Inflation targeting and the dynamics of the transmission mechanism

by

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Abstract

This paper derives closed-form expressions for optimal monetary policy rules when the central bank can influence inflation directly with a one-period lag as well as a two-period lagged effect via the output gap. It turns out that even a modest one-period inflation effect from monetary policy actions has substantial implications for monetary policy that also seem to be a step towards increased realism. For instance, in models where the central bank only can affect inflation with a two-period lag via the output gap, policy becomes more aggressive and the output gap exhibits a tendency to switch sign frequently. This unrealistic oscillating feature can be avoided by allowing the central bank to influence inflation with a one-period lag. The model also illustrates that the nature of empirical (or reduced-form) Phillips curves may reflect monetary policy and the observation that the Phillips curve in recent years has become flatter can in this model be explained by a more counter-cyclical monetary policy.

Keywords: Inflation targeting, optimal monetary policy, the transmission mechanism

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1. *Introduction*

In recent years analysis of small macroeconomic models has lead to important insights concerning the principles of inflation targeting, see e.g. Svensson (1997, 1999a,b) or Orphanides and Wieland (2000). A convenient assumption in these models is that monetary policy actions affect the inflation rate with a fixed lag. This paper deviates from this assumption by letting the central bank influence inflation with a one-period lag as well as a two-period lag, a view that is motivated by empirical as well as theoretical considerations. Indeed, many features of the model used in this study are probably to a large extent incorporated in richer models analyzed in the literature. For instance, the transmission mechanism in open economy models (e.g. Adolfson (2000), or Svensson (2000)) often works through different channels that affect the inflation rate with various lag lengths. Unfortunately, due the complexity of these models numerical methods must be employed.

The advantage of the model analyzed in this study is its simplicity which allows for closed-form solutions and hence a full characterization of the model and, in particular, the relationship between monetary policy actions and dynamic properties of the transmission mechanism. The possibility of an explicit examination of the reduced-form system appears to be a very useful way of assessing the implications of the model. It is shown that even if one-period inflation effects from monetary policy actions are modest the implications for optimal policy are potentially very large. Fortunately, these implications seem to increase the realism of the model. For instance, the presence of one-period effects tends to make monetary policy less aggressive and generate more persistence in the output gap. The model also illustrates that the nature of empirical (or reduced-form) Phillips curves may reflect monetary policy, and the observation that the Phillips curve in recent years has become flatter can in this model be explained by a more counter-cyclical monetary policy (a higher coefficient for the output gap in the instrument rule).

The paper is organized as follows. Sections 2 and 3 present and evaluate the model, respectively, and Section 4 concludes.

2. A simple model

In order to formally analyze the implications of allowing the central bank to influence inflation with a one-period as well as a two-period lag we consider the following model in which the central bank solves the following optimal dynamic problem:

$$\min_{\{i_{t+s}\}} (1-\delta)\sum_{s=0}^{\infty} \delta^s E_t\left[(\pi_{t+s}-\pi^*)^2 + \lambda y_{t+s}^2\right] \quad (1)$$

s.t.

$$\pi_{t+1} = \pi_t + \alpha_y^0 y_{t+1} + \alpha_y^1 y_t + \tilde{\varepsilon}_{t+1} \quad (2)$$

$$y_{t+1} = \beta_y y_t - \beta_r (i_t - \pi_t) + \eta_{t+1} \quad (3)$$

where π_t is the inflation rate, y_t is the output gap, i_t is the policy instrument (a short term interest rate), $\tilde{\varepsilon}_{t+1}$ and η_{t+1} are error terms with mean zero, π^* is the inflation target and where $\delta, \lambda, \alpha_y^0, \alpha_y^1$, β_y and β_r are parameters with, $\delta \leq 1$, $\beta_r > 0$ and $\beta_y < 1$.[1]

The new element in comparison with the other simple models analyzed in the literature is that the Phillips curve includes a contemporaneous as well as a lagged relationship between inflation and the output gap. Notice that the models of Svensson (1997) and Orphanides and Wieland (2000) correspond the special cases $\alpha_y^0 = 0$ and $\alpha_y^1 = 0$ respectively. In theoretical models as well as empirical studies the Phillips curve is often modeled as a purely contemporaneous relationship between inflation and the output gap ($\alpha_y^1 = 0$). This may represent the view that price setters react rapidly to changes in demand. If one believes that price setters react to changes in demand by some lag then a purely lagged relationship ($\alpha_y^0 = 0$) between inflation and the output gap is appropriate. In general one can imagine a mix of these two views and the correct specification is to a large extent an empirical issue. Moreover, by changing α_y^0 or α_y^1 it is possible to analyze changes in how quickly monetary policy actions affect inflation.

