

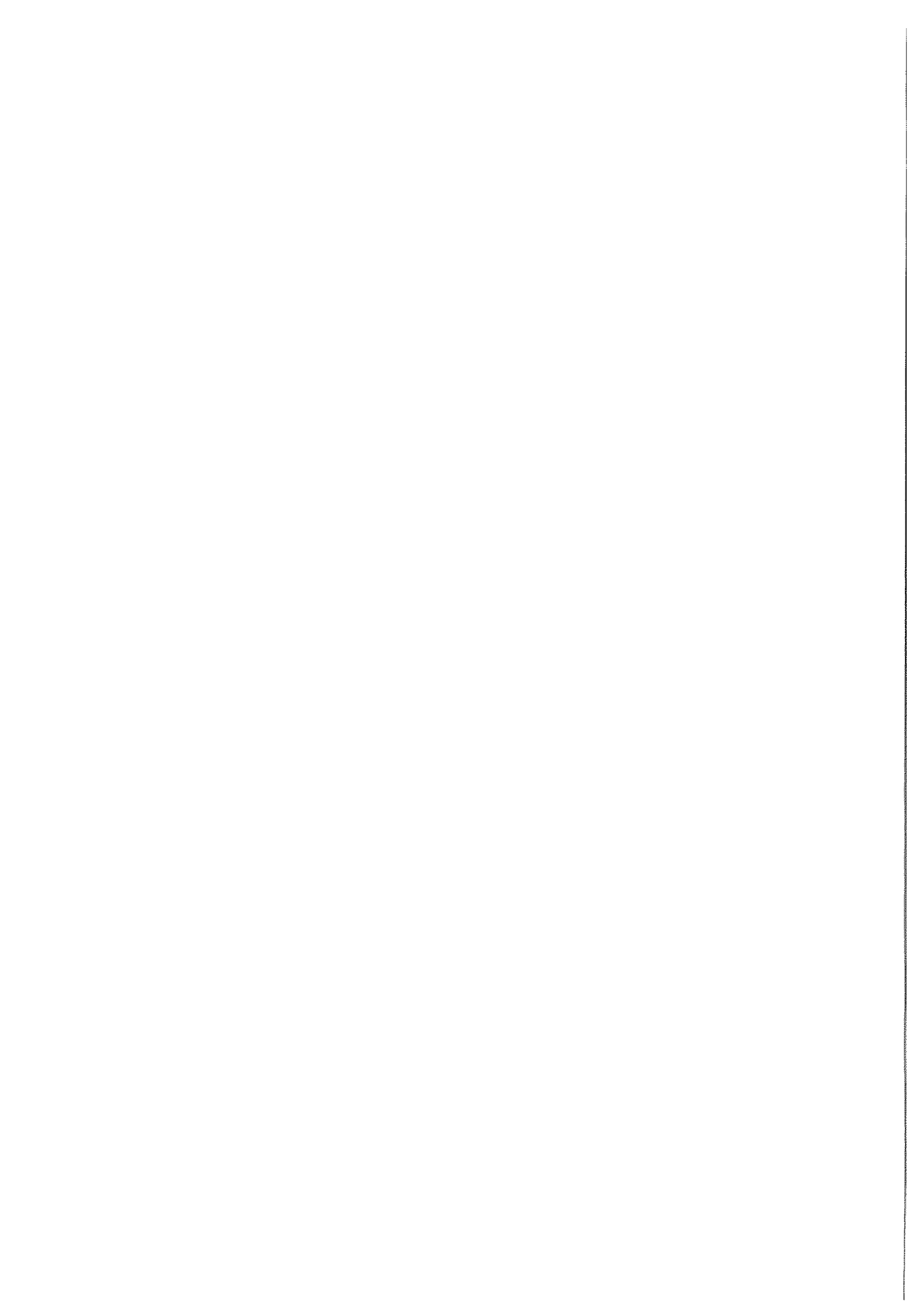
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Target Zone Models and the Intervention Policy: The Swedish Case

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ABSTRACT

The intervention policy of the Swedish Central Bank is studied using daily data on all intervention instruments during the late 1980s. In sharp contrast to the first generation Krugman target zone models, it is found that the interventions occur all over the exchange rate band and almost every day. To capture this feature this paper uses a model with continuous interventions that increase in size when the exchange rate moves from some preferred level, implying that interventions not only takes place at the boundaries. This gives a strong mean reverting behaviour of the fundamental. In addition, time varying devaluation expectations are included in the model. The model is then estimated using the method of simulated moments. The results indicate that this model captures the characteristics of Swedish exchange rate data better than the Krugman model. The estimated degree of mean reversion is substantial.

0. Introduction

The basic target zone model due to Krugman [1991] has been more or less rejected in a number of empirical tests. The Krugman model suggests a non-linear relationship between the exchange rate and some fundamental determinant of the exchange rate, with more pronounced non-linearity close to the boundaries of the target zone. Moreover, this model implies that the exchange rate distribution should be U-shaped. However, no strong evidence of non-linearities nor bi-modality in the exchange rates distribution has been presented. Lindberg and Söderlind [1992] suggest that this, in the Swedish case, could be explained by the presence of mean reverting intra-marginal interventions. The aim of this paper is to formulate a target zone model with a more realistic intervention rule and to study whether it fits Swedish data better than the Krugman model.

Section 1 summarizes some results from attempts to test the Krugman model. Section 2 provides a study of how Swedish intervention policy is conducted, using a unique data set of Swedish central bank interventions. The main conclusion is that there is evidence of exchange rate targeting within the band, exchange rate smoothing, and interest rate smoothing. Section 3 formulates a model with intra-marginal interventions which increases in size when the exchange rate moves from some preferred level — in addition to the Krugman interventions at the boundaries. Furthermore, time varying devaluation expectations are included in the model. In section 4, the model is estimated on daily Swedish data by the method of simulated moments. The results indicate that the model captures the characteristics of Swedish exchange rate data much better than the Krugman model. Finally, in section 5, our conclusions are summarized.

1. What is wrong with the Krugman model?

This section summarizes the Krugman [1991] model and some results from attempts to test the model.

The model starts with the standard asset price relation

$$(1.1) \quad s(t) = f(t) + \alpha E_t[ds(t)]/dt,$$

where $s(t)$ is the logarithm of the exchange rate at time t (that is, the price of the foreign currency in terms of the own currency), $f(t)$ the fundamental determinant of the exchange rate, α a positive parameter and E_t the time t expectations operator. The fundamental can be thought of as

$$(1.2a) \quad f(t) = m(t) + v(t),$$

where $m(t)$ is the logarithm of the money stock, which is controlled by the monetary authority, and $v(t)$ the sum of the logarithms of velocity and other macrovariables exogenous to the exchange rate. For notational convenience, the latter will henceforth be called just velocity. Velocity is assumed to be a Brownian motion,

$$(1.2b) \quad dv(t) = \sigma_v dW_v(t).$$

In (1.2b), σ_v is a constant instantaneous standard deviation and W_v a standard Wiener process.

The central bank keeps the exchange rate to a band, $\underline{s} \leq s(t) \leq \bar{s}$. This is done by restricting the fundamental to a band, $\underline{f} \leq f \leq \bar{f}$. One of the critical assumptions in the Krugman model is that the central bank only intervenes at the boundaries of the exchange rate (and the fundamental) band:

$$(1.3) \quad dm(t) = dL(t) - dU(t), \text{ where } \begin{cases} dL > 0 \text{ only if } f = \underline{f}, \\ dU > 0 \text{ only if } f = \bar{f}, \end{cases}$$

where dL and dU are infinitesimal interventions.

It can be shown that the the exchange rate function $s[f(t)]$ is

$$(1.4) \quad s[f(t)] = f(t) - 2\lambda \sinh[\lambda f(t)],$$

where $\sinh(z)$ denotes the hyperbolic sine function $((e^z - e^{-z})/2)$ and the constant A remains to be determined. In a target zone A can be determined from the "smooth pasting" condition, which requires that $ds(\bar{f})/df=0$. This gives the target zone exchange rate function

$$(1.6a) \quad s[f(t)] = f(t) - \sinh[\lambda f(t)] / [\lambda \cosh(\lambda \bar{f})]$$

where $\cosh(z)$ denotes the hyperbolic cosine function $((e^z + e^{-z})/2)$ and

$$(1.6b) \quad \lambda = [2/(a\sigma_v^2)]^{\frac{1}{2}}.$$

The Krugman model has a number of testable implications. Among these, we note the following. First, the exchange rate is a non-linear function (S-shaped) of the fundamental and the slope is always less than one ("the honeymoon effect"). The non-linearity is more pronounced close to the target zone boundaries.¹ Second, the relation between the interest rate differential and the exchange rate is negative, and weaker for longer maturities.² As a consequence of these two facts, the degree of non-linearity in a prediction equation for the exchange rate decreases with the forecasting horizon. Third, the exchange rate distribution is U-shaped.³

The existence of non-linearities in a target zone context has been studied by several authors. Meese and Rose [1990] compare the in-sample prediction power of a locally weighted regression (LWR) and an OLS of a linearization of (1.6a), for, among others, some EMS currencies. The LWR is a non-parametric approach, which can be regarded as fitting a series of linear approximation to some smooth non-linear function. They use data on money supply and industrial production in order to construct a proxy for the fundamental (f). The results show no strong evidence of non-linearities. Diebold and Nason [1990] reach the same conclusion for some EMS currencies when comparing the out-of-sample prediction power of a random walk and an LWR of an AR(1) structure.

¹This is shown by Krugman [1991].

²This is shown by Svensson [1991b].

³This is shown by Svensson [1991a].

Flood, Rose and Mathieson [1990] cannot either find any out-of-sample evidence of non-linearities in EMS data using a parametric approach. Lindberg and Söderlind [1992] compare an AR(5) and an LWR for the Swedish krona for various out-of-sample forecasting horizons and periods. They find slight evidence of non-linearities during periods when the exchange rate was close to the target zone boundaries. All in all, there is only weak support for non-linearities.

The evidence on the distribution of the exchange rate is fairly clear in its rejection of the Krugman model. Flood, Rose and Mathieson [1990] study EMS currencies and find some but small evidence of bi-modality in the exchange rate distribution for the Belgian franc, Danish krone and French franc and no evidence at all for the Dutch guilders and Italian lira. In very few cases do they find any major clustering of observations close to the boundaries. Lindberg and Söderlind [1992] show that the shape of the distribution of the Swedish krona has some similarities with a normal distribution (and explicit tests cannot reject the hypothesis that the exchange rate distribution is in fact normal).

Any credible exchange rate band with some kind of mean reverting mechanism (that is, as long as the exchange rate band boundaries are not absorbing) should result in a negative correlation between the exchange rate and the interest rate differential. In the Krugman model that relationship is deterministic and the relation becomes weaker for longer maturities. The empirical evidence strongly rejects a deterministic relation between exchange rates and interest rate differentials. This is in itself a rejection of the Krugman model. The empirical evidence on the correlation sign is mixed. Flood, Rose and Mathieson [1990] find no clear pattern in the correlation between the interest rate differential (with 2 days to maturity) and the exchange rate for the EMS currencies. Svensson [1991] studies the Swedish krona and the interest rate differential for 1,3,6 and 12 months and finds negative correlations, with smaller absolute values for longer terms, which is in line with theory. On the other hand, in Lindberg and Söderlind [1992] we get the opposite result, using a somewhat different and longer sample of Swedish data. We

find positive correlations, which increase with longer terms. Kontulainen, Lehmussaari and Suvanto [1990] note that for the Finnish markka, the correlations are negative and more so with longer terms. All these mixed results highlight the importance of the credibility of the exchange rate policy. Rose and Svensson [1991] and Lindberg, Svensson and Söderlind [1991] attempt to quantify the devaluation expectations for the French franc and the Swedish krona, respectively. The results indicate that the devaluation expectations fluctuate substantially over time and that they are occasionally of significant magnitude.

