Online Appendix for

"Risks in macroeconomic fundamentals and excess bond returns predictability"

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A. Controlling for the information in x_t

This appendix presents results on the predictive power of SMRF when controling directly for the information in x_t. In order to guard against the possibility of overfitting in out-of-sample forecasting the information in predictors x_t is summarized by estimating predictors factors, Fx, and a single predictors factor, SFx, by applying PCA to the $T \times 9$ panel of original predictors formed by the six consensus forecasts $z_t^{SPF,h}$, MCEI, 5yTS and BaaCS. Bai and Ng (2002) criteria indicate that this panel is well described by seven factors, from which three (first, third and fifth principal component) were formally chosen using SBIC. These three factors form the vector Fx. Factor SFx is a linear combination of Fx. SFx provides a variable that can directly be used to control for the information in the original predictors, but also guards against the possibility of overfitting in out-of-sample forecasting. The in-sample and out-sample results of this exercise are provided by tables A.1 and A.2. The in-sample results show that, although SFx shows high predictive power for future excess returns, adding SMRF to regressions increases R^2s substantially to levels almost identical to the ones shown by Table 2. In addition, statistical significance of SFx also shifts to SMRF. This result indicates that IQR and IQS add substantial amount of information about the bond risk premia. Out-of-sample evidence are even stronger. As shown by Table A.2, SFx regressions generate very inaccurate forecasts. When including SMRF as an additional predictor, however, we observe dramatic improvements in terms of forecasting accuracy, with R^2_{oos}s in the range of 0.253 to 0.453. These results show that the high predictability found is, to a large extent, due to the extra information obtained from the estimation of Med, IQR and IQS.

B. Alternative estimation procedures for IQR and IQS

The baseline quantile regression estimation used in the paper relies on the modified Barrodale and Roberts algorithm for L_1 (or Least Absolute Deviation) regressions described in Koenker and d'Orey (1987, 1994) in conjunction with the monotone rearrangement procedure of Chernozhukov, Fernandez-Val and Galichon (2010). It is well known in the literature of quantile estimation that regression quantiles can show instabilities at tails due to paucity and sparsity of data (He, 1997; Wang, Li and He, 2012). For this reason I present results for two alternative estimation procedures:

(i) $q_{z_{t,t+4}}(\tau)$ is estimated for $\tau = 0.10$ using the baseline approach. Using $\tau = 0.10$ places less weight on extreme data points and assures more robust estimated quantile regressions;

(ii) $q_{z_{t,t+4}}(\tau)$ is estimated for $\tau = 0.05$ using the approach proposed by Wang, Li and He (2012) (WLH hereafter) which integrates quantile regression and Extreme Value Theory and is suitable for quantile curves at tails. This procedure is explained below.

Wang, Li and He (2012) procedure explained

The estimation is performed without assuming common slopes for $q_{z_{t,t+4}}(\tau)$. I focus here on the procedure for the estimation of conditional high quantiles since a low quantile of $z_{t,t+4}$ can be viewed as a high quantile of $-z_{t,t+4}$. First, I define a sequence $\tau_j = j/(T+1)$, $j = T-b, ..., v$, where $v = T - [T^a]$ with $[T^a]$ denoting the integer part of T^a, $T = 169$ is the sample size and $a > 0$ and $b > a$ are constants fixed as $a = 0.1$, as suggested by Wang, Li and He (2012), and $b = 25$. For each $j = T-b, ..., v$ I estimate $\beta(\tau_j)$ in (6) following the baseline approach. Then, for each $t = 1,...,T$ I define $\widehat{q_j} = \hat{\beta}(\tau_j)' x_t$ and estimate the parameter γ as

$$\widehat{\gamma} = \frac{1}{b - [T^a]} \sum_{j=[T^a]}^{k} log \frac{\widehat{q_{T-j}}}{\widehat{q_{T-b}}} \tag{1}$$

The robust quantile function $q_{z_{t,t+4}}(\tau_s)^\star$ is then obtained as $\widehat{q_{z_{t,t+4}}}(\tau_s)^\star = \left(\frac{1-\tau_{T-b}}{1-\tau_s}\right)^{\widehat{\gamma}} \widehat{q}_{T-b}$, where τ_s is the percentile of interest. In our case, $\tau_s = 0.05$.

Results using alternative estimation procedures

I show results for regressions with the following specifications: (i) MRF, (ii) SMRF, (iii) SMRF and CP, (iv) SMRF and LN and (v) SMRF, CP and LN. I only provide results with asymptotic inference. Tables A.3 and A.4 show results for the baseline approach and percentile equal to 0.10. The optimal vector of macro risk factors selected by SBIC includes the fifth factor now, not the sixth. Statistical significance of MRF and SMRF and R^2s remain very high for all bond maturities and results are comparable to the ones shown in tables 3 and 4. Tables A.5 and A.6 show results for the WLH approach. The SBIC now selects four factors as predictors: MRF1, MRF4, MRF5 and MRF8. Notice that, with the exception of MRF8, all factors are highly statistically significant. The same is true for Wald statistics. The predictive power of MRF and SMRF remains very high with R^2s ranging from 0.17 to 0.30. When comparing the predictive power of SMRF against LN and CP, results are a bit weaker than the ones shown in Table 4, but still highly significant estimates and high R^2s are found, especially from the 5-year maturity. These results corroborate my previous findings.

