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A wake-up call: information contagion and strategic uncertainty*

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Abstract

A financial crisis in one region is a wake-up call for investors in other regions. If the correlation across regional fundamentals is potentially positive but uncertain ex-ante, investors acquire information about this correlation to determine their exposure. Financial contagion can occur in the absence of ex-post exposure, due to elevated strategic uncertainty among informed investors. This novel wake-up call theory of contagion explains how currency crises, bank runs, and debt crises spread across regions without a common investor base, ex-post correlated fundamentals or interconnectedness. Our wake-up call theory generates testable implications for laboratory experiments and new empirical predictions.

Keywords: contagion, information acquisition, wake-up call, mixture distribution.

JEL classifications: C7, D83, G01.

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1 Introduction

The global financial crisis of 2007–09 illustrated that contagion poses an important systemic risk. Historically, there are numerous contagious crises episodes, such as the Asian financial crisis of 1997 and the more recent European sovereign debt crisis. Forbes (2012) distinguishes four different, but not mutually exclusive, channels of contagion: trade, banks and financial institutions, portfolio investors, and wake-up calls. Goldstein (1998) introduced the latter channel, whereby additional information about one region’s fundamental leads to the reappraisal of risk in another region. This wake-up call hypothesis is supported by empirical evidence, such as Bekaert et al. (2014).

Despite the empirical evidence that a wake-up call plays an important role in transmitting crises, there has been surprisingly little theoretical work on this mechanism. Our paper attempts to close this gap by providing a theory of contagion based on the information acquisition after observing a crisis elsewhere, which is a wake-up call to investors. We define contagion as an increase in the probability of a financial crisis in one region due to a crisis in another region.

We develop a model of two regions with initial uncertainty about a potentially positive correlation of fundamentals. In each region, investors play a standard global coordination game of regime change with incomplete information about the fundamental (Morris and Shin (2003)). Consider now a crisis in the first region that is observed by investors in the second region. This wake-up call induces investors to acquire costly information about the actual correlation. Intuitively, informed investors can tailor their strategy to the information obtained, while uninformed investors play an invariant strategy. Consequently, informed investors obtain a higher expected payoff by acting more aggressively upon information that suggests a regime change is more likely.

Our model generates two contagion results. First, in case of unrelated regional fundamentals, contagion can be more likely when speculators are informed about the correlation than when they are uninformed. Since this result holds for a zero realized correlation, we label it ex-post contagion. Second, contagion can be more likely after observing a crisis in the first region and learning about

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1 Bekaert et al. (2014) find that a wake-up call was the key driver of equity market contagion during the global financial crisis. They show that at the core of contagion was the reassessment of risks by investors, rather than financial or trade linkages. See also Didier et al. (2010). Analysing bond market contagion during the Asian crisis in 1997, Basu (2002) finds evidence for contagion based on the reassessment of risks in some South-East Asian countries. Van Rijckeghem and Weder (2003) view the Russian crisis as the outcome of a wake-up call in emerging markets.
a zero correlation than after observing no crisis (ex-ante contagion). If the information cost is sufficiently low, a strategic complementarity in information choices generates a unique equilibrium. Investors acquire information after a wake-up call and contagion occurs in equilibrium.

Ex-post contagion occurs in our model because learning about a zero correlation has two effects on the prior distribution of the second region’s fundamental. First, the mean is higher and this mean effect reduces the probability of a crisis (Morris and Shin (2003)). Second, the variance is higher, which increases the strategic uncertainty among informed investors. This variance effect increases the probability of a crisis if the prior about the second region’s fundamental is strong (Metz (2002)). The variance effect induces an elevated strategic uncertainty among informed investors that makes a given informed investor more concerned about other informed investors attacking, which results in a greater aggressiveness in attacking. We analyze how the interaction of the mean and variance effects leads to contagion if the variance effect is sufficiently strong and link this interaction to the information choice of investors after a wake-up call.

The wake-up call theory of contagion applies to a range of phenomena. Our results attain in a global coordination game of regime change in which agents observe a crisis in another region but are initially uncertain about the links across regions. To illustrate the mechanism, we use a model of speculative currency crises (e.g. Morris and Shin (1998) and Corsetti et al. (2004)). The correlation of fundamentals across regions is initially uncertain because the magnitude of trade or financial links is unknown, as well as the relevance of institutional similarities. Other applications include bank runs (e.g. Rochet and Vives (2004) and Goldstein and Pauzner (2005)) and sovereign debt crises (e.g. Corsetti et al. (2006)). In the first case, uninsured creditors of banks observe a run elsewhere but are uncertain about interbank linkages. In the second case, sovereigns debt holders observe a default elsewhere but are uncertain about the financial and macroeconomic links.

Our theoretical predictions are consistent with documented empirical findings. For example, Eichengreen et al. (1998) find striking evidence that a crisis elsewhere increases the likelihood “of a speculative attack by an economically and statistically significant amount” (p. 2). This observation is consistent with the ex-ante contagion mechanism. Our results also help to rationalize contagion phenomena documented in the empirical literature, such as the unexpected spread from the Russian

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2See also the earlier literature of e.g. Krugman (1979), Flood and Garber (1991), and Obstfeld (1986).
crisis to Brazil in 1998 (Bordo and Murshid (2000) and Forbes (2012)) and similar instances during the Asian crisis in 1997 (Radelet and Sachs (1998) and Corsetti et al. (1999)). The reliance of the ex-post contagion mechanism on the elevated strategic uncertainty among informed investors after a wake-up call is consistent with the view by many “observers [who attribute the spread of the Russian crisis to] ... an enhanced perception of risk” (Van Rijckeghem and Weder (2001), p. 294).

An excellent case study of the wake-up call theory of contagion is documented in section 6.4, where we describe how a banking crisis spread rapidly across unrelated Latvian intermediaries.

Our wake-up call theory of contagion generates a novel set of empirical predictions described in section 6.2. The empirical literature on the contagion of banking and currency crises studies the channels by which contagion spreads and the characteristics of the second region that makes it susceptible to contagion. Our theory suggests that the likelihood of contagion depends in a non-linear way on the characteristics of the first region. In particular, after controlling for the fundamentals of the second region, a crisis in the first region due to extremely low fundamentals is less likely to spread if fundamentals are uncorrelated. Conversely, a crisis in the first region due to moderately low fundamentals is more likely to spread if fundamentals are uncorrelated. Testing these predictions about the non-linear role of the first region’s fundamental promises to improve our understanding of contagion. Existing empirical evidence shows the importance of non-linearities.

Moreover, our theory of contagion has testable implications for laboratory experiments. Since the acquisition of information after a wake-up call can be observed, laboratory experiments are suitable for testing our model’s predictions regarding ex-post contagion and the information choice of investors. Building on the work of Heinemann et al. (2013, 2009), examining contagion within the global games framework in the laboratory is a promising yet little explored avenue for future research. Specifically, we derive three testable implications of our theory in section 6.3.

The existing theoretical literature has explained financial contagion with various mechanisms: the interconnectedness between regions (Allen and Gale (2000) and Dasgupta (2001) for interbank linkages and Kiyotaki and Moore (2002) for balance-sheet contagion), a common discount fac-

\footnotetext[3]{See Glick and Rose (1999) for the role of trade links, Van Rijckeghem and Weder (2001, 2003) for financial links, and Dasgupta et al. (2011) for institutional similarities.}

\footnotetext[4]{For instance, extreme returns can have different implications for the transmission process across markets (Forbes and Rigobon (2002), Bekaert et al. (2013)). As for the propagation of financial shocks, Favero and Giavazzi (2002) contrast contagion with “flight-to-quality” episodes.}
tor channel (Ammer and Mei (1996) and Kodres and Pritsker (2002)), a common investor base (Goldstein and Pauzner (2004) for the increased sensitivity of risk-averse investors to strategic risk, Pavlova and Rigobon (2008) for portfolio constraints, and Oh (2013) for learning about the aggressiveness of other investors), and ex-post correlated regional fundamentals (Acharya and Yorulmazer (2008) and Allen et al. (2012) for asset commonality among banks and Basu (1998) for learning about a common risk factor). We do not rely on the previous mechanisms but provide a novel and complementary theory of contagion based on information acquisition after a wake-up call. We show that contagion can arise even if investors learn that fundamentals are unrelated across regions.

Finally, our paper also makes a technical contribution to the global games literature. The ex-ante uncertainty about the correlation of fundamentals results in heterogeneous priors between informed and uninformed investors that follow a mixture distribution. Our first technical contribution is to analyze the information choice regarding a signal about the correlation of fundamentals in this setup. There is initial uncertainty about a homogeneous prior in Chen and Suen (2013) who study crisis waves based on learning about the prior (mean effect). By contrast, we analyze the information choice of investors after receiving a wake-up call, thereby allowing for heterogeneous priors. Moreover, we study the changes in strategic uncertainty among informed investors (variance effect) that underpins our theory of contagion.

Hellwig and Veldkamp (2009) show that information choices inherit the strategic complementarity or substitutability from the underlying beauty contest game. Our second technical contribution is to show that this “inheritance” result extends to a regime change game with ex-ante uncertainty about the correlation of fundamentals. Furthermore, we study the acquisition of a publicly available signal about this correlation, allow for heterogeneous priors, and offer a novel theory of contagion after a wake-up call. In our model, information about the cross-regional correlation can either increase or decrease the precision of the prior about the fundamental in the second region, which is crucial for the ex-post contagion mechanism. By contrast, Szkup and Trevino (2012b) study a model with a common prior, continuous private information choice and a convex cost of acquiring information. However, strategic complementarity in information choices may not occur.  

Another global games paper that works with mixture distributions is Chen et al. (2012), who develop a theory of rumors in a political regime change. Apart from the different focus, they do not consider information acquisition. Ahnert and Kakhbod (2014) obtain strategic complementarity in information choices in a one-region global coordination game of regime change with a common prior, a discrete private information choice and heterogeneous
This paper is organized as follows. We describe a global coordination game of regime change with ex-ante uncertainty about the correlation of regional fundamentals in section 2. Using mixture distributions, we solve the model in the case of exogenous information in section 3. We establish a novel contagion mechanism in section 4, from both an ex-ante and an ex-post perspective. Allowing for endogenous information in section 5, we show that information choices exhibit strategic complementarity and that our contagion results occur in equilibrium. We describe our theory’s new empirical predictions and testable implications for laboratory experiments in section 6, where we also link our results to the empirical literature on contagion and to the crisis episode in Latvia. Robustness checks and extensions are considered in section 7 and the associated Appendix C. Section 8 concludes. Figures are in Appendix A, while derivations and proofs are in Appendix B.

2 Model

We study a sequential game of speculative currency crises (Morris and Shin (1998)). There are two dates and two countries, both indexed by \( t \in \{1, 2\} \) because currency speculators in country \( t \) only move at date \( t \). Each country is inhabited by a unit continuum of risk-neutral speculators indexed by \( i \in [0, 1] \). A fundamental \( \theta_t \) measures the ability of country \( t \) to defend its currency.

\textbf{Actions and payoffs} At each date, currency speculators move simultaneously. Each speculator either attacks the currency \( (a_{it} = 1) \) or does not attack \( (a_{it} = 0) \). The outcome of the attack depends on both the aggregate attack size, \( A_t = \int_0^1 a_{it} \, di \), and the fundamental, \( \theta_t \). A currency crisis occurs if enough speculators attack relative to the fundamental, \( A_t > \theta_t \). In that case, the attack is successful from the perspective of speculators, since it leads to the abandonment of a currency’s peg. Following Vives (2005), an individual speculator’s benefit from participating in a successful currency attack is \( b > 0 \), while the loss of participating in an unsuccessful attack is \( l > 0 \):

\[ u(a_i = 1, A, \theta) = b \, 1\{A > \theta\} - l \, 1\{A \leq \theta\}. \] (1)

\(^7\)Our theory of contagion only requires the sequential timing of events but not a common investor base.

\(^8\)This may reflect fundamental economic conditions, the soundness of and the commitment to its economic policies, as well as the level of foreign currency and gold reserves held by the monetary authority.
The constant payoff from not attacking is normalized to zero, \( u(a_{it} = 0, A_t, \theta_t) = 0 \). This payoff structure is consistent with currency speculation by short-selling.\(^9\) The payoff differential from attacking increases in the aggregate attack size \( A_t \) and decreases in the fundamental \( \theta_t \). Therefore, the attack decisions of individual speculators exhibit global strategic complementarity.\(^{10}\)

**Information** The key feature of our model is the initial uncertainty about the correlation between fundamentals across countries \( \rho \equiv \text{corr}(\theta_1, \theta_2) \). This correlation captures real, financial, or institutional links between countries.\(^{11}\) The cross-country correlation of fundamentals is zero with probability \( p \in (0, 1) \) or takes the positive value \( \rho_H > 0 \):\(^{12}\)

\[
\rho = \begin{cases} 
0 & \text{w.p. } p \\
\rho_H & \text{w.p. } 1 - p
\end{cases}
\]

Fundamentals in both countries follow a bivariate normal distribution with mean \( \mu_t \equiv \mu \), precision \( \alpha_t \equiv \alpha \in (0, \infty) \), and realized correlation \( \rho \). As in the global games literature pioneered by Carlsson and van Damme (1993), each speculator receives a private signal, \( x_{it} \), about the country’s fundamental before deciding whether to attack:

\[
x_{it} \equiv \theta_t + \epsilon_{it}
\]

where the idiosyncratic noise \( \epsilon_{it} \) is identically and independently normally distributed across speculators and countries with zero mean and precision \( \gamma > 0 \). The random variables for the cross-country correlation, the countries’ fundamentals, and the sequences of idiosyncratic noise are independent.

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\(^9\)In practice, a speculator sells a currency short by borrowing a fixed amount in currency \( X \) to purchase currency \( Y \). Such an operation typically involves transaction costs. If the currency \( X \) is indeed devalued, then the speculator benefits from the reduced value of the debt obligation in terms of currency \( Y \). Otherwise, the speculator incurs a loss.

\(^{10}\)Consequently, complete information about the fundamental leads to multiple equilibria that are sustained by self-fulfilling expectations for interim values of the fundamental \( \theta_t \in (0, 1) \). By contrast, a unique equilibrium exists for extreme values of the fundamental in dominant strategies: speculators attack if the fundamental is low, \( \theta_t \leq 0 \), and do not attack if the fundamental is high, \( \theta_t \geq 1 \).

\(^{11}\)Click and Good (1999) find empirical evidence for the role of geographic proximity and trade in the contagion of currency crises. In contrast, Van Rijckeghem and Weder (2001) and Van Rijckeghem and Weder (2003) find that spillovers through bank lending played a more important role for more recent episodes of currency crises. Finally, Dasgupta et al. (2011) find that institutional similarity to the "ground zero country", from which the wave of crises emerged, is an important determinant for the direction of financial contagion.

\(^12\)We consider the case of negative correlation of fundamentals for robustness in section \( \text{n} \).
If a currency crisis occurs in country 1, the realization of the fundamental $\theta_1$ becomes common knowledge for speculators in country 2. This assumption is motivated by the public scrutiny of the monetary authority after a currency crisis in country 1. In contrast, the realized fundamental $\theta_1$ remains unobserved if no currency crisis occurs in country 1. To ensure that an informed currency speculator of country 2 cannot completely infer the fundamental $\theta_2$ after observing a currency crisis in country 1, we assume $\rho_H < 1$. The information structure is common knowledge.

The currency speculation game in country 2 has two stages. At stage 2, speculators decide simultaneously whether to attack the currency. This coordination stage is preceded by an information stage. We start by studying exogenous information, whereby a commonly known proportion of speculators, $n \in [0,1]$, learns the correlation $\rho$. Next, we consider information acquisition, whereby currency speculators play an information acquisition game after observing a crisis in country 1. At stage 1, each speculator simultaneously decides whether to purchase a perfectly revealing public information about the correlation of the fundamentals at a cost $c > 0$. That is, every speculator can purchase the same signal but observes it privately. Hence, speculators are heterogeneous in their information about both the cross-country correlation and the private information received about their country’s fundamental. The timeline in table 1 summarizes the model.

Date 1:
- The correlation of fundamentals $\rho$ is drawn.
- The fundamentals $\theta_1$ and $\theta_2$ are drawn from a bivariate distribution with correlation $\rho$.
- Speculators receive private signals $x_{i1}$ and simultaneously decide whether to attack.
- Payoffs are realized. If a currency crisis occurs, $\theta_1$ becomes public knowledge.

Date 2: Information stage
- Upon observing a crisis in country 1, speculators simultaneously decide whether to purchase a signal about $\rho$ at cost $c > 0$.

Coordination stage
- Speculators receive private signals $x_{i2}$ and simultaneously decide whether to attack.
- Payoffs are realized.

Table 1: Timeline

---

13 Only the result of Proposition 3 on ex-ante contagion depends on this assumption (see section 4.2).
14 Corsetti et al. (2004) study the impact of a large, and potentially asymmetrically informed, currency speculator. By contrast, our speculators are of equal size and our theory of contagion does not require signalling or herding.
15 For instance, the purchase of an international newspaper that contains information about country 1, its institutional similarities with country 2, as well as information about other factors that influence the likelihood of a cross-country exposure. The international newspaper is costly as it takes money to buy and time to absorb. In terms of wholesale investors or currency speculators, costly information acquisition could be access to Bloomberg and Datastream terminals or the hiring of analysts to understand and interpret the publicly available information.
3 Equilibrium

This section analyzes the existence and uniqueness of an equilibrium in the currency speculation coordination game in both countries. Upon briefly revising the standard equilibrium in country 1, we focus on the equilibrium in country 2 and analyze the case of *exogenous information* about the cross-country correlation of fundamentals. In particular, we study the consequences of public information about the cross-country correlation between fundamentals, $\rho \in \{0, \rho_H\}$, that helps us establish the contagion effects in section 4 and, eventually, the wake-up call theory of contagion in section 5, in which speculators can acquire information about the cross-country correlation.

3.1 Country 1

We start by briefly revising the equilibrium in country 1 that is well established in the literature (e.g. Vives (2005)). A Bayesian equilibrium in the currency attack coordination game in country 1 consists of an attack decision $a(x_{i1})$ for each speculator $i \in [0, 1]$ and an aggregate attack size $A_1$ that satisfies the conditions for both individual optimality of attacking and aggregation:

$$a(x_{i1}) = \max_{a_{i1} \in [0, 1]} E[u(a_{i1}, A_1, \theta_1)|x_{i1}] \forall i \tag{4}$$
$$A_1 = \int_{-\infty}^{+\infty} a(x_{i1}) \sqrt{\gamma} \phi(\sqrt{\gamma}(x_{i1} - \theta_1)) dx_{i1} \equiv A(\theta_1) \tag{5}$$

where $\phi(x)$ is the probability distribution function of the standard Gaussian random variable.

As shown by Morris and Shin (2003), a unique Bayesian equilibrium exists if the private signal is sufficiently precise, $\gamma > \gamma_0 \equiv \frac{\sigma^2}{2\pi} \in (0, \infty)$. The equilibrium is characterized by a threshold of the private signal $x_1^*$ and a threshold of the fundamental $\theta_1^*$. Each individual speculator attacks if and only if his signal falls short of the threshold, $x_{i1} < x_1^*$. A speculative currency crisis occurs if and only if the fundamental is sufficiently weak, $\theta_1 < \theta_1^*$. There are two equilibrium conditions. First, the proportion of attacking speculators equals the fundamental threshold, $A_1^* = \theta_1^*$. Second, a speculator with the threshold signal $x_{1i} = x_1^*$ is indifferent between attacking and not attacking.
This yields one equation that implicitly defines the fundamental threshold \( \theta_1^* \) (see Appendix B.1):

\[
F_1(\theta_1^*) \equiv \Phi \left( \frac{\alpha}{\sqrt{\alpha + \gamma}} (\theta_1^* - \mu) - \sqrt{\frac{\gamma}{\alpha + \gamma}} \Phi^{-1}(\theta_1^*) \right) = \frac{1}{1 + b/l}.
\] (6)

**Lemma 1** (Morris and Shin (2003)) If private information is sufficiently precise, \( \gamma > \gamma_0 \), then a unique Bayesian equilibrium exists in country 1. This equilibrium is in threshold strategies, whereby a speculator attacks if and only if \( x_{i1} < x_1^* \) and a currency attack occurs if and only if \( \theta_1 < \theta_1^* \), where \( \theta_1^* \) is implicitly defined by equation (6).

In Appendix B.1, \( x_1^* \) is defined by equation (B3) and we show the comparative statics of the fundamental threshold \( \theta_1^* \) and the dependence of the ranking of equilibrium thresholds on the strength of the prior about the fundamental.

**Corollary 1** If the prior about the fundamental is strong compared to the relative benefit from attacking, a currency crisis in country 1 only occurs for realized fundamentals \( \theta_1 \) below the prior \( \mu \):

\[
\mu > \Phi \left( \frac{\sqrt{\alpha + \gamma}}{\gamma} \Phi^{-1} \left( \frac{b/l}{b/l + 1} \right) \right) \Rightarrow \theta_1 \leq \theta_1^* < \mu. \tag{7}
\]

### 3.2 Country 2

Suppose there is a currency crisis in country 1, \( \theta_1 < \theta_1^* \), such that speculators in country 2 observe the realized fundamental that satisfies \( \theta_1 < \mu \) by Corollary 1. We start by analyzing the equilibrium in country 2 after a crisis in country 1 in the case of exogenous information, whereby a known fraction of speculators learns the realized cross-country correlation at the beginning of date 2. Let \( d_i \in \{I, U\} \) denote whether speculator \( i \) is informed (I) or uninformed (U) about the cross-country correlation. To demonstrate our contagion results in section 4, it suffices to restrict attention to symmetric information about the cross-country correlation. Hence, currency speculators are either fully informed about the cross-country correlation, \( n = 1 \), or uninformed, \( n = 0 \). However, to establish our wake-up call theory of contagion, we consider asymmetric and endogenous information about the cross-country correlation in section 5.

To distinguish between informed and uninformed speculators in country 2, let \( a_{1I} \equiv a_{12}(d_i = I) \) and \( a_{1U} \equiv a_{12}(d_i = U) \).
I) and \(a_{iU} \equiv a_{i2}(d_i = U)\) denote the individual attacking decision, respectively. Likewise, \(A_{2I}\) and \(A_{2U}\) are the proportions of informed and uninformed speculators who attack. Furthermore, let the equilibrium thresholds be denoted as \(\theta^*_{2I} = \theta^*_{21}(\rho)\) if informed and \(\theta^*_{2U}\) if uninformed. We highlight that the cross-country correlation \(\rho\) is known to informed speculators. Finally, these equilibrium thresholds depend on the observed fundamental \(\theta_1\) if and only if a crisis occurred in country 1.\(^{16}\)

**Bayesian updating of prior about the fundamental**  
Upon observing the currency crisis in country 1, both informed and uninformed speculators update their prior about the fundamental in country 2. First, the update of an informed speculator is affected by both the observed fundamental \(\theta_1\) and the known exposure to the crisis country \(\rho\). The conditional mean is \(\mu_2(\rho, \theta_1) \equiv \mu_2(\rho, \theta_1) = \rho \theta_1 + (1 - \rho) \mu\), while the conditional variance is \(\alpha_2(\rho) \equiv \alpha_2(\rho) = \frac{\alpha}{1 - \rho^2}\), and normality is preserved. An informed speculator forms the following updated prior about the fundamental in country 2:

\[
\begin{align*}
\theta_2|\rho = 0 & \sim \mathcal{N}\left(\mu, \frac{1}{\alpha}\right) \quad (8) \\
\theta_2|\rho = \rho_H, \theta_1 & \sim \mathcal{N}\left(\rho_H \theta_1 + (1 - \rho_H) \mu, \frac{1 - \rho_H^2}{\alpha}\right). \quad (9)
\end{align*}
\]

Relative to the prior, a positive cross-country correlation lowers both the mean and the variance of the updated prior. These reductions are the more pronounced, the larger the correlation, thereby pushing the updated prior towards \(\theta_1\) and away from \(\mu\), as illustrated in Figure 1 in Appendix A.1.

