UIP for short investments in long-term bonds*

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Abstract

The empirical failure of uncovered interest parity (UIP) is one of the best-established facts of international economics. The exchange rates of countries with high nominal interest rates tend to appreciate rather than depreciate as expected from UIP. However, virtually every published test of UIP studies short interest rates. In this paper, UIP is found to hold for carefully calculated returns to investments in long-term bonds and the US dollar – Deutsche mark exchange rate. For the corresponding short interest rates, the standard finding of a significantly negative relationship is confirmed. The results are explained in terms of a small macroeconomic model where the short interest rate is used as a monetary policy instrument to stabilise output and inflation.

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1. Introduction

Uncovered interest parity (UIP) is one of the most frequently tested propositions in international economics. According to the stylised facts, the coefficient β from regressing exchange rates changes on lagged interest differentials is significantly negative - several surveys point out –3 as a typical result (see for instance McCallum, 1994, or Engel, 1996). This finding contrasts blatantly with the +1 coefficient expected from the UIP hypothesis. A striking characteristic of the empirical literature on UIP is the exclusive focus on short interest rates. Before elevating the findings to stylised facts, *long* interest rates should also be investigated. Two recent papers that attempt to fill this void in the literature are Alexius (1998) and Meredith and Chinn (1998). Both study UIP for long investments in long-term bonds and find that the $\beta-$coefficients are typically positive but smaller than one. Similarly, Flood and Taylor (1997) run a UIP test for medium term (three-year) bonds and obtain a $\beta-$coefficient of 0.6.[1] Hence, the few existing results for long interest rates are much more favourable to UIP than the standard finding of a significantly negative coefficient. However, not only is more documentation of the relationship between long interest rates and exchange rates needed *per se*, but there are also serious problems with the data used in previous studies.

When UIP is tested for short interest rates, holding period returns to investments in the two currencies simply equal the short interest rates. Calculating returns to investments in long-term bonds is more complicated since a large part of the profits consists of coupon payments that are made before maturity. The main reason for the flagrant neglect of UIP for long interest

[1] Flood and Taylor (1997) is a general survey of exchange rate economics. Among other things, it contains a regression of three-year interest rates on corresponding exchange rate changes. Their point estimate of β is not significantly different from one or from zero. In contrast, the coefficients in Alexius (1998) and Meredith and Chinn (1998) are significantly positive but also significantly smaller than one.

rates is probably the lack of data on bond prices, coupon payments and term structures of interest rates needed to calculate true returns to investments in long term bonds. This information is certainly not available for the long time series used in Alexius (1998) and Meredith and Chinn (1998). Since coupon payments are normally larger when nominal interest rates are high, the positive regression coefficients documented in these studies could well be a consequence of systematic measurement errors in the data on long interest rates. Alexius (1998) attempts to remove the effects of coupon payments using two different methods: A rough approximation of zero coupon yields and equally rough calculations of the durations of the bonds. The results from testing UIP on the these two data sets and the original yields to maturity suggest that UIP fares better the more careful one is about calculating returns to investments. Meredith and Chinn (1998) do not attempt to take the presence of coupon payments on long term bonds into consideration.

A second source of measurement errors in the data on long investments in long-term bonds is the imprecise information about the true maturities of the bonds. An observation designated as a ten-year bond could well be an eight-, nine- or twelve-year bond. The combined effect of these two types of measurement errors could bias the results from the UIP tests in either direction. The positive correlation between the data on long interest rates and the coupon payments presumably works in favour of a positive regression coefficient. On the other hand, to the extent that the measurement errors are random, they tend to bias the $\beta -$coefficient towards zero.

Both Alexius (1998) and Meredith and Chinn (1998) study large data sets covering long time series for a large number of countries.[2] This paper takes the opposite route and focuses on a small amount of carefully constructed data. By calculating returns to *short* investments in long-term bonds, a large number of observations can be obtained using only the recent period where the availability of data is satisfactory. Even 40 years of data on ten-year bond investments contain only three independent observations. Here, we have almost 300 independent observations on weekly returns. UIP is also tested for corresponding data on short interest rates. Thereby, conclusions may be drawn about (i) whether UIP holds for long interest rates when the returns to investments are constructed rigorously and (ii) whether it is the length of the holding period or the maturity of the instrument that matters for the result.

