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A detrimental feedback loop: deleveraging and adverse selection

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Abstract

Market distress can be the catalyst of a deleveraging wave, as in the 2007/08 financial crisis. This paper demonstrates how market distress and financial sector deleveraging can fuel each other in the presence of adverse selection problems in an opaque asset market segment. At the core of the detrimental feedback loop is investors' desire to reduce their reliance on the distressed opaque market by decreasing their leverage which in turn amplifies adverse selection in the opaque market segment. In the extreme, trade in the opaque asset market segment breaks down. I find that adverse selection is at the root of two inefficiencies: it distorts both investors' long-term leverage choices and investors' short-term liquidity management. I derive implications for central bank policy and highlight the ambiguous role played by transparency.

Keywords: Leverage, endogenous borrowing constraints, financial crisis, liquidity, opacity, private information, central bank policy.

JEL classification: D82, E58, G01, G20.

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At the start of the financial crisis in July 2007, interbank market spreads shot up and subprime asset markets experienced a large drop. Temporarily important market segments dried up completely.[1] At the same time, a pronounced deleveraging wave in the financial sector began. US investment banks drastically cut leverage immediately after the crisis erupted. Data show that US commercial banks as well as EU and UK banks started heavily reducing leverage beginning in 2008. Financial market conditions were the main driver of this deleveraging in 2007 and 2008. Thereafter, the effect of financial market disorder on deleveraging was compounded by regulatory initiatives and a change in economic and policy conditions.[2]

There is a series of academic papers dedicated to the study of the underlying reasons for the market distress at the start of the crisis, and of potential policies that can positively affect economic outcomes in such a scenario. Arguably, credit risk played an important role due to solvency concerns related to the US subprime market. These solvency concerns were fueled by the lack of transparency in the securitization process, highlighting the role played by asymmetric information (Gorton 2008). In addition, liquidity risk was found to be an important contributor to the widening of interest rate spreads (Dick-Nielsen et al. 2012; Schwarz 2014) and was accompanied by hoarding behavior (Heider et al. 2009; Ashcraft et al. 2011; Acharya and Merrouche 2013), a mechanism that has been discussed in the theoretical literature (Gale and Yorulmazer 2013; Malherbe 2014).

The relationship between the exceptional deleveraging in the financial sector (Buttiglione et al. 2014) and the market distress at the start of the crisis still deserves more attention. Here lies the contribution of this paper. It proposes a novel mechanism that draws a connection between financial market distress, reflected in sales of opaque assets at fire sale prices, and deleveraging. This paper presents a model with liquidity management and a leverage choice to analyze the effect of adverse selection problems in an opaque asset market on liquidity provision and leverage. In this framework, I establish a detrimental feedback loop between the intensity of adverse selection problems in the

[1] The spread between LIBOR and the overnight Federal Funds rate for 3-month loans jumped from sub 20 basis point levels before July 2007 to elevated levels between 40 and 100 basis points (Cecchetti 2009, p. 58). A similar picture holds for Europe, where the spread between EURIBOR and the 3-month overnight index swap jumped from below 10 basis points to elevated levels fluctuating around 60 basis points during the year after August 2007. Then, the spread shot up to over 180 basis points in November 2008 (Heider et al. 2009, p. 8). US subprime markets for asset-backed securities and global high-yield corporate bonds were largely affected. In the year after August 2007, the US subprime index fell by over 80% and global high-yield corporate bond spreads climbed to over 60% (see Bank of England Financial Stability Report, April 2008).

[2] Feyen and González del Mazo (2013) provide a detailed account of the deleveraging wave. For US investment banks, leverage ratios (measured as weighted tangible assets over tangible common equity) dropped from around 40% in 2007 to under 30% in 2008, followed by a further drop to under 20% in 2009. Main factors contributing to the deleveraging wave in the initial crisis period till 2008 were the distress in interbank, subprime asset and high-yield corporate bonds markets.

opaque asset market and financial sector deleveraging. In particular, the anticipation of more intense future adverse selection problems in the opaque market provides incentives to investors to reduce their leverage ex-ante. By reducing leverage, investors can build up spare borrowing capacity that allows for a better future access to the prime market segment which is not prone to adverse selection problems. This precautionary behavior, in turn, amplifies the adverse selection problem in the opaque market, generating fire sale prices which, in the extreme, can lead to a breakdown of trade in the opaque market. In particular, I demonstrate that deleveraging and the intensity of an Akerlof (1970) type adverse selection problem in the opaque market are interconnected in a potentially detrimental way through a novel feedback mechanism that has yet to be studied in the existing literature. Furthermore, I investigate the impact of private information problems on efficiency and describe a constrained inefficient liquidity management and leverage choice, which motivates central bank intervention.

This paper takes the view that liquidity management is conducted over a short horizon on a daily basis. In contrast, the leverage choice is part of the medium- to long-term business model and is only adjusted when lucrative investment opportunities arise that require financing or when the market outlook changes drastically. Following this view, I develop a model of liquidity management that relies on the existence of both transparent and opaque asset market segments to share the idiosyncratic liquidity risk of investors. While asset qualities are common knowledge in the transparent market segment, the sellers have private information on the asset qualities in the opaque market segment. The model of liquidity management is augmented with an ex-ante leverage choice that pins down the size of investments in lucrative investment opportunities, as well as the borrowing capacity left available for future trades. This level of "spare" borrowing capacity governs the future access to the prime market segment for collateralized credit. A lower level of leverage is tantamount to a higher level of borrowing capacity, which can entail a worsening of an Akerlof-type adverse selection problem in the opaque asset market. In turn, anticipating future market distress may fuel the incentives to further reduce leverage in order to reduce the necessity to finance in the future through the opaque market at discounted asset prices. However, incentives to reduce leverage are linked to the intensity of the adverse selection problem in a non-trivial way through equilibrium prices that are determined by the endogenous supply of cash-in-the-market (Shleifer and Vishny 1992; Allen and Gale 1994, 2004, 2007).

The endogenous and, hence, imperfectly elastic supply of cash is key to the model mechanics.

Both a higher level of cash and a higher level of leverage are genuinely beneficial for market liquidity. The notion of liquidity used in this paper refers to the cost of converting expected future income into cash. The stronger the adverse selection problem in the opaque market and the less cash available in the economy, the higher the cost. As such, my model incorporates two key reasons for market breakdowns: adverse selection and insufficient financial muscle, both of which are discussed by Tirole (2011). In this paper, cash is modeled as the most "liquid" mean for transactions. Hence, cash is not equal to negative debt (Acharya et al. 2007). This property arises because it is assumed that there is an epsilon-cost of issuing debt, which drives an arbitrarily small wedge between the return of investing cash and the cost of obtaining cash through collateralized credit. This minimal wedge ensures that leverage is only increased when there are positive gains doing so.

The detrimental feedback effect developed in this paper draws a connection between financial market distress and deleveraging. The two key elements of the mechanism are *(i)* adverse selection problems in the opaque asset market that cause market distress and *(ii)* the ability of investors to shield themselves against market distress by adjusting their leverage. Empirically, the link between distress in subprime markets and adverse selection problems can be attributed to a substantial rise in counterparty risk and severe asymmetric information problems in subprime markets at the beginning of the crisis.[3] My model uses the same trigger for market distress. The model then links market distress in a novel way to the financial sector deleveraging wave witnessed during the crisis. At the core of the mechanism is a strategic complementarity in leverage choices that can trigger a deleveraging wave after a small deterioration in the anticipated intensity of adverse selection in the opaque asset market. Deleveraging, which mirrors a quest for unencumbered high quality collateral, has systemic consequences because a reduction in leverage fuels the adverse selection problem.[4] In particular, I demonstrate analytically that there can exist detrimental deleveraging spirals induced by adverse selection which, in the extreme, can lead to a breakdown of pooling or even of all trade in the opaque asset market. In this way, the model draws a compelling connection between the deleveraging wave during the 2007/08 financial crisis and the adverse selection problems in the subprime market. Importantly, the novel feedback mechanism presented in this paper does not rely on

[3]See Gorton (2008) amongst others. Lax screening incentives under the existing securitization procedures may have contributed to the emergence of substantial asymmetric information problems, as argued by Keys et al. (2010).

[4]The emergences of a high demand for unencumbered high quality collateral that can be used for future trades has some features of a "flight to quality". However, the mechanism does not rely on Knightian uncertainty as in Caballero and Krishnamurthy (2008) but has more similarities with the mechanism in Guerrieri and Shimer (2014), which is based on fire sales.

portfolio constraints or margin requirements.[5] Instead, the effect is solely generated by investors' desire to shield themselves from the negative implications of adverse selection in the opaque market.

The strategic complementarity in leverage choices is also at the root of an ("ex-ante") inefficiency, because investors would in many cases be collectively better off if they chose not to reduce their leverage in anticipation of future market distress as this creates a welfare reducing breakdown of pooling in the opaque asset market. Furthermore, I uncover two layers of ("interim") inefficiencies in the liquidity management. First, I analyze under which conditions the private information problem together with incomplete ex-ante risk markets tend to give rise to inefficient under-investment or over-investment in cash. Second, even absent a private information friction, the liquidity management affects ex-ante financing conditions which can result in an inefficient under-investment in cash (Lorenzoni 2008). My result that the economy exhibits an under-investment in cash for a large parameter range contradicts the prescription of models with adverse selection who predict over-investment in cash (e.g. Malherbe 2014). However, this result is in line with the cash-in-the-market pricing literature and is also consistent with Bhattacharya and Gale (1987) who find that moral hazard and adverse selection are associated with under-investment in reserves.

The inefficiencies in the liquidity management suggest immediate policy implications for the regulation of liquidity. Of even more interest is, however, how a policy maker can prevent the emergence of an inefficient deleveraging wave that triggers a breakdown of pooling in the opaque asset market segment. I find that a widening of collateral requirements by the central bank is an effective tool, as well as any policy that makes investors more reliant on financing through the opaque asset market. Allowing for the co-existence of an opaque and transparent asset market segment, I find that more opacity (a large size of the opaque market segment) can be good for market functioning and reduces the incentives for deleveraging.[6]

This paper is most closely related to Bolton et al. (2011) and Malherbe (2014). They find that the anticipation of adverse selection in the future leads to excessive early asset trading and liquidity hoarding, respectively. In Bolton et al. (2011), outside liquidity is the most efficient source of financing for banks with liquidity needs. However, asymmetric information about the quality of bank assets constitutes a detrimental friction when market participants anticipate future liquidity shocks. The authors show that asymmetric information can lead to excessive early asset trading and excessive cash reserves. Moreover, banks reduce their origination of assets in anticipation of fire sale

[5] As it is, for instance, in Brunnermeier et al. (2008) or Geanakoplos (2009).

[6] Other papers also found positive implications of opacity for market functioning (e.g. Dang et al. 2012).

prices due to future liquidity shocks. In Malherbe (2014), information about the quality of assets is also asymmetric. He finds that adverse selection and hoarding behavior can fuel one another.

In contrast to Bolton et al. (2011) and Malherbe (2014), investors in my model anticipate future adverse selection problems and, as a result, seek to reduce their leverage today. This, in turn, can intensify adverse selection in the future and lead to stronger deleveraging today. Unlike Malherbe (2014), the supply of cash is endogenous in my model and, hence, cash holdings do not present a negative externality. My model also differs from Malherbe (2014) in that it focuses on a novel interplay between two frictions: a private information problem and endogenous collateral constraints.

The interplay between these two frictions is also analyzed by Martin and Taddei (2013) and Boissay (2011). However, their setting differs substantially from mine and they consider an environment in which both frictions affect the same asset. Furthermore, Boissay (2011) focuses on self-fulfilling pessimistic beliefs to generate a liquidity dry-up. While Martin and Taddei (2013) find that limited pledgeability exacerbates adverse selection problems, the opposite result arises in my model with *endogenous* borrowing constraints. Nenov (2013) studies advantageous selection and endogenous leverage, highlighting a "debt quality" channel that generates a co-movement between asset prices, aggregate output and credit. In earlier work, Caballero and Krishnamurthy (2001, 2002) analyze the interplay between international and domestic collateral constraints. When the domestic credit market is underdeveloped, domestic agents over-borrow and hold insufficient international collateral. Binding international collateral constraints, in turn, lead to fire sales in domestic markets with negative implications for financial intermediation. Caballero and Krishnamurthy's domestic and international credit market share some similarities with my paper's sub-prime market and its prime market for collateralized credit. Unlike their work, the provision of high quality collateral plays a negative role in my model due to the adverse selection problem in sub-prime markets. Moreover, in my model, the provision of high quality collateral is connected to leverage and investment.

There are several related papers that examine adverse selection problems in macro models following the partial equilibrium model of Eisfeldt (2004). These include Kurlat (2009), Bigio (2014) and Taddei (2010). Kurlat (2009) and Bigio (2014) both extend the framework of Kiyotaki and Moore (2012) by introducing endogenous resaleability through asymmetric information. While Kurlat (2009) focuses on the relationship between liquidity and macroeconomic fluctuations as well as the amplification of shocks through learning, Bigio (2014) adds a labor market friction and analyzes how dispersion shocks to capital quality affect the liquidity of assets and the macroeconomy.

Taddei (2010) rationalizes the positive relationship between aggregate economic activity and the cross-firm divergence of bond yields. Their models contrast with mine in that they abstract from the role of a liquid asset that co-exists with illiquid assets prone to adverse selection problems, which is a key element of the mechanism presented in this paper.[7]

A separate strand of the literature examines adverse selection problems and liquidity in asset markets (Kirabaeva 2011) and in interbank credit markets (Freixas and Holthausen 2004; Heider et al. 2009; Heider and Hoerova 2009). Freixas and Holthausen (2004) is most closely related to this paper. They analyze a model with secured and unsecured credit which is similar to my model in which illiquid assets co-exist with high quality collateral. However, Freixas and Holthausen (2004) consider an *exogenous* change in the income structure that changes the composition between secured and unsecured credit, thereby affecting the intensity of adverse selection in interbank credit markets. Ma (2014) develops a model in which investors can limit their private information on asset qualities by investing in systemic risk assets. This gives rise to a herding behavior that enhances market liquidity, thereby generating a liquidity versus systemic risk trade-off.

The remainder of this paper is organized as follows. Section 1 describes the model. Section 2 proceeds with the equilibrium analysis. It establishes the existence of the detrimental feedback mechanism and contains the efficiency analysis related to liquidity management and the leverage choice. Thereafter, section 3 provides a policy discussion based on these results. Section 4 concludes. All proofs and figures are in the Appendix.

1 The model

The model has four dates, indexed by $t = 0, 1, 2, 3$. It comprises a model of liquidity provision spanning over dates $t = 1, 2, 3$ and an ex-ante leverage choice at date $t = 0$.

1.1 Agents

There are two types of agents: *investors* and *outside financiers.* Investors can be thought of as banks who face idiosyncratic liquidity risk and engage in leveraged investments. Outside financiers, however, can be thought of as fixed income funds or insurers who provide financing to banks.

[7]More recently, Cui and Radde (2014) developed a version of Kiyotaki and Moore (2012) with a liquid asset and search frictions in illiquid asset markets. However, they abstract from adverse selection and focus on the pro-cyclicality of asset liquidity.

Investors There is a continuum of ex-ante identical investors with unit mass who are born at $t = 0$ and consume at dates $t = 2$ and $t = 3$. Similar to Diamond and Dybvig (1983), investors are ex-ante uncertain about whether they would prefer to consume *early* or *late*. The likelihood of an individual investor being either of early type or late type is given by λ and $(1 - \lambda)$, respectively. The preferences are represented by the utility function:

$$u(c_{2i}, c_{3i}) = \beta_i \cdot log(c_{2i}) + (1 - \beta_i) \cdot log(c_{3i}),$$

where c_{ti} is the consumption of a type i investor at date t.[8] A higher relative valuation of consumption at $t = 2$ by early types is reflected in the parameter restriction $1 > \beta_E > \beta_L = 0$.[9]

Outside financiers There are risk-neutral outside financiers who maximize their payoff at $t = 3$ and do not discount time. Their total resources available at $t = 0$ are given by $m_0 > 0$ units of cash.

1.2 Technology

Both investors and outside financiers have access to a risk-less storage technology at each date. Furthermore, investors are endowed with an illiquid long-term investment project that can be leveraged and expanded at $t = 0$. In addition, investors can invest in risky long-term assets at dates $t = 1, 2$.