C. An affine term structure model with macro risk factors

Motivated by the findings that bond risk premia are driven by risks in macroeconomic fundamentals, this appendix examines time variation in term premia. From a monetary policy perspective, understanding time variation in term premia is extremely important as term premia obfuscate the relationship between short-term interest rates that are controlled by central banks and longer-term interest rates, while it also makes it difficult to measure expectations of future short-term rates using the yield curve. I do so by estimating term premia using a Gaussian affine term structure model along the lines of Joslin, Priebsch and Singleton (2014) (JPS hereafter), where state variables are composed by the first three principal components of yields and the three macro risk factors, MRF_1, MRF_4 and MRF_6. I treat the macro risk factors as unspanned in the state vector (Duffee, 2011; Joslin, Priebsch and Singleton, 2014), as I have found that they explain variation in risk premia, but are irrelevant for explaining the cross-sectional variation in the current yields.

Model specification

Following the macro-finance literature since Ang and Piazzesi (2003), I assume that the $p \times 1$ vector of state variables X_t follows a VAR(1) process under the objective probability measure $\mathbb{P}$,

$$X_{t+1} = \mu + \Phi X_t + \Sigma \varepsilon_{t+1} \tag{2}$$

where $\varepsilon_t \sim iid\,N(0, I_p)$, the state vector consists of the first three principal components of yields and MRF_1, MRF_4 and MRF_6, and Σ is an $p \times p$ lower triangular matrix. The pricing kernel is assumed to be conditionally lognormal

$$M_{t+1} = exp\left(-r_t - \frac{1}{2}\lambda_t^{'}\lambda_t - \lambda_t^{'}\varepsilon_{t+1}\right) \tag{3}$$

where $r_t = \delta_0 + \delta_1^{'} X_t$ is the three-month interest rate and the $p \times 1$ vector of risk prices is affine in state variables, $\lambda_t = \lambda_0 + \lambda_1 X_t$. Under the risk-neutral measure $\mathbb{Q}$ the state vector follows the dynamics,

$$X_{t+1} = \mu^{\mathbb{Q}} + \Phi^{\mathbb{Q}} X_t + \Sigma\varepsilon_{t+1} \tag{4}$$

where $\mu^{\mathbb{Q}} = \mu - \Sigma\lambda_0$ and $\Phi^{\mathbb{Q}} = \Phi - \Sigma\lambda_1$.

It then follows that under no-arbitrage bond prices are exponential affine functions of the state variables, $P_t^n = exp\left(A_n + B_n^{'} X_t\right)$, where A_n is a scalar and B_n is an $p \times 1$ vector that satisfy the recursions

$$\begin{aligned} A_{n+1} &= -\delta_0 + A_n + B_n^{'}\mu^{\mathbb{Q}} + \tfrac{1}{2}B_n^{'}\Sigma\Sigma^{'}B_n \\ B_{n+1} &= \Phi^{\mathbb{Q}'}B_n - \delta_1 \end{aligned} \tag{5}$$

which start from $A_1 = -\delta_0$ and $B_1 = -\delta_1$.

I treat the macro risk factors as unspanned factors. To illustrate, partition X_t as $(X_{1t}^{'}, X_{2t}^{'})^{'}$, where X_{1t} and X_{2t} are $p_1 \times 1$ and $p_2 \times 1$ vectors consisting of the first three principal components of yields and MRF_1, MRF_4 and MRF_6, respectively. Set also the last p_2 elements of δ_1 and the upper-right $p_1 \times p_2$ block of $\Phi^{\mathbb{Q}}$ to be equal to zero. Then the last p_2 elements of B_n will be equal to zero and bond prices reduce to

$$P_t^n = exp\left(A_n + B_{1n}^{'} X_{1t}\right) \tag{6}$$

where B_{1n} consists of the first p_1 elements of B_n. The result of this is that factors in X_{2t} are important for forecasting future yields, but only factors in X_{1t} are important for pricing bonds at time t. Model implied yields are then computed as $y_t^n = -n^{-1} log\, P_t^n = -n^{-1}(A_n + B_{1n}^{'} X_{1t})$.

Estimation

The estimation approach follows JPS and Joslin, Singleton and Zhu (2011) with parameters being estimated by MLE. Due to the separation result of the likelihood function derived in Joslin, Singleton and Zhu (2011) parameters in μ and Φ are estimated separately from those governing the risk neutral pricing of bonds, which can be done by a simple OLS. For estimating the remaining identified parameters, i.e. Σ, $\mu_1^{\mathbb{Q}}$, $\Phi_{11}^{\mathbb{Q}}$, δ_0 and $\delta_{1,1}$, where $\mu_1^{\mathbb{Q}}$ and $\delta_{1,1}$ are the $p_1 \times 1$ vectors of $\mu^{\mathbb{Q}}$ and δ_1, and $\Phi_{11}^{\mathbb{Q}}$ is the upper-left $p_1 \times p_1$ block of $\Phi^{\mathbb{Q}}$, it is assumed that observed yields are equal to the model-implied yields plus i.i.d. Gaussian measurement errors. As in JPS, the model is first reparameterized in terms of a $p_1 \times 1$ latent state vector $\mathscr{S}$ that follows a VAR with zero intercept and diagonal slope coefficient matrix equal to $I_1 + \Lambda^{\mathbb{Q}}$, and an equation for the short-rate that assumes the form $r_t = r_\infty^{\mathbb{Q}} + 1\mathscr{S}_t$, where 1 is a line vector of ones. The likelihood is then maximized with respect to these parameters and original parameters' estimates can be retrieved from $r_\infty^{\mathbb{Q}}$ and $\Lambda^{\mathbb{Q}}$ as in JPS.[1] In the estimation I use the three-month interest rate and yields from one to ten years. In order to correct for the possibility of existence of small-sample bias in VAR parameters (Duffee and Stanton, 2004; Kim and Wright, 2005; Kim and Orphanides, 2005) I use the bootstrap approach following Bauer, Rudebusch and Wu (2012).[2]