There are two main explanations for why UIP could be expected to hold for long interest rates but not for short interest rates in tests using data on *ex post* exchange rate changes. It could be a consequence of the long investment horizons used in the previous studies of UIP for long interest rates. Holding periods in studies of short interest rates are always short – three months for studies of three-months interest rates and so on. If it takes time before fundamental relationships affect exchange rates, long investment horizons may be needed to discover that the fundamental UIP hypothesis holds. Flood and Taylor (1997) interpret their results in this manner: "… fundamental things apply as time goes by." Alternatively, the relationship between short interest rates and *ex post* exchange rate changes could be special and hence different from the relationship between long interest rates and *ex post* exchange rate changes. Short interest rates indeed differ from other financial assets in that they constitute the main monetary policy instrument in most industrialised countries with flexible exchange rates. Several authors have tried to explain the negative $\beta-$coefficients in terms of the endogenous

[2] Alexius studies 37 years of data on 14 countries. Meredith and Chinn use 25 years of data for 7 countries.

response of monetary policy to shocks. For instance, McCallum (1994), Meredith and Chinn (1998) and Kugler (2000) construct models including a UIP relationship holds ex ante, but the co-movements of short interest rates and exchange rates as the economy is hit by shocks generate negative $\beta -$coefficients in UIP tests using *ex post* data. If short interest rates are used as a monetary policy instrument in a manner that creates negative $\beta -$coefficients, UIP would hold for long interest rates but not for short interest rates because of the maturity of the instrument *per se* and not because of the length of the holding period. In this paper, a small macroeconomic model is developed along the lines of Meredith and Chinn (1998). The negative relationship between short interest rates and *ex post* exchange rate changes is generated as follows: A demand shock increases output and inflation. Monetary authorities respond by raising the short interest rate. The demand shock also induces an appreciation of the equilibrium exchange rate. This results in a negative correlation between short interest rates and *ex post* exchange rate changes and also a positive correlation between returns to investments in long term bonds and *ex post* exchange rate changes.

2. Data and empirical results

Data on returns to weekly investments in ten-year US and German government bonds have been constructed by Dahlquist, Hördahl and Sellin (1999). The interest rates are collected approximately at closing time of the European markets on Tuesdays (Wednesdays if Tuesdays are holidays). The presence of coupon payments is handled using the Nelson and Siegel (1987) approach. The short interest rates are rolling investments in overnight interest rates from the Riksbank's database. Matching data on the USD/DEM exchange rate is collected from the BIS database and the sample period is October 1993 to November 1998.

The standard test of UIP is to regress *ex post* exchange rate changes on interest differentials and investigate whether $[\alpha, \beta]$ equals $[0,1]$:

$$\text{(1)} \qquad \frac{s_{t+\tau} - s_t}{s_t} = \alpha + \beta\left(i_{t,t+\tau} - i^*_{t,t+\tau}\right) + \varepsilon_{t+\tau},$$

where s_t is the nominal exchange rate, $i_{t,t+\tau}$ is the nominal interest rate between *t* and τ, and τ is the holding period. Table 1 shows the results from applying GMM to (1) for the returns to investments in long term government bonds and for the short interest rates. The properties of the residuals are best studied in the regressions on weekly investments. Since the holding period and the data frequency coincide, these data are not overlapping. The LM tests indicate first order autocorrelation in the residuals from the UIP regressions for weekly investments in long-term bonds and short interest rates. The test statistics for long (short) interest rates are 10.80 (8.87), with p-values of 0.001 (0.003). The Ljung-Box tests for higher order autocorrelation (16 lags) are insignificant for both regressions (18.04 and 18.64, respectively), as are the LM tests for second and higher order autocorrelation. For holding periods above one week, data overlap and there will be $MA(\tau - 1)$ autocorrelation. The Engle (1982) test (not reported) also indicates some heteroscedasticity. The GMM estimation therefore allows for heteroscedasticity, $MA(\tau - 1)$ autocorrelation for holding periods above one week and first order serial correlation in the regressions using data on weekly investments.

Table 1: UIP tests for long term government bond yields and corresponding short interest rates