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[1]An output gap equation of the form: $y_{t+1} = \tilde{\beta}_y y_t - \tilde{\beta}_r\left(i_t - \pi_{t+1|t}\right) + \eta_{t+1}$, in which the real interest rate is defined in term of expected inflation ($\pi_{t+1|t}$) is more appropriate. However, if we take expectations conditional on period t information of this

We will in what follows often rewrite the inflation equation by substituting the output gap equation (3) into (2), which leads to the following inflation equation

$$\pi_{t+1} = \pi_t + \alpha_y y_t - \alpha_r (i_t - \pi_t) + \varepsilon_{t+1} \tag{4}$$

with

$$\alpha_y = \alpha_y^0 \beta_y + \alpha_y^1, \quad \alpha_r = \alpha_y^0 \beta_r, \quad \text{and} \quad \varepsilon_{t+1} = \alpha_y^0 \eta_{t+1} + \tilde{\varepsilon}_{t+1}. \tag{5}$$

When $\alpha_r \neq 0$ we will say that there are *direct* inflation effects (simultaneous with direct output gap effects) from monetary policy actions. Moreover, α_y represents *indirect* inflation effects via the output gap. The presence of the direct effect (i.e. $\alpha_r > 0$) in the inflation equation (4) weakens the link between monetary policy and the output gap, in the sense that monetary policy can affect future inflation using other channels than the one via the output gap. This is important since the strong link between the output gap and monetary policy actions that many models exhibit are hard to reconcile with what can be observed, as will be demonstrated shortly.

Alternative interpretation of the inflation equation (4)

We see from (4) that monetary policy will have a direct effect on inflation the next period—a feature of the model that can be motivated on other grounds than from equation (2). For instance, to the extent monetary policy actions affect inflation expectations it is reasonable this also will affect actual inflation with a relatively short lag. The mechanism that inflation expectations are crucial for the inflation rate itself is highlighted in forward-looking models such as the New-Keynesian Phillips curve.[2] The New-Keynesian Phillips curve has been criticized for implying a negative and unrealistic relationship between the output gap and inflation one period ahead (see Lindé (2001) for an analysis of this feature) but this

expression and the inflation equation (2) and solve for $\pi_{t+1|t}$ we obtain $\pi_{t+1|t} = \pi_t + (\alpha_y^0 \tilde{\beta}_y + \alpha_y^1)\, y_t$, which substituted into the output gap equation above leads to expression (3) with $\beta_y = (\tilde{\beta}_y + \alpha_y^0 \tilde{\beta}_y + \alpha_y^1)/(1+\tilde{\beta}_r)$, and $\beta_r = \tilde{\beta}_r / (1+\tilde{\beta}_r)$.

[2]See Clarida, Gali and Gertler (1999) for a review of this literature.

implication is often actually related to the instrument term appearing in (4).[3] Moreover, there might exist direct effects via asset prices, e.g. monetary policy in an open economy has some effects on the exchange rate and hence imported inflation. Of course if exchange rate effects are considered then it is desirable to model the exchange rate as an endogenous variable. However, it can be shown that under some simplifying assumptions an inflation equation with an endogenous exchange rate can be reduced to an equation similar to (4).[4] Another well-known example occurs when the measure of inflation captures interest expenditures in which case α_r might be negative.[5]

We will in what follows interpret inflation equation (4) *specifically*, i.e. relate it to the particular Phillips curve (2) by using (5) as well as more *generally*, in which case the expressions in (5) do not necessarily hold due to the presence of other direct effects discussed above.

3. Evaluation of the model

One of the main reasons to analyze the case when α_r differs from zero is that, which will be shown below, the special case $\alpha_r = 0$ is very singular in the sense that a small deviation from zero can give rise to large effects on the optimal monetary policy response. A straightforward and drastic way to illustrate this is to consider optimal policy under strict inflation targeting, i.e. when there are no preferences for output stabilization ($\lambda = 0$ in the loss function (1)). It is shown in the appendix that the optimal response in this case can be written as

$$i_t - \pi_t = \frac{1}{\alpha_r}(\pi_t - \pi^*) + \frac{\alpha_y}{\alpha_r} y_t \qquad \text{if } \alpha_r > 0 \text{ and } \lambda = 0 \tag{6}$$

$$i_t - \pi_t = \frac{1}{\alpha_y \beta_r}(\pi_t - \pi^*) + \frac{\beta_y + 1}{\beta_r} y_t \quad \text{if } \alpha_r = 0 \text{ and } \lambda = 0 \tag{7}$$