Second, an uninformed speculator forms an updated prior about \(\theta_2\) by using the ex-ante distribution of the cross-country correlation only. As a result, the updated prior is a mixture distribution: an uninformed speculator believes that \(\theta_2\) is drawn from the distribution described in (8) with probability \(p\) and from the distribution described in (9) with probability \(1 - p\).

Figure 1 depicts the updated prior distributions about the fundamental in country 2. The updated prior distribution of informed speculators who learn that there is no cross-country correlation has the highest mean and variance, while learning that there is correlation leads to an updated prior distribution with the lowest mean and variance. Whereas the mixture distribution can be a unimodal distribution similar to a normal distribution with fat tails (as illustrated in the first

\(^{16}\)This is always the case in this section. However, in section [2] a reference point is sometimes the case of no crisis in country 1.
panel), it may also be bimodal if the mean is strongly updated, i.e. for a sufficiently small $\theta_1$ (as illustrated in the second panel).

### 3.3 Equilibrium if speculators are informed

Observing a currency crisis in country 1 is no news if speculators learn that there is zero cross-country correlation. Then, the analysis of country 1 applies directly, where $\theta_1^*$ is replaced by $\theta_{2I}^*(\rho = 0, \theta_1) = \theta_1^*$. In contrast, if all speculators learn that there is positive correlation, $\rho = \rho_H$, an adaption is required to obtain a corollary of Lemma [1]. The modified threshold for the private signal precision is $\gamma_1 \equiv \frac{\alpha^2}{2\pi(1-\rho_H)^2} \in (\gamma_0, \infty)$. Furthermore, the unique threshold fundamental $\theta_{2I}^* = \theta_{2I}^*(\rho, \theta_1)$ is implicitly defined by:

$$F_2(\theta_{2I}^*, \rho) = \Phi \left( \frac{\alpha_2(\rho) [\theta_{2I}^* - \mu_2(\rho, \theta_1)]}{\sqrt{\alpha_2(\rho) + \gamma}} - \sqrt{\frac{\gamma}{\alpha_2(\rho) + \gamma}} \Phi^{-1}(\theta_{2I}^*) \right) = \frac{1}{1 + b/l} \quad (10)$$

for any realization of the cross-country correlation $\rho \in \{0, \rho_H\}$ and any observed fundamental in the crisis country, $\theta_1 < \theta_1^*$.

**Corollary 2** Suppose speculators observe a crisis in country 1, $\theta_1 < \theta_1^*$, and are informed about the cross-country correlation, $n = 1$. If the private information is precise enough, $\gamma > \gamma_1$, then there exists a unique Bayesian equilibrium in country 2. This equilibrium is in threshold strategies and a currency attack occurs if the realized fundamental is below the threshold $\theta_{2I}^*(\rho, \theta_1)$ defined by equation (10).

We provide a detailed discussion of comparative statics in Appendix B.2 and discuss a specific case in section 4.2. As in country 1, and consistent with the literature, we characterize the strength of the prior about the fundamental in Definition 1. A strong prior about the fundamentals in country 2 (see Definition 1) implies a strong prior in country 1 (see Corollary 1).

**Definition 1** The prior about the fundamental is strong if $\mu \in S_1$ and it is weak if $\mu \in S_2$. This holds independently of the realization of $\rho \in \{0, \rho_H\}$. In contrast, whether the prior is strong or
weak depends on the realization of $\rho$ if $\mu \notin \{S_1, S_2\}$, where:

\[
S_1 = \{\mu, \theta_1, \alpha, \gamma, \rho_H, b, l\} : \mu_2(\rho, \theta_1) > \max\{X(\rho), Y(\rho)\}.
\]

\[
S_2 = \{\mu, \theta_1, \alpha, \gamma, \rho_H, b, l\} : \mu_2(\rho, \theta_1) < \min\{X(\rho), Y(\rho)\}.
\]

\[
X(\rho) = \Phi\left( -\frac{\sqrt{\alpha_2(\rho)} + \gamma}{\sqrt{\gamma}} \Phi^{-1}\left(\frac{1}{1+b/l}\right) \right),
Y(\rho) = \frac{1}{2} - \frac{\sqrt{\alpha_2(\rho)} + \gamma}{\alpha_2(\rho)} \Phi^{-1}\left(\frac{1}{1+b/l}\right).
\]

As shown in Appendix B.2, a weak prior makes attacks on the currency more likely, $\mu_2(\rho, \theta_1) < \theta_{21}^*(\rho, \theta_1) < 1$, while a strong prior makes attacks less likely, $0 < \theta_{21}^*(\rho, \theta_1) < \mu_2(\rho, \theta_1)$.

If speculators learn that the correlation is positive, $\rho = \rho_H$, then both the mean and the variance of the updated prior about $\theta_2$ are lower after a currency crisis in country 1. Therefore, the relative size of these mean and variance effects determines whether the equilibrium threshold increases or decreases relative to the case of zero cross-country correlation, $\theta_{21}^*(\rho_H, \theta_1) \leq \theta_{21}^*(0, \theta_1)$.

Since this ranking of thresholds is crucial for the subsequent analysis, we establish conditions sufficient for $\theta_{21}^*(0, \theta_1) > \theta_{21}^*(\rho_H, \theta_1)$. As stated in Lemma 2, such an ordering of thresholds requires two conditions: (i) a strong prior about the fundamental in country 2; and (ii) an intermediate realization of the fundamental in country 1. As before, $\gamma > \gamma_1$ ensures the existence of unique equilibrium thresholds.

\textbf{Lemma 2} Suppose $\gamma > \gamma_1$ and consider the case of informed investors, $n = 1$. Then, the threshold ranking $\theta_{21}^*(0, \theta_1) > \theta_{21}^*(\rho_H, \theta_1)$ is ensured by a strong prior about the fundamental in country 2 and an intermediate level of the fundamental in country 1, $\theta_1 \in [\theta_1^L, \theta_1^U]$, where the lower bound is

\begin{itemize}
  \item [17] If the relative benefit from attacking is large, i.e. $b > l$, then bounds involving $X(\rho)$ are sufficient. However, a low relative benefit from attacking, i.e. $b \leq l$, requires the strengthening of these bounds involving $Y(\rho)$. Mirroring the definition of the prior, these results hold independently of the realized cross-country correlation. Furthermore, a weak prior is associated with an incidence of attacks above 50% and $\theta_{21}^*(\rho, \theta_1) > 1/2$, while a strong prior is associated with an incidence of attacks below 50% and $\theta_{21}^*(\rho, \theta_1) > 1/2$.
  \item [18] However, no direct conclusion for the overall probability of a currency crisis in country 2 conditional on $\theta_1$ can be drawn since the conditional distribution of $\theta_2$ varies across these cases. In particular, the distribution of $\theta_2 | \rho = \rho_H, \theta_1$ places greater weight on lower realizations than the distribution of $\theta_2 | \rho = 0, \theta_1$.
\end{itemize}
defined as follows:

\[
\theta_1 \equiv \mu + \left( \frac{\theta_1^* - \mu}{\rho H} \right) \left[ 1 - \frac{\alpha + \gamma}{a_2^2(\rho_H)} \right]^{\frac{\alpha_2(\rho_H) + \gamma}{\alpha + \gamma}} + \frac{\sqrt{\gamma} \Phi^{-1}(\theta_1^*) \left[ \frac{\alpha_2(\rho_H) + \gamma}{\alpha + \gamma} - 1 \right]}{\alpha_2(\rho_H) \rho_H} < \mu,
\]

(12)

with \( \theta_1^* \) as defined in equation (11). A necessary condition for \( \theta_1 < \theta_1^* \) is:

\[
\mu < \theta_1 - \frac{\sqrt{\gamma}}{\alpha} \Phi^{-1}(\theta_1) - \frac{\sqrt{\alpha + \gamma}}{\alpha} \Phi^{-1} \left( \frac{1}{1 + b/l} \right).
\]

(13)

If \( b \geq l \), then condition (13) is also sufficient. However, if \( b < l \), then condition (13) and:

\[
\Phi \left( - \frac{\sqrt{\alpha + \gamma}}{\gamma} \Phi^{-1} \left( \frac{1}{1 + b/l} \right) \right) < \theta_1 - \frac{\sqrt{\gamma}}{\alpha} \Phi^{-1}(\theta_1) - \frac{\sqrt{\alpha + \gamma}}{\alpha} \Phi^{-1} \left( \frac{1}{1 + b/l} \right)
\]

(14)

are necessary and sufficient for \( \theta_1 < \theta_1^* \).

**Proof** See Appendix B.3.

Under the sufficient conditions of Lemma 4, there is a positive mass of fundamentals, \( \theta_1 \in [\theta_1, \theta_1^*] \), that is conducive to both a currency crisis in country 1 and the threshold ranking \( \theta_1^* \) in country 2. To avoid that the equilibrium threshold \( \theta_1^* \) is too small, i.e. \( \theta_1^* > \theta_1 \), the prior about country 1’s fundamental must not be too strong, as ensured by equation (13), and the relative benefit from attacking \( b/l \) must not be too small, as ensured by equation (14).

At the core of Lemma 2 is the change in the variance of the updated prior about the fundamental in country 2 as the realized cross-country correlation \( \rho \) changes. We discuss this variance effect in detail in Appendix B.2.2. Under the circumstances described in Lemma 2, a decrease in the relative precision of public signals due to a lower realization of \( \rho \) increases strategic uncertainty and therefore induces speculators to attack more aggressively, \( \theta_1^* \) in country 2. This ranking of equilibrium thresholds is crucial for the contagion effects established in section 4, where we also provide a detailed intuition for the implications of changes in strategic uncertainty.\(^{19}\)

\(^{19}\)Observe that the threshold ranking is reversed for very low realizations of \( \theta_1 \), i.e. \( \theta_1^* \) in country 1 is decreasing in \( \rho \) for all \( \theta_1 < \theta_1^* \). See also the proof of Lemma 4.
3.4 Equilibrium if speculators are uninformed

Consider now the symmetric case of uninformed speculators, $n = 0$. We start by analysing the updating of an uninformed speculator in country 2 after observing a crisis in country 1 and state conditions for the existence of a unique monotone equilibrium in Proposition 1.

**Bayesian updating** Speculators in country 2 have observed a currency crisis in country 1, $\theta_1 < \theta_1^*$, but are uninformed about the realized cross-country correlation of fundamentals $\rho$. Using Bayes’ rule, uninformed speculators use their private signal $x_{i2}$ to form a posterior about the distribution of the cross-country correlation. Let $\hat{p} \equiv \Pr\{\rho = 0|\theta_1, x_{i2}\}$ denote the probability of zero correlation of fundamentals. Its posterior distribution and comparative statics are derived in Appendix B.4.1, where we show that $\frac{d\hat{p}}{dx_{i2}} > 0$ if the private signal is relatively high. Intuitively, a speculator places more weight on the probability of zero cross-country correlation after receiving a relatively good private signal about the fundamental in country 2. Instead, after a low private signal, $\frac{d\hat{p}}{dx_{i2}} > 0$ is not guaranteed. For extremely low signals, an even worse signal makes an uninformed speculator infer that $\rho = 0$ is more likely. In sum, the relationship between the posterior probability of zero cross-country correlation $\hat{p}$ and the private signal $x_{i2}$ is non-monotone.

**Equilibrium analysis** There are once again two equilibrium conditions. First, the critical mass condition states that the proportion uninformed speculators who are attacking equals the fundamental threshold, $A_{2U}^* = \theta_{2U}^*$, where the equilibrium threshold in case of uninformed speculators only depends on the observed fundamental in country 1 after a crisis, $\theta_{2U}^* = \theta_{2U}^*(\theta_1)$:

$$x_{2U}^* = \theta_{2U}^* + \sqrt{\frac{T}{\gamma} \Phi^{-1}(\theta_{2U}^*)}.$$  \hspace{1cm} (15)

Second, an uninformed speculator with the threshold signal $x_{2i} = x_{2U}^*$ is indifferent between attacking and not attacking the currency. As shown in Appendix B.4.2, this yields one equation that implicitly defines the fundamental threshold $\theta_{2U}^*$, where the posterior belief about the no-correlation
probability is evaluated at the threshold signal, such that 
\[ \hat{p}(\theta_{2u}^*) \equiv \Pr\{\theta = 0|\theta_1, x_{i2} = x_{2u}^*(\theta_{2u}^*)\} : \]

\[ G(\theta_{2u}^*, \theta_1) \equiv \hat{p}(\theta_{2u}^*)F_2(\theta_{2u}^*(\theta_1), 0) + (1 - \hat{p}(\theta_{2u}^*))F_2(\theta_{2u}^*(\theta_1), \rho_H) = \frac{1}{1+b/l}. \] (16)

Therefore, \( G(\theta_{2u}^*, \theta_1) \) is a mixture of \( F_2(\theta_{2u}^*(0, \theta_1), 0) \) and \( F_2(\theta_{2u}^*(\rho_H, \theta_1), \rho_H) \) with \( \hat{p}(\theta_{2u}^*) \) as weight. Since the speculators are uninformed about the cross-country correlation, the expression is evaluated at the same fundamental threshold \( \theta_{2u}^* \) throughout.

In contrast to the previous standard analysis, \( G(\theta_{2u}^*, \theta_1) \) is harder to characterize since the weights of the mixture and the posterior beliefs about the probability of cross-country correlation now depend on the threshold signal \( x_{2u}^* \). Therefore, the question arises as to whether or not our focus on monotone equilibria is justified despite the global non-monotonicity of \( \hat{p}(x_{2u}^*(\theta_{2u}^*)) \) in \( x_{2u}^* \) and, hence, \( \theta_{2u}^* \), as established above. Fortunately, the best-response function of an individual speculator \( i \) proves to be strictly increasing in the fundamental thresholds \( \theta_{2u}^* \) used by other speculators:

\[ r' = -\frac{d\Pr\{\theta_2 < \hat{\theta}_{2u}(\hat{x}_2)|\theta_1, x_{i2}\}}{d\hat{x}_2} \frac{dx_2}{d\Pr\{\theta_2 < \hat{\theta}_{2u}(\hat{x}_2)|\theta_1, x_{i2}\}} > 0, \] (17)

where \( \hat{x}_{i2} \) is the critical threshold of the private signal used by player \( i \), \( \hat{x}_2 \) is the threshold used by all other speculators, and \( \hat{\theta}_{2u}(\hat{x}_2) \) is the critical threshold of the fundamental in country 2. This is because \( \Pr\{\theta_2 < \theta_{2u}^*|\theta_1, x_{i2}\} \) is monotonically decreasing in \( x_{i2} \) using a result of \textbf{Milgrom (1981)} (see Appendix B.4.1). Furthermore, given all other speculators use a threshold strategy, \( \Pr\{\theta_2 < \hat{\theta}_{2u}(\hat{x}_2)|\theta_1, x_{i2}\} \) increases in \( \hat{x}_2 \). Following \textbf{Vives (2005)}, the best response of player \( i \) is to use a threshold strategy with attack threshold \( \hat{x}_{i2} \), where \( \Pr\{\theta_2 < \hat{\theta}_{2u}(\hat{x}_2)|\theta_1, \hat{x}_{i2}\} = \frac{1}{1+b/l} \), implying \( r' > 0 \). Therefore, our focus on monotone equilibria is valid and we now determine conditions sufficient for a unique monotone Bayesian equilibrium.

Although the case of uninformed speculators is more complicated because of the presence of the mixture distribution, we are able to show that there exists a unique equilibrium in threshold strategies if the relative precision of the private information is sufficiently high. Formally, \( G(\theta_{2u}^*, \theta_1) \) is monotonically decreasing in \( \theta_{2u}^* \) if \( \gamma \) exceeds a finite threshold \( \gamma_2 \in (0, \infty) \) as summarized by Proposition \( \Pi \).
Proposition 1 **Unique Bayesian equilibrium.** Suppose speculators observe a crisis in country 1, i.e. \( \theta_1 < \theta_1^* \), but are uninformed about the cross-country correlation, \( \gamma = 0 \). If private information is sufficiently precise, \( \gamma > \gamma_2 \), then there exists a unique monotone equilibrium in country 2. Each uninformed speculator attacks if and only if her private signal is smaller than the threshold \( x_{2U}^* \), such that a speculative currency crisis occurs if and only if the realized fundamental in country 2 is smaller than the threshold \( \theta_{2U}^*(\theta_1) \) defined by equation (17).

Furthermore, the equilibrium threshold \( \theta_{2U}^*(\theta_1) \) is a weighted average of the thresholds that prevail if speculators were informed:

\[
\min\{\theta_{2I}^*(0, \theta_1), \theta_{2I}^*(\rho_H, \theta_1)\} < \theta_{2U}^*(\theta_1) < \max\{\theta_{2I}^*(0, \theta_1), \theta_{2I}^*(\rho_H, \theta_1)\}.
\] (18)

**Proof** See Appendix B.5.

This concludes the equilibrium analysis of the coordination stage in country 2 when speculators are endowed with symmetrical information about the cross-country correlation. The equilibrium behavior is summarized in Corollary 2 (if informed) and Proposition 1 (if uninformed). These results, together with the threshold ranking in Lemma 2, allow us to establish our main results on the contagion of financial crises in the subsequent section.

4 Contagion

Contagion is defined as an increase in the probability of a currency crisis in country 2 after a currency crisis in country 1. Here we establish that contagion occurs even if speculators in country 2 learn, prior to their currency attack decision, that the fundamentals of both countries are unrelated. Therefore, a crisis can spread contagiously across countries even if speculators in country 2 are completely insulated from the crisis in country 1. We establish three results on the contagion of financial crises in this section.

Our main result is that contagion can occur ex-post, that is, for a given realization of zero correlation of fundamentals across countries, \( \rho = 0 \). We demonstrate that the probability of a
crisis in country 2 can be higher when speculators are informed about the cross-country correlation than when uninformed. Despite learning supposedly ‘good news’ about the fundamental in country 2, that is no exposure to the crisis country, speculators attack the currency in country 2 more aggressively than without news about the cross-country correlation. Since this effect is obtained for a given realization \( \rho = 0 \), we call it ex-post contagion and analyze it in section 4.1.

Our second result is that contagion also occurs from an ex-ante perspective. The probability of a crisis in country 2 can be higher after observing a crisis in country 1 than after observing no crisis, despite the fact that speculators learn that there is zero exposure to the crisis country. Since this result holds for various cross-country correlations in the no-crisis case, we call it ex-ante contagion and analyze it in section 4.2.

We conclude this section by comparing both of our contagion effects to the previously established information contagion channel. The key difference between our novel contagion effects and an information contagion channel is that the latter is only conditioned on whether or not there is a crisis in country 1. In contrast, we show that contagion can occur after a crisis in country 1 despite speculators learning that there is no exposure to the crisis country.

### 4.1 Ex-post contagion

If there is a currency crisis in country 1, the ability of the monetary authority to defend the currency in country 1 must be low, \( \theta_1 < \theta_1^* < \mu \), where the second inequality follows from Corollary 4. Since the cross-country correlation of fundamentals is potentially positive, country 2’s fundamental may also be low. Hence, learning that the fundamentals are not correlated across countries may be considered as making country 2 more resilient to a currency attack. We show that this conjecture is incorrect and that contagion can occur even when fundamentals are uncorrelated ex-post.

If fundamentals are uncorrelated across countries, \( \rho = 0 \), the likelihood of a currency crisis with informed speculators can be higher than with uninformed speculators. In other words, financial stability – the absence of a speculative currency crisis – improves if speculators do not learn about the (zero) cross-country correlation and believe that a positive correlation, and thus exposure to the crisis country, is possible. More precisely, the likelihood of a currency crisis in country 2 at
the beginning of the second date can be higher when all speculators are informed, i.e. \( n = 1 \), and learn about the cross-country correlation, compared to the case of uninformed speculators, \( n = 0 \). One implication of this *ex-post contagion* result is that no information about the cross-country exposure supports financial stability, which is an uninformed-is-bliss feature.\(^{20}\) As summarized in Proposition 2, ex-post contagion is present for a strong prior about the fundamental and therefore a large degree of *strategic uncertainty* among currency speculators.

**Proposition 2** *Ex-post contagion*. Consider the case in which a currency crisis occurs in country 1, i.e. \( \theta_1 < \theta_1^* \), but in which there is no correlation of fundamentals across countries, i.e. \( \rho = 0 \). Suppose the fundamental in country 2 is strong, the fundamental in country 1 satisfies \( \theta_1 > \theta_1^* \), and \( \gamma > \max\{\gamma_1, \gamma_2\} \). In this case, a currency crisis in country 2 is more likely when speculators are informed about the cross-country correlation than when they are uninformed:

\[
\Pr (\theta_2 < \theta_{2I}^*(\rho, \theta_1)|\rho = 0, \theta_1) > \Pr (\theta_2 < \theta_{2U}^*(\rho, \theta_1)|\rho = 0, \theta_1), \quad \forall \theta_1 \in (\theta_1^*, 1). \tag{19}
\]

**Proof** First, \( \gamma > \max\{\gamma_1, \gamma_2\} < \infty \) meets the sufficient conditions of Corollary and Proposition, so \( \theta_{2I}^*(\rho, \theta_1) \) and \( \theta_{2U}^*(\rho, \theta_1) \) are unique. Second, we have the threshold ranking \( \theta_{2I}^*(0, \theta_1) > \theta_{2I}^*(\rho_H, \theta_1) \) under the sufficient conditions of Lemma, i.e., an intermediate realized fundamental in country 1, \( \theta_1 \in (\theta_1^*, 1) \), and a strong prior about the fundamental in country 2 according to Definition. Third, using the result of the weighted average of Proposition and noting that the weight satisfies \( \bar{\rho} \in (0, 1) \), it follows directly that \( \min\{\theta_{2I}^*(0, \theta_1), \theta_{2I}^*(\rho_H, \theta_1)\} < \theta_{2U}^*(\rho, \theta_1) < \max\{\theta_{2I}^*(0, \theta_1), \theta_{2U}^*(\rho_H, \theta_1)\} \). Combined with the second point, we have the following ranking of thresholds: \( \theta_{2I}^*(0, \theta_1) > \theta_{2U}^*(\rho, \theta_1) \forall \theta_1 \in (\theta_1^*, 1) \). Finally, given that the realized cross-country correlation of fundamentals is \( \rho = 0 \), the ordering of thresholds implies that the probability of a currency crisis in country 2 is higher when speculators are informed than when they are uninformed. (*q.e.d.*)

The key to understanding Proposition is Figure 1. If the cross-country correlation is potentially positive, then a currency crisis in country 1 reduces both the mean and the variance of the

\(^{20}\)More information can lead to adverse outcomes in Hirshleifer (1971). Information acquisition can be privately optimal but has a negative public value, since it makes co-insurance for risk-averse agents infeasible. Instead, Morris and Shin (2007) analyze optimal communication and provide a rationale for coarse information, for instance in credit ratings. Dang et al. (2012) provide an "ignorance-is-bliss" argument, whereby information insensitivity is key to security design in the money market. More transparency can also be harmful in an expert model with career concerns Prat (2005).
updated prior about the fundamental in country 2. The overall effect on the equilibrium threshold depends on the relative size of the mean effect versus the variance effect, since these move in opposite directions. Thus, a strong variance effect, as described in Lemma 2, is at the heart of the ordering of equilibrium thresholds and leads to the ex-post contagion result. Below, we provide additional details of, and intuition for, the mean and variance effects.21

Mean effect  It is well known in the literature (for instance, Morris and Shin (2003), Vives (2005), and Manz (2010)) that adverse public information, a lower prior \( \mu_2(\rho, \theta_1) \), raises the equilibrium threshold. As such, a higher cross-country correlation causes a decrease in the updated prior of the fundamental in country 2, conditional on observing \( \theta_1 < \mu \) as insured by the strong prior (see Corollary I and Definition I). Consequently, observing zero cross-country correlation increases the mean of the updated prior and lowers \( \theta_{2I}^*(0) \) relative to \( \theta_{2I}^*(\rho_H) \), so the mean effect always works against our ex-post contagion result.