	α	β	R^2	$H_0 : \beta = 1$	$H_0 : [\alpha, \beta] = [0,1]$
Long interest rates					
1 week	0.000	0.215	0.024	119.658	120.264
	[0.014]	[2.991]		(0.000)	(0.000)
2 weeks	-0.000	0.258	0.038	76.547	78.682
	[-0.132]	[3.198]		(0.000)	(0.000)
4 weeks	-0.000	0.329	0.062	33.347	34.012
	[-0.135]	[2.831]		(0.000)	(0.000)
8 weeks	-0.001	0.563	0.151	5.800	6.374
	[-0.206]	[3.104]		(0.041)	(0.016)
12 weeks	-0.003	0.831	0.248	0.563	1.033
	[-0.350]	[3.694]		(0.453)	(0.597)
20 weeks	-0.002	0.843	0.251	0.412	0.499
	[-0.160]	[3.441]		(0.521)	(0.779)
30 weeks	0.002	0.931	0.284	0.061	0.061
	[0.077]	[3.336]		(0.805)	(0.970)
Short interest rates					
1 week	-0.000	-1.925	0.002	1.757	1.923
	[-0.330]	[-0.872]		(0.185)	(0.381)
2 weeks	-0.001	-2.900	0.010	4.057	4.221
	[-0.741]	[-1.498]		(0.044)	(0.121)
4 weeks	-0.004	-4.265	0.039	7.316	7.338
	[-1.315]	[-2.191]		(0.007)	(0.025)
8 weeks	-0.009	-4.884	0.096	9.080	9.219
	[-1.373]	[-2.501]		(0.003)	(0.010)
12 weeks	-0.016	-5.604	0.163	11.886	12.177
	[-1.643]	[-2.926]		(0.001)	(0.002)
20 weeks	-0.030	-6.364	0.312	16.699	17.142
	[-2.026]	[-3.531]		(0.000)	(0.000)
30 weeks	-0.048	-7.091	0.470	37.660	39.245
	[-3.358]	[-5.378]		(0.000)	(0.000)

t-values within brackets
p-values within parentheses

As shown in the second column of Table 1, most of the intercepts are insignificantly different from zero. Hence, neither currency has carried a time invariant risk premium over the sample period. The third column contains the slope coefficients. For weekly investments in long-term bonds, the point estimate of β is 0.21. It is significantly larger than zero but also significantly smaller than one. As the holding period is extended, the $\beta-$coefficient rises up to a maximum of 0.94 at investment horizons of 26 weeks. The strict version of the UIP hypothesis, $[\alpha, \beta] = [0,\ 1]$, cannot be rejected from 10 weeks and on. For horizons of 11 weeks or longer, β is above 0.8 and not significantly different from unity. These findings contrast blatantly with previous results from testing UIP on short interest rates. Not only is it confirmed that UIP appears to hold better for long interest rates than for short interest rates, but UIP is not rejected for these data on carefully calculated returns to short investments in long-term bonds. Furthermore, the results are not simply due to low power to reject the null hypothesis as the point estimates of the $\beta -$coefficient are close to unity.

For the short interest rates, the standard finding of a negative $\beta -$coefficient is confirmed. The absolute value of the coefficient increases with the investment horizon and approaches –7, which is larger in absolute value than the typical result. It is not significantly different from zero for weekly investments but becomes significant at four weeks. Figures 1a) and 1b) show the estimated β -coefficients and the five-percent confidence intervals as the investment horizon is increased from one to 32 weeks.

As shown in the fourth column of Table 1, interest differentials explain a minuscule proportion of exchange rate changes for weekly investments. R^2 is only 0.023 for the long interest rates and 0.002 for the short interest rates. However, as the investment horizon is increased, it approaches 0.3 for the long interest rates and 0.5 for the short interest rates. The simple UIP regressions hence have considerable explanatory power for exchange rate changes. In particular, almost half of the variation in exchange rate changes at the 30-week horizon can be explained by minus seven times the movements in the short interest differential. The negative relationship between short interest rates and *ex post* exchange rate changes remains a persistent finding and it is also quantitatively important.

Figure 1a: Point estimates of β and 95 percent confidence intervals for the long interest rates as functions of the investment horizon (number of weeks)