The leveraged investment at date t=0 At the initial date, investors are each endowed with a long-term investment project of size $\kappa > 0$, which yields a deterministic date $t = 3$ return of $\rho > 1$ per unit invested (i.e. constant returns to scale). The long-term investment project can be expanded at $t = 0$ by raising long-term funds from outside financiers at the *endogenous* interest rate r_0, where $\rho \geq r_0 \geq 1$. However, only a fraction $0 < \gamma < 1$ of the income at date $t = 3$ from the investment project is pledgeable and, hence, leverage is limited.[10]

If the investment project is not fully levered up, then some borrowing capacity is available for future periods. Let $\theta_t \geq 0$ be the amount of *spare borrowing capacity*, consisting of the pledgeable return of the long-term investment project that has not been pledged at $t = 0$. Furthermore, let the

[8] Log-utility is used to ensure analytical tractability. More generally, a neoclassical utility function satisfying the Inada conditions is needed.

[9] For the main mechanism of this paper to work, early types have to face a trade-off between consuming in the intermediate period or in the terminal period. Hence, β_E must be strictly smaller than one, as to allow for consumption at $t = 3$. Setting $\beta_L = 0$ simplifies the analysis without affecting the key insights.

[10] The assumption of limited pledgeability could, for instance, be justified by a moral hazard problem (Holmström and Tirole 2010) or by the inalienability of human capital (Hart and Moore 1994).

total return on the leveraged investment project be denoted as $G(\theta_0, r_0)$.

The model of liquidity provision spanning over dates t=1,2,3 Investors enter date $t = 1$ with a predetermined spare borrowing capacity ($\theta_0 \geq 0$). The leverage choices are assumed to be *common knowledge* at $t = 1$. Investors each receive an endowment of one unit of cash at $t = 1$ and no additional endowment thereafter. Furthermore, investors can raise long-term funds from outside financiers or from other investors at dates $t = 1, 2$ at the endogenous interest rates $r_1, r_2 \geq 1$ by pledging their spare borrowing capacity. The decision problem at date $t = 1$ for each investor is to either become an *illiquid investor* who invests all resources in long-term assets, or a *liquid investor* who stores cash. Hence, investors have a discrete liquidity management problem at $t = 1$.[11]

Long-term assets pay off at the terminal date $t = 3$ and can be thought of as a fully diversified portfolio consisting of risky mortgages or corporate loans. At date $t = 2$ each individual loan turns out to be of *bad quality* with probability $0 < \alpha < 1$ and of *good quality* with probability $(1 - \alpha)$. In the latter case, the per unit payoff at date $t = 3$ is $R_G > 1$. In the former case, the per unit payoff at date $t = 3$ is $R_B < R_G$. Let the individual long-term asset returns in each portfolio be independently distributed and also be independent of investors' preferences.

At $t = 2$, risky long-term asset portfolios can be prematurely liquidated using a *private liquidation technology* that yields $\ell_G < R_G$ for *good* loans and $\ell_B = R_B$ for *bad* loans.[12] Alternatively, illiquid investors can securitize their portfolio of risky long-term loans at $t = 2$ and partially or fully sell their long-term asset holdings on the asset market described below.

1.3 Information structure

There are two layers of private information. First, investors learn privately at the beginning of date $t = 2$ whether they are of the early type or of the late type. Second, an exogenous fraction $0 \leq q \leq 1$ of each illiquid investor's risky long-term asset portfolio turns out to be *opaque*, whereas the fraction $(1 - q)$ turns out to be *transparent* (i.e. *non-opaque*). The fundamental value of each individual loan (R_G or R_B) in the *transparent* portfolio is learned *publicly* at the beginning of $t = 2$, while the fundamental value of each individual loan in the *opaque* portfolio of an illiquid investor is learned

[11] Investors can either store all their available resources, or instead, fully invest them in a long-term asset portfolio. Indivisibility of investments at $t = 1$ is a strong assumption. It allows me to derive all results analytically. The indivisibility has the character of an occupational choice. In practice, the indivisibility could for instance be the result of fixed costs for investments in a loan portfolio. Importantly, the key insights of the paper prevail in an economy where investors can select mixed portfolios at $t = 1$.

[12] The private liquidation technology captures the idea of a costly premature project liquidation (Heider et al. 2009).

privately. For simplicity, learning is perfect.

1.4 Market institutions at dates t=1 and t=2

At date $t = 1$, there exists only a collateralized credit market because long-term assets are not yet initiated. Differently, at date $t = 2$, there are two distinct spot markets in which trades take place simultaneously: first, an asset market where illiquid investors can sell risky long-term loans to liquid investors or to outside financiers and, second, a credit market where investors can borrow or lend against the leveraged long-term investment project at the endogenous interest rate r_2.

The asset market at date t=2 The market for long-term assets is an anonymous and competitive spot market comprised of two market segments: first, the *transparent* market segment where good and bad assets are traded at the endogenous prices p_G and p_B, respectively, and, second, the *opaque* market segment where illiquid investors have private information about the asset quality. Since buyers cannot distinguish between *opaque* assets, all are traded at the same endogenous price p.[13] It is assumed that buyers in the *opaque* market segment do not face risk because they purchase a portfolio with a fundamental value corresponding to the average quality traded.[14]

The collateralized credit market at dates t=1 and t=2 In this market, investors can obtain credit up to their predetermined spare borrowing capacity. The borrowing constraint of an investor is given by $\frac{\theta_{t-1}}{r_t}$ at dates $t = 1, 2$.

1.5 Key assumptions

The model is summarized in figure 1 in Appendix A.1. The model section closes with an overview over the assumptions behind the key results of the paper.

Assumption 1: $R_G > 1 > \ell_G > R_B \geq \ell_B = 0$.

Assumption 2: $ER \equiv \alpha \cdot R_B + (1 - \alpha) \cdot R_G > 1$.

Assumption 1 ensures that the possibility of a breakdown of pooling in the opaque market is entertained by allowing for the possibility that the average quality of assets traded can fall short of the

[13] Due to the private information problem, illiquid investors can potentially gain from trading on private information by securitizing their opaque loans of bad quality and selling the "lemons" irrespective of their liquidity needs.

[14] This assumption is common in the literature and is maintained for analytical tractability. It is justified as long as buyers can purchase from multiple sellers at the same time and could also be implemented via an intermediary.

return ℓ_G earned from privately liquidating a good asset. Instead, Assumption 2 guarantees that investments in long-term assets are not dominated by cash. Otherwise, the problem is trivial.

Assumption 3: Investors cannot borrow against their future cash endowments.

Assumption 3 prevents investors from borrowing at $t = 0$ against their future cash endowments. This assumption is easy to justify. Future cash endowments are difficult to pledge as collateral because they are, by their definition, hard to seize.[15]

Assumption 4: There is an arbitrarily small positive cost, $\epsilon > 0$, of issuing collateralized debt.

The epsilon-cost of issuing debt drives an arbitrarily small wedge between the return of investing cash and the cost of obtaining cash in the prime market segment of collateralized credit, reflecting the nature of cash as the most liquid means of transaction. As said earlier, the existence of this wedge ensures that leverage is only increased when there are strictly positive gains doing so.

2 Equilibrium analysis

First, section 2.1 discusses solving the model and provides an equilibrium definition. In section 2.2, I analyze the model of liquidity provision spanning dates $t = 1, 2, 3$. Thereafter, section 2.3 provides an efficiency analysis of liquidity management at $t = 1$, taking the leverage choice as given. Section 2.4 examines the leverage choice at $t = 0$ and compares the symmetric information benchmark (i.e. $q = 0$) to the model with an opaque market segment featuring asymmetric information (i.e. $q > 0$). With asymmetric information, a detrimental feedback loop between adverse selection in the opaque market segment and deleveraging can arise and I discuss how a planner may prevent excessive (inefficient) deleveraging.

The model is solved backwards. At $t = 2$, illiquid and liquid investors face the realization of idiosyncratic liquidity risk. They can use two distinct competitive spot markets at $t = 2$ to share their liquidity risk by trading long-term assets against cash (in both the opaque and transparent market segment), and by borrowing or lending against safe collateral in the credit market. At $t = 1$, investors face the liquidity management problem. Furthermore, investors decide on how much to borrow or lend in the collateralized credit market. Finally, investors decide on leverage at $t = 0$.

[15]Furthermore, future endowments may be stochastic in a richer model and, hence, hard to observe and verify.

The leverage choice plays an important role. Spare borrowing capacity is available for future periods whenever the investment project is not fully levered up at $t = 0$. Given θ_0 and r_0 the leveraged investment's maximum scale, $\overline{\Upsilon}(\theta_0, r_0)$, and the total return, $G(\theta_0, r_0)$, can be derived as:

$$\overline{\Upsilon}(\theta_0, r_0) \equiv \frac{k - \theta_0/r_0}{1 - \gamma \cdot \rho/r_0} \tag{1}$$

$$G(\theta_0, r_0) \equiv \rho \cdot \overline{\Upsilon}(\theta_0, r_0) - \gamma \cdot \rho \cdot \left[\overline{\Upsilon}(\theta_0, r_0) - \frac{\theta_0}{\gamma \cdot \rho}\right] = \frac{\rho \cdot (1-\gamma)}{1 - \gamma \cdot \rho/r_0} \cdot \left(\kappa - \frac{\theta_0}{r_0}\right) + \theta_0. \tag{2}$$

Observe that if it is costly not to fully lever up, i.e. if $\rho > r_0$, then $\frac{\partial G(\theta_0, r_0)}{\partial \theta_0} < 0$ and $\frac{\partial G(\theta_0, r_0)}{\partial r_0} < 0$.

2.1 Equilibrium definition and classification of equilibria

Let a denote the average quality of assets traded in the opaque market segment at $t = 2$.

Definition 1. A competitive equilibrium consists of *(i) asset prices at $t = 2$ in the transparent market segment, p_G^* and p_B^*, an asset price and an average quality of assets traded in the opaque market, p^* and a^*, interest rates r_0^*, r_1^*, r_2^* at which markets clear at dates $t = 0, 1, 2$ that are consistent with the equilibrium measure of liquid investors, f^*, and the leverage choices, (ii) type-dependent decision rules at $t = 2$ as functions of p_G^*, p_B^*,p^*, a^*, r_2^*, and the leverage choice, (iii) investment decisions and financing choices at $t = 1$ as functions of r_1^* and expected future prices, which map into equilibrium measures of liquid investors f^* and illiquid investors $(1 - f^*)$, and (iv) a leverage choice at $t = 0$ as function of r_0^* and expected future prices.*

In the remainder, I refer to a *pooling equilibrium* if illiquid investors of the *early type* are willing to sell their *good quality* long-term assets at date $t = 2$ in the opaque market segment, given the equilibrium asset prices. If, instead, the equilibrium asset price is sufficiently low such that illiquid investors are only willing to sell *bad quality* long-term asset, then I refer to a *breakdown* of pooling in the opaque asset market segment.

2.2 Liquidity management at date t=1 & liquidity provision at date t=2

This section focuses on liquidity management and liquidity provision. Specifically, I analyze the liquidity management at $t = 1$ and market functioning at $t = 2$, taking the leverage choice at $t = 0$ as given. That is, I consider an investors' decision problem at $t = 1$ and her trading decisions at $t = 2$ for all $\theta_0^j \in [0, \gamma \cdot \rho \cdot \kappa]$. Since the leverage choice at $t = 0$ can potentially differ depending on whether investors expect to become liquid or illiquid investors at $t = 1$, it is indexed with the superscripts $j = L$ for liquid ($j = I$ for illiquid).

First, section 2.2.1 analyzes trading decisions at $t = 2$. Then section 2.2.2 derives the average quality of assets traded in the opaque market segment and the market-clearing prices (p_G, p_B, p) at $t = 2$ for given leverage and liquidity choices, establishing a link between leverage and market functioning at $t = 2$. Thereafter, I move to the liquidity management problem at $t = 1$ in section 2.2.3 and present the results on equilibrium existence and characterization.

2.2.1 Trading decisions at date t=2 and supply & demand schedules

Investors enter date $t = 2$ with a predetermined leverage choice summarized in θ_1^j.[16] At the beginning of $t = 2$, investors learn privately if they are of the early or late type. Moreover, the quality of individual assets in the transparent portfolio becomes publicly known, but it is learned privately by illiquid investors for the opaque portfolio.

No-arbitrage

No-arbitrage requires that investments in financial markets yield the same return across all markets:

$$r_2 = \frac{R_G}{p_G} = \frac{R_B}{p_B} = \frac{a}{p}, \tag{3}$$

meaning that one unit of cash invested in the collateralized credit market at date $t = 2$ yields the same return as one unit of cash invested in the transparent or opaque asset market segment.

Liquid investors

Let us start with the decision problem of a *liquid investor*. She enters the period with one unit of cash and may be of either early type or late type. Her problem is to decide on how much cash to consume at $t = 2$ and in what to invest the remainder, which can be either stored or invested in financial markets. Investments in financial markets are preferred over storage whenever $r_2 > 1$. Formally, the problem of a *liquid investor* of type i at $t = 2$ writes:

[16] In section 2.2.2 it will be argued that $\theta_1^j = \theta_0^j$ for $j = L, I$.

$$\begin{aligned}
&\max_{0\leq s^L_{2i}\leq 1, -r_2\leq b^L_{2i}\leq \theta^L_1} && \left\{\beta_i \cdot log\left(c^L_{2i}\right) + (1-\beta_i)\cdot log\left(c^L_{3i}\right)\right\} \\
&s.t. && c^L_{2i} = \left(1+\frac{b^L_{2i}}{r_2}\right)\cdot\left(1-s^L_{2i}\right) \\
& && c^L_{3i} = \left(1+\frac{b^L_{2i}}{r_2}\right)\cdot s^L_{2i}\cdot r_2 - b^L_{2i} + G\left(\theta^L_0, r_0\right),
\end{aligned} \tag{4}$$

where the choice variable s^L_{2i} captures the fraction of available cash resources supplied to the market. The choice variable b^L_{2i} captures the amount borrowed in the collateralized credit market, which takes on a negative value if liquid investors want to lend in the collateralized credit market. The collateral constraint is given by $\frac{\theta^L_1}{r_2}$. As a result, the net supply of cash to the market by a liquid investor is given by $s^L_{2i}\cdot\left(1+\frac{b^L_2}{r_2}\right)-\frac{b^L_2}{r_2}$. Notice that the consumption at $t=3$ includes the return on the leveraged long-term investment project given by $G\left(\theta^L_0, r_0\right)$.

Solving the problem in (4) reveals that liquid investors are indifferent as to how they finance their consumption and investments at $t=2$. Hence, fixing $b^L_{2i}=0$ leads to:[17]

$$s^L_{2i} = (1-\beta_i) - \beta_i\cdot\frac{G\left(\theta^L_0, r_0\right)}{r_2}. \tag{5}$$

The incentive for liquid investors to save part of their available resources increases in the return from investing r_2 and decreases in β_i (which captures the relative utility derived from early consumption). A sufficient condition for $s^L_{2i}>0\ \forall i=E,L$ is given by:

$$\beta_E < \frac{1}{1+G(0,1)}. \tag{6}$$

Illiquid investors

The problem of *illiquid investors* at $t=2$ is more complicated. Illiquid investors must decide on how many long-term assets to sell in the opaque and transparent market segment (or to privately liquidate) to obtain funding, and on how much to borrow in the collateralized credit market. Illiquid investors enter the period with an opaque (transparent) long-term asset portfolio of size q $(1-q)$. Formally, the problem of an *illiquid investor* of type i writes:

[17] Assumption 4 assures that investing borrowed money at $t=2$ is not attractive, i.e. $\frac{b^L_{2i}}{r_2+\epsilon}\cdot r_2 < b^L_{2i}$.

$$\max_{\substack{0 \le s^I_{2i} \le 1, b^I_{2i} \le \theta^I_1, 0 \le d^I_{2i} \le 1 \\ 0 \le d^I_{2iG} \le 1, 0 \le d^I_{2iB} \le 1}} \left\{\beta_i \cdot log\left(c^I_{2i}\right) + (1-\beta_i) \cdot log\left(c^I_{3i}\right)\right\} \tag{7}$$

$$s.t. \quad c^I_{2i} = \begin{pmatrix} (1-q) \cdot \frac{ER}{r_2} \cdot d^I_{2i} + \frac{b^I_{2i}}{r_2} + \\ q \cdot (\alpha \cdot d_{2iB} \cdot \breve{p}_B + (1-\alpha) \cdot d_{2iG} \cdot \breve{p}_G) \end{pmatrix} \cdot \left(1 - s^I_{2i}\right)$$

$$c^I_{3i} = \begin{pmatrix} \begin{pmatrix} (1-q) \cdot \frac{ER}{r_2} \cdot d^I_{2i} + \frac{b^I_{2i}}{r_2} + \\ q \cdot \left(\alpha \cdot d^I_{2iB} \cdot \breve{p}_B + (1-\alpha) \cdot d^I_{2iG} \cdot \breve{p}_G\right) \end{pmatrix} \cdot s^I_{2i} \cdot r_2 + \\ \\ \begin{pmatrix} (1-q) \cdot ER \cdot \left(1 - d^I_{2i}\right) - b^I_{2i} + G\left(\theta^I_0, r_0\right) + \\ q \cdot \left(\alpha \cdot \left(1 - d^I_{2iB}\right) \cdot R_B + (1-\alpha) \cdot \left(1 - d^I_{2iG}\right) \cdot R_G\right) \end{pmatrix} \end{pmatrix},$$

where $\breve{p}_G \equiv max\left\{p = \frac{a}{r_2}, \ell_G\right\}$ and $\breve{p}_B \equiv max\left\{p = \frac{a}{r_2}, \ell_B\right\}$. The choice variable s^I_{2i} captures the fraction of available cash resources supplied to the market by an illiquid investor. The choice variable d^I_{2ih} captures the fraction of opaque long-term assets of quality $h = B, G$ that are sold or privately liquidated by an illiquid investor with preferences $i = E, L$. Similarly, d^I_{2i} captures the fraction of transparent long-term assets that are sold. Finally, the choice variable b^I_{2i} captures the amount borrowed in the collateralized credit market by an illiquid investor.