Variance effect  The equilibrium thresholds also depend on the precisions of the private and public information, as Metz (2002) first analyzed in detail. The variance effect arises because the cross-country correlation \( \rho \) affects the precision of the updated prior \( \sigma_2(\rho) \). If the prior about the fundamental is strong, then the equilibrium threshold is below the mean of the updated prior about the fundamentals in country 2, \( \theta_{2I}^*(\rho, \theta_1) < \mu_2(\rho, \theta_1) \). Given the strong prior about fundamentals (Definition I), speculators in general only attack the currency of country 2 if their private signal is very low. Then, a relative decrease in the precision of public information (because speculators learn \( \rho = 0 \)) increases strategic uncertainty, as reflected by a more dispersed belief about other speculators’ posteriors. Thus, speculators are more concerned about other speculators receiving adverse private signals and attacking the currency. This shift in beliefs about other speculators’ posteriors induces more aggressive speculative attacks and increases \( \theta_{2I}^*(0, \theta_1) \) relative to \( \theta_{2I}^*(\rho_H, \theta_1) \).

If the conditions in Lemma 2 hold, then the variance effect outweighs the mean effect and the ordering of equilibrium fundamental thresholds is \( \theta_{2I}^*(0, \theta_1) > \theta_{2U}^*(\theta_1) \). Given that the true state of the world is \( \rho = 0 \), there is a one-to-one mapping between the ordering of equilibrium thresholds and

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21 In Appendix B.2, we analyze the comparative statics of the equilibrium threshold and the dependence on the mean and variance effects.
the likelihood of a currency crisis. As a consequence, a currency crisis can contagiously spread from one country to another despite the public information about zero cross-country correlation ex-post (Proposition 2). We underscore this result in section 5 by demonstrating that ex-post contagion arises in equilibrium when speculators can acquire costly information about the correlation.

Despite the risk-neutrality of all speculators, the variance effect matters because of strategic uncertainty. Higher-order moments help to predict the equilibrium behavior of other speculators and are therefore payoff-relevant information in the incomplete-information coordination game between speculators. If the variance effect is strong, a currency crisis can contagiously spread from one country to another despite the common knowledge of zero cross-country correlation.

4.2 Ex-ante contagion

After establishing the ex-post contagion result, we show that contagion can also occur ex-ante. A crisis in country 1 spreads contagiously despite speculators learning ‘good news’ about the fundamentals in country 2 in the form of no exposure to the crisis country. More precisely, ‘good news’ about the fundamentals in country 2, $\rho = 0$, after ‘bad news’ (a crisis elsewhere) results in a higher likelihood of a crisis than if no crisis had been observed elsewhere in the first place. This is because observing no crisis elsewhere suggest, that the fundamentals in country 1 must have been rather good and, thus, the fundamentals of country 2 are more likely to be good because the cross-country correlation is potentially positive. The ex-ante contagion result has a wake-up call feature since speculators in country 2 have a private incentive to acquire information about the cross-country correlation after a crisis in country 1, as demonstrated in section 5.

The result is summarized by Proposition 3. Observe that the left-hand side of the inequality in equation (22) is unchanged relative to Proposition 2, while the right-hand side now allows for either realization of the cross-country fundamental $\rho \in \{0, \rho_H\}$ and is only conditional on not observing a crisis in country 1, $\theta_1 \geq \theta_1^*$, Here, we use the assumption that $\theta_1$ is publicly observed only in the case of a currency crisis in country 1.

Using the events $E_1 = \theta_2 < \theta_{2U}^*$ and $E_2 = \theta_1 \geq \theta_1^*$, the ex-ante probability of a crisis in country 2 after not observing a crisis in country 1 is decomposed by the law of total probability as
follows:
\[
\Pr\{E_1|E_2\} = p \Pr\{E_1|\rho = 0, E_2\} + (1 - p) \Pr\{E_1|\rho = \rho_H, E_2\}.
\] (20)

Observe that this result does not require \(\theta_1 > \theta_1\).

**Proposition 3  Ex-ante contagion** Consider the case of a strong fundamental in country 2, a sufficiently precise private signal, and \(\theta_1^* \geq \theta_1\) as assured by conditions (13) and (14) of Lemma 2. If the following sufficient condition holds:
\[
\frac{1 - \rho_H^2}{\alpha} \leq \left[\theta_{21}^*(0) + \rho_H \theta_1 - (1 + \rho_H)\mu\right]^2,
\] (21)

with \(\theta_{21}^*(0)\) defined by equation (14), then a crisis in country 2 is more likely after observing a crisis in country 1 than after observing no crisis in country 1, even if all speculators in country 2 learn \(\rho = 0\):
\[
\Pr\{\theta_2 < \theta_{21}^*|\rho = 0, \theta_1 < \theta_1^*\} > \Pr\{\theta_2 < \theta_{21}^*|\theta_1 \geq \theta_1^*\}.
\] (22)

**Proof** See Appendix B.6.

We can again use the mean and variance effects to gather intuition for this result. First, if there is zero realized cross-country correlation, then the conditioning on a crisis does not matter and the probabilities are the same. Second, if the fundamentals are correlated, then the conditional probability on the right-hand side differs from the left-hand side for two reasons. On the one hand, the public information about the fundamental is more precise, such that strategic uncertainty is reduced under the sufficient conditions of Lemma 2. Therefore, the variance effect always works in favor of the stated inequality. On the other hand, learning that no crisis occurred in country 1 shifts the mean of the updated prior. However, the mean effect is ambiguous in general since realizations \(\theta_1 \in [\theta_1^*, \mu]\) are consistent with both no crisis in country 1 and ‘bad news’ about the fundamental in country 2. The stated sufficient condition ensures that this part of the mean effect for \(\theta_1 \in [\theta_1^*, \mu]\) is always dominated by the variance effect and the other part of the mean effect for \(\theta_1 \in [\mu, \infty)\), which in turn ensures ex-ante contagion. The sufficient condition in equation (21) is more likely to hold, the stronger the prior belief about the fundamentals in country 2 and the less precise public information since both are associated with a low \(\theta_{21}^*(0)\).
Comparison to information contagion  Here we contrast our contagion results from an information contagion channel established by Acharya and Yorulmazer (2008). They show that the funding costs of one bank increases after bad news about another bank if the banks’ loan portfolio returns have a common factor. To avoid such information contagion ex-post, banks optimally herd their investment ex-ante. Allen et al. (2012) compare the impact of information contagion on systemic risk across asset structures, whereby adverse news about aggregate solvency of the banking system leads to runs on multiple banks. While these papers also study the consequences for the ex-ante portfolio choices of banks, we can capture the notion of ex-post information contagion based on correlated fundamentals here with the following simple comparative static in a special case.

Suppose speculators in country 2 are informed about the cross-country correlation and this takes the value \( \rho = \rho_H \). Whenever a crisis occurs in country 1, i.e. \( \theta_1 < \theta^*_1 \), then the probability of a currency crisis in country 2 is higher the worse the realized fundamental in country 1:

\[
\frac{d\theta^*_2(\rho_H, \theta_1)}{d\theta_1} < 0.
\]  

(23)

This comparative static is a simple ex-post information contagion channel based on correlated fundamentals. The result follows directly from the comparative statics derived for country 1 in Appendix B1. A lower observed fundamental in the crisis country implies that the fundamental in country 2 is now more likely to be low as well due to positive cross-country correlation. As a result, speculators attack more aggressively, thereby raising the equilibrium threshold.

Our contagion results contrast sharply with the information contagion channel. Information contagion arises in these papers after adverse news when fundamentals are correlated across countries. In contrast, the contagion results in Propositions 4 and 5 arise when speculators learn that there is zero cross-country correlation. We analyze the implications of ex-ante uncertainty about the cross-country correlation of fundamentals. Crucially, contagion can occur after a crisis in country 1 even if speculators know ex-post that there is no exposure to the crisis country.
5 Information acquisition

In this section, we present a wake-up call theory of contagion. Observing a crisis in country 1 is a wake-up call for speculators in country 2. This induces speculators to acquire information about the cross-country correlation in order to determine the exposure to the crisis country. Contagion can occur even if speculators observe zero cross-country correlation. Therefore, our ex-post and ex-ante contagion results stated in Propositions 2 and 3, respectively, obtain as an equilibrium phenomenon when information is endogenous.

We use Perfect Bayesian Equilibrium (PBE) as a solution concept and formally define it below. Recall that date 2 is divided into an information stage and a coordination stage. To analyze the private incentives to acquire information, we start by extending our previous analysis of the coordination stage to the case of asymmetrically informed speculators. Let \( d_i \in \{I, U\} \) denote whether speculator \( i \) acquires information and becomes informed (I) or whether she does not acquire information and stays uninformed (U). We solve for the optimal attacking rule \( a^*_2 \) in the coordination stage in section 5.1. We prove the existence and uniqueness of such a rule for a given proportion of informed speculators in Lemma 3. Furthermore, we analyze the dependence of the optimal attacking rule on the proportion of informed speculators in Lemma 4. Thereafter, we solve for the optimal information acquisition rule \( d^*_i \) in section 5.2.

Definition 2 A pure-strategy Perfect Bayesian Equilibrium in threshold strategies consists of an information acquisition rule \( d^*_i \in \{I, U\} \) for each speculator \( i \in [0, 1] \), an aggregate proportion of informed speculators \( n^* \in [0, 1] \), an attacking decision rule \( a^*_2(n^*; \theta_1, x_{i2}) \in \{0, 1\} \), and an aggregate proportion of attacking speculators \( A^*_2 \in [0, 1] \) such that:

1. All speculators optimally choose \( d_i \) in the information stage given \( n^* \).

2. The proportion \( n^* \) is consistent with the optimal choices implied by step 1:

\[
n^* = \int_0^1 1\{d^*_i = I\} di. \tag{24}
\]
3. Uninformed speculators make optimal attack decision in the coordination stage:

\[ a_{2U}^*(n^*; \theta_1, x_{i2}) = \arg \max_{a_{2U} \in \{0,1\}} E[u(n^*; a_{2U}, A_2, \theta_2, \theta_1, \rho)|x_{i2}] \] (25)

and informed speculators make optimal attack decision in the coordination stage:

\[ a_{2I}^*(n^*; \theta_1, \rho, x_{i2}) = \arg \max_{a_{2I} \in \{0,1\}} E[u(n^*; a_{2I}, A_2, \theta_2, \theta_1, \rho)|x_{i2}, \rho] \] (26)

for any given realization of \( \rho \in \{0, \rho_H\} \).

4. The aggregate mass of attacking speculators in the coordination stage, \( A_2^* \), is consistent with the optimal individual attacking decisions:

\[
A_2^* = A(n^*; \theta_2, \rho) = n^* \int_{-\infty}^{+\infty} a_{2I}^*(n^*; \theta_1, \rho, x_{i2}) \sqrt{\gamma} \phi(\sqrt{\gamma}(x_{i2} - \theta_2))dx_{i2} \\
+ (1 - n^*) \int_{-\infty}^{+\infty} a_{2U}^*(n^*; \theta_1, x_{i2}) \sqrt{\gamma} \phi(\sqrt{\gamma}(x_{i2} - \theta_2))dx_{i2} \] (27)

for either realization of the cross-country correlation \( \rho \in \{0, \rho_H\} \).

5.1 Stage 2: optimal attacking rules

We start with the coordination stage of country 2 and generalize our analysis of section 3.4 by allowing for asymmetrically informed speculators. That is, a fraction \( 0 \leq n \leq 1 \) of speculators knows the realization of the cross-country correlation \( \rho \) (informed speculators), while the remainder is uninformed. In equilibrium, \( n^* \) is known at the coordination stage and each individual speculator cannot affect it with her decision. Therefore, the assumption of common knowledge about \( n \) made in section 4 is no longer required. Acquiring information is a dominant action given the conditions of Proposition 4.

Maintaining our focus on monotone equilibria, each speculator attacks if her private signal is below a threshold specific to her information choice. There are now three thresholds: one for uninformed speculators \( x_{2U}^*(n, \theta_1) \) and two for informed speculators \( x_{2I}^*(n, \rho, \theta_1) \), that is one for each realization of the cross-country correlation \( \rho \in \{0, \rho_H\} \). Now, the fundamental threshold is
also a function of the proportion of informed speculators and, as before, depends on the realization of $\rho$. Thus, there are two equilibrium fundamental thresholds $\theta_2^*(n, \rho, \theta_1)$. The equilibrium in the coordination stage can be described by two equations in two unknowns $\theta_2^*(n, 0, \theta_1)$ and $\theta_2^*(n, \rho_H, \theta_1)$, which we describe in detail in Appendix B.7. Intuitively, we recover our previous results for the limiting cases of uninformed (informed) speculators as the proportion of informed speculators converges to zero (one).

We now analyze whether an optimal attacking rule exists and when it is unique. This is complicated by the presence of asymmetrically informed speculators. More specifically, we now must keep track of the interaction of uninformed speculators, characterized by a mixture distribution, with informed speculators. However, we are able to prove in Lemma 3 that there exists a unique attacking rule for sufficiently precise private information.

**Lemma 3 Optimal attacking rule in the coordination stage.** Suppose speculators observe a crisis in country 1, i.e. $\theta_1 < \theta_1^*$, and a given proportion $n \in [0, 1]$ is informed about the cross-country correlation $\rho$. If private information is sufficiently precise, then there exists a unique attacking rule. Each uninformed speculator attacks if and only if $x_2 < x_{2U}^*(n, \theta_1)$. Similarly, each informed speculator attacks if and only if $x_2 < x_{2I}^*(n, \rho, \theta_1)$ after learning the realization of $\rho$. A currency crisis occurs if and only if fundamentals are low, $\theta_2 < \theta_2^*(n, \rho, \theta_1)$.

**Proof** See Appendix B.8.

We analyze next how the fundamental thresholds depend on the proportion of informed speculators. This question is crucial for understanding the private incentives to acquire information at the information stage described in section 5.2. We have already established the polar cases of completely uninformed and informed speculators, $n \in \{0, 1\}$. Proposition 1 shows that the fundamental threshold is independent of the realized cross-country correlation if all speculators are uninformed $\theta_{2U}^*(\theta_1)$. The fundamental threshold varies with the realized cross-country correlation if all speculators are informed $\theta_{2I}^*(\theta_1)$. The fundamental threshold is independent of the realized cross-country correlation if all speculators are informed (Corollary 2), where Lemma 2 establishes sufficient conditions for $\theta_{2I}^*(0, \theta_1) > \theta_{2I}(\rho_H, \theta_1)$, including $\theta_1 > \theta_1$. By contrast, $\theta_{2I}^*(0, \theta_1) < \theta_{2I}(\rho_H, \theta_1)$ if $\theta_1 < \theta_1$ and $\theta_{2I}^*(0, \theta_1) = \theta_{2I}^*(\rho_H, \theta_1)$ if $\theta_1 = \theta_1$. 

25
In Lemma 4, we establish conditions sufficient for the fundamental thresholds to evolve continuously and monotonically as the proportion of informed speculators changes. As a consequence, the distance between both fundamental thresholds as well as between both attacking thresholds evolve monotonically as well. More specifically, the distance between the fundamental thresholds, 
 
\[ |\theta^*_2(n, 0, \theta_1) - \theta^*_2(n, \rho_H, \theta_1)|, \]

continuously and monotonically increases in the proportion of informed speculators \( n \). This property is crucial for establishing the strategic complementarity in information acquisition choices in section 5.2 that, in turn, allows us to obtain our two contagion results with endogenous acquisition of information.

**Lemma 4 More informed speculators and the equilibrium thresholds.** Consider the case of a currency crisis in country 1, i.e. \( \theta_1 < \theta^*_1 \), and strong fundamentals in country 2. If private information is sufficiently precise, i.e. \( \gamma < \gamma < \infty \), and public information is sufficiently imprecise, i.e. \( 0 < \alpha < \overline{\alpha} \), then we have the following results:

(A) **Boundedness.** The fundamental thresholds in the polar case of informed speculators bound the fundamental thresholds in the general case of asymmetrically informed speculators:

\[
\begin{align*}
\text{if } \theta_1 &\geq \theta^*_1 : & \theta^*_2(\rho_H, \theta_1) &\leq \theta^*_2(n, \rho, \theta_1) \leq \theta^*_2(0, \theta_1) \quad \forall \rho \in \{0, \rho_H\} & \forall n \in [0, 1] \\
\text{if } \theta_1 &< \theta^*_1 : & \theta^*_2(0, \theta_1) &\leq \theta^*_2(n, \rho, \theta_1) \leq \theta^*_2(\rho_H, \theta_1) \quad \forall \rho \in \{0, \rho_H\} & \forall n \in [0, 1].
\end{align*}
\]

(B) **Monotonicity.** The fundamental threshold in the case of zero (positive) cross-country correlation increases (decreases) in the proportion of informed speculators. Strict monotonicity is attained if and only if the fundamental thresholds are strictly bounded:

\[
\frac{d\theta^*_2(n, 0, \theta_1)}{dn} = \begin{cases} 
> 0 & \text{if } \theta^*_2(\rho_H, \theta_1) < \theta^*_2(n, \rho, \theta_1) < \theta^*_2(0, \theta_1) \\
< 0 & \text{if } \theta^*_2(0, \theta_1) < \theta^*_2(n, \rho, \theta_1) < \theta^*_2(\rho_H, \theta_1) \quad \forall \rho, \quad n \in [0, 1]. \\
= 0 & \text{if } \theta^*_2(\rho, \theta_1) = \theta^*_2(n, \rho, \theta_1)
\end{cases}
\]
\[
\frac{d\theta_2^*(n, \rho_H, \theta_1)}{dn} \begin{cases} 
< 0 & \text{if } \theta^*_2(0, \rho, \theta_1) < \theta^*_2(n, \rho, \theta_1) < \theta^*_2(0, \theta_1) \\
> 0 & \text{if } \theta^*_2(0, \theta_1) < \theta^*_2(n, \rho, \theta_1) < \theta^*_2(\rho_H, \theta_1) \quad \forall \rho, \ n \in [0, 1). \\
= 0 & \text{if } \theta^*_2(\rho, \theta_1) = \theta^*_2(n, \rho, \theta_1)
\end{cases}
\] (29)

(C) **Monotonicity in attacking thresholds.** As a consequence of the monotonicity in fundamentals thresholds:

\[
\frac{d|x^*_2(n, 0, \theta_1) - x^*_2(n, \rho_H, \theta_1)|}{dn} \geq 0 \quad \forall \ n \in [0, 1). \tag{30}
\]

**Proof** See Appendix B.9.

The analytical conditions stated in Lemma 4 are sufficient but not necessary for the results. Indeed, less restrictive conditions are consistent with the monotonicity results, as revealed by a numerical analysis. For example, strict monotonicity obtains for both fundamental thresholds in Figure 2 in Appendix A.2, which uses as parameters a public signal precision of \( \alpha = 1 \) (substantially positive) and a private signal precision of \( \gamma = 1.5 \) (quite small). Then, ex-post contagion occurs as described in Proposition 2, \( \theta^*_2(0, \theta_1) > \theta^*_2(\theta_1) > \theta^*_2(\rho_H, \theta_1) \). As depicted in Figure 2, this threshold ranking holds for any strictly positive proportion of informed speculators.

These results are the consequence of informed speculators capitalizing on their information advantage. While uninformed speculators must use the same attack threshold irrespective of the realized correlation, informed speculators adjust their attack thresholds accordingly. Therefore, the fundamental thresholds for \( \rho = 0 \) and \( \rho = \rho_H \) diverge as the proportion of informed speculators increases. Intuitively, a larger proportion of informed speculators raises the fundamental threshold \( \theta^*_2(n, 0, \theta_1) \) because informed speculators attack more aggressively after learning \( \rho = 0 \), compared with uninformed speculators (Part (a) of Lemma 4; see thick dotted line in Figure 2). The opposite is true in the case of positive correlation, \( \rho = \rho_H \), because informed speculators attack less aggressively compared to uninformed speculators. Consequently, the fundamental threshold \( \theta^*_2(n, \rho_H, \theta_1) \) decreases in the proportion of informed speculators (see thick dashed line in Figure 2), whereas the
difference between the thresholds increases in \( n \) (Part (b) of Lemma \( \mathfrak{I} \)).

### 5.2 Stage 1: optimal information acquisition

Having derived the optimal attacking rule in the coordination stage of country 2, we analyzed the dependence of the fundamental thresholds on the proportion of informed speculators \( n \in [0, 1] \). Next, we turn to the information stage to solve for pure-strategy PBE. We start by showing that the information acquisition choices of speculators exhibit strategic complementarity. This allows us to demonstrate that there exists a unique equilibrium in which all speculators acquire information if the information cost is sufficiently small. This equilibrium features both the ex-post contagion effect (Proposition \( \mathfrak{E} \)) and the ex-ante contagion effect (Proposition \( \mathfrak{F} \)).

We continue to focus on the case in which a currency crisis occurs in country 1, \( \theta_1 < \theta^*_1 \). Thus, speculators in country 2 observe the fundamental \( \theta_1 \) before deciding whether or not to acquire costly information about the cross-country correlation \( \rho \). Such information helps a speculator assess the exposure to the crisis country. Recall that the purchased information is a perfect signal about the realized \( \rho \) and that the additional signal is publicly available to all speculators at a cost. We also maintain the sufficient conditions of Lemma \( \mathfrak{G} \) so \( \theta^*_1 < \theta_1 < \theta^*_1(0) \) and \( \theta^*_1(\rho_H) \).

Consider the information choice of an individual speculator \( i \) who takes the proportion of informed speculators \( n \) as given. She compares the expected payoffs from becoming informed, \( d_i = I \), and from remaining uninformed, \( d_i = U \). The expected utility of an informed speculator comprises the information cost, the benefit from attacking if a currency crisis occurs, and the cost of attacking if no currency crisis occurs. She also takes the possible realizations of the cross-country correlation into account:

\[
\begin{align*}
\text{E}[u(d_i = I, n)] & \equiv EU_I - c \\
& = -c + p \left( b \int_{-\infty}^{\theta^*_2(n,0,\theta_1)} \int_{\rho_2 \leq \rho_2(n,\rho_H,\theta_1)} g(x_i \rho_2|\theta_2) dx_i \rho_2 f(\theta_2|0, \theta_1) d\theta_2 \right) \\
& \quad + (1 - p) \left( b \int_{-\rho_2(0,0,\theta_1)}^{\rho_2(\rho_H, \theta_1)} \int_{\rho_2 \leq \rho_2(n,\rho_H,\theta_1)} g(x_i \rho_2|\theta_2) dx_i \rho_2 f(\theta_2|\rho_H, \theta_1) d\theta_2 \right). 
\end{align*}
\]
where the distribution of the fundamental in country 2 conditional on the realized cross-country correlation, $f(\theta_2|\rho, \theta_1)$, is normally distributed with mean $\mu_2(\rho, \theta_1)$ and with precision $\alpha_2(\rho)$, while the distribution of the private signal conditional on the fundamental, $g(x|\theta_2)$, is normally distributed with mean $\theta_2$ and precision $\gamma$.