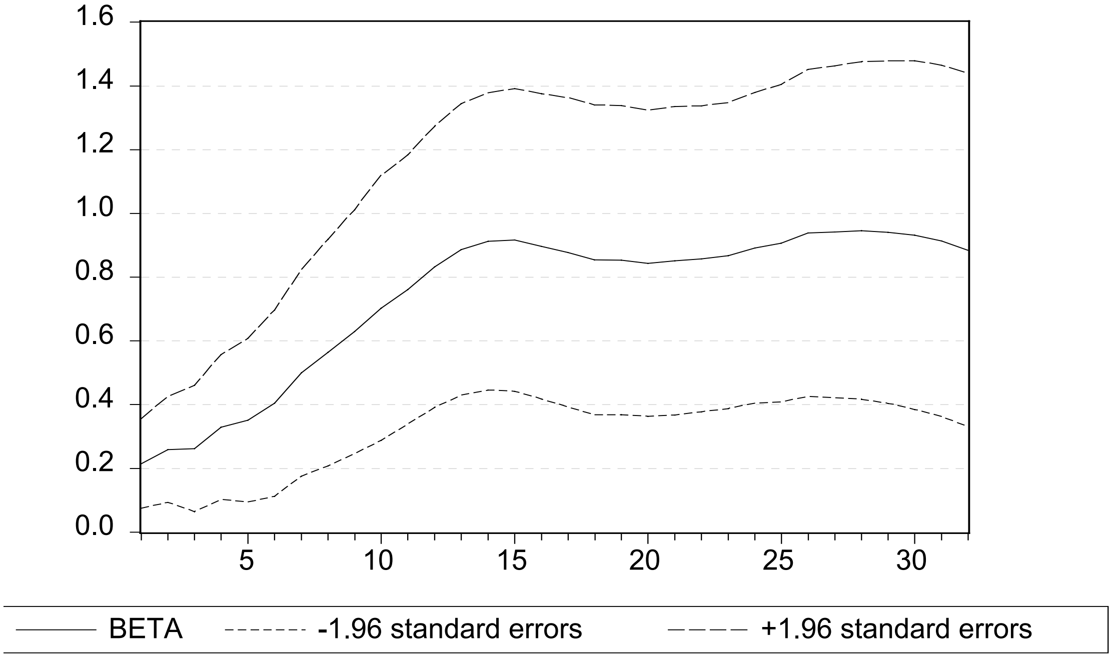
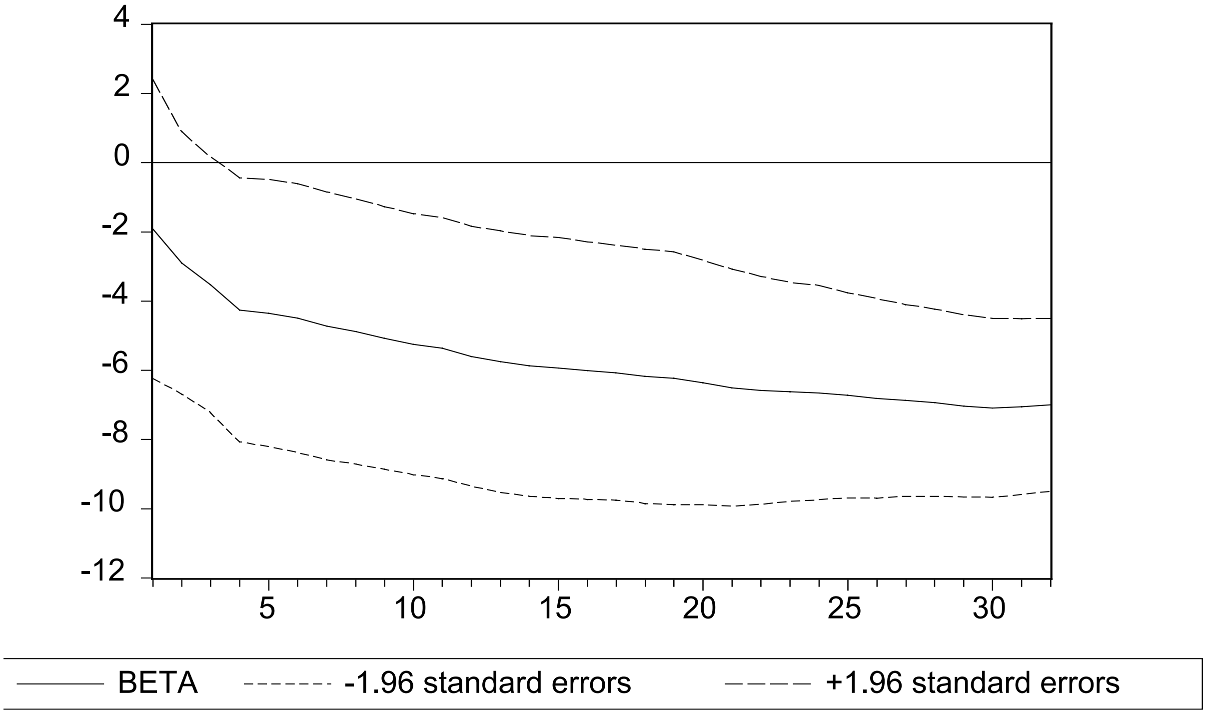


Figure 1b: Point estimates of β and 95 percent confidence intervals for the short interest rates as functions of the investment horizon (number of weeks)



3. Explaining the results

Traditional explanations for the empirical failure of UIP focus either on time varying risk premia, expectational errors and/or "peso problems" (broadly interpreted to include switches between appreciating and depreciating regimes for nominal exchange rates). The standard approaches do not appear likely to provide explanations for the present finding that UIP holds for short investments in long-term bonds while the $\beta -$coefficient is negative (and large) for the corresponding short interest rates.