Given that illiquid investors have the option to either sell or privately liquidate their assets, one has to distinguish between two cases. If $p > \ell_G$, they are willing to sell the *opaque* long-term assets of good quality in the market to raise $p \cdot d^I_{2iG} = \frac{a}{r_2} \cdot d^I_{2iG}$ units of cash. Instead, if $p \le \ell_G$, they weakly prefer private liquidation and raise $\ell_G \cdot d^I_{2iG}$ units of cash. For simplicity, it is assumed that good quality opaque assets are privately liquidated as opposed to securitized and sold in the market if $p = \ell_G$.[18] Hence, pooling in the opaque market segment cannot be supported if $p \le \ell_G$.

The first-order necessary condition associated with the problem in (7) and further derivations can be found in Appendix A.2. Suppose there exists a pooling equilibrium, i.e. $p > \ell_G$, which requires that d^I_{2EG} is interior. It shows that early types prefer to finance through markets not affected by asymmetric information where they do not face a discount. Hence, $d^I_{2EB} = d^I_{2E} = 1$, $b^I_{2E} = \theta^I_1$ and:

[18] This simplification rules out the existence of equilibria with *partial* pooling, where good types are indifferent whether to sell or not. The key insights of the paper are not affected by this simplification. See Bertsch (2012) for a discussion of equilibria with partial pooling in a related model with adverse selection.

$$d_{2EG}^{I}=\frac{\beta_E\cdot\left(q+\frac{G\left(\theta_0^I,r_0\right)-\theta_1^I}{R_G}\right)-q\cdot\alpha-(1-\beta_E)\cdot\left((1-q)\cdot\frac{ER}{a}+\frac{\theta_1^I}{a}\right)}{q\cdot(1-\alpha)}. \tag{8}$$

A sufficient condition for $d_{2EG}^{I}<1$ is given by:

$$\beta_E<\frac{1-q\cdot(1-\alpha)}{1+G\left(0,1\right)/R_G}. \tag{9}$$

Intuitively, β_E and $G\left(\theta_0^I,r_0\right)$ cannot be too large, in order to preserve the trade-off between consuming at dates $t=2$ and $t=3$. Lemma 1 below provides a necessary condition for a pooling equilibrium to exist, which also constitutes a sufficient condition for $d_{EG}^{I}>0$.

In sum, the trade-off between consuming at dates $t=2$ and $t=3$ is preserved for both liquid and illiquid investors of early type if the following condition holds:

Condition 1: $\beta_E<\min\left\{\frac{1}{1+G(0,1)},\frac{1-q\cdot(1-\alpha)}{1+G(0,1)/R_G}\right\}$.

Similarly, it can be shown for late types that $d_{2LB}^{I}=1$ and $d_{2LG}^{I}=d_{2E}=0$. Furthermore, late types do not access the collateralized credit market, i.e. $b_{2E}^{I}=0$, because of the epsilon-cost associated with borrowing (Assumption 4). Hence, they only re-invest their cash, i.e. $s_{2E}^{I}=q\cdot\alpha\cdot\frac{a}{r_2}$.

2.2.2 Financial market equilibria at date t=2

Average quality of assets The average quality of assets traded in the opaque market segment at $t=2$ is defined as:

$$a=\frac{\alpha\cdot R_B+\lambda\cdot(1-\alpha)\cdot R_G\cdot d_{2EG}^{I}}{\alpha+\lambda\cdot(1-\alpha)\cdot d_{2EG}^{I}}. \tag{10}$$

Suppose there exists a pooling equilibrium, then:

$$a\left(\theta_1^I,\theta_0^I,r_0\right)=\frac{\alpha\cdot R_B+\lambda\cdot R_G\cdot\left(\beta_E\cdot\left(1+\frac{G\left(\theta_0^I,r_0\right)-\theta_1^I}{q\cdot R_G}\right)-\alpha-(1-\beta_E)\cdot\frac{(1-q)\cdot ER+\theta_1^I}{q\cdot a}\right)}{\alpha+\lambda\cdot\left(\beta_E\cdot\left(1+\frac{G\left(\theta_0^I,r_0\right)-\theta_1^I}{q\cdot R_G}\right)-\alpha-(1-\beta_E)\cdot\frac{(1-q)\cdot ER+\theta_1^I}{q\cdot a}\right)}. \tag{11}$$

Equation (11) implicitly defines $a\left(\theta_1^I,\theta_0^I,r_0\right)$. It has more than one solution of which the one with the higher value is relevant, since the interest is in a scenario with a pooling equilibrium where $a\left(\theta_1^I,\theta_0^I,r_0\right)>\ell_G$. Interestingly, a does not depend on prices at $t=2$ and the aggregate level of liquidity in the economy provided $a \geq p > \ell_G$. This contrasts with models in which short-term funding is perfectly elastic, such as in Malherbe (2014). In Lemma 1 of Malherbe (2014), the author employs a model with a perfectly elastic supply of cash and demonstrates that investments in storage present a negative externality. This contrasts with my model in which the demand for cash is inelastic for prices above ℓ_G, which is a natural property of models with cash-in-the-market pricing. Intuitively, cash-in-the-market pricing features higher aggregate cash holdings as a force that is typically beneficial for market functioning as opposed to being a negative externality.

In the subsequent analysis, it is critical to understand how the average quality of assets traded depends on the borrowing constraint and key deep parameters of the model. Of particular interest is the dependency of a on the tightness of the borrowing constraint of illiquid investors. Using the implicit function theorem, one can show that a tends to decrease in θ_1^I, θ_0^I and r_0. Conversely, a tends to increase in q and R_B. These results are intuitive: a better ability of illiquid investors to borrow reduces sales of good quality opaque assets and thereby amplifies the adverse selection problem. In the extreme, if the average quality of traded assets is depressed by too much, then a pooling equilibrium cannot exist. The results are summarized formally in Lemma 1.

Lemma 1. Average quality of assets traded in the opaque market segment

(a) *A necessary condition for a pooling equilibrium to exist is given by:*

$$\frac{\alpha \cdot R_B + \lambda \cdot R_G \cdot \left(\beta_E \cdot \left(1 + \frac{G\left(\theta_0^I, r_0\right) - \theta_1^I}{q \cdot R_G}\right) - \alpha - (1 - \beta_E) \cdot \frac{(1-q) \cdot ER + \theta_1^I}{q \cdot \ell_G}\right)}{\alpha + \lambda \cdot \left(\beta_E \cdot \left(1 + \frac{G\left(\theta_0^I, r_0\right) - \theta_1^I}{q \cdot R_G}\right) - \alpha - (1 - \beta_E) \cdot \frac{(1-q) \cdot ER + \theta_1^I}{q \cdot \ell_G}\right)} > \ell_G. \tag{12}$$

(b) *The partial derivatives are* $\frac{\partial a}{\partial \theta_0^I} < 0$, $\frac{\partial a}{\partial \theta_1^I} < 0$, $\frac{\partial a}{\partial r_0} < 0$, $\frac{\partial a}{\partial q} > 0$, *and* $\frac{\partial a}{\partial R_B} > 0$ *provided* α *is sufficiently small.*

(c) *The result in (b) holds independent of* α, *provided* q *is sufficiently large and* $R_G \leq 2 \cdot \ell_G$.

Proof. See Appendix A.3.

Result *(b)* of Lemma 1 prescribes that $\frac{\partial a}{\partial \theta_0^I} < 0$. This result shows to be a crucial element of the detrimental feedback loop between deleveraging and adverse selection. Intuitively, a better access to alternative markets makes illiquid investors less reliant on raising funding in the opaque market segment. Similarly, a lower quality of lemons (a smaller R_B) and a larger size of the transparent market segment (a smaller q) also amplifies the adverse selection problem. Notably, the sufficient conditions in *(b)* and *(c)* show to be mild and are satisfied for a large parameter range. Intuitively, they ensure that the adverse selection problem is not too strong and that the possibility of a breakdown of pooling has bite (i.e. a value of ℓ_G that is not too low relative to R_G).

Market-clearing at date t=2 Taken together, the trading decisions derived in section 2.2.1 yield a market-clearing condition. Given the co-existence of different asset market segments, it is useful to express everything in terms of units of cash. For markets to clear, the supply of cash must be weakly larger than the demand: $S(p) \geq D(p)$. Both supply and demand depend on f, the endogenous fraction of liquid investors, and on m_2, the cash held by outside financiers at date $t = 2$:

$$\begin{aligned} & \overbrace{m_2 + f \cdot \left[\lambda \cdot s^L_{2E} + (1-\lambda) \cdot s^L_{2L}\right]}^{\text{(net) supply of cash}} \\ = \quad & \underbrace{(1-f) \cdot \lambda \cdot \left[(1-q) \cdot \frac{ER}{r_2} \cdot d^I_{2E} + q \cdot \left(\alpha \cdot \frac{a}{r_2} \cdot d^I_{2EB} + (1-\alpha) \cdot \frac{a}{r_2} \cdot d^I_{2EG}\right) + b^I_{2E}\right]}_{\text{(net) demand for cash}} \end{aligned} \tag{13}$$

After solving for r_2, the results can be summarized as follows.

Lemma 2. Market-clearing at date t=2

(a) *If a pooling equilibrium exists (i.e. Condition 1 holds and $a \geq p > \ell_G$), then the market-clearing interest rate is:*

$$r_2 = \max \left\{ 1, \frac{G\left(\theta^I_0, r_0\right) \cdot \frac{a}{R_G} + \left(1 - \frac{a}{R_G}\right)\theta^I_1 + q \cdot a + (1-q) \cdot ER + \frac{f \cdot G\left(\theta^L_0, r_0\right)}{1-f}}{\left(m_2 + f \cdot (1 - \lambda \cdot \beta_E)\right) / \left(\lambda \cdot \beta_E \cdot (1-f)\right)} \right\}. \tag{14}$$

(b) *If the solution is interior (i.e. $a > p > \ell_G$), then the partial derivatives are $\frac{\partial r_2}{\partial \theta^I_0} < 0$, $\frac{\partial r_2}{\partial \theta^L_0} < 0$, $\frac{\partial r_2}{\partial \theta^I_1} > 0$, $\frac{\partial r_2}{\partial r_0} < 0$, $\frac{\partial r_2}{\partial q} < 0$, $\frac{\partial r_2}{\partial R_B} = 0$, and $\frac{\partial r_2}{\partial a} > 0$. Provided m_2 is sufficiently small, then:*

$$\begin{cases} \frac{\partial r_2}{\partial f} < 0,\ \frac{\partial p}{\partial f} > 0 & if\ p \in [\ell_G, a) \\ \frac{\partial r_2}{\partial f}, \frac{\partial p}{\partial f} = 0 & if\ p = a. \end{cases}$$

Proof. Equation (14) follows from (13) after plugging in the demand and supply schedules. If the solution to the pricing function is such that $p = \frac{a}{r_2} > a \Leftrightarrow r_2 < 1$, then the market-clearing prices are given by $r_2 = 1 \Leftrightarrow p = a$. Instead, if $p \leq \ell_G$, then a pooling equilibrium cannot be supported because $d^I_{2EG} = 0$. See Appendix A.4 for the proof of result *(b)* when $p(r_2, a) > \ell_G$.

Following Allen and Gale (2007), I refer to cash-in-the-market pricing when the equilibrium asset prices are below the "fundamental values" of assets (i.e. if $p < a$) due to a shortage of aggregate liquidity in the economy.[19] Cash-in-the-market pricing arises if $\frac{dr_2}{df} < 0$ because the cash available in the economy is endogenous and, therefore, the supply of short-term funding is limited (low elasticity; bounded supply). Again, the sufficient condition shows to be mild and satisfied for a large parameter range. Intuitively, the level of cash m_2 held by outside financiers at date $t = 2$ cannot be too large, in order to preserve cash-in-the-market pricing. Otherwise, there would be no incentive for private liquidity supply by investors (i.e. $f = 0$ and $r_2 = 1$).

Figure 2 in Appendix A.1 gives a graphical illustration of the market-clearing at $t = 2$ under pooling (left graph). Notice that the demand for cash is increasing in p as financing becomes cheaper. Conversely, the supply of cash is decreasing in p. Furthermore, when p falls short of ℓ_G the supply vanishes for all $p \in (R_B, \ell_G]$. This is because no good quality opaque assets are traded if $p \leq \ell_G$.[20]

2.2.3 Liquidity management at date t=1

Investors have to decide whether they want to become *liquid investors*, who (together with the outside financiers) are the natural providers of cash at $t = 2$, or *illiquid investors,* who are the natural demanders of cash at $t = 2$. This paper focuses on rational expectations equilibria in which investors form correct perceptions about future prices (p, p_G, p_B and r_2) and the average quality of assets traded in the opaque market segment at $t = 2$. First, the $t = 1$ decision problem is analyzed. Hereafter, the results on the existence and characterization of equilibria are discussed.

The problem at date t=1 Investors face a discrete choice at $t = 1$ described by $x \in \{0, 1\}$. They either become *liquid investors* and store their entire endowment ($x = 0$) or they become *illiquid investors* and fully invest their resources at $t = 1$ in risky long-term assets ($x = 1$). In other words, investors operate at the extensive margin. A problem that mirrors an occupational choice.

[19] Cash-in-the-market pricing means that long-term assets trade at a discount when compared to their expected return in the final period, which is itself determined by the average quality of assets traded ("fundamental value").

[20] Recall the simplifying assumption in section 2.2.1 that good quality opaque assets are privately liquidated if $p = \ell_G$.

Both, liquid and illiquid investors have access to the collateralized credit market at $t = 1$ where they can borrow additional cash as far as their spare borrowing capacity $\left(\theta_0^L, \theta_0^I\right)$ allows for it, i.e. select $b_1^L \leq \frac{\theta_0^L}{r_1}$ and $b_1^I \leq \frac{\theta_0^I}{r_1}$. In addition, liquid investors can decide to repay a part of their liabilities from financing the leveraged investment at $t = 0$, i.e. select $b_1^L < 0$. Consistent with the occupational choice character, it is assumed that illiquid investors must fully invest their available resources at $t = 1$ in risky long-term assets, which precludes a repayment of credit, i.e. $b_1^I \geq 0$.[21] As a result, the $t = 1$ problem can be simplified by setting $b_1^I = 0$. This is because $b_1^I > 0$ would result in an undesirable increase in the exposure of illiquid investors to utility-reducing illiquid investments.