Crucially, an informed speculator varies her attacking threshold with the observed cross-country correlation using the thresholds $x_{2I}^*(n; 0, \theta_1)$ and $x_{2I}^*(n; \rho_H, \theta_1)$. By contrast, an uninformed speculator cannot tailor her attacking strategy and must use the same attacking threshold $x_{2U}^*(n, \theta_1)$ irrespective of the realized cross-country correlation. Therefore, the expected utility of an uninformed speculator who does not pay the information cost reads as:

$$E[u(d_i = U, n)] \equiv EU_U$$

$$= p \left( \int_{x_{2U}^*(n, \theta_1)} b f_{\theta_2}^{(-\infty, n, 0, \theta_1)}(\theta_2|0, \theta_1) d\theta_2 \right)$$

$$+ (1 - p) \left( \int_{x_{2U}^*(n, \rho_H, \theta_1)} b f_{\theta_2}^{(+\infty, n, \rho_H, \theta_1)}(\theta_2|\rho_H, \theta_1) d\theta_2 \right).$$

Next, we show that the monotonicity in attack thresholds established in Lemma 4 (C) leads to strategic complementarity in information acquisition choices: a speculator’s incentive to acquire information increases in the proportion of informed speculators. Recall that informed speculators tailor their attack strategy by using different thresholds for each realization of the cross-country correlation, while the uninformed speculators must use the same attack threshold throughout. Optimality at the information stage requires that a given speculator acquires information if the expected utility differential $EU_I - EU_U$ is no smaller than the information cost:

$$d_i^* = I \Leftrightarrow EU_I - EU_U \geq c.$$ 

In other words, it pays to acquire information about the cross-country correlation if the benefit from using tailored attacking thresholds covers at least the information cost. Let the highest possible information cost $\bar{c}(n, \theta_1) \equiv EU_I - EU_U$ make a speculator indifferent between becoming informed
and remaining uninformed about the cross-country correlation.

\[
\bar{c}(n, \theta_1) = p \left( \int^\theta_2_{(n,0,\theta_1)} \int^{x_{2U}^*(n,0,\theta_1)}_\psi g(x_{12}|\theta_2) dx_{12} f(\theta_2|0, \theta_1) \theta_2 - \int^{\psi^*}_{x_{2U}^*(n,0,\theta_1)} \int^{\theta_2}_{-\infty} g(x_{12}|\theta_2) dx_{12} f(\theta_2|0, \theta_1) \theta_2 \right) - (1-p) \left( \int^\theta_2_{\theta_2(n,\rho_H,\theta_1)} \int^{x_{2U}^*(n,\rho_H,\theta_1)}_{\psi^*} g(x_{12}|\theta_2) dx_{12} f(\theta_2|\rho_H, \theta_1) \theta_2 - \int^{\psi^*}_{x_{2U}^*(n,\rho_H,\theta_1)} \int^{\theta_2}_{\theta_2(n,\rho_H,\theta_1)} g(x_{12}|\theta_2) dx_{12} f(\theta_2|\rho_H, \theta_1) \theta_2 \right),
\]

To gain intuition, consider the benefit from using a tailored attacking threshold. The marginal benefit of an informed speculator from increasing her attack threshold \( \hat{x}_{2I}(n, \rho, \theta_1) \) is given by:

\[
b \int^{\theta^*_2(n,\rho,\theta_1)}_{-\infty} g(\hat{x}_{2I}(n, \rho, \theta_1)|\theta_2) f(\theta_2|\rho, \theta_1) \theta_2 - l \int^{+\infty}_{\theta^*_2(n,\rho,\theta_1)} g(\hat{x}_{2I}(n, \rho, \theta_1)|\theta_2) f(\theta_2|\rho, \theta_1) \theta_2,
\]

which is zero when evaluated at \( \hat{x}_{2I}(n, \rho, \theta_1) = x_{2I}^*(n, \rho, \theta_1) \forall \rho \in \{0, \rho_H\} \). Furthermore, equation (35) is monotonically decreasing in \( \hat{x}_{2I}(n, \rho, \theta_1) \) because:

\[
dg(d\hat{x}_{2I}(n, \rho, \theta_1)|\theta_2) = \begin{cases} > 0 & \text{if } \hat{x}_{2I}(n, \rho, \theta_1) < \theta_2 \\ < 0 & \text{if } \hat{x}_{2I}(n, \rho, \theta_1) > \theta_2 \end{cases}
\]

and \( \lim_{\gamma \to \infty} x_{2I}^*(n, \rho, \theta_1) = \theta^*_2(n, \rho, \theta_1) \forall \rho \in \{0, \rho_H\} \).

Recall that \( x_{2I}^*(n, 0, \theta_1) > x_{2U}^*(n, \theta_1) > x_{2I}^*(n, \rho_H, \theta_1) \) when \( \theta_{2I}(0) > \theta^*_2(\rho_H) \). Hence, the marginal benefit from increasing \( x_{2I}^*(n, 0, \theta_1) \) above \( x_{2U}^*(n, \theta_1) \) is:

\[
p \left( b \int^{\theta^*_2(n,0,\theta_1)}_{-\infty} g(x_{2U}^*|\theta_2) f(\theta_2) \theta_2 - l \int^{\theta^*_2(n,0,\theta_1)}_{-\infty} g(x_{2U}^*|\theta_2) f(\theta_2) \theta_2 \right) > 0
\]

while the marginal benefit from increasing \( x_{2I}^*(n, \rho_H, \theta_1) \) above \( x_{2U}^*(n, \theta_1) \) is given by:

\[
(1-p) \left( b \int^{\theta^*_2(n,\rho_H,\theta_1)}_{-\infty} g(x_{2U}^*|\theta_2) f(\theta_2|\rho_H, \theta_1) \theta_2 - l \int^{+\infty}_{\theta^*_2(n,\rho_H,\theta_1)} g(x_{2U}^*|\theta_2) f(\theta_2|\rho_H, \theta_1) \theta_2 \right) < 0.
\]

These expressions are best understood in terms of type-I and type-II errors. Let the null hypothesis be that there is a crisis in country 2, such that \( \theta_2 < \theta^*_2 \). Each of the expressions in equations (37) and...
Lemma 4 together with Proposition 1 imply the following. After a crisis in country 1, we have
for strong fundamentals in country 2, a sufficiently precise private information, and a sufficiently
imprecise public information that $\theta_{2I}(n, \rho_H, \theta_1) < \theta_{2I}(n, 0, \theta_1) \forall \ n \in [0, 1]$ if $\theta_1 \in (\theta_1, \theta_1^*)$. Hence,
the marginal benefit from increasing $x_{2I}(n, 0, \theta_1)$ above $x_{2U}(n, \theta_1)$ is positive because the type-I
error is more costly than the type-II error. By contrast, the marginal benefit from decreasing
$x_{2I}(n, \rho_H, \theta_1)$ below $x_{2U}(n, \theta_1)$ is positive because the type-II error is relatively more costly.

How is the threshold information cost affected as the proportion of informed speculators increases? Under the sufficient conditions of Lemma 2, we find that $\theta_{2I}(0) > \theta_{2I}(\rho_H)$. Therefore, an
increase in the proportion of informed speculators is associated with a (weak) increase in both $\theta_2^*(0)$
and $x_2^*(0)$ as well as a (weak) decrease in both $\theta_2^*(\rho_H)$ and $x_2^*(\rho_H)$. Furthermore, $x_{2U}(n, \theta_1)$ is unaffected. An increase in $n$ leads to a relative increase of the benefit component in the first summand
of equation (34) and a relative increase of the loss component in the second summand. For this
reason, the left-hand side of equation (34) increases in $n$, giving rise to a strategic complementarity
in information choices. The result is generalized and stated formally in Lemma 5.

Lemma 5 Strategic complementarity in acquiring information. Consider the case of a
currency crisis in country 1, i.e. $\theta_1 < \theta_1^*$, and strong fundamentals in country 2. If private
information is sufficiently precise, i.e. $\gamma < \gamma < \infty$, and public information is sufficiently imprecise,
$0 < \alpha < \alpha$, then the incentives to acquire information increase in the proportion of informed
speculators $n$:

$$\frac{dc(n, \theta_1)}{dn} \geq 0 \ \forall \ \theta_1 < \theta_1^*.$$  

Furthermore, for all $n \in [0, 1]$:

$$c(n, \theta_1) \begin{cases} > 0 & \forall \ \theta_1 \neq \theta_1 \\ = 0 & \text{if} \ \theta_1 = \theta_1. \end{cases}$$
Proof See above for the proof if $\theta_1 < \theta_1 < \theta_1^*$. For $\theta_1 = \theta_1$, there are no benefits from acquiring information because $x_{21}^*(n, \rho, \theta_1) = x_{21U}^*(n, \theta_1) \forall \rho$. Hence, $\tilde{c}(n, \theta_1) = 0 \forall n \in [0, 1]$. In Appendix B.10, we extend the argument to the case in which $\theta_1 < \theta_1$.

Strategic complementarity in information choices arises in many global coordination games with information acquisition (e.g. Hellwig and Veldkamp (2009)). Intuitively, information supports coordination, which we obtain in a model with discrete information acquisition choice and publicly available signals. Lemma 5 describes the strategic complementarity in information acquisition choices. As a result, for any $\theta_1 \in (\theta_1, \theta_1^*)$, there exists a range of positive information costs, $0 < c < \bar{c}(0, \theta_1)$, such that each speculator acquires information about the cross-country correlation irrespective of the proportion of informed speculators, $d_i^* = I$. That is, the acquisition of information is a dominant strategy for all speculators. Hence, all speculators acquire information, i.e. $n^* = 1$. Thus, the strategic complementarity in information acquisition choices established in Lemma 5 allows us to find simple conditions that guarantee the existence of a unique pure-strategy PBE. Furthermore, we demonstrate that the ex-post contagion and ex-ante contagion results previously described for exogenous information are an equilibrium phenomenon in a model with endogenous information acquisition whenever the information cost is sufficiently low. These results are summarized in Proposition 4.

Proposition 4 Existence of a unique PBE with ex-post and ex-ante contagion. Consider the case of a currency crisis in country 1, i.e. $\theta_1 < \theta_1^*$, and strong fundamentals in country 2. Suppose that private information is sufficiently precise, i.e. $\gamma < \gamma < \infty$, and public information is sufficiently imprecise, i.e. $0 < \alpha < \alpha$. If the information cost is sufficiently small, i.e. $0 < c < \bar{c}(0, \theta_1)$, then there exists a unique pure-strategy PBE in which all investors acquire information about the cross-country correlation, $n^* = 1$, and subsequently use the attacking threshold $x_{21}^*(1, \rho, \theta_1)$ for $\rho \in \{0, \rho_H\}$.

Furthermore, both contagion results are obtained despite each speculator learning that countries are not correlated, $\rho = 0$.

1. Ex-post contagion arises if $\theta_1 > \theta_1$ without further restrictions on the information cost $c$. 

32
2. **Ex-ante contagion** arises if \( c \leq \int_{-\infty}^{\theta_1^*} \tilde{c}(0, \theta_1)f(\theta_1)d\theta_1 \) without further restrictions on the realized fundamental in country 1, \( \theta_1 \).

**Proof** See Appendix B.11.

Contagion arises in *Calvo and Mendoza (2000)* since globalization shifts the incentives of investors from costly information acquisition to imitation and detrimental herding. By contrast, financial contagion arises in our theory because investors have an incentive to acquire information after a wake-up call.

## 6 Testable implications

In this section, we develop testable implications of the wake-up call theory of contagion and link our results to the existing empirical and experimental literatures. Section 6.1 relates our results to the empirical literature on contagion and interdependency. We develop empirical predictions in section 6.2 and testable predictions for laboratory experiments in section 6.3. Finally, section 6.4 documents the aforementioned Latvian crisis episode of 2011, which is an excellent case study of our wake-up call of contagion.

### 6.1 Empirical literature

There is a large literature on contagion, and more generally, interdependencies, in both financial economics and in international finance.\(^{22}\) The approaches differ and range from probability models (e.g. *Eichengreen et al.* (1996)) to the correlation analysis (e.g. *Forbes and Rigobon* (2002)) as well as the closely related VAR models (e.g. *Favero and Giavazzi* (2002)), latent factor/GARCH models (e.g. *Beetsma et al.* (2014)), and extreme value analysis (e.g. *Bae et al.* (2003)). Of particular interest to our paper is the empirical literature that investigates the channels of contagion and how these depend on the fundamental characteristics of the second region (see *Glick and Rose* (1999), *Van Rijckeghem and Weder* (2001, 2003), and *Dasgupta et al.* (2014)). This literature suggests that

\(^{22}\)See *Forbes* (2012) for an excellent recent literature survey.
the likelihood of a spread of a crisis is higher with stronger trade and financial links, as well as with a higher institutional similarity. The cross-regional correlation in our model is meant to capture such factors. As described in section 6.2, we propose contributing to this literature by accounting for the non-linear effects of the fundamental in the initially affected region.

6.2 Empirical predictions

As outlined earlier, a large part of the empirical literature studies the channels by which contagion spreads and the characteristics of regions that are potential victims and make it susceptible to contagion. Our theory complements this literature by studying how the likelihood of contagion depends on the realized fundamental in the first region. The empirical predictions are a direct implication of Proposition 2 and Lemma 2.

Let \( P_0 \) be a parameter that captures the intensity of trade links, financial links, and institutional similarities with the crisis region. \( P \) is zero if there are no links and increases with the intensity of links. Consistent with our model, let \( \theta_1 \) be the fundamental in the first region affected by a crisis and let \( 1(\theta_2, P) \) be an indicator function that takes the value of 1 if another region with the characteristics \( \theta_2 \) (fundamentals of the other region, such as key macro variables) and \( P \) (intensity of links to first region) is affected by a contagious crisis. Two empirical predictions from our model can be formally stated as follows:

\[ \frac{d1(\theta_2, P)}{d\theta_1} < 0. \tag{41} \]

**Empirical prediction 1.**

\[ \frac{d1(\theta_2, P)}{d\theta_1} < 0. \]

**Empirical prediction 2.** Under the conditions of Proposition 2, there exists some \( \theta_1 \) such that:

\[ \frac{d1(\theta_2, P)}{dP} < 0 \quad \text{if} \quad \theta_1 > \theta_1, \]

\[ \frac{d1(\theta_2, P)}{dP} > 0 \quad \text{if} \quad \theta_1 < \theta_1. \tag{42} \]

The first empirical prediction, equation (41), states that a crisis in the first region due to a worse realization of the fundamental \( \theta_1 \) is more likely to spread contagiously, the stronger the expo-
sure observed by the empiricist (that is a higher $P$ such as financial links). This prediction hinges on the mean effect. In combination with the variance effect, our ex-post contagion mechanism (Proposition 2) emerges, giving rise to the second empirical prediction in equation (12). In particular, after controlling for the contemporaneous fundamentals of the potential victims (the realization of $\theta_2$), a crisis in the first region due to moderately low fundamentals is more likely to spread if the empiricist observes few or no linkages, that is, $P = 0$. This is because $\theta_{2I}^*(0) > \theta_{2I}^*(\rho_H) \forall \theta_1 \in (\theta_1, \theta_1^*)$. By contrast, given equation (11), a crisis in the first region due to extremely low fundamentals is less likely to spread if the empiricist observes few or no linkages, that is, $P = 0$. This is because $\theta_{2I}^*(0) < \theta_{2I}^*(\rho_H) \forall \theta_1 < \theta_1^*$.

The apparent non-linearity of our theoretical results in $\theta_1$ underlines the importance of discriminating between moderate and extreme fundamental realizations in the first region, alongside the characteristics of the second region, when studying the channels of contagion. The empirical implications of our model could help to explain crisis episodes where contagion cannot be attributed to direct or indirect exposure, such as in the Latvian example.

6.3 Predictions testable in laboratory experiments

Our wake-up call theory of contagion provides three testable implications for laboratory experiments. In particular, our novel ex-post contagion result and the implications related to the information choice are suitable for a laboratory environment, in which the acquisition of information after a wake-up call can be observed. The literature on laboratory experiments features several experimental studies on financial contagion as well as on bank runs 23 While there has been a substantial interest in studying global games models in the laboratory following Heinemann et al. (2013, 2008), contagion within the global games framework has only recently begun to attract attention 24

First, one can study the role of strategic uncertainty, along the lines of Proposition 4, by contrasting the cases in which all subjects are either informed or uninformed about the actual correlation. Can the variance effect (elevated strategic uncertainty) overturn the mean effect,

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23 See e.g. Cipriani and Guarino (2008) and Cipriani et al. (2013) for an experiment on financial contagion and Schotter and Yorulmazer (2009) and Garratt and Keister (2009) for an experiment on bank runs.

24 To our knowledge, Trevino (2013) is the only experimental study on contagion within the global games framework. The author investigates a fundamental and a social learning channel of contagion.
as predicted by the ex-post contagion mechanism? If so, contagion can occur even if all investors observe that there are no exposures or interconnectedness ex-post, relative to uninformed investors.

Second, one can investigate our theory’s predictions concerning the acquisition of public information about the actual correlation $\rho$.\(^{25}\) Does there exist a positive information cost $\bar{c}$ such that all participants want to purchase public information, as predicted by Proposition 4? Furthermore, do the incentives to acquire information increase in the number of informed participants (when an individual participant knows this number), as predicted by Lemma 4?

Third, one can study the role of the fundamental in the first region in the two previous experiments. In particular, it would be interesting to test our results regarding the role played by the threshold $\theta_1$ (see Proposition 2 and Lemma 2) and the resulting non-linearity.

6.4 The Latvian crisis of 2011

The Latvian banking crisis in late 2011 provides a compelling example of the wake-up call theory of contagion. Here, we connect the facts surrounding the Latvian crisis episode (see introduction) to our theoretical results in greater detail. The ex-ante contagion mechanism established in Proposition 3 suggests that a bank run against the Latvian subsidiary of Swedbank is more likely after observing the failure of Krajbanka even if all the depositors of Swedbank learn that Swedbank is not exposed to Krajbanka. According to our theory, this result holds for strong fundamentals of the potential victim (Definition 1), a characteristic that was arguably appropriate for Swedbank. Swedbank had an A+ rating by Standard&Poors. It was considered systemically important for Sweden and was the largest Swedish bank by number of customers. Moreover, a strong commitment by Swedbank to support its Latvian subsidiary was likely, not only because of the interest to maintain its leading market position in Latvia, but also because of the international commitment to assist Latvia.\(^{26}\)

\(^{25}\)While there is some experimental work on the acquisition of private signals e.g. by Szkup and Trevino (2012a) the acquisition of publicly available signals is still unexplored.

\(^{26}\)The international support for Latvia was also reflected in the USD 2.5 billion in December 2008, which was part of a larger package for Latvia supported by the EU and others, totalling USD 10.5 billion. In addition, the international support is also documented in the Vienna Initiative from 2009 who states on their website the key objective to prevent “a large-scale and uncoordinated withdrawal of cross-border bank groups from the region” (see http://vienna-initiative.com/vienna-initiative-part-1/).
Furthermore, a significant direct exposure of Swedbank to Krajbanka was not indicated, which is a local lender that is one ninth of the size of the Latvian subsidiary of Swedbank when measured by total assets. The root cause for the failure of Krajbanka had to do with its owners, Bankas Snoras, a Lithunian bank that had been nationalized in November 2011, and the Russian banker Vladimir Antonov. Hence, it was hardly related to Swedbank but was rather idiosyncratic. Consequently, the depositors of Swedbank were unlikely to be concerned about the possibility of direct exposure or interconnectedness. Instead, they experienced a wake-up call that led to contagion based on elevated strategic uncertainty, which could only be stopped after official intervention. On Sunday, December 11 the level of ATM withdrawals reached 10 million lats. During a weekend, limited ATM withdrawals are the only option to receive cash. Only after official intervention and press statements by Latvian and Swedish authorities did the run stop on Monday, December 12.

7 Robustness and extensions

Appendix contains a detailed discussion of the robustness of our results and interesting extensions of our model. The key insights of our paper are robust to several variations of our model, including regional asymmetries, incomplete learning about the cross-country correlation $\rho$, and negative cross-regional correlation. Interestingly, the strength of our contagion effect is likely to increase when the first region is more prone to a crisis than the second region. While changes in timing can matter in our model, both regional asymmetries and, in particular, an extension of our model that allows for cross-regional correlation through an aggregate shock overcome the sensitivity to timing without affecting our key insights.

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27 Source: Financial reports by Swedbank and KPMG Baltics who managed the liquidation of Krajbanka. The Latvian assets of Swedbank represented only 2.5% of the total assets held by its Swedish parent bank. Moreover, the funding to Krajbanka did not come from Swedbank. Until November 2011, the Russian Vladimir Antonov was a majority shareholder and chairman of leading Lithuanian bank Bankas Snoras (who held big stakes in Krajbanka) and of Krajbanka. He was a member of the Supervisory Board in both banks.

28 Source: BBC news on Dec. 12, 2011, “Panic fuels Latvian run on bank”. The withdrawal size was verified ex-post and is consistent with information form official sources. This corresponds to 4% of Latvian households deposits in Swedbank (Source: Swedbank’s Annual Report 2011).

29 First statements from Latvian regulators and the Swedish Financial Supervisory Authority regulators were given to Latvian media on Sunday evening. See also press release from the Latvian regulator Financial and Capital Market Commission on Dec. 12, 2011.
8 Conclusion

We have proposed a wake-up call theory of financial contagion. When the correlation across regional fundamentals is potentially positive but uncertain ex-ante, a crisis in one region is a wake-up call for investors in another region. To determine their exposure, investors acquire information. Due to a strategic complementarity in information choices, all investors acquire information about the cross-regional correlation if the information cost is sufficiently low.

We show that contagion can be more likely when speculators are informed about the zero cross-regional correlation than when they are uninformed (ex-post contagion). The key driver of this result is elevated strategic uncertainty among informed investors. We also show that contagion can be more likely after observing a crisis in the first region and learning about a zero correlation than after observing no crisis (ex-ante contagion).

Our theory helps to improve our understanding of “unexpected contagion” across regions with seemingly unrelated fundamentals and no interconnectedness. We discuss a previously unexplained banking crisis in Latvia that is an excellent example of our theory. We also provide novel predictions for the empirical literature on contagion as well as testable implications for laboratory experiments.

While we focused on establishing the wake-up call theory of contagion and its testable predictions, we plan to investigate the value of information and policy implications in subsequent work.
References


A Figures

A.1 Updated Prior distributions

Figure 1: The updated prior distributions about the fundamental in country 2: the dashed brown and dotted blue lines represent the updated prior distribution for an informed speculator who observes no correlation and positive correlation, respectively. The solid red line represents an uninformed speculator’s updated prior distribution. Parameters: $\mu = 0.6$, $\alpha = 1$, $p = 0.7$, $\rho_H = 0.8$ and $\theta_1 = 0.07$ ($\theta_1 = -1$) for the first (second) panel.

A.2 The incentives to acquire information

Figure 2: The fundamental thresholds as a function of the proportion of informed speculators $n$. Parameters are $\mu = 0.6$, $\alpha = 1$, $\gamma = 1.5$, $b/l = 1/3$, $p = 0.7$, $\rho_H = 0.8$ and $\theta_1 = 0.07$. 
B Appendix

B.1 Bayesian equilibrium in country 1

The first equilibrium condition states that the equilibrium proportion of attacking speculators equals the critical fundamental threshold below which a currency crisis occurs:

\[ A(\theta_i^*) = \Pr\{x_{i1} < x_i^* | \theta_i^*\} = \Phi(\sqrt{\gamma}(x_i^* - \theta_i^*)) = \theta_i^* \]

\[ \Rightarrow x_i^* = \theta_i^* + \frac{1}{\sqrt{\gamma}} \Phi^{-1}(\theta_i^*). \] (43)

The second equilibrium condition is an indifference condition for currency speculators. Upon receiving the threshold private signal \( x_{i1} = x_i^* \), a speculator is indifferent between attacking and not attacking the currency:

\[ b \Pr\{\theta_1 < \theta_i^* | x_{i1} = x_i^*\} - l \Pr\{\theta_1 > \theta_i^* | x_{i1} = x_i^*\} = 0 \] (44)

where:

\[ \Pr\{\theta_1 < \theta_i^* | x_{i1}\} = \Phi\left(\frac{\theta_i^* - \mathbb{E}[\theta_1 | x_{i1}]}{\sqrt{\text{Var}[\theta_1 | x_{i1}]}\sqrt{\gamma}}\right) = \Phi\left(\sqrt{\alpha + \gamma}\left[\theta_i^* - \frac{\alpha \mu + \gamma x_{i1}}{\alpha + \gamma}\right]\right). \]

which is decreasing in \( x_{i1} \). Therefore, it is indeed optimal for a speculator to attack if and only if \( x_{i1} < x_i^* \). Combining the two equilibrium conditions leads to equation (12), in which the right-hand side is constant and the left-hand side changes according to:

\[ \frac{dF_1(\theta_1)}{d\theta_1} = \frac{\phi(\cdot)}{\sqrt{\alpha + \gamma}} \left[\alpha - \frac{\sqrt{\gamma}}{\phi(\Phi^{-1}(\theta_1))}\right]. \] (45)

In the limit, as \( \theta_1 \to 0 \), then \( F(\theta_1) \to 1 \). Likewise, as \( \theta_1 \to 1 \), then \( F_1(\theta_1) \to 0 \). Given that \( 0 < \frac{1}{1 + \gamma \alpha} < 1 \), a sufficiently precise private signal, \( \gamma > \gamma_0 \equiv \frac{\alpha^2}{2\pi} \), ensures \( \frac{dF_1(\theta_1)}{d\theta_1} < 0 \) and thus the existence and uniqueness of \( \theta_i^* \).

