varieties of the two intermediate goods G and F, and firms invest to discover the next generation of each variety. Within each sector, the innovations all draw on the same pool of technological knowledge, but the arrival rates for different varieties are independent of each other. There are, however, no spill-overs between the two research sectors.

At any time t, sector $j \in \{G, F\}$ features a worldwide leading-edge technology parameter: $\bar{A}_t^j$. An innovation of variety i in sector j then allows the innovator to start producing variety i in sector j by using the leading-edge parameter in j. A country's relative productivity in sector $j \in \{G, F\}$ is defined as its average productivity over the leading edge, i.e., $a_t^j \equiv A_t^j / \bar{A}_t^j$.

The outcomes of R&D investments are stochastic and a firm that aims at improving a specific variety of an intermediate good chooses the probability that maximizes the expected payoff from R&D. Higher probabilities require more resources. Specifically, innovations at rates $\mu_{i,t}^G$ and $\mu_{i,t}^F$ are respectively assumed to be governed by the following cost functions:

$$\Theta_{i,t}^G = \left(\eta_1 \mu_{i,t}^G + \frac{\nu_1}{2}\left(\mu_{i,t}^G\right)^2\right)\frac{Y_t}{a_t^G} \tag{18}$$

$$\Theta_{i,t}^F = \left(\eta_2 \mu_{i,t}^F + \frac{\nu_2}{2}\left(\mu_{,i,t}^F\right)^2\right)\frac{Y_t}{a_t^F}, \tag{19}$$

where η_1, ν_1, η_2, ν_2 are all positive constants. The cost functions have three components each and two of them are standard. First, the costs are increasing (quadratically) in their respective research effort μ_i^j. Second, the costs are both assumed to grow with final output Y_t. The third component is less standard and it implies that a given innovation probability becomes more expensive the further away from the frontier a country is (i.e., the lower a country's relative productivity). This assumption captures the idea of increasing complexity that makes technologies increasingly difficult to master and to adapt to local settings. This term thus introduces a disadvantage of backwardness. It is only of importance for the result in Proposition 5 and it is discussed further in section 4.3.

The timing of the R&D process is as follows. In each period, exactly one R&D firm is assigned to each variety of machine in each sector. These firms then choose the innovation probabilities based on the associated costs and profits. The firms finance their research projects by issuing shares. If a research project is successful, the resulting next period profits are paid out to the shareholders in proportion to the shares held. This allows for all agents to hold a balanced portfolio of shares in R&D firms. By the law of large numbers, the return to this portfolio is risk free.[25] Finally, if no new innovation is made on a particular variety, the existing patent is randomly assigned to some firm.

Our modeling of the research process takes some shortcuts in order to make the model as tractable as possible. These shortcuts merit a discussion. First, by assuming that varieties are assigned to firms before research investments are made, we abstract from the risks of multiple research projects resulting in innovations on the same variety. Second, since patents only last for one period, the decision to invest in research only takes the expected next-period profits into account. If, instead, patents were assumed to last until the next innovation on the same variety occurs, the research investment would be made based on the expected value of discounted profits over the stochastic and endogenous lifetime of the patent. Third, if patented technologies were made freely available, rather than randomly assigned, once they expire, we would have to distinguish between machine varieties for which there are valid patents (that are produced monopolistically) and machine varieties for which there are no valid patents (that are produced competitively). The first two modeling assumptions are also made by Aghion et al. (2005), and all three assumptions are made by Acemoglu et al. (2012) (although they use a

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[25] All R&D firms are owned in equal shares by all households. We thus assume complete financial markets. This is not fully realistic but since our focus is on the potential importance of energy, it is a natural first step.

fixed pool of scientists rather than investments as input to research).

Using more realistic assumptions for these aspects of the research process would clearly affect the exact conditions for parameter combinations under which the results hold. Given that our main results are qualitative in nature, however, we believe that the alternative assumptions are unlikely to change our results in any important way. This is also confirmed for the third assumption in an online appendix to Acemoglu et al. (2012) in which they derive conditions under the alternative assumption that varieties for which there was no innovation in the current period are produced competitively rather than monopolistically. The exact conditions change somewhat, but the results remain qualitatively similar.

When combined, our modeling assumptions imply that investment in innovation on each variety is made to maximize the expected net value of the project using the risk-free rate of return to discount expected future profits. Furthermore, the profits generated by patents are shared equally among all households. The resulting equilibrium probabilities are given by[26]

$$\mu_t^G \equiv \mu_{i,t}^G = \frac{1-\alpha}{\nu_1 \alpha}\frac{k_{t+1}}{Y_t} a_t^G \bar{A}_{t+1}^G - \frac{\eta_1}{\nu_1} \tag{20}$$

$$\mu_t^F \equiv \mu_{i,t}^F = \frac{(1-\gamma)\gamma}{\nu_2 \alpha^2} L_{F,t+1} k_{t+1}^{1-\alpha} \frac{p_{F,t+1}}{p_{G,t+1}} \frac{E_{t+1}^{\gamma}}{Y_t \left(A_{t+1}^F\right)^{\gamma}} a_t^F \bar{A}_{t+1}^F - \frac{\eta_2}{\nu_2}. \tag{21}$$

Equations (20) and (21) show that R&D investments are constant across varieties within each respective research sector. By the law of large numbers, the laws of motions for average productivities A_{t+1}^G and A_{t+1}^F are then respectively given by

$$A_{t+1}^j = \mu_t^j \bar{A}_{t+1}^j + (1-\mu_t^j) A_t^j, \quad j \in \{G, F\}. \tag{22}$$

The frontiers $\bar{A}^G$ and $\bar{A}^F$ are assumed to evolve through spillovers from research efforts

$$\bar{A}_{t+1}^j = \bar{A}_t^j \left(1 + \mu_t^j\right), \quad j \in \{G, F\}. \tag{23}$$

Section 2 describes the endogeneity problem that is present when trying to empirically estimate the relationship between the fossil share and economic growth. Note then from equation (21) that this is true in the model. Specifically, the growth rate of output between period t and $t+1$ is a function of the growth rate of A^F. This growth rate then depends on the innovation probability μ^F which, in turn, depends on future fossil-fuel use $L_{F,t+1}$. Hence, both the growth rate and the

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[26] See Appendix A.3 for more details.

fossil share are determined simultaneously, thus implying that one variable is not causing the other.

4 Results

The dynamic model has now been fully specified and we are ready to derive the results. We start with the case in which no fossil energy exists.

4.1 Stagnation without fossil energy

As shown in section 3.7, final output is exclusively produced with the stagnant energy input when the stock of fossil energy is zero ($R = 0$). Proposition 1 below then states that sustained growth is not possible when final-good production only takes place with the stagnant energy input.

Proposition 1. *If $\sigma < 1$, there cannot be sustained growth in output and consumption without fossil energy. If $\alpha(1-\alpha)\beta < \eta_1$ and the initial level of A^G is low enough, there will be a transient period with positive research investments and growth, but A^G is bounded in the long run. There exist a continuum of steady states that can be indexed by the level of A^G and among these steady states production is increasing in the associated constant technology level A^G.*

Proof. See Appendix A.4. ☐

The properties of the steady state are described in Appendix A.4. The steady state in Proposition 1 is broadly consistent with the pre-industrial period of stagnation and temporary growth. It will be referred to as the pre-industrial steady state.