The simplified investors' maximization problem reads:

$$\max_{x \in \{0,1\}, b_1^L \leq \frac{\theta_0^L}{r}, \; c_{ti}^k \geq 0 \; \forall t,i,k} \left\{ x \cdot V\left(c_{2E}^I, c_{3E}^I, c_{3L}^I\right) + (1-x) \cdot W\left(c_{2E}^L, c_{3E}^L, c_{3L}^L\right) \right\} \tag{15}$$

s.t.

$$V(\cdot) = \lambda \left[\beta_E \cdot log\left(c_{2E}^I\right) + (1-\beta_E) \cdot log\left(c_{3E}^I\right)\right] + (1-\lambda) \cdot log\left(c_{3L}^I\right)$$
$$W(\cdot) = \lambda \left[\beta_E \cdot log\left(c_{2E}^L\right) + (1-\beta_E) \cdot log\left(c_{3E}^L\right)\right] + (1-\lambda) \cdot log\left(c_{3L}^L\right)$$

$$c_{2E}^I = (1-q) \cdot \frac{ER}{r_2} + q \cdot \left(\alpha \cdot \breve{p}_B + (1-\alpha) \cdot d_{2EG}^I \cdot \breve{p}_G\right) + \frac{b_{2E}^L}{r_2}$$
$$c_{3E}^I = G\left(\theta_0^I, r_0\right) + q \cdot (1-\alpha) \cdot \left(1 - d_{2EG}^I\right) \cdot R_G - b_{2E}^L$$
$$c_{3L}^I = G\left(\theta_0^I, r_0\right) + (1-q) \cdot ER + q \cdot (\alpha \cdot a + (1-\alpha) \cdot R_G)$$
$$c_{2E}^L = \left(1 + \frac{b_1^L}{r_1}\right) \cdot \left(1 - s_E^L\right)$$
$$c_{3E}^L = \left(1 + \frac{b_1^L}{r_1}\right) \cdot s_E^L \cdot r_2 + G\left(\theta_0^L, r_0\right) - b_1^L$$
$$c_{3L}^L = \left(1 + \frac{b_1^L}{r_1}\right) \cdot r_2 + G\left(\theta_0^L, r_0\right) - b_1^L.$$

Recall that the superscripts $j = I, L$ correspond to *illiquid investors* and *liquid investor*s, respectively. Preferences are captured by the subscripts $i = E, L$. The supply and demand schedules (s_E^L, d_{iG}^I) are derived in section 2.2.1. After plugging in the consumption terms, b_1^L drops out because liquid investors are indifferent between storing cash and lending. Hence, we can simply set $b_1^L = 0$, which implies that all the cash carried over into $t = 1$ by outside financiers can be made available

[21] This assumption follows directly from the restriction to the occupation choice-type problem at $t = 1$. A modeling approach chosen to simplify the analysis. Without this additional assumption illiquid investors could overcome the restriction imposed by the occupation choice problem by (partially) repaying their credit from $t = 0$ and, thereby, effectively varying their portfolio liquidity continuously.

Note that in a model with continuous portfolio choice this issue does not arise. Here the equilibrium interest rate is such that investors are indifferent (at the intensive margin) between investing more resources into long-term assets, as opposed to cash. Given such an interest rate nobody would want to partially repay their credit from date $t = 0$, provided collateralized credit comes at ϵ cost while storage is costless.

for $t = 2$, i.e. $m_1 = m_2$. At $t = 1$, θ_0^L and θ_0^I are predetermined and taken as given. They can potentially differ for investors who expect to become an *illiquid* or *liquid investor* at $t = 1$.[22] Thus, we can set $\theta_1^L = \theta_0^L$ and $\theta_1^I = \theta_0^I$. Finally, notice that the resources available for investments at $t = 1$ depend on b_0^L and b_0^I, the amounts borrowed at $t = 0$ from outside financiers.

In equilibrium, it must be true that $r_1 = r_2$.[23] Furthermore, in a pooling equilibrium, investors are indifferent between becoming an *illiquid* or *liquid investor* at $t = 1$. The equilibrium fraction of *liquid investors* f must solve:

$$V\left(r_2\left(f\right)\right) = W\left(r_2\left(f\right)\right). \tag{16}$$

Let $\widehat{f}$ be the solution to equation (16).

Lemma 3. Uniqueness

Provided α or β_E is sufficiently small and provided that a pooling equilibrium in the date $t = 2$ market exists (i.e. $a \geq p > \ell_G$), then the solution $\widehat{f}$ to equation (16) *is unique.*

Proof. See Appendix A.5.

The proof rests on a single-crossing property. Illiquid investors gain from a higher level of liquidity in the economy, while liquid investors loose, i.e. $\frac{\partial V}{\partial f} > 0$ and $\frac{\partial W}{\partial f} < 0$. As before, the result is guaranteed to hold if either α or β_E is sufficiently small. The sufficient condition shows to be mild and satisfied for a large parameter range.

Three types of equilibria can be distinguished:[24] *(i)* a *"pooling equilibrium"*, where both *good* and *bad types* sell in the opaque market segment. Pooling equilibria feature cash-in-the-market pricing, i.e. $\ell_G < p \leq a$. Existence requires that the pricing function in (14) yields an internal solution when evaluated at $\widehat{f}$. Moreover, there *always* exists either *(ii)* equilibria characterized by a *"breakdown"* of pooling where the good types prefer to privately liquidate their high quality assets instead of selling them in the opaque market, i.e. $p \leq \ell_G$, or *(iii)* equilibria characterized by *"liquidity hoarding"* where investors only store cash at $t = 1$, i.e. $f^* = 1$.[25] Proposition 4 summarizes the results.

Proposition 4. Existence of a date t=1 equilibrium and characterization

For a given θ_0^I, θ_0^L, $G\left(\theta_0^I, r_0\right)$, $G\left(\theta_0^L, r_0\right)$ and m_1 there:

[22] This is because investors can correctly anticipate at $t = 0$ what their desired liquidity choice will be at $t = 1$ and, thus, adjust their leverage choice accordingly depending on their type j. Notably, the potential heterogeneity in θs does not arise in an alternative setup, where the indivisibility assumption in the $t = 1$ liquidity choice is relaxed. However, the key insights are unaffected.

[23] If $r_1 > r_2$ then both outside financiers and investors do not want to hold any cash at $t = 1$. Conversely, if $r_1 < r_2$, then the $t = 1$ credit market does not clear because outside financiers would like to borrow unbounded amounts.

[24] As said in section 2.2.1, I abstract from equilibria with partial pooling which does not affect the key insights.

[25] An equilibrium characterized by liquidity hoarding is unlikely to occur and becomes impossible when q is small.

(a) *exists a pooling equilibrium if and only if* $\ell_G < p = \frac{a\left(\theta_1^I,\theta_0^I,r_0\right)}{r_2\left(a,\hat{f};\theta_1^I,r_0,\theta_0^I,\theta_0^L\right)} \leq a\left(\theta_1^I,\theta_0^I,r_0\right)$*; provided* α *or* β_E *sufficiently small, it is characterized by* $f^* = \widehat{f}$ *and* $p_G^* = p_B^* \cdot \frac{R_G}{R_B} = \frac{R_G}{r_2\left(a,\hat{f};\theta_1^I,r_0,\theta_0^I,\theta_0^L\right)} \leq R_G$*, which holds with strict inequality if* $p < a\left(\theta_1^I,\theta_0^I,r_0\right)$

(b) *exists a liquidity hoarding equilibrium if* $\frac{a\left(\theta_1^I,\theta_0^I,r_0\right)}{r_2\left(a,\hat{f};\theta_1^I,r_0,\theta_0^I,\theta_0^L\right)} > a\left(\theta_1^I,\theta_0^I,r_0\right) \geq \ell_G$*, characterized by* $f^* = 1$ *and* $p_G^* = p_B^* \cdot \frac{R_G}{R_B} = R_G$

(c) *always exists an equilibrium where pooling in the opaque asset market segment breaks down, characterized by* $f^* \in [0,1]$ *and* $p^* \in [0, R_B]$*,*

where $a\left(\theta_1^I,\theta_0^I,r_0\right)$ *is implicitly defined by equation (11).*

Proof. The proof of result *(a)* follows from Lemma's 1, 2 and 3. See Appendix A.6 for the proof of results *(b)* and *(c)*.

Next, I take the competitive equilibrium derived in section 2.2 and analyze the efficiency of the liquidity management at $t = 1$.

2.3 Efficiency at date t=1

This section analyzes "interim" efficiency, that is, the efficiency of the liquidity choice at $t = 1$ for a given leverage choice at $t = 0$. The efficiency of the leverage choice will be discussed in section 2.4. In what follows, I consider the problem of a constrained planner who can manipulate f at $t = 1$ but who cannot infer with markets at $t = 2$. Recall the pricing formula in equation (14) and from Lemma 2 that $\frac{\partial r_2}{\partial f} > 0$. Using an envelope-type argument, I examine whether a constrained planner would select a level of f that is different from the one found in the competitive equilibrium. The result is summarized in Proposition 5.

Proposition 5. Efficiency at date t=1

For a given leverage choice at date $t = 0$*, pooling equilibria are generically not constrained efficient provided* $p\left(\hat{f}\right) < a$ *and:*

$$\left\{-\frac{\lambda \cdot \beta_E}{r_2(f)} + \frac{f}{r_2(f) + G}\right\} \cdot \left.\frac{\partial r_2(f)}{\partial f}\right|_{f=\widehat{f}} \neq 0.$$

A larger (smaller) supply of cash from outside financiers (m_1) *is more likely to be associated with an inefficient under-investment (over-investment) in cash.*

Proof. See Appendix A.7.

Intuition. Equilibria are "typically" constrained inefficient. The private information problem, in combination with incomplete ex-ante risk markets, is at the root of this "interim" inefficiency.[26] Due to a trading-on-private-information motive, illiquid investors with bad quality assets can gain from private information even if they do not have a liquidity need. Instead, liquid investors and illiquid investors with a liquidity need loose. When making their choice at $t = 1$, investors do not take into account that their choice affects future market prices and the average quality of assets traded. The direction of the inefficiency depends crucially on the comparison between these gains and losses.

If the provision of liquidity by outside financiers is large (high m_1), then a tendency for under-investment in cash prevails. In other words, the liquidity provision generated by investors ("insiders") is insufficient. Instead, it cannot be ruled out analytically that the opposite tendency may arises when the provision of liquidity by outside financiers vanishes (small m_1).

2.4 Leverage choice at date t=0

This section discusses the leverage choice at $t = 0$. The aim is to understand the interplay of this leverage choice with the intensity of adverse selection in the opaque asset market segment at $t = 2$. In section 2.4.1, I start with a discussion of the special case where all long-term assets are transparent, i.e. $q = 0$. Here, the private information problem does not exist. Thereafter, in section 2.4.2, I move to the general case, i.e. $q > 0$, and discuss the emergence of a tension between leverage and adverse selection. Section 2.4.2 also establishes the main results of the paper on the existence of a detrimental feedback loop between deleveraging and adverse selection. Finally, section 2.4.3 relates the analytical results to the global financial crisis of 2007/08 and concludes with a numerical example that illustrates the feedback loop.

2.4.1 The special case: q=0

In the special case where $q = 0$, all long-term assets are transparent and individual asset qualities are publicly know. Thus, there is no adverse selection. The previous analysis of dates $t = 2$ and $t = 1$ is greatly simplified. The main results are summarized in Proposition 6.

[26]See also Gale and Yorulmazer (2013) or Bertsch (2012) for papers featuring a constrained inefficient liquidity choice due to a combination of incomplete ex-ante risk markets and private information.

Proposition 6. Existence, uniqueness and efficiency of a date t=0 equilibrium if q=0

Provided Condition 1 holds and:

$$\frac{\gamma \cdot \rho \cdot \kappa}{ER - \gamma \cdot \rho} \leq m_0 < \frac{\gamma \cdot \rho \cdot \kappa}{ER - \gamma \cdot \rho} + \lambda \cdot \beta_E \cdot \left(1 + \frac{G(0, ER)}{ER}\right) \tag{17}$$

is satisfied, then there exists a unique equilibrium characterized by $\theta_0^{L,I*} = 0$, $r_2^* = ER$, *and:*

$$f^* = \min\left\{0, \lambda \cdot \beta_E \cdot \left(1 + \frac{G(0, ER)}{ER}\right) - \left(m_0 - \frac{\gamma \cdot \rho \cdot \kappa}{ER - \gamma \cdot \rho}\right)\right\} < 1. \tag{18}$$

While the leverage choice $\theta_0^{L,I*} = 0$ *is always efficient, the liquidity management is constraint efficient under the sufficient condition:*

$$\left[\frac{\rho^2 \cdot (1 - \gamma)}{(1 - \gamma \cdot \rho)^2} \cdot \frac{ER^2}{\lambda \cdot \beta_E \cdot (1 - \lambda \cdot \beta_E) \cdot (ER + G(0, r_0))}\right] \cdot \gamma \cdot \kappa \leq ER - 1. \tag{19}$$

Proof. See Appendix A.8.

Condition 1 ensures the existence of an inter-temporal trade-off for illiquid investors of the early type at $t = 2$. Furthermore, the right-hand side of inequality (17) ensures a scarcity of liquidity in the aggregate (cash-in-the-market pricing) while the left-hand side of (17) ensures that the resources of outside financiers are sufficient to finance all investments at $t = 0$. Hence, investors face an *inter-temporal trade-off* and a *liquidity trade-off*. If $m_1 = m_2 = m_0 - \frac{\gamma \cdot \rho \cdot \kappa}{ER - \gamma \cdot \rho}$ is large, implying that the supply of liquidity by outside financiers is abundant, then inequality (17) is violated and $f^* = 0$ from equation (18). Notice that $f^* < 1$ is guaranteed from Condition 1.

Due to the inability of investors to borrow against future endowments (Assumption 3), a constrained planner may, in principle, be able to improve upon the allocation by reducing the interest rate r_0 with the help of a subsidy on cash holdings that aims to engineer an increase in f. This is because constrained investors do not internalize at $t = 1$ how their individual liquidity management affects the financing conditions at $t = 0$ (Lorenzoni 2008, Korinek 2012). To see this, consider a constrained planner who can subsidize or tax investments at dates $t = 0, 1$ by honoring a balanced intra-period budget. The equilibrium is constrained efficient if the leveraged investments are not too large. Inequality (19) reveals that this is the case if $\gamma \cdot \kappa$ is sufficiently small relative to the loss, $ER - 1$, from forgone investments in long-term assets at $t = 1$ due to the increase in f. Conversely, if inequality (19) is violated, then a tendency for an inefficient under-provision of liquidity prevails.

Interestingly, the results of Proposition 6 change drastically when an opaque market segment with private information is introduced. This is discussed in the next section.

2.4.2 The general case: q>0

Section 2.2 contains the analysis of dates $t = 1, 2, 3$ for the general case. What remains is the analysis of the leverage choice at $t = 0$. The key trade-off is highlighted in section 2.4.2.1 and the results on leverage are summarized in section 2.4.2.2. Thereafter, section 2.4.2.3 establishes the detrimental feedback loop between deleveraging and adverse selection.

2.4.2.1 Solution to the problem at date t=0 Recall that the return on the leveraged long-term investment project is decreasing in the spare borrowing capacity and in the interest rate at $t = 0$. In a rational expectations equilibrium, investors correctly anticipate the future measure f of liquid investors at $t = 1$, as well as market prices and the average quality of assets traded at $t = 2$. Furthermore, they anticipate their individual liquidity choice at $t = 1$. As a result, investors who expect to become liquid investors at $t = 1$ select $\theta_0^{L*} = 0$ as long as Condition 1 holds. Instead, investors who expect to become illiquid investors may reduce leverage at $t = 0$ because of future benefits from $\theta_0^I > 0$. To see this, consider their problem at $t = 0$:

$$\max_{0 \leq \theta_0^I \leq \gamma \cdot \kappa} \left(\begin{array}{l} \lambda \cdot \beta_E \cdot log \left[\beta_E \cdot \left(\frac{R_G - a}{R_G} \cdot \frac{\theta_1^I}{a} + \frac{G(\theta_0^I, r_0)}{R_G} + q + \frac{(1-q) \cdot ER}{a} \right) \cdot \frac{a}{r_2} \right] \\ + \lambda \cdot (1 - \beta_E) \cdot log \left[(1 - \beta_E) \cdot \left(\frac{R_G - a}{R_G} \cdot \frac{\theta_1^I}{a} + \frac{G(\theta_0^I, r_0)}{R_G} + q + \frac{(1-q) \cdot ER}{a} \right) \cdot R_G \right] \\ + (1 - \lambda) \cdot log \left[G\left(\theta_0^I, r_0\right) + (1 - q) \cdot ER + q \cdot (\alpha \cdot a + (1 - \alpha) \cdot R_G) \right] \end{array} \right) \tag{20}$$

Notice that the payoffs in the different contingencies are the same as in section 2.2.3.