**Equilibrium properties** If the private signal is sufficiently precise, i.e. \( \gamma > \gamma_0 \), then the equilibrium threshold \( \theta_i^* \) decreases in the mean of the fundamental \( \mu \) but increases in the relative gain
from attacking \( b/l \). Thus, the probability of a currency crisis increases if either the prior about the fundamental is lower or the relative benefit from attacking is higher:

\[
\frac{d\theta^*_1}{d\mu} = \frac{\alpha}{\alpha - \frac{\sqrt{\gamma}}{\phi(\Phi^{-1}(\theta^*_1))}} < 0 \quad (46)
\]

\[
\frac{d\theta^*_1}{d(b/l)} = -\frac{\sqrt{\alpha + \gamma} \left[ \alpha - \frac{\sqrt{\gamma}}{\phi(\Phi^{-1}(\theta^*_1))} \right]}{\phi(\cdot)(1 + \frac{b}{l})^2} > 0. \quad (47)
\]

There are two possible rankings of the equilibrium thresholds depending on the prior about the fundamental. The prior is defined as strong if it is high compared to the relative gain from attacking \( b/l \), which corresponds to the condition in Corollary $\Box$:

\[
\mu > \bar{\mu} \equiv \Phi\left( \frac{\sqrt{\alpha + \gamma}}{\gamma} \Phi^{-1}\left( \frac{b/l}{b/l + 1} \right) \right).
\]

A strong prior prevents speculative currency attacks for realized fundamentals close to but below the prior: \( 0 < \theta^*_1 < \mu \). In contrast, the prior about the fundamental \( \mu \) is weak if the above inequality is reversed. A weak prior leads to speculative currency attacks for realized fundamentals close to but above the prior: \( \mu < \theta^*_1 < 1 \). In the simplifying case of \( b = l \), a strong prior means \( \frac{1}{2} < \mu \), while a weak prior means \( \mu < \frac{1}{2} \). This implies \( 0 < x^*_1 < \theta^*_1 < \frac{1}{2} < \mu \) for a strong prior and \( \mu < \frac{1}{2} < \theta^*_1 < x^*_1 < 1 \) for a weak prior.

### B.2 Informed speculators

This section establishes the comparative static results when all speculators are informed about the realization of the cross-country correlation after observing a crisis in country 1, i.e. \( \theta_1 < \theta^*_1 \leq \mu \).

We discuss in turn the role of the public and private signal precision in section [B.2.1] and the implications for the ordering of equilibrium thresholds for either realization of the cross-country correlation in section [B.2.2].

Definition $\Box$ formalizes the distinction between a weak and a strong prior belief about the
fundamental. The sets $S_1$ and $S_2$ can be derived by reformulating equation (11) to:

$$\Phi^{-1}(\theta_{2l}^*) - \frac{\alpha_2(\rho)}{\sqrt{\gamma}}(\theta_{2l}^* - \mu_2(\rho, \theta_1)) = -\frac{\sqrt{\alpha_2(\rho) + \gamma}}{\sqrt{\gamma}}\Phi^{-1}\left(\frac{1}{1 + b/l}\right). \quad (48)$$

Then, $X(\rho)$ can be derived by setting $\theta_{2l}^* = \mu_2(\rho, \theta_1)$ and isolating $\mu_2(\rho, \theta_1)$. A sufficient condition assuring that strong (weak) prior beliefs are associated with a low (high) incidence of attacks below (above) 50% are derived from equation (15) by setting $\theta_{2l}^* = \frac{1}{2}$, which leads to $Y(\rho)$.

### B.2.1 Comparative statics 1: precision of public and private signals

The subsequent discussion draws in parts from Bannier and Heinemann (2005). We have:

$$\frac{d\theta_{2l}^*}{d\alpha} \begin{cases} < 0 \text{ if } \theta_{2l}^* < \mu_2(\rho, \theta_1) + \frac{1}{2\sqrt{\alpha_2(\rho) + \gamma}}\Phi^{-1}\left(\frac{1}{1 + b/l}\right) \\ \geq 0 \text{ otherwise} \end{cases}$$

and:

$$\frac{d\theta_{2l}^*}{d\gamma} \begin{cases} > 0 \text{ if } \theta_{2l}^* < \mu_2(\rho, \theta_1) + \frac{1}{2\sqrt{\alpha_2(\rho) + \gamma}}\Phi^{-1}\left(\frac{1}{1 + b/l}\right) \\ \leq 0 \text{ otherwise}. \end{cases}$$

If $b \leq l$, then a prior belief that fundamentals are relatively strong, i.e. $\theta_{2l}^*(\rho, \theta_1) < \mu_2(\rho, \theta_1) \forall \rho \in \{0, \rho_H\}$, implies that $\frac{d\theta_{2l}^*}{d\alpha} < 0$ and $\frac{d\theta_{2l}^*}{d\gamma} > 0$. If $b > l$, then a prior belief that fundamentals are relatively weak, i.e. $\theta_{2l}^*(\rho, \theta_1) > \mu_2(\rho, \theta_1) \forall \rho \in \{0, \rho_H\}$, implies that $\frac{d\theta_{2l}^*}{d\alpha} > 0$ and $\frac{d\theta_{2l}^*}{d\gamma} < 0$.

Instead, if $b > l$, then $\theta_{2l}^*(\rho, \theta_1) < \mu_2(\rho, \theta_1) \forall \rho \in \{0, \rho_H\}$ does not necessarily imply that $\frac{d\theta_{2l}^*}{d\alpha} < 0$ and $\frac{d\theta_{2l}^*}{d\gamma} > 0$. In other words, the inequalities involving $X(\rho)$ in Definition 11 are no longer sufficient if $b > l$. However, Definition 11 provides a more restrictive definition of a strong (weak) prior about fundamentals by imposing additional conditions involving $Y(\rho)$, which assure that a strong (weak) prior belief is associated with a low (high) incidence of attacks below (above) 50%. In this way, Definition 11 also ensures that a strong prior belief implies that $\frac{d\theta_{2l}^*}{d\alpha} < 0$ and $\frac{d\theta_{2l}^*}{d\gamma} > 0$ even if $b > l$. Similarly, Definition 11 ensures that a weak prior belief implies that $\frac{d\theta_{2l}^*}{d\alpha} > 0$ and
\[ \frac{d\theta^*_2}{d\gamma} < 0 \text{ even if } b \leq l. \]

### B.2.2 Comparative statics 2: Ordering of the equilibrium thresholds

The aim of this section is to shed light on the interplay between the *mean effect* and the *variance effect*, which crucially influences the ordering of equilibrium thresholds \( \theta^*_2(t_0, \theta_1) \) and \( \theta^*_2(t_1, \theta_1) \). Here, our focus is exclusively on the ordering of equilibrium thresholds and not on the ordering of likelihoods of successful currency attacks.\(^{30}\)

One of the first papers that examine the dependence of equilibrium thresholds on the precision of the private signal \( \gamma \) and the public signal \( \alpha \) was [Metz (2002)](https://doi.org/10.1007/s10690-002-0064-3). An inspection of equation (10) for the special case \( b = l = \frac{1}{2} \) reveals that the equilibrium threshold \( \theta^*_2(t_0, \theta_1) \) increases (decreases) in the precision of the private signal \( \gamma \) when the prior belief is that fundamentals are strong (weak). This result is consistent with the findings of [Rochet and Vives (2004)](https://doi.org/10.1017/CBO9780511537223.010). A related result is that the above relationship is opposite when considering a change in the precision of the public signal \( \alpha \).

Table 2 summarizes the effects of an increase in the cross-country correlation \( \rho \) if \( \theta_1 < \theta^*_1 \). Both the mean \( \mu_2(\rho, \theta_1) \) and the precision \( \alpha_2(\rho) \) of the updated prior about the fundamental in country 2 are affected by a change in \( \rho \). The effect of an increase in \( \rho \) on \( \theta^*_2(t_1, \theta_1) \) and its impact on the ordering of equilibrium thresholds depends on the prior about the fundamentals. Therefore, the cases where the mean effect (ME) and the variance effect (VE) go in opposite directions are emphasized in bold. If the cross-country correlation is potentially positive, i.e. \( \rho_H > 0 \), this requires a strong prior about the fundamental.

To understand the mechanics behind the results presented in table 2 recall that \( \frac{d\alpha_2(\rho)}{d|\rho|} > 0. \) As a result, the precision of the public signal is lowest in the state where there is no correlation (\( \alpha < \alpha_2(\rho_H) \)). Consequently, the variance effect tends to decrease (increase) \( \theta^*_2(t_1, \theta_1) \) if the prior belief is that fundamentals are strong (weak). For a prior belief that fundamentals are strong, there is a clear tension between the mean and the variance effect if \( \rho_H > 0 \), which plays a crucial role in Lemma 2. Furthermore, this tension vanishes if \( \theta_1 \geq \theta^*_1 \) since the mean and variance effects go in

---

\(^{30}\)As mentioned in the main text, there is no one-to-one mapping between the ordering of equilibrium thresholds and the ordering of likelihoods of a currency crisis since the realization of the cross-country correlation affects the conditional distribution of the fundamental, \( \theta_2|\rho \).
Finally, prior, then if private information is sufficiently precise, then the same direction. This last result is used in the proof of Proposition 3.

<table>
<thead>
<tr>
<th>Prior belief</th>
<th>Effect of an increase in $\rho$ on $\theta^*_2I(\rho, \theta_1)$</th>
<th>Ordering of thresholds</th>
</tr>
</thead>
<tbody>
<tr>
<td>strong</td>
<td>$\frac{d\theta^*_2I(\rho, \theta_1)}{d\rho} &gt; 0$</td>
<td>$\theta^<em>_2I(\rho_H, \theta_1) &lt; \theta^</em>_2I(0, \theta_1)$</td>
</tr>
<tr>
<td>weak</td>
<td>$\rho \in (-1, 1)$</td>
<td>$\theta^<em>_2I(\rho_H, \theta_1) &lt; \theta^</em>_2I(0, \theta_1)$</td>
</tr>
<tr>
<td></td>
<td>$\frac{d\theta^*_2I(\rho, \theta_1)}{d\theta_1} &lt; 0$</td>
<td>$\theta^<em>_2I(\rho_H, \theta_1) &gt; \theta^</em>_2I(0, \theta_1)$</td>
</tr>
<tr>
<td></td>
<td>$\frac{d^2\theta^*_2I(\rho, \theta_1)}{d\rho d\theta_1}  &gt; 0$</td>
<td>$\theta^<em>_2I(\rho_H, \theta_1) &gt; \theta^</em>_2I(0, \theta_1)$</td>
</tr>
</tbody>
</table>

Table 2: Effect of an increase in $\rho$ on $\theta^*_2I(\rho, \theta_1)$ and the ordering of equilibrium thresholds given $\theta_1 < \theta^*_1 \leq \mu$.

the same direction. This last result is used in the proof of Proposition 3.

B.3 Proof of Lemma 2

If private information is sufficiently precise, then $F_2(\theta^*_2I, \rho)$ decreases in $\theta^*_2I$ for a given $\rho$. Hence, equation (VII) implies that $\theta^*_2I(0, \theta_1) > \theta^*_2I(\rho_H, \theta_1)$ if $F_2(\theta^*_2I(0, \theta_1), 0) > (\theta^*_2I(0, \theta_1), \rho_H)$, where $\alpha(0) = \alpha$ and $\mu(0, \theta_1) = \mu$:

$$\frac{\alpha}{\sqrt{\alpha + \gamma}} [\theta^*_2I(0, \theta_1) - \mu] - \sqrt{\frac{\gamma}{\alpha + \gamma}} \Phi^{-1}(\theta^*_2I(0, \theta_1)) > 0$$

Solving this inequality for $\theta_1$, which is implicit in $\mu(\rho_H, \theta_1)$, results in condition (VII).

Next, $\theta_1 < \mu$ arises because $\theta^*_2I < \mu$, $[1 - \frac{\alpha}{\alpha(\rho_H)}] \sqrt{\frac{\alpha(\rho_H) + \gamma}{\alpha + \gamma}} > 0$, and $[\sqrt{\frac{\alpha(\rho_H) + \gamma}{\alpha + \gamma}} - 1] > 0$. Finally, $\Phi^{-1}(\theta^*_2I(0, \theta_1)) < 0$ if $\mu(\rho, \theta_1) < Y(\rho) \forall \rho \in \{0, \rho_H\}$. Therefore, $\theta_1 \in [\theta_1, \mu]$ is non-empty and the inequality in equation (VII) follows.\footnote{If we were to use just $X$, and not also $Y$, in the definition of the bounds that characterize a weak or a strong prior, then $[\theta_1, \mu]$ may be empty under some parameter values.}

We now present a condition to ensure $\theta_1 \in [\theta_1, \theta^*_1]$ is non-empty. This requires intermediate values of $\theta^*_1$. From equation (VII), $\theta_1 < \theta^*_1$ if:

$$\frac{\alpha}{\sqrt{\alpha + \gamma}} (\theta_1 - \mu) - \sqrt{\frac{\gamma}{\alpha + \gamma}} \Phi^{-1}(\theta_1) > \Phi^{-1}\left(\frac{1}{1 + b/l}\right),$$

(50)

49
which can be reformulated to equation (13).

Finally, we must verify that the constraint in equation (13) is consistent with Definition 1, which requires:

\[ \mu > \Phi\left(-\sqrt{\frac{\alpha + \gamma}{\gamma}} \Phi^{-1}\left(\frac{1}{1 + b/l}\right)\right) \]  
\[ \mu > \frac{1}{2} - \sqrt{\frac{\alpha + \gamma}{\alpha}} \Phi^{-1}\left(\frac{1}{1 + b/l}\right). \]  

(51)

(52)

First, inequality (52) is more restrictive than inequality (51) if \( -\sqrt{\frac{\alpha + \gamma}{\gamma}} \Phi^{-1}\left(\frac{1}{1 + b/l}\right) > 0 \) or \( b > l \). A combination of inequality (52) and inequality (13) leads to:

\[ \theta_1 - \frac{\sqrt{\gamma}}{\alpha} \Phi^{-1}(\theta_1) - \sqrt{\frac{\alpha + \gamma}{\alpha}} \Phi^{-1}\left(\frac{1}{1 + b/l}\right) > \frac{1}{2} - \sqrt{\frac{\alpha + \gamma}{\alpha}} \Phi^{-1}\left(\frac{1}{1 + b/l}\right). \]  

(53)

The above inequality is always satisfied since the left-hand side is decreasing in \( \theta_1 \) given our assumption on \( \gamma \) for uniqueness. Furthermore, we have already established that \( \theta_1 < 0.5 \) because of \( \mu > Y(0) \) in Definition 1.

Second, inequality (51) is more restrictive than inequality (52) if \( b < l \). A combination of inequality (51) and inequality (13) leads to the condition in equation (14), which holds if \( b/l \) is sufficiently high.  \( \text{(q.e.d.)} \)

B.4 Uninformed speculators

B.4.1 Bayesian updating

Uninformed speculators use Bayes’ rule to form a posterior belief about the probability of uncorrelated fundamentals:

\[ \hat{p} \equiv \Pr\{\rho = 0|\theta_1, x_{i2}\} = \frac{p \Pr\{x_{i2}|\theta_1, \rho = 0\}}{p \Pr\{x_{i2}|\theta_1, \rho = 0\} + (1 - p) \Pr\{x_{i2}|\theta_1, \rho = \rho_H\}. \]  

(54)
We now compute \( \Pr\{x_{i2}|\theta_1, \rho\} \) for both realizations of \( \rho \). Since the variance terms are unconditional on \( \theta_1 \), we find the sum of \( \text{Var}[\epsilon_{i2}] \) and \( \text{Var}[\theta_2] \), which is \( \frac{1}{\alpha} \) and \( \frac{1-\rho^2}{\alpha} \). Therefore:

\[
\Pr\{x_{i2}|\theta_1, \rho = 0\} = \frac{1}{\sqrt{\text{Var}[x_{i2}|\rho = 0]}} \phi\left( \frac{x_{i2} - \text{E}[x_{i2}|\theta_1, \rho = 0]}{\sqrt{\text{Var}[x_{i2}|\rho = 0]}} \right) = \left( \frac{1}{\alpha} + \frac{1}{\gamma} \right)^{-\frac{1}{2}} \phi\left( \frac{x_{i2} - \mu}{\sqrt{\frac{1}{\alpha} + \frac{1}{\gamma}}} \right)
\]

\[
\Pr\{x_{i2}|\theta_1, \rho = \rho_H\} = \frac{1}{\sqrt{\text{Var}[x_{i2}|\rho = \rho_H]}} \phi\left( \frac{x_{i2} - \text{E}[x_{i2}|\theta_1, \rho = \rho_H]}{\sqrt{\text{Var}[x_{i2}|\rho = \rho_H]}} \right) = \left( \frac{1-\rho_H^2}{\alpha} + \frac{1}{\gamma} \right)^{-\frac{1}{2}} \phi\left( \frac{x_{i2} - [\rho_H \theta_1 + (1-\rho_H) \mu]}{\sqrt{\frac{1-\rho_H^2}{\alpha} + \frac{1}{\gamma}}} \right).
\]

(55)

(56)

Since we maintain \( \rho_H > 0 \) throughout this paper (with the exception of section 7), we find the following derivatives for the posterior belief about the no-correlation probability \( \hat{\rho} \).

\[
\frac{d\hat{\rho}}{d\theta_1} \begin{cases} 
\ge 0 & \text{if } x_{i2} \le \rho_H \theta_1 + (1-\rho_H) \mu \\
< 0 & \text{otherwise.}
\end{cases}
\]

(57)

If the private signal about the fundamental in country 2, \( x_{i2} \), is sufficiently low, an increase in the fundamental of country 1, \( \theta_1 \), leads uninformed speculators to put a larger probability on uncorrelated fundamentals across countries.\( ^{32} \)

How does \( \hat{\rho} \) vary with the private signal \( x_{i2} \)?

\[
\frac{d\hat{\rho}}{dx_{i2}} = \frac{p(1-p)}{\left[ p\left( \frac{\sqrt{\frac{1}{\alpha} + \frac{1}{\gamma}}} \right)^{-1} \phi\left( \frac{x_{i2} - \mu}{\sqrt{\frac{1}{\alpha} + \frac{1}{\gamma}}} \right) + (1-p)\left( \frac{\sqrt{1-\rho_H^2}}{\alpha} + \frac{1}{\gamma} \right)^{-1} \phi\left( \frac{x_{i2} - [\rho_H \theta_1 + (1-\rho_H) \mu]}{\sqrt{\frac{1-\rho_H^2}{\alpha} + \frac{1}{\gamma}}} \right) \right]^2}
\]

(58)

To determine its sign, note that the denominator of equation (58) is strictly positive, while the

\( ^{32} \)The signs of the derivative are reversed for \( \rho_H < 0 \).
numerator is weakly positive if:

\[
\left( \frac{\alpha + \gamma (1 - \rho_H^2)}{\alpha + \gamma} \right) (\mu - x_{i2}) > [\rho_H \theta_1 + (1 - \rho_H) \mu] - x_{i2}
\]  

and negative otherwise. Therefore, we can state:

\[
\frac{d\hat{p}}{dx_{i2}} = \begin{cases} 
> 0 & \text{if } \rho_H > 0 \text{ and } x_{i2} \geq \rho_H \theta_1 + (1 - \rho_H) \mu \\
< 0 & \text{if } \rho_H < 0 \text{ and } x_{i2} \leq \rho_H \theta_1 + (1 - \rho_H) \mu \\
\leq 0 & \text{otherwise}.
\end{cases}
\]  

Speculators in country 2 observed a currency crisis in country 1, that is, \( \theta_1 \leq \theta_1^* < \mu \), but are uninformed about the cross-country correlation between fundamentals which potentially is positive. Therefore, after receiving a relatively good private signal, that is, \( x_{i2} \geq \rho_H \theta_1 + (1 - \rho_H) \mu \), a speculator places more weight on the probability of zero cross-country correlation.

However, the relationship between \( \hat{p} \) and \( x_{i2} \) is non-monotone after an uninformed speculator observes a relatively bad private signal, that is, \( x_{i2} < \rho_H \theta_1 + (1 - \rho_H) \mu \). If the private signal takes an intermediate value, \( \frac{d\hat{p}}{dx_{i2}} > 0 \) still holds. Instead, if the private signal is very low, then \( \frac{d\hat{p}}{dx_{i2}} \leq 0 \) due to the more dispersed prior distribution if \( \rho = 0 \). For the same reason, an extremely high or low private signal induces uninformed speculators to believe that fundamentals are uncorrelated across regions:

\[
\lim_{x_{i2} \to +\infty} \hat{p} = 1 = \lim_{x_{i2} \to -\infty} \hat{p}.
\]  

B.4.2 Equilibrium analysis

Two equilibrium conditions have to be satisfied. First, \( A_{2U}^* = \theta_{2U}^* \), which leads to the first equilibrium condition given by equation (15). Second, a speculator with the threshold signal \( x_{2U}^* \) is indifferent between attacking the currency or not, given \( \theta_{2U}^* \):

\[
b \Pr\{\theta_2 \leq \theta_{2U}^* | \theta_1, x_{2U}^*\} - l \Pr\{\theta_2 > \theta_{2U}^* | \theta_1, x_{2U}^*\} = 0
\]  

52
where:

\[
\Pr\{\theta_2 \leq \theta_{2U}^* | \theta_1, x_{2U} \} = \hat{p}(\theta_{2U}^*) \Phi_I(\rho = 0, \theta_{2U}^*) + (1 - \hat{p}(\theta_{2U}^*)) \Phi_I(\rho = \rho_H, \theta_{2U}^*). \tag{63}
\]

The indifference condition is a mixture between the indifference conditions for the informed speculators that observe no correlation, \( n = 1 \) and \( \rho = 0 \), and the informed speculators that observe positive correlation, \( n = 1 \) and \( \rho = \rho_H \). Combining equations (15) and (62) leads to the equilibrium condition stated in equation (16).