4.2 Sustained balanced growth with fossil energy

In this section, we show that, under some restrictions on the parameters, there exist a balanced growth path (BGP), in which final output exclusively sources energy from fossil fuel. The resource constraint in the final-output-producing country is given by

$$Y_t = C_t + K_{t+1} + \Theta_t^G + \Theta_t^F + p_{E,t}E_t. \tag{24}$$

We are now looking for a balanced growth path where all the terms in the resource constraint (24) are growing at a constant rate. Since the consumption of the fossil-fuel producers is equal to $p_{E,t}E_t$, their consumption will then also grow at the same constant rate. With the Euler equation given by

$$\frac{C_{t+1}}{C_t} = \beta r_{t+1}, \tag{25}$$

the constant gross growth rate has to equal βr. To simplify the notation, the following definition is employed

$$\tilde{y}_t \equiv \frac{\left(A_t^F\right)^{1-\gamma} R_t^\gamma}{A_t^G}. \tag{26}$$

The definition of $\tilde{y}_t$ takes into account that the supply of fossil fuel in each period is given by $E_t = (1-\beta)R_t$. The properties of the BGP require that both k and $\tilde{y}$ are constant. Theses and other properties, are formally listed in Definition 1.

Defnition 1. *A balanced growth path for the economy described in sections 3-3.8 is defined as to satisfy the following properties*

- k, $\tilde{y}$, r, μ^G, μ^F, p_G, *and* p_F *are all constant.*
- A^G, $\bar{A}^G$, C, K, Y, $p_E E$, Θ^G *and* Θ^F *all grow at the same net rate* g_{A^G}.
- A^F *and* $\bar{A}^F$ *both grow at net rate* $g_{A^F,t}$.

A necessary condition for k to be constant is that A^G and K are both growing at the same rate, g_{A^G}. The Euler equation (25) reveals that this growth rate must be

$$g_{A^G} = \beta r - 1. \tag{27}$$

Since A^G grows at rate g_{A^G} and R “grows” at the rate $\beta - 1$, $\tilde{y}$ being constant requires that

$$g_{A^F} = \beta r^{\frac{1}{1-\gamma}} - 1. \tag{28}$$

Equations (27) and (28) together imply that $g_{A^F} > g_{A^G}$ as long as $r > 1$. This is intuitive because A^F must grow faster than A^G to compensate for R falling over time. The interest rate on the BGP can be derived by combining (11), (12) and (14). Formally, r is given by

$$r = \alpha^2 \left[1 + \left((1-\beta)^\gamma \frac{\tilde{y}}{k^\alpha}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} \left(\frac{1}{k}\right)^{1-\alpha}, \tag{29}$$

which is constant if k and $\tilde{y}$ are both constant. The prices p_G and p_F are respectively given by $\left[(\tilde{y}(1-\beta)^{\gamma}/k^{\alpha})^{\frac{\sigma-1}{\sigma}}+1\right]^{\frac{1}{\sigma-1}}$ and $\left[(k^{\alpha}/\tilde{y}(1-\beta)^{\gamma})^{\frac{\sigma-1}{\sigma}}+1\right]^{\frac{1}{\sigma-1}}$, and they are both constant if k and $\tilde{y}$ are constant.

From (23) and Definition 1, it follows that $\mu^G = g_{A^G}$ and $\mu^F = g_{A^F}$ are constants. In order for both innovation probabilities to lie between 0 and 1, we need both growth rates g_{a^G} and g_{A^F} to be between 0 and 1.[27] A necessary condition for this to be true is

$$\beta \geq 2^{\frac{\gamma-1}{\gamma}} \in (0,1). \tag{30}$$

From (22) it follows that the relative technology levels must be $a^G = (1+g_{A^G})/2$ and $a^F = (1+g_{A^F})/2$. It is straightforward to show that with constant k and $\tilde{y}$, production Y grows at the same rate as A^G. With both μ^G and μ^F being constant, it follows from (18) and (19) that Θ^G and Θ^F are both growing at the rate g_{A^G}.

It remains now to verify that all the relevant conditions can be fulfilled by a constant k and $\tilde{y}$ in such a way that $r \geq 1/\beta$ and that the innovation probabilities lie between 0 and 1. Substituting production, Y_t, and $\mu^G = g_{A^G}$ from (27) into (20), and using (29) delivers the following expression

$$\beta r - 1 = \frac{\alpha(1-\alpha)\beta}{\nu_1}\frac{1}{1+\left((1-\beta)^{\gamma}\frac{\tilde{y}}{k^{\alpha}}\right)^{\frac{\sigma-1}{\sigma}}} - \frac{\eta_1}{\nu_1}. \tag{31}$$

Similarily, substituting production and $\mu^F = g_{A^F}$ from (28) into (21), and using (29) delivers

$$\beta r^{\frac{1}{1-\gamma}} - 1 = \frac{\beta\gamma(1-\gamma)}{\nu_2}\frac{\left((1-\beta)^{\gamma}\frac{\tilde{y}}{k^{\alpha}}\right)^{\frac{\sigma-1}{\sigma}}}{1+\left((1-\beta)^{\gamma}\frac{\tilde{y}}{k^{\alpha}}\right)^{\frac{\sigma-1}{\sigma}}} - \frac{\eta_2}{\nu_2}. \tag{32}$$

We now have three equations: (29), (31) and (32) in the three unknowns r, k and $\tilde{y}$. A solution to this system of equations corresponds to a balanced growth path. This system can at most have one solution. The conditions under which this solution exists are given in Proposition 2.

Proposition 2. *A necessary condition for a balanced growth path to exist is that* $\eta_1 \leq \alpha(1-\alpha)\beta$. *Additional requirements are provided by the following bullet points.*

- *If* $\nu_2 + \eta_2 < \beta\gamma(1-\gamma)$, *then there exists a balanced growth path if and only*

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[27] There might also be balanced-growth paths corresponding to corner solutions where the marginal value of increasing the innovation probability remains strictly higher than the marginal cost as the probability approaches 1. Our analysis abstracts from this possibility.

if both inequalities

$$\frac{\eta_1}{\alpha(1-\alpha)\beta}+\frac{\left(\beta^{-\frac{\gamma}{1-\gamma}}-1\right)\nu_2+\eta_2}{\beta\gamma(1-\gamma)} \leq 1 \tag{33}$$

$$\frac{\left(\beta^{\gamma}2^{1-\gamma}-1\right)\nu_1+\eta_1}{\alpha(1-\alpha)\beta}+\frac{\nu_2+\eta_2}{\beta\gamma(1-\gamma)} \geq 1. \tag{34}$$

are fulfilled.

- *If* $\beta^{-\frac{\gamma}{1-\gamma}}\nu_2+\eta_2 \leq \beta\gamma(1-\gamma) \leq \nu_2+\eta_2$, *then there is a balanced growth path if and only if inequality* (33) *is fulfilled.*
- *If* $\beta^{-\frac{\gamma}{1-\gamma}}\nu_2+\eta_2 > \beta\gamma(1-\gamma)$, *then there is no balanced growth path.*

Proof. See the Appendix A.5. ☐

On the balanced growth path, the innovation probabilities must lie between 0 and 1. Inequality (33) then ensures that research costs are not too high, while inequality (34) ensures that research costs are not too low. It is straightforward to verify that, for given values of α, β, γ and σ (where β fulfills inequality (30)), there exist values for η_1, η_2, ν_1 and ν_2 such that both inequalities (33) and (34) are fulfilled. In particular, inequality (30) implies that $\beta^{-\frac{\gamma}{1-\gamma}}-1 \in [0,1]$ and $\beta^{\gamma}2^{1-\gamma}-1 \geq 0$ which, in turn, implies that the left-hand side of (33) is smaller than the left-hand side of (34). Hence, for given values of α, β and γ there are always values of η_1, η_2,ν_1 and $\nu 2$ such that (33) and (34) are simultaneously fulfilled.