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[3]To see this notice that the New-Keynesian Phillips curve: $\pi_t = \beta\pi_{t+1|t} + \kappa y_t$, ($\pi_{t+1|t}$ denotes expected inflation in period *t+1* conditional on period *t* information) implies $\pi_{t+1} = \beta^{-1}\pi_t - \beta^{-1}\kappa y_t + u_{t+1}$ (i), where $u_{t+1} = \pi_{t+1} - \pi_{t+1|t}$ is an expectational error. Moreover, the output gap is assumed to be the central bank's instrument. If this is interpreted as a simple relationship between the interest rate (actual instrument) and the output gap of the form $y_t = -c(i_t - \pi_t)$ then (i) can also take the form of (4). However, since the error term is u_{t+1} endogenous (depends on expected monetary policy) this inflation equation can not be seen as a variant of the inflation process (3), for which the error term is exogenous.

[4]This is possible in an open economy model where deviations from purchasing power parity depend solely on domestic monetary policy (no foreign shocks or exchange rate risk premium shocks).

[5]See Nessén and Söderström (2001) for an analysis of direct interest rate effects stemming from interest rate expenditures in CPI.

As seen from (6), in the case of strict inflation targeting the monetary policy response does not converge to (7) but becomes aggressive without bounds when α_r approaches zero from the positive side. Clearly, it can be potentially very misleading to use the approximation $\alpha_r = 0$ in expressions for the monetary policy response even if one has good reasons to believe that $\alpha_r = 0$ indeed is a good approximation.

The example above demonstrates that the instrument rules derived by Svensson (1997) are not always robust with respect to small changes in the model. One should, however, bear in mind that the example above is based on strict inflation targeting, which is considered to be a quite unrealistic special case. In the case of flexible inflation targeting ($\lambda > 0$) it is shown in the appendix that the optimal instrument rule takes the form

$$i_t - \pi_t = g_\pi (\pi_t - \pi^*) + g_y y_t \tag{8}$$

where

$$g_\pi \equiv \frac{1 - c(\lambda, \lambda_0)}{a_y \beta_r}, \quad g_y \equiv \frac{\beta_y}{\beta_r} + \gamma g_\pi, \tag{9}$$

$$c(\lambda, \lambda_0) \equiv \frac{\lambda_1 - a_y \lambda_0}{\lambda_1 + k(\lambda, \lambda_0)}, \qquad 10)$$

$$\lambda_1 \equiv \lambda + \lambda_0^2, \quad \lambda_0 \equiv \frac{\alpha_r}{\beta_r}, \quad \gamma \equiv \alpha_y - \lambda_0 \beta_y, \quad a_y \equiv \lambda_0 + \gamma \tag{11}$$

$$k(\lambda, \lambda_0) \equiv \frac{1}{2}\left[1 - \frac{(1-\delta)\lambda_1}{\delta a_y^2} - \frac{2\lambda_0}{a_y} + \sqrt{\left(1 - \frac{(1-\delta)\lambda_1}{\delta a_y^2} - \frac{2\lambda_0}{a_y}\right)^2 + \frac{4\lambda}{\delta a_y^2}}\right] \qquad (12),$$

Closer inspection of the instrument rule (8) reveals that when α_r approaches zero it actually converges to the rule derived by Svensson (1997) in the flexible inflation targeting case ($\lambda >$ 0). Thus, under the realistic assumption of flexible inflation targeting the consequences of adding an instrument term to the inflation equation may be modest. In order to examine this issue further we numerically evaluate the instrument rule (8) for different parameter values for α_r and λ assuming the following values for the other parameters:

$$\alpha_y = 0.34, \; \beta_y = 0.77, \; \beta_r = 0.4 \text{ and, } \delta = 0.95 \tag{13}$$

This set of parameter values is taken from Nessén (1999) who relied on estimates of equations (2) and (3) on annual data 1976–1998 for the Euro area provided by Orphanides and Wieland (2000).

As seen from Table 1 the addition of an instrument term in the inflation equation has several notable implications. *First*, the presence of one period effects ($\alpha_r > 0$) does lead to substantial reductions in the response coefficients g_π and g_y, which is natural since interest changes have larger impact on future inflation. However, the optimal policy rule still appears to be aggressive relative to the standard Taylor rule ($g_\pi = g_y = 0.5$), which suggests that other elements such as parameter uncertainty or interest rate smoothing may have to be added in order to take further steps towards realism.