The model shows a good fit. Fitting errors measured in terms of MSE are small and equal to 0.0039, indicating that the first three principal components together are able to account for almot all the cross-sectional variation in yields and that no other factor is required for this purpose.

It is also worth computing a Wald statistic testing the hypothesis that macro risk factors do not enter matrix Φ in (15). The hypothesis is highly rejected with Wald statistic equal to 2065.9 and p-value virtually equal to zero, indicating that MRF_1, MRF_4 and MRF_6 do help to predict future interest rates, and motivating their inclusion in the state vector under the $\mathbb{P}$ measure.

Table A.7 shows the full set of parameter estimates for the affine model, including λ_0 and λ_1, which govern expected excess returns. Bootstraped standard errors are in parentheses.[3] The prices of level (PC1) and slope (PC2) risks have a significant negative constant component, implying that investors on average require positive expected excess returns for holding the level and slope portfolios. In addition to level and slope risks being nonzero unconditionally, I find that the level risk varies significantly as a function of MRF_4 and MRF_6. The loading on these factors have positive and negative coefficients, respectively, implying that shocks to MRF_4 (MRF_6) have a negative (positive) impact on risk premia. Another finding is that the slope carries a significant price of risk. The level factor, the slope factor itself, as well as the curvature (PC3) factor all significantly affect the price of slope risk over time. Coefficients on the level and curvature factors are positive, indicating that expected excess returns on the slope portfolio is decreasing in the level and curvature of yields. Contrary, the coefficient on the slope factor shows a negative coefficient.

Decomposing long term yields

Long term yields can be represented as the sum of future nominal short-rate expectations plus a term premium defined as the average of risk premia of declining maturities.[4] After estimating the affine model parameters, the n-year term premium can be computed as the difference between the n-year implied yield under $\mathbb{Q}$ and the average of expected short-rate up to year n under the $\mathbb{P}$ measure.

This decomposition is ilustrated by Figure A.1 for the 10-year yield. Two aspects are noteworthy. First, the long-term term premium implied by risks in macroeconomic fundamentals have a marked countercyclical behaviour showing declines during expansions and increases during recessions. Observe that increases have been more pronounced since the early 1990s, even though the 10-year yield has shown a decreasing pattern

[1] Appendix B in Joslin, Priebsch and Singleton (2010) specifies how to do this.

[2] I also test the indirect inference approach proposed by Bauer, Rudebusch and Wu (2012), which is supposed to remove higher-order bias. I found that the two methods deliver almost the same results in the particular case of this paper. I use the bootstrap approach due to its simplicity and faster computation.

[3] The method used to bootstrap standard errors is as follows. First I resample the OLS residuals in the state equation and randomly choose a starting value among the T observations to construct a bootstrap sample for state variables using the original state equation parameters. Then, using the maximum likelihood estimates of the parameters, I simulate a path of the term structure for the whole sample and estimate the model based on these simulated data. These steps are repeated 1000 times delivering empirical probability distributions for all parameters from which bootstrap standard deviations can be easily computed.

[4] y_t^n can be decomposed as $y_t^n = \frac{1}{n}E_t(r_t + r_{t+1} + ... + r_{t+n-1}) + \frac{1}{n}[E_t(rx_{t+1}^n) + E_t(rx_{t+2}^{n-1}) + ... + E_t(rx_{t+n-1}^2)]$.

since then. Second, these movements have been closely followed by decreases in short-rate expectations, indicating that the two components seem to move in opposite directions, in particular, during bad times. For instance, notice that while the term premium rose sharply during the the late 2000s recession, short-rate expectations declined abruptly to levels close to zero. The correlation between the two series is high and negative: -51%.

Consistent with previous findings concerning SMRF (the return risk premia), term premium is highly countercyclical. Figure A.1 - Panel B shows lead/lag relations between the term premium and growth rates for real GDP, industrial production and unemployment rates. Contemporaneous correlations with real GDP, industrial production and unemployment growth rates are -32%, -27% and 32%, respectively, and are highly statistically significant. In addition, cross-correlations turn negative and positive as macro variables are led/lagged, indicating that bond premia implied by risks in macroeconomic fundamentals are closely related to movements in the real economy. This result is suggested by theory (Campbell and Cochrane, 1999; Wachter, 2006; Bansal and Yaron, 2004; Rudebusch and Swanson, 2009).

Impulse response analysis

Duffee (2011) points out that factors whose impacts on term premium and short-rate expectations cancel each other may be considered unspanned, as shocks have no impact on current bond yields. As Figure A.1 suggests, term premium and short-rate expectations implied by expected macro risks move in opposite directions. It is then worth verifying how shocks to macro risk factors affect these two terms separately and, consequently, yields.