We believe that the empirical performance of the Krugman model can be much improved by the explicit introduction of time-varying devaluation expectations and mean-reverting intra-marginal interventions. In order to motivate the introduction of intra-marginal interventions, we shall first present some empirical facts about the Swedish intervention policy.

2. Description of the Swedish intervention policy

In August 1977 Sweden withdrew from the then existing European system for exchange rate collaboration (known as the snake) and pegged the krona unilaterally vis-à-vis a trade weighted currency basket. The krona was devalued in November 1981 and October 1982. In June 1985 the bandwidth was officially declared to be $\pm 1.5\%$ around the benchmark value, the central parity. For the earlier period Sveriges Riksbank (the central bank of Sweden) claims to have been defending an unofficial zone of $\pm 2.25\%$. In May 1991 the Riksbank abandoned the currency basket and pegged the Swedish krona to the theoretical ecu. This measure was not accompanied by any realignment of the krona and the basic characteristics of the exchange rate system prevailed. The ecu peg is unilateral and the bandwidth is unchanged.

The Swedish exchange rate and credit market regulations were gradually dismantled

during the 1980s.⁴ Since December 1985 Sweden has relied on a market oriented system for the implementation of monetary and exchange rate policy. The key element of the institutional framework is a predetermined supply function for borrowed reserves, that is, discount window borrowing of the commercial banks from the Riksbank.^{5 6} The commercial banks have unlimited access to discount window borrowing. However, the marginal borrowing rate is increased, in a predetermined step by step fashion, when discount window borrowing rises. The amount of borrowing permitted for an individual bank at each marginal rate is related to the capital base of the bank. Moreover, the Riksbank uses lagged reserve accounting to fix the demand for reserves by the banks.⁷ Thus, to the extent that the demand for total reserves is sensitive to the interest rate it is due to the elasticity of the currency demand of the public. This means that the demand for total reserves is more or less insensitive to interest rate changes in the very short run, which implies that the supply of total reserves is neither an instrument nor an operative target for Swedish monetary policy.

In the Swedish case, the overnight interest rate is treated as the operative (intermediate) target for monetary policy whereas the supply of non-borrowed reserves is the principal monetary instrument. By adjusting the supply of non-borrowed reserves, using non-sterilized interventions in the foreign exchange market or the domestic market for government bills, the Riksbank is able to force the banks to borrow at the preferred marginal borrowing rate. As the demand for reserves by the banks is fixed and the banks have free access to discount window borrowing at an increasing rate, non-sterilized interventions can operationally be defined as central bank operations that affect the

⁴See, for instance, Englund [1990] for a review of the deregulation process.

⁵For a discussion of the institutions, targets and instruments in Swedish monetary policy see Hörngren and Westman—Mårtensson [1991].

⁶The similarities to the German system, described by Batten et al [1990], are obvious. A comparison between the Swedish, the US and the German system is made by Freedman [1990].

⁷The accounting period is one month and required reserves is adjusted with a two month lag.

amount of discount window borrowing and thus the marginal borrowing rate. The central bank has almost complete control of the overnight interest rate, as no bank is willing to pay more for overnight funds than the marginal rate offered by the central bank.⁸ By following this procedure the central bank is also able to influence interest rates on longer maturities and to control the exchange rate. Of course, the Riksbank is not able to control the exchange rate independently of the interest rates. A simplified balance sheet of the Riksbank and some definitions are provided in Appendix 1.

Let us now take a closer look at how Sveriges Riksbank has designed its intervention policy and make some comparisons with the intervention rule of the Krugman model (1.3). We will proceed by making a cursory visual inspection of the interventions across the exchange rate band and finally by looking at some simple stylized facts, that is, how interventions are correlated to exchange rates and interest rates. The intervention data, made available by Sveriges Riksbank, are daily and cover the period January 1988 to December 1990, that is, 746 observations. The data have been purged from a few observations coinciding with changes in the shape of the supply curve of borrowed reserves since these changes were accompanied by interventions that were made to offset any effects on the marginal borrowing rate. Interest rates, that is, Euro-deposit rates quoted at around 10 a.m. Central European time, were obtained from the Bank for International Settlements. The exchange rate is the official fixing rate at 10.40 a.m. in units of kronor per unit currency basket. Diagram 2.1 shows the exchange rate as the percentage deviation from central parity for the period January 1988 to December 1990. The exchange rate is ranging from -1.5 to 0.8 during this period and the mean is -0.6 .

In the very short run, the Riksbank stabilizes the exchange rate with the help of spot interventions in the foreign exchange market. The interventions in the spot market take place quite frequently. The Riksbank sterilizes some of the interventions with the

⁸See Englund et al. [1989] for an analysis of the operative characteristics of the system.

help of currency swaps, repurchase agreements and reverse repurchase agreements.⁹ During the sample period, the Riksbank acted as a net seller of foreign currency in 15% of the business days and as a net buyer in 33% of the business days, while the krona was at the (lower) boundary only during less than 1% (6 days) of the trading days. This is clearly not in line with the intervention rule in the Krugman model, where interventions are assumed to take place only at the boundaries of the exchange rate band.

Diagram 2.2 shows the intensity of the spot interventions across the exchange rate band as net purchase of foreign currency per day in 0.2% intervals of the band. Near the strong edge of the band, to the left in the diagram, the Riksbank buys foreign currency as expected. Moreover, net purchases are smaller closer to the center and for some intervals the Riksbank acts as a net seller. This supports the idea of a mean reverting policy rule with intra-marginal interventions. In that case, one would also expect to find net sales of foreign currency in the intervals close to the weakest position of the krona, but we actually observe the opposite, that is, large net purchases of foreign currency. These observations reflect two short periods of huge interventions in the market for government bills in February 1990 and October 1990, respectively. On both occasions, the interest rates were increased sharply by the Riksbank in response to a supply pressure on the krona. Consequently a demand pressure for Swedish kronor followed, which was in line with the aim to reverse foreign exchange flows. The appreciation of the krona was then partly counteracted by the Riksbank that bought foreign currency spot. This illustrates that other types of interventions than in the foreign exchange market are of importance for the control of the exchange rate.

The most important monetary instruments are repurchase and reversed repurchase agreements that the Riksbank makes with market makers in Swedish government bills.

⁹A repurchase agreement is a combination of a open market purchase and a forward contract to resell the assets at some future date. A currency swap is an agreement to sell (buy) foreign currency at one date and to buy (sell) it from (to) the same counterpart at a future date.

These agreements normally cover a period of one to seven days. During the sample period, repurchase or reversed repurchase agreements were made by the Riksbank in 30% of the business days. Together with the Riksbank's activity in the foreign exchange market, this gives the picture of a central bank that intervenes almost continuously. On some occasions the Riksbank also intervenes directly in the spot market for government bills, that is, makes open market operations. The change in the Riksbank's holdings of government bills across the exchange rate band are shown in Diagram 2.3.¹⁰ It is obvious that intra-marginal interventions is the rule rather than an exception. It is also worth noting that the Riksbank has decreased its holdings of government bills substantially in some of the weaker intervals of the band to defend the exchange rate.

The spot interventions in the foreign exchange market and the change in the holdings of government bills can be viewed as different components of dm in equation (1.3), while the change of non-borrowed reserves corresponds closely to dm itself.¹¹ Therefore, let us check how the supply of non-borrowed reserves has been managed by the Riksbank in order to get a better overall picture of the intervention policy. Diagram 2.4 shows the change in non-borrowed reserves across the exchange rate band. The diagram makes it evident that the Riksbank adjusts the amount of non-borrowed reserves to control the exchange rate intra-marginally.

In comparison with the Krugman model we must conclude that an intervention rule with nothing more than infinitesimal interventions at the boundaries of the exchange rate band is ill suited as a description of the Swedish intervention policy. It is obvious that the intervention rule should include intra-marginal interventions. However, the question is then how the intra-marginal interventions should be modelled. The choice of model should of course be based on the stylized facts. The inverted U-shape of the Swedish exchange

¹⁰The Riksbank's holdings of government bills are also influenced by overnight government borrowing at the Riksbank.

¹¹See appendix 1 for a definition of non-borrowed reserves.

rate distribution, rather than a U-shape as predicted by the Krugman model, indicates that fundamentals are mean reverting, which could arise from a mean reverting intra-marginal policy rule. So far, the evidence presented in diagrams 2.2–4 is mixed. Therefore, it might be fruitful to us to study this in more detail.

Table 2.1 shows the correlations between interventions and a selected set of variables. The significant negative correlations between the interventions and the first difference of the exchange rate are quite interesting, since they indicates that preferences for exchange rate smoothing may have influenced the intervention policy of the Riksbank. If there is no drift in velocity, an intervention rule that only includes mean reverting interventions would produce positive correlations to the first difference of the exchange rate. Therefore, we must conclude that there is evidence in favour of exchange rate smoothing. There is also a strong positive correlation between the change in non-borrowed reserves and the first difference of the 6-month interest rate. This indicates that preferences for interest rate smoothing, to some extent, influence the intervention policy of the Riksbank. This is not surprising, since interest rate smoothing is a well-known behaviour of central banks.¹²

¹²See, for instance, Batten et al. [1990].

Table 2.1 : Correlations between interventions and a set of macro variables.†

	Net spot purchase of foreign currency	Change in government bills holdings	Change in non-borrowed reserves
Exchange rate	0.03 (0.83)	-0.04 (0.22)	-0.01 (0.64)
First difference of exchange rate	-0.28 (0.00)	-0.04 (0.35)	-0.10 (0.03)
Interest rate differential ^a	0.28 (0.01)	-0.06 (0.22)	0.02 (0.37)
Interest rate ^a	0.19 (0.03)	-0.04 (0.21)	0.03 (0.26)
First difference of interest rate ^a	0.06 (0.45)	0.10 (0.15)	0.13 (0.06)

†The probability values within parentheses were estimated using the method Newey—West (1987) with 10 lags.

^a 6-month interest rates on Euro-deposits.

Now, let us check if there is any evidence of a mean reverting policy rule. The correlation coefficient between the level of the exchange rate and the change in government bills holdings is negative, which supports the idea of a mean reverting policy rule. However, there is a contradicting positive sign of the correlation between spot interventions, that is, net purchases of foreign currency, and the exchange rate. This is not of major importance since there is an overall support for a mean reverting policy rule due to the negative correlation between the change in non-borrowed reserves and the exchange rate. However, it is worth noting that none of the correlation coefficients to the level of the exchange rate are significant. This might indicate that the target level of the exchange

rate within the band has been changed during the sample period. However, exchange rate and interest rate smoothing add a lot of noise to the intervention data, which makes it more difficult to detect any distinct evidence of exchange rate targeting within the band in daily data. To check the latter possibility we used a data set ranging from January 1987 to December 1990 with monthly averages of the exchange rate and monthly interventions. The correlations to the exchange rate level turn out to be much stronger in monthly data. The correlation coefficient between net spot purchase of foreign currency and the exchange rate is -0.08 . The correlation coefficient between the change in non-borrowed reserves and the exchange rate is -0.16 . Thus, a study of monthly data strengthen the case for a mean reverting policy rule.