An important characteristic of short interest rates is that they are used as the principal monetary policy instruments in most industrialised countries with flexible exchange rates. The approach with the greatest potential to explain the present puzzle appears to be models where a) a UIP relationship is included *ex ante* but b) the co-movements of short interest rates and exchange rates in response to shocks generate the observed negative relationship between short interest rates and *ex post* exchange rate changes. McCallum (1994) and Meredith and Chinn (1998) provide two examples of how such models can be constructed. Both models however contain some puzzling features. The model developed here avoids some of the pitfalls of McCallum (1994) and Meredith and Chinn (1998), but may well introduce some new ones.

McCallum (1994) showed that the relationship between interest rates and *ex post* exchange rate changes can be negative if the interest rate is used as a monetary policy instrument to stabilise the nominal exchange rate. He postulates a reaction function where monetary policy responds to shocks to the exchange rate risk premium. His reaction function has been criticised on theoretical as well as empirical grounds. Monetary authorities in major economies presumably care more about domestic developments (output and inflation) than about the nominal exchange rate. Empirical studies have also been unable to detect a significant policy response of the type modelled by McCallum (see for instance Mark and Wu, 1996).

Meredith and Chinn (1998) incorporate the key mechanism from McCallum's (1994) paper into a small macroeconomic model where output and inflation are functions of the real exchange rate. They postulate a more realistic monetary policy reaction function in the form of a Taylor rule, i.e., assume that monetary policy responds to movements in output and inflation rather than in the nominal exchange rate. They also include a long interest rate that is not used as a monetary policy instrument but determined by the expectations theory of the term structure. When the model is simulated, it yields co-movements of interest rates and exchange rate changes that are roughly consistent with their empirical findings. The average $\beta-$coefficient is –0.50 for the short interest rates and 0.82 for the long interest rates. The specific mechanism that generates the negative relationship between short interest rates and ex post exchange rate changes is rather complicated. A temporary increase in the risk premium initially depreciates the exchange rate. Inflation and output increase since they are functions of the exchange rate. The central bank raises the short interest rate to push output and inflation back towards their target levels. Hence, an exchange rate depreciation and a high short interest rate are observed in the first period. However, in the second period, the risk premium shock dissipates and the exchange rate appreciates. Inflation, output and the short interest rate fall. The exchange rate appreciation occurs in spite of the high lagged short interest rate, which is why β can be negative the for short interest rates. For the long horizons captured in UIP tests for long term investments in government bonds, the fundamental UIP relationship dominates.

The models in McCallum (1994) and Meredith and Chinn (1998) are driven by shocks to the UIP relationship. A main weakness of the models is the interpretation of these shocks. What are they and where do they come from? Sometimes they are interpreted as shocks to the exchange rate risk premium. But a true risk premium implies a higher *expected* return to investments in a currency. Furthermore, the existence of a risk premium *per se* is a contradiction of UIP. As the simulations of Meredith and Chinn demonstrate, the variance of these shocks has to be very large to generate the desired negative relationship between *ex post* exchange rate changes and short interest differentials. Models of the exchange rate risk premium have however been unable to generate substantial endogenous risk premia. A second weakness of the Meredith and Chinn (1998) model is that monetary policy reacts to the shocks to the risk premium because output and inflation are functions of the real exchange rate. It has however been difficult to document significant such effects for large economies and for the United States in particular.[3]

The present model is a different elaboration on the idea that the endogenous co-movements of the variables in response to shocks may generate a negative $\beta -$coefficient in *ex post* data even though UIP holds *ex ante*. There are two major differences compared to the model of Meredith and Chinn. UIP is defined as an *ex ante* relationship between expected exchange rate changes and expected interest differentials. Not only the nominal exchange rate but also the nominal interest rates are stochastic variables. This assumption is natural in the case of short investments to long term bonds, since such investments are indeed risky. It is perhaps less natural to assume that short interest rates are not known in advance. However, if unexpected movements in the exchange rate are to be negatively correlated with unexpected movements

[3] Indeed, the real exchange rate is excluded from the output and inflation equations even in MULTIMOD, from which Meredith and Chinn take their parameter values, excludes the real exchange rate from the output and

in the short interest rates, there must obviously be unexpected movements also in the latter. The data on returns to rolling investments in short interest rates in the previous section are consistent with the assumption that short interest rates are stochastic.

The mechanism that generates the negative $\beta -$coefficients in this model is simple and intuitive. Both monetary policy (i.e. the short interest rate) and the exchange rate respond to shocks. An unexpected demand shock increases output and inflation. The central bank reacts by raising the short interest rate. The higher output gap also appreciates the equilibrium exchange rate. Hence, high short interest rates and exchange rate appreciations are observed simultaneously, but this is not due to a causal relationship between the two variables. There are also supply shocks and exchange rate shocks. However, as will be shown in a moment, interest rates and exchange rates move in the same direction in response to these shocks. The mechanism that generates negative $\beta -$coefficients in Meredith and Chinn (1998) hence does not go through in this model.