Figure A.2 shows impulse response functions (IRFs) for the term premium, short-rate expectations and the 10-year yield to one-standard deviation shocks in MRF_1, MRF_4 and MRF_6. Notice that shocks in all macro factors cause off-setting movements in the term premium and expected short-term interest rates, leaving current yields statistically unaffected. While shocks in MRF_1 and MRF_4 (MRF_6) drive term premium down (up), they bring the expectations component up (down). Following Duffee (2011), these results provide even stronger evidence that macro risk factors are indeed unspanned by the yield curve.

It is also worth interpreting the impulse response functions shown by Figure A.2. Notice that a positive shock in MRF_1 drives term premium down by about 10 basis points after which it gradually reverts back. This is consistent with the notion that higher expected economic activity leads to lower risk premium. A similar but stronger effect is observed for MRF_4 with term premium decreasing by about 33 basis points. A shock in MRF_6, on the other hand, causes an increase in term premium of about 20 basis points, which is consistent with the idea that higher upside inflation risks raise risk premium for long-term bonds as investors will demand a higher premium to compensate for inflation risk. The magnitude of the impacts over the expectations component are lower and mostly statistically insignificant.

D. Recursive R^2_{oos} and utility gains in real-time

In order to check the over time stability of the results shown in Table 9, Figure A.3 shows recursive R^2_{oos}s and utility gains computed against the constant model of no-predictability. Notice that R^2_{oos}s for SMRF regressions show high stability and statistical significance against the constant model over the full period of evaluation. The SMRF+CP specification performs quite well up to the early 2000's, from when its R^2_{oos}s decline substantially. Recursive utility gains obtained from SMRF regressions are a bit downward trended, but still quite high over the full period. R^2_{oos}s are particularly high in the beggining of the period, with utility gains achieving levels around 6% up to 1995. Results achieved by the portfolio of model SMRF+CP are more stable and, although lower, still positive over the full period. The most successful model in real-time is SMRF.

Table A.1: In-sample predictability - SMRF and SFx

Notes: This table shows the predictive power of SMRF and SFx. t-stats computed using Newey-West standard errors with six lags are reported in parentheses and $\overline{R}^2$ refers to the adjusted-R^2.

	$SMRF$	SFx	$\overline{R}^2$
rx^2		0.225	0.126
		(3.157)	–
	0.224	0.027	0.206
	(2.911)	(0.291)	–
rx^3		0.436	0.144
		(3.243)	–
	0.423	0.063	0.230
	(3.045)	(0.361)	–
rx^5		0.816	0.180
		(3.567)	–
	0.748	0.154	0.277
	(3.478)	(0.541)	–
rx^7		1.196	0.208
		(3.914)	–
	1.003	0.310	0.302
	(3.517)	(0.792)	–
rx^{10}		1.722	0.232
		(4.078)	–
	1.328	0.548	0.319
	(3.607)	(1.080)	–

Table A.2: Out-of-sample predictability - SMRF and SFx

Notes: Panel A shows R^2_{oos} statistics for predictor based models against a constant, with $R^2_{oos} > 0$ indicating outperformance of predictor based models. Panel B reports R^2_{oos} of models with ("unrestricted") and without ("restricted") SMRF with $R^2_{oos} > 0$ indicating outperformance of models augmented with SMRF ("unrestricted" models). In both panels, ($\star$), ($\star\star$), ($\star\star\star$) indicate statistical significance according to the MSPE-adjusted test of Clark and West (2007) at 10%, 5% and 1%, respectively. (†), (††), († † †) indicate statistical significance according to the MSE-F test of McCracken (2007) at 10%, 5% and 1%, respectively. The R^2_{oos} statistic is defined as

$$R^{2,j}_{oos} = 1 - \frac{\sum_{t=R}^{T}\left(rx^n_{t,t+4} - \widehat{rx}^{n,j}_{t,t+4}\right)^2}{\sum_{t=R}^{T}\left(rx^n_{t,t+4} - \widehat{rx}^{n,b}_{t,t+4}\right)^2},$$

where $\widehat{rx}^{n,j}_{t,t+4}$ is a forecast generated from model $j = SMRF, SFx, SMRF + SFx$ and $\widehat{rx}^{n,b}_{t,t+4}$ is the forecast generated from the benchmark, with $b = constant, SFx$.

	Panel A - against constant				Panel B - against "restricted" model	
	1990Q1-2011Q4		1990Q1-2007Q4		1990Q1-2011Q4	1990Q1-2007Q4
	$SMRF$	SFx	$SMRF$	SFx	$SMRF + SFx\,vs\,SFx$	$SMRF + SFx\,vs\,SFx$
rx^2	$-0.099^{\star}$	-0.661	$-0.039^{\star}$	-0.523	$0.253^{\star\star\star\dagger\dagger\dagger}$	$0.264^{\star\star\star\dagger\dagger\dagger}$
rx^3	$-0.011^{\star\star}$	-0.571	$0.014^{\star\star}$	-0.491	$0.286^{\star\star\star\dagger\dagger\dagger}$	$0.291^{\star\star\star\dagger\dagger\dagger}$
rx^5	$0.134^{\star\star\star\dagger\dagger\dagger}$	-0.401	$0.133^{\star\star\star\dagger\dagger\dagger}$	-0.403	$0.348^{\star\star\star\dagger\dagger\dagger}$	$0.354^{\star\star\star\dagger\dagger\dagger}$
rx^7	$0.219^{\star\star\star\dagger\dagger\dagger}$	-0.317	$0.205^{\star\star\star\dagger\dagger\dagger}$	-0.335	$0.393^{\star\star\star\dagger\dagger\dagger}$	$0.391^{\star\star\star\dagger\dagger\dagger}$
rx^{10}	$0.292^{\star\star\star\dagger\dagger\dagger}$	-0.253	$0.278^{\star\star\star\dagger\dagger\dagger}$	-0.269	$0.435^{\star\star\star\dagger\dagger\dagger}$	$0.432^{\star\star\star\dagger\dagger\dagger}$