Finally, it follows from (17) that a sufficient condition to keep producing exclusively using the fossil-energy input is that $\left(A^F\right)^{\frac{1-\gamma}{\gamma}}R$ does not decrease over time. Note, however, that $\left(A^F\right)^{\frac{1-\gamma}{\gamma}}R$ can be written as $(1-\beta)\left(A^G\tilde{y}\right)^{\frac{1}{\gamma}}$. With $\tilde{y}$ being constant and with A^G increasing over time, $\left(A^F\right)^{\frac{1-\gamma}{\gamma}}R$ will, in fact, increase along the balanced growth path.

The steady state in Proposition 2 features balanced growth and is, thus, consistent with the post-industrial period in the Western world. We will refer to this steady state as the fossil steady state.

4.3 Heterogeneous final-output producing countries

In this section, we analyze what the model has to say about the Great Divergence. The assumption of homogenous final-good-producing countries is therefore dropped and countries are instead allowed to differ with respect to their initial technology levels. The main difference compared to the setting in sections 3.3-3.8 is that the technological frontiers are now assumed to evolve according to $\bar{A}^j_{t+1} = \bar{A}^j_t \left(1 + \sum_{m=1}^{M} \varrho_m \mu^j_{m,t}\right)$, $j \in \{F, G\}$, where $\varrho_m \geq 0$ is a spillover coefficient and M denotes the discrete number of final-goods-producing countries. Hence, as in Howitt (2000), spillovers from R&D are now linking the different countries and the steady-state world growth rates of the frontiers depend on each country's investment rate in the two sectors. To keep the model as simple as possible, we abstract from international capital movements.

The bottom graph in Figure 1 shows that the gap between the richest region (Western world) and the poorest region (Africa) has increased continuously over the roughly 200 years that has passed since the Industrial Revolution. Pritchett (1997) argues that if we accept (i) the current estimates of relative income between countries; (ii) the estimates of historical growth rates; and (iii) a lower bound for income of $250, then we cannot escape the conclusion that the last 150 years have seen substantial divergence.[28]

According to Maddion (2001), the ratio of income in the richest set of countries relative to in the poorest grew from roughly two in 1820 to around 20 in 2001. In the data described in Section 2, the difference in the logarithm of GDP/capita between the U.S. and the ten poorest countries increased from 3.56 in 1974, to 3.79 in 1984, to 4.08 in 1994 and to 4.11 in 2004. Hence, the divergent trend between the richest and the poorest set of countries seems to have continued into the 2000s.[29]

Overall, these facts suggest that growth rates can differ substantially across countries for long periods of time. In addition, the magnitude of the difference in income between the richest and the poorest set of countries corresponds to a situation in which the rich countries have been growing at an annual rate of two percent for 200 years, whereas the poor countries instead have been growing at a rate of about zero during the same period. Under these circumstances, the rich countries should be around four log units richer than the poor countries.

The question is why some countries would not be growing despite the possibility of international technology transfer and, with the words of Gerschenkron (1962), the "advantage of backwardness".[30] Specifically, Gerschenkron argues that back-

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[28] Acemoglu (2009) reaches a similar conclusion. Note, however, from the bottom graph in Figure 1 that the gaps between the Western world and India and China both have decreased in the 20th century.

[29] Young (2012), however, uses alternative data to measure consumption and growth and argues that growth has been relatively high in sub-Saharan countries for the last two decades.

[30] As mentioned in footnote 3, several theories on this subject exist.

ward countries may experience episodes of rapid growth that is driven by rapid productivity catch-up. In the model, one advantage of backwardness is found in the fact that the costs for R&D are proportional to the country-specific income (Y), whereas the resulting profits from a successful innovation benefit from growing international spillovers. This fact incentivizes backward countries to invest heavily in R&D. As a result, without any disadvantage of backwardness, they grow at relatively faster rates than countries at the front. Because of the spillovers from innovation, all countries then also share the same long-run growth rate at the BGP.

One reason for a lack of growth in some countries could be that, even though there are advantages of backwardness, there may also be some disadvantages. For instance, Pritchett (1997) argues that the cases in which, particularly, the most backward countries actually gain significantly on the leader are historically rare and that backwardness, in fact, seems to carry severe disadvantages. These disadvantages are captured in (18) and (19) in that the research costs for a specific country also increase with the distance to the frontier. Our specification of the research costs thus combines advantages and disadvantages of backwardness. Before considering the Great Divergence dynamically, the next section first analyzes how a country's choice between the different energy inputs is determined.

4.3.1 The choice between the two energy inputs

Proposition 3 identifies the key factors that influence a country's choice between the different energy inputs at a given point in time.

Proposition 3. *A country such that*

$$\left[\left(A_t^G k_t^\alpha\right)^{\frac{\sigma-1}{\sigma}} + 1\right]^{\frac{1}{\sigma-1}} \leq \frac{p_{E,t}}{\left(A_t^F\right)^{\frac{1-\gamma}{\gamma}}} \frac{1}{\gamma^2(1-\gamma)^{\frac{1-\gamma}{\gamma}}} \tag{35}$$

will satisfy its energy demand exclusively from the stagnant energy input. A country such that

$$\left[\left((1-\gamma)\, A_t^G k_t^\alpha\right)^{\frac{\sigma-1}{\sigma}} + 1\right]^{\frac{1}{\sigma-1}} \geq \frac{p_{E,t}}{\gamma^2(1-\gamma)^{\frac{1-\gamma}{\gamma}} \left(A_t^F\right)^{\frac{1-\gamma}{\gamma}}} \tag{36}$$

will satisfy its energy demand exclusively from the fossil-energy input. The remaining countries will use both energy inputs.

Proof. See Appendix A.6. □

Note that even though country-specific values for A_t^G, k_t, A_t^F are endogenous from each country's perspective, these variables are all given within a specific period. In particular, values in period t were decided on prior to period t. In addition, because countries are price takers in the fossil-fuel market, $p_{E,t}$ is considered exogenous by all countries.

The left hand side of (35) denotes the marginal product of intermediate good F when $L_F = 0$. The right hand side instead denotes the marginal cost of F. As expected, the marginal cost is increasing in the price of fossil fuel and it is decreasing in the level of the fossil-energy efficiency $\left(A^F\right)$.

With the marginal product of F given by $\left[\left(\frac{(1-\gamma)A_t^G k_t^\alpha}{1-\gamma+\gamma L_{F,t}}\right)^{\frac{\sigma-1}{\sigma}}+1\right]^{\frac{1}{\sigma-1}}$, the only variable that can be used to influence this marginal product within a period is L_F. Note, however, that the marginal product is strictly decreasing in L_F. Hence, if the inequality in (35) is strict, the marginal cost of F is strictly higher than the marginal product of F, even though $L_F = 0$. In this case, the stagnant energy input strictly dominates fossil energy in final-output production. Fossil energy is then simply too expensive an input to be efficient.

In (36), the marginal cost of fossil energy is instead lower than the marginal product even though only fossil energy is used. In this case, fossil energy strictly dominates the stagnant input. Proposition (3) thus provides conditions for which of the two energy inputs that are used in a country at a given point in time. It also identifies how this choice depends on technology levels and the price of fossil fuel.

4.3.2 The Great Divergence

We now illustrate that differences in initial conditions can generate divergence in the model. Specifically, we consider a setting with two final–goods-producing countries that initially differ with respect to their level of technological advancement. One country is initially technologically advanced, whereas the other country is initially less technologically advanced. For simplicity, we assume that the less advanced country is small so that we can abstract from the potential effects of its actions on the technology frontier and the fossil-fuel price.

The Great Divergence is modeled as a steady state due to the fact that the gap between the Western world and the very poorest countries seems to have continued to increase over roughly 200 years. Since some countries, in fact, have been able to escape from their "poverty traps", we discuss possible ways to leave the state of poverty in Section 4.3.3.