The model consists of six equations. Subindex $t, t+\tau$ is used to describe the time period from t to $t+\tau$, where τ is the investment horizon. $\Delta s_{t,t+\tau}$ hence denotes the exchange rate realised at $t+\tau$ minus the exchange rate in *t*. This non-standard notation is used to avoid confusion as to how the variables are matched. For instance, Δs_t normally denotes $s_t - s_{t-1}$, while the same time index *t* in the case of interest rates, i_t, denotes the interest rate from *t* to *t+1*. All variables are defined as the domestic variable minus the foreign variable. Foreign variables are assumed to be constant throughout this section.[4]

inflation equations of the US since it is insignificant (Masson, Symanski and Meredith, 1990).

[4] This assumption is fairly innocent as long as domestic shocks and foreign shocks are independent. Since the exchange rate would move also in response to foreign interest rates and foreign output, the relationship between

The inflation rate depends on the output gap, lagged inflation, the real exchange rate and a supply shock:

(2) $$\pi_{t,t+\tau} = \rho^{\pi}\pi_{t-\tau,t} + \alpha^{y} y_{t,t+\tau} + \alpha^{q} q_{t,t+\tau} + \varepsilon^{\pi}_{t,t+\tau}$$

The output gap is a function of the real interest rate, the lagged output gap, the real exchange rate and a demand shock:

(3) $$y_{t,t+\tau} = \rho^{y} y_{t-\tau,t} + \delta^{i}(i_{t-\tau,t} - \pi_{t-\tau,t}) + \delta^{q} q_{t,t+\tau} + \varepsilon^{y}_{t,t+\tau}$$

The two parameters linking the real exchange rate to inflation and output, α^{q} and δ^{q}, are included only for comparison to the Meredith and Chinn (1998) model. As discussed above, it has been difficult to document significant effects of the real exchange rate on output and inflation for the US in particular.

The equilibrium real exchange rate depends on the output gap:

(4) $$q_{,t+\pi} \equiv s_{t,t+\tau} + p^{*}_{t,t+\tau} - p_{t,t+\tau} + \varepsilon^{q}_{t,t+\tau} = -\gamma y_{t,t+\tau} + \varepsilon^{q}_{t.t+\tau}$$

Such a short run relationship is used in the so called DEER (Desired Equilibrium Exchange Rate) and FEER (Fundamental Equilibrium Exchange Rate) approaches to the equilibrium real exchange rate frequently used by the IMF and various central banks. There is also a large

domestic interest rates and the exchange rate would be weaker (β would be lower, as would R^2) if foreign literature on the relationship between relative productivity and the real exchange rate, but it is less relevant here since it focuses on long run productivity trends in output rather than output gaps.

The real short interest rate is set according to a Taylor rule, i.e. as a linear function of inflation and the output gap:

(5) $$i_{t,t+\tau} - E_{t-\tau}\pi_{t,t+\tau} = \chi^{\pi}\pi_{t,t+\tau} + \chi^{y} y_{t,t+\tau}$$

The long interest rate (yield to maturity, ytm_t) on a bond with maturity T is determined by expected future short interest rates:

(6) $$ytm_t = \frac{1}{T} E_t\left(i_t^S + i_{t+1}^S + \ldots + i_{t+T}^S\right).$$

For simplicity, there are no coupon payments. Bond prices and returns to investments in bonds of maturity T, $i_{t,t+\tau}^L$, are given by (7):

(7) $$P_t = \frac{1}{\left(1 + ytm_t\right)^T},\ i_{t,t+\tau}^L = \frac{P_{t+\tau} - P_t}{P_t}$$

The timing is as follows. First, expectations about the exchange rate change and the nominal interest rates are formed. Then, the shocks are realised. Monetary policy (the short interest rate) is chosen after the central bank has observed the shocks. Inflation, output and the

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variables were explicitly modelled as well.

exchange rate are realised. The term structure adjusts as soon as the market has observed the short interest rate.

UIP is interpreted to imply that expected returns to all investments are equal:

$$E_{t-1}\left[\Delta s_{t,t+\tau}\right] = E_{t-1}\left[i^{S}_{t,t+\tau} - i^{S*}_{t,t+\tau}\right] = E_{t-1}\left[i^{L}_{t,t+\tau} - i^{L*}_{t,t+\tau}\right] \tag{8}$$

By assuming that expected returns to investments in domestic bonds, domestic short interest rates, foreign currency bonds and foreign short interest rates are equal, several types of risk premia and term premia are set to zero. For instance, Svensson (1993) identifies an exchange rate risk premium, an inflation risk premium, a forward term premium, a rollover term premium and a holding period term premium. Disregarding these premia is motivated by the belief that they do not contain the key to understanding why UIP would hold for short investments in long term bonds but not for the corresponding short interest rates. A different approach is explored here. Modelling risk premia as well would only complicate the analysis and obscure the main issue: Whether a small macroeconomic model with endogenous responses to shocks and endogenous monetary policy can explain the deviations from UIP.