**B.4.3 Monotonicity**

The conditional density function \( f(x|\theta) \) is normal with mean \( \theta \) and satisfies the monotone likelihood ratio property (MLRP). That is, for all \( x_i > x_j \) and \( \theta' > \theta \), we have:

\[
\frac{f(x_i|\theta')}{f(x_i|\theta)} \geq \frac{f(x_j|\theta')}{f(x_j|\theta)} \iff \frac{\phi\left(\sqrt{\gamma} (x_i - \theta')\right)}{\phi\left(\sqrt{\gamma} (x_i - \theta)\right)} \geq \frac{\phi\left(\sqrt{\gamma} (x_j - \theta')\right)}{\phi\left(\sqrt{\gamma} (x_j - \theta)\right)}. \tag{64}
\]

Using Proposition 1 of Milgrom (1981), we conclude that \( \Pr\{\theta_2 \leq \theta_{2U}^* | \theta_1, x_{12}\} \) monotonically decreases in \( x_{12} \). Furthermore, notice that \( \frac{d\Pr\{\theta_2 \leq \theta_{2U}^* | \theta_1, x_{12}\}}{d\theta_{2U}} > 0 \). From equation (63) we can derive:

\[
0 \leq \frac{d\hat{\theta}_2(\bar{\theta}_2)}{d\bar{\theta}_2} \leq \frac{1}{1 + \sqrt{2\pi}}. \tag{65}
\]

**B.5 Proof of Proposition 1**

The result in Proposition 1 is proven in three steps. First, we show that \( G(\theta_{2U}, \theta_1) \rightarrow 1 > \frac{1}{1+b/l} \) as \( \theta_{2U} \rightarrow 0 \), as well as \( G(\theta_{2U}, \theta_1) \rightarrow 0 < \frac{1}{1+b/l} \) as \( \theta_{2U} \rightarrow 1 \). Second, we show that \( \frac{dG(\theta_{2U}, \theta_1)}{d\theta_{2U}} < 0 \) for some sufficiently high but finite values of \( \gamma \), such that \( G \) strictly decreases in \( \theta_{2U} \). Therefore, if \( \theta_{2U}^* \) exists, it is unique. Third, by continuity, there exists a \( \theta_{2U}^* \) that solves \( G(\theta_{2U}, \theta_1) = \frac{1}{1+b/l} \).

**Step 1: limiting behavior** Observe that \( G(\theta_{2U}, \theta_1) \) is a weighted average of \( F_2(\theta_{2U}^*(\theta_1), 0) \) and \( F_2(\theta_{2U}^*(\theta_1), \rho_H) \). As \( \theta_{2U} \rightarrow 0 \), then \( F_2(\theta_{2U}^*(\theta_1), \rho) \rightarrow 1 \) for any \( \rho \in \{0, \rho_H\} \), so \( G(\theta_{2U}, \theta_1) \rightarrow 1 > \frac{1}{1+b/l} \). Likewise, as \( \theta_{2U} \rightarrow 1 \), then \( F_2(\theta_{2U}^*(\theta_1), \rho) \rightarrow 0 \) for any \( \rho \in \{0, \rho_H\} \), so \( G(\theta_{2U}, \theta_1) \rightarrow 0 < \frac{1}{1+b/l} \)
**Step 2: strictly negative slope** Using the indierence condition of uninformed speculators (15) to substitute \( x_{2U}^* \), we arrive at equation (16). Thus, the total derivative of \( G \) is:

\[
\frac{dG}{d\theta_{2U}} = \hat{p}(\theta_{2U}) \frac{dF_2(\theta_{2U}^*(\theta_1),0)}{d\theta_{2U}} + (1 - \hat{p}(\theta_{2U})) \frac{dF_2(\theta_{2U}^*(\theta_1),\rho_H)}{d\theta_{2U}} + \frac{d\hat{p}(\theta_1,x_{2U}(\theta_{2U}))}{dx_{2U}} \frac{dx_{2U}}{d\theta_{2U}} \left[ F_2(\theta_{2U}^*(\theta_1),0) - F_2(\theta_{2U}^*(\theta_1),\rho_H) \right].
\]

The proof proceeds by inspecting the individual terms of equation (66).

We know from our analysis of the case of informed speculators that \( \frac{dF_2(\theta_{2U}^*(\theta_1),0)}{d\theta_{2U}} < 0 \) if \( \gamma > \gamma_0 \) and that \( \frac{dF_2(\theta_{2U}^*(\theta_1),\rho_H)}{d\theta_{2U}} < 0 \) if \( \gamma > \gamma_1 \). Moreover, these derivatives are also strictly negative in the limit as private noise vanishes:

\[
\lim_{\gamma \to \infty} \frac{dF_2(\theta_{2U}^*(\theta_1),0)}{d\theta_{2U}} = \lim_{\gamma \to \infty} \frac{dF_2(\theta_{2U}^*(\theta_1),\rho_H)}{d\theta_{2U}} = -1 < 0.
\]

Thus, the first two components of the sum are negative and finite in the limit of vanishing private noise. By continuity, these terms are also negative for a sufficiently high but finite private noise by continuity.

The sign of the third summand in equation (66) is ambiguous: \( F_2(\theta_{2U}^*(\theta_1),0) \leq F_2(\theta_{2U}^*(\theta_1),\rho_H) \) whenever \( \theta_{21}^*(0) \leq \theta_{21}^*(\rho_H) \) and \( F_2(\theta_{2U}^*(\theta_1),0) > F_2(\theta_{2U}^*(\theta_1),\rho_H) \) otherwise. However, the difference vanishes in the limit:

\[
\lim_{\gamma \to \infty} \left[ F_2(\theta_{2U}^*(\theta_1),0) - F_2(\theta_{2U}^*(\theta_1),\rho_H) \right] = 0.
\]

The last term to consider is \( \frac{d\hat{p}(\theta_1,x_{2U}(\theta_{2U}))}{dx_{2U}} \frac{dx_{2U}}{d\theta_{2U}} \). Given the previous sufficient conditions on the relative precision of the private signal:

\[
0 < \frac{dx_{2U}}{d\theta_{2U}} = 1 + \frac{1}{\sqrt{\gamma}} \frac{1}{\phi(\Phi^{-1}(\theta_{2U}))} < 1 + \frac{\sqrt{2\pi}}{\alpha}.
\]

Finally, from section 5.3.4.1, we know that the sign of \( \frac{d\hat{p}}{dx_{2U}} \) is ambiguous. However, the derivative is finite for \( \gamma \to \infty \). Taken together with the zero limit of the first factor of the third term, this term vanishes in the limit.
As a result, by continuity, there must exist a finite level of precision \( \gamma > \gamma_2 \in (0, \infty) \) such that \( \frac{dG(\theta_{2U}; \theta_1)}{d\theta_{2U}} < 0 \) for all \( \gamma > \gamma_2 \). This concludes the second step of the proof and therefore the overall proof of Proposition 3. (q.e.d.)

B.6 Proof of Proposition 3

Recall that speculators observe \( \theta_1 \) only if there was a crisis in country 1, i.e., if \( \theta_1 < \theta_1^* \). Recall our notation, where \( \theta_{2U}^* \equiv \theta_{2U}^*|\theta_1 \geq \theta_1^* \) denotes the equilibrium threshold of country 2 after no crisis in country 1 is observed and \( \theta_{2U}^*(\theta_1) \equiv \theta_{2U}^*|\theta_1 < \theta_1^* \) if a crisis in 1 is observed.

The proof is constructed in five steps. First, it is most intuitive to decompose the right-hand side of equation (22) for \( E_3 \equiv \theta_2 < \theta_{2U}^* \) by the law of total probability:

\[
\Pr\{E_3|\theta_1 \geq \theta_1^*\} = p \Pr\{E_3|\rho = 0, \theta_1 \geq \theta_1^*\} + (1 - p) \Pr\{E_3|\rho = \rho_H, \theta_1 \geq \theta_1^*\}. \tag{69}
\]

Since \( p \in (0, 1) \), it then suffices to show the following two conditions: (i) \( \Pr\{\theta_2 < \theta_{2U}^*(0)|\rho = 0, \theta_1 < \theta_1^*\} > \Pr\{\theta_2 < \theta_{2U}^*|\rho = 0, \theta_1 \geq \theta_1^*\} \); and (ii) \( \Pr\{\theta_2 < \theta_{2U}^*(0)|\rho = 0, \theta_1 < \theta_1^*\} > \Pr\{\theta_2 < \theta_{2U}^*|\rho = \rho_H, \theta_1 \geq \theta_1^*\} \), which we do below. In other words, we construct sufficient conditions without resorting to the ex-ante probability of positive cross-country correlation.

**Inequality (i)** Second, since the sufficient conditions of Corollary 2 and Proposition 3 are met (sufficiently precise private information), \( \theta_{2U}^*(\theta_1) \) and \( \theta_{2U}^*(0, \theta_1) \) are unique. Furthermore, since the sufficient conditions of Lemma 2 are met (intermediate realized fundamental in country 1 and strong prior about the fundamental in country 2), \( \theta_{2U}^*(\theta_1) < \theta_{2U}^*(0, \theta_1) \) for all \( \theta_1 \in (\bar{\theta}_1, \bar{\theta}_1^*) \), where the interval is non-empty given the sufficient conditions in Lemma 2. As a result, the true distribution is the same, while the thresholds are ranked as stated. Therefore, the stated inequality (i) follows directly.

**Inequality (ii)** Third, following an argument similar to the proof of Proposition 3, it can be shown that \( \theta_{2U}^* \) is unique, which is sketched here. Absent a crisis in country 1, the realization of \( \theta_1 \) is unobserved, so the indifference condition of speculators integrates over all possible \( \theta_1 \geq \theta_1^* \).
weighted by an updated distribution of the fundamental in country 1 that uses the private signal, \( f(\theta_1|x_2^*) \):

\[
\begin{align*}
\Pr\{\theta_1|\rho = 0, x_2^*(\theta_{2U}^*)\} &= \alpha \phi(\sqrt{\alpha}(\theta_1 - \mu)) \\
\Pr\{\theta_1|\rho = \rho_H, x_2^*(\theta_{2U}^*)\} &= \frac{1}{\sqrt{\text{Var}[\theta_1|\rho = \rho_H, x_2^*(\theta_{2U}^*)]}} \phi\left(\frac{\theta_1 - \text{E}[\theta_1|\rho = \rho_H, x_2^*(\theta_{2U}^*)]}{\sqrt{\text{Var}[\theta_1|\rho = \rho_H, x_2^*(\theta_{2U}^*)]}}\right) \\
&= \frac{1}{\alpha + 1 - \rho_H^2} - \frac{1}{\gamma} \phi\left(\frac{\theta_1 - [\rho_H x_2^*(\theta_{2U}^*) + (1 - \rho_H)\mu]}{\sqrt{\frac{1}{\alpha + 1 - \rho_H^2}}\gamma}\right).
\end{align*}
\]

Thus \( \theta_{2U}^* \) solves:

\[
\int_{\theta_1^*}^{\infty} \left(\hat{p}(\theta_{2U}^*) \Pr\{\theta_1|\rho = 0, x_2^*(\theta_{2U}^*)\} F_2(\theta_{2U}^*, 0) + (1 - \hat{p}(\theta_{2U}^*)) \Pr\{\theta_1|\rho = \rho_H, x_2^*(\theta_{2U}^*)\} F_2(\theta_{2U}^*, \rho_H)\right) f(\theta_1|x_2^*(\theta_{2U}^*)) d\theta_1 = \frac{1}{1+b/l}.
\]

Following the steps in the proof of Proposition \( \text{H} \), one can show that there exists a unique \( \theta_{2U}^* \) that solves the above system if \( \gamma \) is sufficiently high. The integration over \( \theta_1 \), which effectively constructs a weighted average, does not alter the analysis qualitatively.

Fourth, how do the thresholds compare? We want to show that \( \theta_{2U}^* < \theta_{2I}^*(0) \) if \( \theta_1 \geq \theta_1^* \), which is sketched here. For each realization of \( \theta_1 > \theta_1^* \), the \( \theta_{2U}^* \) that solves

\[
\hat{p}(\theta_2) F_2(\theta_{2U}^*, 0) + (1 - \hat{p}(\theta_2)) F_2(\theta_{2U}^*, \rho_H) = \frac{1}{1+b/l}
\]

is smaller than \( \theta_{2I}^*(0) \) since \( \theta_{2I}^*(\rho_H, \theta_1) < \theta_{2I}^*(0) \) \( \forall \theta_1 > \theta_1^* \) by a generalization of Lemma \( \text{H} \) that allows for \( \theta_1 > \theta_1^* \) (see also comparative statics in Appendix \( \text{B.2.2} \)). As a result, the integration over all different possible realizations of \( \theta_1 \), which is \( \theta_1 > \theta_1^* \), must yield \( \theta_{2U}^* < \theta_{2I}^*(0) \). Thus, we have established the same ranking of equilibrium thresholds as for inequality \( \text{(i)} \). However, the true distribution differs between the left-hand and right-hand sides of inequality \( \text{(ii)} \), which we discuss below.

Fifth, observe that \( \theta_2 \) is drawn from a less favorable distribution whenever \( \rho = \rho_H \) and \( \theta_1 \in [\theta_1, \mu] \). Likewise, the distribution is more favorable if \( \rho = \rho_H \) and \( \theta_1 > \mu \) holds. The argument below relies on cancelling out the part \( \theta_1 \in [\theta_1, \mu] \), which works against inequality \( \text{(ii)} \),
with $\theta_1 \in [\mu, \mu + (\mu - \theta_1^*)]$, which works for it. This will allow us to provide a condition sufficient for inequality (ii), where $\theta_{2U}^*$ is replaced by $\theta_{2I}^*(0)$.

Because of the threshold ranking $\theta_{2U}^* < \theta_{2I}^*(0)$ (step 4), the following condition is sufficient for inequality (ii):

$$\Pr\{\theta_2 < \theta_{2I}^*(0) | \rho = 0, \theta_1 < \theta_1^*\} \geq \Pr\{\theta_2 < \theta_{2I}^*(0) | \rho = \rho_H, \theta_1 \geq \theta_1^*\}. \quad (74)$$

Since the crisis probability in country 2 is low for a high fundamental in country 1, $\rho = 2\mu - \theta_1^*$, because of positive cross-country correlation, $\rho = \rho_H$, we have yet another sufficient condition:

$$\Pr\{\theta_2 < \theta_{2I}^*(0) | \rho = 0, \theta_1 < \theta_1^*\} \geq \Pr\{\theta_2 < \theta_{2I}^*(0) | \rho = \rho_H, \theta_1^* \leq \theta_1 \leq 2\mu - \theta_1^*\}. \quad (75)$$

The idea of this reformulation is to generate a condition, $\theta_1^* \leq \theta_1 \leq 2\mu - \theta_1^*$, that is symmetric around $\theta_1$, and focusing on $\theta_1^*$ (which is fine, given that $\theta_1^* \geq \theta_1^*$). This allows us to disregard the ex-ante distribution of $\theta_1$ in the subsequent proof. More precisely, for any $\theta_1^- \in [\theta_1^*, \mu]$, there exists a $\theta_1^+ \equiv 2\mu - \theta_1^- \in [\mu, 2\mu - \theta_1^*]$ such that both of these values are equally likely, $\phi(\sqrt{\alpha}[\theta_1^+ - \mu]) = \phi(\sqrt{\alpha}[\theta_1^- - \mu])$. Also, the left-hand side is $\Pr\{\theta_2 < \theta_{2I}^*(0) | \rho = 0, \theta_1 < \theta_1^*\} = \phi(\sqrt{\alpha}[\theta_2 - \mu])$.

Then, inequality \( (75) \) must hold if for any pair $(\theta_1^+, \theta_1^-)$:

$$\frac{\phi\left(\sqrt{\alpha_2(\rho_H)}[\theta_2 - \mu_2(\rho_H, \theta_1^+)]\right) + \phi\left(\sqrt{\alpha_2(\rho_H)}[\theta_2 - \mu_2(\rho_H, \theta_1^-)]\right)}{2} \leq \phi\left(\sqrt{\alpha}[\theta_2 - \mu]\right) \forall (\theta_1^+, \theta_1^-). \quad (76)$$

By construction, $\theta_1^+$ and $\theta_1^-$ are equidistant to $\mu$, so the above inequality holds if $m(\theta_1)$ strictly decreases and is weakly convex, where:

$$m(\theta_1) \equiv \sqrt{\frac{\alpha_2(\rho_H)}{2\pi}} \exp\left\{-\left(\frac{\alpha_2(\rho_H)}{2}\theta_{2I}^*(0) - [\rho_H \theta_1 + (1 - \rho_H)\mu] \right)^2\right\}. \quad (77)$$

By the Lemma 2, $m'(\theta_1) < 0$ for all $\theta_1 \in (\theta_1^*, 2\mu - \theta_1^*)$. Furthermore, $m''(\theta_1) \geq 0$ for all $\theta_1^- \in [\theta_1^*, \mu]$ if the inequality in equation \( (71) \) of Proposition 3 holds. This is because $\theta_1^* \geq \theta_1$, which completes the proof. \( q.e.d. \)
B.7 Derivations for the coordination stage in country 2

Recall that $\theta_1$ is observed by speculators in country 2 if and only if there is a currency crisis in country 1, $\theta_1 < \theta_1^*$. Furthermore, a fraction $n$ is informed about the cross-country correlation $\rho$, where $n$ is chosen optimally at the information stage. As before, the equilibrium conditions comprise an indifference condition and a critical mass condition. First, the equilibrium proportion of attacking speculators $A_2^*$ equals the fundamental threshold $\theta_2^*(\rho)$, where we use the short-hand $\theta_2^*(\rho) \equiv \theta_2^*(n, \rho, \theta_1)$. Another short-hand notation is $x_{2I}(\rho) \equiv x_{2I}(n, \rho, \theta_1)$ and $x_{2U}^* \equiv x_{2U}^*(n, \theta_1)$ for the attacking thresholds of informed and uninformed speculators, respectively.

First, for all $\rho \in \{0, \rho_H\}$ the critical mass condition is:

$$\theta_2^*(\rho) = n \phi(\sqrt{\gamma}[x_{2I}(\rho) - \theta_2^*(\rho)]) + (1 - n) \phi(\sqrt{\gamma}[x_{2U}^* - \theta_2^*(\rho)]).$$ (78)

Second, in contrast to the cases of $n \in \{0, 1\}$, the critical mass condition can no longer be used to express the attacking threshold as a function of the fundamental threshold. In these polar cases, we inserted this relationship in the indifference condition, which then simplified to the function $F_1$, $F_2$, and $G$ that fully characterized the equilibrium threshold $\theta^*$. By contrast, the interaction between informed and uninformed speculators implies that the attacking threshold of uninformed investors, $x_{2U}^*$, cannot be separated. Therefore, we define the following useful short-hand $\Psi$ to characterize the indifference conditions:

$$\Psi(\theta_2^*, x_{2d}^*, \rho) \equiv \phi\left(\theta_2^* \sqrt{\alpha_2(\rho) + \gamma - \frac{\alpha_2(\rho) \mu_2(\rho, \theta_1) + \gamma x_{2d}^*}{\sqrt{\alpha_2(\rho) + \gamma}}}ight)$$ (79)

for $d \in \{I, U\}$ and $\rho \in \{0, \rho_H\}$. Therefore, the indifference condition for an uninformed speculator is:

$$J(n, \theta_2^*(0), \theta_2^*(\rho_H), x_{2U}^*) \equiv \hat{p} \Psi(\theta_2^*(0), x_{2U}^*, 0) + (1 - \hat{p}) \Psi(\theta_2^*(\rho_H), x_{2U}^*, \rho_H) = \frac{1}{1 + b/l}$$ (80)

where $\hat{p} = \hat{p}(\theta_1, x_{2U}^*)$ and an indifference condition for an informed speculator for each realization...
of the cross-country correlation:

\[ \Psi (\theta_2^*(\rho), x_{21}^*(\rho), \rho) = \frac{1}{1 + \beta / \gamma} \quad \forall \rho \in \{0, \rho_H \}. \] (81)

These are related to the previous functions since \( \Psi \) is the generalization of \( F_2 \), while \( J \) generalizes \( G \).

We have five equation in five unknowns. In the simplest case, in country 1, we had two thresholds \( x_1^* \) and \( \theta_1^* \). There, the objective was to establish aggregate behavior by inserting the critical mass condition, which states \( x_1^* \) in terms of \( \theta_1^* \), into indifference condition. This yields one equation implicit in \( \theta_1^* \). We pursue a modified strategy here, solving this system of equations in order to express the equilibrium in terms of \( \theta_2^*(0) \) and \( \theta_2^*(\rho_H) \) only.

We also use the following insight. Since uninformed speculators do not observe the realized cross-country correlation, the attacking threshold must be identical across these realizations, \( x_{2U}^*(\rho = 0) = x_{2U}^*(\rho = \rho_H) \). In the following steps, we derive this threshold for either realization of \( \rho \) by using the fundamental threshold \( \theta_2^*(\rho) \) and equalize both expressions.

First, we use the critical mass condition in equation (78) for \( \theta_2^*(0) \) to express \( x_{2U}^* \) as a function of \( \theta_2^*(0) \) and \( x_{21}^*(0) \). Second, we use the indifference condition of informed speculators in case of \( \rho = 0 \), equation (81), to obtain \( x_{21}^*(0) \) as a function of \( \theta_2^*(0) \). Third, we use the critical mass condition in equation (78) for \( \theta_2^*(\rho_H) \) to express \( x_{2U}^* \) as a function of \( \theta_2^*(\rho_H) \) and \( x_{21}^*(\rho_H) \). Then, we use the indifference condition of informed speculators in case of \( \rho = \rho_H \), equation (81), to obtain \( x_{21}^*(\rho_H) \) as a function of \( \theta_2^*(\rho_H) \). Thus, we arrive at:

\[ x_{2U}^*(\rho) = \theta_2^*(\rho) + \frac{1}{\sqrt{\gamma}} \Phi^{-1}\left( \frac{\theta_2^*(\rho) - n \Phi\left( \frac{\alpha_2(\rho)(\theta_2^*(\rho) - \mu_2(\rho, \theta_1)) - \sqrt{\alpha_2(\rho) + \gamma} \Phi^{-1}(1 + \gamma) \right)}{1 - n} \right) \quad \forall \rho. \] (82)

For further reference, for all \( \rho \in \{0, \rho_H\} \), a sufficient condition for the partial derivatives with respect to the equilibrium thresholds to be strictly positive is \( \gamma > \gamma_1 \):

\[ \frac{dx_{2U}^*(\rho)}{d\theta_2^*(\rho)} > 0. \] (83)
Since the attacking threshold is the same for an uninformed speculator, subtracting equation (82) evaluated at \( \rho = 0 \) from the same equation evaluated at \( \rho = \rho_H \) must yield zero. This yields the first of two implicit relationships between \( \theta_2^*(0) \) and \( \theta_2^*(\rho_H) \):

\[
K(n, \theta_2^*(0), \theta_2^*(\rho_H)) \equiv x_{2U}^*(0) - x_{2U}^*(\rho_H) = 0. \tag{84}
\]

Now, we construct the second implicit relationship between the two aggregate thresholds \( \theta_2^*(0) \) and \( \theta_2^*(\rho_H) \) in two steps. First, insert equation (82) evaluated at \( \rho = 0 \) in \( J(n, \theta_2^*(0), \theta_2^*(\rho_H), x_{2U}^*) \) and in \( \hat{p} \) as used in \( J(n, \theta_2^*(0), \theta_2^*(\rho_H), x_{2U}^*) \). Second, insert equation (82) evaluated at \( \rho = \rho_H \) in \( J(n, \theta_2^*(0), \theta_2^*(\rho_H), x_{2U}^*) \). Combining both expressions yields:

\[
L(n, \theta_2^*(0), \theta_2^*(\rho_H)) \equiv J(n, \theta_2^*(0), \theta_2^*(\rho_H), x_{2U}^*(0), x_{2U}^*(\rho_H)) - \frac{1}{1 + b/l} = 0. \tag{85}
\]

For further reference, the partial derivatives of \( L \) with respect to the equilibrium thresholds are:

\[
\frac{\partial L}{\partial \theta_2^*(0)} = \frac{d\hat{p}(\theta_1, x_{2U}^*(0))}{d\theta_2^*(0)} \frac{dx_{2U}^*(0)}{d\theta_2^*(0)} [\Psi(\theta_2^*(0), x_{2U}^*(0), 0) - \Psi(\theta_2^*(\rho_H), x_{2U}^*(\rho_H), \rho_H)] + \hat{p}(\theta_1, x_{2U}^*(0)) \frac{\partial \Psi(\theta_2^*(0), x_{2U}^*(0), 0)}{\partial \theta_2^*(0)}, \tag{86}
\]

\[
\frac{\partial L}{\partial \theta_2^*(\rho_H)} = (1 - \hat{p}(\theta_1, x_{2U}^*(0))) \frac{d\Psi(\theta_2^*(\rho_H), x_{2U}^*(\rho_H), \rho_H)}{d\theta_2^*(\rho_H)}. \tag{87}
\]

### B.8 Proof of Lemma 3

This proof builds heavily on the description of the coordination stage in the case of potentially asymmetrically informed speculators described in Appendix B.7. The equilibrium is characterized by two equations, (S2) and (S3), in two unknowns, \( \theta_2(0) \) and \( \theta_2(\rho_H) \). Here we show existence and uniqueness of the pair \((\theta_2^*(0), \theta_2^*(\rho_H))\). Then, the attacking thresholds are uniquely backed out from \((\theta_2^*(0), \theta_2^*(\rho_H))\).