Table 1. Policy response coefficients (g_π, g_y) for different combinations of α_r and λ

	λ = 0.1	**λ = 0.2**	**λ = 0.3**	**λ = 0.5**	**λ = 1**
α_r **= 0.0**	(6.91 , 4.27)	(6.65 , 4.18)	(6.45 , 4.12)	(6.15 , 4.02)	(5.64 , 3.84)
α_r **= 0.1**	(5.82 , 2.78)	(5.47 , 2.73)	(5.26 , 2.70)	(4.95 , 2.66)	(4.48 , 2.59)
α_r **= 0.2**	(4.49 , 1.72)	(4.27 , 1.73)	(4.12 , 1.74)	(3.90 , 1.75)	(3.54 , 1.77)
α_r **= 0.3**	(3.27, 1.15)	(3.19, 1.17)	(3.13 , 1.18)	(3.01 , 1.21)	(2.78 , 1.27)
α_r **= 0.4**	(2.47 , 0.86)	(2.44, 0.88)	(2.41 , 0.89)	(2.35 , 0.92)	(2.20 , 0.98)
α_r **= 0.5**	(1.98, 0.69)	(1.96 0.70)	(1.93 , 0.72)	(1.89 , 0.75)	(1.79 , 0.81)

Second, the response coefficients are quite insensitive to changes in λ, whereas changes in α_r have substantial effects. In other words the nature of the transmission mechanism (especially the size of α_r) appears to be more important quantitatively than the extent to which the policy maker has preferences for output stabilization (the size of λ). *Third*, when $\alpha_r = 0$ (or very small) an increase in λ always leads to a less aggressive policy, i.e. reductions in both g_π and g_y. However, when α_r is sufficiently large we see that an increase in λ implies a larger output gap coefficient and a smaller inflation gap coefficient, which might be more in line with intuition. We see from (9) that this happens when $\gamma = \alpha_y - \lambda_0 \ \beta_y < 0$, i.e. when $\alpha_r >$ $\alpha_y \ \beta_r / \beta_y = 0.18$. As we will se shortly the sign of the parameter γ is crucial in several respects.

We have so far evaluated the implications of having direct inflation effects in the inflation equation (4) for a fixed value of α_y without relating them to the inflation equation (2). We will now examine the effects of changing the parameters α_y^0 and α_y^1 in equation (2), which is something different. In order to obtain reasonable parameter values equation (2) is estimated using annual data provided by OECD, see Table 2.[6] Notice first that during the period 1964–1999 the contemporaneous effect from the output gap on inflation is stronger than the lagged effect, which is statistically significant only at the 10 percent level.

Table 2. Estimation of the generalized Phillips curve: $\pi_{t+1} = \pi_t + \alpha_y^0 y_{t+1} + \alpha_y^1 y_t + \tilde{\varepsilon}_{t+1}$ **and implied parameter values** (α_y, α_r) **and response coefficients** (g_π, g_y)

	1964–1999	*1964–1980*	*1981–1999*
$\hat{\alpha}_y^0$	**0.3201** (2.43)	0.1674 (0.86)	**0.6252** (4.98)
$\hat{\alpha}_y^1$	0.2500 (1.90)	**0.5685** (2.86)	–0.1765 (–1.43)
$\bar{R}^2$	0.425	0.451	0.641
α_y	0.50	0.70	0.30
α_r	0.13	0.07	0.25
(g_π, g_y), λ=0.1	(4.26 , 2.99)	(3.38 , 3.85)	(3.97 , 1.22)
(g_π, g_y), λ=1	(2.92 , 2.66)	(2.24 , 3.20)	(3.40 , 1.33)

Inflation is measured as the annual percentage change of CPI and estimates of the output gap are taken from OECD. t-values are given within parenthesis and significance at the 5 percent level is indicated by bold face.