Thus, in the Swedish case, there are signs of preferences for exchange rate and interest rate smoothing, a sort of general "leaning against the wind" policy. It is also possible to argue that the Riksbank has a preferred exchange rate level in the band and to some extent tries to minimize deviations from that level, that is, to minimize the asymptotic (unconditional) standard deviation of the exchange rate. A number of intervention rules could most certainly be considered in light of these facts. However, we choose to make a very simple extension of the intervention rule in the Krugman model. We allow for continuous interventions that increase in size when the exchange rate moves from some preferred level, implying that interventions are made not only at the boundaries. This intervention rule is consistent with the observed correlations between interventions and the exchange rate level. We will also briefly discuss how the modelling of the intervention rule should be modified to account for the correlations between interventions and the first difference of the exchange rate.

3. Devaluation expectations and intra-marginal interventions

In Sections 1 and 2 we argued that there is plenty of evidence of devaluation

expectations and that the Riksbank has pursued a very active intervention policy. The effect of the former is a non-deterministic relation between the exchange rate and the interest rate differential, and for some periods, a positive correlation sign between the exchange rate and the interest rate differential. The effect of the latter is an exchange rate rate distribution with much more mass at the interior of the band than predicted by the Krugman model. To capture these features in a formal exchange rate model suitable for estimation, we propose a combination of the Bertola–Svensson [1990] model for time varying devaluation expectations and the mean reverting interventions discussed in, among others, Delgado and Dumas [1991].

A. Stochastic devaluation risk

This subsection presents a version of the Bertola–Svensson [1990] model for stochastic devaluation risk.

The logarithm of the exchange rate $s(t)$ can be decomposed into the exchange rate within the band $x(t)$ (approximately the percentage deviation from central parity) and the logarithm of the central parity $c(t)$

$$(3.1a) \quad s(t) \equiv x(t) + c(t).$$

In a similar fashion, we decompose the money supply $m(t)$ as

$$(3.1b) \quad m(t) \equiv n(t) + c(t).$$

The $n(t)$ component can be thought of as an instrument for governing the current exchange rate band. The $c(t)$ component, the central parity, is constant except during devaluations when it takes a jump. We assume that $n(t)$ and $c(t)$ are independent stochastic processes and that the width of the exchange rate band is constant. This means that $[\underline{s}(t), \bar{s}(t)] = [\underline{x} + c(t), \bar{x} + c(t)]$ holds, where $[\underline{x}, \bar{x}]$ is the constant band for $x(t)$, and $x(t)$ does not jump at a devaluation. Using this notation the asset pricing relation (1.1) can be written as

$$(3.2) \quad x(t) + c(t) = v(t) + n(t) + c(t) + \alpha E_t[dx(t) + dc(t)]/dt.$$

Let $g(t)$ denote the expected *rate* of devaluation,

$$(3.3a) \quad g(t) \equiv E_t[dc(t)]/dt,$$

and $h(t)$ a *composite* fundamental,

$$(3.3b) \quad h(t) = v(t) + n(t) + \alpha g(t).$$

This enables us to express (3.2) as

$$(3.4) \quad x(t) = h(t) + \alpha E_t[dx(t)]/dt,$$

which indeed looks similar to (1.1).

Similarly to (1.2b) we assume that $g(t)$ follows

$$(3.5) \quad dg(t) = \sigma_g dW_g(t),$$

where W_g is a standard Wiener process.¹³ Furthermore, we assume that $v(t)$ and $g(t)$ are independent, which means that the sum of $v(t)$ and $\alpha g(t)$ (the exogenous continuous processes) follows

$$(3.6a) \quad d[v(t) + \alpha g(t)] = \sigma dW(t),$$

where dW is a standard Wiener process and

$$(3.6b) \quad \sigma \equiv \sqrt{\sigma_v^2 + \alpha^2 \sigma_g^2}.$$

Note the formal similarity between (3.6a) and (1.2b). In the case with only marginal interventions, Bertola and Svensson [1990] show that the solution of the model for $x(t)$ specified by (1.3) and (3.3–3.6) is formally the same as in the Krugman model (1.6) with $h(t)$ and σ replacing $f(t)$ and σ_v . This means that the exchange rate band is now defined in terms of x and h instead of s and f .

B. Continuous and mean reverting interventions

This subsection formalizes the notion of continuous and mean reverting interventions by making use of the Ornstein–Uhlenbeck process, previously analyzed by, among others, Delgado and Dumas [1991]. The distributional effects of this intervention rule are derived

¹³Bertola and Svensson [1990] allow for both drift and non-zero correlation between the standard fundamental (velocity) and the expected rate of devaluation.

in Appendix 2.

According to (3.1b) interventions $dm(t)$ take two forms: occasional jumps in $c(t)$, that is, devaluations (of either sign) and infinitesimal changes in $n(t)$, denoted $dn(t)$, in order to keep $x(t)$ within the band $[\underline{x}, \bar{x}]$. The policy rule for devaluations is not an issue here, so we focus on $n(t)$.¹⁴

In addition to the Krugman interventions (1.3) we assume continuous interventions of increasing size as the exchange rate moves way from some preferred level. This can be formalized as

$$(3.7) \quad dn(t) = -\rho[h(t) - h_0]dt + dL(t) - dU(t), \quad \text{where} \begin{cases} dL > 0 \text{ only if } h = \underline{h} \\ dU > 0 \text{ only if } h = \bar{h}, \end{cases}$$

where \underline{h} and \bar{h} are the boundaries of the band for $h(t)$. In (3.7) h_0 corresponds to a preferred exchange rate level within the band $x_0 = x(h_0)$, and ρ is a constant positive policy parameter. Combining (3.6a) and (3.7), we find that in the interior of the band the composite fundamental follows the Ornstein–Uhlenbeck process

$$(3.8) \quad dh(t) = -\rho[h(t) - h_0]dt + \sigma dW(t).$$

Compared with the formulation in the Krugman model (1.2b) or the Bertola–Svensson model (3.6a), (3.8) means that we have introduced a variable drift parameter. The drift becomes negative when $h(t)$ exceeds the preferred value h_0 , and vice versa. This tends to make the $h(t)$ process mean reverting also without the interventions at the boundaries, that is, reverting to the value h_0 . At the boundaries, we have the infinitesimal marginal interventions as before.

It is shown in Appendix 2 that the density function for the composite fundamental is

¹⁴See Bertola and Svensson [1990] for a study of the statistical implications of various stochastic processes for $g(t)$, Lindberg, Svensson and Söderlind [1991] for an attempt to identify the driving variables of $g(t)$, and Edin and Vredin [1991] for an attempt to estimate the actual devaluation policy rule of the central banks in the Nordic countries.

$$(3.9) \quad \psi(h) = K \exp[-\rho(h-h_0)^2/\sigma^2], \quad h \in [\underline{h}, \bar{h}]$$

with the constant K determined by the requirement that the integral over $[\underline{h}, \bar{h}]$ equals unity. Hence, the distribution of the regulated Ornstein–Uhlenbeck process is simply a truncated normal distribution. In contrast, the fundamental in the case with a Brownian motion without drift is uniformly distributed on $[\underline{h}, \bar{h}]$.

Using (3.3b), (3.4), (3.7) and applying Ito's lemma, we have that the function for the exchange rate within the band $x[h(t)]$ must fulfill the second order differential equation

$$(3.10a) \quad y \frac{d^2 x(y)}{dy^2} + \left(\frac{1}{2} - y\right) \frac{dx(y)}{dy} - \frac{1}{2\alpha\rho} x(y) + \frac{h}{2\alpha\rho} = 0, \text{ with}$$

$$(3.10b) \quad y = \rho[h_0 - h(t)]^2/\sigma^2.$$

The homogeneous equation associated with (3.10a) is the so called Kummer's equation.

The general solution is

$$(3.11a) \quad x(h) = \frac{h + \alpha\rho h_0}{1 + \alpha\rho} + A M\left[\frac{1}{2\alpha\rho}, \frac{1}{2}, \frac{\rho(h_0 - h)^2}{\sigma^2}\right] + B M\left[\frac{1 + \alpha\rho}{2\alpha\rho}, \frac{3}{2}, \frac{\rho(h_0 - h)^2}{\sigma^2}\right] \frac{\sqrt{\rho}(h_0 - h)}{\sigma},$$

where A and B are two constants and $M(\cdot)$ is

$$(3.11b) \quad M(a, b, y) = 1 + \frac{ay}{b} + \frac{a(a+1)y^2}{b(b+1)2!} + \frac{a(a+1)(a+2)y^3}{b(b+1)(b+2)3!} + \dots,$$

the so-called Kummer's function.¹⁵

The constants A and B are determined by the smooth pasting conditions. Furthermore, the boundaries and the mean of the composite fundamental (h) can be determined from the boundaries (\underline{x}, \bar{x}) and the mean (x_0) of the exchange rate.¹⁶ Hence, $A, B, \underline{h}, \bar{h}$ and h_0 can be obtained from

$$(3.12a-e) \quad \begin{aligned} dx(\underline{h})/dh &= 0 \\ dx(\bar{h})/dh &= 0 \\ \underline{x} &= x(\underline{h}) \\ \bar{x} &= x(\bar{h}) \\ x_0 &= x(h_0) \end{aligned}$$

¹⁵See Abramowitz and Stegun [1972] or Pearson [1990].

¹⁶This solution has been derived by Froot and Obstfeld [1991] and Delgado and Dumas [1991].

The resulting exchange rate function will henceforth be called the regulated exchange rate within the band.

A freely floating exchange rate would amount to no interventions, which implies that $\rho=A=B=0$, and (3.11) becomes $x(h)=h$ and, by (3.1a), $s(t)=h(t)+c(t)$. The case with mean reverting interventions ($\rho>0$), but without any exchange rate boundaries (implying $A=B=0$) could be interpreted as a managed floating exchange rate, with $s(t)=[h(t)+\alpha\rho h_0]/(1+\alpha\rho)+c(t)$.

The density function of the exchange rate within the band can be found by applying the lemma on change of variable in density functions

$$(3.13) \quad \Psi(x) = \psi[x^{-1}(x)] \left| \frac{dx^{-1}(x)}{dx} \right| \quad x \in (\underline{x}, \bar{x}),$$

where $x^{-1}(\cdot)$ denotes the inverse of the exchange rate function (3.11) and $\psi(\cdot)$ the density function for the composite fundamental given in (3.9). Note that the density function is defined only in the interior of the exchange rate band due to the smooth pasting conditions. Unfortunately, there is no explicit form for this density function, but it can easily be calculated numerically.

The function for exchange rate within the band and the distribution are illustrated in Diagrams 3.1–3.4. For these diagrams, we have used the parameter values $\alpha=3$ years, $\sigma=0.1$ per $\sqrt{\text{year}}$, $\underline{x}=-1.5\%$, $\bar{x}=1.5\%$, $x_0=0$ and $c(t)=0$. In Diagrams 3.1–3.2 $\rho=1$ per year and in Diagrams 3.3–3.4 $\rho=3$ per year. In Diagrams 3.1 and 3.3, the freely floating exchange rate and managed floating exchange rate are also illustrated. It is clear that with these parameter values, the intramarginal interventions do much of the job to stabilize the exchange rate, since the managed floating exchange rate is very close to the regulated exchange rate. However, the managed floating exchange rate will eventually show extreme values since there are no boundaries.

The degree of mean reversion is also illustrated by Diagrams 3.5–3.6, which show the expected exchange rate within the band after 1,3,6 and 12 months, for $\rho=1$ per year and $\rho=3$ per year, respectively. These expected values for finite horizons are obtained by

solving Kolmogorov's backward equation, with suitable initial and boundary conditions, as discussed in Svensson [1991]. It is clear from Diagrams 3.5–3.6 that a higher degree of mean reversion (higher ρ) implies that the expected exchange rate within the band will be closer to the unconditional mean ($x_0=0$ in the diagram). Two things are worth noting. First, for high mean reversion ($\rho=3$) and a long (for instance, 12 month) forecasting horizons, the expected exchange within the band is virtually a constant. This is in line with the empirical findings in Lindberg, Svensson and Söderlind [1991]. Second, the expected exchange rate within the band is almost a linear function of the current exchange rate, which is consistent with the lack of nonlinearity found by, among others, Meese and Rose [1990], Diebold and Nason [1990], Flood, Rose and Mathieson [1990], Lindberg and Söderlind [1992] and Lindberg, Svensson and Söderlind [1991].