Variables in the technologically less-advanced country are denoted with subscript S (for stagnant), whereas variables in the advanced country are denoted as before, i.e., they do not carry a country-specific subscript. Assume then that the conditions in Proposition 2 are fulfilled so that a BGP exist. Proposition 4 then provides conditions for when the Great Divergence could arise.

Proposition 4. *Assume that, at time $t = t_0$, the technologically advanced country is initially in the fossil steady state, i.e., it exclusively produces with fossil energy and experiences balanced growth, whereas the technologically less advanced country is initially in the pre-industrial steady state, i.e., it exclusively produces with the stagnant energy input and does not grow. The technologically advanced country continuing to grow and the less technologically advanced country remaining in the pre-industrial steady-state is always an equilibrium.*

Proof. See the Appendix section A.7. □

The intuition for the proposition comes from the fact that, in each period, the choice of whether to use fossil fuel or not depends on the domestic state of technology A_S^F in relation to the fossil-fuel price. That price, in turn, depends on the level A^F in the fossil-energy-using country. If that country is sufficiently more advanced, it drives up the international fossil-fuel price so much that the less advanced country does not find it profitable to use fossil fuel at all. In addition, from (15), it follows that the price of fossil fuel increases over time as a result of technical change in the developed country, as well as from the depletion of the stock of fossil fuel. Without investments in sector F, fossil energy thus becomes increasingly more expensive relative to the stagnant energy input in country S as time passes.

Getting out of the pre-industrial steady-state requires a coordinated effort to advance the average fuel efficiency within the stagnant country, which is the reason for why there always is an equilibrium where the stagnant country remains stagnant. Individual agents are not alone able to increase the average to make fossil energy profitable and take the country out of the stagnant steady state.

In general, there may, however, be additional steady-states in which a coordinated effort would take the country out of stagnation. Proposition 5 states that if the country in the pre-industrial steady-state is sufficiently far behind in terms of the fossil technology, this equilibrium is unique.

Proposition 5. *Make the same assumptions as in Proposition 4. Assume, furthermore, that $\sigma < 1$ and that the technology level A_S^F is sufficiently low in the* *country in the pre-industrial steady-state compared to the level in the growing country, A^F, then the equilibrium described in Proposition 4 is unique.*

Proof. See the Appendix section A.8. $\square$

What Proposition 5 adds to Proposition 4 is that it shows that there are conditions under which we know that there is no equilibrium where the stagnant country can start investing in research in the F sector. In particular, the proof shows that if A^F_{S,t_0} is sufficiently small relative to $A^F_{t_0}$, country S could not afford the costs required to improve A^F_S to the point where it becomes profitable to use fossil fuel. There is likely a significantly larger set of circumstances where country S could potentially afford to leave the stagnant steady state but where it would still not be an equilibrium to do so.[31]

Note that the Great Divergence does not just follow trivially from the assumption that costs are increasing in the distance to the frontier. In fact, the existence of two different energy inputs and the fact that fossil fuel is sold on a world market are also necessary assumptions for this result. If only the fossil technology was available, the marginal product of fossil fuel would go to infinity when E goes to zero within a country (because energy is essential for production). Since the investments that aims to improve the fossil-energy efficiency are increasing in this marginal product (through p_F in (21)), there would eventually be positive investments in this sector even with the disadvantage of backwardness. The presence of two energy inputs puts an upper bound on the marginal product of fossil energy. Similarly, if the price of fossil fuel is not set on a world market but, instead, only depend on conditions within country S, then the equilibrium price would be sufficiently low for fossil fuel to always be used (as shown in Section 3.7).

The argument for divergence in the model reflects those in Basu and Weil (1998) and Acemoglu and Zilibotti (2001), i.e., that the technologies that are developed at the frontier are not appropriate for less developed countries. In our setting, this is because the frontier focuses, among other things, on developing the efficiency of fossil energy, which is an energy source that the less developed country does not use.

4.3.3 Escaping the poverty trap

Even though the last two hundred years have witnessed some divergence, it is also clear that individual countries have been able to leave the state of poverty

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[31] The proposition does not say anything about why the countries initially would be different, only that if they are sufficiently different, divergence can persist.

and have begun to converge to the Western world. Examples include Hong Kong, Singapore, South Korea, Taiwan, China and Botswana. The model in this paper is deterministic, implying that a country that starts out in a poverty trap will stay there forever. It seems clear, however, that exogenous shocks could potentially allow countries to escape from the poverty trap.

As shown in the proof to Proposition 5, costs of research can be prohibitively high. Hence, a sufficiently large income shock, such as the discovery of valuable natural resources, should be able to generate enough resources for the necessary additional investments.[32] The effects from income shocks are left for future research.

4.4 Income and the choice of energy in the model

Let us now return to the correlations displayed in Section 2 and see what the model predicts about income, growth and the choice of energy. Energy can either be derived directly from land with the technology in (5), or from fossil energy with the technology in (7). A measure of the share of energy that comes from fossil energy is thus given by[33]

$$f \equiv \frac{E}{L_B + E}.$$

Note that this measure is zero in the pre-industrial steady state in Proposition 1, and is equal to one in the fossil steady state in Proposition 2. It immediately follows that these steady-state predictions about income, growth and energy use are qualitatively in line with the stylized facts in section 2. Specifically, in the pre-industrial steady state, income is low and does not grow and only the stagnant energy input is used. In the fossil steady state, income is high (because it is growing) and only fossil energy is used.

Consider now a more general setting. Specifically, assume that there exist a large number of countries that are all in the hybrid regime, i.e., they are producing final output with both energy inputs. Assume further that these countries differ with respect to their initial values for k, A^G and A^F. For the fossil share, the variations in k and A^G only matter through their effects on G. It thus suffices to consider countries that differ in their levels of G and A^F. The following expressions

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[32] Given that the country is not affected by the natural resource curse. One of the reasons for the growth miracle of Botswana is claimed to be large discoveries of diamonds.

[33] We here define the fossil share in terms of inputs. An alternative definition, in terms of the intermediate energy goods, $\frac{F}{B+F}$ would have given predictions that were even more in line with the data.

can then be derived

$$\frac{\partial f}{\partial G} = \frac{1-\gamma}{\gamma}\frac{L_B f}{L_B+E}\left[\frac{L_B+F}{F}+\frac{L_B+F}{L_B}\right]\frac{1}{G}>0, \tag{37}$$

$$\frac{\partial f}{\partial A^F} = \frac{1-\gamma}{\gamma}\frac{L_B f}{L_B+E}\left[\sigma\frac{1-\gamma}{\gamma}\frac{L_B+H}{\varphi_G}\left(\frac{1}{L_F}+\frac{1}{L_B}\right)-1\right]\frac{1}{A^F}=? \tag{38}$$

where $\varphi_G \equiv \frac{G^{\frac{\sigma-1}{\sigma}}}{G^{\frac{\sigma-1}{\sigma}}+(B+F)^{\frac{\sigma-1}{\sigma}}} \in (0,1)$.[34] The first derivative (37) is unambiguously positive but the second derivative (38) is ambiguous when $\sigma < 1$. The implication is that the fossil share does not increase unambiguously with income in the hybrid regime. Instead, it depends on *why* countries differ with respect to income. Specifically, if countries differ because some countries have higher capital stocks and/or a higher level of the capital/labor efficiency, i.e., A^G which is similar in concept to the standard measure of total factor productivity, then we should observe that the fossil share is increasing in income. However, if some countries are richer because they have higher energy-saving technologies, then this may or may not be the case.