However, the dynamic properties of the Phillips curve appear to have changed over time. Until 1980, only the lagged, and not the contemporaneous, output gap enters significally in the Phillips curve equation, whereas the opposite holds for the period 1981–1999. Indeed one can not rule out the possibility that the lagged output gap since 1981 actually has had a negative impact on inflation. This is important since the sign of the parameter has several important implications. First, it is easily verified that if the model is evaluated in terms of the inflation equation (2), then the crucial parameter γ coincides with α_y^1, which indicates the judgement that γ (α_y^1) actually has been negative in recent years. Moreover, if one allows for additional direct effects discussed earlier this judgement is reinforced further. Second, the New Keynesian Phillips curve implies a negative relationship between inflation and the lagged

[6]The Phillips curve was estimated by OLS, which is justified by the fact that the residuals from this regression exhibit a very low

output gap (see footnote 3)—a feature of the New Phillips curve that has been criticized for being unrealistic. However, the negative estimate of α_y^1 for the period 1981–1999 does provide some (weak) evidence against this criticism.

Moreover, the implied estimate of α_y is quite large for the full and first half of the sample but since 1981 the estimate is close to the value used in Table 5 (0.34). Direct inflation effects measured by the implied estimate of α_r are present mainly since 1981. The overall impression is that the response coefficients are of the same magnitude as in Table 1 and quite insensitive to changes in λ. This is true in particular for g_y, which, on the other hand, reacts strongly to changes in α_y^1.

The reduced-form system

Our view of the empirical relationships between macro variables is often based on estimates of reduced-form equations such as non-structural VAR systems even if there often are disagreements about how to give structural interpretations to such relationships. It is therefore interesting to examine the implications of a reduced-form system derived from a particular model. If we substitute the instrument rule (8) into equations (2) and (3) we obtain the following reduced-form dynamic VAR type expressions for inflation and the output gap:

$$\begin{pmatrix} \pi_{t+1} \\ y_{t+1} \end{pmatrix} = h + A \begin{pmatrix} \pi_t \\ y_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{t+1} \\ \eta_{t+1} \end{pmatrix} \tag{14}$$

where

$$h \equiv \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \alpha_r g_\pi \pi^* \\ \beta_r g_\pi \pi^* \end{pmatrix} = \frac{1}{a_y} \begin{pmatrix} \lambda_0 (1 - c(\lambda, \lambda_0)) \pi^* \\ (1 - c(\lambda, \lambda_0)) \pi^* \end{pmatrix} \tag{15}$$

$$A \equiv \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} 1 - \alpha_r g_\pi & \alpha_y - \alpha_r g_y \\ -\beta_r g_\pi & \beta_y - \beta_r g_y \end{pmatrix}$$

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correlation with the residuals obtained from an OLS estimation of the output gap equation (3).

$$= \{\text{if } \lambda > 0\}^{7} = \begin{pmatrix} 1 & \gamma \\ 0 & 0 \end{pmatrix} - \frac{1-c(\lambda,\lambda_0)}{a_y} \begin{pmatrix} \lambda_0 & \lambda_0\gamma \\ 1 & \gamma \end{pmatrix} \tag{16}$$

where the parameters λ_0, γ and a_y are given by (11). Several interesting observations can be made. In contrast to the "structural" equation (2), the reduced-form equation for inflation has a constant term that captures the inflation target, π^*. Indeed, when α_r is non-zero the inflation target equals $(1-A_{11})/h_1$, which can be estimated. In practice such estimation can be problematic if π^* varies over time, a problem that is avoided in the case $\alpha_r = 0$. The coefficient for lagged inflation, A_{11}, in the reduced-form inflation equation is strictly less than unity when $\alpha_r > 0$, which is in accordance with empirical evidence even if it is often hard to reject a unit root (the case $\alpha_r = 0$).

More important is the observation that a positive value of α_r weakens the relationship between the inflation rate and the lagged output gap. This may explain the evidence of a flatter Phillips curve[8] as a result of a more counter-cyclical monetary policy (a higher value of the response coefficient g_y).[9] Indeed, in the case of strict inflation targeting (with $g_y = \alpha_y / \alpha_r$) the Phillips curve is dead, i.e. there exists no reduced-form relationship between inflation and the output gap. It is more surprising that the Phillips curve may become flatter when the policy maker puts more weight on output stabilization (higher λ). To realize this remember when α_r is sufficiently large (such that the variable γ, see (11), becomes negative) the response coefficient g_y is increasing in λ, and we see from (16) that the Phillips curve tends to be flatter the higher is the response coefficient g_y. Calculations based on parameters values used in Table 1 or Table 2 often produce a very weak or even negative reduced-form relationship between the inflation rate and the lagged output gap.[10]

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[7] This expression is correct also in the case when $\lambda = \alpha_r = 0$. When $\lambda = 0$ and α_r differs from zero the reduced-form system takes the form $\pi_{t+1} = \pi^* + \varepsilon_{t+1}$, $y_{t+1} = \lambda_0^{-1}\pi^* - \lambda_0^{-1}\pi_t - \lambda_0^{-1}\gamma y_t + \eta_{t+1}$

[8] In this paragraph the term Phillips curve refers to the popular empirical specification in which the inflation rate depends on the lagged output gap, i.e. equation (4) with $\alpha_r = 0$.