4. Estimating the Swedish exchange rate process

In this section, the model presented in Section 3 is estimated using daily Swedish exchange rate data for the period between June 27, 1985 and November 15, 1990.

A Estimation of $\{\alpha, \sigma, \rho\}$

The model summarized by (3.6b), (3.7) and (3.11) has 9 parameters $\{\alpha, \sigma_f, \sigma_g, \rho, \underline{h}, \bar{h}, h_0, A, B\}$. Since we use only exchange rate data for the estimation, the components σ_v and σ_g in (3.6b) are not identified, only σ is. In the next subsection we use interest rate differential data in order to give a rough estimate of the split of σ into σ_v and σ_g . For the moment, that leaves us with 8 parameters $\{\alpha, \sigma, \rho, \underline{h}, \bar{h}, h_0, A, B\}$, but we know that the boundaries for $x(t)$ are $[\underline{x}, \bar{x}] = [-1.5\%, 1.5\%]$, and we let the mean x_0 be estimated by the mean of the sample which is -0.63% . By exploiting the relations in (3.12a–e) the parameters $\{\underline{h}, \bar{h}, h_0, A, B\}$ are defined implicitly by $\{\alpha, \sigma, \rho\}$ and the knowledge of $\{\underline{x}, \bar{x}, x_0\}$.

Henceforth, the discussion will focus on estimating the parameters $\{\alpha, \sigma, \rho\}$ and it should be understood that this also involves solving for the implicitly defined parameters.

The simulated moments estimator (SME) for $\{\alpha, \sigma, \rho\}$ is given by minimizing

$$(4.1a) \quad G(\alpha, \sigma, \rho)'(\Sigma^*)^{-1}G(\alpha, \sigma, \rho),$$

where

$$(4.1b) \quad G(\alpha, \sigma, \rho) = 1/T^* \sum_{t=1}^{T^*} M_t^* - 1/T \sum_{t=1}^T M_t(\alpha, \sigma, \rho).$$

In (4.1a) $G(\cdot)$ is a $[q \times 1]$ vector of differences between empirical and simulated moments defined in (4.1b), where q is the number of moments used. In (4.1b) M_t^* and M_t are $[q \times 1]$ vectors of moment generating functions for empirical and simulated data, respectively, with T^* denoting the sample size of data and T the length of the simulations. Σ^* is the $[q \times q]$ covariance matrix of M_t^* , which is estimated using the method of Newey–West [1987]. This choice of the weight matrix gives the most efficient SME.¹⁷ Σ^* is estimated using 10 non-zero autocovariances, but since the correct number of autocovariances is unknown we will also report some results from a sensitivity experiment using 100 non-zero autocovariances. If we let $\mu_k(y)$ denote the k^{th} central moment of some variable y , then we have used the following 8 moments in (4.1): $\mu_2(x_t)$, $\mu_2(\Delta x_t)$, $\mu_4(x_t)$, $\mu_1(x_t x_{t-1})$, $\mu_1(\Delta x_t \Delta x_{t-1})$, $\mu_1(\Delta x_t \Delta x_{t-2})$, $\mu_2(\Delta x_t \Delta x_{t-1})$ and $\mu_2(\Delta x_t \Delta x_{t-2})$.

The algorithm for the SME used is as follows.

- (i) Generate a discrete approximation to the standard Wiener process $\{\Delta W_t\} = \epsilon_t \sqrt{\Delta t}$, where ϵ_t is drawn from a standard normal distribution and Δt is the fraction of a year of the sampling interval. In the simulations $\Delta t = 1/264$, which corresponds to daily observations. The sample length of simulations (T) is 11230, while the empirical sample length (T^*) is 1240.
- (ii) Generate a series of composite fundamentals $\{h_t\}$ from

¹⁷See Duffie and Singleton [1989] and Ingram and Lee [1991] who investigate the SME in detail.

$$(4.2) \quad h_{t+1} = \begin{cases} \bar{h} - (h_t + \sigma \Delta W_{t+1} - \bar{h}) & \text{if } h_t - \rho(h_t - h_0)\Delta t + \sigma \Delta W_{t+1} \geq \bar{h} \\ \underline{h} - (h_t + \sigma \Delta W_{t+1} - \underline{h}) & \text{if } h_t - \rho(h_t - h_0)\Delta t + \sigma \Delta W_{t+1} \leq \underline{h} \\ h_t - \rho(h_t - h_0)\Delta t + \sigma \Delta W_{t+1} & \text{otherwise,} \end{cases}$$

where h_0 is used as the initial value of h_t .

(iii) For each point $\{\alpha_i, \sigma_j, \rho_k\}$ in a three dimensional (subscripts i, j, k indicate the division in each dimension) grid generate an exchange rate series $\{x_t\}$ by (3.11).

(iv) The SME is given by the triple $\{\alpha_i, \sigma_j, \rho_k\}$ that minimizes (4.1).

All in all, 25050 different grid points has been used with the grid covering the domain $\alpha \in [0.15, 0.64]$, $\sigma \in [0.01, 0.05]$ and $\rho \in [1, 5]$.

Duffie and Singleton [1989] and Ingram and Lee [1991] show that, under some regularity conditions, the estimates of the parameters are consistent and normally distributed, with the covariance matrix

$$(4.3a) \quad (1 + \tau)(D_0'(\Sigma^*)^{-1}D_0)^{-1}/T^*,$$

where $\tau = T^*/T$ and D_0 the expected value of the Jacobian of $M_w(\alpha, \sigma, \rho)$, which can be estimated by

$$(4.3b) \quad \hat{D}_0 = \frac{\partial \left(\frac{1}{T} \sum_{t=1}^T M_s(\hat{\alpha}, \hat{\sigma}, \hat{\rho}) \right)}{\partial \{\alpha, \sigma, \rho\}}.$$

In practice, the derivatives in (4.3b) have been approximated by finite differences. One of the regularity conditions is that D_0 should have full column rank. This condition has been investigated numerically at different $\{\alpha, \sigma, \rho\}$ values including the point estimates reported below, and it seems to be fulfilled. Furthermore, the estimated differences between the simulated and the empirical moments $G(\hat{\alpha}, \hat{\sigma}, \hat{\rho})$ are also normally distributed with covariance matrix $(1 + \tau)\Sigma^*/T^*$. A measure of goodness of fit is

$$(4.4) \quad T^*G(\hat{\alpha}, \hat{\sigma}, \hat{\rho})'(\Sigma^*)^{-1}G(\hat{\alpha}, \hat{\sigma}, \hat{\rho}) \sim \chi^2(q-3),$$

which should be interpreted as that the hypothesis that the model is correctly specified

(able to fit data) should be rejected if the left-hand side exceeds the $\chi^2(q-3)$ -value at, say, the 5% significance level.

The estimates and the asymptotic standard errors are given in Table 4.1.

Table 4.1: Estimated parameter and asymptotic standard errors [†]				
	Empirical	Parameter	Std error	Significance level
α		0.353571	0.274451	0.197646
σ		0.031263	0.014027	0.025825
ρ		3.684211	0.376635	0.000000
A		0.000154	0.000161	0.337715
B		0.000145	0.000072	0.045518
\underline{h}		-0.031636	0.011568	0.006241
\bar{h}		0.045445	0.022010	0.038948
h_0		-0.006530		
$\mu_2(x_t)$	0.259393	0.255796	0.026014	0.890047
$\mu_2(\Delta x_t)$	0.007129	0.006369	0.000615	0.216397
$\mu_4(x_t)$	0.148594	0.157520	0.030642	0.770829
$\mu_1(x_t x_{t-1})$	0.255362	0.252612	0.025898	0.915452
$\mu_1(\Delta x_t \Delta x_{t-1})$	-0.000942	-0.000104	0.000383	0.028645
$\mu_1(\Delta x_t \Delta x_{t-2})$	-0.000048	-0.000066	0.000310	0.953147
$\mu_2(\Delta x_t \Delta x_{t-1})$	0.000049	0.000000	0.000024	0.039152
$\mu_2(\Delta x_t \Delta x_{t-2})$	0.000015	0.000001	0.000008	0.099340
Qfit		12.590532		0.027533

[†]The asymptotic standard errors for the implicitly defined parameters $\{h, \bar{h}, h_0, A, B\}$ have been calculated using the delta method. See, for instance, Judge et al. [1985]. In the table, x_t is measured as percentage deviation from central parity.

These results could, for instance, be compared with the results from estimating the Krugman model in Lindberg and Söderlind [1992], where we were unable to find a unique minimum of the loss function due to its flatness. The multiple minima were located along

a line passing through $\{\hat{\alpha}, \hat{\sigma}\} = \{1.03, 0.0028\}$ with an approximative slope of $\Delta\hat{\alpha}/\Delta\hat{\sigma} = 17000$. The introduction of the ρ parameter enable us to find a unique minimum of the loss function. The σ and ρ parameters are significantly different from zero at the 3% and 1% significance levels, respectively, while α only at the 20% significance level. Thus, the loss function is still somewhat flat in the α -dimension. The resulting function for the exchange rate within the band is illustrated in Diagram 4.1. It is indeed hard to detect the non-linearity, except very close to the exchange rate band boundaries. The results further indicate that λ is not significantly different from zero. This is interesting since in (3.11) λ multiplies the non-symmetric term. Hence, we cannot reject the hypothesis that the function is symmetric around $x_0 = -0.63\%$. Of course, we know that this is not literally true, but in practice it might have worked that way since the Riksbank intervened in order to keep the exchange rate at a certain distance from the upper band boundary. This is also seen in Diagram 4.2 which plots a kernel density estimate of the empirical density function and the density function implied by the point estimate.¹⁸ Notice that the empirical density is very low above 0.5% and virtually zero above 1%. The empirical and estimated density functions are located in about the same range and with most of the probability mass around the mean x_0 . However, the empirical density function has several peaks in the interior of the range: one around $x = -1\%$ and another around $x = -0.3\%$, in contrast to the estimated density which has one peak in the interior at $x = -0.6$ (by construction). One possible, model related interpretation, is that our sample might contain two (or more) regimes with different values of x_0 . Furthermore, the estimated density function shows a peak at the lower boundary. Since the smooth pasting conditions are in operation, there will always be such a tendency. We argued in conjunction with Diagrams 3.2 and 3.4 that this tendency can be reduced to being invisible (as in Diagram 3.4) if the the mean reversion in fundamentals is strong enough. Here we have the case that, even if the mean

¹⁸See Silverman [1982]. Here, the estimate is calculated at 1000 different points, using a Gaussian kernel and a window size of $1.06 \cdot \text{standard deviation} / \text{sample size}^{1/5}$.

reversion is very strong ($\rho=3.68$), the mean x_0 is fairly close to the lower boundary. Therefore, there is still a significant portion of probability mass of the composite fundamental close to the lower (but not the upper) boundary. In Diagram 4.2 this shows up as a peak, although a very narrow one, in the exchange rate density function.