Let us now analyze what the model predicts for the relationship between the fossil share and economic growth. Since we are not able to solve analytically for the relationship between GDP-growth and the state variables, we instead focus on consumption growth which is derived from the Euler equation (25). Here, the partial derivative of the interest rate with respect to G is zero (due to intratemporal adjustments). Variations in A^G and K that leave k unchanged will thus not affect the interest rate. The effects of changes in k and A^F are then given by the following partial derivatives

$$\frac{\partial g_c}{\partial k} = -(1-\alpha)\frac{r}{k}<0, \tag{39}$$

$$\frac{\partial g_c}{\partial A^F} = \frac{1-\varphi_G}{\varphi_G}\frac{1-\gamma}{\gamma}\frac{r}{A^F}>0. \tag{40}$$

Hence, if two countries only differ in that one country has a higher value of A^G (i.e., a lower k), the model prediction is that this country should also have a higher fossil share, higher income and a higher growth rate of consumption. This is consistent with the results in Section 2. The model also predicts a positive relation between A^F and g_c.

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[34] See Appendix A.9 for explanations of exactly how (37) and (38) are derived.

4.5 The hybrid regime: production with both energy inputs

In this section, we again return to the assumption of one representative final-good-producing country to say something about economic growth in this regime. The reason for the return to the representative agent is that the hybrid regime is more complicated than the other two regimes because no steady state exist in this regime. Instead, the hybrid regime is a transitional phase.

It was shown in section 3.7 that fossil energy is always used in production when positive quantities of fossil energy (in efficiency units) exist. Hence, if the stock of fossil energy in efficiency units, which is denoted by $\overline{R} \equiv \left(A^F\right)^{\frac{1-\gamma}{\gamma}} R$, would increase exogenously from zero to a positive number, the final-good-producing economy would immediately transition from the pre-industrial regime to either the hybrid regime or the fossil regime. Whether it would end up in the former or the latter regime depends, according to (17), on the new value of $\overline{R}$. Since the stock of fossil fuel is growing at the rate β, R will never become fully depleted after that. Hence, fossil energy is forever used after the discovery.

The question is then if a positive amount of fossil energy is a sufficient condition for the economy to transition into a state of sustained economic growth. While it is not possible to analytically characterize the full transition in the hybrid regime, Proposition 6 establishes that sustained growth is not possible when the stock of fossil fuel, in efficiency units, is "too small".

Proposition 6. *When $\sigma < 1$, sustained positive growth is not possible in the hybrid regime if $\overline{R}$ is smaller than a critical value.*

Proof. See Appendix A.10. ☐

Since $\overline{R}$ can be written as $L_F / \left((1-\beta)(1-\gamma)^{\frac{1}{\gamma}} \right)$, the intuition for Proposition 6 comes from the fact that $\overline{R}$ enters as a scale effect in the R&D arbitrage equation for energy (21). Hence if $\overline{R}$ is sufficiently small, the market for improving the energy efficiency is too small to be profitable. The energy efficiency will then not grow. If capital and energy are complements, the capital-augmenting technology will also eventually cease to grow. Without technological progress, the economy cannot continue to experience positive growth.

Without energy R&D investments $\overline{R}_t$ is falling at the rate $\beta < 1$, which implies that final output is, to a larger and larger extent, produced with the stagnant energy input. In fact, asymptotically, all final output is produced with the stagnant energy input. Proposition 6 thus states that it is not sufficient to find fossil fuel to embark on a path towards sustained growth.

The transitional properties associated with an $\overline{R}$ that is larger than the critical value are potentially somewhat involved, and they are left for future research.

5 Conclusions

This paper analyzes the potential importance of one specific factor in simultaneously accounting for the pre-industrial period of stagnation, the post-industrial period of balanced growth, as well as the Great Divergence: accessibility and/or affordability of fossil energy. We first use data on specific energy use for 134 countries over the time period 1960-2012 to derive two stylized facts: (i) countries with higher income derive a relatively larger share of their energy from fossil energy and (ii) countries that use a relatively larger share of fossil energy are also growing at higher rates.

We then set up an endogenous-growth model with two types of countries: those that extract and sell fossil fuel, and those that buy and use fossil fuel as an input to produce final goods. With this model, we derive three different steady states. First, abstracting from heterogeneity we show that if the stock of fossil fuel in efficiency units is zero, then there exists a stagnant steady state where final output is exclusively produced with the pre-industrial energy input.

Second, if some parameter restrictions are satisfied, there exists balanced growth path where growth is constant and output is exclusively produced with fossil energy.

Third, introducing heterogeneity in the distribution of initial technology levels, we show that the model can produce the Great Divergence. Specifically, with sufficiently large differences in initial technology levels, the model features a steady state in which a technologically advanced country experiences sustained growth and endogenously produces only with fossil energy, whereas the less advanced country instead endogenously chooses the pre-industrial energy input and does not grow. All three steady-states are consistent with the two stylized facts.

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A Appendix

A.1 The first order conditions for final and intermediate producers

Given that B and F are perfect substitutes, it is possible to get corner solutions where either $B = 0$ or where $F = 0$. The first-order conditions in the final-output sector yield

$$\begin{aligned} p_G &= (Y/G)^{\frac{1}{\sigma}}, & (41)\\ \kappa &\geq \left(\frac{Y}{B+F}\right)^{\frac{1}{\sigma}}, & (42)\\ p_F &\geq \left(\frac{Y}{B+F}\right)^{\frac{1}{\sigma}}. & (43)\end{aligned}$$

where the inequalities are equalities whenever strictly positive amounts of the corresponding inputs are used.

The first-order conditions for the competitive producers of G and F give the following inverse demand functions

$$\begin{aligned}
W &= p_G(1-\alpha)\frac{G}{N}, && (44)\\
p_i^G &= p_F N^{1-\alpha}\alpha A_i^G \left(x_i^G\right)^{\alpha-1}, && (45)\\
\kappa &= p_F(1-\gamma)\frac{F}{L_F}, && (46)\\
p_i^F &= p_F\gamma L_F^{1-\gamma} A_i^F \left(x_i^F\right)^{\gamma-1}. && (47)
\end{aligned}$$

A.2 Derivation of equation (12)

Maximizing π_i^F with respect to x_i^F delivers the profit-maximizing quantity

$$x_i^F = \left(\gamma^2 \frac{p_F}{p_E}\right)^{\frac{1}{1-\gamma}} L_F. \tag{48}$$

The total amount of fossil fuel required to produce all the machines is then the integral over all varieties, i.e.,

$$E = \int_0^1 A_i^F x_i^F di = \left(\gamma^2 \frac{p_F}{p_E}\right)^{\frac{1}{1-\gamma}} A^F L_F. \tag{49}$$

Using (49), in (48) gives (12).

A.3 Equilibrium research

The equilibrium innovation probabilities maximize the net expected value of the research investment. The equilibrium condition for innovation in sector $j \in \{F, G\}$ is

$$\frac{d\Theta_t^j}{d\mu_t^j} = \frac{\bar{\pi}_{t+1}^j}{r_{t+1}}. \tag{50}$$

In order to derive the equilibrium condition for research in sector G we start by computing the profit generated by a patent on a machine with productivity $\bar{A}_{t+1}^G$. Combining (8), (10), (11) and (14) gives

$$\bar{\pi}_{t+1}^G = \frac{1-\alpha}{\alpha} r_{t+1} k_{t+1} \bar{A}_{t+1}^G.$$

Substituting this and the research cost (18) into (50) and rewriting, we arrive at (20).

Similarly, the profit generated by a patent on a machine in sector F with productivity $\bar{A}^F_{t+1}$ is given by (8). Combining this expression with (48) and (49) gives

$$\bar{\pi}^F_{t+1} = (1-\gamma)\gamma L^{1-\gamma}_{F,t+1} p_{F,t+1} \frac{\bar{A}^F_{t+1} E^{\gamma}_{t+1}}{\left(A^F_{t+1}\right)^{\gamma}}.$$

Substituting the above expression into the research arbitrage condition (50), and using the research cost (19) provides an expression for the equilibrium innovation probability. Rewriting this using (11) and (14) delivers (21).