Key trade-off Both period profits of early types are increasing in θ_1^I if $\left(\frac{R_G - a}{R_G / a} + \frac{\partial G / \partial \theta_0^I}{R_G} \right) > 0$. In other words, illiquid investors are willing to install spare borrowing capacity at $t = 0$, i.e. select $\theta_0^I > 0$, as long as the benefit of reducing leverage $\frac{R_G - a}{R_G / a} > 0$ outweighs the cost $\frac{\partial G / \partial \theta_0^I}{R_G} < 0$. The formal condition is stated in Lemma 7.

2.4.2.2 Results on leverage

Lemma 7. Leverage choice

Provided the sufficient conditions of either Lemma 1(b) or Lemma 1(c) are met, and provided *a pooling equilibrium exists at date $t = 2$, then, for a given r_0, individual investors optimally select $\theta_0^I > 0$ whenever the cost of reducing leverage is not too high, i.e. if:*

$$-\frac{\partial G(0, r_0)}{\partial \theta_0^I} \leq \frac{\lambda \cdot \left(1 - \frac{a(0,0;G(0,r_0))}{R_G}\right)}{1 - \lambda \cdot \left(1 - \frac{a(0,0;G(0,r_0))}{R_G}\right)}, \tag{21}$$

where $a(0, 0; G(0, r_0))$ is implicitly defined by equation (11).

Proof. See Appendix A.9.

Observe that the sufficient condition in Lemma 7 is more relaxed, the higher the value of λ. Intuitively, a higher likelihood of being the early type makes it more important for illiquid investors to shield themselves against adverse selection in the opaque asset market segment by holding spare borrowing capacity. As said earlier, the term $\left(1 - \frac{a}{R_G}\right)$ captures the benefit from being able to reduce high quality asset sales in the opaque market by having better access to alternative financing. Notice that r_0 in inequality (21) is determined from the problem at $t = 1$ and from market clearing, after imposing $\theta_0^I = \theta_0^L = 0$.

It turns out that the incentive for an individual illiquid investor to select a positive spare borrowing capacity is increasing in the leverage choice of other investors. In other words, I find a strategic complementarity in leverage choices which is stated formally in Proposition 8. Let $\hat{\theta}_{0,n}^I\left(\theta_{0,-n}^I\right)$ be the optimal $\theta_{0,n}^I$ chosen by investor n as a function of other investors' choice $\theta_{0,-n}^I$.

Proposition 8. Strategic complementarity in leverage choices

Provided a pooling equilibrium exists at date $t = 2$, and provided the inequality in Lemma 7 holds, then there is a strategic complementarity in leverage choices, i.e.:

$$\begin{cases} \frac{d\hat{\theta}^I\left(\theta_{-n}^I, k_{-n}^I\right)}{d\theta_{-n}^I} > 0 & \forall\, \hat{\theta}^I < \theta_{max}^I \equiv \gamma \cdot \kappa \cdot \rho \\ \frac{d\hat{\theta}^I\left(\theta_{-n}^I, k_{-n}^I\right)}{d\theta_{-n}^I} = 0 & if\ \hat{\theta}^I = \theta_{max}^I \end{cases} \tag{22}$$

if κ, β_E sufficiently small, q sufficiently large, and $R_G \leq 2 \cdot \ell_G$.

Proof. See Appendix A.10.

The idea of the proof is to establish the existence of a strategic complementarity in leverage choices that builds on the results from Lemmas 1, 2 and 7. Although the proof is analytically involved, the set of sufficient conditions allows for a very intuitive explanation of the underlying mechanism. For the desired result to arise, a sufficiently high intensity of the adverse selection problem is needed in order to provide incentives for investors to deleverage. This is achieved with the help of the restrictions on κ, β_E and q. A sufficiently large value of q ensures that the opaque market segment is sufficiently large and, hence, future distress (a low average quality and, hence, a low price) in this market segment is a relevant concern for investors at $t = 0$. Furthermore, small values of κ and β_E guarantee that the adverse selection problem is sufficiently strong by lowering the quantity of high quality assets sold, i.e. lowering d_{2EG}. Finally, the possibility of a breakdown of pooling in the opaque market needs to be entertained, which is ensured by $R_G \leq 2 \cdot \ell_G$.

The strategic complementarity in leverage choice lays the foundations for an inefficient leverage choice. Individuals reduce leverage by too much and do not take into account that this can lead to a breakdown of pooling in the opaque market. How such a scenario can arise is discussed next.

2.4.2.3 Deleveraging and the severity of adverse selection at date t=2 A main insight of this paper is the existence of a detrimental feedback loop between deleveraging and the intensity of adverse selection in the opaque asset market segment. I find that a more severe adverse selection causes a reduction in leverage, provided the existence of the strategic complementarity in leverage choices established in Proposition 8. The result is formally stated in Proposition 9 below and the focus is on symmetric equilibria, i.e. equilibria where $\theta^I_{0,n} = \theta^I_{0,-n} = \theta^I_0$ for all n.

Proposition 9. Detrimental deleveraging

Consider a pooling equilibrium and suppose that the adverse selection in the opaque asset market segment worsens due to a reduction in R_B. A lower value of R_B causes a:

(a) *reduction in leverage, i.e. $\frac{d\theta^{I*}}{dR_B} < 0$, which further amplifies the adverse selection provided that κ, β_E sufficiently small, q sufficiently large, and $R_G \leq 2 \cdot \ell_G$, and provided that the inequality in Lemma 7 holds after the reduction of R_B.*

(b) *breakdown of pooling for a sufficiently strong amplification of adverse selection. Formally, if the p solving market-clearing, when evaluated at θ^{I*}, falls short of ℓ_G.*

Proof. See Appendix A.11.

Proposition 9 establishes the existence of a detrimental feedback loop. It demonstrates that a stronger adverse selection in the opaque market due to a decrease in R_B causes deleveraging. However, the deleveraging itself triggers a further reduction in the average quality of assets traded, as shown in Lemma 1(a). This in turn amplifies deleveraging and creates downward pressure on the price of opaque assets, because the relative return for investors purchasing opaque assets has to be sufficiently attractive, when compared to the return on investing in non-opaque assets (see equation (3)). In the extreme, this detrimental feedback mechanism can lead to a breakdown of pooling in the opaque market when the price falls short of ℓ_G. The pooling equilibrium ceases to exist due to a drastic deleveraging, causing a substantial welfare loss. The set of conditions is the same as for Proposition 8 and the same interpretation applies. Notably, the stated conditions are sufficient but not necessary for the result in Proposition 9 to hold.

To conclude, this section demonstrates the existence of the detrimental feedback loop. Deleveraging intensives adverse selection and can make it impossible to support a pooling equilibrium that would exist absent deleveraging. In such a scenario, the leverage choice is clearly constrained inefficient. Finally, recall that the interest of this paper is to analyze under what conditions a pooling equilibrium can exist. However, provided a pooling equilibrium exists, it is always the case that a welfare inferior equilibrium without pooling co-exists (coordination failure). This is due to the adverse selection problem and the strategic complementarity in leverage choices.

2.4.3 The detrimental feedback loop: a numerical example

The above mechanism explains how a deterioration in the quality of subprime mortgage-backed securities triggered both a breakdown in opaque subprime markets and a deleveraging wave in the financial crisis of 2007/08. After realizing that subprime markets may come under distress, banks started to adjust their business models. In search of unencumbered high quality collateral, banks began deleveraging. This enabled them to reduce their dependency on refinancing through opaque markets. However, the simultaneous exit further amplified the distress in those markets.

To illustrate the previous analytical results, consider a numerical example using the parameters given in table 1. This numerical example will also support the discussion of policy implications in section 3. The parameters are selected such that there exists a pooling equilibrium. In particular, the adverse selection problem is assumed to be relatively mild (a low α), while the relative size of the opaque market segment is assumed to be large (a high q).

Variable	β_E	λ	α	R_G	R_B	ℓ_G	ρ	κ	γ	m_0	q
Value	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{10}$	$\frac{7}{5}$	$\frac{1}{5}$	$\frac{2}{3}$	$\frac{7}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{3}{10}$	$\frac{9}{10}$

Table 1: Model parameters

Given the result from Proposition 4, there exists a pooling equilibrium characterized by $\theta^* = \theta^{I,F*} = 0$, $r^* \approx 1.27$ and $a^* \approx 0.99$. The corresponding market-clearing prices are given by $p^* \approx 0.78$, $p_G^* \approx 1.10$ and $p_B^* \approx 0.16$, respectively. As long as adverse selection is relatively mild, investors do not have an incentive to install spare borrowing capacity at $t = 0$, i.e., to select a positive θ. This changes when the adverse selection problem is stronger, as suggested by Proposition 9.

Let us examine what happens in a crisis scenario triggered by a deterioration in the quality of subprime assets. Consider an increase in the intensity of adverse selection caused by a small drop in the value of lemons from $R_B = 0.2$ to $R_B' = 0$. In this case, the expected return drops only slightly from $ER = 1.26$ to $ER' = 1.24$. However, there no longer exists a pooling equilibrium because the incentives to deleverage by selecting a strictly positive θ^I are growing too large (Lemma 7, Proposition 9). As a result, the opaque asset market segment breaks down in equilibrium. Intuitively, the more intense adverse selection incentivizes investors to increase θ^I which in turn amplifies adverse selection, eventually pushing p below ℓ_G. This is illustrated in figure 2 in Appendix A.1 (right graph): the supply of cash drops sharply when R_B falls and adverse selection gets more intense, such that no pooling equilibrium can be supported. In summary, a detrimental feedback loop evolves, leading to a complete breakdown in the opaque market and an equilibrium characterized by $\theta^{I*} = \gamma \cdot \kappa \cdot \rho \approx 0.12$, $r^* \approx 1.13$, $f^* \approx 0.08$ and $p^* = 0$. Interestingly, the pooling equilibrium prevails, despite $R_B' = 0$, if a social planner forces investors to select $\theta^I = 0$. In the latter case, $r^* \approx 1.24$, $a^* \approx 0.91$ and $p^* \approx 0.73 > \ell_G = \frac{2}{3}$. This highlights the detrimental effect of deleveraging on the asset market.

3 Policy implications

The previous analysis reveals several immediate policy implications. Firstly, a policy maker can counteract the inefficient liquidity management (Proposition 5) by manipulating f through taxing investments in risky long-term assets. Secondly, a policy maker can prevent an inefficient detrimental deleveraging spiral that arises due to a deterioration in the asset quality (Proposition 9) by making deleveraging less attractive at $t = 0$. Thirdly, a policy maker can intervene at $t = 2$ and provide liquidity to markets by purchasing long-term assets trading at fire sale prices.

While the computation of a Pigovian tax to counteract the inefficient liquidity management is standard, the relevant policies to influence the leverage choice at $t = 0$ and to provide liquidity at $t = 2$ demand further discussion. In the remainder, I explore policies that aim to directly manipulate the leverage choice and the liquidity provision. Of particular interest is how a policymaker who faces a deleveraging wave in anticipation of a deterioration in asset qualities, such as in the financial crisis of 2007/08, can prevent the emergence of the detrimental feedback loop described in section 2.4.2.2?

I present three different options available to the policy maker that all aim to increase the costs of reducing leverage, with the goal of preventing excessive deleveraging (as a reaction to anticipated future market distress, which in turn amplifies distress in the opaque market segment).

First, the policy maker can widen the collateral requirements for refinancing at $t = 0$. This amounts to an increase in γ, which increases the individual costs of deleveraging $-\frac{\partial G(0,r_0)}{\partial \theta^I_{0,n}}$ and, thereby, discourages deleveraging. From equation (29) in the Proof of Lemma 7, a necessary and sufficient condition that prevents deleveraging is given by:

$$\begin{aligned} \frac{\partial}{\partial \theta^I_{0,n}}\Big|_{\theta^I_{0,n}=0} = \quad & \lambda \cdot \frac{\frac{R_G - a(0,0;G(0,r_0))}{a(0,0;G(0,r_0))} + \frac{\partial G(0,r_0)}{\partial \theta^I_{0,n}}}{G(0,r_0) + q \cdot R_G + (1-q) \cdot ER \cdot \frac{R_G}{a(0,0;G(0,r_0))}} \\ & + (1-\lambda) \cdot \frac{\frac{\partial G(0,r_0)}{\partial \theta^I_{0,n}}}{G(0,r_0) + q \cdot (\alpha \cdot a(0,0;G(0,r_0)) + (1-\alpha) \cdot R_G) + (1-q) \cdot ER} < 0. \end{aligned} \tag{23}$$

The first-order effect of an increase in the cost of deleveraging, $\frac{\partial G(0,r_0)}{\partial \theta^I_{0,n}} > 0$, works in favor of inequality (23). However, the second-order effects via general equilibrium prices and the quality of assets traded, as well as the effects through $\frac{\partial G(0,r_0)}{\partial \gamma} > 0$, are more difficult to assess. In the framework of the numerical example from section 2.4.3, the first-oder effects prevail. To see this, consider the case of $R'_B = 0$ where the pooling equilibrium ceases to exist. Now, consider a widening of collateral requirements from $\gamma = 1/3$ to $\gamma' = 1/2$. Such an intervention restores pooling by preventing the detrimental feedback loop from materializing: $r^* \approx 1.24$, $a^* \approx 0.96$ and $p^* \approx 0.77 > \ell_G = \frac{2}{3}$.

Second, if the policy maker has any means by which to reduce either the asymmetric information problem at $t = 2$ or the number of lemons in the market, then a credible commitment to achieving these goals can prevent a detrimental deleveraging wave. Again, this is because deleveraging at $t = 0$ is discouraged (see equation (21) in Lemma 7) as the benefit from installing a positive spare borrowing capacity is smaller, the higher a. The first-order effect of such a policy is a decrease in $\frac{R_G - a}{a}$, which works in favor of inequality (30). Again, second-order effects via general equilibrium prices are difficult to assess. However, in the framework of the numerical example from 2.4.3, the first-oder effect prevails. To see this, consider the case of $R'_B = 0$ and an increase in the size of the opaque market segment from $q = \frac{9}{10}$ to $q' = 1$. Again, the pooling equilibrium is restored by preventing the detrimental feedback loop from materializing: $r^* \approx 1.24$, $a^* \approx 0.95$ and $p^* \approx 0.76 > \ell_G = \frac{2}{3}$.

This highlights the ambitious role played by transparency. Here, more opacity (less transparency) is beneficial, as it helps to restore a pooling equilibrium. This contrasts with the negative implications of opacity related to the constrained inefficiency of the liquidity management (Proposition 5 and 6).

Third, the policy maker can also discourage deleveraging by influencing market prices (i.e. by reducing r_0 and thus increasing $-\frac{\partial G(0,r_0)}{\partial \theta_{0,n}^I}$) through liquidity provisions at dates $t = 0$ or $t = 2$. If fully anticipated, the timing of such a liquidity provision is not relevant. However, observe that any public liquidity provision is exactly offset by a reduction in private liquidity provision. A date $t = 1$ equilibrium with a strictly positive measure of liquid investors demands that $V(f) = W(f)$. Since $dV(f)/df > 0$ and $dW(f)/df < 0$, any public liquidity provision must trigger an increase in the number of illiquid investors that completely offsets the intervention. Consequently, a central bank can only restore market functioning if it fully crowds out the private liquidity supply, thereby "becoming in effect the lender of first resort" (Gale and Yorulmazer 2013, page 291).

4 Conclusion

This paper presents a novel feedback mechanism which explains how deleveraging and the intensity of adverse selection in opaque asset markets can fuel each other, as in the global financial crisis of 2007/08. The view taken by this paper is that the leverage choice is a decision about the medium to long-term business model, that is adjusted only occasionally when lucrative investment opportunities arise or when the economic outlook changes. Conversely, liquidity management is a day-to-day business that is conducted over a short horizon. Nevertheless, the leverage choice and liquidity management are intertwined. In particular, at the core of this paper's mechanism, there is an interplay between adverse selection in opaque asset markets and the incentives to reduce leverage. These incentives for deleveraging arise because illiquid investors want to become less reliant on opaque asset markets for their liquidity management when the intensity of adverse selection is expected to be higher.

Adverse selection is at the root of both an ("interim") inefficiency in the liquidity management (short-term) and an ("ex-ante") inefficiency in the leverage choice (long-term). The first inefficiency is caused by a distortion of the liquidity management due to a combination of incomplete markets for ex-ante risk-sharing and a private information friction. The second inefficiency occurs because of a negative externality in borrowing capacity choices which is in turn rooted in the private information friction and can potentially generate a detrimental feedback loop. I discuss several central bank policies that have been used during the crisis and analyze their effectiveness. Within the modeling framework of this paper, both a widening of collateral requirements and public liquidity provision can be effective in preventing the emergence of a detrimental feedback loop between deleveraging and adverse selection. Furthermore, there is scope for liquidity regulation due to the constraint inefficiency of the liquidity management choice.