The hypothesis that the simulated moments are equal to the empirical moments can only be rejected at the 5% significance for two of the moments, namely $\mu_1(\Delta x_t \Delta x_{t-1})$ and $\mu_2(\Delta x_t \Delta x_{t-1})$. This means that the model has some problems in capturing the first autocorrelation of the change in the exchange rate within the band. This is responsible for the fairly poor measure of fit on the last row of Table 4.1. At the 3%, but not the 2%, significance level it is possible to reject the hypothesis that the model fits the data. What is behind this result? In Section 2 we noted that there is evidence of an exchange rate smoothing behaviour of the Riksbank. To the extent that the smoothing (interventions) takes place instantaneously when a disturbance occurs, our model is able to capture this. To illustrate this point, rewrite (3.6a) as

$$d[v(t)+ag(t)] = (1+\gamma)\sigma dW(t),$$

and assume that (3.8) is as before. This implies that the interventions must be

$$dn(t) = -\rho[h(t) - h_0]dt + dL(t) - dU(t) - \gamma\sigma dW(t),$$

with dL and dU defined as before. Here, the instantaneous standard deviation of the exogenous process $v(t)+ag(t)$ has been increased with the factor $\gamma\sigma$, while the interventions are assumed to instantaneously counteract this.¹⁹ This leaves the evolution of $h(t)$ in (3.8) unchanged. Obviously, this will give a negative correlation between $dn(t)$ and $dx(t)$, as found for data in Table 2.1, but it can not explain the high negative autocorrelation of $dx(t)$ found in data. However, if some of the smoothing is carried out by lagged interventions, for instance taking place the day after the shock, then this will give a negative autocorrelation in $dx(t)$. Therefore, our partial conclusion is that even if

¹⁹As usual we leave all considerations about optimality of the intervention rule aside. In particular, we assume that the central bank, for some reason, chooses not to counteract all disturbances.

our model is consistent with instantaneous smoothing (which surely occurs in reality) it might still lack a mechanism for exchange rate smoothing which operates with a lag. It is also worth noting that if $\gamma = 0$, then the correlation between the exchange rate level, $x(t)$, and the interventions, $dn(t)$, is close to minus one. However, with $\gamma > 0$ this correlation tends to zero, because it is swamped by noisy smoothing intervention. Though, as shown in section 2, the negative correlation between the exchange rate and interventions is uncovered in time averages of data.

We have also run exactly the same algorithm but using an estimate of Σ^* based on 100 non-zero autocovariances. The estimated parameters $\{\alpha, \sigma, \rho\}$ (and standard deviations) are then 0.420 (0.300), 0.0347 (0.015) and 3.643 (0.691), respectively. Hence, the point estimates differ somewhat, but not significantly so. The fitted moments are correspondingly similar. The only marked difference is that the fit of $\mu_1(\Delta x_t \Delta x_{t-1})$ and $\mu_2(\Delta x_t \Delta x_{t-1})$ is now even more strongly rejected. Hence, the overall fit of the model can now be rejected at the 1% significance level.

Additional insight in the working of the model and how it behaves in relation to data and the Krugman model can be gained from a small Monte-Carlo experiment. Using the estimated $[\hat{\alpha}, \hat{\sigma}, \hat{\rho}]$, 100 samples with 1240 observations of the exchange rate within the band were generated and for each of these a number of statistics were computed. The initial values of the composite fundamental (h) were drawn from the distribution given by (3.9). Table 4.2 shows the average of these 100 samples. The counterparts for the Krugman model, from a Monte-Carlo experiment in Lindberg and Söderlind [1992], are given in column 2. Statistics for the official index series are given in column 3.

Table 4.2: Comparison simulated and empirical exchange rate[†]

	Simulated	Krugman	Empirical
Mean	-0.606 (0.171)	-0.004	-0.611
Cond std deviation ^a	0.079 (0.002)	0.018	0.082
Min	-1.472 (0.060)	-0.457	-1.515
Max	0.556 (0.334)	0.464	0.818
Skewness ^b	0.205 (0.297)	0.012	0.367
Normality ^c	47.959 (73.530)	92.417	67.264
Homoskedasticity ^d	26.245 (10.916)	17.287	78.708*
Unit root ^e	-3.168 (0.653)	-1.590	-3.013*

[†]Rejection on the 5% significance level, for skewness, normality, homoskedasticity and unit root tests, is denoted by *. These significance levels are only reported for data.

^aThe conditional standard deviation is for one step-ahead forecasts error based on a fifth order AR.

^bThe asymptotic distribution for the skewness measure (α_3) is such that $\alpha_3 \sim N(0, 6/T)$, where T is the number of observations, given that the observations are independent. But, here the observations are strongly autoregressive. Therefore, the 5% critical value has been calculated by means of a Monte Carlo experiment. This amounted to fitting an AR(5) to each series and sample. Simulating this AR(5) process (which is normally distributed) 250 times and calculating the statistics gives an estimate of the variance, which usually is larger than $6/T$.

^cBera—Jarque's test for normality. The asymptotic distribution for the statistics is $\chi^2(2)$, given that the observations are independent. Due to autoregression, the 5% critical value has been calculated by means of a Monte Carlo experiment (see ^b).

^dWhite's test of conditional homoskedasticity. The test statistics is asymptotically distributed as $\chi^2(m)$, where m is the number of quadratic terms in the auxiliary regression (here 10). See, for instance, Spanos [1986] for a description. The variables used in the auxiliary regression are 5 lags of the series studied.

^ePerron's [1988] $Z(t_\mu)$ test (using 25 lags). The statistics should be lower than -2.86 in order to reject the hypothesis of unit root (non-stationarity) on the 5% significance level. See Fuller [1976].

The introduction of a preferred exchange rate within the band x_0 does of course mean that the simulated distribution has a mean in accordance with data. The conditional

standard deviation (that is, the standard deviation of Δx_t conditional on x_{t-1}, x_{t-2}, \dots) is four times as high as in the Krugman model and now well in line with data. Since the negative mean is now fairly well fitted, the minimum values of the simulations are very close to the lower boundary. But, due to the high degree of mean reversion, the maximum is too low compared with data. In contrast to the simulations with the Krugman model, the present simulations show some weak evidence of skewness. As before and in accordance with data, there is only very weak evidence of non-normality and conditional heteroskedasticity. Furthermore, due to the increased mean reversion the hypothesis of a unit root (which is known to be false for the simulations) can now be rejected. The combined evidence of Tables 4.1 and 4.2 suggests that although the model is statistically rejected in Table 4.1, the introduction of a mean reverting composite fundamental helps us to fit a number of features (moments) of the exchange rate data much more closely than with the Krugman model. We regard this as a step in the right direction.

B Estimation of devaluation expectations and further identification of parameters

The existence of significant devaluation expectations for the Swedish krona has been documented by, among others, Lindberg, Svensson and Söderlind [1991]. The most obvious sign of devaluation expectations is a non deterministic relation and, during some periods, a positive correlation between the exchange rate within the band and the interest rate differential. Does this model improve upon the Krugman model which predicts a deterministic negative relationship between the exchange rate and the interest rate differential?

As shown in (3.6b) the existence of devaluation expectations affects the interpretation of the estimated instantaneous standard deviation of the composite fundamental $\hat{\sigma}$. In order to split the estimated $\hat{\sigma}$ into its components $\hat{\sigma}_v$ and $\hat{\sigma}_g$, we need more information about either velocity $v(t)$ or the expected rate of devaluation $g(t)$. The probably most efficient way to proceed is to use the estimated parameters together with

data on the exchange rate and the interest rate differential to form an estimate of the expected rate of devaluation $\hat{g}(t)$. Based on this estimate, an estimate of σ_g can be obtained, which by using (3.6b), $\hat{\sigma}$ and \hat{a} gives an estimate of σ_v .

Assuming uncovered interest rate parity, we have the expected rate of devaluation at time t during the period $[t, t+\tau]$, $g(t, \tau)$, as

$$(4.5) \quad g(t, \tau) = \delta(t, \tau) - E_t[x(t+\tau) - x(t)]/\tau,$$

where $\delta(t, \tau)$ is the interest rate differential between a τ -period riskless asset denominated in Swedish kronor and a similar asset denominated in the foreign currency basket. We use daily data on the Swedish official exchange rate index (expressed as a percentage deviation from central parity) and the interest rate differential on Euro-deposits with 1 month to maturity. Hence, in terms of (4.5) $\tau=1/12$ year, since we choose the time unit to be years (the interest rates are expressed as yearly rates).

By using the estimated coefficients from Table 4.1, the relation between $E_t[x(t+\tau)]$ and $x(t)$ can be estimated by solving Kolmogorov's backward equation (as was done in order to produce Diagram 3.6). The results are shown in Diagrams 4.3–4. The former shows the estimated expected exchange rate within the band after 1 month as a function of the current exchange rate within the band, while the latter shows the estimated expected rate of depreciation of the exchange rate within the band. It is worth noting that the expected change is zero when the current exchange rate is -0.6% , which corresponds to x_0 .

Diagram 4.5 displays the estimated expected rate of depreciation of the exchange rate within the band, constructed by applying the function in Diagram 4.4 on data for the exchange rate within the band. The interest rate differential is also shown. Finally, the estimated expected rate of devaluation over a one month horizon $\hat{g}(t, 1/12)$ is shown in Diagram 4.6. The diagram is similar to the empirical results in Lindberg, Svensson and Söderlind [1991] who formed an estimate of $E_t[x(t+\tau)]$ by invoking rational expectations and regressing $x(t+\tau)$ on $x(t)$ using ex post data.

According to the random walk assumption about $g(t)$ in (3.5) the theoretical finite

expected rate of devaluation $g(t, \tau)$ equals the theoretical instantaneous rate of expected devaluation $g(t)$.²⁰ It is then correct to treat our estimate of the finite $g(t, 1/12)$ as an estimate of the instantaneous $g(t)$, as well. Based on this, $\hat{\sigma}_g = 0.0588$ per (year)^{3/2}. Then since $\hat{a} = 0.35$ year and $\hat{\sigma} = 0.0313$ per $\sqrt{\text{year}}$ (3.6b) gives $\hat{\sigma}_v = 0.0233$ per $\sqrt{\text{year}}$. Will this produce the positive correlation on 0.197 between $x(t)$ and $\delta(t, 1/12)$ observed in data? Bertola and Svensson [1990] shows that the larger $a\sigma_g$ is relative to σ_v the higher is the correlation between the exchange rate and the interest rate differential. With σ_g high enough, the correlation can even become positive, in sharp contrast to the Krugman model. A simple Monte-Carlo experiment with 100 samples of length 1123 (as in our data) using these parameter estimates gives an average correlation of -0.018 with a standard deviation of 0.277. This is different from the data but not significantly so, and moreover it is definitely different from the prediction of the Krugman model. This is partial evidence of the ability of the Bertola-Svensson model of devaluations expectations to fit the data.

It is legitimate to ask whether the random walk assumption about $g(t)$ in (3.5) is reasonable. In this context it is interesting to note that a Perron [1988] $Z(t_\mu)$ test of the estimated expected rate of devaluation gives the test statistics -2.55 , which is larger than the critical value -2.86 . Hence, the hypothesis of a unit root (random walk) in the expected rate of devaluation cannot be rejected, which gives some support for the assumption in (3.5).

5. Summary and Conclusions

In the Swedish case, interventions occur all over the exchange rate band and almost every day. Thus, an appropriate interventions rule should include intra-marginal

²⁰The results in Lindberg, Svensson and Söderlind [1991] give some support for the notion that the expected rate of devaluation is equal across maturities.

interventions modelled as occurring on a continuous basis. The inverted U-shape of the empirical exchange rate distribution suggest that fundamentals are mean reverting and thus, possibly, that interventions occur in a mean reverting fashion. Our study of Swedish intervention data provides some support for this idea. Therefore, we formulated a model where interventions follows an Ornstein–Uhlenbeck process in the interior of the band, that is, continuous interventions that increase in size when the exchange rate moves from some preferred level, in addition to the interventions at the boundaries. Moreover, the presence of time-varying devaluation expectations is undoubtedly a salient feature of the Swedish krona and was therefore included in the model in a way proposed by Bertola and Svensson [1990]. The model was then estimated on Swedish data and its performance in relation to data and the Krugman model was evaluated in Monte–Carlo experiments.