First, we analyze the relationship between \( \theta_2(0) \) and \( \theta_2(\rho_H) \) as governed by \( K \). Using equations (S2) and (S3), \( \frac{\partial K}{\partial \theta_2^*(0)} > 0 \) and \( \frac{\partial K}{\partial \theta_2(\rho_H)} < 0 \). Hence, \( \frac{\partial \theta_2^*(0)}{\partial \theta_2(\rho_H)} > 0 \) by the implicit function
Second, we analyze the relationship between \( \theta_2(0) \) and \( \theta_2(\rho_H) \) as governed by \( L \). It can be shown that \( \gamma > \gamma_1 \) is sufficient for \( \frac{\partial L}{\partial \theta_2(\rho_H)} < 0 \). Thus, one can show that \( \frac{dL}{d\theta_2(0)} < 0 \) holds for a sufficiently high but finite value of \( \gamma \). This is proven by generalizing the argument of the proof of Proposition \( \text{[1]} \) so \( \lim_{\gamma \to \infty} |\Psi(\theta_2^*(0), x_{2U}, 0) - \Psi(\theta_2^*(\rho_H), x_{2U}, \rho_H)| = 0 \). Hence, \( \frac{d\theta_2(0)}{d\theta_2(\rho_H)} < 0 \) in the limit. By continuity, there exists a finite precision of private information that guarantees the inequality as well.

Taken both of these points together, \((\theta_2^*(0), \theta_2^*(\rho_H))\) is unique if it exists. This arises from the established strict monotonicity and the opposite sign.

Third, we establish existence of \((\theta_2^*(0), \theta_2^*(\rho_H))\) by making two points: (i) for the highest permissible value of \( \theta_2(0) \), the value of \( \theta_2(\rho_H) \) prescribed by \( K \) is strictly larger than the value of \( \theta_2(\rho_H) \) prescribed by \( L \); and (ii) for the lowest permissible value of \( \theta_2(0) \), the value of \( \theta_2(\rho_H) \) prescribed by \( K \) is strictly smaller than the value of \( \theta_2(\rho_H) \) prescribed by \( L \).

To make these points, consider the following auxiliary step. For any \( \theta_2(\rho) \geq \theta_{21}^*(\rho) \), it can be shown that:

\[
\frac{\partial \Phi^{-1}\left(\frac{\theta_2^*(\rho) - n\Phi\left(\frac{\alpha_2(\rho)(\theta_2^*(\rho) - \mu_2(\rho, \theta_1)) - \sqrt{\alpha_2(\rho) + \gamma} \Phi^{-1}\left(\frac{1}{1+b\gamma}\right)}{1-n}\right)}{1-n}ight)}{\partial \rho} \geq 0 \tag{88}
\]

because \( F_2(\theta_2(\rho), \rho) \leq \frac{1}{1+b\gamma} \) for any \( \rho \in \{0, \rho_H\} \). Note that both the previous expression and the partial derivative hold with strict inequality if \( \theta_2(\rho) > \theta_{21}^*(\rho) \).

Inspecting the inside of the inverse of the cdf, \( \Phi^{-1} \), we define the highest permissible value of \( \theta_2(\rho) \) that is labelled \( \overline{\theta}_2(\rho, n) \) for all \( \rho \):

\[
1 = \frac{\overline{\theta}_2(\rho, n) - n\Phi\left(\frac{\alpha_2(\rho)(\overline{\theta}_2(\rho, n) - \mu_2(\rho, \theta_1)) - \sqrt{\alpha_2(\rho) + \gamma} \Phi^{-1}\left(\frac{1}{1+b\gamma}\right)}{\sqrt{\gamma}}\right)}{1-n}\tag{89}
\]

Therefore, \( 1 \geq \overline{\theta}_2(\rho, 1) \geq \theta_{21}^*(\rho) \forall \rho \), where the first (second) inequality binds if and only if \( n = 0 \) (\( n = 1 \)).

Next, evaluate \( K \) at the highest permissible value, \( \theta_2(0) = \overline{\theta}_2(0, n) \), which yields \( \theta_2(\rho_H) = \ldots \)
\( \tilde{\theta}_2(\rho_H, n) \). Likewise, evaluate \( L \) at the highest permissible value, \( \theta_2(0) = \tilde{\theta}_2(0, n) \), which yields \( \theta_2(\rho_H) < \tilde{\theta}_2(\rho_H, n) \). This proves point (i).

We now proceed with point (ii). For any \( \theta_2(\rho) \leq \theta_{2I}^*(\rho) \) it can be shown that:

\[
\frac{\partial \Phi^{-1}\left(\frac{\theta_2^*(\rho) - \alpha \Phi^{-1}(\frac{\alpha(\theta_2^*(\rho) - \mu_2(\rho, \theta_1)) - \sqrt{\alpha(\theta_2^*(\rho) + \gamma)} \Phi^{-1}(\frac{1}{1+\theta_1})}{\sqrt{\alpha}}}{1 - n}\right)}{\partial n} \leq 0 \tag{90}
\]

because \( F_2(\theta_2(\rho), \rho) \geq \frac{1}{1+\theta_1} \) for any \( \rho \in \{0, \rho_H\} \). Note that both the previous expression and the partial derivative hold with strict inequality if \( \theta_2(\rho) < \theta_{2I}^*(\rho) \).

Inspecting the inside of the inverse of the cdf, \( \Phi^{-1} \), we define the lowest permissible value of \( \theta_2(\rho) \), which is labelled \( \theta_2(\rho, n) \) for all \( \rho \):

\[
0 = \frac{\theta_2(\rho, n) - n \Phi^{-1}\left(\frac{\alpha(\theta_2(\rho, n) - \mu_2(\rho, \theta_1)) - \sqrt{\alpha(\theta_2(\rho) + \gamma)} \Phi^{-1}(\frac{1}{1+\theta_1})}{\sqrt{\alpha}}}{1 - n}\right)}{1 - n}. \tag{91}
\]

Therefore, \( 0 \leq \theta_2(\rho, 1) \leq \theta_{2I}^*(\rho) \) \( \forall \rho \), where the first (second) inequality binds if and only if \( n = 0 \) \( (n = 1) \).

Next, evaluate \( K \) at the lowest permissible value, \( \theta_2(0) = \theta_2(0, n) \), which yields \( \theta_2(\rho_H) = \tilde{\theta}_2(\rho_H, n) \). Likewise, evaluate \( L \) at the lowest permissible value, \( \theta_2(0) = \theta_2(0, n) \), which yields \( \theta_2(\rho_H) > \tilde{\theta}_2(\rho_H, n) \). This proves point (ii) and completes the proof. \( \text{(q.e.d.)} \)

### B.9 Proof of Lemma 4

We prove the results of Lemma 4 in turn. A general observation is that the updated belief on the probability of positive cross-country correlation becomes degenerate: \( \hat{p} \rightarrow p \) for \( \alpha \rightarrow 0 \). Results (A) and (B) are closely linked, so we start by proving them below.

**Results (A) and (B).** This prove has three steps.

**Step 1:** We show in the first step that both fundamental thresholds in the case of asymmetrically informed speculators lie either within these bounds or outside of them. As a consequence
of $\hat{p} \to p$, condition $L(n, \theta^*_2(0), \theta^*_2(\rho_H)) = 0$ prescribes that, for any $n$, the thresholds $\theta^*_2(0)$ and $\theta^*_2(\rho_H)$ are either simultaneously within or outside of the two bounds given by the fundamental thresholds if all speculators are informed, $\theta^*_{2I}(0)$ and $\theta^*_{2I}(\rho_H)$. This is proven by contradiction. First, suppose that $\theta^*_2(\rho_H) < \theta^*_{2I}(\rho_H)$ and $\theta^*_2(0) < \theta^*_{2I}(0)$. This leads to a violation of $L(\cdot) = 0$ because $J(n, \theta^*_2(0), \theta^*_2(\rho_H)) > 0$. Second, suppose that $\theta^*_2(\rho_H) > \theta^*_{2I}(\rho_H)$ and $\theta^*_2(0) > \theta^*_{2I}(0)$. Again, this leads to a violation because $J(n, \theta^*_2(0), \theta^*_2(\rho_H)) < 0$.

**Step 2:** We now derive the derivatives of the fundamental thresholds with respect to the proportion of informed speculators, $\frac{d\theta^*_2(\rho)}{dn}$ and $\frac{d\theta^*_{2I}(\rho)}{dn}$. Applying the implicit function theorem for simultaneous equations, we obtain these derivatives:

$$
\frac{d\theta^*_2(n, 0, \theta_1)}{dn} = \frac{1}{|M|} \left| \begin{array}{c} -\frac{\partial K}{\partial n} - \frac{\partial K}{\partial \theta_2(0, \rho, \theta_1)} \\ -\frac{\partial L}{\partial n} - \frac{\partial L}{\partial \theta_2(0, \rho, \theta_1)} \\ \frac{\partial K}{\partial \theta_2(0, 0, \theta_1)} - \frac{\partial K}{\partial \theta_2(0, \rho, \theta_1)} \\ \frac{\partial L}{\partial \theta_2(0, 0, \theta_1)} - \frac{\partial L}{\partial \theta_2(0, \rho, \theta_1)} \end{array} \right| = |M_1|
$$

where $|M| \equiv \det(M)$. We also find that:

$$
\frac{d\theta^*_2(n, \rho_H, \theta_1)}{dn} = \frac{1}{|M|} \left| \begin{array}{c} \frac{\partial K}{\partial \theta_2(n, 0, \theta_1)} - \frac{\partial K}{\partial \theta_2(n, \rho_H, \theta_1)} \\ \frac{\partial L}{\partial \theta_2(n, 0, \theta_1)} - \frac{\partial L}{\partial \theta_2(n, \rho_H, \theta_1)} \end{array} \right| = |M_2|
$$

To find $|M|$, recall from the proof of Lemma $\mathbf{3}$ that $\frac{\partial K}{\partial \theta_2(0)} > 0$ and $\frac{\partial K}{\partial \theta_2(\rho_H)} < 0$. Furthermore, $\frac{\partial L}{\partial \theta_2(0)} < 0$ and $\frac{\partial L}{\partial \theta_2(\rho_H)} < 0$ for a sufficiently high but finite value of $\gamma$. As a result, $|M| < 0$ for a sufficiently high but finite value of $\gamma$.

The proof proceeds by analyzing $|M_1|$ and $|M_2|$. To do this, we first examine the derivatives
\[ \frac{\partial K}{\partial n} \quad \text{and} \quad \frac{\partial L}{\partial n} \]. Thereafter, we combine the results to obtain the signs of \(|M_1|\) and \(|M_2|\):

\[ \frac{\partial K}{\partial n} = \frac{\partial x_{2U}(0)}{\partial n} - \frac{\partial x_{2U}(\theta_2(\rho H))}{\partial n} = \sqrt{\frac{1}{\gamma}} \frac{\Phi\left(\frac{1}{\gamma} \Phi^{-1}(\frac{1}{1+b_1}) \right) - \Phi\left(\frac{1}{\gamma} \Phi^{-1}(\frac{1}{1+b_1}) \right)}{(1-n)^2 \phi(\Phi^{-1}(\ldots))}. \] (94)

To evaluate this partial derivative, we can use the optimality condition in the case of symmetrically informed speculators, \(n = 1\). That is, \(\theta_{2I}(\rho)\) is defined as the solution to \(F_2(\theta_{2I}(\rho), \rho) = 0\), where uniqueness requires that \(F_2\) is strictly decreasing in the first argument. This implies:

\[ \theta_2(\rho) - \Phi\left(\frac{\alpha_2(\rho, \theta_1)(\theta_2(\rho) - \mu_2(\rho, \theta_1)) - \sqrt{\alpha_2(\rho, \theta_1) + \gamma} \Phi^{-1}(\frac{1}{1+b_1})}{\sqrt{\gamma}}\right) \leq 0 \quad \text{if} \quad \theta_2(\rho) \lesssim \theta_{2I}(\rho). \]

Therefore, we obtain:

\[ \frac{\partial K}{\partial n} = \begin{cases} < 0 & \text{if} \theta_2^*(n, 0, \theta_1) < \theta_{2I}^*(0) \land \theta_2^*(n, \rho H, \theta_1) > \theta_{2I}^*(\rho H) \\ > 0 & \text{if} \theta_2^*(n, 0, \theta_1) > \theta_{2I}^*(0) \land \theta_2^*(n, \rho H, \theta_1) < \theta_{2I}^*(\rho H) \\ = 0 & \text{if} \theta_2^*(n, 0, \theta_1) = \theta_{2I}^*(0) \land \theta_2^*(n, \rho H, \theta_1) = \theta_{2I}^*(\rho H) \end{cases} \quad \forall \quad n \in [0, 1). \]

After having found the partial derivative for one equilibrium condition \((K)\), we turn to the other equilibrium condition \((L)\). Here, we can invoke the envelope theorem in order to obtain \(\frac{\partial L}{\partial n} = 0\). The idea is the following. Since \(L\) represents the indifference condition of an uninformed speculator, the proportion of informed speculators enters only indirectly via \(x_{2U}^*\). Therefore, we can write:

\[ \frac{\partial L}{\partial n} = \frac{\partial J}{\partial x_{2U}^*} \frac{\partial x_{2U}^*}{\partial n} + \frac{\partial J}{\partial n}. \] (95)

Since \(x_{2U}^*\) is the optimal attacking threshold of an uninformed speculator, it satisfies \(J(\cdot, x_{2U}^*) = \frac{1}{1+b_1}\). Thus, we must have \(\frac{\partial J}{\partial x_{2U}^*} = 0\), which corresponds to a first-order optimality condition. (This implicitly uses the result that the equilibrium is unique.)
Third, we obtain the derivatives of the fundamental thresholds for sufficiently small but
positive values of $\alpha$. We find that:

$$
\frac{d\theta^*_{2}(n,0,\theta_1)}{dn} = \begin{cases} 
> 0 & \text{if } \theta^*_2(n,0,\theta_1) < \theta^*_2(0) \land \theta^*_2(n,\rho_H,\theta_1) > \theta^*_2(\rho_H) \\
< 0 & \text{if } \theta^*_2(n,0,\theta_1) > \theta^*_2(0) \land \theta^*_2(n,\rho_H,\theta_1) < \theta^*_2(\rho_H) \\
= 0 & \text{if } \theta^*_2(n,0,\theta_1) = \theta^*_2(0) \land \theta^*_2(n,\rho_H,\theta_1) = \theta^*_2(\rho_H)
\end{cases} \quad \forall \ n \in [0,1).
$$

and:

$$
\frac{d\theta^*_2(n,\rho_H,\theta_1)}{dn} = \begin{cases} 
< 0 & \text{if } \theta^*_2(n,0,\theta_1) < \theta^*_2(0) \land \theta^*_2(n,\rho_H,\theta_1) > \theta^*_2(\rho_H) \\
> 0 & \text{if } \theta^*_2(n,0,\theta_1) > \theta^*_2(0) \land \theta^*_2(n,\rho_H,\theta_1) < \theta^*_2(\rho_H) \\
= 0 & \text{if } \theta^*_2(n,0,\theta_1) = \theta^*_2(0) \land \theta^*_2(n,\rho_H,\theta_1) = \theta^*_2(\rho_H)
\end{cases} \quad \forall \ n \in [0,1).
$$

Step 3: In this final step, we combine the results from the previous two steps to show
both boundedness and monotonicity. In particular, we use the result that the derivative of the
fundamental threshold w.r.t. the proportion of informed speculators is zero once the boundary is
hit. Therefore, the thresholds in the general case of asymmetrically informed speculators are always
bounded, which proves Result (A). The distinction between the two cases arises because:

$$
\theta^*_2(0) = \begin{cases} 
> \theta^*_2(\rho_H) & \text{if } \theta_1 > \theta_1 \\
< \theta^*_2(\rho_H) & \text{if } \theta_1 < \theta_1 \\
= 0 & \text{if } \theta_1 = \theta_1.
\end{cases} \quad (96)
$$

Given boundedness, in turn, the derivatives of the fundamental threshold can be clearly signed,
yielding Result (B).

Now, for the case of $\theta_1 \geq \theta_1$, we prove that $\theta^*_2(\rho_H) \leq \theta^*_2(\rho_H), \theta^*_2(0) \leq \theta^*_2(0)$ for all $n$ if $\alpha$
sufficiently small. First, $\theta^*_2(\rho_H) < \theta^*_2(0) = \theta^*_2(\rho_H)$ if $n = 0$, while $\theta^*_2(0) = \theta^*_2(0)$
and $\theta^*_2(\rho_H) = \theta^*_2(\rho_H)$ if $n = 1$. Second, $\frac{d\theta^*_2(0)}{dn}\big|_{n=0} > 0$ and $\frac{d\theta^*_2(0)}{dn}\big|_{n=1} = 0$. Third, by continuity
$\theta^*_{2U} < \theta^*_2(0) < \theta^*_2(0)$ and $\frac{d\theta^*_2(n,\theta_1,0)}{dn} > 0$ for small values of $n$. Fourth, if for any $\hat{n} \in (0,1]$
\( \theta_2^*(0) \searrow \theta_2^*(0) \) when \( n \to \hat{n} \), then for sufficiently small but positive values of \( \alpha \) – it has to be true that \( \theta_2^*(\rho) \searrow \theta_2^*(\rho) \) when \( n \to \hat{n} \). This is because of the result in step 1. Fifth, given \( \frac{d\theta_2^*(n,0,\theta_1)}{dn} < 0 \) if \( \theta_2^*(0) > \theta_2^*(0) \) and \( \theta_2^*(\rho_H) < \theta_2^*(\rho_H) \), it follows by continuity that \( \theta_2^*(0) = \theta_2^*(0) \) and \( \theta_2^*(\rho_H) = \theta_2^*(\rho_H) \) for all \( n \geq \hat{n} \). In conclusion, \( \theta_2^*(\rho_H) \leq \theta_2^*(\rho_H), \theta_2^*(0) \leq \theta_2^*(0) \) for all \( n \in [0,1] \) if \( \alpha \) sufficiently small.

For the case \( \theta_1 < \theta_1 \) it can be proven that \( \theta_2^*(\rho_H) \geq \theta_2^*(\rho_H), \theta_2^*(0) \geq \theta_2^*(0) \) for all \( n \) if \( \alpha \) is sufficiently small using a similar argument (all signs in relation to fundamental thresholds flip).

**Result (C).** From equation (91),

\[
x_2^*(\rho) = \frac{\alpha_2(\rho, \theta_1) + \gamma \theta_2^*(\rho) - \frac{\alpha_2(\rho, \theta_1)}{\gamma} \mu_2(\rho, \theta_1)}{\gamma} - \frac{\sqrt{\alpha_2(\rho, \theta_1) + \frac{\gamma}{2}}}{\gamma} \Phi^{-1}(\frac{1}{1 + b/l})
\]  

(97)

we see that:

\[
\frac{\gamma}{\alpha_2(\rho, \theta_1) + \gamma} \frac{dx_2^*(\rho)}{dn} = \frac{d\theta_2^*(\rho)}{dn}
\]

(98)

Therefore, by continuity, there exists a sufficiently small but positive value of \( \alpha \) that implies the required inequality, taking into account the monotonicity of the fundamental thresholds. Therefore, the distance between the fundamental thresholds is monotone for any \( n > 0 \), which implies \( \frac{d((x_2^*(0) - x_2^*(\rho_H)))}{dn} > 0 \). *(q.e.d.)*

**B.10 Proof of Lemma 5**

It remains to consider the case of \( \theta_1 < \theta_1 \). Here, we have \( \theta_2^*(0) < \theta_2^*(\rho_H) \) and \( \theta_2^*(0, \theta_1) \leq \theta_2^*(n, \rho, \theta_1) \leq \theta_2^*(\rho_H, \theta_1) \) \( \forall \rho \in \{0,1\} \). We will prove that \( \frac{dx(n, \theta_1)}{dn} \geq 0 \) \( \forall \theta_1 < \theta_1 \) and \( \sigma(n, \theta_1) > 0 \) \( \forall \theta_1 < \theta_1 \).

Again, it is optimal to purchase information if the differential expected payoff \( EU_1 - EU_U \) is
positive (observe the difference to equation (99)):

\[-p\left(\int_{\theta_1}^{\theta_2} \int_{\theta_1}^{\theta_2} g(x_{12} | \theta_2) dx_{12} f(\theta_2 | 0, \theta_1) d\theta_2 + \int_{\theta_1}^{\theta_2} \int_{\theta_1}^{\theta_2} g(x_{12} | \theta_2) dx_{12} f(\theta_2 | 0, \theta_1) d\theta_2 \right) + (1 - p)\left(\int_{\theta_1}^{\theta_2} \int_{\theta_1}^{\theta_2} g(x_{12} | \theta_2) dx_{12} f(\theta_2 | \rho_H, \theta_1) d\theta_2 + \int_{\theta_1}^{\theta_2} \int_{\theta_1}^{\theta_2} g(x_{12} | \theta_2) dx_{12} f(\theta_2 | \rho_H, \theta_1) d\theta_2 \right) - c \geq 0. \quad (99)\]

Given that \( \theta_1 < \theta_2 \), the first two summands in (99) are strictly positive and, thus, \( \overline{\sigma}(n, \theta_1) > 0 \forall \theta_1 < \theta_2 \). Furthermore, an increase in \( n \) is associated with a (weak) decrease in \( \theta_1 \) and \( x_2 \), and a (weak) increase in \( \theta_2 \) and \( x_1 \). For this reason, an increase in \( n \) leads to a relative increase of the loss component in the first summand of equation (99) and a relative increase in the benefit component in the second summand. As a result, we have that the left-hand side of equation (99) increases in \( n \). Thus, \( \frac{\partial \sigma(n, \theta_1)}{\partial n} \geq 0 \forall \theta_1 < \theta_2 \), which concludes the proof. \((q.e.d.)\)

**B.11 Proof of Proposition 4**

The first result follows from Lemma 3 in combination with Lemma 4. Lemma 4 gives conditions such that the interval \( (\theta_1, \theta_2) \) is non-empty. From Lemma 4 there exists a strictly positive cost level such that information acquisition occurs if \( \theta_1 < \theta_2 \) for all \( c \leq c(0, \theta) \). Hence, there does exist a unique pure-strategy PBE in which the ex-post contagion effect arises if the private signal is sufficiently precise and the public signal sufficiently imprecise. More specifically, there exist unique optimal attacking rules at the coordination stage and a unique information acquisition rule at the information stage.

The second result follows from Lemma 5 in combination with Proposition 3. From Lemma 5 there exists a strictly positive cost level such that information acquisition occurs for all \( \theta_1 \neq \theta_2 \). Conditional on \( \rho = 0 \), the distribution of \( \theta_1 \) is described by the pdf \( f(\theta_1) \), which places a strictly positive probability mass on \( \theta_1 \neq \theta_2 \). As a result, there exists a strictly positive cost level such that information acquisition occurs if \( c \leq \int_{-\infty}^{\theta_2} \overline{\sigma}(0, \theta_1) f(\theta_1) d\theta_1 \). Hence, there exists a unique pure-strategy PBE where the ex-ante contagion effect arises if private signal is sufficiently precise and
the public signal sufficiently imprecise. (q.e.d.)