[9] Beaudry and Doyle (2000) provide empirical evidence that the Phillips curves for Canada and the U.S. have become flatter in the 1990s, and they also explain this result by changes in central bank behavior.

[10] Notice, however, that the estimate of α_y (which is crucial for the slope of the empirical Phillips curve) often is based on an equation where α_r is restricted to zero, which leads to a downward bias of α_y if α_r is positive.

The coefficient A_{21} is always negative implying that inflation is stable even if $\alpha_r = 0$. The presence of a negative inflation term in the reduced-form equation for aggregate demand does not mean that disinflation permanently will raise aggregate demand since it is the inflation rate relative to the target (π^*) that matters.[11] Finally we have that A_{22} is strictly less than unity but otherwise its sign and magnitude is undetermined. The sign of A_{22} is related to the problem of *oscillation* in the output gap that monetary policy models of this kind often exhibits, and once again we see from equation (16) that it is the sign of γ that matters. Notice that $\alpha_r = 0$ implies that $A_{22} < 0$, i.e. a positive shock to the output gap in period t will reduce the output gap in period $t+1$ and vice versa. To understand this let us consider a positive shock, η_t, to the output gap in a neutral macroeconomic state (i.e. $\pi_t = \pi^*$ and $y_t = 0$). This will give rise to an inflation forecast above the target two periods ahead if monetary policy is left unchanged. The only way to counteract these inflationary impulses is to tighten monetary policy such that a negative output gap arises in the following period. This is true even if the policy-maker cares a lot of output stability (λ large), but in this case the change in the output gap will be less drastic.[12] Moreover, the larger the output gap is in period t, the larger output gap of the opposite sign is needed in the subsequent period. This oscillating feature of the output gap that is implied by the optimal policy response is hard to reconcile with actual developments since the output gap appears to be a quite persistent process.

The oscillating behavior of the output gap can, however, disappear if we allow for a positive value of α_r such that the parameter γ becomes negative. For instance, the parameter values suggested in (13) together with the assumptions $\lambda = 0.5$ and $\alpha_r = 0.2$ give $A_{22} = 0.31$, implying some degree of persistence in the output gap. Once again the presence of a positive instrument term ($\alpha_r > 0$) in the inflation equation seems to be a step towards increased realism.[13]

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[11] It is possible to rewrite (13) in terms of the inflation gap as $z_{t+1} = A z_t + u_{t+1}$, where $z_t = (\pi_t - \pi^*, y_t)'$ and $u_{t+1} = (\varepsilon_{t+1}, \eta_{t+1})'$.

[12] When $\alpha_r = 0$ strict inflation targeting ($\lambda = 0$) implies $A_{22} = -1$ whereas under flexible inflation targeting ($\lambda > 0$) A_{22} approaches 0 below as λ tends to infinity (strict output targeting).

[13] It can be noticed that the presence of interest rate smoothing reduces the problems in the sense that it contributes to more persistence in the dynamic system. However, interest smoothing does not remove the unrealistic feature (of the model with $\alpha_r =$ 0) that a positive shock to the output gap in period t has a negative impact on the output gap in period $t+1$.

4. Conclusions

This paper analyzes optimal monetary policy rules when the central bank can influence inflation directly by a one-period lag as well as two-period lagged effect via the output gap (which the central bank can influence with a one-period lag). It is shown that direct inflation effects can be derived from a Phillips curve that allows for a contemporaneous relationship between inflation and the output gap but other arguments for direct inflation effects are also provided.

Closed-form expressions for optimal instrument rules are derived and it turns out that the introduction of direct inflation effects from monetary policy actions of modest magnitude has quite drastic implications that also are steps towards increased realism. The restricted model, in which changes in the instrument only affects inflation with a two-period lag, implies an unreasonably aggressive policy with an unrealistic oscillating behavior of the output gap.