Table A.3: Predictive Power of Macro Risk Factors - baseline approach with $\tau = 0.10$

Notes: This table shows the predictive power of MRF and SMRF when $q_{z_{t,t+4}}(\tau)$ is estimated using $\tau = 0.10$. t-stats computed using Newey-West standard errors with six lags are reported in parentheses. Wald statistics were also computed using Newey-West variance-covariance matrices with six lags.

	MRF_1	MRF_4	MRF_5	$SMRF$	$\overline{R}^2$	Wald
	0.397	-0.494	0.447		0.192	0.000
rx^2	(2.002)	(-2.341)	(2.034)		–	–
				0.228	0.182	
				(4.080)	–	
	0.803	-0.864	0.903		0.214	0.000
rx^3	(2.302)	(-2.366)	(2.357)		–	–
				0.445	0.210	
				(4.444)	–	
	1.682	-1.223	1.729		0.255	0.000
rx^5	(2.921)	(-2.250)	(2.772)		–	–
				0.832	0.262	
				(5.306)	–	
	2.605	-1.416	2.487		0.282	0.000
rx^7	(3.371)	(-1.929)	(2.998)		–	–
				1.195	0.291	
				(5.803)	–	
	3.954	-1.633	3.502		0.310	0.000
rx^{10}	(3.756)	(-1.686)	(3.188)		–	–
				1.700	0.315	
				(5.879)	–	

Table A.4: Predictive Power of SMRF, CP and LN factors - baseline approach with $\tau = 0.10$

Notes: This table shows the predictive power of CP, LN and SMRF factors when $q_{z_{t,t+4}}(\tau)$ is estimated using $\tau = 0.10$. t-stats computed using Newey-West standard errors with six truncation lags are reported in parentheses. Regressions in which LN is included in the set of predictors are estimated using the sample 1968Q4 - 2007Q4.

	SMRF	*CP*	$\overline{R}^2$	*SMRF*	*LN*	$\overline{R}^2$	*SMRF*	*CP*	*LN*	$\overline{R}^2$
		0.242	0.215		0.304	0.213		0.195	0.190	0.354
rx^2		(5.205)	–		(4.957)	–		(3.897)	(2.704)	–
	0.121	0.164	0.245	0.180	0.202	0.321	0.092	0.145	0.167	0.374
	(2.058)	(3.286)	–	(3.681)	(3.374)	–	(1.613)	(2.557)	(2.364)	–
		0.468	0.242		0.545	0.208		0.384	0.321	0.375
rx^3		(5.338)	–		(5.1045)	–		(4.353)	(2.771)	–
	0.240	0.314	0.279	0.362	0.338	0.342	0.192	0.280	0.272	0.400
	(2.158)	(3.227)	–	(4.155)	(3.175)	–	(1.827)	(2.726)	(2.279)	–
		0.814	0.261		0.863	0.189		0.692	0.459	0.384
rx^5		(5.159)	–		(5.595)	–		(4.299)	(2.989)	–
	0.516	0.486	0.321	0.684	0.472	0.361	0.395	0.476	0.359	0.423
	(2.974)	(2.927)	–	(4.957)	(3.461)	–	(2.689)	(2.822)	(2.317)	–
		1.185	0.296		1.182	0.188		1.004	0.596	0.406
rx^7		(5.669)	–		(5.579)	–		(4.507)	(3.126)	–
	0.724	0.723	0.360	0.991	0.615	0.380	0.572	0.692	0.451	0.449
	(2.940)	(3.069)	–	(5.242)	(3.268)	–	(2.911)	(2.975)	(2.334)	–
		1.686	0.321		1.559	0.172		1.445	0.715	0.411
rx^{10}		(6.018)	–		(5.012)	–		(4.768)	(2.918)	–
	1.029	1.029	0.390	1.435	0.739	0.385	0.835	0.990	0.504	0.460
	(2.955)	(3.275)	–	(5.203)	(2.682)	–	(2.897)	(3.205)	(2.007)	–

Table A.5: Predictive Power of Macro Risk Factors - WLH approach

Notes: This table shows the predictive power of *MRF* and *SMRF* when $q_{z_{t,t+4}}(\tau)$ is estimated using $\tau = 0.05$ and the WLH approach. t-stats computed using Newey-West standard errors with six lags are reported in parentheses. Wald statistics were also computed using Newey-West variance-covariance matrices with six lags.