The results indicate that a model with mean reverting intra-marginal interventions and time-varying devaluation expectations captures the basic characteristics of the Swedish exchange rate band much better than the Krugman model. First, the Bertola–Svensson model of devaluation expectations has the ability to explain the positive correlation between the exchange rate and the interest rate differential found in Swedish data. Second, the estimated degree of mean reversion is substantial. This explains the inverted U-shape of the Swedish exchange rate distribution and the difficulties to capture any non-linearities in the exchange rate.

However, the appropriate design of the intervention rule is not self-evident. Swedish intervention data indicate that exchange rate smoothing is of importance. In simulations our model has some difficulties in capturing the first autocorrelation of the change in the exchange rate. A partial conclusion is therefore that the model lacks a mechanism for exchange rate smoothing, which operates with a lag. Intervention data also suggest that preferences for interest rate smoothing influence Swedish intervention policy. An interesting topic for future research is to illustrate the dependence between policy preferences, intervention rule, and the behaviour of exchange rate and interest rates.

Another interesting topic is the role of sterilized interventions and signalling in an exchange rate band.

Appendix 1. The Balance Sheet of the Riksbank and the Market for Reserves

A simplified balance sheet of the Riksbank can be written as

Assets	Liabilities
FX	CU
GB	RR
BR	NW

where

- FX = foreign exchange reserves,
- GB = holdings of government bills (incl. other government borrowing),
- BR = borrowed reserves, i.e. discount window borrowing,
- CU = currency held by the public,
- RR = required reserves, and
- NW = the net worth of the Riksbank.

The supply of non-borrowed reserves NBR, is defined as

$$(A1.1) \quad NBR = FX + GB - NW.$$

Thus, the supply of total reserves R^s is

$$(A1.2) \quad R^s = NBR + BR.$$

The demand for total reserves is given by

$$(A1.3) \quad R^d = CU + RR.$$

The equilibrium condition in the market for reserves is

$$(A1.4) \quad R^s = R^d,$$

which we can write as

$$(A1.5) \quad NBR + BR = CU + RR.$$

Thus, we are able to write the change in non-borrowed reserves as

$$(A1.6) \quad \Delta NBR = \Delta CU + \Delta RR - \Delta BR.$$

Appendix 2. The asymptotic distribution of the regulated Ornstein–Uhlenbeck process

This appendix shows the derivation of the asymptotic distribution of the exchange rate, given that the fundamental follows an Ornstein–Uhlenbeck process with zero mean. The approach of the derivation follows that of Bertola and Caballero [1992].

The Ornstein–Uhlenbeck process with zero mean has the differential

$$(A2.1) \quad df(t) = -\rho f(t) dt + \sigma dz(t),$$

where z is a standard Wiener process, and we let f denote the fundamental. This process has the discrete time approximation

$$(A2.2) \quad f_{t+\Delta t} = \begin{cases} f_t + \Delta f & \text{with probability } q \\ f_t - \Delta f & \text{with probability } 1-q. \end{cases}$$

According to (A2.1) $E df(t) = -\rho f(t) dt$, which gives that the probability q must satisfy

$$(A2.3a) \quad -\rho f_t \Delta t = q \Delta f + (1-q)(-\Delta f),$$

which gives

$$(A2.3b) \quad q = \frac{1}{2}(1 - \rho f_t \Delta t / \Delta f).$$

For this approximation to converge to the process (A2.1) as $(\Delta f, \Delta t) \rightarrow 0$, the rates of convergence of $(\Delta f, \Delta t)$ must be such that the variance obeys

$$\sigma^2 \Delta t = q(\Delta f + \rho f \Delta t)^2 + (1-q)(-\Delta f + \rho f \Delta t)^2,$$

which by using (A2.3b) can be rearranged as

$$\sigma^2 = (\Delta f)^2 / \Delta t - \rho^2 f^2 \Delta t.$$

The last term will definitely go towards zero, as Δf and Δt do. The resulting requirement on the rates of convergence is that

$$(A2.4) \quad \sigma^2 = (\Delta f)^2 / \Delta t$$

should hold.

Now, to get to the state f (were the time index has been dropped for convenience),

we can either start in $f+\Delta f$ or in $f-\Delta f$. The probabilities in each of the two cases of reaching f is

$$\begin{cases} f+\Delta f: \frac{1}{2}[1 + \rho(f+\Delta f)\Delta t/\Delta f] \text{ (requires an increment of } -\Delta f) \\ f-\Delta f: \frac{1}{2}[1 - \rho(f-\Delta f)\Delta t/\Delta f] \text{ (requires an increment of } \Delta f). \end{cases}$$

The asymptotic (steady state) probability density function $\psi(f)$ for this discrete process must obey the the following equation around the state f

$$(A2.5) \quad \psi(f) = \psi(f+\Delta f)\frac{1}{2}[1 + \rho(f+\Delta f)\frac{\Delta t}{\Delta f}] + \psi(f-\Delta f)\frac{1}{2}[1 - \rho(f-\Delta f)\frac{\Delta t}{\Delta f}],$$

which can be rearranged to

$$(A2.6) \quad 0 = [\psi(f+\Delta f) - \psi(f)] - [\psi(f) - \psi(f-\Delta f)] + \frac{\rho f \Delta t}{\Delta f} \{ [\psi(f+\Delta f) - \psi(f)] + [\psi(f) - \psi(f-\Delta f)] \} + p \Delta t [\psi(f+\Delta f) + \psi(f-\Delta f)].$$

Multiplying (A2.6) with $1/(\Delta f)^2$ and evaluating the limiting expression as $(\Delta f, \Delta t) \rightarrow 0$, we have

$$(A2.7) \quad \frac{d^2[\psi(f)]}{df^2} + \frac{d[\psi(f)]}{df} \frac{dt}{(df)^2} - 2\rho f + \psi(f) \frac{dt}{(df)^2} - 2\rho = 0,$$

where dt and df denotes the limiting Δt and Δf . Using the relation (A2.4) and multiplying with $\sigma^2/2$, this can be written

$$(A2.8) \quad \frac{\sigma^2}{2} \frac{d^2[\psi(f)]}{d^2f} - \frac{d[-\rho f \psi(f)]}{df} = 0,$$

which is the Kolmogorov forward equation (in steady state).

Karlin and Taylor [1981] show the solution to this 2nd order differential equation. In short, the solution is found in the following standard way. Integrating (A2.8) once and multiplying with $2/\sigma^2$, gives

$$\frac{d[\psi(f)]}{df} + \psi(f) \frac{2\rho f}{\sigma^2} = C_1,$$

where C_1 is some constant. Now, multiply with the integrating factor $\exp(\rho f^2/\sigma^2)$, the resulting expression can be written

$$\frac{d[\exp(\rho f^2/\sigma^2)\psi(f)]}{df} = \exp(\rho f^2/\sigma^2)C_1.$$

Integrating once more and multiplying the result with $\exp(-\rho f^2/\sigma^2)$ gives the solution to (A2.8) as

$$(A2.9) \quad \psi(f) = \exp(-\rho f^2/\sigma^2) C_1 \int_0^f \exp(\rho \varphi^2/\sigma^2) d\varphi + C_2 \exp(-\rho f^2/\sigma^2).$$

As usual the density function $\psi(f)$ must fulfill the following two conditions

$$(A2.10a) \quad \int \psi(f) = 1,$$

where the integration is over the entire state space, and

$$(A2.10b) \quad \psi(f) \geq 0 \text{ for all } f \text{ in the state space.}$$

In the case with no boundaries, it can be shown that $C_1=0$ in order to ensure that (A2.10b) is fulfilled. Then (A2.9) is nothing but the normal density function (see Karlin and Taylor [1981]).

What if there are reflecting boundaries? Denote the lower and upper boundaries with \underline{f} and \bar{f} , respectively. At the upper boundary, the state \bar{f} can be reached either from \bar{f} itself, since a positive increment Δf would immediately be counteracted by an intervention of the same size, or from $\bar{f}-\Delta f$. The probabilities in each of the two cases of reaching \bar{f} is

$$\begin{cases} \bar{f} & : \frac{1}{2}[1 - \rho \bar{f} \Delta t / \Delta f] \quad (\text{an increment of } \Delta f \text{ counteracted by intervention}) \\ \bar{f}-\Delta f & : \frac{1}{2}[1 - \rho(\bar{f}-\Delta f) \Delta t / \Delta f] \quad (\text{an increment of } \Delta f). \end{cases}$$

Hence, around \bar{f} the asymptotic density function must satisfy

$$(A2.11) \quad \psi(\bar{f}) = \psi(\bar{f}) \frac{1}{2} [1 - \rho \bar{f} \Delta t / \Delta f] + \psi(\bar{f}-\Delta f) \frac{1}{2} [1 - \rho(\bar{f}-\Delta f) \Delta t / \Delta f],$$

which can be rearranged as

$$0 = [\psi(\bar{f}) - \psi(\bar{f}-\Delta f)] + \rho \bar{f} \Delta t [\psi(\bar{f}) + \psi(\bar{f}-\Delta f)] - \Delta t \rho \psi(\bar{f}-\Delta f).$$

Multiplying with $1/\Delta f$, using (A2.4) and evaluating the limiting expression as $(\Delta f, \Delta t) \rightarrow 0$, we have

$$0 = \frac{d[\psi(\bar{f})]}{d\bar{f}} + \frac{2\rho \bar{f}}{\sigma^2} \psi(\bar{f}) - \lim \frac{\Delta t \rho}{\Delta f} \psi(\bar{f}-\Delta f).$$

Since the rates of convergence are such that (A2.4) is fulfilled and that eventually $1/(\Delta f)^2 \gg 1/\Delta f$, we have that $\lim \Delta t/\Delta f = 0$. As a result, the boundary condition at the upper boundary is

$$(A2.12) \quad 0 = \frac{d[\psi(\bar{f})]}{d\bar{f}} + \frac{2\rho\bar{f}}{\sigma^2}\psi(\bar{f}),$$

and analogous at the lower boundary \underline{f} . This is a version of the result in Cox and Miller [1968].

Using (A2.9) in (A2.12) and simplifying gives the requirement $C_1=0$ (except in the degenerate case when $\underline{f}=\bar{f}=0$). Hence, the density function simplifies to

$$(A2.13a) \quad \psi(f) = C_2 \exp(-\rho f^2/\sigma^2),$$

with C_2 determined by the analogue of (A2.10a)

$$(A2.13b) \quad \frac{1}{\int_{\underline{f}}^{\bar{f}} \exp(-\rho f^2/\sigma^2) df} = C_2.$$

Hence, the distribution of the regulated Ornstein-Uhlenbeck process is simply a truncated normal distribution.

In the case with non-zero mean of the process

$$df(t) = -\rho[f(t) - f_0]dt + \sigma dz(t)$$

a simple change of variable establishes the distribution as,

$$(A2.14) \quad \psi(f) = K \exp[-\rho(f-f_0)^2/\sigma^2] \text{ with}$$

with the constant K determined by the requirement that the integral over $[\underline{f}, \bar{f}]$ equals unity.