Finally, this paper uncovers the ambiguous role of market transparency. On one hand, a larger size of the transparent asset market segment has direct positive implications for liquidity risk-sharing and, hence, social welfare. On the other hand, it can also be harmful because it amplifies the adverse selection problem in the opaque market segment and, thereby, may provide incentives for investors to reduce leverage. For future research, a detailed welfare analysis of the role played by market transparency could be of interest.

The results of this paper show to be robust to several variations of the model. For tractability, I assumed throughout this paper that the liquidity choice at $t = 1$ is binary, meaning that the investment decision at $t = 1$ is indivisible. When relaxing this assumption by allowing for mixed portfolios, the key qualitative insights of this paper appear to be unaltered. Similarly, the assumption that a private information problem only exists for the opaque asset market and not for the leveraged long-term investment project started at $t = 0$ can be relaxed without affecting the key insights. What matters is that there exist spot markets at $t = 2$ with a varying degree of adverse selection problems and that the adverse selection in the opaque asset market is strongest. Also, the fixed shares of investments in transparent and opaque assets is a model simplification that is not crucial for the key insight, which prevails even when the transparent market segment vanishes ($q = 1$).

Finally, the feature of a breakdown in pooling hinges on Assumption 1. A richer economy with more than two possible payoffs of the risky long-term asset (for instance a continuum approximation) would require a more complicated parameter assumption in order to generate a breakdown in pooling and preserve the existence of the detrimental feedback loop derived in Proposition 9. In particular, there must be a relatively large probability mass for low return realizations. For the application to the global financial crisis of 2007/08, this distributional feature is arguably realistic. The same is true for the private information on asset qualities. Prior to the crisis, financial market participants with superior information, such as US investment banks, were more than happy to off-load opaque bad quality subprime assets at high prices to less informed banks, such as the German Landesbanken.

A Appendix

A.1 Figures

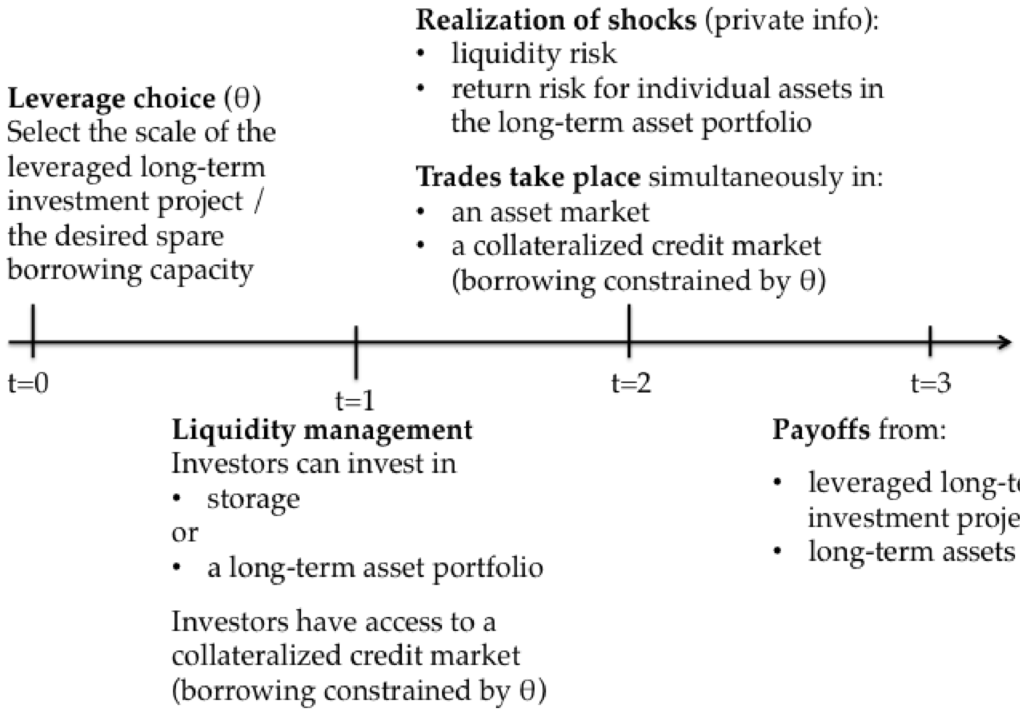

Figure 1: Timeline

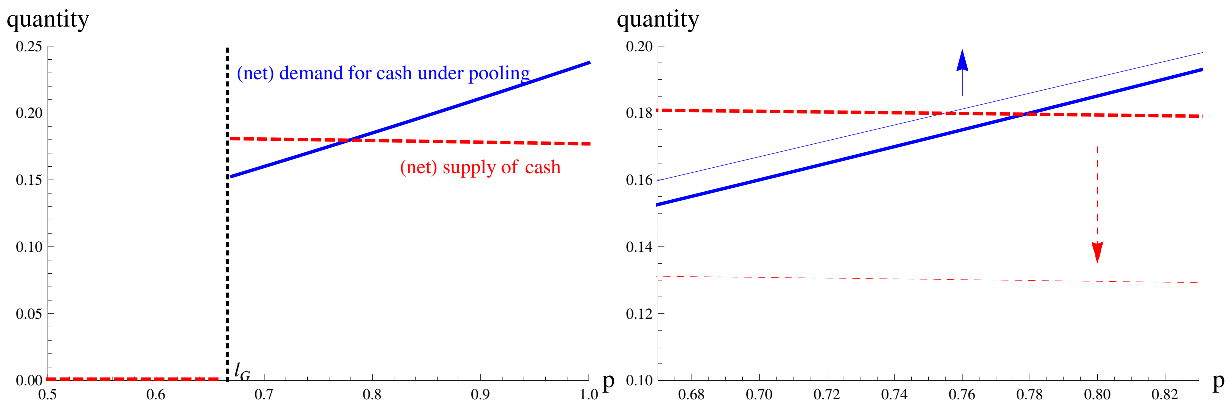

Figure 2: Market-clearing at $t = 2$: demand and supply from equation (13). Everything is expressed in terms of p. The parameters are the same as in the baseline example of section 2.4.3. Left: a (unique) pooling equilibrium exists ($R_B = 0.2$). Right: no pooling equilibrium exists ($R_B = 0$).

A.2 Derivations of the trading decisions at date $t = 2$

The first-order necessary conditions associated with the problem in (7) write:

$$
\begin{aligned}
b_{2i}^I: \quad & \tfrac{\beta_i}{c_{2i}^I} \cdot \tfrac{1-s_{2i}^I}{r_2} + \tfrac{1-\beta_i}{c_{3i}^I} \cdot \left(s_{2i}^I - 1\right) - \mu_3 = 0 \\
d_{2i}: \quad & \tfrac{\beta_i}{c_{2i}^I} \cdot (1-q) \cdot \tfrac{ER}{r_2} \cdot \left(1 - s_{2i}^I\right) + \tfrac{1-\beta_i}{c_{3i}^I} \cdot (1-q) \cdot ER \cdot \left(s_{2i}^I - 1\right) + \mu_8 - \mu_9 = 0 \\
d_{2iG}: \quad & \tfrac{\beta_i}{c_{2i}^I} \cdot q \cdot (1-\alpha) \cdot \breve{p}_G \cdot \left(1 - s_{2i}^I\right) \\
& + \tfrac{1-\beta_i}{c_{3i}^I} \cdot q \cdot (1-\alpha) \cdot \left(\breve{p}_G \cdot r_2 \cdot s_{2i}^I - R_G\right) + \mu_4 - \mu_5 = 0 \\
d_{2iB}: \quad & \tfrac{\beta_i}{c_{2i}^I} \cdot q \cdot \alpha \cdot \breve{p}_B \cdot \left(1 - s_{2i}^I\right) \\
& - \tfrac{1-\beta_i}{c_{3i}^I} \cdot q \cdot (1-\alpha) \cdot \left(\breve{p}_B \cdot r_2 \cdot s_{2i}^I - R_B\right) + \mu_7 - \mu_8 = 0 \\
s_{2i}^I: \quad & -\tfrac{\beta_i}{c_{2i}^I} \cdot \begin{pmatrix} (1-q) \cdot \frac{ER}{r_2} \cdot d_{2i}^I + \frac{b_{2i}^I}{r_2} + \\ q \cdot \left(\alpha \cdot d_{iB}^I \cdot \breve{p}_B + (1-\alpha) \cdot d_{iG}^I \cdot \breve{p}_G\right) \end{pmatrix} \\
& + \tfrac{1-\beta_i}{c_{3i}^I} \cdot \begin{pmatrix} (1-q) \cdot \frac{ER}{r_2} \cdot d_{2i}^I + \frac{b_{2i}^I}{r_2} + \\ q \cdot \left(\alpha \cdot d_{2iB}^I \cdot \breve{p}_B + (1-\alpha) \cdot d_{2iG}^I \cdot \breve{p}_G\right) \end{pmatrix} \cdot r_2 + \mu_1 - \mu_2 = 0
\end{aligned}
$$

where μ_1, μ_2,...,μ_9 are the multipliers on the first, second,..., ninth inequality constraint, respectively.

Suppose d_{2EG}^I is interior. From the third and fourth first-order condition, $d_{2EB}^I = 1$ follows. The third first-order condition together with the first (second) condition implies, that $b_{2E}^I = \theta_1^I$ ($d_{2E}^I = 1$), and together with the fifth condition that $s_{2E}^I = 0$. Hence, equation (8) follows.

A.3 Proof of Lemma 1

The results of Lemma 1 are proven in turn.

(a) The left-hand side and the right-hand side of equation (11) are continuous and increasing in a. As a result, the larger root of (11) only exceeds ℓ_G if inequality (12) holds.

(b) and (c) The average quality a is implicitly defined by:

$$
F_1\left(a; \theta_1^I, r_0, \theta_0^I\right) \equiv \alpha \cdot \left(R_B \cdot a - a^2\right) + \lambda \cdot (R_G - a) \cdot \begin{pmatrix} \beta_E \cdot \left(a + \frac{G\left(\theta_0^I, r_0\right) - \theta_1^I}{q \cdot R_G / a}\right) - \alpha \cdot a \\ -(1-\beta_E) \cdot \frac{(1-q) \cdot ER + \theta_1^I}{q} \end{pmatrix} = 0. \quad (24)
$$

By application of the implicit function theorem:

$$\begin{aligned}
\frac{\partial F_1\left(a;\theta_1^I,r_0,\theta_0^I\right)}{\partial\theta_0^I} &= \lambda\cdot(R_G-a)\cdot\frac{\beta_E}{q\cdot R_G/a}\cdot\frac{dG\left(\theta_0^I,r_0\right)}{d\theta_0^I}<0\\
\frac{\partial F_1\left(a;\theta_1^I,r_0,\theta_0^I\right)}{\partial\theta_1^I} &= \lambda\cdot(R_G-a)\cdot\left(-\frac{\beta_E}{q\cdot R_G/a}-\frac{1-\beta_E}{q}\right)<0\\
\frac{\partial F_1\left(a;\theta_1^I,r_0,\theta_0^I\right)}{\partial r_0} &= \lambda\cdot(R_G-a)\cdot\frac{\beta_E}{q\cdot R_G/a}\cdot\frac{dG\left(\theta_0^I,r_0\right)}{dr_0}<0
\end{aligned}$$

and:

$$\begin{aligned}
\frac{\partial F_1\left(a;\theta_1^I,r_0,\theta_0^I\right)}{\partial q} &= \lambda\cdot(R_G-a)\cdot(1-\beta_E)\cdot\left(ER+\theta_1^I\right)\cdot\frac{1}{q^2}>0\\
\frac{\partial F_1\left(a;\theta_1^I,r_0,\theta_0^I\right)}{\partial R_B} &= \alpha\cdot a-\lambda\cdot(R_G-a)\cdot(1-\beta_E)\cdot\frac{1-q}{q}\cdot\alpha.
\end{aligned}$$

Furthermore:

$$\begin{aligned}
\frac{\partial F_1\left(a;\theta_1^I,r_0,\theta_0^I\right)}{\partial a} = &-\alpha\cdot a+\lambda\cdot(1-\beta_E)\cdot\frac{(1-q)\cdot ER+\theta_1^I}{q}\\
&+\lambda\cdot(R_G-2\cdot a)\cdot\left(\beta_E\cdot\left(1+\frac{G\left(\theta_0^I,r_0\right)-\theta_1^I}{q\cdot R_G}\right)-\alpha\right).
\end{aligned}$$

Observe that $\frac{\partial^2 F_1}{\partial R_B\partial q}>0$ and $\frac{\partial F_1}{\partial R_B}\Big|_{q\to 1}>0$. By continuity and differentiability $\frac{\partial F_1}{\partial R_B}>0$ for a sufficiently large q. Moreover, $\frac{\partial F_1}{\partial a}<0$ for q sufficiently large and $R_G\leq 2\cdot\ell_G$. Similarly, it can be shown that $\frac{\partial F_1}{\partial R_B}>0$ and $\frac{\partial F_1}{\partial a}>0$ for α sufficiently small. To see this, notice that $a\nearrow R_G$ for $\alpha\searrow 0$ and, hence, $\frac{\partial F_1}{\partial R_B}>0$ and $\frac{\partial F_1}{\partial a}<0$ for $\alpha\searrow 0$. As a result, $\frac{\partial a}{\partial\theta_0^I}<0$, $\frac{\partial a}{\partial\theta_1^I}<0$, $\frac{\partial a}{\partial r_0}<0$, $\frac{\partial a}{\partial q}>0$, and $\frac{\partial a}{\partial R_B}>0$ provided α is sufficiently small or provided q is sufficiently large and $R_G\leq 2\cdot\ell_G$. *(q.e.d.)*

A.4 Proof of Lemma 2

Provided a pooling equilibrium exists, the market-clearing interest rate is implicitly defined by:

$$\begin{aligned}
&F_2\left(r_2,a,f;\theta_1^I,r_0,\theta_0^I,\theta_0^L\right)\\
\equiv\ &r_2-\frac{\lambda\cdot\beta_E\cdot(1-f)}{m_2+f\cdot(1-\lambda\cdot\beta_E)}\cdot\left(\frac{G\left(\theta_0^I,r_0\right)-\theta_1^I}{R_G/a}+q\cdot a+(1-q)\cdot ER+\theta_1^I+\frac{f\cdot G\left(\theta_0^L,r_0\right)}{1-f}\right)
\end{aligned}\tag{25}$$

By application of the implicit function theorem:

$$\begin{aligned}
\frac{\partial F_2\left(r_2,a,f;\theta_1^I,r_0,\theta_0^I,\theta_0^L\right)}{\partial\theta_0^I} &= -\frac{\lambda\cdot\beta_E\cdot(1-f)}{m_2+f\cdot(1-\lambda\cdot\beta_E)}\cdot\frac{a}{R_G}\cdot\frac{\partial G\left(\theta_0^I,r_0\right)}{\partial\theta_0^I}>0\\
\frac{\partial F_2\left(r_2,a,f;\theta_1^I,r_0,\theta_0^I,\theta_0^L\right)}{\partial\theta_0^L} &= -\frac{\lambda\cdot\beta_E\cdot(1-f)}{m_2+f\cdot(1-\lambda\cdot\beta_E)}\cdot\frac{f}{1-f}\cdot\frac{\partial G\left(\theta_0^L,r_0\right)}{\partial\theta_0^L}>0\\
\frac{\partial F_2\left(r_2,a,f;\theta_1^I,r_0,\theta_0^I,\theta_0^L\right)}{\partial\theta_1^I} &= -\frac{\lambda\cdot\beta_E\cdot(1-f)}{m_2+f\cdot(1-\lambda\cdot\beta_E)}\cdot\left(-\frac{a}{R_G}+1\right)<0\\
\frac{\partial F_2\left(r_2,a,f;\theta_1^I,r_0,\theta_0^I,\theta_0^L\right)}{\partial r_0} &= -\frac{\lambda\cdot\beta_E\cdot(1-f)}{m_2+f\cdot(1-\lambda\cdot\beta_E)}\cdot\left(\frac{a}{R_G}\cdot\frac{\partial G\left(\theta_0^I,r_0\right)}{\partial r_0}+\frac{f}{1-f}\cdot\frac{\partial G\left(\theta_0^L,r_0\right)}{\partial r_0}\right)>0
\end{aligned}$$

and:

$$\begin{aligned}\frac{\partial F_2\left(r_2,a,f;\theta_1^I,r_0,\theta_0^I,\theta_0^L\right)}{\partial q} &= -\frac{\lambda\cdot\beta_E\cdot(1-f)}{m_2+f\cdot(1-\lambda\cdot\beta_E)}\cdot(a-ER)>0\\ \frac{\partial F_2\left(r_2,a,f;\theta_1^I,r_0,\theta_0^I,\theta_0^L\right)}{\partial R_B} &= -\frac{\lambda\cdot\beta_E\cdot(1-f)}{m_2+f\cdot(1-\lambda\cdot\beta_E)}\cdot(1-q)\cdot\alpha<0.\end{aligned}$$