A.4 Proof of Proposition 1

By combining (4), (11), (14), (20) and (41) we can characterize the equilibrium when only the pre-industrial energy input is used by the following equations

$$\begin{aligned}
Y_t &= [(A^G_t k^{\alpha}_t)^{\frac{\sigma-1}{\sigma}} + 1]^{\frac{\sigma}{\sigma-1}}, && (51)\\
r_t &= \alpha^2 \left[1 + \left(\frac{1}{A^G_t k^{\alpha}_t}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} \left(\frac{1}{k_t}\right)^{1-\alpha}, && (52)\\
\mu^G_t &\geq \frac{1-\alpha}{\nu_1 \alpha} \frac{k^{1-\alpha}_t}{\left[1 + \left(\frac{1}{A^G_t k^{\alpha}_t}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}} \frac{k_{t+1}}{k_t} \frac{\bar{A}^G_{t+1}}{\bar{A}^G_t} - \frac{\eta_1}{\nu_1}, && (53)
\end{aligned}$$

where (53) holds with equality whenever there is positive investment in research. Furthermore, the equilibrium must fulfill the Euler equation (25). The production function (51) implies that $Y_t \leq 1$ for all t which, in turn, implies that also $C_t \leq 1$ and $K_{t+1} \leq 1$ for all t. We will first prove the existence of steady states and then rule out that Y_t asymptotically will go to either zero or one.

In steady state, $k_{t+1} = k_t$ and $\bar{A}^G_{t+1} = \bar{A}^G_t$. Furthermore, the Euler equation (25) says that in steady state $r_t = \frac{1}{\beta}$. Substituting this and (52) in (53) yields

$$\mu^G_t \geq \left[\frac{\alpha(1-\alpha)\beta}{1 + (A^G_t k^{\alpha}_t)^{\frac{1-\sigma}{\sigma}}} - \eta_1\right] \frac{1}{\nu_1}.$$

The right-hand side is decreasing in $A^G_t k^{\alpha}_t$ and negative if $A^G_t k^{\alpha}_t$ is large. If $\alpha(1-\alpha)\beta > \eta_1$, then the right-hand side will be positive for small values of $A^G_t k^{\alpha}_t$. A combination of A^G and k thus constitute a steady state if it fulfills (52) with $r = \frac{1}{\beta}$ and if $A^G k^{\alpha}$ is sufficiently large. The right-hand side of (52) depends negatively on both A^G and k. A larger steady state value of A^G must thus be associated with a lower value of k. However, the difference in k has to be sufficiently small so that $A^G k^\alpha$ is still larger for the larger value of A^G. Hence $A^G k^\alpha$ and, consequently, Y are larger in steady states associated with higher values of A^G. Furthermore, if $\alpha(1-\alpha)\beta > \eta_1$ there is a cutoff value of Y such that there are no steady states associated with Y-values lower than this cutoff. Since K must be below one, this cutoff value in terms of Y can be associated with a cutoff value in terms of A^G such that there can be no steady state with A^G below this cutoff value. The derived parameter condition is identical to the necessary condition for the existence of a balanced growth path (see Proposition 2). This concludes the proof of existence of steady states.

We will now rule out that Y asymptotically goes to either zero or one. For Y to go to zero, $A^G k^\alpha$ must go to zero. Furthermore, since K must go to zero while A^G is non-decreasing, k must go to zero. Equation (52) then implies that the interest rate must go to infinity. From the Euler equation it then follows that consumption should grow, which is not possible since $C \leq Y$. Consequently C must go to zero. It is thus not possible that production asymptotically goes to zero.

Consider instead the case where Y asymptotically goes to one. Since K is bounded from above by one, and $A^G k$ must go to infinity, this requires that A^G goes to infinity and consequently that the innovation probability remains positive. Solving equation (52) for $k_t^{1-\alpha}$ and substituting the resulting expression in (53) yields

$$\mu_t^G \geq \frac{\alpha(1-\alpha)}{\nu_1}\frac{1}{r_t}\frac{1}{1+\left(A_t^G k_t^\alpha\right)^{\frac{1-\sigma}{\sigma}}}\frac{k_{t+1}}{k_t}\frac{\bar{A}_{t+1}^G}{\bar{A}_t^G} - \frac{\eta_1}{\nu_1}.$$

Both ratios $\frac{k_{t+1}}{k_t}$ and $\frac{\bar{A}_{t+1}^G}{\bar{A}_t^G}$ are bounded. For the probability to remain positive as $A_t^G k_t^\alpha$ goes to infinity, the interest rate must go to zero. The Euler equation then implies that consumption must go to zero at an accelerating speed. While we cannot rule out such an equilibrium, it is straightforward to show that such an equilibrium would, at some point, end up in a situation where a coordinated effort in one period would take the economy to a steady state where consumption is higher in that and all subsequent periods compared to remaining on the path where Y goes to one and C goes to zero. In particular, as Y goes to one, A^G goes to infinity and the required capital stock in the steady state associated with that A^G goes to zero. The associated steady-state level of consumption, thus, goes to one. We therefore conclude that such a potential equilibrium does not seem relevant in practice.

A.5 Proof of Proposition 2

We are looking for conditions under which equations (29), (31) and (32) have a solution that implies innovation probabilities between 0 and 1. This corresponds to growth rates g_{A^G} and g_{A^F} between 0 and 1. From (27) and (28) it follows that we must find a solution with an r that fulfills

$$r \in \left[\frac{1}{\beta}, \frac{2}{\beta}\right] \cup \left[\left(\frac{1}{\beta}\right)^{1-\gamma}, \left(\frac{2}{\beta}\right)^{1-\gamma}\right] = \left[\frac{1}{\beta}, \left(\frac{2}{\beta}\right)^{1-\gamma}\right]. \tag{54}$$

This intervall is nonempty if and only if condition (30) is fulfilled. To simplify the notation we define

$$z \equiv \left((1-\beta)^{\gamma}\frac{\tilde{y}}{k^{\alpha}}\right)^{\frac{\sigma-1}{\sigma}}.$$

For any given value of $z > 0$, this definition provides a one-to-one relationship between k and $\tilde{y}$. The equilibrium conditions can therefore be expressed in terms of r, k and z. A unique solution with an r that fulfills (54) corresponds to a unique balanced growth path. Substituting z in (29), (31) and (32) delivers the following system of equations.

$$\begin{aligned} r &= \alpha^2 \left[1+z\right]^{\frac{1}{\sigma-1}} \frac{1}{k^{1-\alpha}} \\ \beta r - 1 &= \frac{\alpha(1-\alpha)\beta}{\nu_1}\frac{1}{1+z} - \frac{\eta_1}{\nu_1} \\ \beta r^{\frac{1}{1-\gamma}} - 1 &= \frac{\beta\gamma(1-\gamma)}{\nu_2}\frac{z}{1+z} - \frac{\eta_2}{\nu_2}. \end{aligned}$$

The last two equations only contain r and z. The second equation gives a negative relationship between the variables while the third equation gives a positive relationship. This sub-system can thus have at most one solution. Given such a solution (providing values of r and z) the first equation gives a unique value of k that, in turn, implies a unique value of $\tilde{y}$. The system in k, r and z can thus have at most one solution and if the sub-system consisting of the second and third equations has a solution with a value of r that fulfills (54) and a positive value of z, then there is a unique balanced growth path.