As long as initial values are zero, UIP holds trivially since expected interest rate differentials as well as expected exchange rate changes are zero. Assuming that expected exchange rate changes and expected relative returns to short investments in long-term bonds are zero may be reasonable since they are notoriously difficult to predict.[5] However, *short* interest rate

[5] The return to short investments in long term bonds consists of two parts: The long interest differential and price changes due to movements in the long interest rate. The first part is known in advance but the second is not, since movements in the long interest rate are unpredictable. For the short investments discussed here, the second, unpredictable part is much larger then the first. Hence, returns to short bond investments can reasonably be assumed to be unpredictable.

differentials are highly autocorrelated. If the American short interest rate is higher than the German short interest rate today, it is likely to be so tomorrow as well. By allowing autocorrelation in the output gap, autocorrelated short interest rates can be generated in this model as well. The dynamics work in the direction of a positive $\beta-$coefficient since the exchange rate is expected to depreciate when the short interest rate is expected to be high. This will be discussed further below. For now, initial values of the output gap and the inflation rate are assumed to be zero.

The strategy of Meredith and Chinn (1998) is to calibrate and simulate their model to show that it is capable of generating negative $\beta-$coefficients. Here, the $\beta-$coefficients can easily be calculated using the expressions for the nominal interest rates and the nominal exchange rate. The $\beta-$coefficient in the simple UIP tests for short interest rates equals the covariance of interest differentials and the nominal exchange rate change divided by the variance of the interest differential:

$$\beta = \frac{\operatorname{cov}\left(\left(i_{t,t+\tau}^{S} - i_{t,t+\tau}^{S*}\right), \Delta s_{t,t+\tau}\right)}{\operatorname{var}\left(i_{t,t+\tau}^{S} - i_{t,t+\tau}^{S*}\right)} \tag{9}$$

The $\beta-$coefficient emerging from the model as interest rates and exchange rates move in response to unexpected shocks can be expressed as follows:

$$\beta = \frac{A_1 \operatorname{var}\left(\varepsilon_{t,t+\tau}^{\pi}\right) + A_2 \operatorname{var}\left(\varepsilon_{t,t+\tau}^{y}\right) + A_3 \operatorname{var}\left(\varepsilon_{t,t+\tau}^{q}\right)}{\operatorname{var}\left(i_{t,t+\tau}^{S} - i_{t,t+\tau}^{S*}\right)} \tag{10}$$

The constants A_1, A_2 and A_3 are functions of the model parameters, as is the expression for the variance of the short interest rate. First, it can be shown that A_3 is positive, implying that shocks to the exchange rate lead to movements in exchange rates and short interest rates of the same sign (i.e. the sign consistent with UIP).

(11) $$A_3 = \frac{\left(1+\alpha^q+\alpha^y\delta^q\right)\left(\alpha^q\chi^\pi+\delta^q\left(\alpha^y\chi^\pi+\chi^y\right)\right)}{\left(1+\delta^q\gamma\right)^2}\operatorname{var}\left(\varepsilon_t^q\right)$$

The finding that A_3 is positive contrasts to the model of Meredith and Chinn, where exchange rate shocks generate opposite movements in interest rates and exchange rates. However, the contemporaneous correlation is positive in their model as well: As the exchange rate depreciates, output and inflation increase and the central bank responds by raising the short interest rate. It is the particular dynamics that yields the negative relationship in their model. It is clear that the Meredith and Chinn effect is not present here.

Since it has been difficult to document a significant relationship between the real exchange rate on one hand and inflation and output on the other for large economies, α^q and δ^q are set to zero from here and on. They complicate the calculations substantially and were included only to enable comparisons to the Meredith and Chinn (1998) model. Allowing the real exchange rate to affect inflation and output complicates the analysis of the other effects considerably while adding little of interest. Hence, for α^q and δ^q equal to zero, we have:

(12) $$A_1 = \chi^\pi$$

(13) $$A_2 = \left(\alpha^y\chi^\pi+\chi^y\right)\left(\alpha^y-\gamma\right)$$

and

(14) $$\mathrm{var}\left(i^S_{t,t+\tau}\right) = \chi^{\pi^2}\,\mathrm{var}\left(\varepsilon^{\pi}\right) + \left(\chi^{y^2} + \delta^{i^2}\chi^{\pi^2} + 2\delta^i\chi^{\pi}\chi^{y}\right)\mathrm{var}\left(\varepsilon^y_t\right)$$

A_1 is obviously positive, implying that the covariance of interest rates and exchange rate changes is positive in response to supply shocks. A_2 is negative if the effect of output on the real exchange rate is larger than the effect of output on the inflation rate, i.e. if the nominal as well as the real exchange rate appreciates in response to a positive demand shock. The mechanism here is that as the output gap and hence inflation increase, the central bank raises the short interest rate while the equilibrium exchange rate appreciates due to the high domestic demand.