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Diagram 2.1 Exchange rate

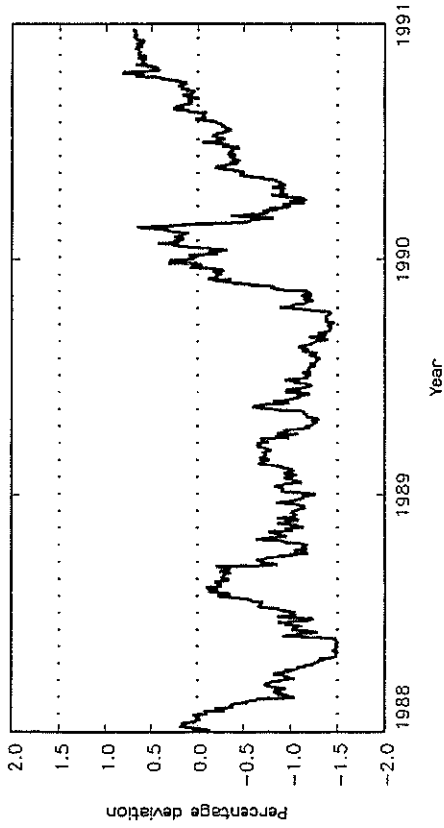


Diagram 2.2 Spot interventions

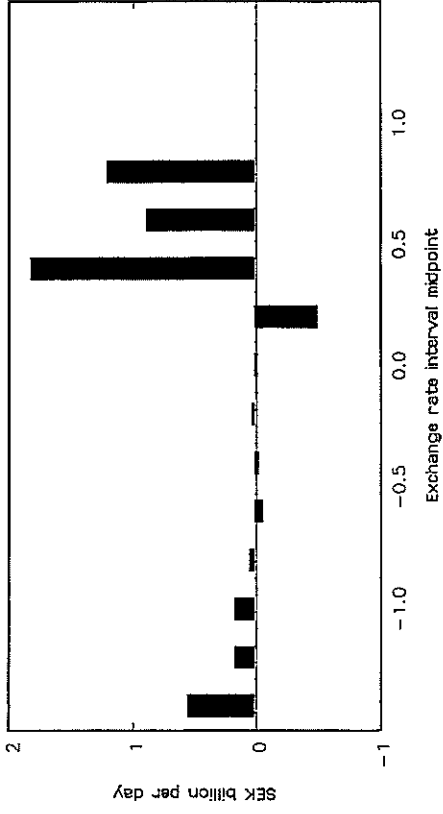


Diagram 2.3 Change in government bills holdings

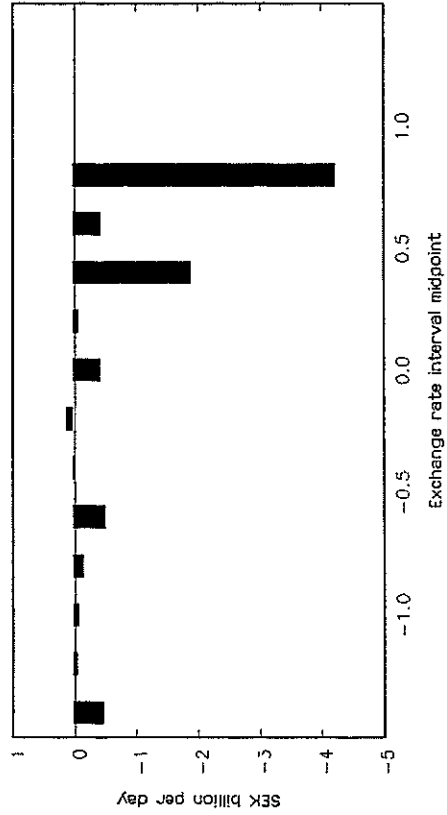


Diagram 2.4 Change in non-borrowed reserves

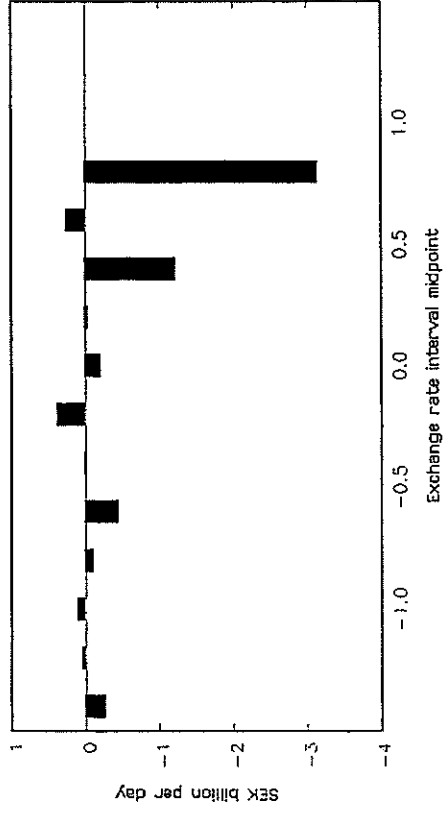


Diagram 3.1: $\rho=1$

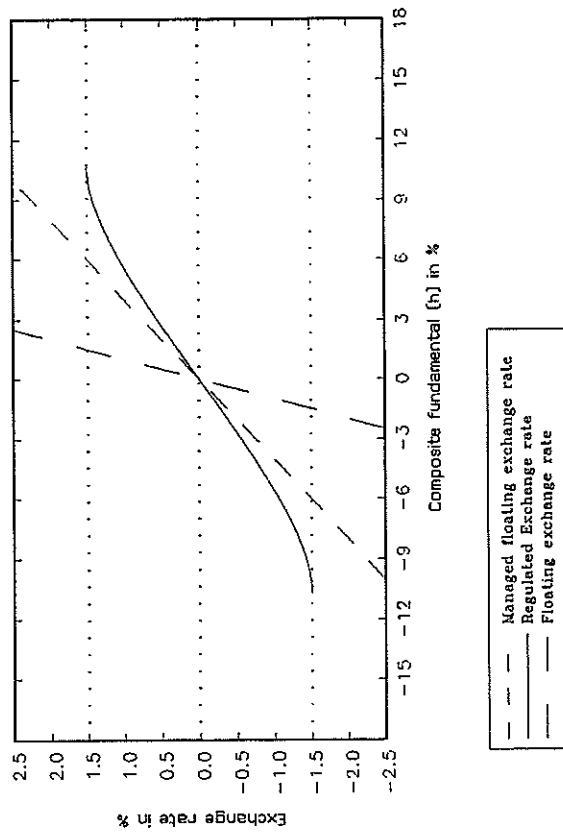


Diagram 3.2: $\rho=1$

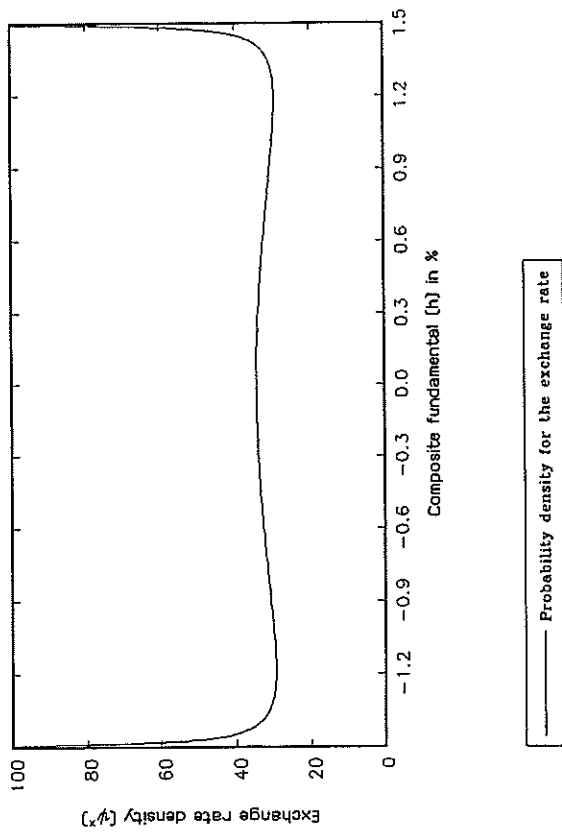


Diagram 3.3: $\rho=3$

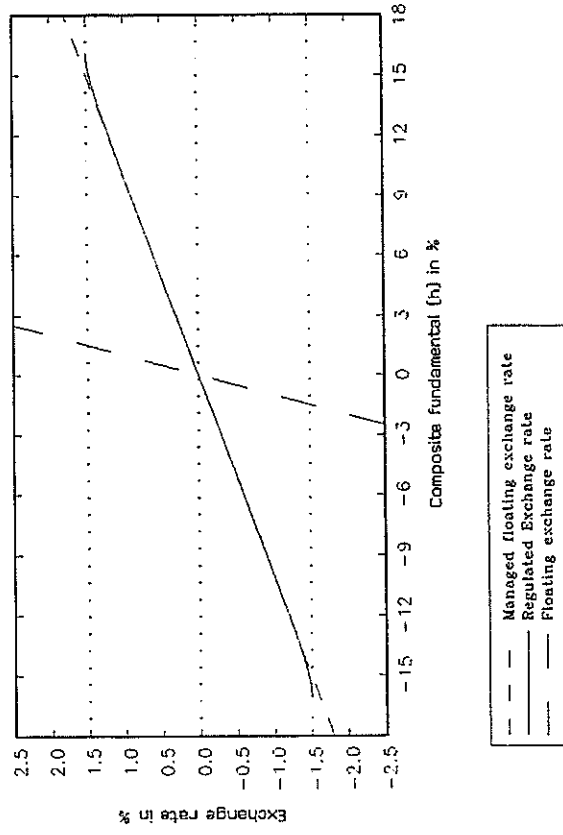


Diagram 3.4: $\rho=3$

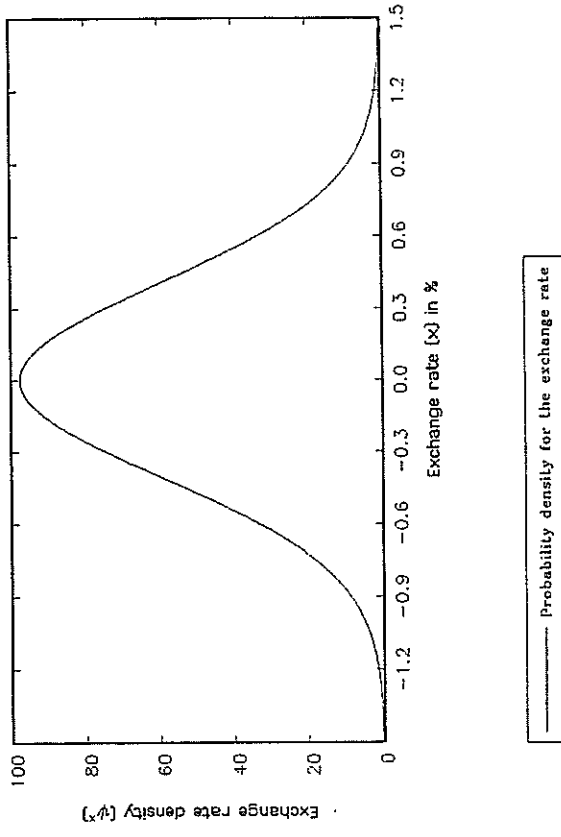


Diagram 3.5 Expected exchange rate: $\rho = 1$

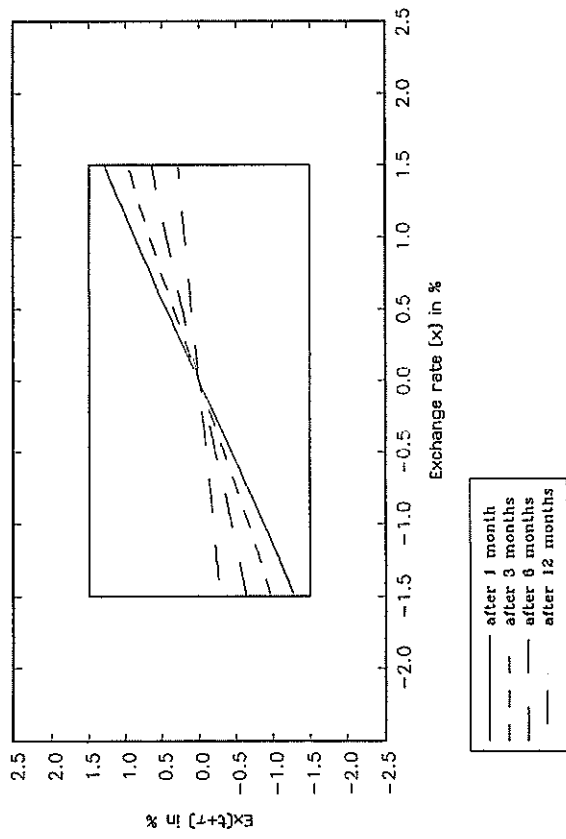