C Extensions and robustness

Here we discuss in detail the robustness of our key insights to important variations of our model and the extensions listed in section 7.

C.1 Heterogeneity across countries: a numerical illustration

In our model, both countries are identical if $\rho = 0$. This is because $\mu_1 = \mu_2$ and $b_1/l_1 = b_2/l_2$. However, in practice the ground-zero country is often more prone to a crisis. To accommodate for this fact, we can extend our framework to the asymmetric case in which country 2 has a strong prior about the fundamental (as before), while the crisis country 1 has a weak prior. Interestingly, this heterogeneity in countries allows for a much stronger contagion effect. The strengthening of the contagion effect arises because a crisis occurs in a weak country 1 already with higher realizations of $\theta_1$ for which the mean effect is smaller. Hence, $\frac{d(\theta_2^*(0) - \theta_2^*(\rho_H, \theta_1))}{d\theta_1} > 0$.

To see this, consider first a numerical example with two symmetric countries using the parameters from Figure 2 in Appendix A.2: $\alpha = 1$ (substantially positive), $\gamma = 1.5$ (quite small), $\mu_1 = \mu_2 = \mu = 0.6$, $b_1/l_1 = b_2/l_2 = b/l = 1/3$, $p = 0.7$, $\rho_H = 0.8$. For these parameter values, the critical fundamental threshold in country 1 is $\theta_1^* = 0.1$. Suppose a crisis occurs in country 1 after a fundamental realization of $\theta_1 = 0.07 < \theta_1^*$ that is bad, but not too bad. This leads to more aggressive attacks in country 2 when speculators learn that $\rho = 0$, i.e., $\theta_2^*(0) = 0.1 > \theta_2^*(\rho_H) = 0.09$. Here, the variance effect outweighs the mean effect.

Next, let us revisit the previous numerical example with the only change that $b_1/l_1 = 1/3 < b_2/l_2 = 2$ instead of $b_1/l_1 = b_2/l_2 = 1/3$. Now $\theta_1^*$ increases from 0.1 to 0.75 because the ground-zero crisis country 1 is weaker. While the aggressiveness of attacks in country 2 is unaltered when $\rho = 0$, i.e. $\theta_2^*(0) = 0.1$, the aggressiveness of attacks in country 2 can be much weaker when $\rho = \rho_H$. This is because a crisis in country 1 happens already for higher realizations of $\theta_1$. For instance now a relatively high $\theta_1 = 0.5$ already triggers a crisis in country 1. However, $\theta_1 = 0.5$ implies weaker
attacks in country 2, i.e. \( \theta^*_{2I}(\rho_H, \theta_1 = 0.5) = 0.01 < \theta^*_{2I}(\rho_H, \theta_1 = 0.1) = 0.09 \). As a result, the difference in equilibrium fundamental thresholds, and hence the strength of the ex-post contagion effect, increases from \( \theta^*_{2I}(0) - \theta^*_{2I}(\rho_H, 0.1) = 0.01 \) to \( \theta^*_{2I}(0) - \theta^*_{2I}(\rho_H, 0.5) = 0.09 \).

### C.2 Incomplete learning about the correlation

The key insights of our model are robust to incomplete learning about \( \rho \). Suppose, for instance, that speculators purchase a noisy signal about \( \rho \) that is correct with a certain probability only. The only change is that we now have to work with mixture distributions also for the case of informed speculators. In other words, when moving from the polar case of complete learning about \( \rho \) to incomplete learning, informed speculators and, hence, their attacking thresholds become more similar to the ones of uninformed speculators. As a result, the quantitative difference between the thresholds used by informed and uninformed shrinks, but our qualitative results are unaffected.

### C.3 Negative correlation of fundamentals

With negative cross-country correlation of fundamentals, i.e. \( \rho_H < 0 \), a low fundamental realization in country 1 is associated with an improvement of public information about country 2 when speculators learn that \( \rho = \rho_H \) (an increase in the prior \( \mu_2(\rho_H, \theta_1) > \mu \)). As a result, the key threshold rankings are reversed, as can be seen in Table 2 in Appendix section B.2.2. Hence, the mirror image of the ex-post contagion effect established in section 4.1 can arise when \( \rho_H < 0 \). After a crisis in country 1, the incidence of a crisis in country 2 can be higher if speculators learn the good news about country 2’s fundamental that \( \rho = \rho_H < 0 \) instead of staying uninformed about the realization of \( \rho \).

### C.4 The sensitivity our specification of ex-ante uncertainty

What is the role of \( p \) and \( \rho_H \)? A variation in \( p \) has a quantitative effect on our results only. In particular, a change in \( p \) leaves \( \theta_1 \) unaltered. However, an increase (decrease) in \( p \) reduces (increases) the difference between \( \theta^*_{2U} \) and \( \theta^*_{2I}(0) \). As a result, the ex-post contagion effect in
Proposition 2 is weakened (strengthened) when \( p \) increases (decreases) but our qualitative results are unaffected.

The effect of changing \( \rho_H \) is harder to understand because it affects both \( \theta_1 \) and the prior in country 2. However, we show that our model exhibits *ex-post stability*. That is, the ex-post contagion result holds with opposite inequality if \( \rho = \rho_H \):

\[
\text{Pr}(\theta_2 < \theta^*_2(\rho, \theta_1) | \rho = \rho_H, \theta_1) < \text{Pr}(\theta_2 < \theta^*_2U(\theta_1) | \rho = \rho_H, \theta_1) \quad \forall \theta_1 \in (\theta_1, \theta^*_1) \quad (100)
\]

\[
\text{Pr}(\theta_2 < \theta^*_2(\rho, \theta_1) | \rho = 0, \theta_1) < \text{Pr}(\theta_2 < \theta^*_2U(\theta_1) | \rho = 0, \theta_1) \quad \forall \theta_1 < \theta^*_1. \quad (101)
\]

### C.5 Different timing assumption and aggregate macro shocks

In our baseline model with symmetric countries, it is the case that country 2 is identical to the crisis country 1 if \( \rho = 0 \). We acknowledge that this feature may not seem fully plausible, but we emphasize that it can easily be overturned when allowing countries to be asymmetric as demonstrated in section C.1. In particular, assuming relatively weaker fundamentals for country 1 leads to the plausible result that a crisis in country 2 is less likely if \( \rho = 0 \) than a crisis in country 1.

Furthermore, the fact that a reversal of the timing leads to an outcome in country 2 that is the same as the outcome in country 2 without the reversal in timing and \( \rho = 0 \) is a consequence of the correlation structure used in our model. We argue below that this feature would disappear in an alternative model with a macro shock without affecting our key insights. In our paper, we do not use a modelling approach with an aggregate shock because it would unnecessarily complicate the analysis by adding one additional layer of Bayesian updating.

Suppose we extend our model by introducing an observed aggregate macro shock that is potentially correlated to the fundamentals in countries 1 and 2. As before, speculators are uncertain about the actual correlation and only speculators in the second country have the opportunity to learn more about the correlation of their country’s fundamentals with the macro shock. Now, a reversal of the timing leads to an outcome in country 2 that is no longer the same as the outcome in country 2 without the reversal in the timing and \( \rho = 0 \). This is because being the first country implies that speculators cannot learn about the actual correlation to the observed aggregate shock.
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<table>
<thead>
<tr>
<th>Title</th>
<th>Authors</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimation of an Adaptive Stock Market Model with Heterogeneous Agents</td>
<td>Henrik Amilon</td>
<td>2005:177</td>
</tr>
<tr>
<td>Some Further Evidence on Interest-Rate Smoothing: The Role of Measurement Errors in the Output Gap</td>
<td>Mikael Apel and Per Jansson</td>
<td>2005:178</td>
</tr>
<tr>
<td>Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through</td>
<td>Malin Adolfson, Stefan Laséen, Jesper Lindé and Mattias Villani</td>
<td>2005:179</td>
</tr>
<tr>
<td>Are Constant Interest Rate Forecasts Modest Interventions? Evidence from an Estimated Open Economy DSGE Model of the Euro Area</td>
<td>Malin Adolfson, Stefan Laséen, Jesper Lindé and Mattias Villani</td>
<td>2005:180</td>
</tr>
<tr>
<td>Inference in Vector Autoregressive Models with an Informative Prior on the Steady State</td>
<td>Mattias Villani</td>
<td>2005:181</td>
</tr>
<tr>
<td>Bank Mergers, Competition and Liquidity</td>
<td>Elena Carletti, Philipp Hartmann and Giancarlo Spagnolo</td>
<td>2005:182</td>
</tr>
<tr>
<td>Exploring Interactions between Real Activity and the Financial Stance</td>
<td>Tor Jacobson, Jesper Lindé and Kasper Roszbach</td>
<td>2005:184</td>
</tr>
<tr>
<td>Two-Sided Network Effects, Bank Interchange Fees, and the Allocation of Fixed Costs</td>
<td>Mats A. Bergman</td>
<td>2005:185</td>
</tr>
<tr>
<td>Real Exchange Rate and Consumption Fluctuations following Trade Liberalization</td>
<td>Kristian Jönsson</td>
<td>2005:187</td>
</tr>
<tr>
<td>Bayesian Inference of General Linear Restrictions on the Cointegration Space</td>
<td>Mattias Villani</td>
<td>2005:189</td>
</tr>
<tr>
<td>Forecasting Performance of an Open Economy Dynamic Stochastic General Equilibrium Model</td>
<td>Malin Adolfson, Stefan Laséen, Jesper Lindé and Mattias Villani</td>
<td>2005:190</td>
</tr>
<tr>
<td>Forecast Combination and Model Averaging using Predictive Measures</td>
<td>Jana Eklu and Sune Karlsson</td>
<td>2005:191</td>
</tr>
<tr>
<td>A Simultaneous Model of the Swedish Krona, the US Dollar and the Euro</td>
<td>Hans Lindblad and Peter Sellin</td>
<td>2006:193</td>
</tr>
<tr>
<td>Efficient Bayesian Inference for Multiple Change-Point and Mixture Innovation Models</td>
<td>Paolo Giordani and Robert Kohn</td>
<td>2006:196</td>
</tr>
<tr>
<td>Derivation and Estimation of a New Keynesian Phillips Curve in a Small Open Economy</td>
<td>Karolina Holmberg</td>
<td>2006:197</td>
</tr>
<tr>
<td>Monetary Policy and Staggered Wage Bargaining when Prices are Sticky</td>
<td>Mikael Carlsson and Andreas Westermark</td>
<td>2006:199</td>
</tr>
<tr>
<td>The Swedish External Position and the Krona</td>
<td>Philip R. Lane</td>
<td>2006:200</td>
</tr>
<tr>
<td>Title</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>Price Setting Transactions and the Role of Denominating Currency in FX Markets</td>
<td>2007:201</td>
<td></td>
</tr>
<tr>
<td>by Richard Friberg and Fredrik Wilander</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The geography of asset holdings: Evidence from Sweden</td>
<td>2007:202</td>
<td></td>
</tr>
<tr>
<td>by Nicolas Coeurdacier and Philippe Martin</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Evaluating An Estimated New Keynesian Small Open Economy Model</td>
<td>2007:203</td>
<td></td>
</tr>
<tr>
<td>by Malin Adolfsson, Stefan Laséen, Jesper Lindé and Mattias Villani</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The Use of Cash and the Size of the Shadow Economy in Sweden</td>
<td>2007:204</td>
<td></td>
</tr>
<tr>
<td>by Gabriela Guibourg and Björn Segendorf</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bank supervision Russian style: Evidence of conflicts between micro- and macro-prudential concerns</td>
<td>2007:205</td>
<td></td>
</tr>
<tr>
<td>by Sophie Claeyts and Koen Schoors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimal Monetary Policy under Downward Nominal Wage Rigidity</td>
<td>2007:206</td>
<td></td>
</tr>
<tr>
<td>by Mikael Carlsson and Andreas Westermak</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financial Structure, Managerial Compensation and Monitoring</td>
<td>2007:207</td>
<td></td>
</tr>
<tr>
<td>by Vittoria Cerasi and Sonja Daltung</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financial Frictions, Investment and Tobin's q</td>
<td>2007:208</td>
<td></td>
</tr>
<tr>
<td>by Guido Lorenzoni and Karl Walentin</td>
<td></td>
<td></td>
</tr>
<tr>
<td>by Mathias Trabandt</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Acquisition versus greenfield: The impact of the mode of foreign bank entry on information and bank lending rates</td>
<td>2007:210</td>
<td></td>
</tr>
<tr>
<td>by Sophie Claeyts and Christa Hainz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nonparametric Regression Density Estimation Using Smoothly Varying Normal Mixtures</td>
<td>2007:211</td>
<td></td>
</tr>
<tr>
<td>by Mattias Villani, Robert Kohn and Paolo Giordani</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The Costs of Paying – Private and Social Costs of Cash and Card</td>
<td>2007:212</td>
<td></td>
</tr>
<tr>
<td>by Mats Bergman, Gabriella Guibourg and Björn Segendorf</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Using a New Open Economy Macroeconomics model to make real nominal exchange rate forecasts</td>
<td>2007:213</td>
<td></td>
</tr>
<tr>
<td>by Peter Sellin</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Introducing Financial Frictions and Unemployment into a Small Open Economy Model</td>
<td>2007:214</td>
<td></td>
</tr>
<tr>
<td>by Lawrence J. Christiano, Mathias Trabandt and Karl Walentin</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Earnings Inequality and the Equity Premium</td>
<td>2007:215</td>
<td></td>
</tr>
<tr>
<td>by Karl Walentin</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bayesian forecast combination for VAR models</td>
<td>2007:216</td>
<td></td>
</tr>
<tr>
<td>by Michael K. Andersson and Sune Karlsson</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Do Central Banks React to House Prices?</td>
<td>2007:217</td>
<td></td>
</tr>
<tr>
<td>by Daria Finocchiaro and Virginia Queijo von Heideken</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The Riksbank’s Forecasting Performance</td>
<td>2007:218</td>
<td></td>
</tr>
<tr>
<td>by Michael K. Andersson, Gustav Karlsson and Josef Svensson</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Macroeconomic Impact on Expected Default Frequency</td>
<td>2008:219</td>
<td></td>
</tr>
<tr>
<td>by Per Asberg and Hovick Shahnazarian</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monetary Policy Regimes and the Volatility of Long-Term Interest Rates</td>
<td>2008:220</td>
<td></td>
</tr>
<tr>
<td>by Virginia Queijo von Heideken</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Governing the Governors: A Clinical Study of Central Banks</td>
<td>2008:221</td>
<td></td>
</tr>
<tr>
<td>by Lars Frisell, Kasper Roszbach and Giancarlo Spagnolo</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The Monetary Policy Decision-Making Process and the Term Structure of Interest Rates</td>
<td>2008:222</td>
<td></td>
</tr>
<tr>
<td>by Hans Dillén</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How Important are Financial Frictions in the U S and the Euro Area</td>
<td>2008:223</td>
<td></td>
</tr>
<tr>
<td>by Virginia Queijo von Heideken</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Block Kalman filtering for large-scale DSGE models</td>
<td>2008:224</td>
<td></td>
</tr>
<tr>
<td>by Ingvar Strid and Karl Walentin</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimal Monetary Policy in an Operational Medium-Sized DSGE Model</td>
<td>2008:225</td>
<td></td>
</tr>
<tr>
<td>by Malin Adolfsson, Stefan Laséen, Jesper Lindé and Lars E. O. Svensson</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm Default and Aggregate Fluctuations</td>
<td>2008:226</td>
<td></td>
</tr>
<tr>
<td>by Tor Jacobson, Rikard Kindell, Jesper Lindé and Kasper Roszbach</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Title</td>
<td>Author(s)</td>
<td>Year:Volume</td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td>---------------------------------------------------------------------------</td>
<td>-------------</td>
</tr>
<tr>
<td>Re-Evaluating Swedish Membership in EMU: Evidence from an Estimated Model</td>
<td>Ulf Söderström</td>
<td>2008:227</td>
</tr>
<tr>
<td>Expectation Driven Business Cycles with Limited Enforcement</td>
<td>Karl Walentin</td>
<td>2009:229</td>
</tr>
<tr>
<td>Effects of Organizational Change on Firm Productivity</td>
<td>Christina Hakanson</td>
<td>2009:230</td>
</tr>
<tr>
<td>Flexible Modeling of Conditional Distributions Using Smooth Mixtures of Asymmetric Student T Densities</td>
<td>Feng Li, Mattias Villani and Robert Kohn</td>
<td>2009:233</td>
</tr>
<tr>
<td>Forecasting Macroeconomic Time Series with Locally Adaptive Signal Extraction</td>
<td>Paolo Giordani and Mattias Villani</td>
<td>2009:234</td>
</tr>
<tr>
<td>Evaluating Monetary Policy</td>
<td>Lars E. O. Svensson</td>
<td>2009:235</td>
</tr>
<tr>
<td>Risk Premiums and Macroeconomic Dynamics in a Heterogeneous Agent Model</td>
<td>Ferre De Graeve, Maarten Dossche, Marina Emiris, Henri Sneessens and Raf Wouters</td>
<td>2010:236</td>
</tr>
<tr>
<td>Picking the Brains of MPC Members</td>
<td>Mikael Apel, Carl Andreas Claussen and Petra Lennarstdotter</td>
<td>2010:237</td>
</tr>
<tr>
<td>Housing collateral and the monetary transmission mechanism</td>
<td>Karl Walentin and Peter Sellin</td>
<td>2010:239</td>
</tr>
<tr>
<td>The Discursive Dilemma in Monetary Policy</td>
<td>Carl Andreas Claussen and Øistein Reisland</td>
<td>2010:240</td>
</tr>
<tr>
<td>Monetary Regime Change and Business Cycles</td>
<td>Vasco Cúrdia and Daria Finocchiar</td>
<td>2010:241</td>
</tr>
<tr>
<td>Bayesian Inference in Structural Second-Price common Value Auctions</td>
<td>Bertil Wegmann and Mattias Villani</td>
<td>2010:242</td>
</tr>
<tr>
<td>Equilibrium asset prices and the wealth distribution with inattentive consumers</td>
<td>Daria Finocchiaro</td>
<td>2010:243</td>
</tr>
<tr>
<td>Identifying VARS through Heterogeneity: An Application to Bank Runs</td>
<td>Ferre De Graeve and Alexei Karas</td>
<td>2010:244</td>
</tr>
<tr>
<td>Modeling Conditional Densities Using Finite Smooth Mixtures</td>
<td>Feng Li, Mattias Villani and Robert Kohn</td>
<td>2010:245</td>
</tr>
<tr>
<td>The Output Gap, the Labor Wedge, and the Dynamic Behavior of Hours</td>
<td>Luca Sala, Ulf Söderström and Antonella Trigari</td>
<td>2010:246</td>
</tr>
<tr>
<td>Anticipated Alternative Policy-Rate Paths in Policy Simulations</td>
<td>Stefan Laséen and Lars E. O. Svensson</td>
<td>2010:248</td>
</tr>
<tr>
<td>The Effects of Endogenous Firm Exit on Business Cycle Dynamics and Optimal Fiscal Policy</td>
<td>Lauri Vilmi</td>
<td>2011:250</td>
</tr>
<tr>
<td>Parameter Identification in a Estimated New Keynesian Open Economy Model</td>
<td>Malin Adolfson and Jesper Lindé</td>
<td>2011:251</td>
</tr>
<tr>
<td>Title</td>
<td>Authors</td>
<td>Year:Volume</td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------</td>
<td>-------------</td>
</tr>
<tr>
<td>Wage Adjustment and Productivity Shocks</td>
<td>Mikael Carlsson, Julián Messina and Oskar Nordström Skans</td>
<td>2011:253</td>
</tr>
<tr>
<td>Taking the Twists into Account: Predicting Firm Bankruptcy Risk with Splines of Financial Ratios</td>
<td>Paolo Giordani, Tor Jacobson, Erik von Schedvin and Mattias Villani</td>
<td>2011:256</td>
</tr>
<tr>
<td>Collateralization, Bank Loan Rates and Monitoring: Evidence from a Natural Experiment</td>
<td>Geraldo Cerqueiro, Steven Ongena and Kasper Roszbach</td>
<td>2012:257</td>
</tr>
<tr>
<td>Labor-Market Frictions and Optimal Inflation</td>
<td>Mikael Carlsson and Andreas Westermark</td>
<td>2012:259</td>
</tr>
<tr>
<td>Output Gaps and Robust Monetary Policy Rules</td>
<td>Roberto M. Billi</td>
<td>2012:260</td>
</tr>
<tr>
<td>The Information Content of Central Bank Minutes</td>
<td>Mikael Apel and Marianna Blix Grimaldi</td>
<td>2012:261</td>
</tr>
<tr>
<td>The Cost of Consumer Payments in Sweden</td>
<td>Björn Segendorf and Thomas Jansson</td>
<td>2012:262</td>
</tr>
<tr>
<td>Trade Credit and the Propagation of Corporate Failure: An Empirical Analysis</td>
<td>Tor Jacobson and Erik von Schedvin</td>
<td>2012:263</td>
</tr>
<tr>
<td>Structural and Cyclical Forces in the Labor Market During the Great Recession: Cross-Country Evidence</td>
<td>Luca Sala, Ulf Söderström and Antonella Trigari</td>
<td>2012:264</td>
</tr>
<tr>
<td>Pension Wealth and Household Savings in Europe: Evidence from SHARELIFE</td>
<td>Rob Alessie, Viola Angelini and Peter van Santen</td>
<td>2013:265</td>
</tr>
<tr>
<td>Long-Term Relationship Bargaining</td>
<td>Andreas Westermark</td>
<td>2013:266</td>
</tr>
<tr>
<td>Using Financial Markets To Estimate the Macro Effects of Monetary Policy: An Impact-Identified FAVAR*</td>
<td>Stefan Pitschner</td>
<td>2013:267</td>
</tr>
<tr>
<td>DYNAMIC MIXTURE-OF-EXPERTS MODELS FOR LONGITUDINAL AND DISCRETE-TIME SURVIVAL DATA</td>
<td>Matias Quiroz and Mattias Villani</td>
<td>2013:268</td>
</tr>
<tr>
<td>Conditional euro area sovereign default risk</td>
<td>André Lucas, Bernd Schwaab and Xin Zhang</td>
<td>2013:269</td>
</tr>
<tr>
<td>Nominal GDP Targeting and the Zero Lower Bound: Should We Abandon Inflation Targeting?*</td>
<td>Roberto M. Billi</td>
<td>2013:270</td>
</tr>
<tr>
<td>Un-truncating VARS*</td>
<td>Ferre De Graeve and Andreas Westermark</td>
<td>2013:271</td>
</tr>
<tr>
<td>Housing Choices and Labor Income Risk</td>
<td>Thomas Jansson</td>
<td>2013:272</td>
</tr>
<tr>
<td>Identifying Fiscal Inflation*</td>
<td>Ferre De Graeve and Virginia Quejo von Heideken</td>
<td>2013:273</td>
</tr>
<tr>
<td>On the Redistributive Effects of Inflation: an International Perspective*</td>
<td>Paola Boel</td>
<td>2013:274</td>
</tr>
<tr>
<td>Business Cycle Implications of Mortgage Spreads*</td>
<td>Karl Walentin</td>
<td>2013:275</td>
</tr>
<tr>
<td>Approximate dynamic programming with post-decision states as a solution method for dynamic economic models</td>
<td>Isaiah Hull</td>
<td>2013:276</td>
</tr>
<tr>
<td>A detrimental feedback loop: deleveraging and adverse selection</td>
<td>Christoph Bertsch</td>
<td>2013:277</td>
</tr>
<tr>
<td>Distortionary Fiscal Policy and Monetary Policy Goals</td>
<td>Klaus Adam and Roberto M. Billi</td>
<td>2013:278</td>
</tr>
<tr>
<td>Predicting the Spread of Financial Innovations: An Epidemiological Approach</td>
<td>Isaiah Hull</td>
<td>2013:279</td>
</tr>
</tbody>
</table>