	MRF_1	MRF_4	MRF_5	MRF_8	$SMRF$	$\overline{R}^2$	Wald
rx^2	0.432	-0.555	0.263	0.058		0.175	0.000
	(2.304)	(-3.550)	(1.977)	(0.312)		–	–
					0.225	0.166	
					(3.802)	–	
rx^3	0.860	-0.958	0.534	0.242		0.187	0.000
	(2.457)	(-3.800)	(2.309)	(0.704)		–	–
					0.436	0.190	
					(4.075)	–	
rx^5	1.758	-1.451	0.987	0.768		0.230	0.000
	(3.010)	(-4.072)	(2.556)	(1.346)		–	–
					0.823	0.242	
					(4.718)	–	
rx^7	2.700	-1.809	1.537	1.117		0.262	0.000
	(3.545)	(-3.928)	(2.937)	(1.529)		–	–
					1.195	0.275	
					(5.245)	–	
rx^{10}	4.076	-2.286	2.196	1.610		0.292	0.000
	(3.983)	(-3.728)	(3.155)	(1.739)		–	–
					1.711	0.302	
					(5.405)	–	

Table A.6: Predictive Power of SMRF, CP and LN factors - WLH approach

Notes: This table shows the predictive power of CP, LN and SMRF factors when $q_{z_{t,t+4}}(\tau)$ is estimated using $\tau = 0.05$ and the WLH approach. t-stats computed using Newey-West standard errors with six truncation lags are reported in parentheses. Regressions in which LN is included in the set of predictors are estimated using the sample 1968Q4 - 2007Q4.

	SMRF	*CP*	$\overline{R}^2$	*SMRF*	*LN*	$\overline{R}^2$	*SMRF*	*CP*	*LN*	$\overline{R}^2$
rx^2		0.242	0.215		0.304	0.213		0.195	0.190	0.354
		(5.205)	–		(4.957)	–		(3.897)	(2.704)	–
	0.111	0.175	0.240	0.179	0.212	0.309	0.0805	0.154	0.173	0.368
	(1.972)	(3.852)	–	(4.100)	(3.500)	–	(1.581)	(2.855)	(2.495)	–
rx^3		0.468	0.242		0.545	0.208		0.384	0.321	0.375
		(5.338)	–		(5.104)	–		(4.353)	(2.771)	–
	0.217	0.337	0.272	0.350	0.365	0.320	0.155	0.304	0.288	0.390
	(2.006)	(3.787)	–	(4.222)	(3.349)	–	(1.602)	(3.318)	(2.471)	–
rx^5		0.814	0.261		0.863	0.189		0.692	0.459	0.384
		(5.159)	–		(5.595)	–		(4.299)	(2.989)	–
	0.483	0.524	0.313	0.666	0.519	0.336	0.333	0.519	0.388	0.409
	(2.717)	(3.466)	–	(4.785)	(3.499)	–	(2.410)	(3.334)	(2.511)	–
rx^7		1.185	0.296		1.182	0.188		1.004	0.596	0.406
		(5.669)	–		(5.579)	–		(4.507)	(3.126)	–
	0.698	0.765	0.354	0.959	0.686	0.350	0.473	0.758	0.494	0.433
	(2.774)	(3.533)	–	(4.835)	(3.347)	–	(2.516)	(3.541)	(2.580)	–
rx^{10}		1.686	0.321		1.559	0.172		1.445	0.715	0.411
		(6.018)	–		(5.012)	–		(4.768)	(2.918)	–
	1.012	1.077	0.386	1.405	0.833	0.355	0.717	1.074	0.562	0.443
	(2.839)	(3.695)	–	(4.742)	(2.849)	–	(2.562)	(3.795)	(2.292)	–

Table A.7: Affine model - full parameter estimates

Notes: This table shows the full parameters estimates for the affine model with macro risk factors MRF1, MRF4 and MRF6. Bootstrap standard errors are shown in parentheses.

μ'	0.497	−0.245	0.231	−0.535	0.169	0.297	$\mu^{\mathbb{Q}'}$	2.398	0.757	0.079			
	(0.988)	(0.310)	(0.127)	(0.197)	(0.274)	(0.265)		(0.880)	(0.222)	(0.223)			
Φ	0.957	0.055	0.261	−0.205	0.339	−0.538	$\Phi^{\mathbb{Q}}$	0.991	0.242	−0.348			
	(0.035)	(0.242)	(0.623)	(0.352)	(0.214)	(0.217)		(0.010)	(0.016)	(0.024)			
	0.015	0.752	0.843	0.194	−0.059	0.081		−0.008	0.888	0.514			
	(0.010)	(0.074)	(0.192)	(0.104)	(0.065)	(0.066)		(0.005)	(0.025)	(0.016)			
	−0.001	0.042	0.592	−0.069	0.006	−0.001		0.004	0.009	0.805			
	(0.004)	(0.031)	(0.080)	(0.044)	(0.027)	(0.028)		(0.002)	(0.004)	(0.023)			
	0.002	0.113	0.157	0.808	−0.034	0.051							
	(0.007)	(0.048)	(0.125)	(0.070)	(0.041)	(0.045)							
	0.005	0.021	−0.396	0.075	0.833	0.077							
	(0.009)	(0.062)	(0.166)	(0.090)	(0.056)	(0.057)							
	−0.018	−0.114	0.516	0.179	−0.090	0.727							
	(0.009)	(0.065)	(0.166)	(0.094)	(0.055)	(0.061)							
Σ	4.204	−1.012	0.026	−0.288	0.184	0.175	δ_0	0.016					
	(2.146)	(0.833)	(0.090)	(0.246)	(0.295)	(0.264)		(0.000)					
	−1.012	0.974	−0.237	0.414	−0.389	−0.075	δ_1'	0.083	−0.155	0.144			
	(0.833)	(0.315)	(0.035)	(0.099)	(0.105)	(0.096)		(0.007)	(0.008)	(0.008)			
	0.026	−0.237	0.115	−0.136	0.140	0.007	λ_0'	−0.927	−1.708	−1.461			
	(0.090)	(0.035)	(0.020)	(0.036)	(0.039)	(0.038)		(0.440)	(0.714)	(1.059)			
	−0.288	0.414	−0.136	0.313	−0.193	−0.079	λ_1	−0.017	−0.091	0.297	−0.100	0.165	−0.262
	(0.246)	(0.099)	(0.036)	(0.660)	(0.347)	(0.210)		(0.011)	(0.066)	(0.216)	(0.080)	(0.075)	(0.078)
	0.184	−0.389	0.140	−0.193	0.436	0.022		0.017	−0.212	0.557	0.017	0.026	−0.057
	(0.295)	(0.105)	(0.039)	(0.347)	(0.987)	(0.352)		(0.007)	(0.074)	(0.225)	(0.076)	(0.071)	(0.070)
	0.175	−0.075	0.007	−0.079	0.022	0.342		0.000	−0.116	−0.324	−0.106	0.056	−0.065
	(0.263)	(0.096)	(0.038)	(0.210)	(0.352)	(0.532)		(0.013)	(0.092)	(0.325)	(0.116)	(0.114)	(0.111)