Furthermore:

$$\begin{aligned}\frac{\partial F_2\left(r_2,a,f;\theta_1^I,r_0,\theta_0^I,\theta_0^L\right)}{\partial r_2} &= 1\\ \frac{\partial F_2\left(r_2,a,f;\theta_1^I,r_0,\theta_0^I,\theta_0^L\right)}{\partial a} &= -\frac{\lambda\cdot\beta_E\cdot(1-f)}{m_2+f\cdot(1-\lambda\cdot\beta_E)}\cdot\left(\frac{G\left(\theta_0^I,r_0\right)-\theta_1^I}{R_G}+q\right)<0\\ \frac{\partial F_2\left(r_2,a,f;\theta_1^I,r_0,\theta_0^I,\theta_0^L\right)}{\partial f} &= \frac{\lambda\cdot\beta_E\cdot\left[(m_2+1-\lambda\cdot\beta_E)\cdot\left(\frac{G\left(\theta_0^I,r_0\right)-\theta_1^I}{R_G/a}+q\cdot a+(1-q)\cdot ER+\theta_1^I\right)-m_2\cdot G\left(\theta_0^L,r_0\right)\right]}{(m_2+f\cdot(1-\lambda\cdot\beta_E))^2}.\end{aligned}$$

By continuity and differentiability, $\frac{\partial F_2}{\partial f}>0$ provided that m_2 is sufficiently small. As a result, $\frac{\partial r_2}{\partial\theta_0^I}<0$, $\frac{\partial r_2}{\partial\theta_0^L}<0$, $\frac{\partial r_2}{\partial\theta_1^I}>0$, $\frac{\partial r_2}{\partial r_0}<0$, $\frac{\partial r_2}{\partial q}<0$, $\frac{\partial r_2}{\partial R_B}=0$, and $\frac{\partial r_2}{\partial a}>0$.

Moreover, $\frac{\partial r_2}{\partial f}<0$ provided m_2 is sufficiently small. Let $\widetilde{f}$ be the solution to the pricing function. If it exceeds the maximum price paid by buyers, i.e. if $p\left(\widetilde{f}\right)>a$, then the market-clearing interest rate does not depend on f. This is because, first, from Lemma 1, the average quality a does not depend on f and, second, because of the pricing function $\frac{\partial r_2}{\partial f}<0\ \Leftrightarrow\ \frac{\partial p}{\partial f}>0$. Hence, the solution remains non-interior for all $f>\widetilde{f}$. Instead, if the solution to the pricing function falls short of the minimum price accepted by sellers of opaque good quality assets, i.e. if $p\left(\widetilde{f}\right)\leq\ell_G$, then a pooling equilibrium cannot be sustained. *(q.e.d.)*

A.5 Proof of Lemma 3

The proof consists of three steps. First, insert the demand and supply schedules derived in section 2.2.1 for the case of pooling:

$$V\left(r_2\left(f\right)\right)\equiv\left(\begin{array}{l}\lambda\cdot\beta_E\cdot log\left[\frac{\beta_E\cdot\left(q+\frac{G\left(\theta_0^I,r_0\right)-\theta_0^I}{R_G}\right)\cdot a+\beta_E\cdot(1-q)\cdot ER-(1-\beta_E)\cdot\theta_1^I+b_{2E}^I}{r_2}\right]\\ +\lambda\cdot(1-\beta_E)\cdot log\left[\frac{\left(q\cdot a+\frac{G\left(\theta_0^I,r_0\right)-b_{2E}^L}{R_G/a}\right)-\beta_E\cdot\left(q\cdot a+\frac{G\left(\theta_0^I,r_0\right)-\theta_1^I}{R_G/a}\right)+(1-\beta_E)\cdot\left((1-q)\cdot ER+\theta_1^I\right)}{a/R_G}\right]\\ +(1-\lambda)\cdot log\left[G\left(\theta_0^I,r_0\right)+(1-q)\cdot ER+q\cdot(\alpha\cdot a+(1-\alpha)\cdot R_G)\right]\end{array}\right)$$

$$W\left(r_2\left(f\right)\right)\equiv\left(\begin{array}{c}\lambda\cdot\beta_E\cdot log\left[\beta_E\cdot\left(1+\frac{G\left(\theta_0^L,r_0\right)}{r_2}\right)\right]+\lambda\cdot\left(1-\beta_E\right)\cdot log\left[\left(1-\beta_E\right)\cdot\left(r_2+G\left(\theta_0^L,r_0\right)\right)\right]\\ \\ +\left(1-\lambda\right)\cdot log\left[r_2+G\left(\theta_0^L,r_0\right)\right]\end{array}\right)$$

where $b_{2E}^I=\theta_1^I=\theta_0^I$, provided there exists a pooling equilibrium in the date $t=2$ market. Recall that b_1^L cancels out. Henceforth, we set $b_1^L=0$. Second, observe that $\frac{\partial V}{\partial f}>0$ and $\frac{\partial W}{\partial f}<0$. Third, given the results of Lemma 2 the function W (V) takes on its lowest (highest) value for the highest permissible value of $\widetilde{f}$ where $r_2\left(\widetilde{f}\right)=1$. Provided α or β_E is sufficiently small $V\left(r_2\left(\widetilde{f}\right)=1\right)>W\left(r_2\left(\widetilde{f}\right)=1\right)$. Given that $W\left(r_2\right)\big|_{r_2\rightarrow\infty}>V\left(r_2\right)\big|_{r_2\rightarrow\infty}$, it follows from differentiability and continuity that, for a given average quality of opaque assets traded, there exists a unique $\widehat{f}$ solving equation (16). The pooling equilibrium at date $t=2$ exists if $\frac{a}{r_2\left(\hat{f}\right)}>\ell_G$. *(q.e.d.)*

A.6 Proof of Proposition 4

Results *(b)* and *(c)* of Proposition 4 are proven in turn. First, notice that $\frac{a\left(\theta_1^I,\theta_0^I,r_0\right)}{r_2\left(a,\hat{f};\theta_1^I,r_0,\theta_0^I,\theta_0^L\right)}>a\left(\theta_1^I,\theta_0^I,r_0\right)\geq\ell_G$ implies that becoming a liquid investor is more attractive than becoming an illiquid investor, despite the interest rate in the date $t=2$ market being at its lower bound $r_2=1$. The previous inequality implies that $V\left(r_2=1\right)<W\left(r_2=1\right)$. Hence, it is optimal for all investors to become liquid investors, i.e. the outcome is a collective cash hoarding ($f^*=1$). This is true despite the absence of cash-in-the-market pricing, resulting in a high market valuation that reflects the fundamental value of assets traded in the opaque market segment. From no-arbitrage, $p_G^*=p_B^*\cdot\frac{R_G}{R_B}=R_G$ follows, concluding the proof of result *(b)*.

Second, it remains to be shown that there always exists an equilibrium where pooling in the opaque market segment breaks down, which is characterized by $f^*\in[0,1]$ and $p^*\in[0,R_B]$. This equilibrium can be constructed as follows. Suppose investors believe at date $t=1$ that there will be a breakdown of pooling in the opaque market segment at $t=2$, i.e. they believe that $p\leq\ell_G$. Notice that at date $t=2$ such an equilibrium can always be supported by any $p\in[0,R_B]$. The characterization of this breakdown equilibrium requires re-visiting market-clearing (equation (13)):

$$\begin{aligned}&m_2+f\cdot\left(1-\lambda\cdot\beta_E\cdot\left(1+\frac{G\left(\theta_0^L,r_0\right)}{r_2}\right)\right)-\left(1-f\right)\cdot\lambda\cdot\frac{\theta_1^I}{r_2}\\=\;&\left(1-f\right)\cdot\lambda\cdot\left(\left(1-q\right)\cdot\frac{ER}{r_2}+q\left(\alpha\cdot\frac{R_B}{r_2}+\left(1-\alpha\right)\cdot\frac{\ell_G}{r_2}\cdot d_{2iG}^I\right)\right)\end{aligned}\tag{26}$$

where:

$$d_{2iG}^{I}\left(l_B,l_G\right)=\frac{\beta_E\cdot\left(q+\frac{G\left(\theta_0^I,r_0\right)-\theta_1^I}{R_G}\right)-q\cdot\alpha\cdot\left(\beta_E+(1-\beta_E)\cdot\frac{R_B}{\ell_G}\right)-(1-\beta_E)\cdot\frac{(1-q)\cdot ER+\theta_1^I}{\ell_G\cdot r_2}}{q\cdot(1-\alpha)}.$$

Let $\hat{r}_2\left(f\right)$ be the solution to equation (26). Then, the market clearing interest rate is given by $r_2^*=max\left\{1,\hat{r}_2\left(f\right)\right\}$. Similar to the pooling case, it can be shown that $\hat{r}_2\left(f\right)$ is decreasing in f. The liquidity choice problem at $t=1$ if investors anticipate a breakdown of pooling is constructed similarly to before. If an interior solution exists, then $f^*\in[0,1)$ and $p^*\in[0,R_B]$. At the corner solution, the breakdown equilibrium exhibits liquidity hoarding, i.e. $f^*=1$, with $r^*=1$ and $p^*\in[0,R_B]$. This concludes the proof of result *(c)*. *(q.e.d.)*

A.7 Proof of Proposition 5

The proof proceeds by analyzing efficiency if the solution is interior ($\ell_G<p\left(\widehat{f}\right)\leq a$) and if the solution is in one of the corners.

(a) Assuming an interior solution, the problem of the constrained social planner reads:

$$\max_{0\leq f\leq 1}\left\{(1-f)\cdot V\left(r_2\left(f\right)\right)+f\cdot W\left(r_2\left(f\right)\right)\right\}.$$

Given a pooling equilibrium exists, the derivative with respect to f writes:

$$-V\left(r_2\left(f\right)\right)+W\left(r_2\left(f\right)\right)+\left\{-(1-f)\cdot\frac{\lambda\cdot\beta_E}{r_2}+f\cdot\frac{1-\lambda\cdot\beta_E\cdot\left(1+\frac{G}{r_2}\right)}{r_2+G}\right\}\cdot\frac{\partial r_2\left(f\right)}{\partial f}\tag{27}$$

Using an envelope-type argument equation (27) simplifies when evaluated at $\widehat{f}$:

$$\left\{-\frac{\lambda\cdot\beta_E}{r_2}+\frac{f}{r_2+G}\right\}\cdot\left.\frac{\partial r_2\left(f\right)}{\partial f}\right|_{f=\widehat{f}}.\tag{28}$$

Hence, the equilibrium is constrained inefficient if equation (28) is non-zero.

Recall that, $m_1=m_2$ and that $\widehat{f}$ is smaller (larger) when m_2 is larger (smaller). This is because from Lemma 3 there is only one price solving $V\left(r_2\right)=W\left(r_2\right)$, while we know from the pricing function that f (supply of liquidity by investors) and m_2 (supply of liquidity by outside financiers) are substitutes. Hence, given continuity and monotonicity, a larger (smaller) m_2 tends to be associated with an inefficient under-investment (over-investment) in cash:

$$\left\{-\frac{\lambda \cdot \beta_E}{r_2} + \frac{f}{r_2 + G}\right\} \cdot \left.\frac{\partial r_2(f)}{\partial f}\right|_{f=\widehat{f}} > 0 \ (< 0).$$

(b) Next, assume a corner solution with $p\left(\hat{f}\right) > a$. Here, $f^* = f^*_{SP} = 1$. Finally, assume a corner solution with $p\left(\hat{f}\right) \leq \ell_G$, then $f^* = f^*_{SP} \in [0,1]$. Hence, the equilibrium is efficient. *(q.e.d.)*

A.8 Proof of Proposition 6

The proof proceeds by analyzing the problems at dates $t = 0, 1, 2$ in three steps. Thereafter, the efficiency analysis follows. First, consider the trading decisions at date $t = 2$. The solution to the problem of liquid investors stays unaltered if $q = 0$. Suppose d_{2E} takes on an interior solution, then:

$$\frac{\beta_E}{c^I_{2E}} \cdot \frac{ER}{r_2} = \frac{1-\beta_E}{c^I_{3E}} \cdot ER \quad \Leftrightarrow \quad \frac{\beta_E}{\frac{ER}{r_2} \cdot d_{2E} + \frac{b^I_{2E}}{r_2}} \cdot \frac{ER}{r_2} = \frac{1-\beta_E}{ER \cdot (1-d_{2E}) - b^I_{2E} + G(\theta^I_1, r_0)} \cdot ER.$$

Illiquid investors are indifferent between borrowing or selling assets. Setting $b^I_{2E} = 0$ yields:

$$d_{2E} = \beta_E \cdot \left(1 + \frac{G(\theta^I_1, r_0)}{ER}\right).$$

Hence, the interiority of d_{2E} is guaranteed if Condition 1 holds.

Second, consider the liquidity management problem at $t = 1$ to obtain (18). The unique interest rate making investors indifferent is $r_2 = ER$. From market-clearing, the corresponding proportion of liquid investors f solves:

$$f = (1-f) \cdot \lambda \cdot \beta_E \cdot \left(1 + \frac{G(\theta^I_0, r_0)}{ER}\right) + f \cdot \lambda \cdot \beta_E \left(1 + \frac{G(\theta^L_0, r_0)}{ER}\right) - m_2.$$

Third, consider the leverage choice problem at $t = 0$. As there is no benefit from leaving spare borrowing capacity, investors fully lever up, i.e. they select $\theta^L_0 = \theta^I_0 = 0$. The resources of outside financiers are sufficient to finance all investments if $m_0 \geq \frac{\gamma \cdot \rho \cdot \kappa}{ER - \gamma \cdot \rho}$. Following the same argument as in section 2.2.3, it can be argued that $r_0 = r_1 = r_2 \equiv r$. Hence,

$$\begin{aligned} r = &\quad ER \\ f = &\quad \lambda \cdot \beta_E \cdot \left(1 + \tfrac{G(0,ER)}{ER}\right) - m_2, \end{aligned}$$

where $f > 0$ requires that the second inequality of (17) holds. Taken together, inequality (18) follows. The equilibrium if $q = 0$ is unique given the uniqueness of the interest rate that makes investors indifferent at date $t = 1$ (Lemma 3).