Returns to investments to long-term bonds always move in the opposite direction from unexpected movements in the short interest rate. The yield to maturity in (6) rises with the short interest rate for two reasons. First, the short interest rate $i^S_{t,t+\varsigma}$ enters with a weight of one over the maturity of the bond. Second, expected future interest rates also rise because output and inflation are positively autocorrelated. As yield to maturity rises, the price of the bond falls, as does the return to bond investments. The size of this effect depends on two factors working in opposite directions. The longer the maturity T of the bond is relative to the holding period τ , the smaller is the direct effect of the short interest in period *t*. A shock in period *t* also affects future output and inflation and thereby future short interest rates. This effect is larger the more autocorrelated output and inflation are.

Unexpected demand shocks hence generate a negative correlation between short interest rates and *ex post* exchange rate changes and a positive correlation between long interest rates and *ex* *post* exchange rate changes. In this model, the observed pattern where β is negative for short interest rates and positive for returns to investments in long term bonds would only be observed if demand shocks have dominated over the sample period.[6] The responses to supply shocks are inconsistent with the empirical findings.

Up to now, the discussion has focused on unexpected movements in the variables in response to unexpected shocks. *Expected* short interest differentials are positively correlated with expected movements in the exchange rate, i.e. work in favour of a positive $\beta-$coefficient. Ignoring the dynamics of the model hence helped isolating the mechanism capable of generating negative $\beta-$coefficients. However, if initial values are non-zero, there will also be expected interest rate differentials and expected exchange rate movements. For instance, if a positive output gap is observed in the previous period, the output gap is expected to be positive in period *t* as well. The central bank is then expected to set a high short interest rate to push down inflation and the output gap. Since the output gap is expected to diminish, the exchange rate is also expected to depreciate. The *signs* of the expected exchange rate movements and expected short interest rate differentials are hence consistent with UIP. However, the expected nominal depreciation from (5) is exactly equal to the expected interest differential in (2) only if the parameters obey the implied restriction.

In terms of the standard decomposition of the deviations from UIP into expectational errors on one hand and risk premia on the other, this model is a special case of the former. It is the correlation between unexpected movements in interest rates and unexpected movements in exchange rates that generates the negative $\beta-$coefficients.

[6] The required ratio of the variance of the demand shocks to the variance of the supply shocks can easily be calculated from (10) to (12).

4. Concluding remarks

A standard test of UIP has been performed for carefully constructed returns to short investments in long-term bonds and the corresponding short interest rates. For weekly investments in long term government bonds, the $\beta-$coefficient is significantly positive but small (0.21). As the investment horizon is increased, the point estimate rises up to a maximum of 0.94 for 26 weeks investments. It is above 0.8 for investments longer than 11 weeks. The hypothesis that β equals one cannot be rejected for horizons of 11 weeks or longer. The joint hypothesis $[\alpha, \beta] = [0,1]$ is not rejected from 10 weeks on. Hence, the standard test indicates that UIP holds for these data on short investments in long-term bonds. This result is in stark contrast to the well documented finding of a large and significantly negative $\beta-$coefficient. The latter result is confirmed for the short interest rates here as well.

Alexius (1998) and Meredith and Chinn (1998) study UIP for long interest rates and find that the $\beta-$coefficients are typically significantly positive but also significantly below unity. The results are explained in terms of the long horizon needed to capture the fundamental UIP relationship. Here, however, UIP is found to hold for *short* investments in long term bonds. It appears to be the maturity of the instrument *per se* and not the investment horizon that lies behind the now slightly less tentative finding that UIP holds better for long interest rates than for short interest rates.

A small macroeconomic model that is capable of generating negative $\beta-$coefficients for short interest rates and positive $\beta-$coefficients for long interest rates is developed along the lines of Meredith and Chinn (1998). It is the endogenous co-movements of monetary policy (i.e. the short interest rate) and the exchange rate as they respond to demand shocks that generates the negative relationship. Demand shocks increase output and inflation, which induce higher short interest rates and exchange rate appreciations. High short interest rates and exchange rate appreciations are therefore observed simultaneously, although this is not due to a causal relationship between the variables.

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