Figure A.1: 10-year yield decomposition

Notes: Panel A shows NBER-dated recessions and time series for the 10-year yield (solid black), 10-year short-rate expectations (dotted blue) and 10-year term premium (solid blue) estimated from the affine term structure model with macro risk factors. The term premium is smoothed using exponential-weighted moving average. In Panel B graphs show lead/lag correlations between the non-smoothed 10-year term premium and growth rates of key economic activity indicators. The term premium is at date t while growth rates are at time t + l, where l refers to lead (if negative) and lags (if positive). Leads and lags are shown in annual frequency.

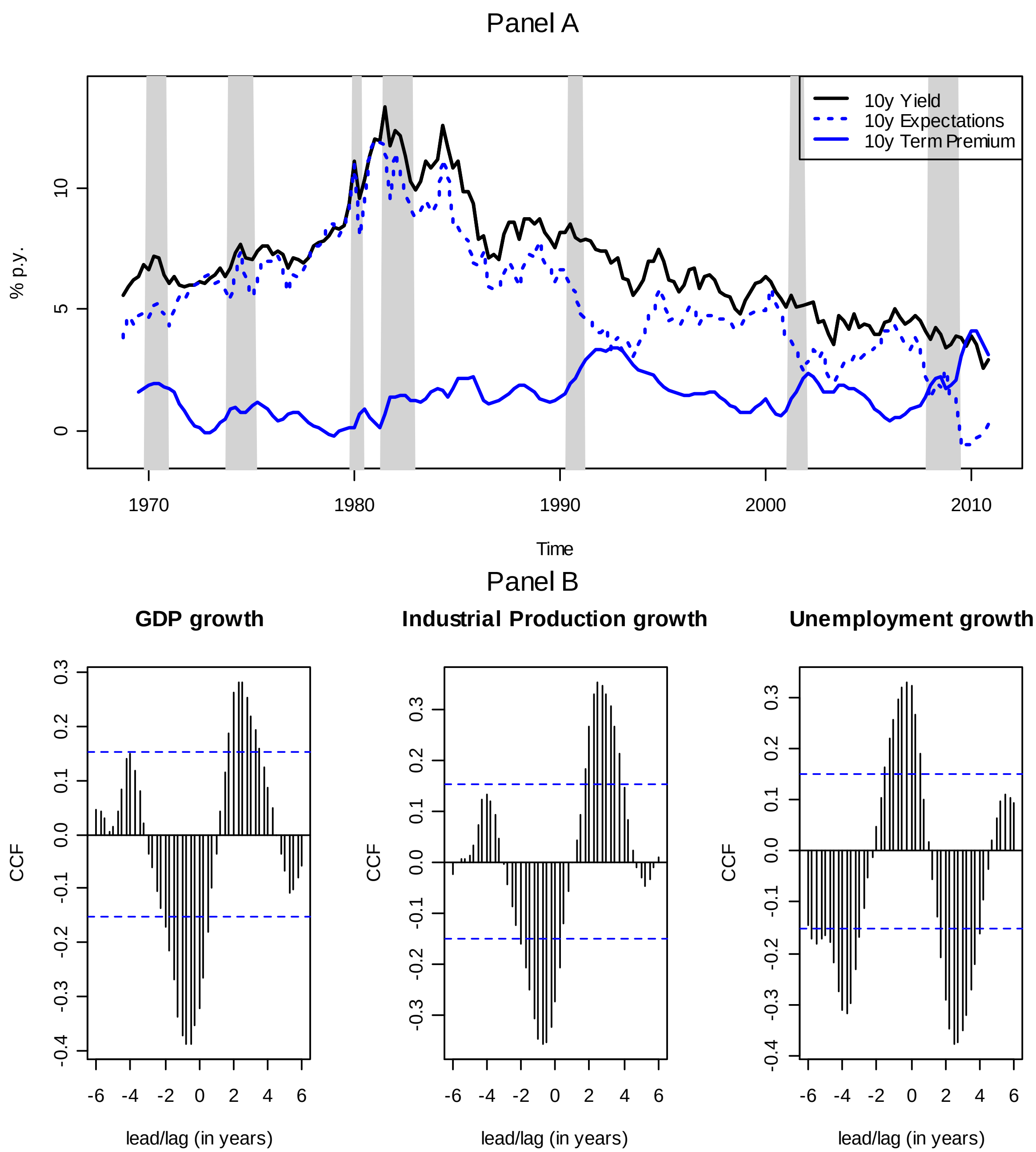

Figure A.2: Impulse response functions