The second and third equations can be rewritten as

$$\begin{aligned} z &= \frac{\alpha(1-\alpha)\beta}{(\beta r-1)\,\nu_1+\eta_1}-1 \equiv z_1(r) \\ z &= \frac{\beta\gamma(1-\gamma)}{\beta\gamma(1-\gamma)-\left[\nu_1\left(\beta r^{\frac{1}{1-\gamma}}-1\right)+\eta_2\right]}-1 \equiv z_2(r). \end{aligned}$$

The function z_1 is decreasing in r. If $\eta_1 \geq \alpha(1-\alpha)\beta$, it is (weakly) negative for all $r \geq \frac{1}{\beta}$ and there is no solution with positive z. Hence, $\eta_1 \leq \alpha(1-\alpha)\beta$ is a necessary condition. As long as this is fulfilled, z_1 is positive for r sufficiently close to $\frac{1}{\beta}$.

As long as the denominator of the ratio of z_2 is positive, z_2 is positive and increasing in r. If the denominator is negative for some r it will be negative also for all larger values of r and when the denominator is negative, so is z_2. When $r = \left(\frac{2}{\beta}\right)^{1-\gamma}$, $\beta r^{\frac{1}{1-\gamma}} = 2$. The denominator will then be positive if $\eta_2+\nu_2 < \beta\gamma(1-\gamma)$. Under this assumption, the denominator will thus be positive for all relevant r and there is a balanced growth path if and only if

$$z_1\left(\frac{1}{\beta}\right) \geq z_2\left(\frac{1}{\beta}\right) \text{ and } z_1\left(\left(\frac{2}{\beta}\right)^{1-\gamma}\right) \leq z_1\left(\left(\frac{2}{\beta}\right)^{1-\gamma}\right). \tag{55}$$

When $r = \frac{1}{\beta}$, $\beta r^{\frac{1}{1-\gamma}} = \beta^{-\frac{\gamma}{1-\gamma}} \in (1,2]$. If the denominator is negative for this value of r, z_2 will be negative for all relevant r and there is no solution. The denominator is (weakly) negative for $r = \frac{1}{\beta}$ if

$$\left(\beta^{-\frac{\gamma}{1-\gamma}}-1\right)\nu_2+\eta_2 \geq \beta\gamma(1-\gamma).$$

In the case

$$\left(\beta^{-\frac{\gamma}{1-\gamma}}-1\right)\nu_2+\eta_2 < \beta\gamma(1-\gamma) \leq \nu_2+\eta_2$$

z_2 will be positive for $r = \frac{1}{\beta}$ and then diverge to plus infinity for some $r \in \left(\frac{1}{\beta}, \left(\frac{2}{\beta}\right)^{1-\gamma}\right]$. This implies that there will be a solution if $z_1\left(\frac{1}{\beta}\right) \geq z_2\left(\frac{1}{\beta}\right)$. This covers the last possible case. Remains to express the inequalities (55) in terms of parameters. This requires the following expressions

$$\begin{aligned}
z_1\left(\frac{1}{\beta}\right) &= \frac{\alpha(1-\alpha)\beta}{\eta_1} - 1 \\
z_2\left(\frac{1}{\beta}\right) &= \frac{\beta\gamma(1-\gamma)}{\beta\gamma(1-\gamma) - \nu_2\left(\beta^{-\frac{\gamma}{1-\gamma}} - 1\right) - \eta_2} - 1 \\
z_1\left(\left(\frac{2}{\beta}\right)^{1-\gamma}\right) &= \frac{\alpha(1-\alpha)\beta}{\left(\beta^{\gamma}2^{1-\gamma} - 1\right)\nu_1 + \eta_1} - 1 \\
z_2\left(\left(\frac{2}{\beta}\right)^{1-\gamma}\right) &= \frac{\beta\gamma(1-\gamma)}{\beta\gamma(1-\gamma) - \nu_2 - \eta_2} - 1.
\end{aligned}$$

Using these function values, the inequalities (55) can be rewritten as[35]

$$z_1\left(\frac{1}{\beta}\right) \geq z_2\left(\frac{1}{\beta}\right) \Leftrightarrow \frac{\eta_1}{\alpha(1-\alpha)\beta} \leq 1 - \frac{\left(\beta^{-\frac{\gamma}{1-\gamma}} - 1\right)\nu_2 + \eta_2}{\beta\gamma(1-\gamma)}$$

and

$$z_1\left(\left(\frac{2}{\beta}\right)^{1-\gamma}\right) \leq z_1\left(\left(\frac{2}{\beta}\right)^{1-\gamma}\right) \Leftrightarrow \frac{\left(\beta^{\gamma}2^{1-\gamma} - 1\right)\nu_1 + \eta_1}{\alpha(1-\alpha)\beta} \geq 1 - \frac{\nu_2 + \eta_2}{\beta\gamma(1-\gamma)}.$$

A.6 Proof of Proposition 3

First, combine (4), (16), (43), and the fact that $L_B = 1 - L_F$ to get an expression for p_F. This expression is a measure of the marginal product of good F. Second, combine (12), (16) and (49) to arrive at another expression for p_F. This expression is a measure of the marginal cost of good F.

Equating the two expressions for p_F delivers the following implicit expression for the equilibrium value of L_F in the hybrid regime:

$$\left[\left(\frac{(1-\gamma)\,A^G k^{\alpha}}{1-\gamma+\gamma L_F}\right)^{\frac{\sigma-1}{\sigma}} + 1\right]^{\frac{1}{\sigma-1}} = \frac{1}{\gamma^2\,(1-\gamma)^{\frac{1-\gamma}{\gamma}}}\frac{p_E}{\left(A^F\right)^{\frac{1-\gamma}{\gamma}}} \qquad (56)$$

The left hand side of (56) is strictly decreasing in L_F. Hence, a country will only produce with the pre-industrial energy input if the right-hand side is larger than the left-hand side, even though $L_F = 0$. Setting $L_F = 0$ in (56) gives the first inequality in Proposition 3.

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[35] The rewriting requires certain expressions to be positive, which they will be in the cases when the inequalities are relevant.

A country will only produce with fossil energy if the right hand side is less than the left hand side, even though $L_F = 1$. Setting $L_F = 1$ in (56) yields the second inequality in the Proposition.

A.7 Proof of proposition 4

We have assumed that the less developed country produces solely using the pre-industrial energy input, implying that technology factor A^F_{S,t_0} fulfills inequality (35). Individual agents cannot invest enough in research to improve the economy-wide state of the fossil-fuel-intensive technology. If A^F_S, A^G_S and k_S do not change while the fossil-fuel price p_E increases, inequality (35) will hold also in subsequent periods and fossil-fuel use in the stagnant country will be zero. The profit associated with a patent in that country will then be zero and no agent will invest in research. Remaining in the pre-industrial steady-state is thus an equilibrium.

A.8 Proof of proposition 5

Consider inequality (35). By assumption this is fulfilled in period t_0. Assume that it is also fulfilled in period $t{>}t_0$. Since the expression on the left-hand side is smaller than one when $\sigma < 1$, it will be fulfilled in period $t+1$ as well if

$$A^F_{S,t+1} \leq \left(\frac{p_{E,t+1}}{\gamma^2 (1-\gamma)^{\frac{1-\gamma}{\gamma}}} \right)^{\frac{\gamma}{1-\gamma}} = \frac{p_{E,t+1}^{\frac{\gamma}{1-\gamma}}}{\gamma^{\frac{2\gamma}{1-\gamma}} (1-\gamma)} \equiv \tilde{A}^F_{S,t+1}.$$

If, in period t there has not been any innovation in sector F since period t_0, the technology factor is $A^F_{S,t} = A^F_{S,t_0}$. From (22) we have that in order to get to $A^F_{S,t+1} = \tilde{A}^F_{S,t+1}$ the required innovation probability is

$$\tilde{\mu}^F_{S,t} = \frac{\tilde{A}^F_{S,t+1} - A^F_{S,t_0}}{\bar{A}^F_{t+1} - A^F_{S,t_0}}.$$