The latter feature, which appears to be a typical implication of many small-scale monetary policy models, can be avoided by introducing one-period inflation effects from monetary policy actions. More specifically, it is shown that *only* if the one-period inflation effects from monetary policy actions are sufficiently large (i.e. if the parameter α_r in inflation equation is larger than some critical value) this will imply: (i) the output gap will exhibit persistence, i.e. a shock to the output gap in period t will increase the output gap in period $t+1$; (ii) The coefficient for the output gap (g_y) in the optimal instrument rule will be an increasing function of the weight for output stabilization (λ) in the central bank's loss function; and (iii) the reduced-form relationship between inflation and the lagged output gap will weaken the more the central bank cares about output stabilization. The first and second implications seem to be in line with intuition. The third implication, which is more surprising albeit a direct consequence of the second, also suggests that a more counter-cyclical monetary policy may have contributed to a flatter Phillips curve in recent years.

Finally, the paper demonstrates that the dynamic specification of the transmission mechanism appears to be a more crucial aspect than the weight the central bank attaches to output stabilization. This observation underscores the main message of the paper, viz. that the dynamic nature of the transmission mechanism and its implication for optimal monetary policy is a field of research that deserves more attention.

Appendix: Derivation of optimal policy

To solve for optimal policy in the dynamic optimization problem stated in equations (1)–(3) we make the following substitutions of variables:

$$z_t = \pi_t - \lambda_0 y_t - \pi^*, \quad \lambda_0 = \alpha_r / \beta_r \tag{A1}$$

$$r_t = i_t - \pi^* \tag{A2}$$

In terms of these new variables the optimization problem can be written as

$$\min_{\{r_{t+s}\}} (1-\delta)\sum_{s=0}^{\infty} \delta^s \left[z_{t+s}^2 + 2\lambda_0 z_{t+s} y_{t+s} + \lambda_1 y_{t+s}^2\right] \tag{A3}$$

s.t.

$$z_{t+1} = z_t + a_y y_t + e_{t+1} \tag{A4}$$

$$y_{t+1} = b_y y_t - \beta_r (r_t - z_t) + \eta_{t+1} \tag{A5}$$

where

$$\lambda_1 = \lambda + \lambda_0^2, \quad a_y = \alpha_y + \lambda_0 (1-\beta_y), \quad b_y = \beta_y + \alpha_r, \quad e_{t+1} = \varepsilon_{t+1} - \lambda_0 \eta_{t+1} \tag{A6}$$

Inspection of the optimization problem (A3)–(A5) reveals that it has almost the same mathematical structure as the problem analyzed by Svensson (1997). The only difference is that the term $2\lambda_0 z_{t+s} y_{t+s}$ has been added in the loss function. There are three cases;

Case 1: $\lambda > 0$. Inspired by the methodology in Svensson (1997) we first consider the following problem

$$V(z_t) = \min_{y_t} \left\{ \tfrac{1}{2}\left[z_t^2 + 2\lambda_0 z_t y_t + \lambda_1 y_t^2\right] + \delta E_t [V(z_{t+1})] \right\} \tag{A7}$$

$$\text{s.t. } z_{t+1} = z_t + a_y y_t + e_{t+1} \tag{A8}$$

where the output gap is considered as control variable. Next we assume that the indirect loss function, $V(z_t)$, is quadratic,

$$V(z_t) = k_0 + \tfrac{1}{2} k z_t^2 \tag{A9}$$

To determine the coefficients k_0 and k we examine the first order condition (using (A9))

$\lambda_0 z_t + \lambda_1 y_t + \delta a_y k z_{t+1|t} = \{\text{using (A8)}\} =$

$\lambda_0 z_t + \lambda_1 y_t + \delta a_y k(z_t + a_y y_t)$ implying that optimal control, $\hat{y}_t$, fulfills

$$\hat{y}_t = -\frac{(\lambda_0 + \delta a_y k)}{\lambda_1 + \delta a_y^2 k} z_t \tag{A10}$$

To identify k we use the envelope theorem, i.e.

$$V_z(z_t) \equiv k z_t \equiv z_t + \lambda_0 \hat{y}_t + \delta k z_{t+1|t} = \{\text{using (A8)}\} = (1+\delta k)\, z_t + (\delta a_y k + \lambda_0)\, \hat{y}_t$$

$$= \{\text{using (A10)}\} = \left[(1+\delta k) - \frac{(\delta a_y k + \lambda_0)^2}{\lambda_1 + \delta a_y^2 k}\right] z_t \qquad \text{(A11)}$$