Notes: This figure shows impulse response functions for the 10-year term premium, 10-year average short-rate expectations and 10-year yield to one standard-deviation shocks in MRF1, MRF4 and MRF6. 95% percent confidence intervals are shown as dotted lines.

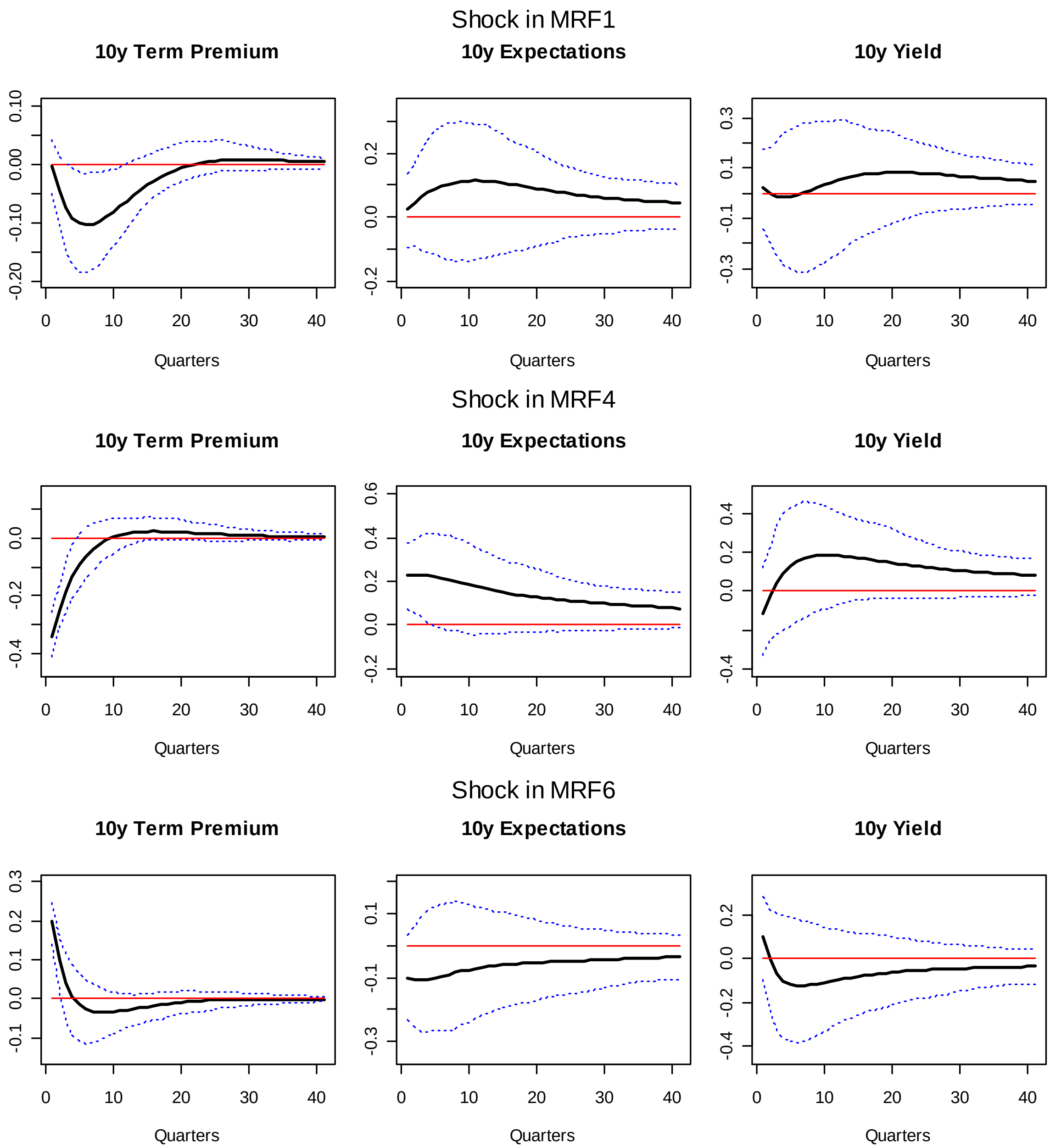

Figure A.3: Recursive estimates of R^2_{oos} and utility gains in real-time

Notes: Panel A shows recursive R^2_{oos} computed for the period 1995Q1-2011Q4 in a fully real-time exercise. Asterisks indicate statistical significance at 5% according to the MSPE-adjusted statistic of Clark and West (2007). Panel B shows recursive utility gains accrued by an investor investing in a portfolio of US government bonds. R^2_{oos}s and utility gains are computed against a constant model of no-predictability.