Finally, the equilibrium is constrained efficient if inequality (19) holds. To see this, observe that:

$$\frac{\partial r_2}{\partial f} = -\lambda \cdot \beta_E \cdot \frac{ER \cdot m_2 + (ER + G(0, r_0)) \cdot (1 - \lambda \cdot \beta_E)}{(m_2 + f \cdot (1 - \lambda \cdot \beta_E))^2} < 0$$

is smallest if $m_2 = 0$. If the constrained planner induces a higher f by subsidizing cash holdings at $t = 1$, then the marginal benefit for investors is $\frac{dG(\theta, r_0)}{dr_0} \cdot \frac{\partial r_2}{\partial f}\Big|_{f=f^*}$, while the social cost is $ER - 1$. On the other hand, taxing cash holdings at $t = 1$ is clearly welfare decreasing. Hence, constrained efficiency is guaranteed if inequality (19) holds. *(q.e.d.)*

A.9 Proof of Lemma 7

The incentives to select a positive θ_0^I can be understood by analyzing the problem in (20). Recall that $\theta_0^I = \theta_1^I$ and $r_0 = r_1 = r_2$. The first-order condition of (20) writes:

$$\begin{aligned} \frac{\partial}{\partial \theta_{0,n}^I} = \quad & \lambda \cdot \frac{\frac{R_G - a}{a} + \frac{\partial G\left(\theta_{0,n}^I, r_0\right)}{\partial \theta_{0,n}^I}}{\frac{R_G - a}{a} \cdot \theta_1^I + G\left(\theta_{0,n}^I, r_0\right) + q \cdot R_G + (1-q) \cdot ER \cdot \frac{R_G}{a}} + \\ & (1 - \lambda) \cdot \frac{\frac{\partial G\left(\theta_{0,n}^I, r_0\right)}{\partial \theta_{0,n}^I}}{G\left(\theta_{0,n}^I, r_0\right) + q \cdot (\alpha \cdot a + (1-\alpha) \cdot R_G) + (1-q) \cdot ER}. \end{aligned} \tag{29}$$

Provided the conditions from Lemma 1(b) or Lemma 1(c) hold, a sufficient condition for the derivative of the objective function in (20) to be positive, i.e. $\frac{\partial}{\partial \theta_0^I} > 0$, is given by inequality (21). *(q.e.d.)*

A.10 Proof of Proposition 8

This proof analyzes the change in the first-order condition of (20) when $\theta_{0,-n}^I$ increases:

$$\frac{\partial \left(\frac{\partial}{\partial \theta_{0,n}^I}\right)}{\partial \theta_{0,-n}^I} = \overbrace{\frac{\partial \left(\frac{\partial}{\partial \theta_{0,n}^I}\right)}{\partial a}}^{\equiv A} \cdot \frac{da}{d\theta_{0,-n}^I} + \overbrace{\frac{\partial \left(\frac{\partial}{\partial \theta_{0,n}^I}\right)}{\partial r_0}}^{\equiv B} \cdot \frac{dr_0}{d\theta_{0,-n}^I}. \tag{30}$$

A strategic complementarity in leverage choices exists if (30) has a positive sign. Sufficient conditions for this to be true are derived in the remainder of the proof. Evaluating the partial derivatives yields:

$$A = \left[\begin{array}{c} \frac{\lambda\cdot\frac{R_G}{a^2}\cdot\left(-\left(G\left(\theta_0^I,r_0\right)+q\cdot R_G+(1-q)\cdot ER\right)+\left(\theta_1^I+(1-q)\cdot ER\right)\cdot\frac{\partial G\left(\theta_0^I,r_0\right)}{\partial\theta_0^I}\right)}{\left(G\left(\theta_0^I,r_0\right)+\theta_1^I\cdot\left(\frac{R_G}{a}-1\right)+q\cdot R_G+(1-q)\cdot ER\cdot\frac{R_G}{a}\right)^2} \\ -\frac{(1-\lambda)\cdot q\cdot\alpha\cdot\frac{\partial G\left(\theta_0^I,r_0\right)}{\partial\theta_0^I}}{\left(G\left(\theta_0^I,r_0\right)+q\cdot(\alpha\cdot a+(1-\alpha)\cdot R_G)+(1-q)\cdot ER\right)^2} \end{array}\right]$$

$$B = \left[\begin{array}{c} \frac{\lambda\cdot\frac{\partial^2 G\left(\theta_0^I,r_0\right)}{\partial\theta_0^I\partial r_0}\cdot\left(G\left(\theta_0^I,r_0\right)+\theta_1^I\cdot\left(\frac{R_G}{a}-1\right)+q\cdot R_G+(1-q)\cdot ER\cdot\frac{R_G}{a}\right)-\left(\left(\frac{R_G}{a}-1\right)+\frac{\partial G\left(\theta_0^I,r_0\right)}{\partial\theta_0^I}\right)\cdot\frac{\partial G\left(\theta_0^I,r_0\right)}{\partial r_0}}{\left(G\left(\theta_0^I,r_0\right)+\theta_1^I\cdot\left(\frac{R_G}{a}-1\right)+q\cdot R_G+(1-q)\cdot ER\cdot\frac{R_G}{a}\right)^2} \\ +\frac{(1-\lambda)\cdot\left(\frac{\partial^2 G\left(\theta_0^I,r_0\right)}{\partial\theta_0^I\partial r_0}\cdot\left(G\left(\theta_0^I,r_0\right)+q\cdot(\alpha\cdot a+(1-\alpha)\cdot R_G)+(1-q)\cdot ER\right)-\frac{\partial G\left(\theta_0^I,r_0\right)}{\partial\theta_0^I}\cdot\frac{\partial G\left(\theta_0^I,r_0\right)}{\partial r_0}\right)}{\left(G\left(\theta_0^I,r_0\right)+q\cdot(\alpha\cdot a+(1-\alpha)\cdot R_G)+(1-q)\cdot ER\right)^2} \end{array}\right]$$

where $\frac{\partial G\left(\theta_0^I,r_0\right)}{\partial r_0} = \frac{\rho\cdot(1-\gamma)}{(r_0-\gamma\cdot\rho)^2}\cdot\left(\theta_0^I-\gamma\cdot\rho\cdot\kappa\right) = \frac{\partial^2 G\left(\theta_0^I,r_0\right)}{\partial\theta_0^I\partial r_0}\cdot\left(\theta_0^I-\gamma\cdot\rho\cdot\kappa\right) < 0$. Notice that A is guaranteed to be negative provided that the result of Lemma 7 holds and κ is sufficiently small, while q is sufficiently large. Furthermore, B is guaranteed to be positive for a sufficiently small κ (which implies that $\frac{\partial G\left(\theta_0^I,r_0\right)}{\partial r_0}$ is small).

Suppose that $\frac{da}{d\theta_0^I} < 0$. Given that a A is negative and B is positive, the sign of (30) is positive if either $\frac{dr_0}{d\theta_0^I} > 0$ or $\left(\frac{dr_0}{d\theta_0^I}/\frac{da}{d\theta_0^I}\right) < -\frac{A}{B}$. The proof proceeds by deriving conditions such that $\frac{da}{d\theta_0^I} < 0$ and either $\frac{dr_0}{d\theta_0^I} > 0$ or $\left(\frac{dr_0}{d\theta_0^I}/\frac{da}{d\theta_0^I}\right) < -\frac{A}{B}$ holds.

First, the implicit function theorem for simultaneous equations is used to derive $\frac{da}{d\theta_0^I}$ and $\frac{dr_0}{d\theta_0^I}$ (recall that $\theta_0^I = \theta_1^I$ and $r_0 = r_1 = r_2$). In Lemmas 1 and 2, the two optimality conditions stemming from date $t = 2$ and the comparative statics are derived. It remains to analyze the optimality condition stemming from date $t = 1$:

$$F_3\left(r_2, a, f; r_0, \theta_0^I, \theta_0^L\right) \equiv V\left(r_2\left(f\right)\right) - W\left(r_2\left(f\right)\right) = 0. \tag{31}$$

By application of the implicit function theorem:

$$\frac{\partial F_3\left(r_2, a, f; r_0, \theta_0^I, \theta_0^L\right)}{\partial\theta_0^I} = \left(\begin{array}{c} \frac{\lambda\cdot\left(\frac{\partial G\left(\theta_0^I,r_0\right)}{\partial\theta_0^I}+\left(\frac{R_G}{a}-1\right)\right)}{G\left(\theta_0^I,r_0\right)+\theta_0^I\cdot\left(\frac{R_G}{a}-1\right)+(1-q)\cdot ER\cdot\frac{R_G}{a}+q\cdot R_G} \\ \\ +\frac{(1-\lambda)\cdot\frac{\partial G\left(\theta_0^I,r_0\right)}{\partial\theta_0^I}}{G\left(\theta_0^I,r_0\right)+(1-q)\cdot ER+q\cdot(\alpha\cdot a+(1-\alpha)\cdot R_G)} \end{array}\right) > 0 \; given \; (21)$$

$$\frac{\partial F_3\left(r_2,a,f;r_0,\theta_0^I,\theta_0^L\right)}{\partial \theta_0^L} = -\frac{\frac{\partial G\left(\theta_0^L,r_0\right)}{\partial \theta_0^L}}{r_0+G\left(\theta_0^L,r_0\right)} > 0$$

$$\begin{aligned}\frac{\partial F_3\left(r_2,a,f;r_0,\theta_0^I,\theta_0^L\right)}{\partial r_0} = & \frac{\lambda\cdot\frac{\partial G\left(\theta_0^I,r_0\right)}{\partial r_0}}{G\left(\theta_0^I,r_0\right)+q\cdot R_G+(1-q)\cdot ER\cdot\frac{R_G}{a}+\theta_1^I\cdot\left(\frac{R_G}{a}-1\right)}+ \\ & \frac{(1-\lambda)\cdot\frac{\partial G\left(\theta_0^I,r_0\right)}{\partial r_0}}{G\left(\theta_0^I,r_0\right)+(1-q)\cdot ER+q\cdot(\alpha\cdot a+(1-\alpha)\cdot R_G)} - \frac{1+\frac{\partial G\left(\theta_0^L,r_0\right)}{\partial r_0}}{r_0+G\left(\theta_0^L,r_0\right)} < 0\ if\ \kappa\ suff.\ small.\end{aligned}$$

Furthermore:

$$\begin{aligned}\frac{\partial F_3\left(r_2,a,f;r_0,\theta_0^I,\theta_0^L\right)}{\partial a} = & \begin{pmatrix}\frac{\lambda\cdot\beta_E}{a} \\ -\frac{\lambda\cdot\left((1-q)\cdot\frac{ER}{a}+\frac{\theta_1^I}{a}\right)\cdot\frac{R_G}{a}}{G\left(\theta_0^I,r_0\right)+(1-q)\cdot ER\cdot\frac{R_G}{a}+q\cdot R_G+\theta_0^I\cdot\left(\frac{R_G}{a}-1\right)} \\ +\frac{(1-\lambda)\cdot q\cdot\alpha}{G\left(\theta_0^I,r_0\right)+(1-q)\cdot ER+q\cdot(\alpha\cdot a+(1-\alpha)\cdot R_G)}\end{pmatrix} > 0\ if\ q\ large\ and\ \kappa\ small \\ \frac{\partial F_3\left(r_2,a,f;r_0,\theta_0^I,\theta_0^L\right)}{\partial f} = & 0.\end{aligned}$$

As a result, provided inequality (21) (Lemma 7) holds. By continuity and differentiability:

$$\frac{da}{d\theta_{0,-n}^I} = \frac{da}{d\theta_0^I} = \frac{|J_2|}{|J|} < 0\ if\ \kappa\ small,\ q\ large\ and\ R_G \leq 2\cdot\ell_G.$$

Furthermore, provided inequality (21) holds. By continuity and differentiability:

$$\frac{dr_0}{d\theta_{0,-n}^I} = \frac{dr_0}{d\theta_0^I} = \frac{|J_1|}{|J|} < 0$$

if κ small, q large, $R_G \leq 2\cdot\ell_G$, and in addition $|J_1| = -\frac{\partial F_1}{\partial a}\cdot\frac{\partial F_2}{\partial f}\cdot\frac{\partial F_3}{\partial\theta_0^I} + \frac{\partial F_3}{\partial a}\cdot\frac{\partial F_2}{\partial f}\cdot\frac{\partial F_1}{\partial\theta_0^I} < 0$.

If the last inequality is violated, $|J_1| \geq 0$, then the sign of equation (30) is positive provided κ sufficiently small, q sufficiently large, and $R_G \leq 2\cdot\ell_G$. Instead, if $|J_1| < 0$, then:

$$\frac{\frac{dr_0}{d\theta_0^I}}{\frac{da}{d\theta_0^I}} = \frac{|J_1|/|J|}{|J_2|/|J|} = \frac{-\frac{\partial F_1}{\partial a}\cdot\frac{\partial F_2}{\partial f}\cdot\frac{\partial F_3}{\partial \theta_0^I}+\frac{\partial F_3}{\partial a}\cdot\frac{\partial F_2}{\partial f}\cdot\frac{\partial F_1}{\partial \theta_0^I}}{-\frac{\partial F_1}{\partial \theta_0^I}\cdot\frac{\partial F_2}{\partial f}\cdot\frac{\partial F_3}{\partial r_2}+\frac{\partial F_3}{\partial \theta_0^I}\cdot\frac{\partial F_2}{\partial f}\cdot\frac{\partial F_1}{\partial r_2}}$$

$$< \frac{-\left(\begin{array}{c}\frac{\lambda\cdot\beta_E}{a}-\frac{\lambda\cdot\left((1-q)\cdot ER+\theta_1^I\right)\cdot\frac{R_G}{a^2}}{G\left(\theta_0^I,r_0\right)+\theta_0^I\cdot\left(\frac{R_G}{a}-1\right)+q\cdot R_G+(1-q)\cdot ER\cdot\frac{R_G}{a}}\\ \\ +\frac{(1-\lambda)\cdot q\cdot\alpha}{G\left(\theta_0^I,r_0\right)+q\cdot(\alpha\cdot a+(1-\alpha)\cdot R_G)+(1-q)\cdot ER}\end{array}\right)}{\left(\begin{array}{c}\frac{\lambda\cdot\frac{\partial G\left(\theta_0^I,r_0\right)}{\partial r_0}}{G\left(\theta_0^I,r_0\right)+\theta_1^I\cdot\left(\frac{R_G}{a}-1\right)+q\cdot R_G+(1-q)\cdot ER\cdot\frac{R_G}{a}}\\ \\ +\frac{(1-\lambda)\cdot\frac{\partial G\left(\theta_0^I,r_0\right)}{\partial r_0}}{G\left(\theta_0^I,r_0\right)+q\cdot(\alpha\cdot a+(1-\alpha)\cdot R_G)+(1-q)\cdot ER}-\frac{1+\frac{\partial G\left(\theta_0^L,r_0\right)}{\partial r_0}}{r_0+G\left(\theta_0^L,r_0\right)}\end{array}\right)}$$

Provided κ is sufficiently small, the expression is arbitrarily close to a weakly negative value provided that β_E is sufficiently small, which implies low values of α to assure that $0 < d_{2EG} < 1$ is satisfied. At the same time, A and B are not a function of β_E. Hence, $\left(\frac{dr_0}{d\theta_0^I}/\frac{da}{d\theta_0^I}\right) < -\frac{A}{B}$ is guaranteed to hold for a sufficiently small β_E provided κ sufficiently small, q sufficiently large, and $R_G \leq 2\cdot\ell_G$. Under the same conditions $\frac{d\hat{\theta}^I}{d\theta_{-n}^I} = 0$ if $\hat{\theta}^I = \theta_{max}^I$. *(q.e.d.)*

A.11 Proof of Proposition 9

The proof of Proposition 9 analyzes the change in the first-order condition of (20) when R_B increases:

$$\frac{\partial\left(\frac{\partial}{\partial\theta_{0,n}^I}\right)}{\partial R_B} = -\left(\begin{array}{c}\frac{\lambda\cdot\left[\left(\frac{R_G}{a}-1\right)+\frac{\partial G\left(\theta_{0,n}^I,r_0\right)}{\partial\theta_{0,n}^I}\right]\cdot(1-q)\cdot\alpha\cdot\frac{R_G}{a}}{\left(G\left(\theta_{0,n}^I,r_0\right)+\theta_1^I\cdot\left(\frac{R_G}{a}-1\right)+q\cdot R_G+(1-q)\cdot ER\cdot\frac{R_G}{a}\right)^2}\\ +\frac{(1-\lambda)\cdot\frac{\partial G\left(\theta_{0,n}^I,r_0\right)}{\partial\theta_{0,n}^I}\cdot(1-q)\cdot\alpha}{\left(G\left(\theta_{0,n}^I,r_0\right)+q\cdot(\alpha\cdot a+(1-\alpha)\cdot R_G)+(1-q)\cdot ER\right)^2}\end{array}\right)+A\cdot\frac{da}{dR_B}+B\cdot\frac{dr_0}{dR_B}. \tag{32}$$

Given the strategic complementarity in leverage choices from Proposition 8, a lower R_B increases θ_0^I if (32) has a negative sign. Following the same analysis as in the Proof of Proposition 8, we find that:

$$\frac{dr_0}{dR_B} = \frac{|J_1|}{|J|} > 0\ if\ \kappa\ small,\ q\ large\ and\ R_G \leq 2\cdot\ell_G$$
$$\frac{da}{dR_B} = \frac{|J_2|}{|J|} > 0\ if\ \kappa\ small,\ q\ large\ and\ R_G \leq 2\cdot\ell_G.$$

Hence, the sign of (32) is guaranteed to be negative if $\left(\frac{dr_0}{dR_B}/\frac{da}{dR_B}\right) < -\frac{A}{B}$. The desired result arises by application of the same argument as in the Proof of Proposition 8 provided that κ, β_E sufficiently small, q sufficiently large, and $R_G \leq 2\cdot\ell_G$. *(q.e.d.)*

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