Identification of the coefficient for z_t gives $k = \left[(1+\delta k) - \frac{(\delta a_y k + \lambda_0)^2}{\lambda_1 + \delta a_y^2 k}\right]$, which after rearrangements lead us to the following quadratic equation in k:

$$k^2 - \frac{1}{\delta a_y^2}\left(\delta a_y^2 - (1-\delta)\lambda_1 - 2\lambda_0 \delta a_y\right)k - \frac{\lambda}{\delta a_y^2} = 0 \qquad \text{(A12)}$$

Since we know that the product of the roots to the quadratic equations is $-\lambda/(\delta a_y^2) < 0$, there is one negative root and one positive. Since k must be non-negative the appropriate k is the positive root of the quadratic equation (A12):

$$k = k(\lambda, \lambda_0) = \frac{1}{2}\left[1 - \frac{(1-\delta)\lambda_1}{\delta a_y^2} - \frac{2\lambda_0}{a_y} + \sqrt{\left(1 - \frac{(1-\delta)\lambda_1}{\delta a_y^2} - \frac{2\lambda_0}{a_y}\right)^2 + \frac{4\lambda}{\delta a_y^2}}\right] \qquad \text{(A13)}$$

where we remember that $\lambda_1 = \lambda + \lambda_0^2$. For future reference we also notice that (A8) and (A10) together imply

$$z_{t+1|t} = c(\lambda, \lambda_0)\, z_t \qquad \text{(A14)}$$

where

$$c(\lambda, \lambda_0) = \frac{\lambda_1 - a_y \lambda_0}{\lambda_1 + k(\lambda, \lambda_0)} \qquad \text{(A15)}$$

A careful comparison with appendix B.2 in Svensson (1997) reveals that $k(\lambda, \lambda_0)$, $c(\lambda, \lambda_0)$, z_t and r_t play the same role as $k(\lambda)$, $c(\lambda)$, $(\pi_t - \pi^*)$ and i_t respectively in Svensson's analysis, and hence the optimal policy rule can be written as

$$r_t - z_t = f_z\, z_t + f_y\, y_t \qquad \text{(A16)}$$

where

$$f_z = \frac{1 - c(\lambda, \lambda_0)}{a_y \beta_r} = \frac{1 - c(\lambda, \lambda_0)}{(\alpha_y + \lambda_0(1-\beta_y))\beta_r},$$

$$f_y = \frac{b_y + 1 - c(\lambda, \lambda_0)}{\beta_r} = \frac{\beta_y + \alpha_r + 1 - c(\lambda, \lambda_0)}{\beta_r} \qquad \text{(A17)}$$

If we use (A1) and (A2) to express the optimal instrument rule in terms of i_t and $(\pi_t - \pi^*)$ instead of r_t and z_t it is straightforward to show that

$$i_t - \pi_t = g_\pi (\pi_t - \pi^*) + g_y \, y_t \quad \text{(A18)}$$

where

$$g_\pi = f_z, \quad g_y = \frac{\beta_y}{\beta_r} + \gamma g_\pi, \quad \gamma = \alpha_y - \lambda_0 \beta_y \quad \text{(A19)}$$

In the case $\alpha_r = \lambda_0 = 0$ we see that the optimal rule (A18) simplifies to the corresponding expression derived by Svensson (see equation (2.14) in Svensson (1997)).

Case 2, $\lambda = 0$, $\alpha_r > 0$. In this case the optimal rule is given by

$$i_t - \pi_t = g_\pi (\pi_t - \pi^*) + g_y \, y_t \quad \text{(A20)}$$

where

$$g_\pi = 1/\alpha_r, \quad g_y = \alpha_y / \alpha_r \quad \text{(A21)}$$

The intuition behind this rule is that under strict inflation targeting it should be optimal to control the expected inflation in the next period such that it coincides with the target, i.e. $(\pi_{t+1|t} = \pi^*)$. When $\alpha_r > 0$ the instrument rule according to (A20) delivers this. Formally it is also possible to come to this conclusion by following the analysis above, but in this case one has to select the solution $k = 0$ to the quadratic equation (A12), which is not necessarily the same root as in (A13). [14]

Case 3, $\lambda = 0$, $\alpha_r = 0$. In this case we are back to the model in Svensson (1997) and the optimal rule can be obtained by inserting $\lambda = \alpha_r = 0$ (implying $c(\lambda, \lambda_0) = 0$) in (A18).

[14] Inspection of (A13) reveals that $\lambda = 0$ implies a zero root only if $1-(1-\delta)\,\lambda_1 / (\delta a_y^2) - 2\,\lambda_0 / a_y \leq 0$.

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