Diagram 3.6 Expected exchange rate: $\rho = 3$

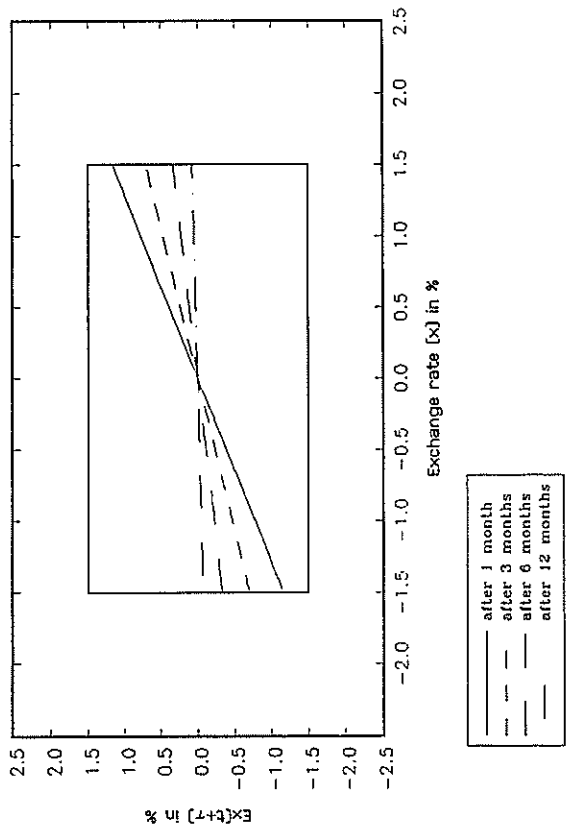


Diagram 4.1 Exchange rate function

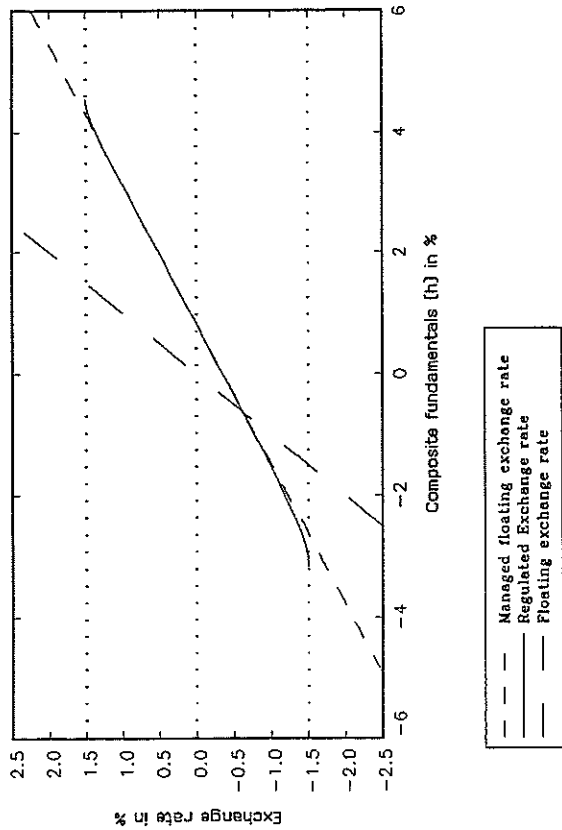


Diagram 4.2 Exchange rate distribution

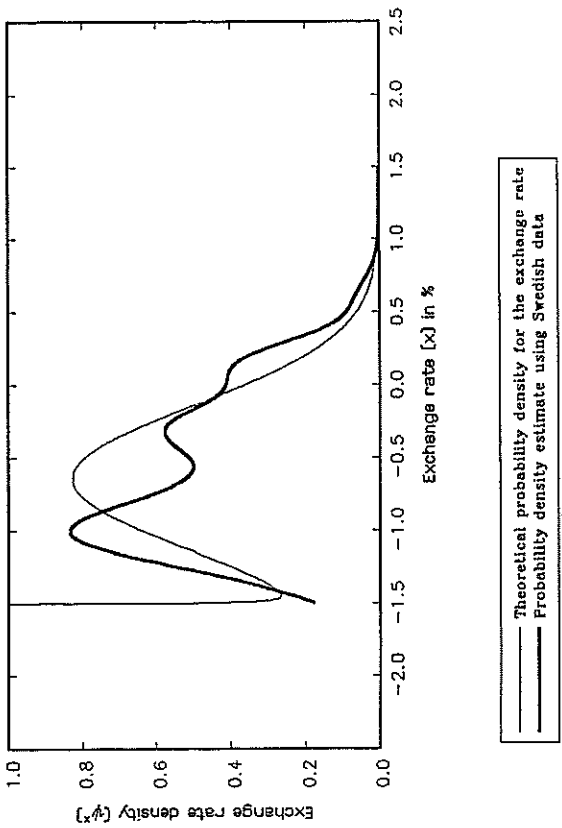


Diagram 4.3 Expected exchange rate for estimated parameters

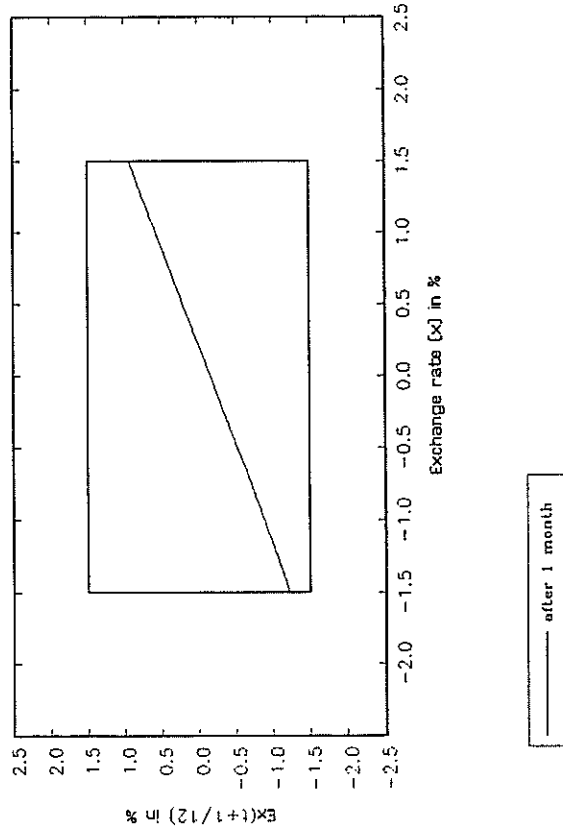
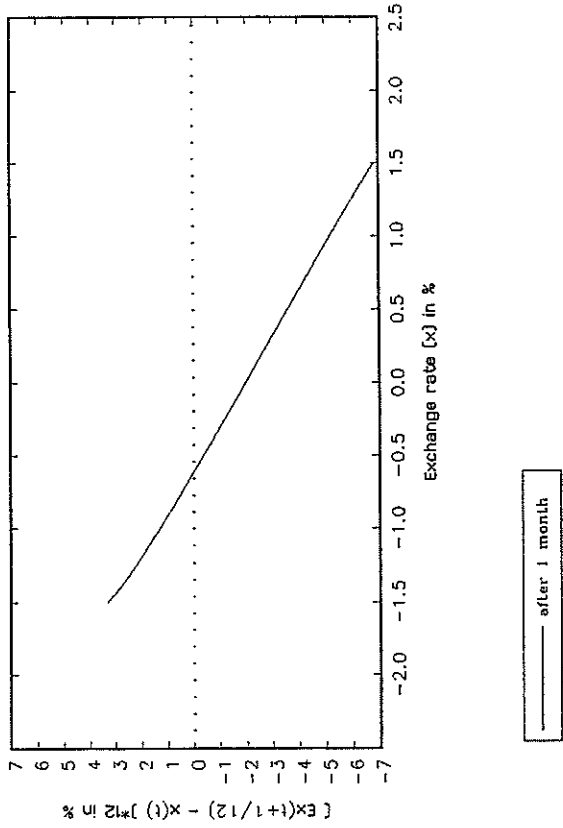


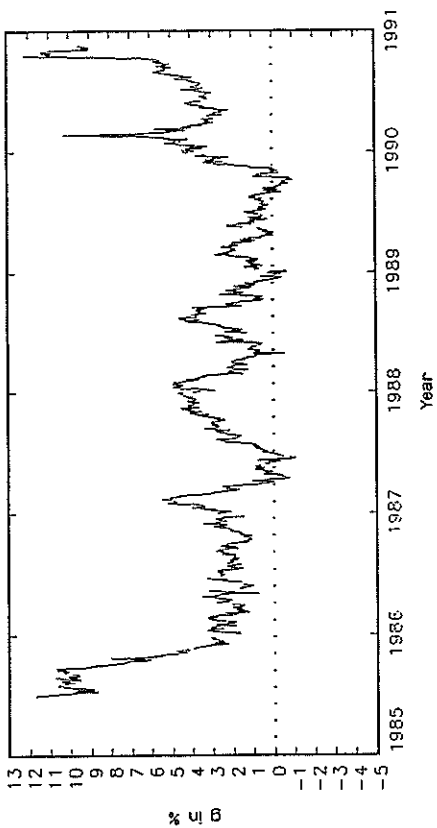
Diagram 4.4 Expected rate of change of the exchange rate



after 1 month

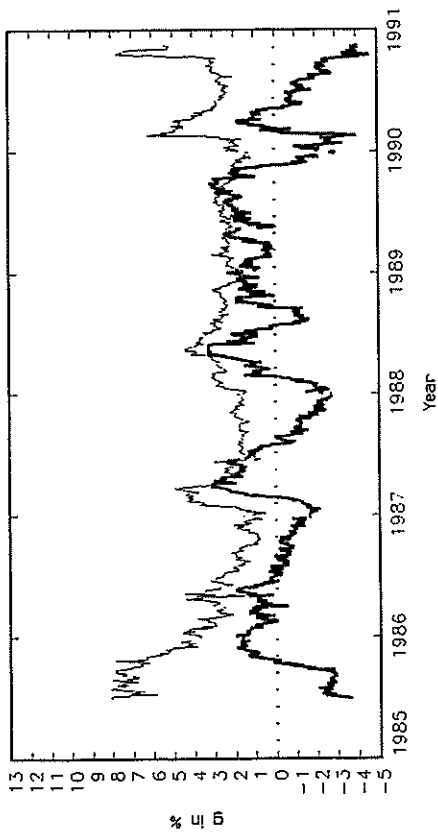
after 1 month

Diagram 4.6



Expected rate of devaluation

Diagram 4.5



Expected rate of change of the exchange rate within the band
Interest rate differential 1 month