Substituting this probability into (19) delivers the the associated research cost:

$$\tilde{\Theta}^F_{S,t} = \left[\eta_2 \frac{\tilde{A}^F_{S,t+1} - A^F_{S,t_0}}{\bar{A}^F_{t+1} - A^F_{S,t_0}} + \nu_2 \left(\frac{\tilde{A}^F_{S,t+1} - A^F_{S,t_0}}{\bar{A}^F_{t+1} - A^F_{S,t_0}} \right)^2 \right] Y_{S,t} \frac{\bar{A}^F_t}{A^F_{S,t_0}},$$

where $Y_{S,t}$ is production in the stagnant country. The limit as A^F_{S,t_0} goes to zero is

$$\lim_{A^F_{S,t_0}\to 0} \tilde{\Theta}^F_{S,t} \equiv \lim_{A^F_{S,t_0}\to 0} \left[\eta_2 + \nu_2 \frac{\tilde{A}^F_{S,t+1}}{\bar{A}^F_{t+1}}\right] \frac{\bar{A}^F_t}{\bar{A}^F_{t+1}} \frac{\tilde{A}^F_{S,t+1}}{A^F_{S,t_0}} Y_{S,t} = \infty.$$

The expression within the bracket is strictly larger than 0; the first ratio outside the bracket lies between $\frac{1}{2}$ and 1; and the second ratio diverges to infinity. Furthermore, the numerator of the last ratio increases over time since the price of fossil fuel increases. The research cost required for the stagnant country to start using fossil fuel can thus be prohibitively large.

A.9 Derivations of equations (37) and (38)

Implicit differentiation of equation (56) gives the derivatives $\partial L_B/\partial G$ and $\partial L_B/\partial A^F$. Implicit differentiation of (15) and (43) then gives the derivatives $\partial E/\partial G$ and $\partial E/\partial A^F$. Combining these expressions then gives equation (37) and (38).

A.10 Proof of Proposition 6

Assume that fossil energy is discovered at time T. Output in the hybrid regime is given by

$$Y^h_T = \frac{A^G_T k^\alpha_T}{1-\gamma} \left[\left(\frac{1-\gamma}{1-\gamma+\overline{R}_T\theta} \right)^{\frac{\sigma-1}{\sigma}} + \left(\frac{1}{A^G_T k^\alpha_T} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \left(1-\gamma+\gamma\overline{R}_T\theta\right),$$

where $\overline{R}_T \equiv \left(A^F_T\right)^{\frac{1-\gamma}{\gamma}} R_T$, and $\theta \equiv (1-\beta)(1-\gamma)^{\frac{1}{\gamma}}\gamma$. Assume then that $\mu^F_{T+s} = 0$ $\forall s \geq 0$, which implies that $A^F_{T+s} = A^F_T$, $\forall s \geq 0$. Without R&D investments to improve A^F, it follows from (3) that $\overline{R}$ is growing at the rate $\beta < 1$. Aggregate income then converges asymptotically to

$$\lim_{s\to\infty} Y^h_{T+s} = \left[\left(A^G_{T+s} k^\alpha_{T+s}\right)^{\frac{\sigma-1}{\sigma}} + 1 \right]^{\frac{\sigma}{\sigma-1}}.$$

Hence, final output is asymptotically produced exclusively in the pre-industrial sector. If A^G has been growing for a number of periods, $A^G_{T+s} > A^G_{T-1}$. However, since the expressions for aggregate income and prices in the hybrid regime are asymptotically the same as in the land-based regime, Proposition 1 implies that sustained growth is not possible in the hybrid regime when $\mu^F_{T+s} = 0$ $\forall s \geq 0$. It remains to show under what conditions μ^F_{T+s} is zero for all $s \geq 0$.

Since R is assumed to be relatively small, we may assume that we are in the hybrid (rather than the fossil) regime. The composite energy input can then be computed by combining (6) and (16):

$$B_t + F_t = 1 + \frac{\gamma}{1-\gamma} L_{F,t}. \tag{57}$$

Conbining (12), (16), the fact that $E_t = R_t (1-\beta)$, and the definition of $\overline{R}_t$ yields

$$\frac{1}{1-\gamma} L_{F,t}^{\gamma} = \overline{R}_t^{\gamma} (1-\beta)^{\gamma}. \tag{58}$$

Let $\tilde{\mu}_t^F$ denote the right-hand side of (21), i.e. $\mu_t^F = \tilde{\mu}_t^F$ whenever $\tilde{\mu}_t^F$ is positive, otherwise $\mu_t^F = 0$. Using (57) and (58) in (21) yields

$$\tilde{\mu}_t^F = \frac{\gamma}{\nu_2 \alpha^2} L_{F,t+1}^{1+\gamma} k_{t+1}^{1-\alpha} \frac{\left(\frac{A_{t+1}^G k_{t+1}^{\alpha}}{1+\frac{\gamma}{1-\gamma} L_{F,t+1}}\right)^{\frac{1}{\sigma}}}{\left[\left(A_t^G k_t^{\alpha}\right)^{\frac{\sigma-1}{\sigma}} + \left(1+\frac{\gamma}{1-\gamma} L_{F,t}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}} \frac{A_t^F}{A_{t+1}^F} \frac{\bar{A}_{t+1}^F}{\bar{A}_t^F} - \frac{\eta_2}{\nu_2}.$$

For all t, $L_{F,t} \in [0,1]$. This impiles

$$\tilde{\mu}_t^F \leq \frac{\gamma}{\nu_2 \alpha^2} L_{F,t+1}^{1+\gamma} k_{t+1}^{1-\alpha} \frac{\left(A_{t+1}^G k_{t+1}^{\alpha}\right)^{\frac{1}{\sigma}}}{\left[\left(A_t^G k_t^{\alpha}\right)^{\frac{\sigma-1}{\sigma}} + 1\right]^{\frac{\sigma}{\sigma-1}}} \frac{A_t^F}{A_{t+1}^F} \frac{\bar{A}_{t+1}^F}{\bar{A}_t^F} - \frac{\eta_2}{\nu_2}.$$

Under the assumption $\mu_t^F = 0$ for all t, $A_t^F = A_{t+1}^F$ and $\bar{A}_t^F = \bar{A}_{t+1}^F$. Hence,

$$\tilde{\mu}_t^F \leq \frac{\gamma}{\nu_2 \alpha^2} L_{F,t+1}^{1+\gamma} k_{t+1}^{1-\alpha} \frac{\left(A_{t+1}^G k_{t+1}^{\alpha}\right)^{\frac{1}{\sigma}}}{\left[\left(A_t^G k_t^{\alpha}\right)^{\frac{\sigma-1}{\sigma}} + 1\right]^{\frac{\sigma}{\sigma-1}}} - \frac{\eta_2}{\nu_2}.$$

Proposition 1 says that k_t and $A_t^G k_t^{\alpha}$ are both bounded from above and that $A_t^G k_t^{\alpha}$ is bounded from below by some number strictly larger than zero. We can thus conclude that for each t, $\tilde{\mu}_t^F \leq \Gamma L_{F,t+1}^{1+\gamma} - \frac{\eta_2}{\nu_2}$, for some bounded Γ. Equation (58) tells us that $L_{F,t}$ is increasing in $\bar{R}_t$ and that it goes to zero as $\bar{R}_t$ goes to zero. Furthermore, $\bar{R}_t$ decreases towards 0 as t increases without innovation in the F-sector. The conclusion is that as long as $\bar{R}_T$ is small enough so that $L_{F,T+1}^{1+\gamma} \leq \frac{1}{\Gamma}\frac{\eta_2}{\nu_2}$, there will be no innovation in the F-sector and there will not be any long-run growth.

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